EFB25 (European conference on few-body problems in physics)

Dynamical vortex production and quantum turbulence in perturbed Bose-Einstein condensates

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Dynamical vortex production and quantum turbulence in perturbed Bose-Einstein condensates

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Outline

- Introduction
- Two kind of BEC systems under stirring potential
- Binary mass-imbalanced mixture with trap perturbed by eliptic deformed time-dependent interaction
- Characterization of a turbulent regime in the time evolution
- Dipolar system under periodic circularly moving obstacle
- Dynamical vortex production and turbulence
- Results and main remarks

Da Silva et al, Phys. Rev. A 107, 033314 (2023) [also in arXiv:2205.14654]

S. Sabari, R. Kishor Kumar and LT, in progress (2023).

Vortex, superfluid and turbulence dynamics

- **Turbulent fluids is discussed in the Feynman Lectures on Physics [pgs. 3-9, Vol. I (1963)] as a very old problem that has not been solved till now, because in physics no one has been able to analyze it from first principles.**
- **The connection of turbulence with a superfluid via the quantized vortex lines was also first suggested by Feynman.**
- Now, due to some similarities found with the corresponding classical theory, a lot of expectation exists that Quantum Turbulence (QT) can shed some light on the general solution of such an old classical problem.
- QT actually is the name given to the turbulent flow of a fluid at high flow rates, such as superfluids.

Classical and Quantum Turbulence

- Quantum turbulence is an apparently random tangle of vortex lines inside a quantum fluid, as indicated by experiments and numerical solutions.

- Some examples of quantum fluids include superfluid helium (^4He and Cooper pairs of 3He), Bose-Einstein condensates (BECs), polariton condensates.

- It is being noticed that quantum fluids exist at temperatures below the critical temperature at which Bose-Einstein condensation takes place.

- Two main questions in the study of quantum turbulence:
  - Are vortex tangles really random, or do they contain some characteristic organised structures?
  - How far one can compare quantum turbulence with classical turbulence?

Binary mass-imbalanced BEC systems under stirring potential

- Vortex and turbulence generated by a stirring time-dependent interaction in a two-species coupled mass-imbalanced condensates.
- For the perturbation we have considered a slightly modified elliptically periodic potential.
- The approach is suggested to the experimentally accessible binary mixtures ^{85}Rb-^{133}Cs and ^{85}Rb-^{87}Rb, which allow us to verify the effect of mass differences in the dynamics.

Laser stirred Binary BEC

In our approach, the Gross-Pitaevskii coupled stirred model is given by,

$$\mathrm{i}\frac{\partial \psi_i}{\partial t} = \left[\frac{-m_1}{2m_i}\nabla^2 + V_0(x,y) + V_s(x,y,t) + \sum_j g_{ij}|\psi_j|^2 \right] \psi_i$$

where $-\mathrm{i}\nabla \equiv -\mathrm{i}\left(\hat{e}_x \frac{\partial}{\partial x} + \hat{e}_y \frac{\partial}{\partial y}\right)$ is the 2D momentum operator, with g_{ij} being the two-body contact interactions, related to the intra-species and inter-species scattering lengths, respectively, a_{ii} and $a_{ij} (j \neq i)$. It is given by

$$g_{ij} \equiv \sqrt{2\pi\lambda}\frac{m_1 a_{ij} N_j}{\mu_{ij} l_\rho},$$

where $\mu_{ij} \equiv m_i m_j/(m_i + m_j)$ is the reduced mass.

Laser stirred Binary BEC

Laser stirring is represented by laboratory-frame time-dependent elliptical trap perturbations in both $x-$ and $y-$directions:

$$V_s(x,y,t) = \frac{\epsilon}{2}\left[(x\cos(\Omega_E t) - y\sin(\Omega_E t))^2 - (x\sin(\Omega_E t) - y\cos(\Omega_E t))^2\right]$$
$$= \frac{\epsilon}{2}\left[(x^2 - y^2)\cos(2\Omega_E t) - 2xy\sin(2\Omega_E t)\right],$$

with Ω_E being the stirring laser frequency and ϵ the strength.

N. G. Parker and C. S. Adams, Emergence and Decay of Turbulence in Stirred Atomic BEC Phys. Rev. Lett. **95**, 145301 (2005).

Dynamics of stirred vortex formation

In general, quantum gases are compressible fluids, such that their corresponding density can change when submitted to a force. This is true to a certain degree, as part of the fluid can behave as an incompressible fluid, similar as a liquid. In our present case, the condensate is submitted to a time-dependent stirring potential, associated to a torque, which is mainly due to a part of the rotational kinetic energy, that we can call as the compressible one.

For each component of the mixture, the total energies $E_i(t)$ are

$$\begin{aligned} E_i(t) &= \int d^2\mathbf{r} \left[\frac{m_1}{2m_i} |\nabla \psi_i|^2 + V_i(x,y,t) n_i(x,y,t) \right] \\ &+ \frac{1}{2} \sum_{j=1,2} g_{ij} \int d^2\mathbf{r}\, n_i(x,y,t) n_j(x,y,t), \end{aligned}$$

where $n_{i=1,2}(x,y,t) \equiv |\psi_i|^2$ are the time-dependent densities.

Dynamics of stirred vortex formation

With the current densities $\mathbf{j}_i(x, y, t)$ in terms of the respective densities and velocity fields $\mathbf{v}_i(x, y, t)$, such that $\mathbf{j}_i(x, y, t) = n_i \; \mathbf{v}_i(x, y, t)$,

$$\mathbf{v}_i(x, y, t) = \frac{1}{2\mathrm{i} \; |\psi_i|^2} \left[\psi_i^\star \nabla \psi_i - \psi_i \nabla \psi_i^\star \right].$$

The associated kinetic energies,

$$E_i^K(t) = \frac{m_1}{2m_i} \int d^2\mathbf{r} \, n_i(x, y, t) |\mathbf{v}_i(x, y, t)|^2,$$

can be decomposed in compressible (c) and incompressible (nc), $E_i^{c,nc}(t)$ parts.

Dynamics of stirred vortex formation

For the decomposition, the density-weighted velocity field, $\mathbf{u}_i(x,y,t) \equiv \sqrt{n_i(x,y,t)}\mathbf{v}_i(x,y,t)$, can be split:
Incompressible part, $\mathbf{u}_i^{(nc)} \equiv \mathbf{u}_i^{(nc)}(x,y,t)$ satisfying $\nabla.\mathbf{u}_i^{(nc)} = 0$;
and
Compressible one, $\mathbf{u}_i^{(c)} \equiv \mathbf{u}_i^{(c)}(x,y,t)$, satisfying $\nabla \times \mathbf{u}_i^{(c)} = 0$.

Therefore, with $\mathbf{u}_i(x,y,t) = \mathbf{u}_i^{(nc)}(x,y,t) + \mathbf{u}_i^{(c)}(x,y,t)$, the corresponding kinetic energies are

$$
\begin{aligned}
E_i^K(t) &= E_i^{nc}(t) + E_i^c(t) \\
&\equiv \frac{m_1}{2m_i}\int d^2\mathbf{r}\left[|\mathbf{u}_i^{(nc)}|^2 + |\mathbf{u}_i^{(c)}|^2\right].
\end{aligned}
$$

In 2D momentum space, with $\mathbf{k} = (k_x, k_y)$ and $d^2k = dk_x dk_y$, and $(\alpha) \equiv (c), (nc)$, we have

$$E_i^{(\alpha)}(t) = \frac{m_1}{2m_i} \int d^2\mathbf{k} |\mathcal{F}_i^{(\alpha)}(\mathbf{k}, t)|^2,$$

$$= \frac{m_1}{8\pi^2 m_i} \int d^2\mathbf{k} \left| \int d^2\mathbf{r}\, e^{-\mathrm{i}\mathbf{k}.\mathbf{r}} \mathbf{u}_i^{(\alpha)} \right|^2.$$

Both compressible and incompressible parts are used to verify the sound waves and vorticity.
The torque experienced by the time-dependent stirring potential, can be obtained through the operator,

$$\tau_z(\mathbf{r}, t) = -\frac{\partial}{\partial\theta} V_s(r, \theta, t) = \epsilon\, r^2 \sin(2\theta + 2\Omega_E t),$$

corresponding to a rotation in the elliptical time-dependent part of the potential, with $2\Omega_E t \to 2\Omega_E t - \pi/2$.

It follows the expected values of the induced angular momenta, $\langle L_z(t)\rangle_i$, and respective moment of inertia, $\langle I(t)\rangle_i$:

$$\langle L_z(t)\rangle_i \;=\; -\mathrm{i}\int d^2r\,\psi_i^\star\frac{\partial}{\partial\theta}\psi_i \;\text{ and }\; \langle I(t)\rangle_i = \int d^2r|\psi_i|^2\,r^2,$$

with the associated classical rotational velocity being

$$\Omega_i(t) \equiv \frac{\langle L_z(t)\rangle_i}{\langle I(t)\rangle_i}.$$

The rotating-frame velocity field, at a given frequency Ω is $\mathbf{v}_\Omega = \mathbf{v} - (\boldsymbol{\Omega} \times \mathbf{r})_z$. The associated particle current $\mathbf{j}_{i,\Omega}(\mathbf{r}, t) \equiv n_i(\mathbf{r}, t)\mathbf{v}_{i\Omega}(\mathbf{r}, t)$ satisfies the continuity equation

$$\frac{\partial n_i(\mathbf{r}, t)}{\partial t} + \nabla \cdot \mathbf{j}_{i\Omega}(\mathbf{r}, t) = 0.$$

When considering a rotating frame, at a given frequency Ω,

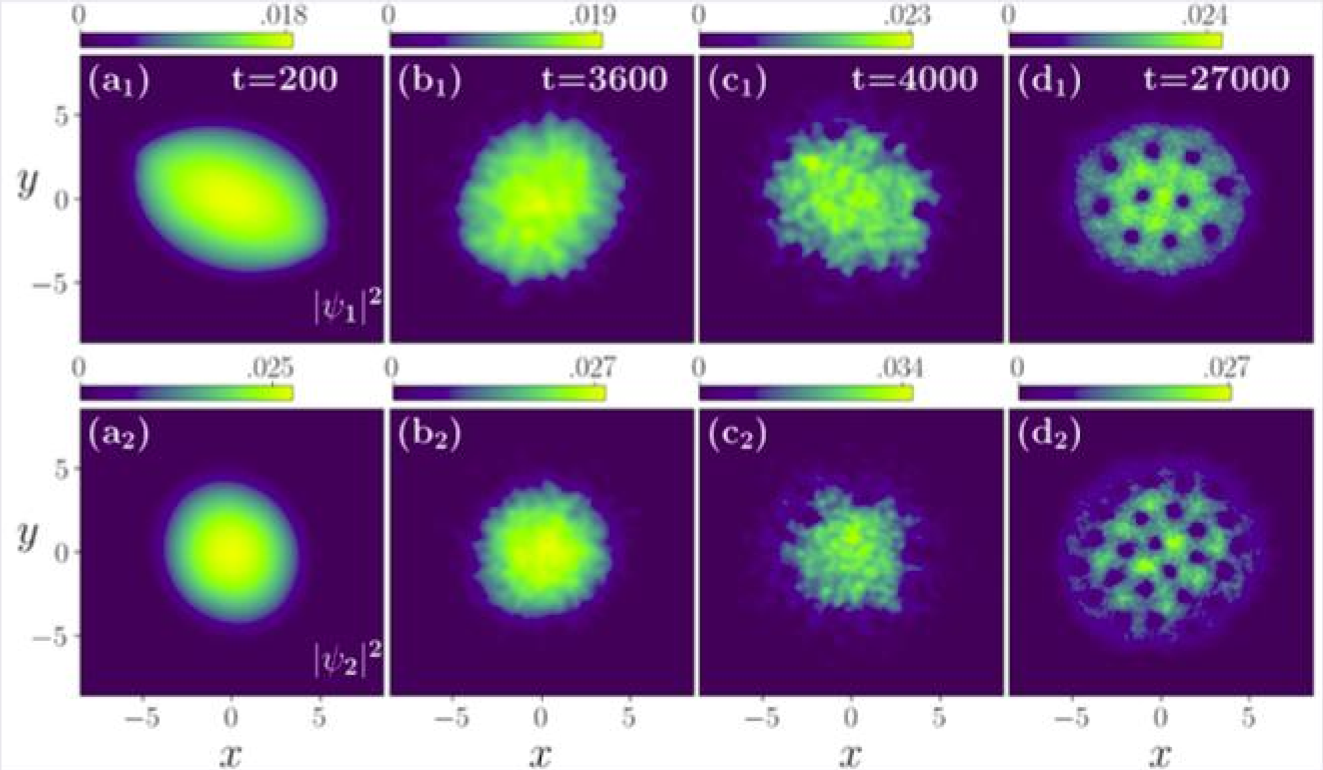
$$E_{i,\Omega}^{(\alpha)}(t) = \frac{m_1}{2m_i} \int d^2\mathbf{k} |\mathcal{F}_{i,\Omega}^{(\alpha)}(\mathbf{k}, t)|^2 \equiv \frac{m_1}{m_i} \int_0^\infty dk\, E_{i,\Omega}^{(\alpha)}(k, t),$$

which defines the *velocity power spectral density* in k space:

$$E_{i,\Omega}^{(\alpha)}(k, t) = k \int_0^{2\pi} d\theta_k |\mathcal{F}_{i,\Omega}^{(\alpha)}(\mathbf{k}, t)|^2.$$

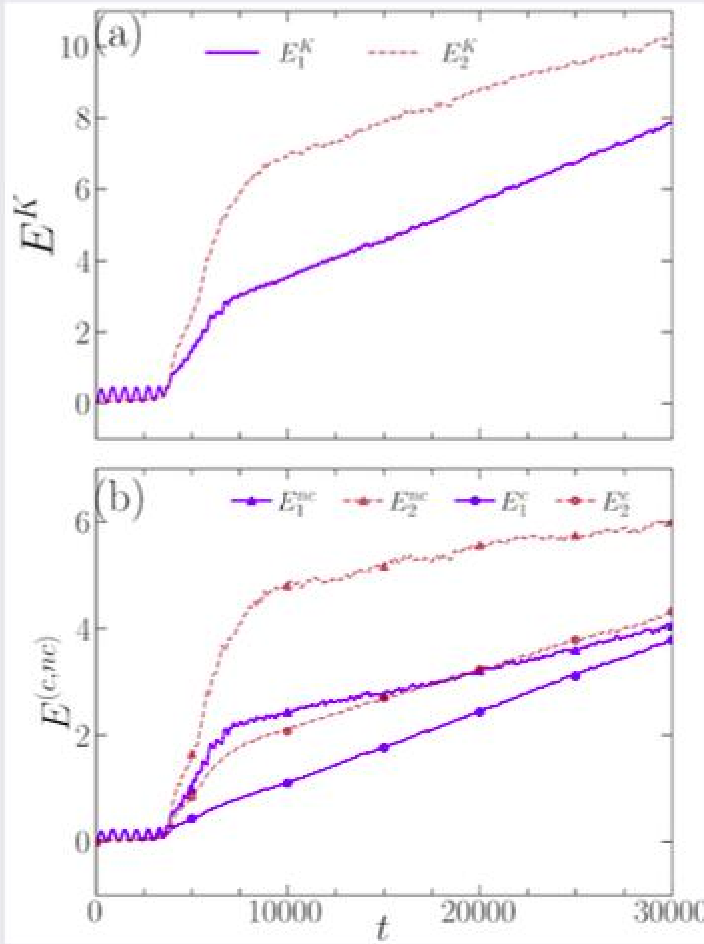
For computational methods for power spectra, see Bradley, Kumar, Pal and Yu, Phys. Rev. A 106, 043322 (also in arXiv:2112.04012).

Binary ^{85}Rb$-^{133}$Cs mixture



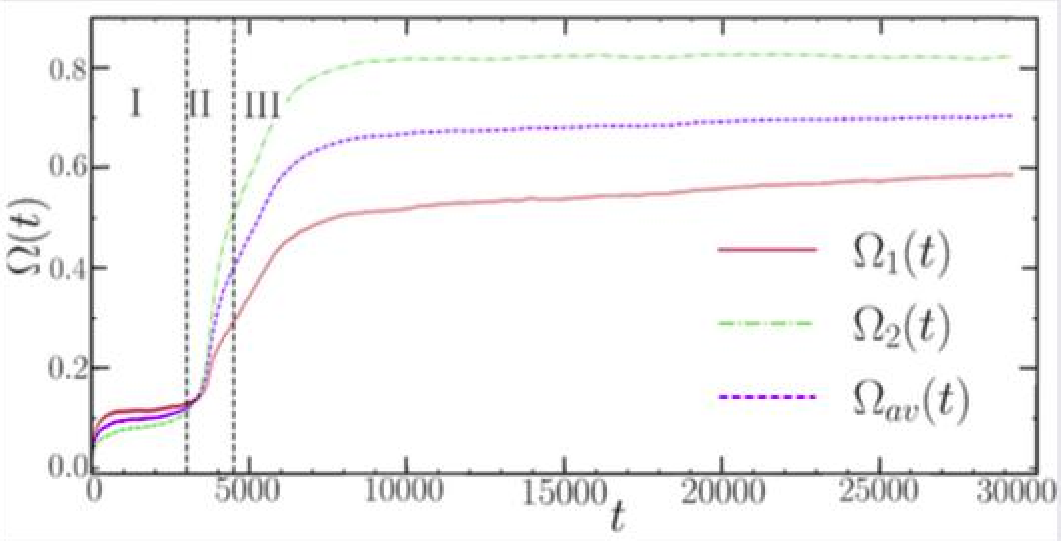
Time evolution of the densities of ^{85}Rb-^{133}Cs mixture. The upper panels are for the ^{85}Rb (component 1), with the lower panels for ^{133}Cs (component 2). Scattering lengths are $a_{ii} = 60a_0$ and $a_{12} = 30a_0$, $\Omega_E = 1.25$ with $\epsilon = 0.025$.

Binary $^{85}Rb-^{133}Cs$ mixture

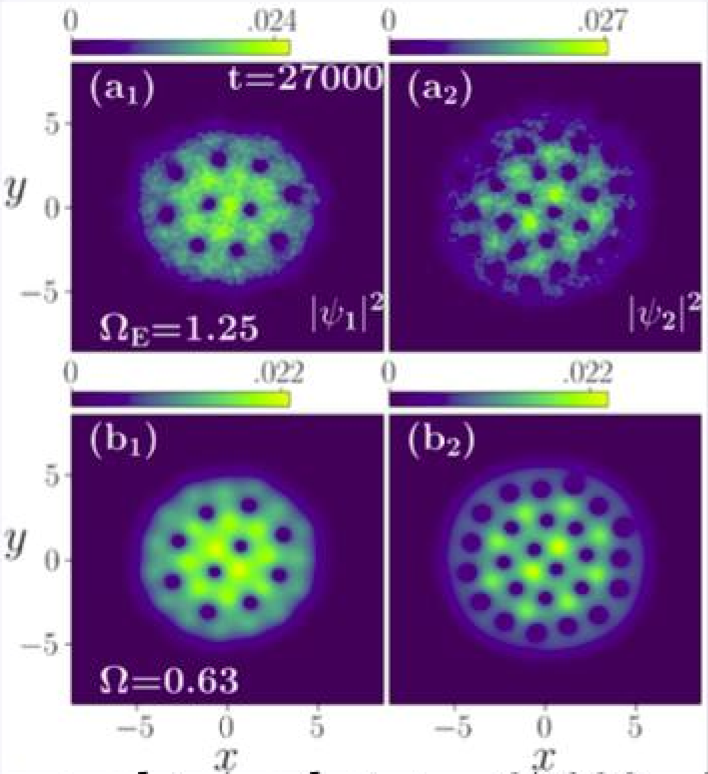


The time-evolutions are presented (in the lab frame) for the total kinetic energies [panel (a)] and corresponding compressed (E_i^c, indicated with bullets) and uncompressed E_i^{nc}, indicated with triangles) parts [panel (b)]. The evolution is shown till stable vortex patterns are verified.

Binary ^{85}Rb$-^{133}$Cs mixture



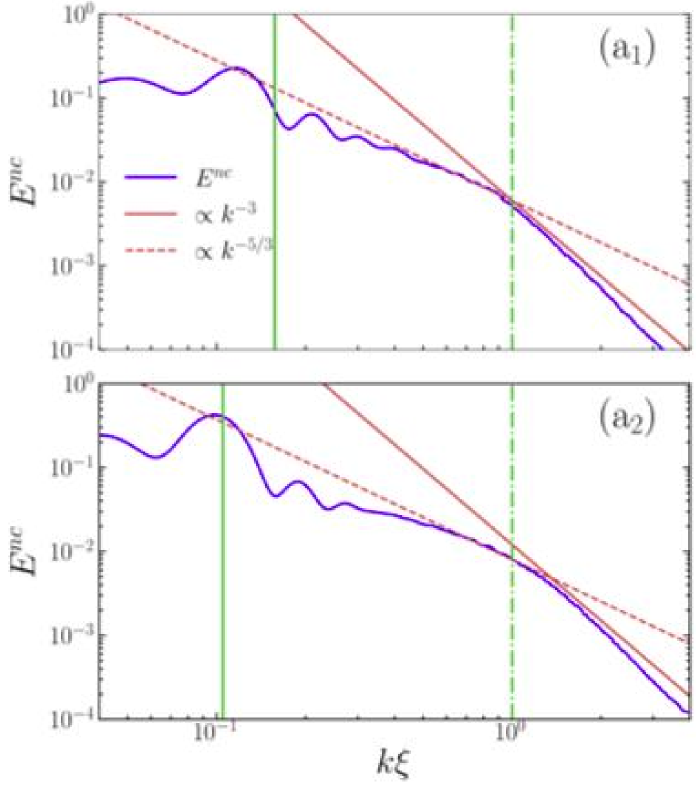
Classical rotational velocity, $\Omega_i(t)$, for ^{85}Rb (solid-red line) and ^{133}Cs (dot-dashed-green line), with the corresponding averaged result, $\Omega_{av}(t) \equiv [\Omega_1(t) + \Omega_2(t)]/2$ (dashed-blue line). The vertical lines are approximately separating three time intervals in the evolution: (I) shape deformations; (II) turbulent regime, with vortex nucleations; and (III) with vortex lattices estabilized.



Vortex-pattern solutions, obtained at $t = 27000$ with the stirring potential, are compared with ground-state solutions of the GP equation, by replacing in the formalism the stirring potential by an effective constant rotational frequency $\Omega_0 = 0.63$: $V_s(x, y, t) \to \Omega_0 L_z = -\mathrm{i}\Omega_0 \frac{\partial}{\partial\theta}$.

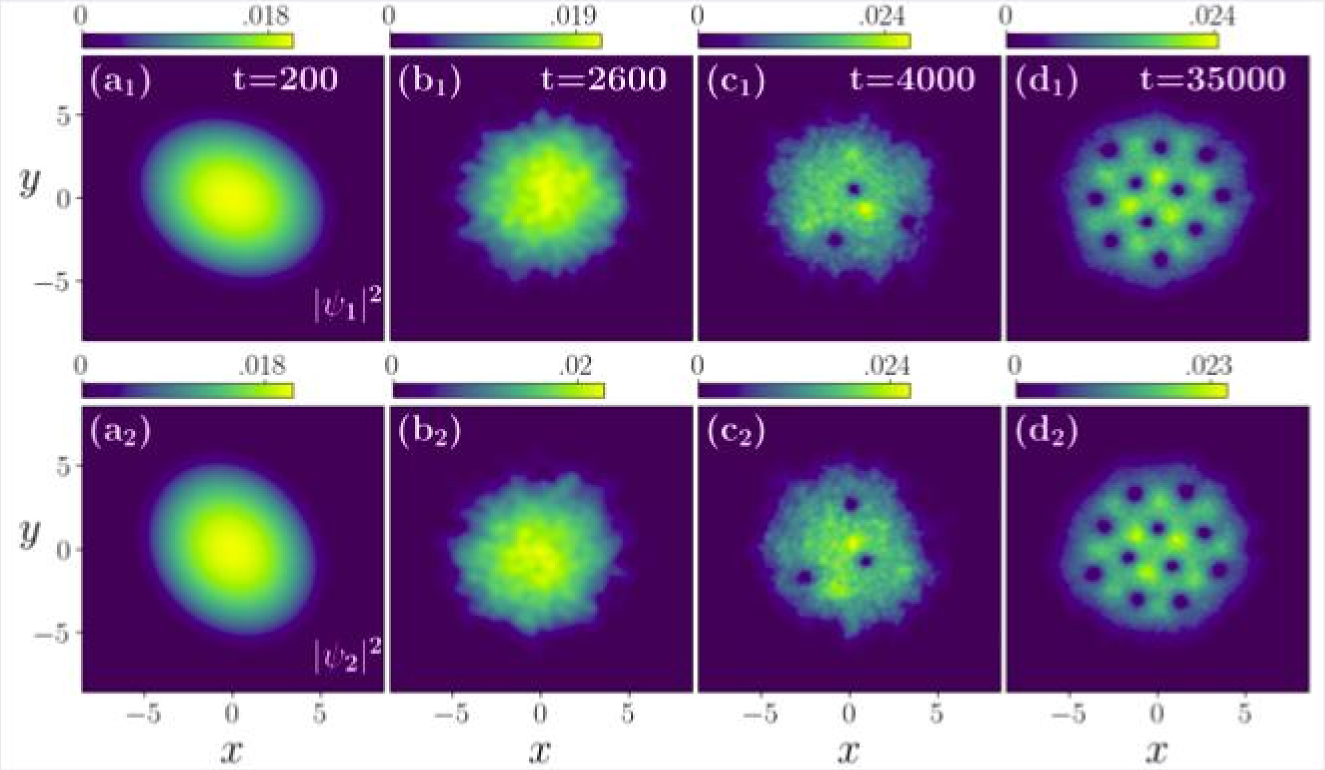
Binary ^{85}Rb$-^{133}$Cs mixture

Average Kinetic energy spectrum from Turbulent regime



Incompressible kinetic energy spectra, $E^{nc} \equiv E^{nc}(k,t)$, for the ^{85}Rb-^{133}Cs mixture, obtained by averaging over 50 samples in the turbulent time-interval regime, confirming the Kolmogorov $k^{-5/3}$ power law behavior in this interval.

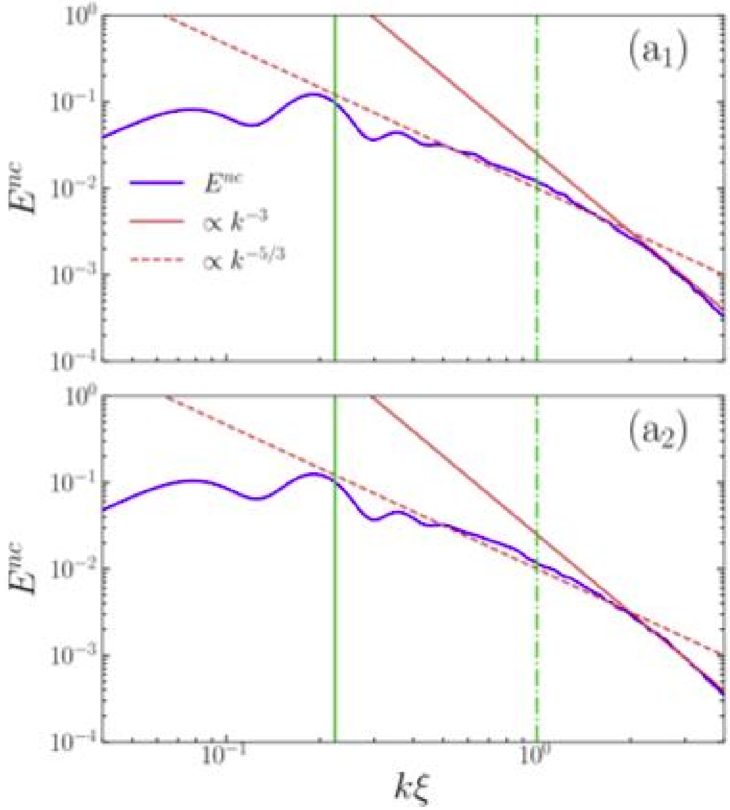
Binary ^{85}Rb$-^{87}$Rb mixture



Time evolution of the densities ^{85}Rb-^{87}Rb mixture. The upper panels are for the ^{85}Rb (component 1), with the lower panels for ^{87}Rb (component 2). $\Omega_E = 1.25$ with $\epsilon = 0.025$.

Binary ^{85}Rb$-^{87}$Rb mixture

Average Kinetic energy spectrum from Turbulent regime



Incompressible kinetic energy spectra, $E^{nc} \equiv E^{nc}(k,t)$, for the ^{85}Rb-^{87}Rb mixture, by averaging over 50 samples in the turbulent regime.

Main remarks

- Vortex patterns, produced dynamically by time-dependent elliptical laser stirring, together with associated turbulent flow behaviors, are studied for two kinds of mass-imbalanced coupled Bose-Einstein condensates.
- The first stage is the shape deformation introduced by the stirring potential. This is followed by a symmetry-breaking turbulent regime, in which vortex nucleations start to be generated at the surface of the two-species condensates, which approximately agrees with the classical Kolmogorov law. A final regime happens with cristallization of stable vortex patterns, associated with the assumed rotational frequency of the stirring potential.

For more details, see the preprint A. N. da Silva, R. K. Kumar, A. Bradley and L.T., Vortex generation in stirred binary Bose-Einstein condensates, Phys. Rev. A **107**, 033314 (2023) [see also in arXiv:2205.14654].

Dipolar system under periodic moving obstacle

- Vortex dynamics associated with moving Gaussian obstacles, in pancake-like trapped dipolar BECs, leading to vortex-antivortex and quantum turbulence
- The critical velocities, to produce vortex-antivortex pairs and vortex clusters, for given repulsive dipolar-dipolar interactions, are obtained by solving a nonlocal 2D GP formalism in real-time.

GP 3D Formalism

$$i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t} = \Big(-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r},t) + g_{3D}\left|\Psi(\mathbf{r},t)\right|^2 \Big)\Psi(\mathbf{r},t)$$
$$+ N\int U_{\rm dd}(\mathbf{r}-\mathbf{r}')\left|\Psi(\mathbf{r}',t)\right|^2 d\mathbf{r}'\Psi(\mathbf{r},t),$$

where $\int d\mathbf{r}|\Psi(\mathbf{r},t)|^2 = 1$, with N =total number of atoms having masses m, $g_{3D} = 4\pi\ \hbar^2 a_s N/m$, ($a_s = s-$wave scattering length) is the strength of the two-body contact interaction, with $U_{dd}(\mathbf{r})$ being the dipolar interaction.

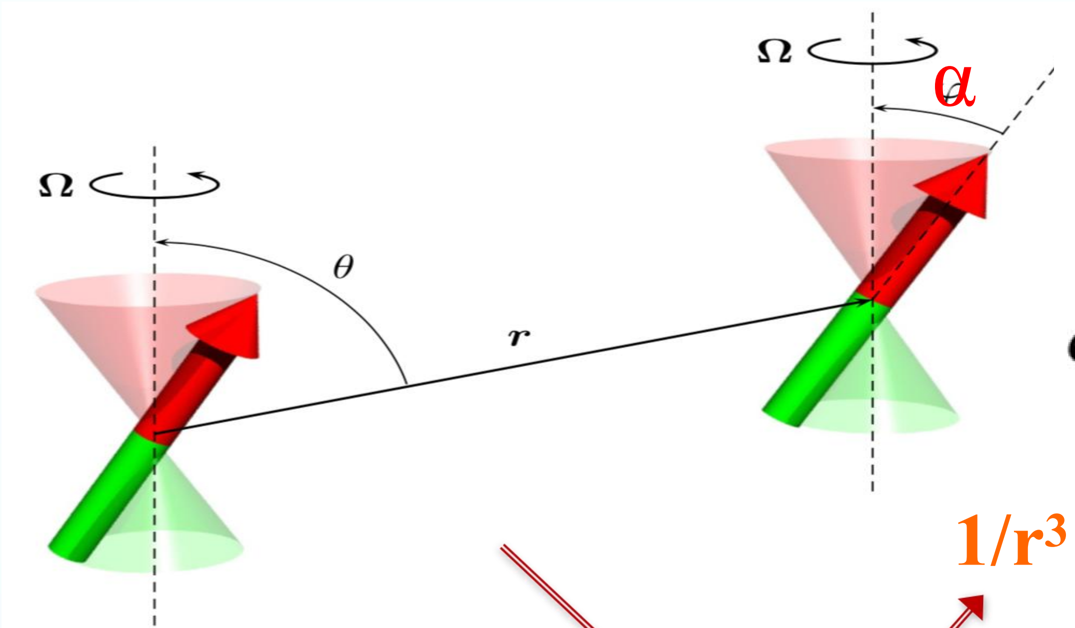
The trap will be assumed with pancake-like cylindrical symmetric harmonic shape perturbed in the transversal direction by a Gaussian-shaped time-dependent interaction, as $V(\mathbf{r},t) \equiv V_{ho}(x,y,z) + V_G(x,y,t)$, where $V_{ho}(x,y,z) = (m\omega_\rho^2/2)(x^2+y^2+\lambda^2 z^2)$, where the trap aspect ratio $\lambda \gg 1$ will help us simplify the formalism.

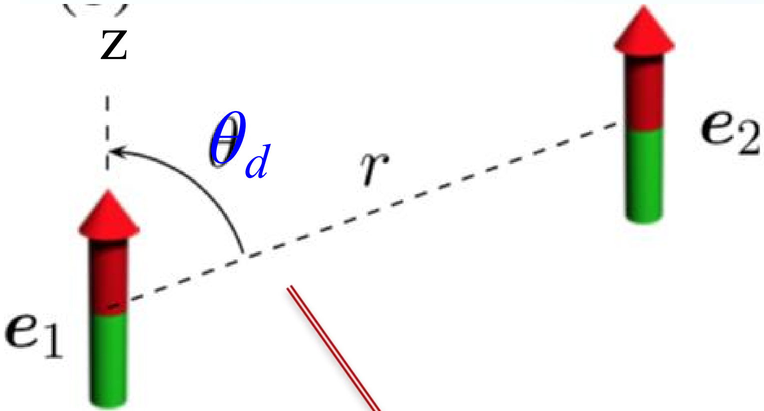
Dipole-dipole interaction

The tunability is performed by time-dependent magnetic fields with dipoles rapidly rotating around the $z-$axis. The magnetic field is given by the combination of a static part along the $z-$direction and a fast rotating part in the $(x, y)-$plane, with frequency such that the atoms are not significantly moving during each period. Once performed a time averaging of the DDI within a period, the 3D averaged interaction between the coupled dipolar atoms with magnetic dipoles $\mu_1 = \mu_2 = \mu$ given in terms of the Bohr magneton μ_B, can be written as

$$\left\langle V_{3D}^{(d)}(\mathbf{r}-\mathbf{r}')\right\rangle = \frac{\mu_0\mu^2}{\hbar\omega_1 l_\perp^3}\left(\frac{1-3\cos^2\theta}{|\mathbf{r}-\mathbf{r}'|^3}\right)\left(\frac{3\cos^2\alpha-1}{2}\right).$$

Tuning of the dipole-dipole interaction





$1/r^3$ → +ve $\theta_d = \pi/2$

Where, α varies from **0** to **π/2**

For --- $\alpha = 0$ → $U_{dd}(r) = +$ ve

$\alpha = \pi/2$ → $U_{dd}(r) = -$ ve

The new form of the dipole-dipole interaction is

$$U_{\mathrm{dd}}(\mathbf{r}) = \frac{C_{dd}}{4\pi}\frac{(1-3\cos^2\theta_d)}{|\mathbf{r}|^3}\frac{(3\cos^2\alpha-1)}{2}$$

T. Lahaye et. Al., Rep. Prog. Phys. **72**, 126401 (2009)

▪ Moving circularly a laser beam

$$V_G = A(t)\exp\left(-\frac{[x - x_0(t)]^2 + [y - y_0(t)]^2}{2W_0^2}\right)$$

$x_0(t) = r_0 \sin(\nu\, t)$

$y_0(t) = r_0 \cos(\nu\, t)$

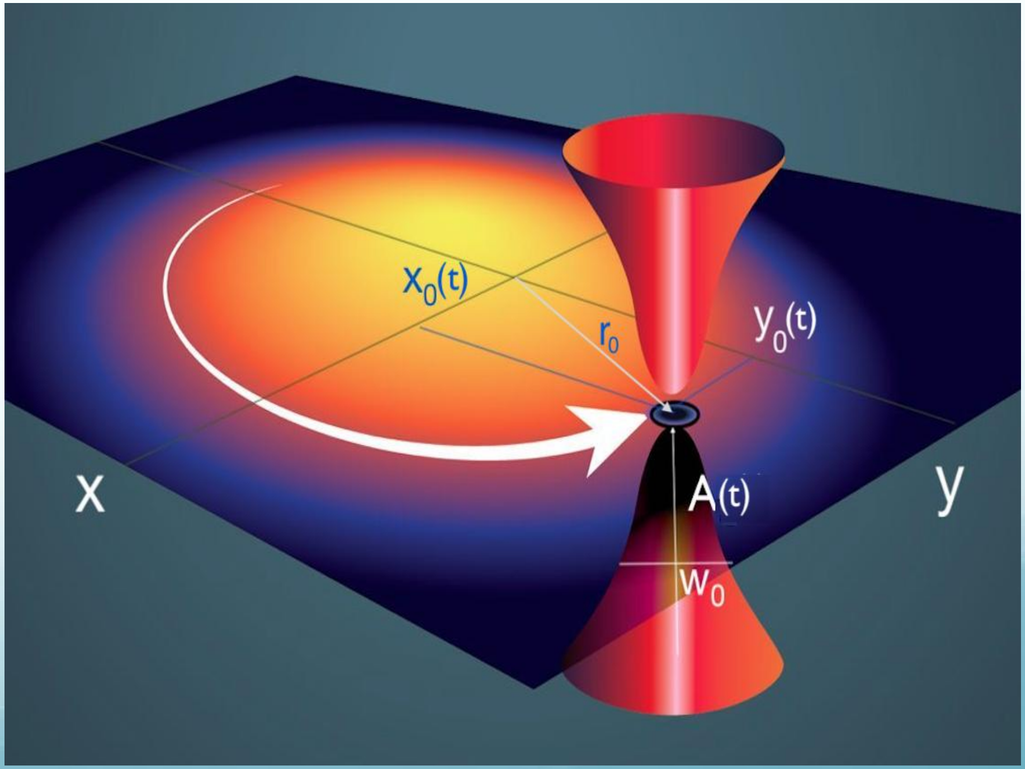
$A(t) = A_0\,[1 + \varepsilon_A \sin(\omega_A\, t)]$

$\varepsilon_A = 0.4$ and $\omega_A = 2$

When $\omega_A = 0$, $A(t) = A_0 = 36$ (90% of μ)

$r_0 = 3.5$

$W_0 = 1.5$



Velocity of the Gaussian potential

$V = r_0\, \nu$

By varying $r_0\, \nu$, we can find the critical velocity Vc for the vortex nucleation in the BECs.

Moving circularly a laser beam

$$V_G = A(t)\exp\left(-\frac{[x-x_0(t)]^2+[y-y_0(t)]^2}{2W_0^2}\right)$$

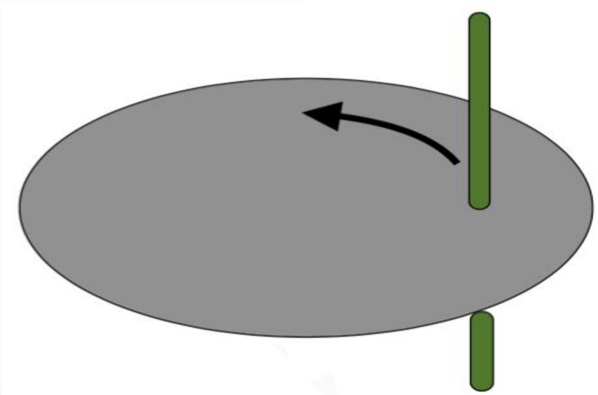
$x_0(t) = r_0 \sin(\nu t)$

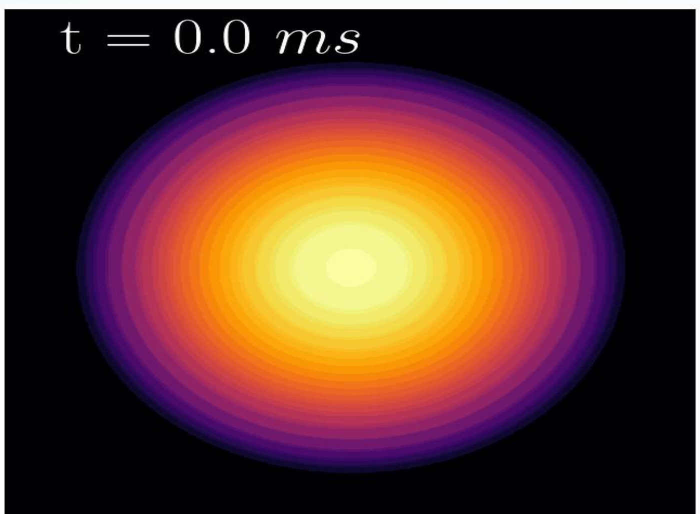
$y_0(t) = r_0 \cos(\nu t)$

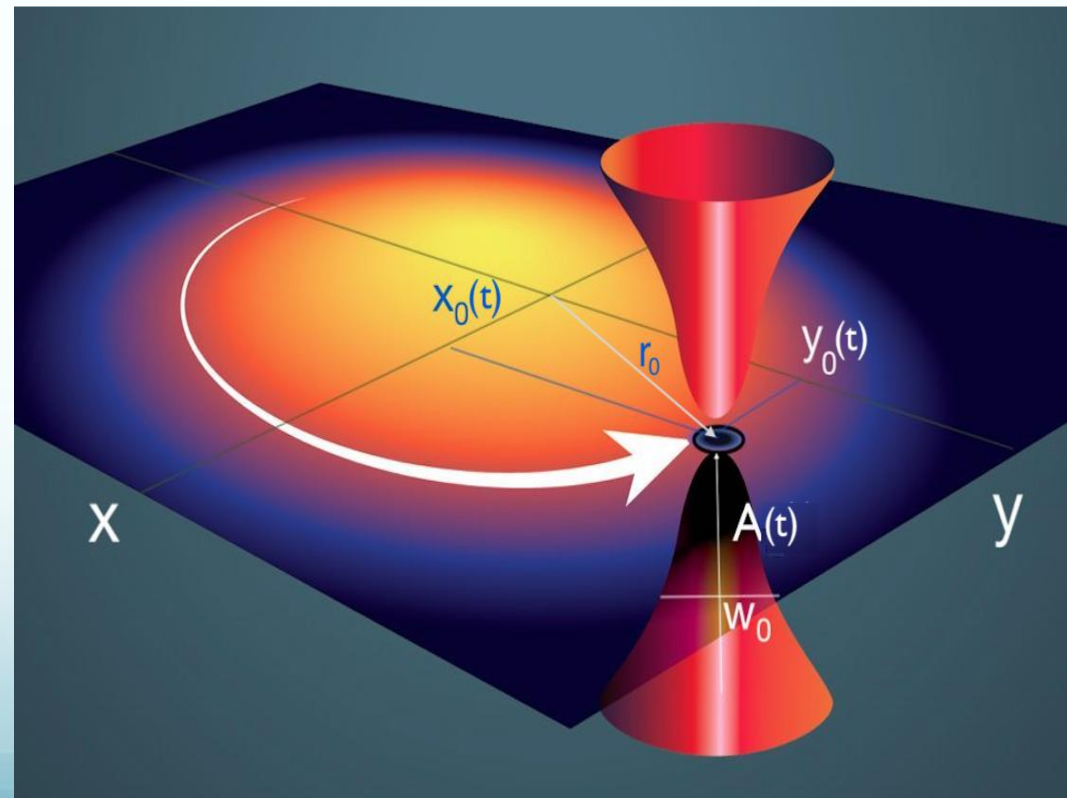
$A(t) = A_0\,[1 + \varepsilon_A \sin(\omega_A t)]$

$\varepsilon_A = 0.4$ and $\omega_A = 2$

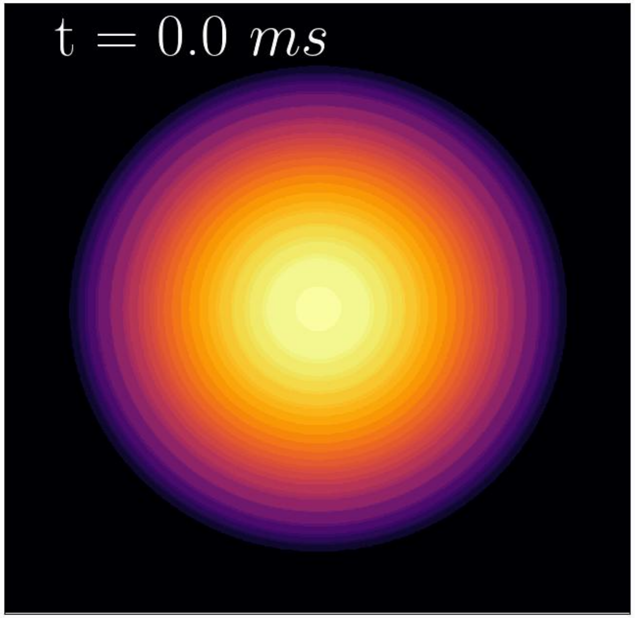
When $\omega_A = 0$, $A(t) = A_0 = 36$ (90% of μ)



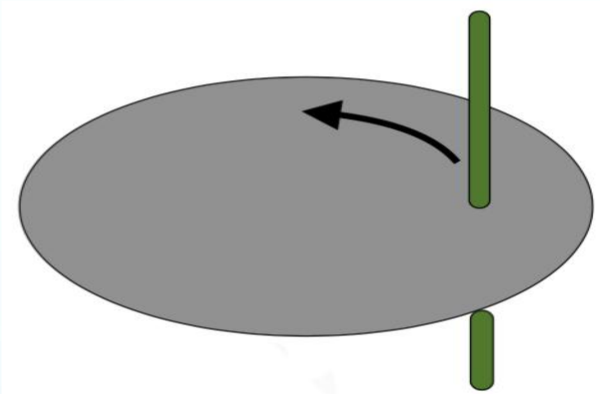




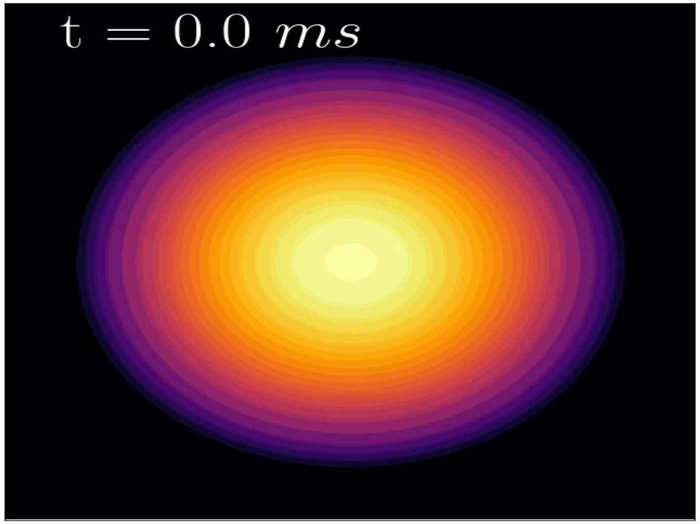
▪ Moving circularly a laser beam



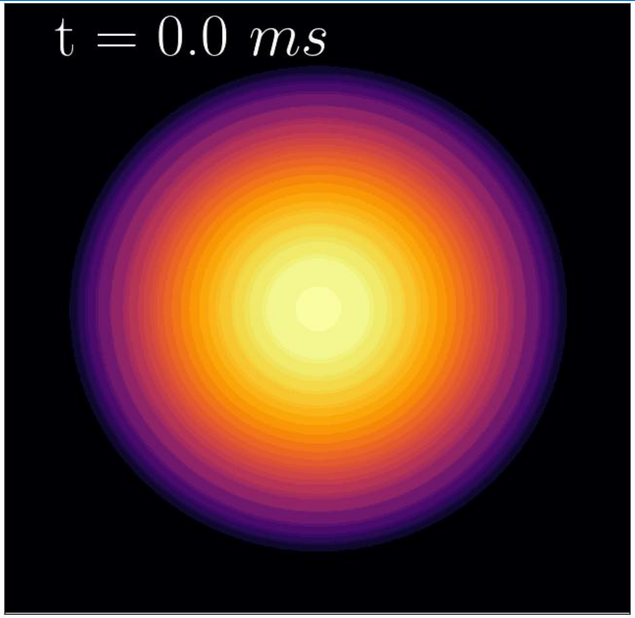
$\alpha = 0$, $v = 0.5$



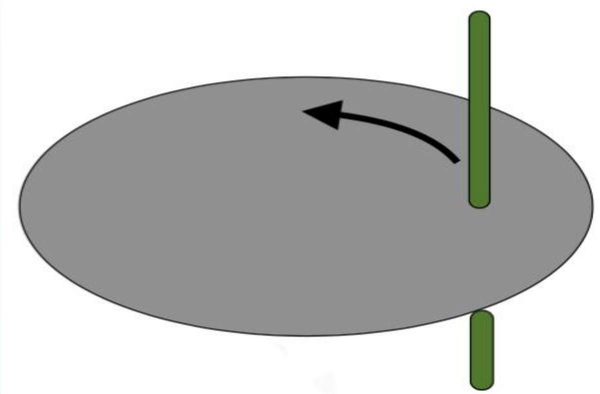
$\alpha = 0$, $v = 0.8$



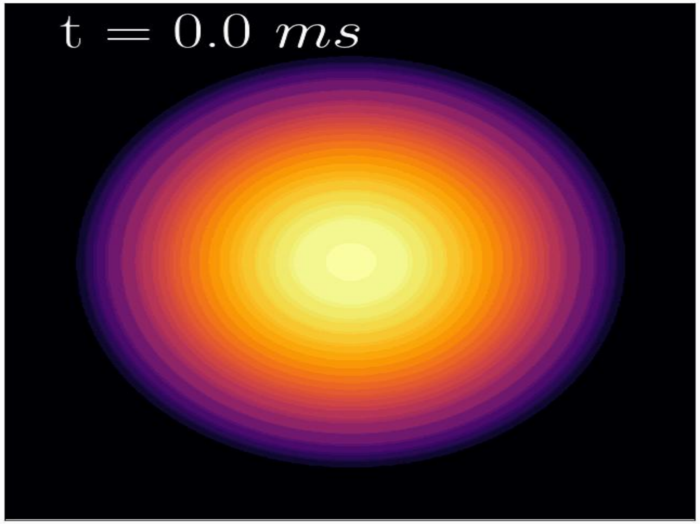
▪ Moving circularly a laser beam

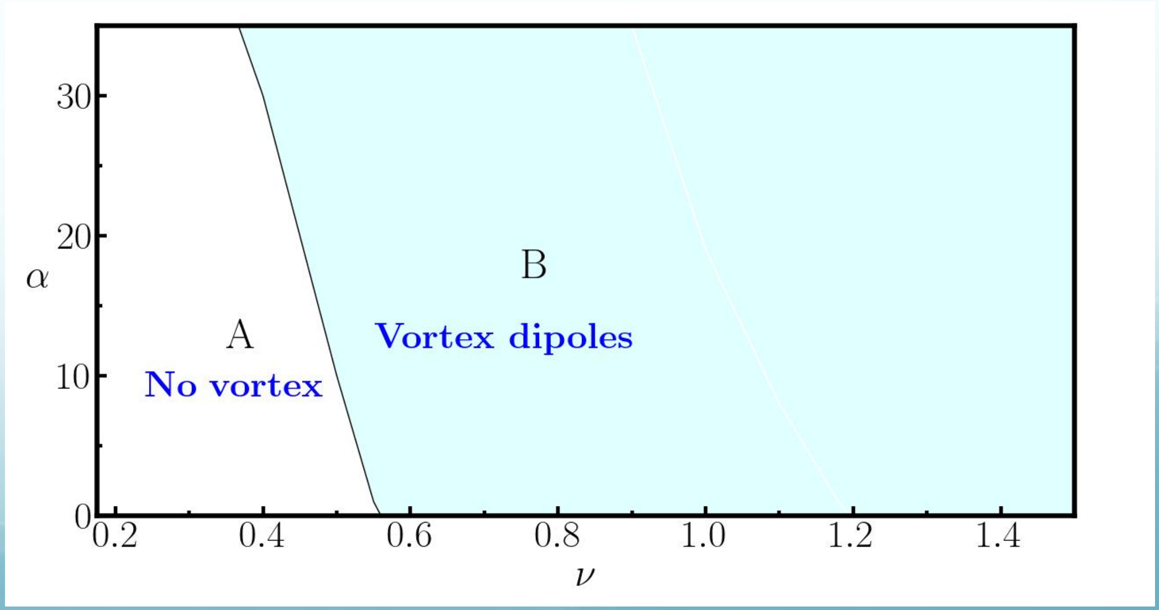


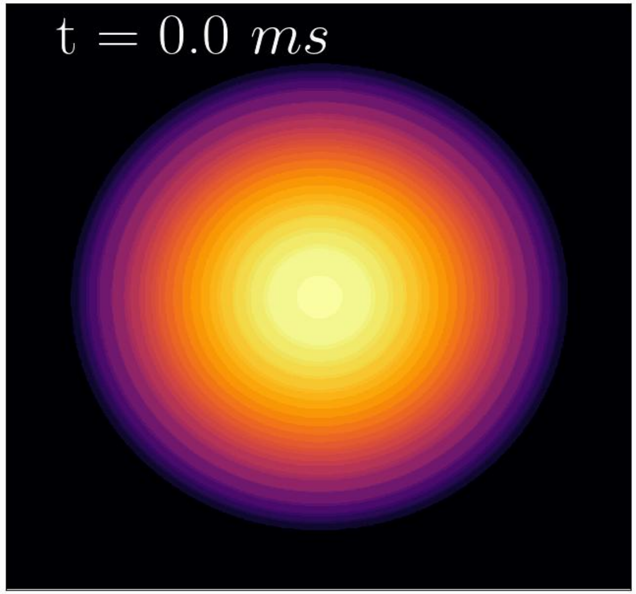
α = 0, ν = 0.5

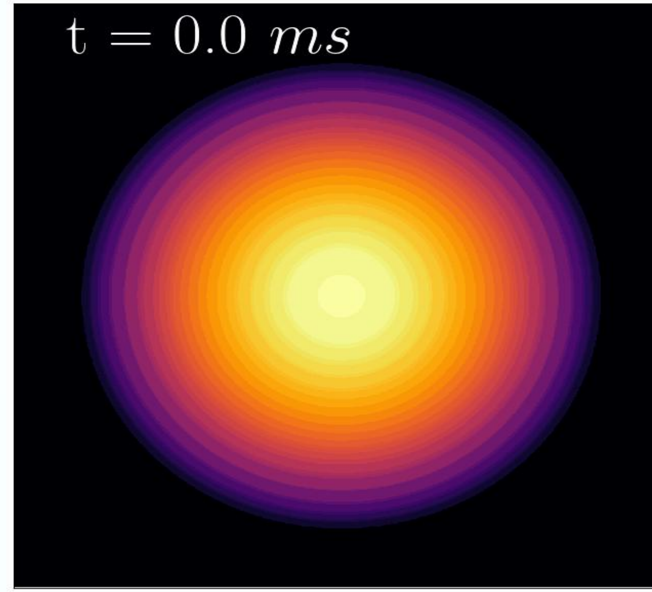


α = 0, ν = 0.8





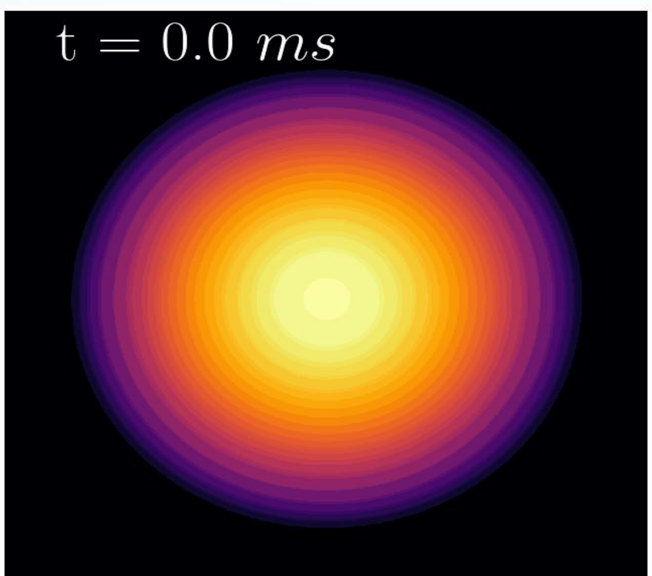


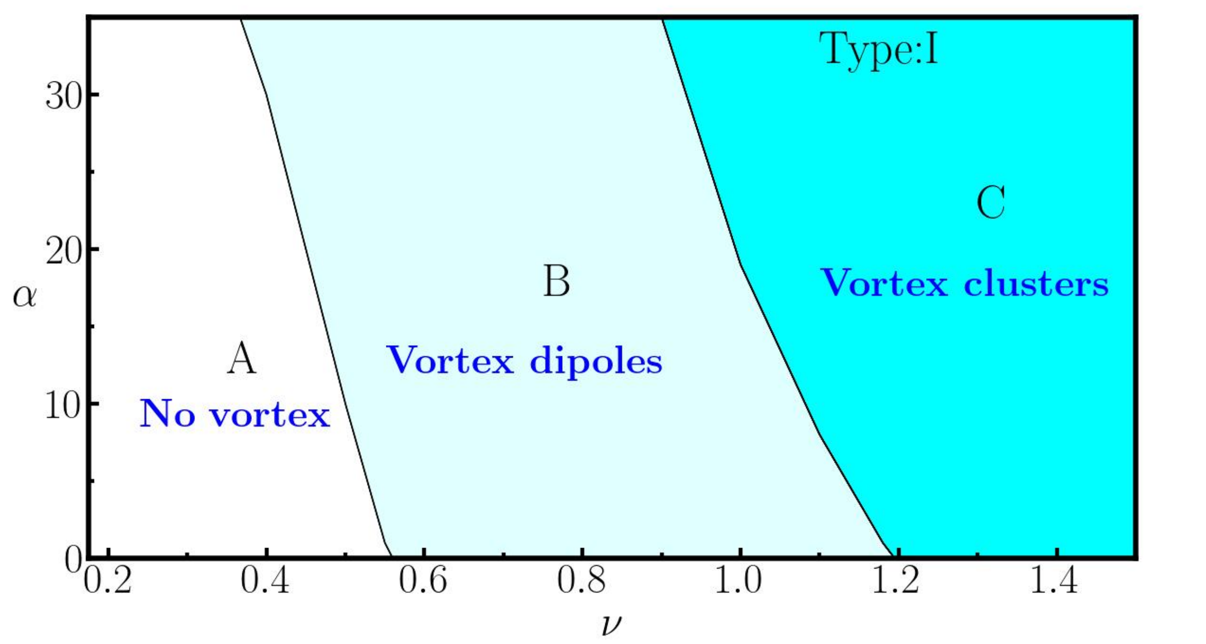


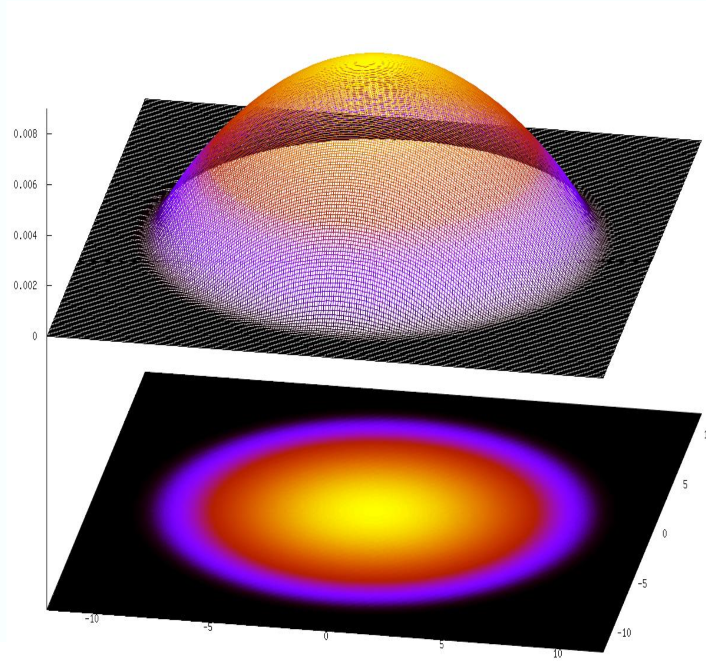
$\alpha = 0, \nu = 0.8$

$\alpha = 0, \nu = 1.5$

$\alpha = 0, \nu = 0.5$

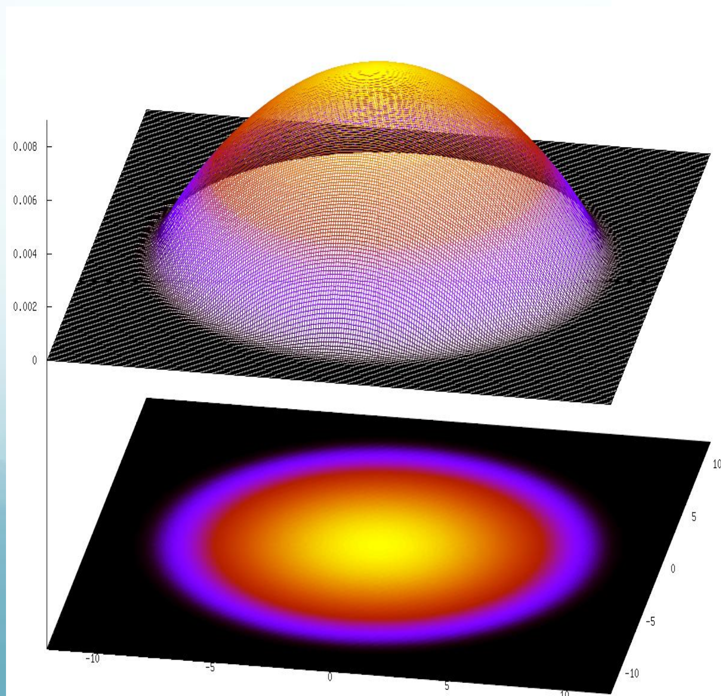




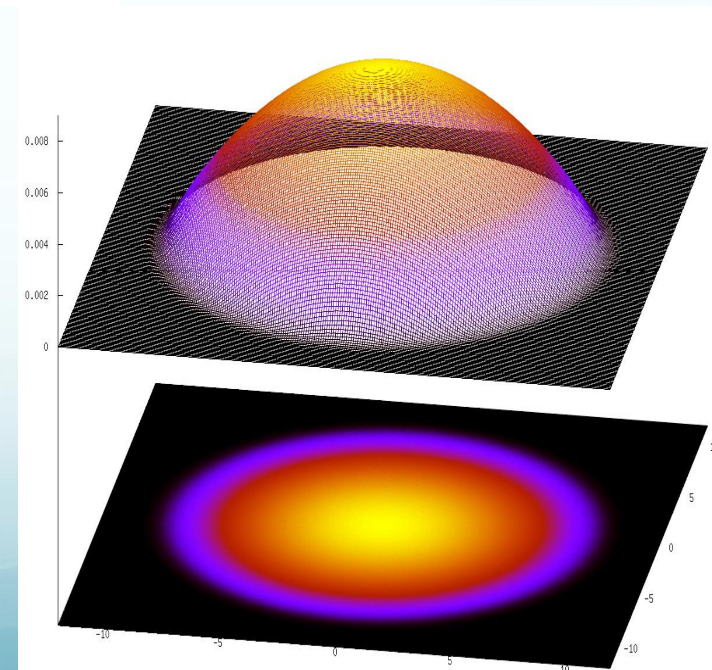


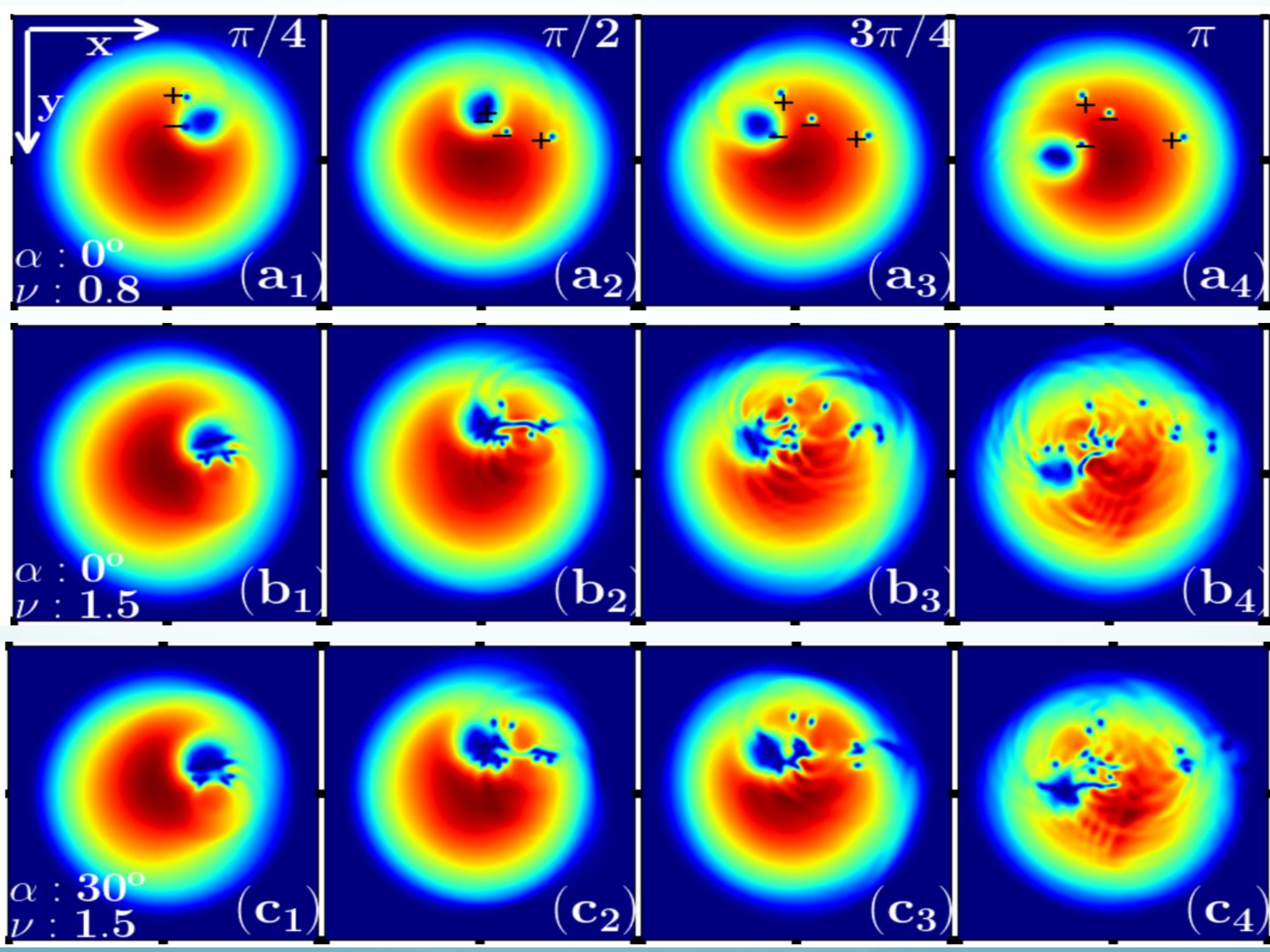
$\alpha = 0, \nu = 0.5$

$\alpha = 0, \nu = 1.5$



$\alpha = 0, \nu = 0.8$





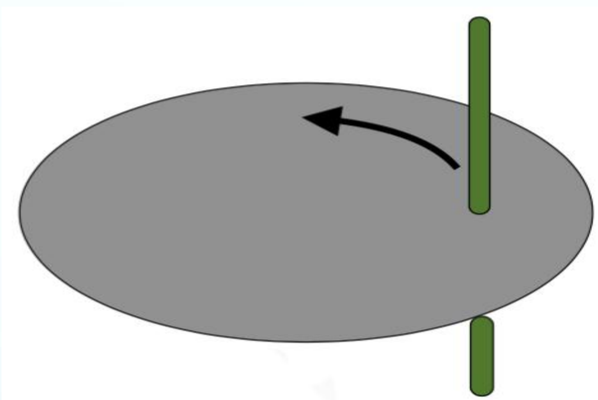
x
y
π/4
π/2
3π/4
π
α : 0°
ν : 0.8
(a1)
(a2)
(a3)
(a4)
α : 0°
ν : 1.5
(b1)
(b2)
(b3)
(b4)
α : 30°
ν : 1.5
(c1)
(c2)
(c3)
(c4)

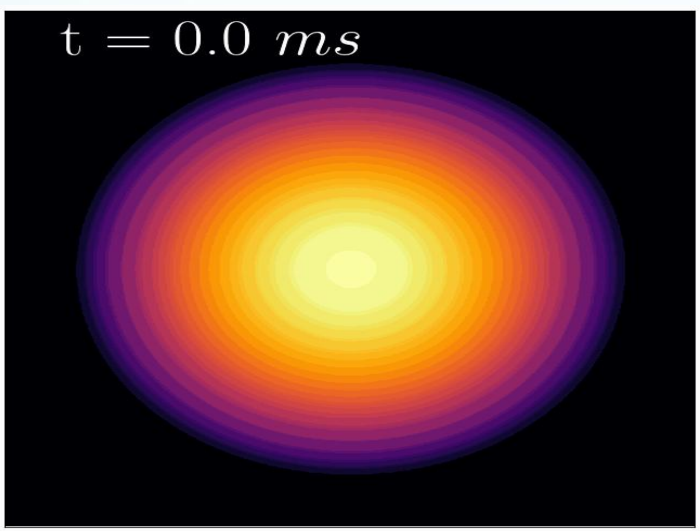
Circular and linear movement comparison

$$V_G = A(t)\exp\left(-\frac{[x - x_0(t)]^2 + [y - y_0(t)]^2}{2W_0^2}\right)$$

$x_0(t) = r_0 \sin(v\,t)$

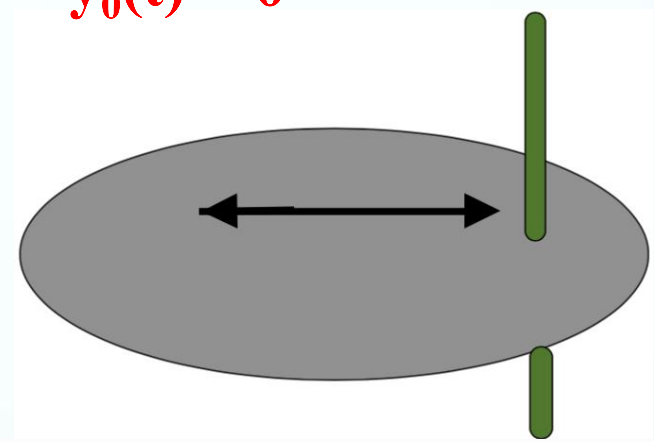
$y_0(t) = r_0 \cos(v\,t)$

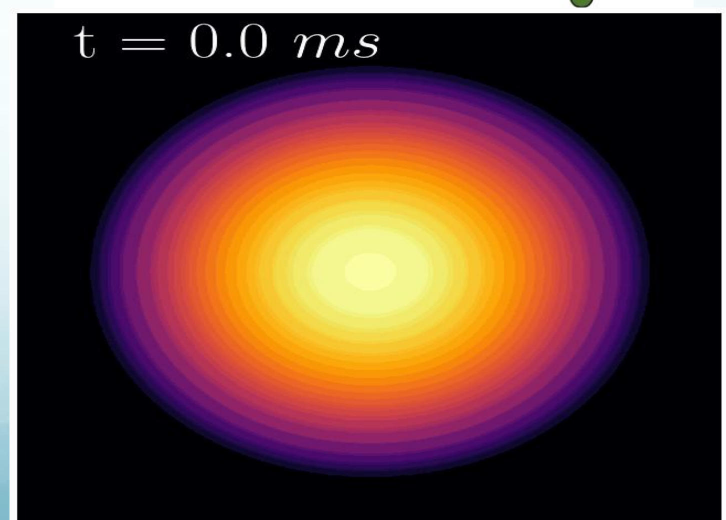




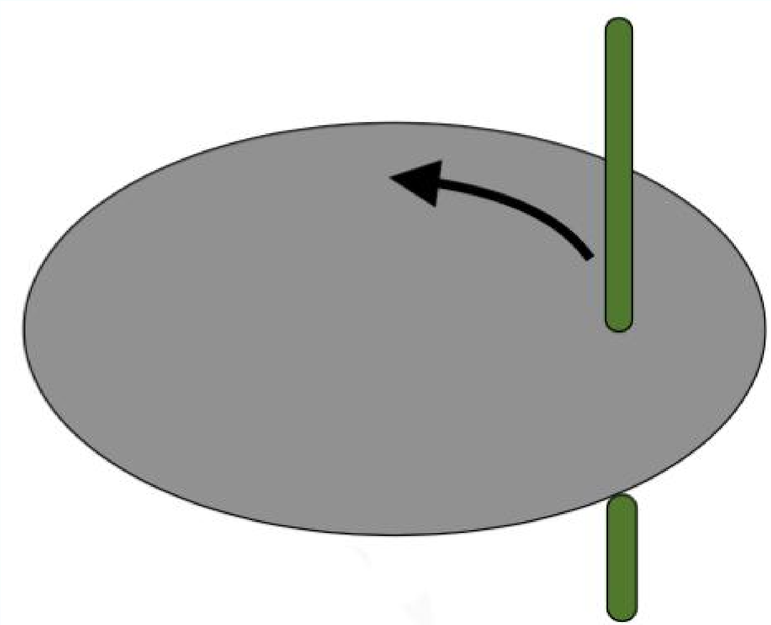
$x_0(t) = r_0 \sin(v\,t)$

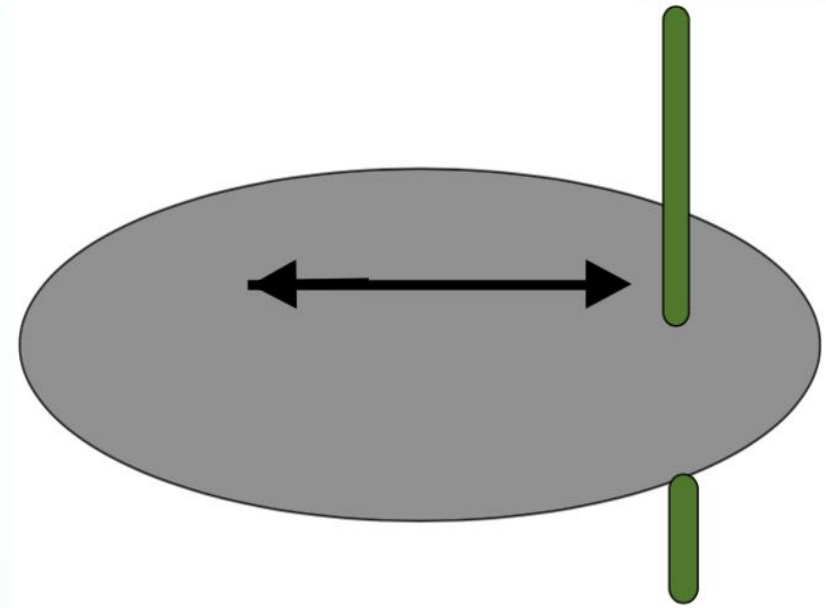
$y_0(t) = 0$

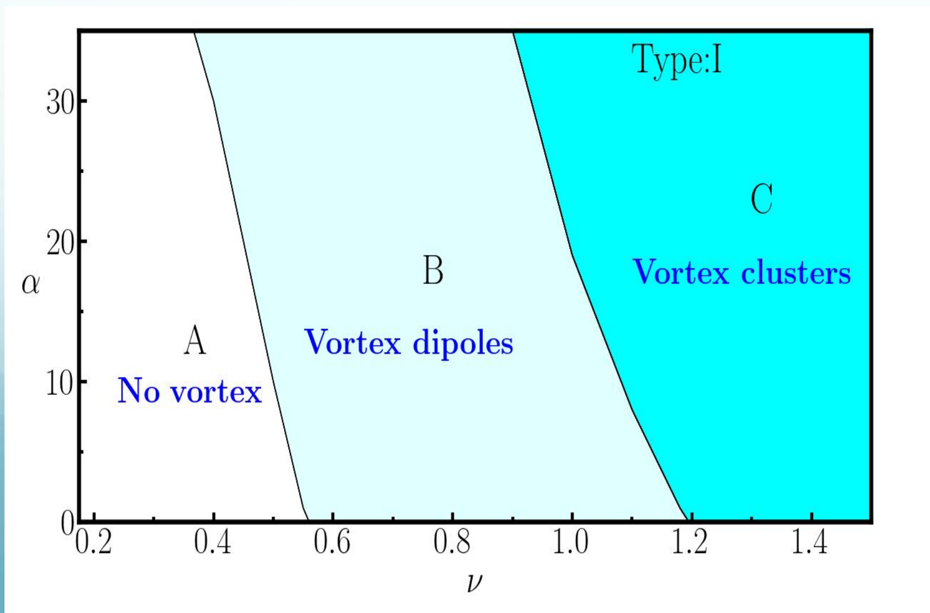


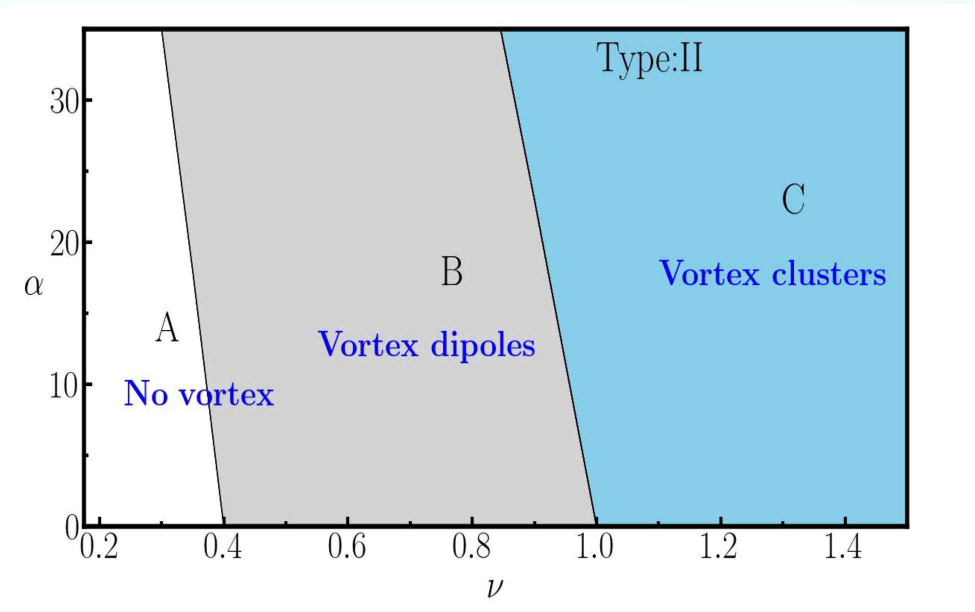


Case(b) - $\alpha = 0 - \pi/2$ $\theta_d = \pi/2$









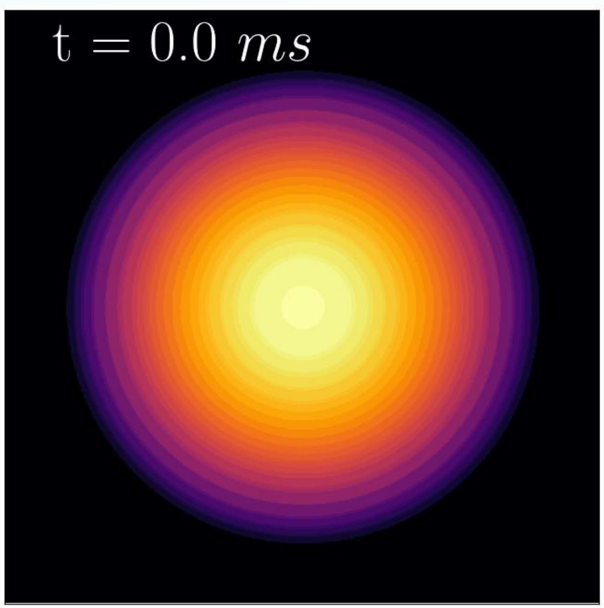
Vortex-pair nucleation - linear movement

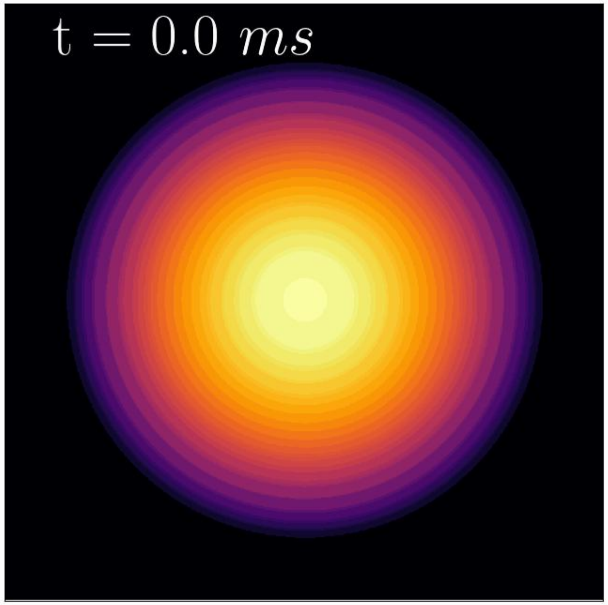
$\theta_d = \pi/2$

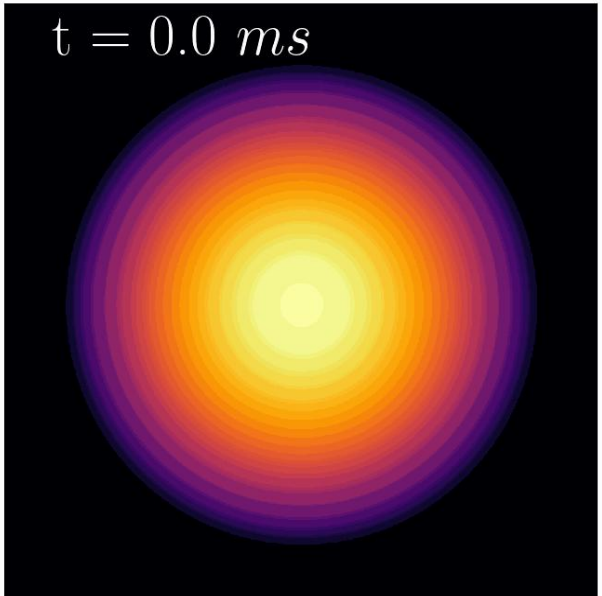
$\alpha = 0$

$$U_{\rm dd}(\mathbf{r}) = \frac{C_{dd}}{4\pi}\frac{(1-3\cos^2\theta_d)}{|\mathbf{r}|^3}\frac{(3\cos^2\alpha - 1)}{2}$$

Vp < Vc **Vp = Vc** **Vp ≥ Vc**







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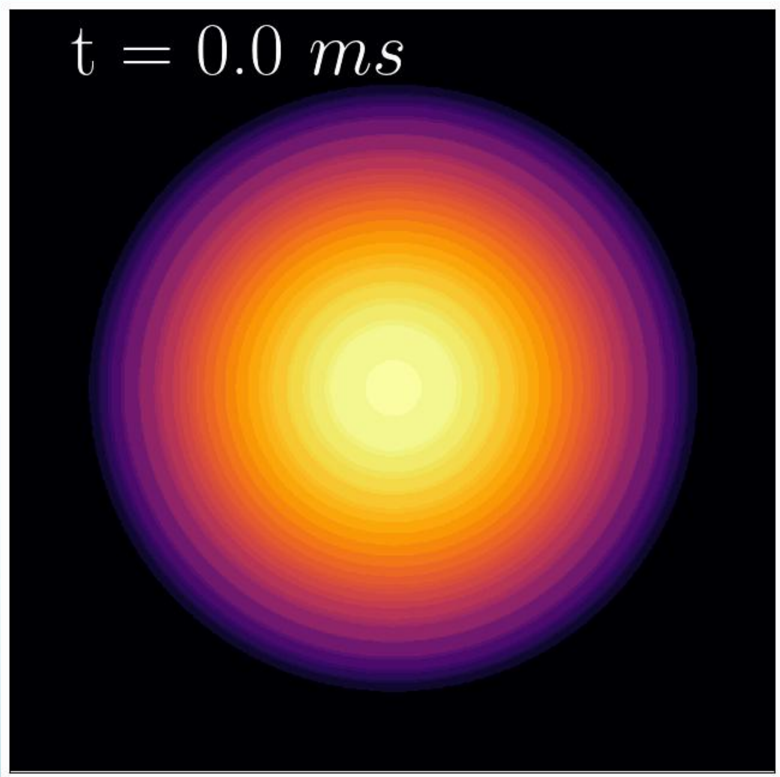
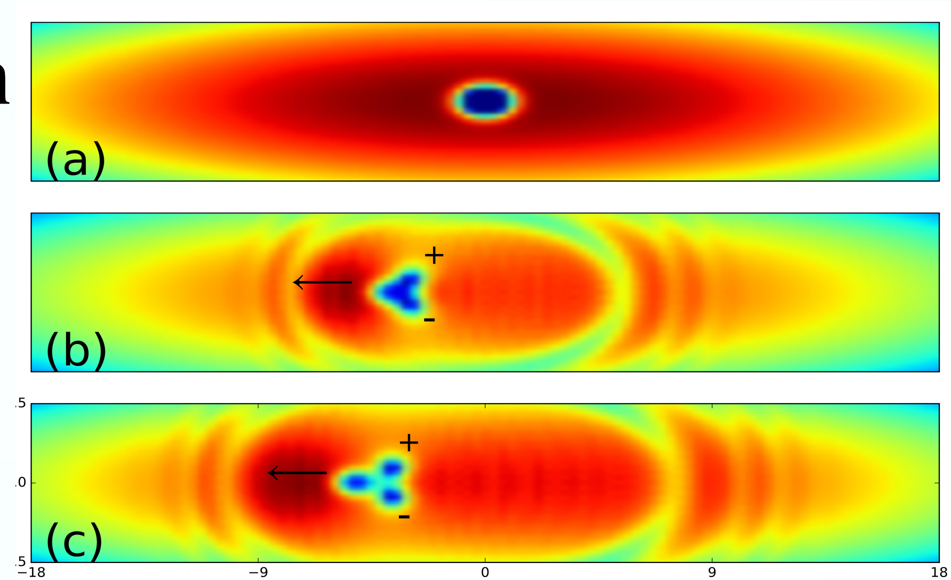
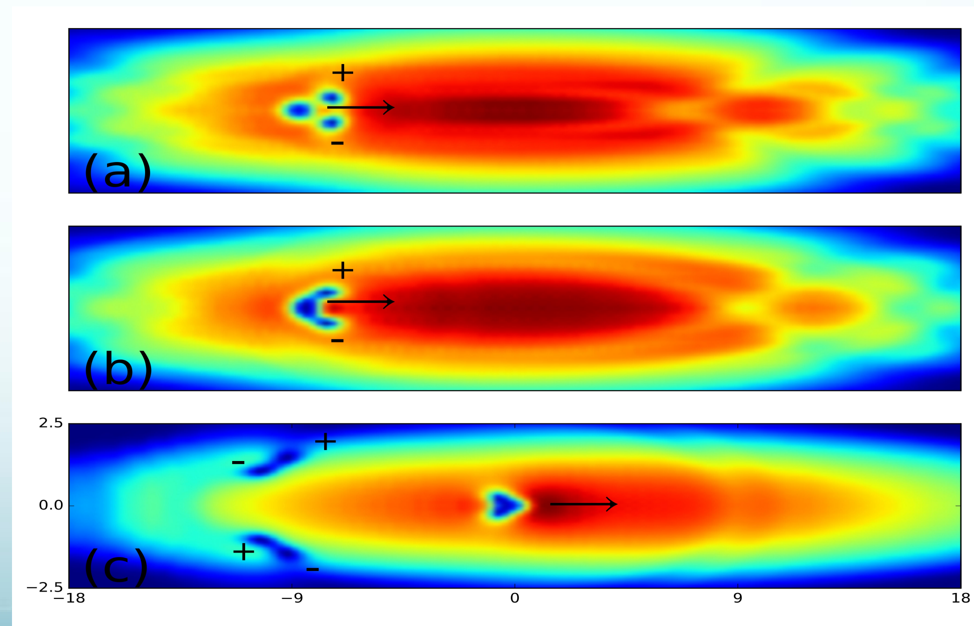
Vortex-pair nucleation

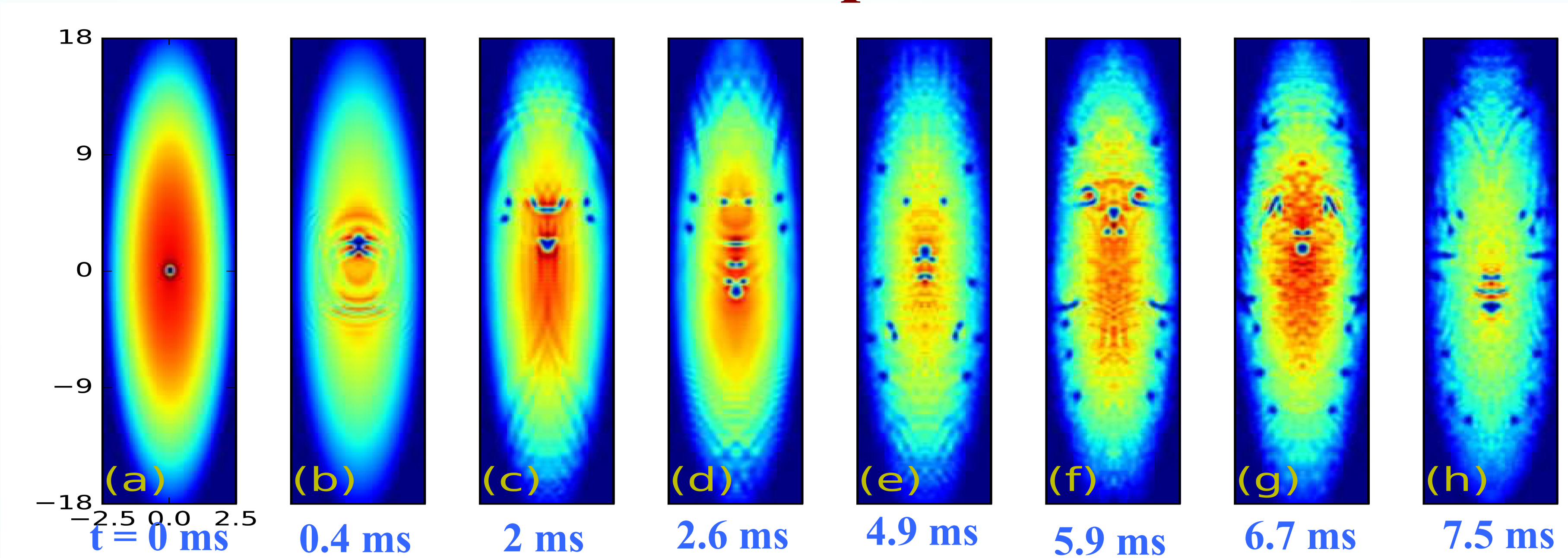


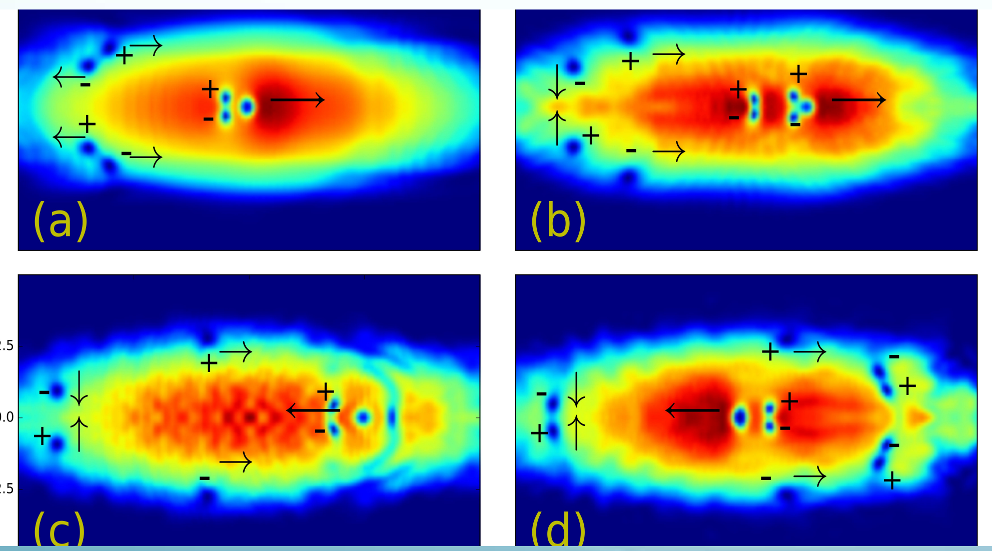


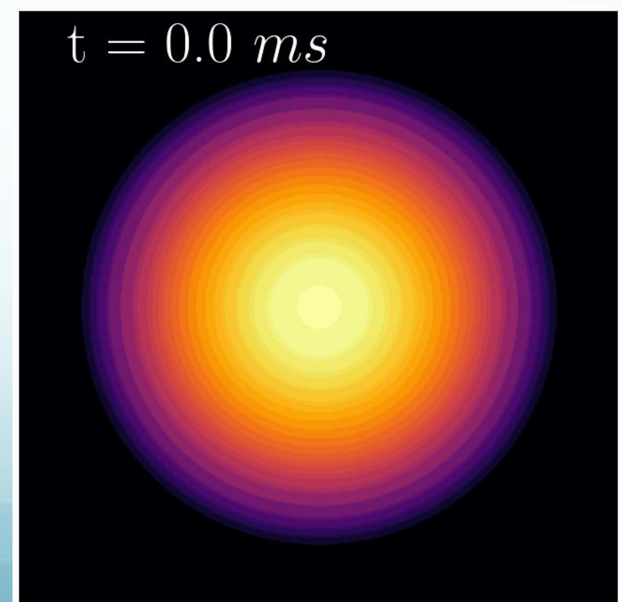


S. Sabari and **R. KishorKumar**, *Eur. Phys. J. D* **72**, 48 (2018)

Nucleation of rarefaction pulses







Final remarks *on dipolar BEC under periodic moving obstacle*

- It was shown a study on vortex dynamical production under circularly moving obstacle, in a trapped dipolar BEC [such as ^{168}Er or ^{164}Dy].
- The critical velocities to produce vortex-antivortex pairs, as well as to produce clusters of vortices were verified that increase by increasing the strength of the DDI.
- In case of linear oscillating obstacle, rarefaction pulses are observed due to interaction of dynamically migrating vortex dipoles.
- With the obstacle moving at fixed radius, it is not observed vortex-antivortex cancellations, as it can happen in the linear case (when the movement is in the $x-$direction with $y = 0$). In this case, one should notice that the vortex and antivortex emerge in parts of the fluid having not exactly the same density.

Actually, in our study on vortex production under periodic stirring potential, we are also characterizing manifestation of quantum turbulence, from analysis of the incompressible part of the corresponding kinetic energy.

Thanks for the attention!

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Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) (Proc. 2017/05660-0) and Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) (Proc. 304469-2019-0).