Two-body double pole and three-body bound states: physical and unphysical quark masses

V. S. Timóteo (UNICAMP, Brazil) and U. van Kolck (IJCLab, France)

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Acknowledgements

Outline

* Introduction / Motivation

* Lattice QCD predictions

* Two-body potentials

* Three-body bindings

* Final Remarks

Introduction / Motivation

Verify if lattice QCD predictions for two and three nucleons
are supported by effective calculations with separable potentials

for a review of 3N system, see works
from Bochum, Krakow, Ohio groups

Lattice QCD ⟷ Separable 2N potentials

Lüscher formula ⟷ Faddeev equation

S.R.Beane *et al*. [NPLQCD], Phys. Rev. C 88 (2013) 024003 (two-nucleons)

S.R.Beane *et al*. [NPLQCD], Phys. Rev. D 87 (2013) 034506 (three-nucleons)

Lattice QCD predictions

$m_\pi = 806$ MeV $\qquad$ $m_N = 1.634\ (0)\ (0)\ (18)$ GeV

triplet: $a_{21} = 1.82^{+0.14+0.17}_{-0.13-0.12}$ fm, $\quad r_{21} = 0.906^{+0.068+0.068}_{-0.075-0.084}$ fm

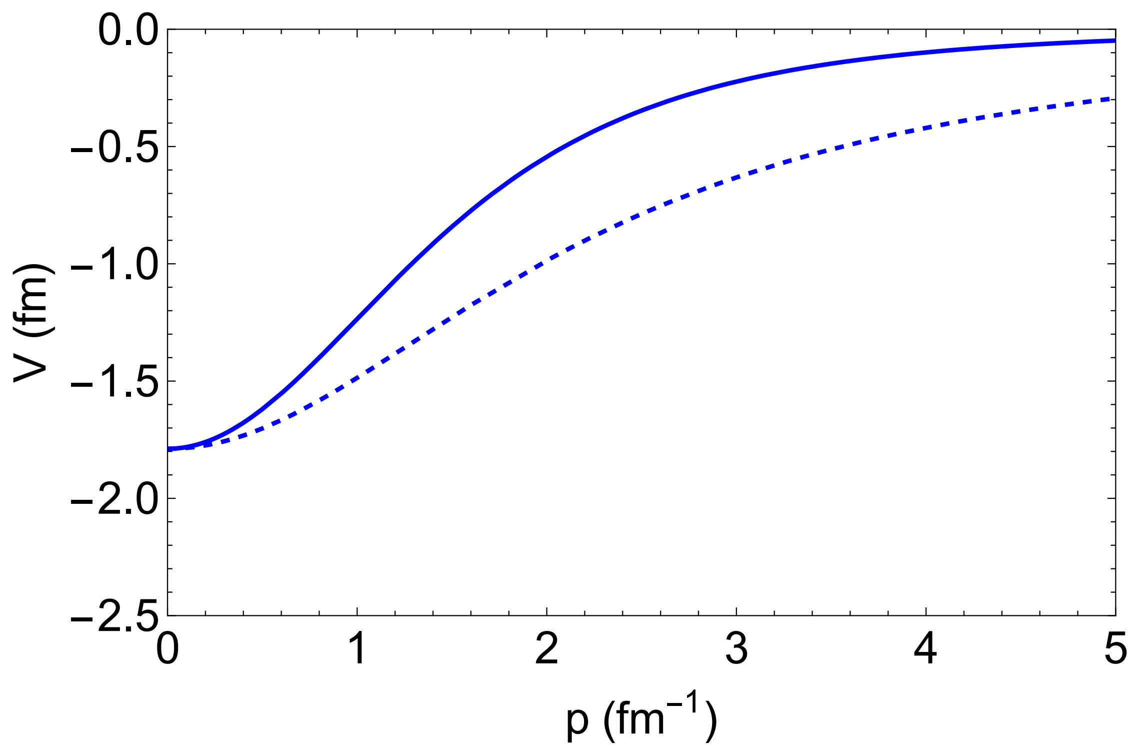
singlet: $a_{20} = 2.33^{+0.19+0.27}_{-0.17-0.20}$ fm, $\quad r_{20} = 1.130^{+0.071+0.059}_{-0.077-0.063}$ fm

$$a \sim 2\,r$$

S.R.Beane *et al*. [NPLQCD], Phys. Rev. C 88 (2013) 024003

2N separable potentials

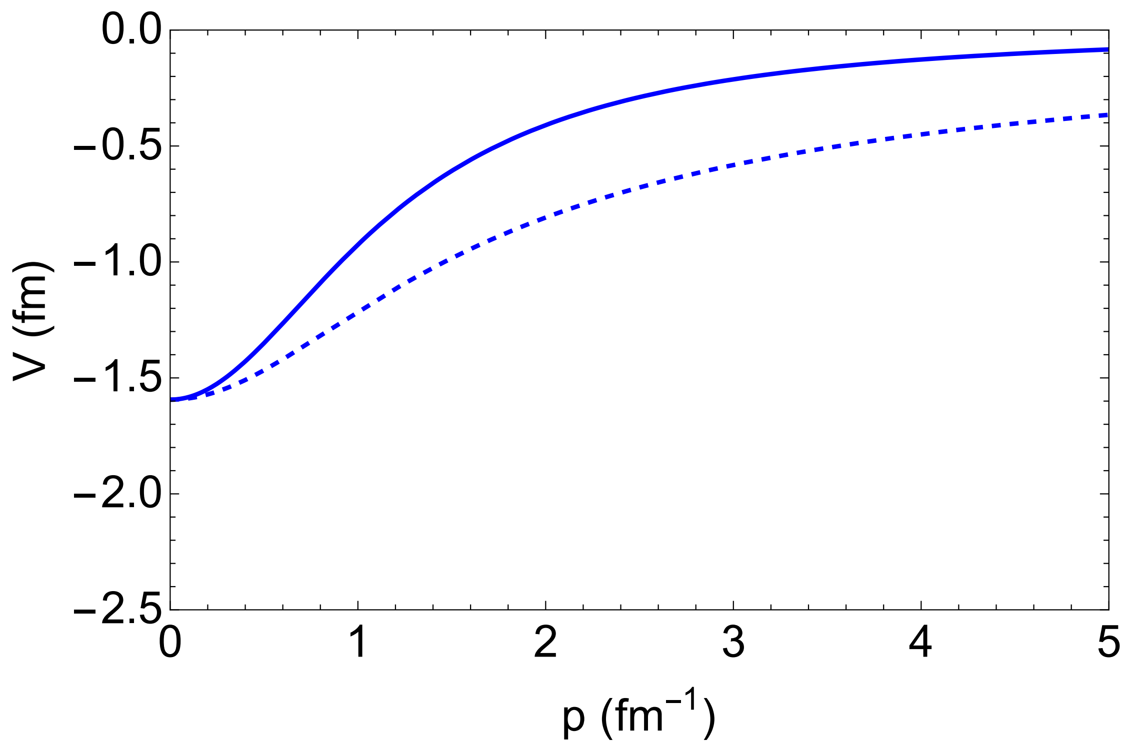
$$g(p) = \left(1 + \frac{p^2}{\alpha^2}\right)^{-1}$$



Separable two-nucleon potentials

$$V_2(p', p) = \frac{4\pi}{m} \lambda \, g(p') \, g(p)$$

$$g(p) = \left(1 + \frac{p^2}{\alpha^2}\right)^{-1/2}$$

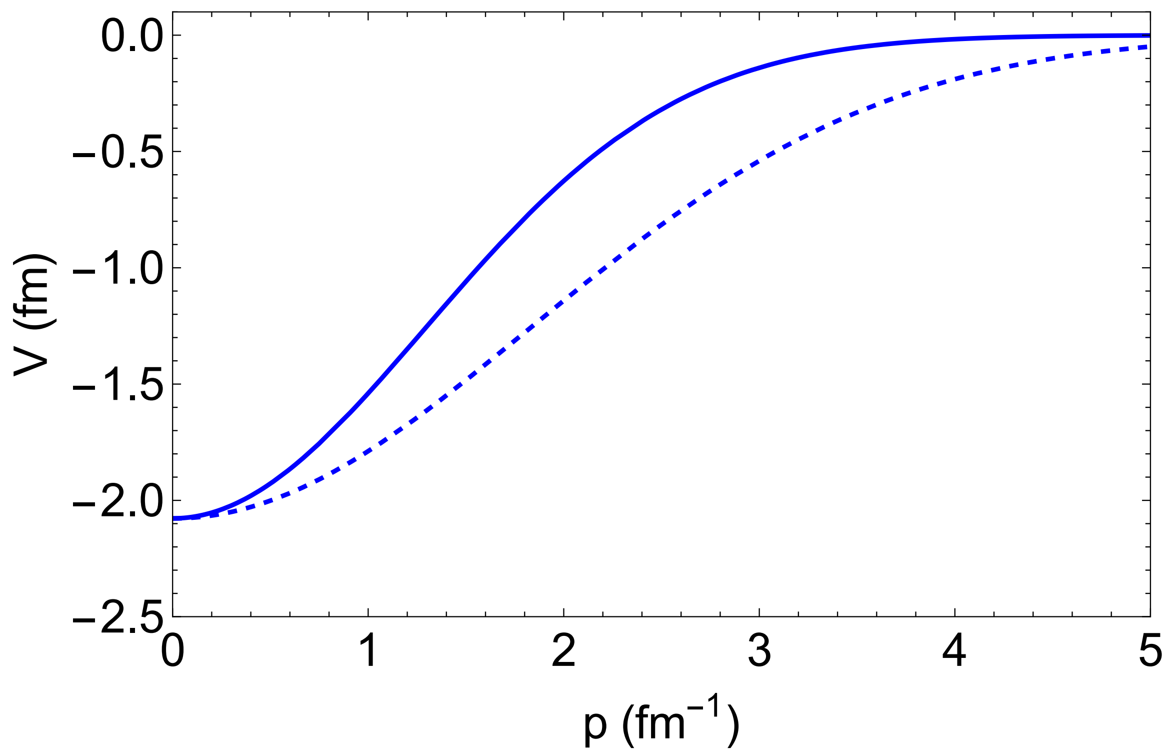


$\alpha \, , \lambda$

$a \;=\; 2\, r$ (LQCD)

$a_{s,t} \, , \, r_{s,t}$ (empirical)

$$g(p) = e^{-p^2/\alpha^2}$$



Y. Yamaguchi, Phys. Rev. 95 (1954) 1628

2N T-matrix

Analytical T-matrix:

$$T_2(p',p;k) = \frac{4\pi}{m}\frac{g(p')g(p)}{\Lambda^{-1}(-ik)-\lambda^{-1}} = \frac{4\pi}{m}\frac{g(p')g(p)}{g^2(k)}\left[-ik-\frac{1}{a_2}+R(k)\,k^2\right]^{-1}$$

with

$$\frac{1}{a_2} = \frac{1}{\lambda}+\frac{2}{\pi}\int_0^\infty dl\, g^2(l) \qquad R(k) = \frac{1}{a_2k^2}\left(g^{-2}(k)-1\right)+\frac{i}{k}-\frac{2}{\pi}g^{-2}(k)\int_0^\infty dl\,\frac{g^2(l)}{l^2-k^2-i\varepsilon}$$

$$\Lambda^{-1}(-ik) = -\frac{2}{\pi}\int_0^\infty dl\, g^2(l) - ik + R(k)k^2$$

Pole:

$$k = i\,\kappa_2 \qquad \lambda = \Lambda(\kappa_2)$$

Double pole

$$g(p) = \left(1 + \frac{p^2}{\alpha^2}\right)^{-1/2} \qquad \frac{1}{a_2} = \frac{1}{\lambda} + \alpha \qquad 2R(k) = r_2 = -\frac{2}{\lambda \alpha^2}$$

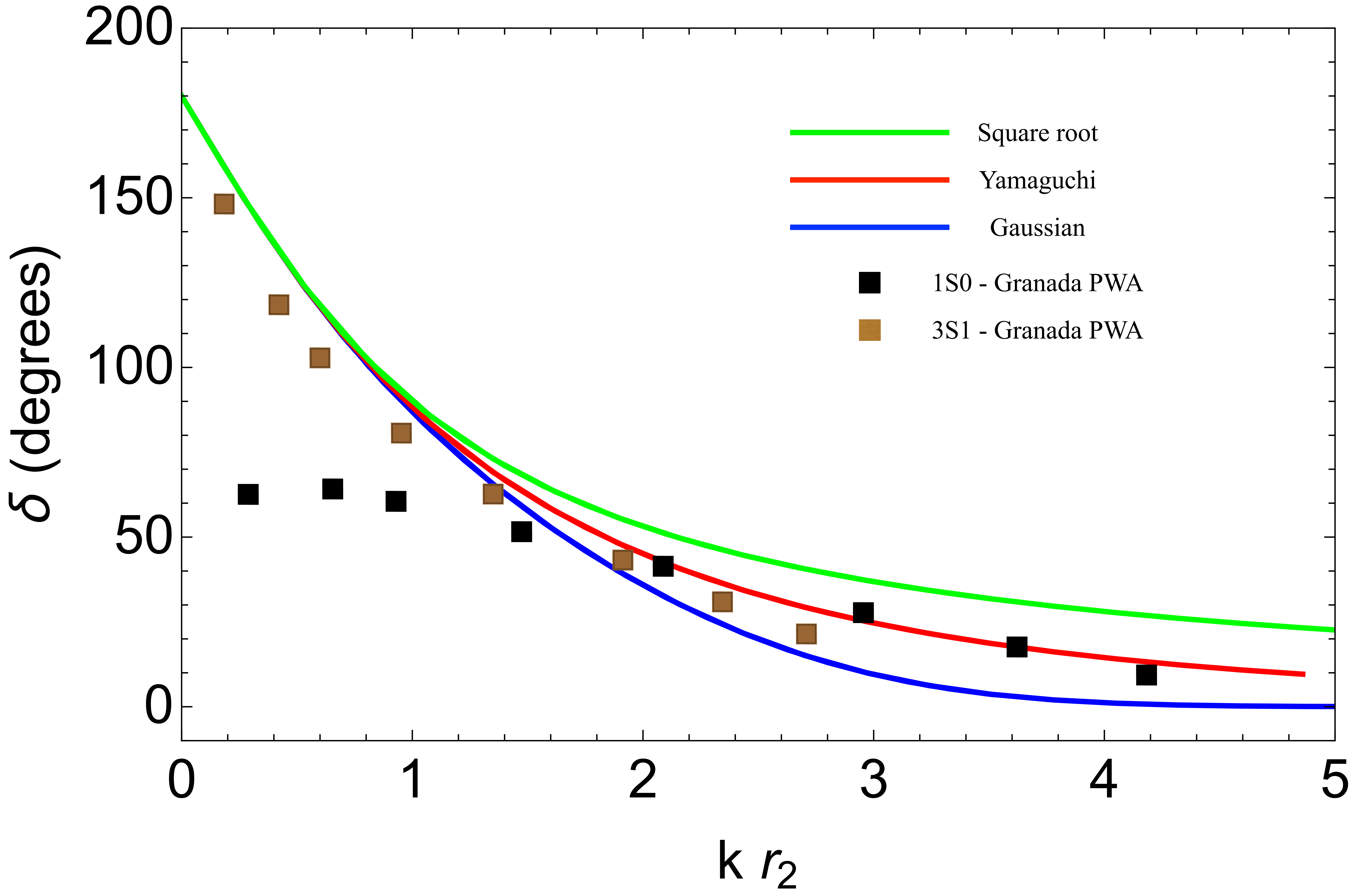
For $\lambda < -\frac{1}{\alpha}$ the separable potential with the above g(p) generates two poles:

$$\kappa_2 = \alpha$$ (independent of λ)

$$\frac{1}{\lambda} = -\frac{\alpha^2}{\kappa_2 + \alpha}$$

2N phase-shifts

$a \sim 2\,r$



Spinless three-body system

A. G. Sitenko and V. F. Kharchenko, Nucl. Phys. 49 (1963) 15

V. S. Timóteo, Ann. Phys. 432 (2021) 168573

Jacobi momenta

$$\vec{k}_{ij} = \frac{1}{2}\left(\vec{p}_i - \vec{p}_j\right), \quad \vec{k}_i = \frac{1}{3}\left(2\vec{p}_i - \vec{p}_j - \vec{p}_k\right)$$

Wave function

$$\psi(p,q) = -\frac{\lambda g(p)}{\kappa_3^2 + p^2 + 3q^2/4}\, a(q)$$

$$\det\left[\delta_{ij} - \bar{\mathscr{K}}(q_i, q_j'; \kappa_3)\right] = 0$$

Profile function

$$a(q) = \frac{2}{\pi}\int_0^\infty dq'\, q'^2\, \mathscr{K}(q,q';\kappa_3)\, a(q')$$

Kernel

$$\mathscr{K}(q,q';\kappa_3) = \left[\Lambda^{-1}\left(\sqrt{\kappa_3^2 + \frac{3q^2}{4}}\right) - \lambda^{-1}\right]^{-1} \int_{-1}^{1} dy\, \frac{g(\pi_2)\, g(\pi_1)}{\kappa_3^2 + q^2 + q'^2 + qq'y}$$

$$\pi_1 = \sqrt{q^2/4 + q'^2 + qq'y}, \quad \pi_2 = \sqrt{q^2 + q'^2/4 + qq'y}$$

Three-nucleon system

Wave functions

$$\begin{pmatrix} \psi_1 \\ \psi_0 \end{pmatrix} = -\frac{1}{\kappa_3^2 + p^2 + 3q^2/4} \begin{pmatrix} \lambda_1 g_1(p) a(q) \\ \lambda_0 g_0(p) b(q) \end{pmatrix}$$

Profile functions

$$\begin{pmatrix} a(q) \\ b(q) \end{pmatrix} = \frac{1}{2\pi} \int_0^\infty dq' q'^2 \begin{pmatrix} \mathscr{K}_{11}(q,q';\kappa_3) & 3\mathscr{K}_{10}(q,q';\kappa_3) \\ 3\mathscr{K}_{01}(q,q';\kappa_3) & \mathscr{K}_{00}(q,q';\kappa_3) \end{pmatrix} \begin{pmatrix} a(q') \\ b(q') \end{pmatrix}$$

Kernel

$$\mathscr{K}_{ss'}(q,q';\kappa_3) = \left[\Lambda_s^{-1}\left(\sqrt{\kappa_3^2 + 3q^2/4}\right) - \lambda_s^{-1}\right]^{-1} \int_{-1}^{1} dy \, \frac{g_s(\pi_2) g_{s'}(\pi_1)}{\kappa_3^2 + p^2 + q'^2 + qq'y}$$

$$\det \begin{pmatrix} \mathbf{1} - \bar{\mathscr{K}}_{11}(q,q';\kappa_3) & -3\bar{\mathscr{K}}_{10}(q,q';\kappa_3) \\ -3\bar{\mathscr{K}}_{01}(q,q';\kappa_3) & \mathbf{1} - \bar{\mathscr{K}}_{00}(q,q';\kappa_3) \end{pmatrix} = 0$$

A. G. Sitenko and V. F. Kharchenko, Nucl. Phys. 49 (1963) 15

V. S. Timóteo, Ann. Phys. 432 (2021) 168573

Two-Nucleon Binding Energies

Lattice QCD ER parameters, unphysical pion mass

	Square-Root	Yamaguchi	Gaussian	LQCD
B_{21}/MeV	25.3	19.5	18.4	19.5 (3.6) (3.1) (0.2)
B_{20}/MeV	12.7	11.1	10.7	15.9 (2.7) (2.7) (0.2)

Empirical ER parameters, unphysical pion mass

	Square-Root	Yamaguchi	Gaussian	experiment
B_{21}/MeV	13.4949	9.26145	8.69714	2.224575(9)
B_{20}/MeV	5.66342	3.88675	3.64993	—

Three-Nucleon Binding Energies

Lattice QCD ER parameters, unphysical pion mass

	Square-Root	Yamaguchi	Gaussian	LQCD
B_3/MeV	56.5	56.6	56.5	53.9 (7.1) (8.0) (0.6)

Empirical ER parameters, physical pion mass

	Square-Root	Yamaguchi	Gaussian	experiment
B_3/MeV	7.496939	8.945608	8.397675	8.481798(2)

Final Remarks

* The results with unphysical masses are close to the lattice QCD calculations
* The results with physical masses are close to the experimental values
* Our calculations with effective potentials support the lattice QCD results