Benchmarking electron-nucleus scattering within coupled cluster theory on ^{4}He

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EFB25, Mainz, 1/08/2023

Cluster of Excellence PRISMA+ Precision Physics, Fundamental Interactions and Structure of Matter

JG|U

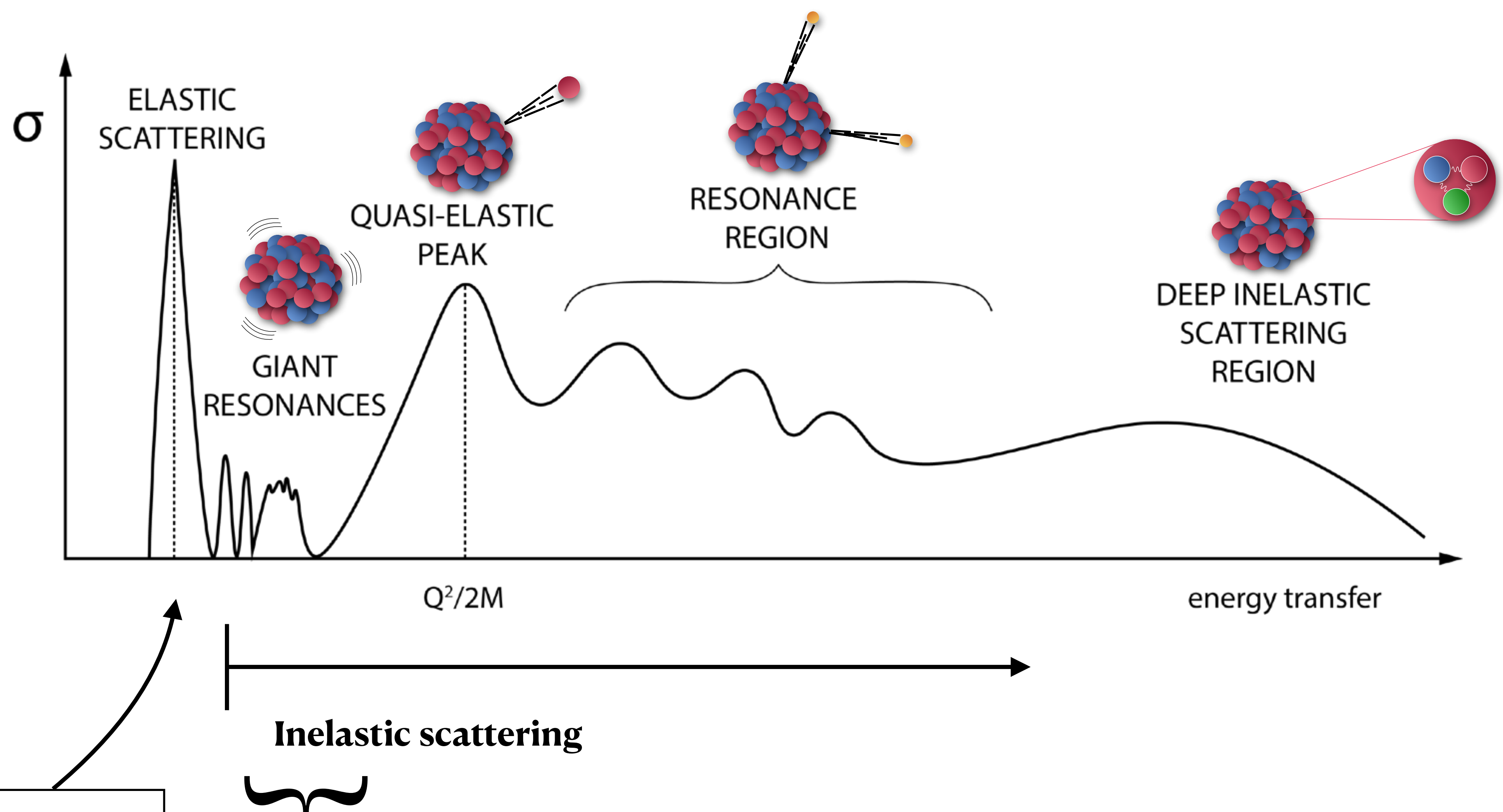
Alexander von Humboldt Stiftung/Foundation

MARIE CURIE ACTIONS

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 101026014

Motivation

Electro(weak) nuclear responses



Elastic scattering: CEvNS (coherent elastic neutrino-nucleus scattering)

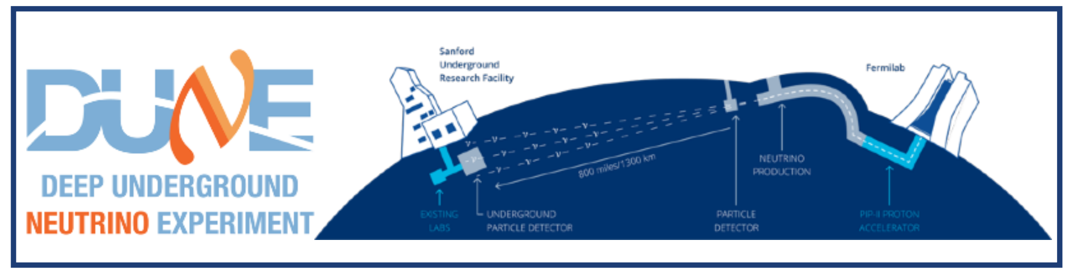
e.g. Supernovae neutrinos

Long-baseline experiments (DUNE, HyperK)

Quasielastic response

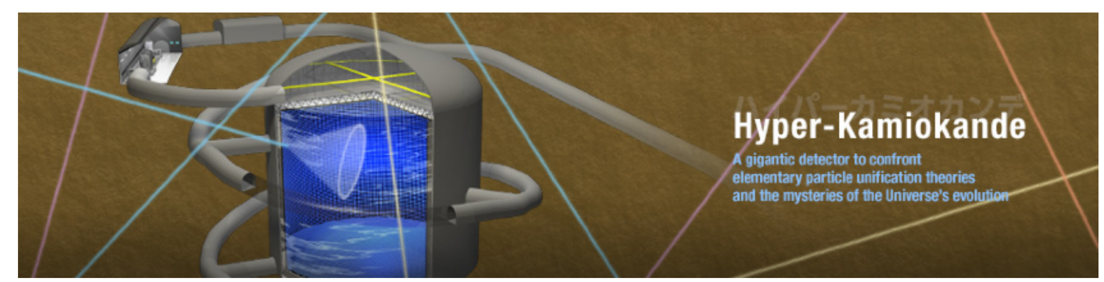
Long-baseline ν experiments

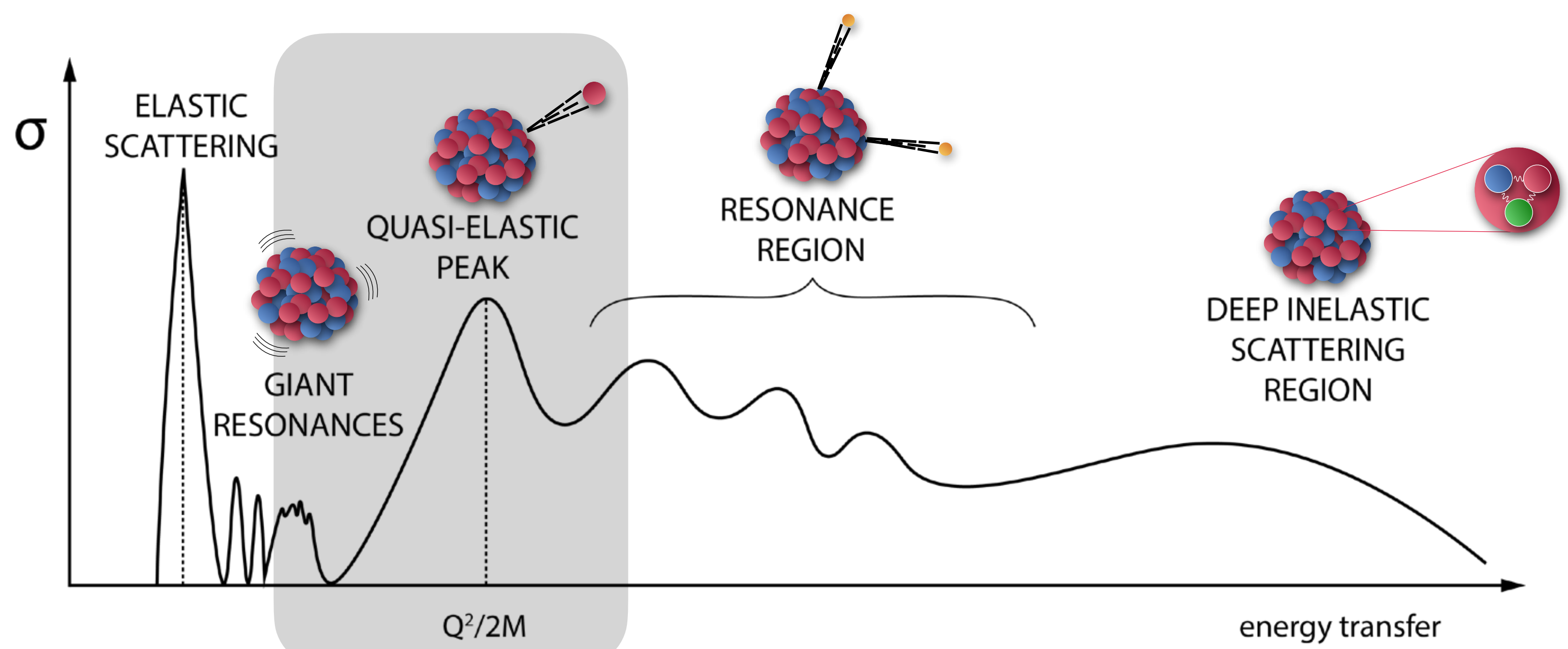
- ✓ Momentum transfer ~hundreds MeV
- ✓ Upper limit for ab initio methods
- ✓ Important mechanism for HyperK, DUNE
- ✓ Role of final state interactions
- ✓ Role of 1-body and 2-body currents



^{40}Ar

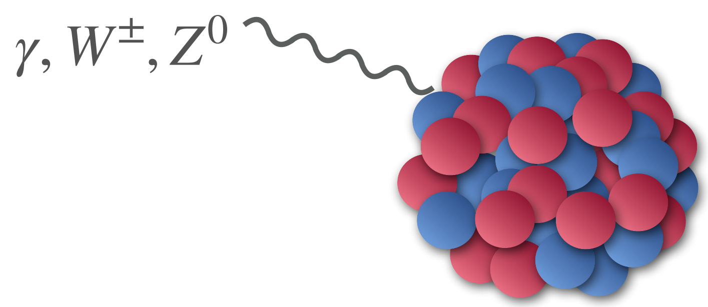
^{16}O





Nuclear response

$J_\mu = (\rho, \vec{j})\, |\, \Psi \rangle$



$\sigma \propto L^{\mu\nu} \; R_{\mu\nu}$

lepton tensor — nuclear responses

$$R_{\mu\nu}(\omega, q) = \sum_f \langle \Psi \,|\, J_\mu^\dagger(q) \,|\, \Psi_f \rangle \langle \Psi_f \,|\, J_\nu(q) \,|\, \Psi \rangle \delta(E_0 + \omega - E_f)$$

$$\left.\frac{d\sigma}{d\omega dq}\right|_{\nu/\bar{\nu}} = \sigma_0 \left(v_{CC} R_{CC} + v_{CL} R_{CL} + v_{LL} R_{LL} + v_T R_T \pm v_{T'} R_{T'} \right)$$

$$\left.\frac{d\sigma}{d\omega dq}\right|_{e} = \sigma_M \left(v_L R_L + v_T R_T \right)$$

← Rosenbluth separation

Coupled cluster method

Reference state (Hartree-Fock): $|\Psi\rangle = a_i^\dagger a_j^\dagger \dots a_k^\dagger |0\rangle$

Include correlations through e^T operator

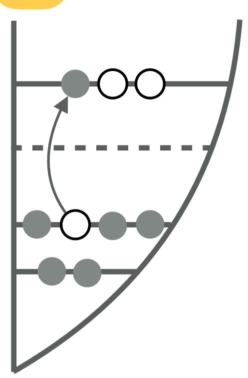
$$\mathscr{H}_N e^T |\Psi\rangle = E e^T |\Psi\rangle$$

1p1h

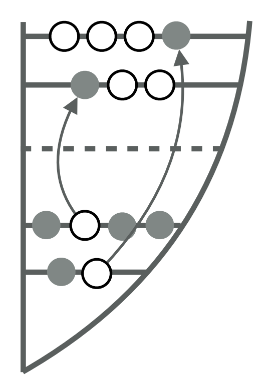
2p2h

Expansion: $T = \sum t_a^i a_a^\dagger a_i + \frac{1}{4} \sum t_{ab}^{ij} a_a^\dagger a_b^\dagger a_i a_j + \dots$

singles



doubles



coefficients obtained through coupled cluster equations

Coupled cluster method

Reference state (Hartree-Fock): $|\Psi\rangle = a_i^\dagger a_j^\dagger \ldots a_k^\dagger |0\rangle$

Include correlations through e^T operator

$$\mathscr{H}_N e^T |\Psi\rangle = E e^T |\Psi\rangle$$

- ✓ Controlled approximation through truncation in T
- ✓ Polynomial scaling with A (predictions for ^{132}Sn and ^{208}Pb)

1p1h

2p2h

$$\text{Expansion: } T = \sum t_a^i a_a^\dagger a_i + \frac{1}{4} \sum t_{ab}^{ij} a_a^\dagger a_b^\dagger a_i a_j + \ldots$$

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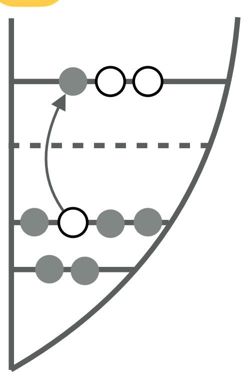

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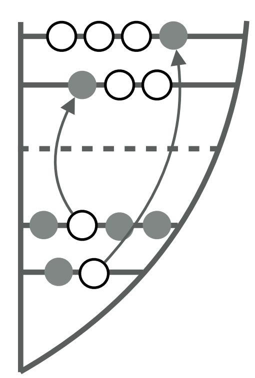
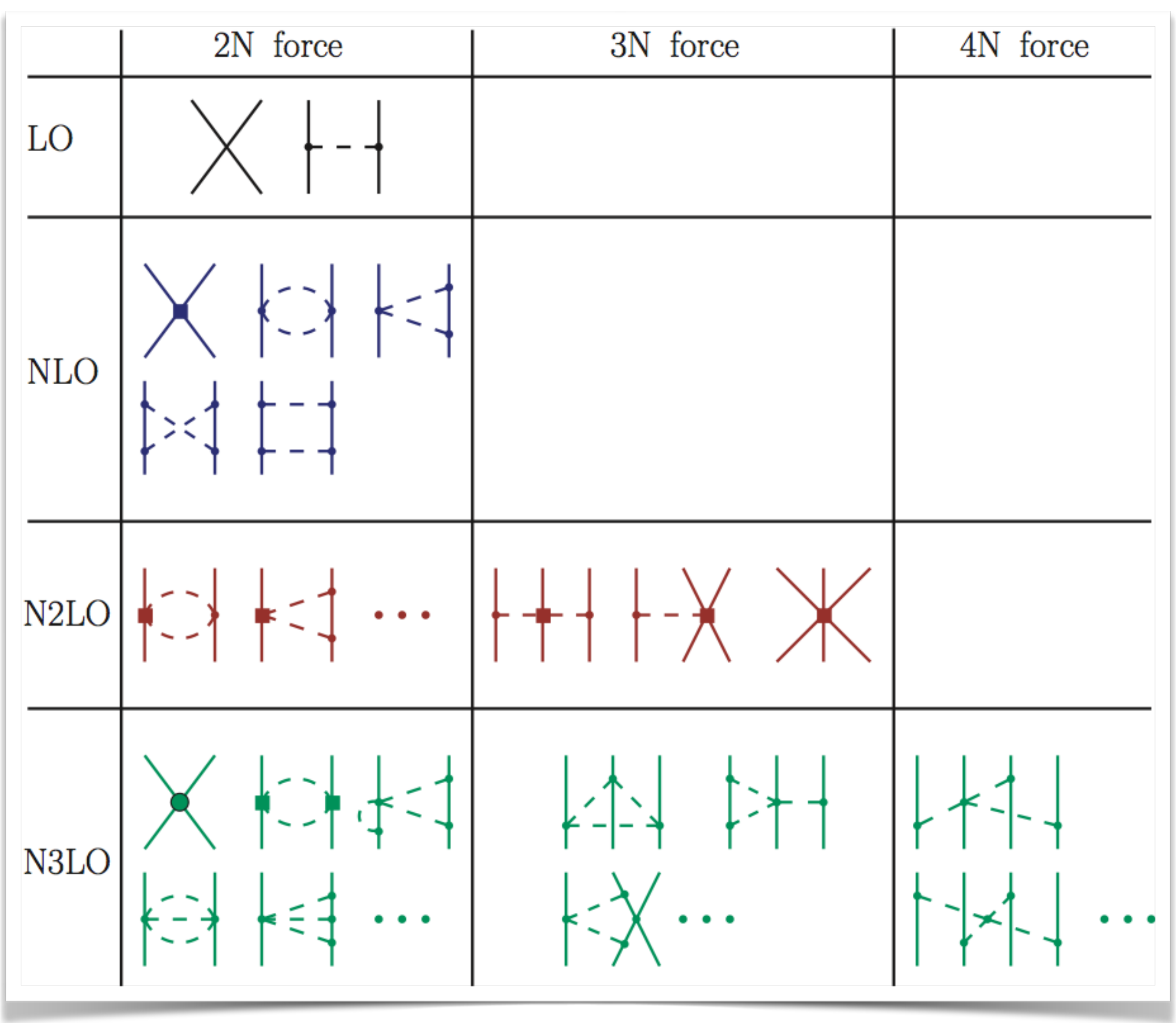


coefficients obtained through coupled cluster equations

Hamiltonian & electroweak currents

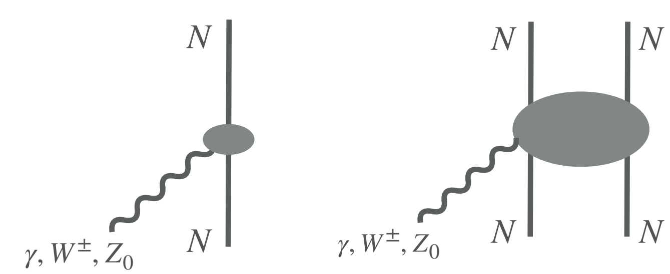
Chiral Hamiltonians

$$\mathscr{H} = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$



Electroweak current (consistent with the Hamiltonian)

$$J = \sum_i j_i + \sum_{i<j} j_{ij} + \dots$$



known to give significant contribution for neutrino-nucleus scattering

Hamiltonian & electroweak currents

Chiral Hamiltonians

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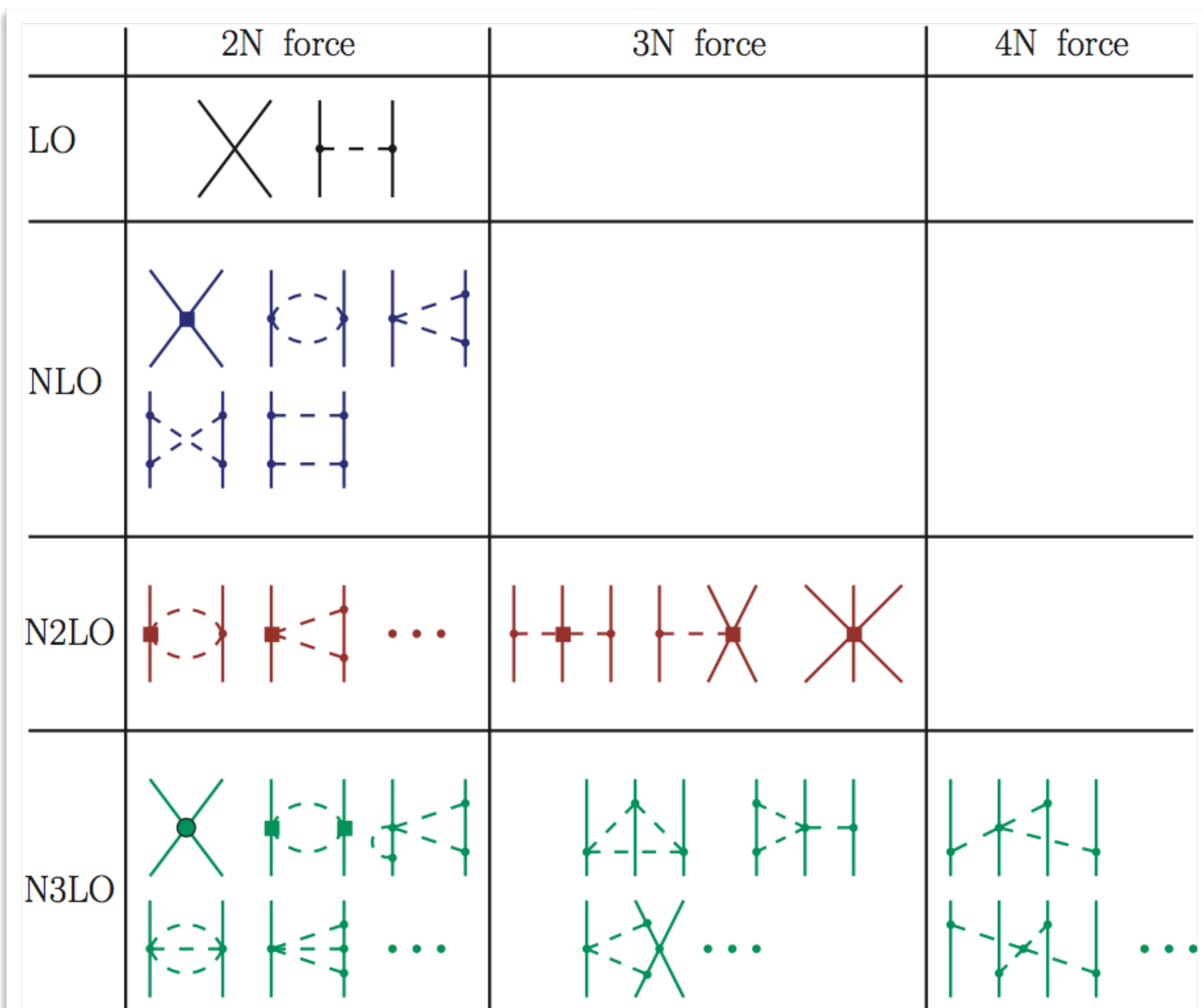
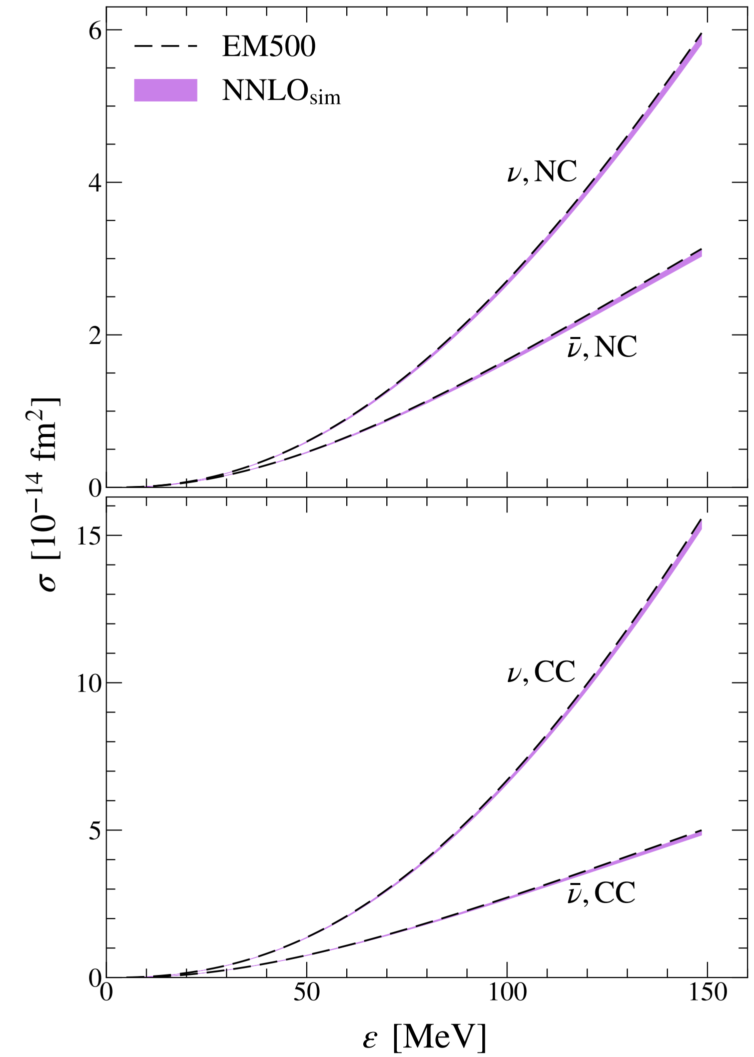


$\nu(\bar{\nu}) + d \to \mu^{\pm} + X$



Multipole decomposition for 1- and 2-body EW currents

B. Acharya, S. Bacca

Phys.Rev.C 101 (2020) 1, 015505

Coulomb sum rule

$$m_0(q) = \int d\omega R_L(\omega, q) = \sum_{f \neq 0} |\langle \Psi_f | \hat{\rho} | \Psi \rangle|^2 = \langle \Psi | \hat{\rho}^\dagger \hat{\rho} | \Psi \rangle - |F_{el}(q)|^2$$

easier to calculate since we do not need $|\Psi_f\rangle$

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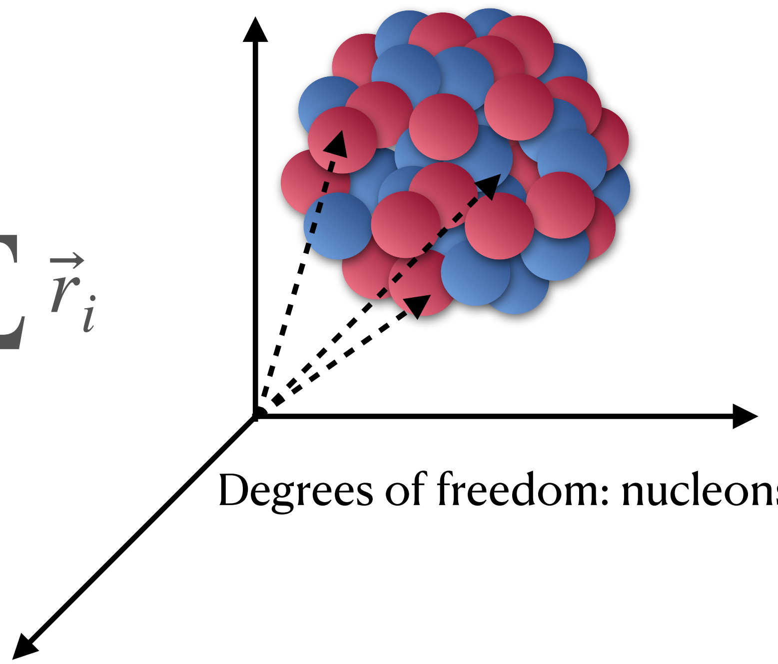
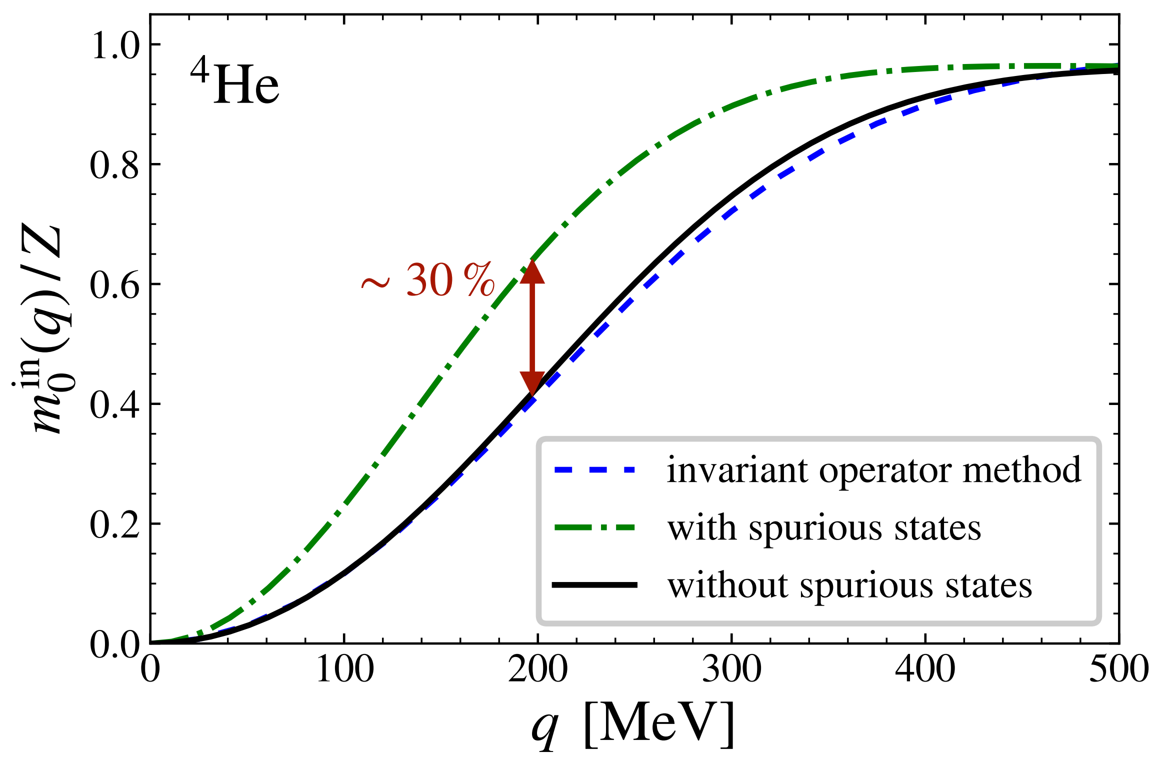
center of mass problem

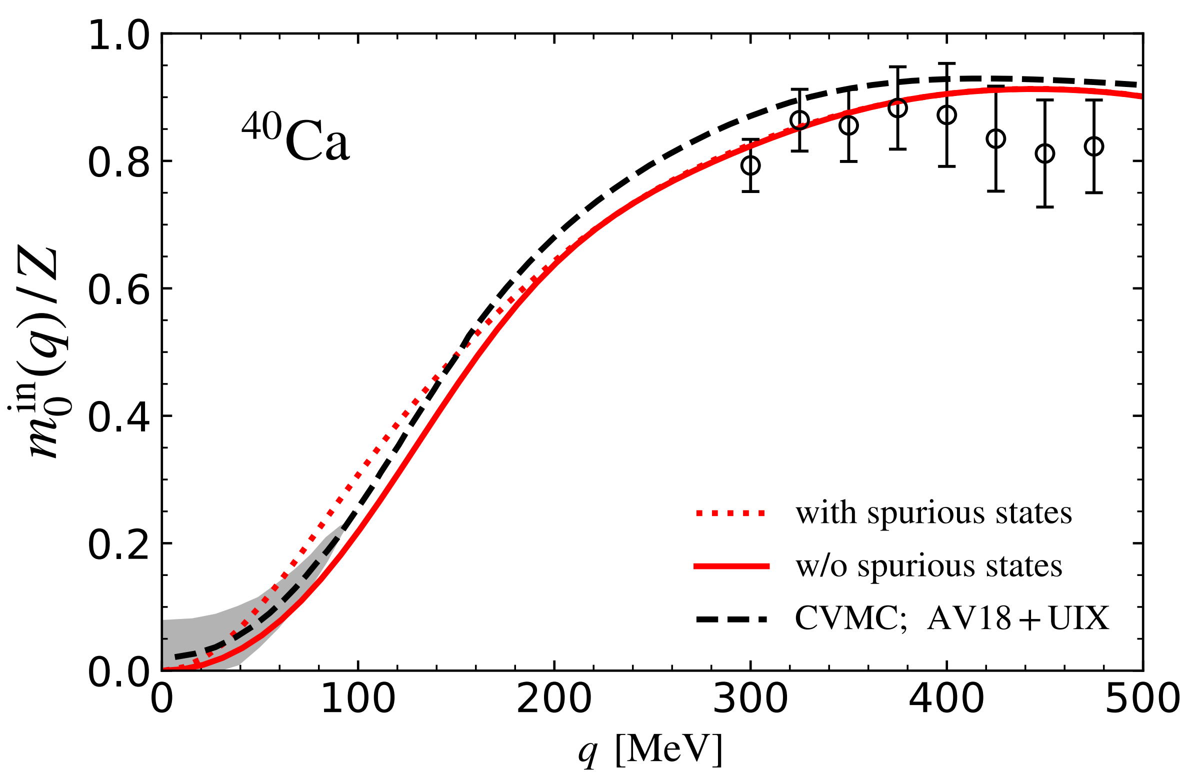
$$|\Psi\rangle \text{ has 3A coordinates} \rightarrow \text{3(A-1) } \overset{\text{intrinsic}}{\text{coordinates}} + \vec{R} = \frac{1}{A}\sum_i^A \vec{r}_i$$



With translationally non-invariant operators we may excite spurious states

Coulomb sum rule





J.E.S. B. Acharya, S.Bacca, G. Hagen
Phys.Rev.C 102 (2020) 064312

CoM spurious states dominate for light nuclei

Electromagnetic responses

Lorentz Integral Transform + Coupled Cluster

$$R_{\mu\nu}(\omega, q) = \sum\!\!\!\!\!\!\int_f \langle \Psi | J_\mu^\dagger | \Psi_f \rangle \langle \Psi_f | J_\nu | \Psi \rangle \delta(E_0 + \omega - E_f)$$

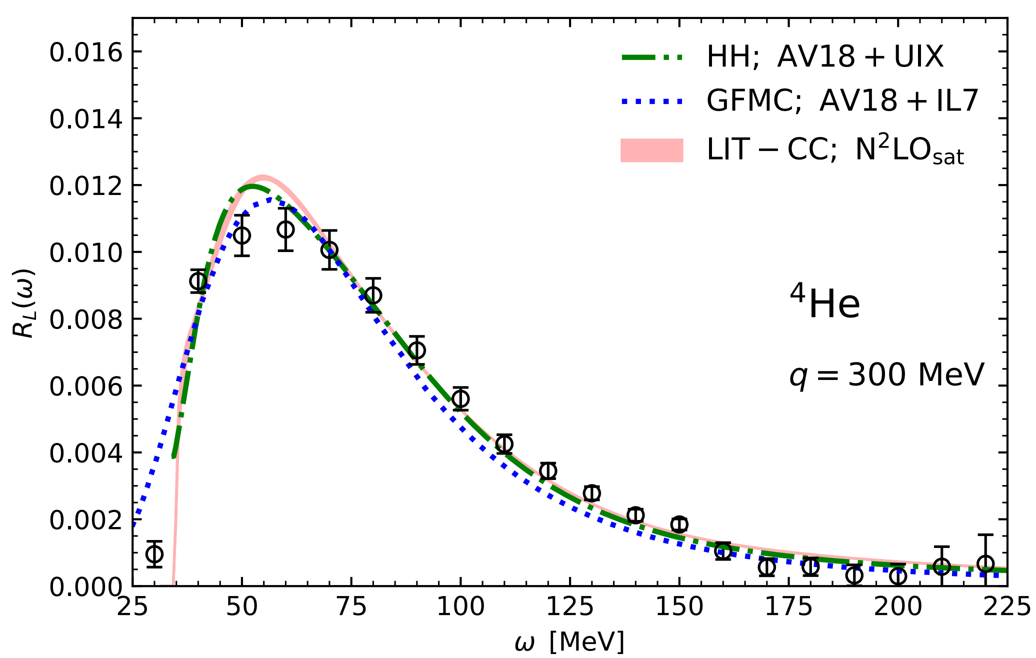
Consistent treatment of final state interactions.

Integral transform:

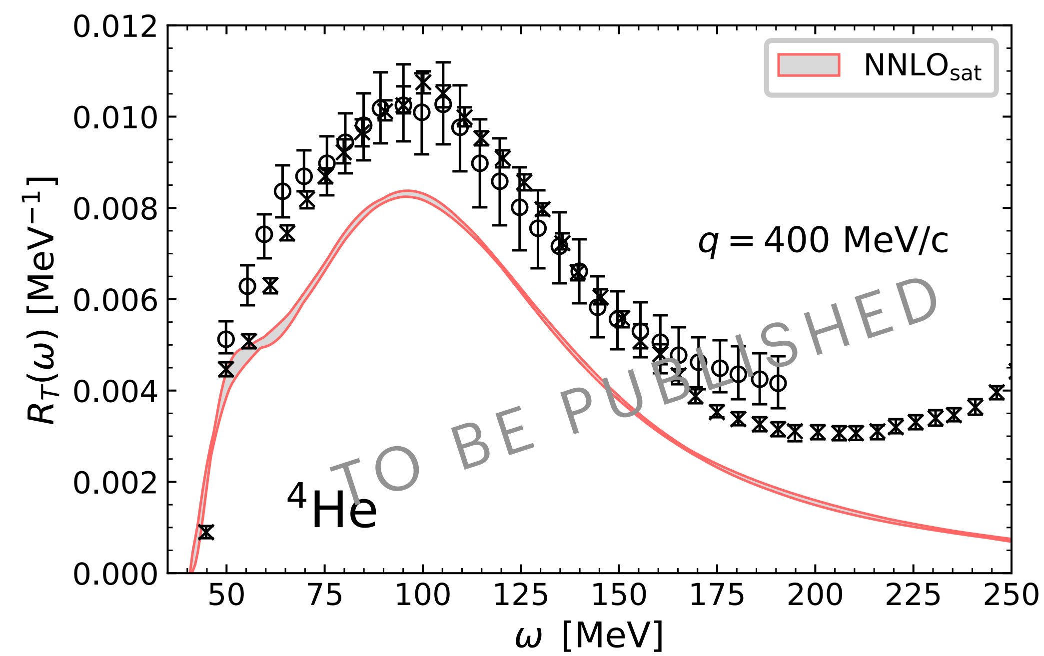
$$S_{\mu\nu}(\sigma, q) = \int d\omega K(\omega, \sigma) R_{\mu\nu}(\omega, q) = \langle \Psi | J_\mu^\dagger K(\mathscr{H} - E_0, \sigma) J_\nu | \Psi \rangle$$

Lorentzian kernel:

$$K_\Gamma(\omega, \sigma) = \frac{1}{\pi} \frac{\Gamma}{\Gamma^2 + (\omega - \sigma)^2}$$



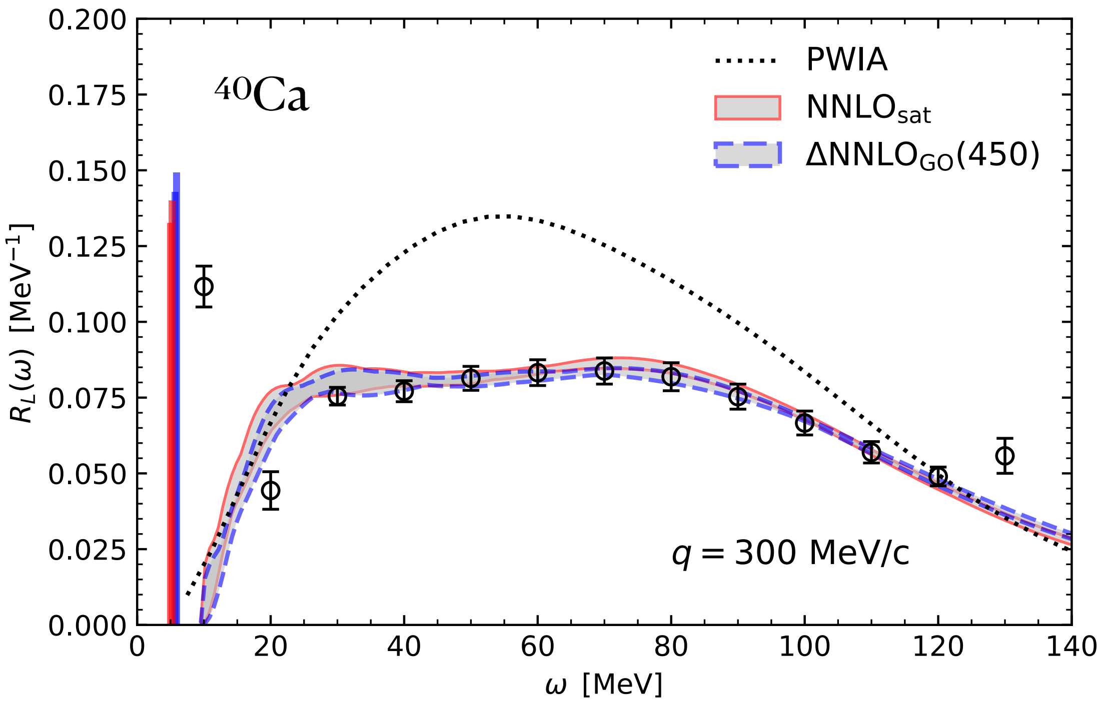
JES, B. Acharya, S. Bacca, G. Hagen; *PRL* 127 (2021) 7, 072501



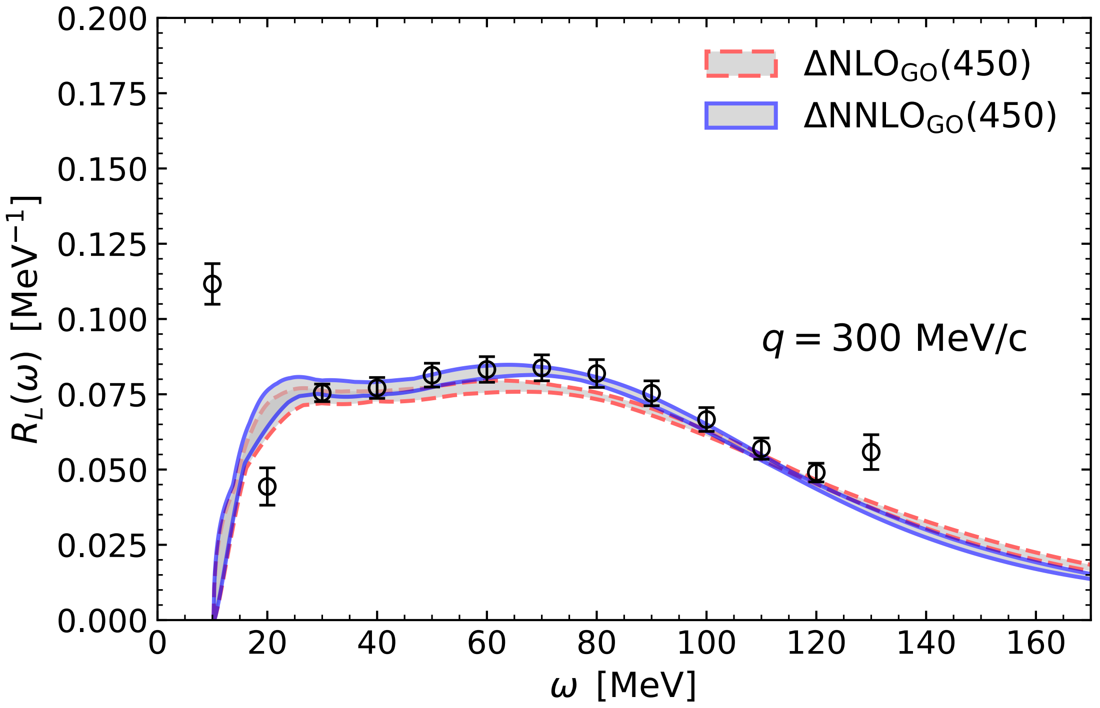
(Only 1-body current)

Longitudinal response ^{40}Ca

Lorentz Integral Transform + Coupled Cluster



JES, B. Acharya, S. Bacca, G. Hagen; *PRL* 127 (2021) 7, 072501



B. Acharya, S. Bacca, JES et al. Front. Phys. 1066035(2022)

- ✓ CC singles & doubles
- ✓ varying underlying harmonic oscillator frequency
- ✓ two different chiral Hamiltonians
- ✓ *inversion procedure*

First ab-initio results for many-body system of 40 nucleons

Exclusive cross-sections

- LIT-CC calculations for $q \lesssim 450$ MeV
- Inclusive cross sections
- No pion production

$$\Downarrow$$

- Ideas (and approximations) needed to address relevant physics for ν oscillation experiments

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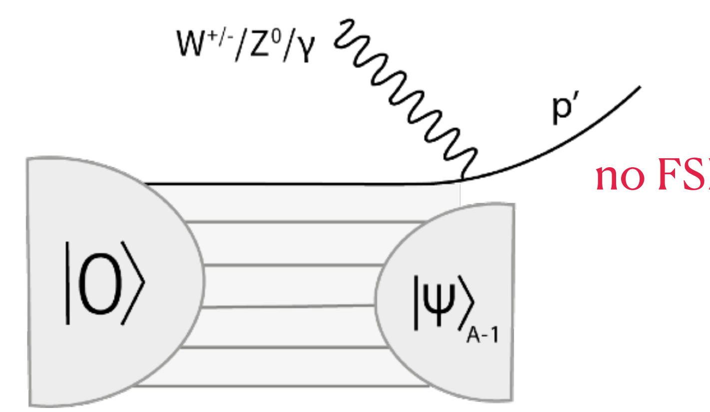
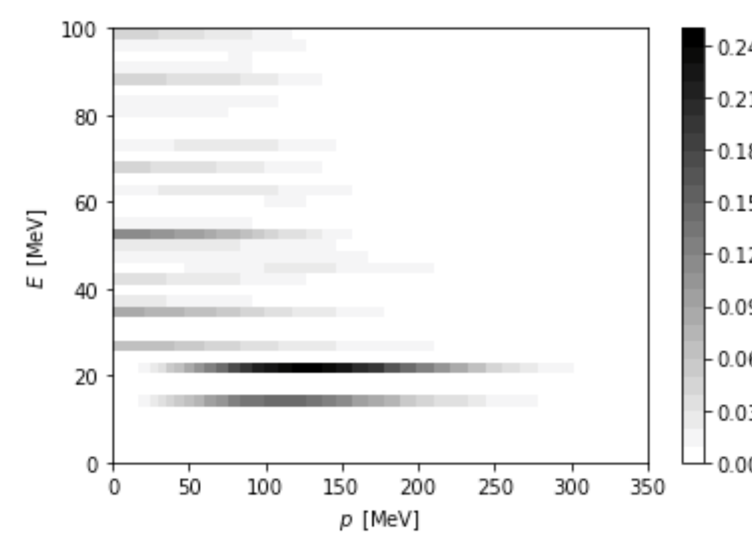
SPECTRAL FUNCTION



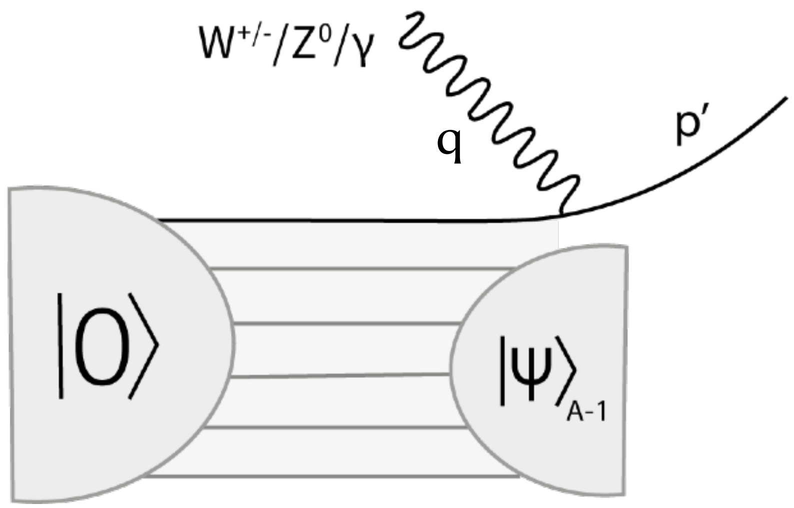
Impulse Approximation



Probability density of finding nucleon $(E, \mathbf{p})$ in ground state nucleus

Spectral functions

Coupled Cluster + ChEK method

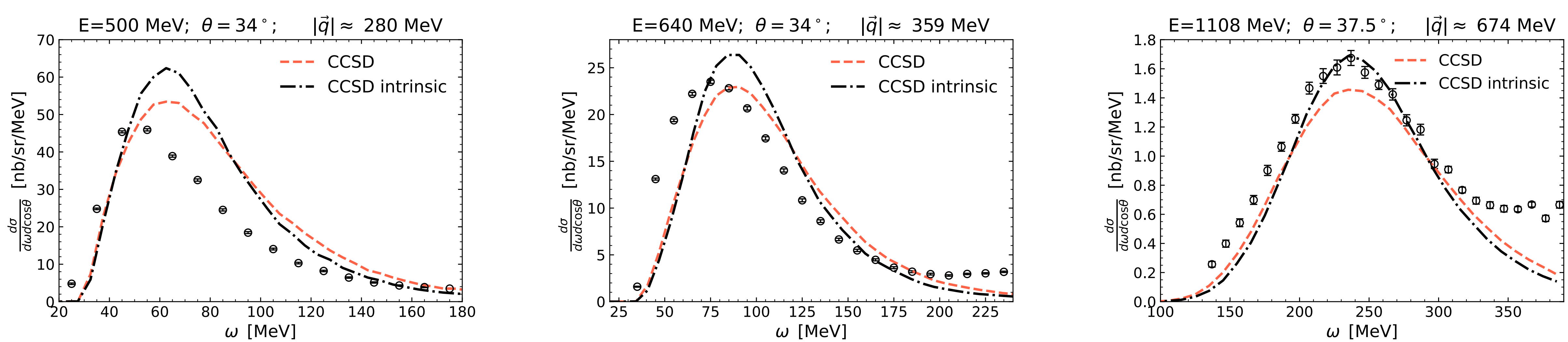


$$\sigma \propto |\mathscr{M}|^2 S(E,p)$$

Factorized interaction vertex (relativistic, pion production...)

Spectral function - nuclear information

growing **q** momentum transfer → final state interactions play minor role



JES, S. Bacca, G. Hagen, T. Papenbrock *Phys.Rev.C* 106 (2022) 3, 034310

Outlook

- LIT-CC benchmark for electron scattering → ready for neutrino
- Role of 2-body currents for medium-mass nuclei
- Extending the response calculation to ^{40}Ar
- Spectral functions (within Impulse Approximation):
    - Relativistic regime
    - Semi-inclusive processes
    - Further steps: 2-body spectral functions, accounting for FSI

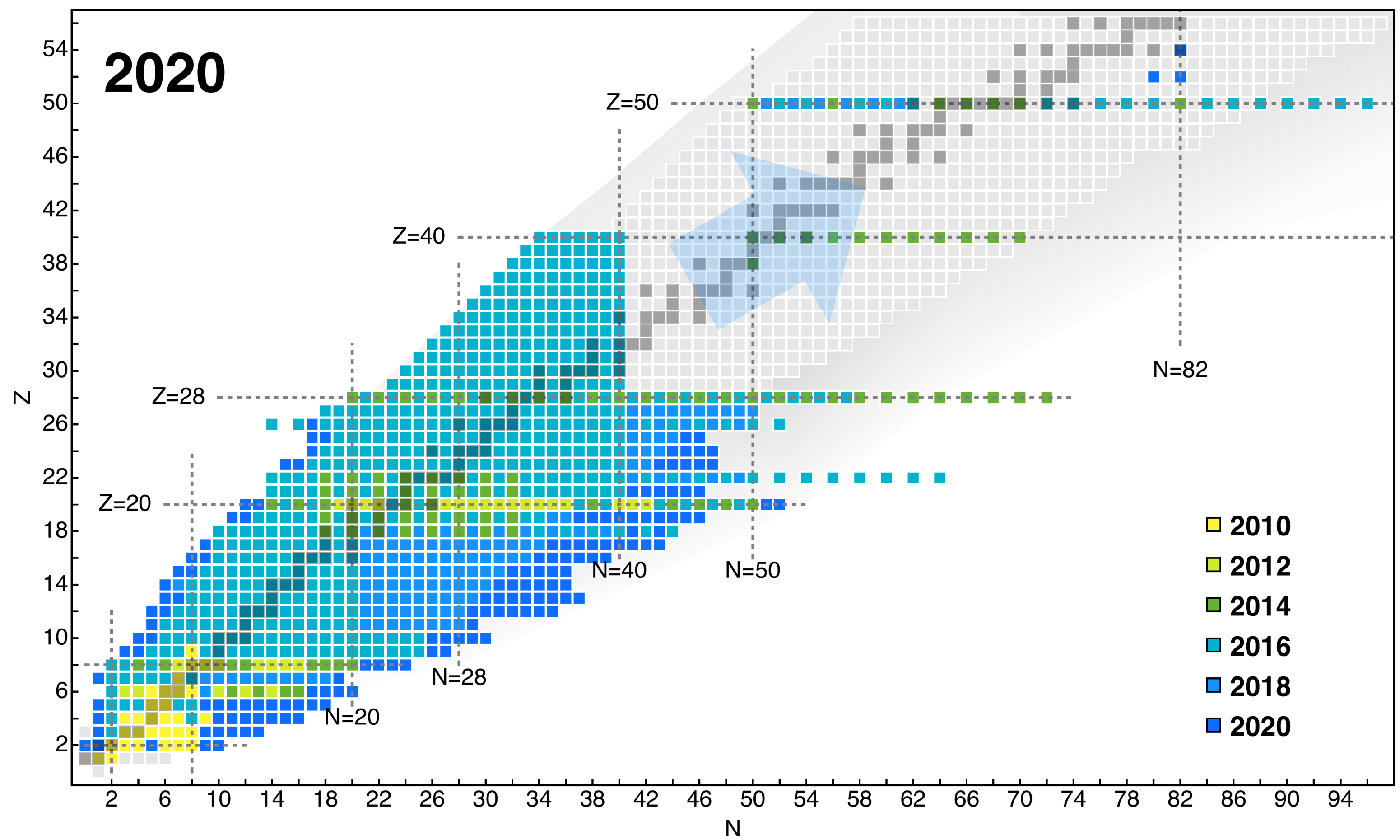
Backup

An initio nuclear methods

$$\mathscr{H}\,|\Psi\rangle = E\,|\Psi\rangle$$

“we interpret the ab initio method to be a systematically improvable approach for quantitatively describing nuclei using the finest resolution scale possible while maximizing its predictive capabilities.”

A. Ekström et al, *Front. Phys.*11 (2023) 29094



H. Hergert, *Front.in Phys.* 8 (2020) 379

- ➡ Developments on the side of many body methods (IMSRG, CC, SCGF, QMC, etc.)
- ➡ Developments of chiral nuclear forces (->faster convergence)

Lorentz Integral Transform (LIT)

$$R_{\mu\nu}(\omega, q) = \sum\!\!\!\!\!\!\int_f \langle \Psi | J_\mu^\dagger | \Psi_f \rangle \langle \Psi_f | J_\nu | \Psi \rangle \delta(E_0 + \omega - E_f)$$

continuum spectrum

Integral transform

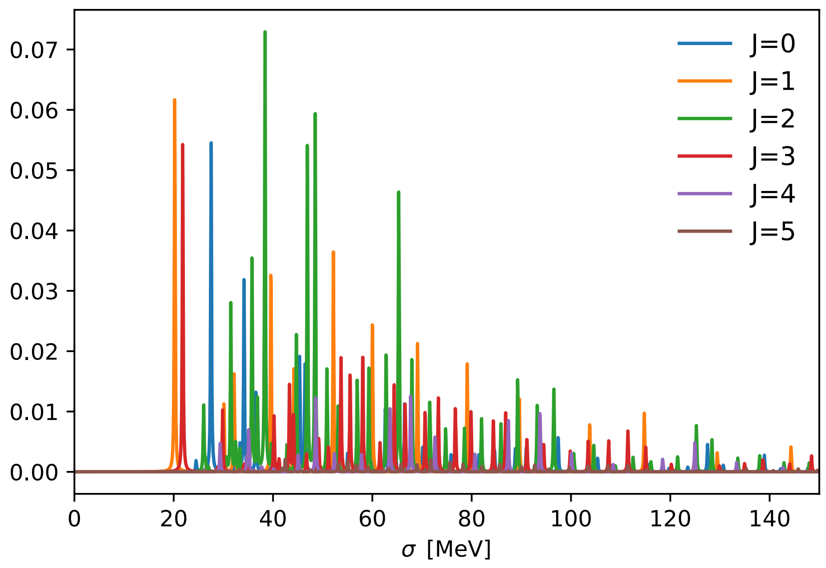
$$S_{\mu\nu}(\sigma, q) = \int d\omega K(\omega, \sigma) R_{\mu\nu}(\omega, q) = \langle \Psi | J_\mu^\dagger K(\mathscr{H} - E_0, \sigma) J_\nu | \Psi \rangle$$

Lorentzian kernel:

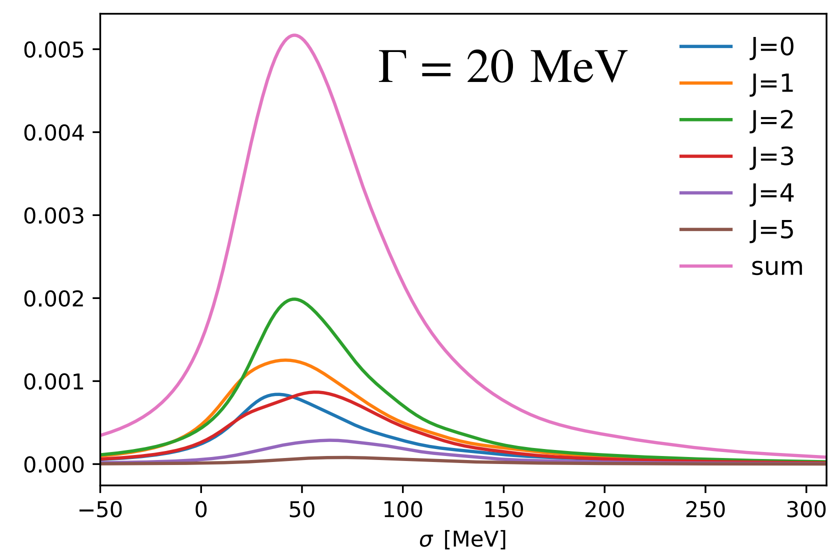
$$K_\Gamma(\omega, \sigma) = \frac{1}{\pi} \frac{\Gamma}{\Gamma^2 + (\omega - \sigma)^2}$$

$S_{\mu\nu}$ has to be inverted to get access to $R_{\mu\nu}$

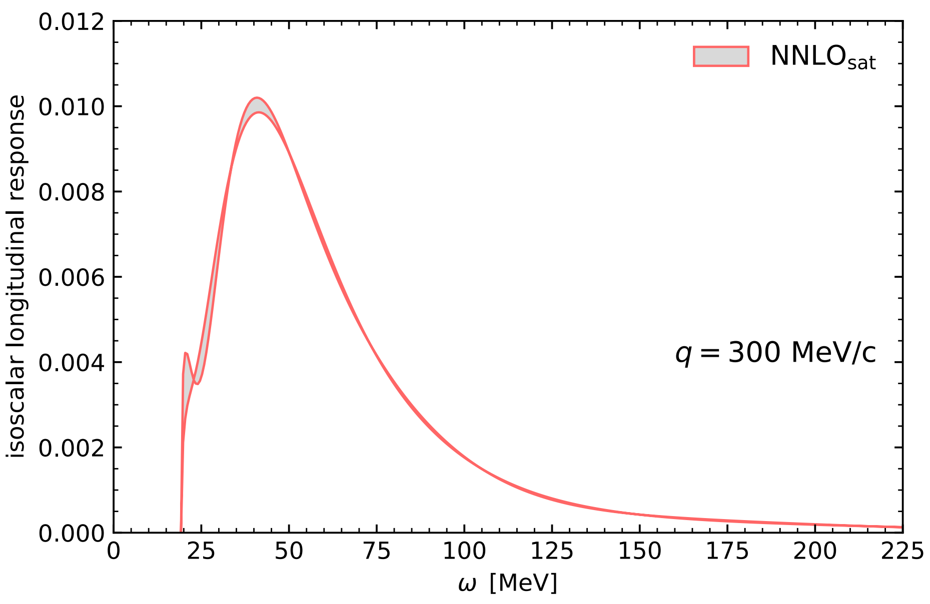
Lorentz Integral Transform



Integral transform



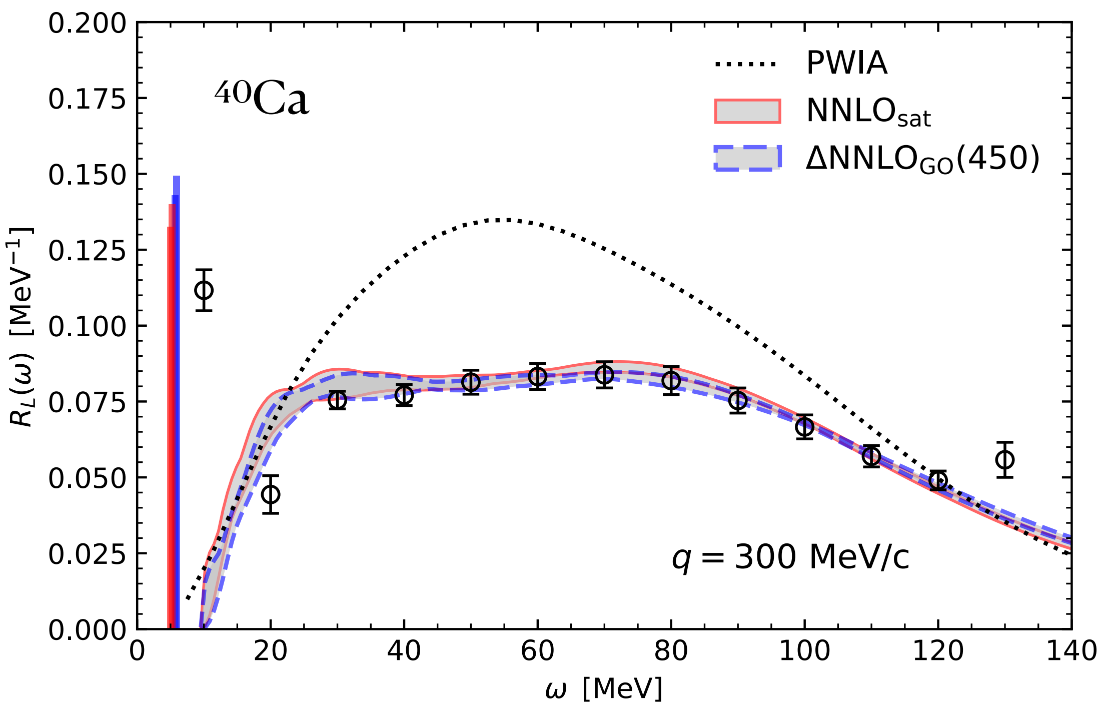
Inversion

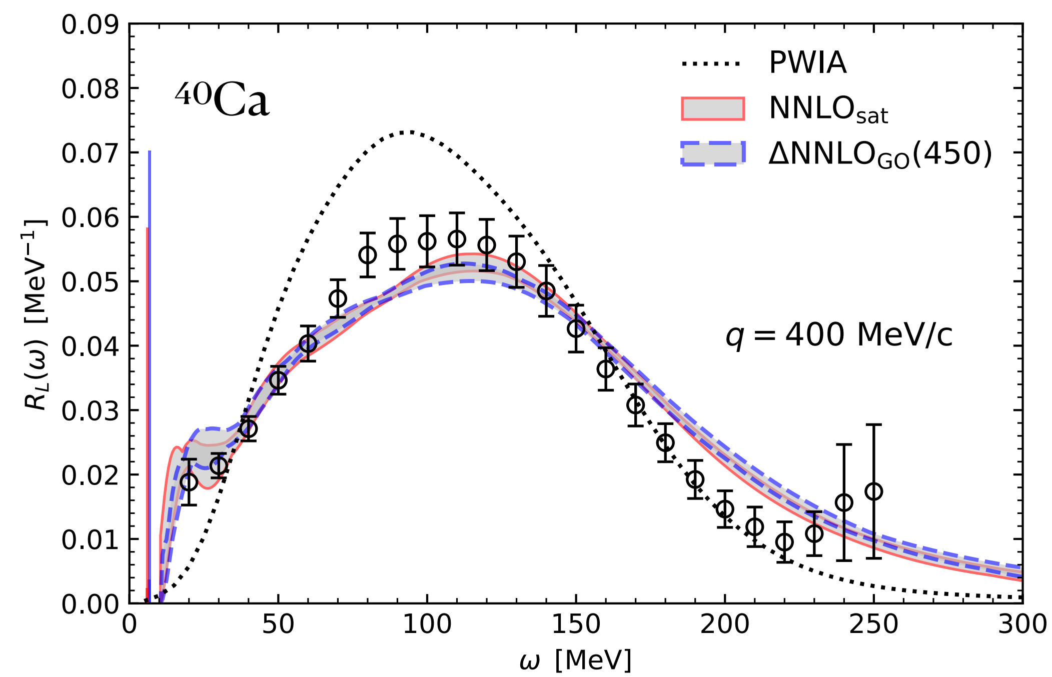


Longitudinal isoscalar response on ^{4}He at q=300 MeV

Longitudinal response ^{40}Ca

Lorentz Integral Transform + Coupled Cluster





JES, B. Acharya, S. Bacca, G. Hagen; *PRL* 127 (2021) 7, 072501