

# Tetraquarks in the unified covariant approach of QFT.

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- Tetraquarks need relativistic description in the QFT
- In QFT the tetraquark eqs are for the four-body ( $2q2\bar{q}$ ) system where transitions to two-body states,  $2q2\bar{q} \rightarrow q\bar{q}$ , are allowed
- Framework: 4D covariant approach **based on the analysis of the Feynman diagrams**

- History of the covariant equations for tetraquarks:
  - (i) 1992, Kvi., Khvedelidze, *Theor. Math. Phys.* **90**, 62, 4-body covariant equations without annihilation (like a 4 quark system, no transition to  $2q$ )
  - (ii) 2012, Heupel, Eichmann, Fischer, *Phys.Lett.* **B718**, 545, numerical solution of the 4-body covariant equations
  - (iii) 2014, Kvi., Blankleider, *Phys.Rev.* **D90**, 045042, inclusion of the effect of quark-antiquark annihilation, coupling to the 2-body, quark-antiquark, channel
  - (iv) 2020, Santowsky, Eichmann, Fischer, Wallbott, Williams, *Phys.Rev.* **D102**, 5, another, "phenomenological", attempt to include the coupling to the 2-body,  $q\bar{q}$ , channel
  - (v) 2022, Kvi., Blankleider, *Phys.Rev.* **D106**, 5, the general exact equations for tetraquarks

# 1992, Kvi., Khvedelidze, Theor. Math. Phys. **90**, 62, 4-body covariant equations

- 4-body covariant equations like a 4 quark system,  
$$\Psi = G_0^{(4)} K^{(4)} \Psi$$

- Challenges in derivation:

- (i) The problem of overcounting even for no coupling to two-body channel -sum of pair interaction kernels overcounts therefore **subtractions** are needed

$$K^{(4)} = \sum K_{ij}^{(2)} - K_{12}^{(2)} K_{34}^{(2)} - K_{13}^{(2)} K_{24}^{(2)} - K_{14}^{(2)} K_{23}^{(2)},$$

\*Such subtractions are not canceled within  $K^{(4)}$ , cancelation happens between different iterations of  $K^{(4)}$ .

$$K^{(4)} + K^{(4)} K^{(4)} = \dots - K_{12}^{(2)} K_{34}^{(2)} + K_{12}^{(2)} K_{34}^{(2)} + K_{34}^{(2)} K_{12}^{(2)} = \dots + K_{12}^{(2)} K_{34}^{(2)}$$

\*Unusual structure of  $K^{(4)}$  for using Faddeev-Yakubovsky rearrangement

- (ii) In the TOPT approach 4q kernels even cannot be expressed in terms of 2q kernels  $K_{ij}^{(2)}$

- To rearrange the equation  $\Psi = G_0^{(4)} K^{(4)} \Psi$  break the kernel into three parts,  $K^{(4)} = K_1 + K_2 + K_3$ , where

$$K_3 = K_{12}^{(2)} + K_{34}^{(2)} - K_{12}^{(2)} K_{34}^{(2)},$$

$$K_1 = K_{13}^{(2)} + K_{24}^{(2)} - K_{13}^{(2)} K_{24}^{(2)}, \quad (K_2 \text{ similarly}),$$

and define  $T_i$ ,  $T_i = K_i + K_i G_0^{(4)} T_i$  with the solutions,

$$T_3 = T_{12}^{(2)} + T_{34}^{(2)} + T_{12}^{(2)} T_{34}^{(2)},$$

$$T_1 = T_{13}^{(2)} + T_{24}^{(2)} + T_{13}^{(2)} T_{24}^{(2)}, \quad (T_2 \text{ similarly}),$$

where  $T_{ij}^{(2)}$  are two-body off-shell scattering amplitudes,

$$T_{ij}^{(2)} = K_{ij}^{(2)} + K_{ij}^{(2)} G_0^{(2)} T_{ij}^{(2)}.$$

Then the rearranged equation is

$$\Psi = \sum_i \Psi_i. \text{ where } \Psi_i = G_0^{(4)} T_i \sum_{j \neq i} \Psi_j$$

- Three goals achieved. **No analog in QM for two of them:**
  - (i) kernels  $T_i$  are expressed in terms of two-body t-matrices  $T_{ij}^{(2)}$   
(usual result of Faddeev rearrangement in Quantum Mechanics)
  - (ii) got rid of **subtraction** terms present before rearrangement  
(**subtractions do not exist in the 4-body potential of QM**)
  - (iii) In the 3D quasipotential and TOPT approaches the rearrangement is **necessary** at least for the kernels to be expressed in terms of functions defined in the two-body subsystems  $T_1 + T_2 + \int dz T_1(E - z) T_2(z)$   
(such a problem does not exist in Quantum Mechanics, equations' kernels are clear **before rearrangement as well**, the sum of pair interaction potentials)

# 1992, Kvi., Khvedelidze, Theor. Math. Phys. **90**, 62, 4-body covariant equations

- That time (in 1992) we even **did not dream** that this sort of equations could be solved numerically, we have been attracted only by the mathematical challenges they pose. But, alas, **in 20 years since then they are being solved!** (see next slides)
- (i) This fact,  
(ii) the advances in the methods of numerical solutions,  
(iii) the well elaborated input functions offered on the market  
(iv) calculations carried out by now  
**revived our interest**

- The first results for tetraquarks in a covariant continuum approach based on our equation of 1992 for four quarks, but applied to a system of 2 quarks+2 antiquarks,  $2q2\bar{q}$ ,

$$\Psi_i = G_0^{(4)} T_i \sum_{j \neq i} \Psi_j,$$

where

$$T_3 = T_{12}^{(2)} + T_{34}^{(2)} + T_{12}^{(2)} T_{34}^{(2)},$$

$$T_1 = T_{13}^{(2)} + T_{24}^{(2)} + T_{13}^{(2)} T_{24}^{(2)}, \quad (T_2 \text{ similarly}),$$

- This approach is "similar in spirit to the quark-diquark model of the nucleon" elaborated in G. Eichmann, I R. Alkofer, et al., Phys. Rev. C 79 (2009) 012202,



- Using 4q eqs for the system of  $2q+2\bar{q}$  implies that in the  $2q2\bar{q}$  Green function  $G^{(4)}$   $q\bar{q}$  annihilation diagrams are neglected, i.e. its  $q\bar{q}$ -irreducible part  $G_{ir}^{(4)}$  is considered, see below
- Further approximations are made to reduce  $2q2\bar{q}$  space to  $MM + D\bar{D}$ , i.e. to come to a picture of tetraquark as composed of two mesons,  $MM$ , or of diquark-antidiquark,  $D\bar{D}$ :  
(i)  $T_3 \sim T_{12}^{(2)} T_{34}^{(2)}$ ,  $T_1 \sim T_{13}^{(2)} T_{24}^{(2)}$ , ( $T_2$  similarly),

(ii) reduction to a two-body problem proceeds by assuming that the two-body T-matrices  $T_{ij}^{(2)}$  are dominated by meson and diquark pole contributions,

$$T_{ij}^{(2)} = i\Gamma_{ij}D_{ij}\bar{\Gamma}_{ij},$$

where  $D_{ij}(P_{ij}) = 1/(P_{ij}^2 - m_{ij}^2)$  is the propagator for the bound particle (diquark, antidiquark, or meson) in the two-body channel  $ij$

- **Physical results in Phys.Lett. B718, 545 (2012):**

For the lightest scalar tetraquark a mass of the order of 400 MeV and a wave function dominated by the pion-pion constituents is found.

Both results are in agreement with a meson molecule picture for the  $f_0(600)$ .

The results suggest the presence of a potentially narrow all-charm tetraquark in the mass region 5 – 6 GeV

- So far mathematically pure four-quark system was considered.
- But in Quantum Field Theory (QFT) the number of particles is not conserved.
- The  $2q2\bar{q}$  Green function  $G^{(4)}$  in terms of its  $q\bar{q}$ -irreducible part  $G_{ir}^{(4)}$  and its  $q\bar{q}$ -reducible part  $M_{ir}^{(4-2)} G^{(2)} M_{ir}^{(2-4)}$ .  
The exact relation:

$$G^{(4)} = G_{ir}^{(4)} + M_{ir}^{(4-2)} G^{(2)} M_{ir}^{(2-4)}$$

# 2014, Kvi., Blankleider, Phys.Rev. **D90**, 045042, simplest coupling to 2-body, quark-antiquark, channel

- First attempt to account for coupling to two-body  $q\bar{q}$  state  
2014, Kvi., Blankleider, Phys.Rev. **D90**, 045042

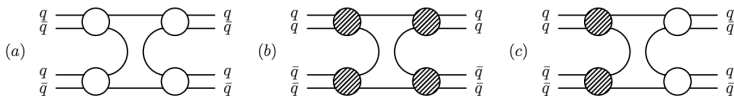


FIG. 1. Examples of terms involving  $q\bar{q}$  annihilation contributing to the tetraquark amplitude within a model involving only meson, diquark and anti-diquark constituents: (a)  $MM$  scattering, (b)  $D\bar{D}$  scattering, (c)  $D\bar{D} \leftarrow MM$  transition. Such terms are not taken into account when the covariant four-body equations of KK [3] are used to describe the  $2q2\bar{q}$  system.



- this **disconnected** diagrams are used as (**unusual!**) parts of the input  $2q$  T-matrices, but **in a sophisticated way** to avoid non-physical/inexistent contributions to the four-body eqs

- Equations with minimal coupling to  $q\bar{q}$  channel were obtained

$$\begin{aligned}
 \text{---} \Phi_M \text{---} &= \frac{1+\mathcal{P}}{2} \left( \text{---} \text{---} \text{---} \Phi_M \text{---} - 2 \text{---} \text{---} \text{---} \Phi_D \text{---} + \text{---} \text{---} \text{---} \Gamma^* \text{---} \right) \\
 \text{=} \Phi_D &= - \text{---} \text{---} \text{---} \Phi_M \text{---} + \text{---} \text{---} \text{---} \Gamma^* \text{---} \\
 \text{---} \Gamma^* &= \frac{1}{2} \left( \text{---} \text{---} \text{---} \Phi_M \text{---} + \text{---} \text{---} \text{---} \Phi_D \text{---} \right)
 \end{aligned}$$

Timidly criticised for "disconnected part" of the input  $q\bar{q}$  T-matrix with  $\delta$ -function.

Nothing to criticise since **the kernel of the eqs does not involve  $\delta$ -function**, although used in the derivation.

2020, Santowsky, Eichmann, Fischer, Wallbott, Williams,  
 Phys.Rev.**D102**, 5, "phenomenological" attempt to include  
 coupling to  $q\bar{q}$  channel

- Phenomenologically motivated equations

$$\begin{aligned} \Phi_1 &= \text{crossed loop with } \Phi_2 + \text{crossed loop with } \Phi_3 + \text{quark loop with } \Gamma^* \\ \Phi_3 &= \text{crossed loop with } \Phi_1 + \text{crossed loop with } \Phi_2 + \text{quark loop with } \Gamma^* \\ \Gamma^* &= K^{(2)} \Gamma^* + K^{(2)} \Phi_1 + K^{(2)} \Phi_3 \end{aligned}$$

where  $K^{(2)}$  is one-gluon-exchange  $q\bar{q}$  kernel.

Making an educated ("phenomenologically motivated") guess one must be prepared to face the possibility that these guessed equations may be inconsistent with the rules of QFT?

# 2022, Kvi., Blankleider, Phys.Rev. **D106**, 5, the general exact equations for tetraquarks

- We start with the two-body eq **although** for the tetraquark,

$$\Gamma^* = K^{(2)} G_0^{(2)} \Gamma^*$$

where the kernel  $K^{(2)} = \Delta + \bar{N} G_{ir}^{(4)} N$  is complicated by the infinite series for  $G_{ir}^{(4)}$ ,

$$G_{ir}^{(4)} = G_0^{(4)} + G_0^{(4)} K^{(4)} G_0^{(4)} + G_0^{(4)} K^{(4)} G_0^{(4)} K^{(4)} G_0^{(4)} + \dots \quad (\text{B})$$

- This complication is handled by using the equation for  $G_{ir}^{(4)}$ ,  
$$G_{ir}^{(4)} = G_0^{(4)} + G_0^{(4)} K^{(4)} G_{ir}^{(4)}$$
to derive coupled channel set of 2q-4q equations with a simpler kernel  $K^{(4)}$ .

- Indeed the 2q eq  $\Gamma^* = [\Delta + \bar{N}G_{ir}^{(4)}N] G_0^{(2)}\Gamma^*$  can be written as  $\Gamma^* = \Delta G_0^{(2)}\Gamma^* + \bar{N}G_0^{(4)}\Phi$ , where  $\Phi = G_0^{(4)-1}G_{ir}^{(4)}NG_0^{(2)}\Gamma^*$ . This relation defines  $\Phi$  as the 4q component of the tetraquark. Using the eq for  $G_{ir}^{(4)}$  we get the eq  $\Phi = K^{(4)}G_0^{(4)}\Phi + NG_0^{(2)}\Gamma^*$ , which in combination with  $\Gamma^* = \Delta G_0^{(2)}\Gamma^* + \bar{N}G_0^{(4)}\Phi$ , represents the set of eqs with "simple" kernels  $K^{(4)}, N, \Delta, \bar{N}$ .



- In the above mentioned pole approximation for input 2q t-matrices these 4q eqs become mathematically 2-body eqs in  $q\bar{q}$ - $MM$  space,

$$\Phi = VG_0^M\Phi + NG_0^{(2)}\Gamma^*.$$

$$\Gamma^* = \Delta G_0^{(2)}\Gamma^* + \bar{N}G_0^M\Phi$$

where  $G_0^M$  are the two-body 2M and  $D\bar{D}$  propagators;

$V$  is a  $2 \times 2$  matrix whose elements are the kernels for the transitions between 2M and  $D\bar{D}$  states.

# 2022, Kvi., Blankleider, Phys.Rev. **D106**, 5, the general exact equations for tetraquarks

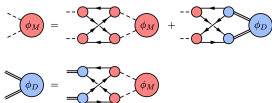
- The final equations in diagrams:

$$\begin{aligned}
 \text{---} \Phi_M &= \frac{1+\mathcal{P}}{2} \text{---} \Phi_M - 2 \text{---} \Phi_D + \text{---} \Gamma^* \\
 \text{=} \Phi_D &= - \text{---} \Phi_M + \text{---} \Gamma^* \\
 \text{=} \Gamma^* &= \text{---} \Delta \text{---} \Gamma^* + \frac{1}{2} \text{---} \Phi_M + \text{---} \Phi_D
 \end{aligned}$$

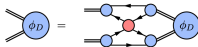
These are **general exact** four-body eqs (even if  $N, K^{(4)}$  are **approximate** in the case where  $4q-2q$  transitions are allowed. Here is important that  $\Delta$  (and  $N$ ) are clearly specified:  $\Delta$  is the sum of all  $q\bar{q}$  irreducible diagrams except those included in  $\bar{N}G_{ir}^{(4)}N$ .

- **Currently there are two different seemingly unrelated eqs:**

(i) The Giessen Group tetraquark eq in the double scattering approximation,  $T_3 \sim T_{12}^{(2)} T_{34}^{(2)}$ , W. Heupel, G. Eichmann, C. S. Fischer, Phys.Lett.B 718, 545 (2012)



(ii) The Moscow Group tetraquark eq, D. Ebert, R. N. Faustov, and V. O. Galkin, E. M. Savchenko, Phys. Lett. B 634, 214 (2006), Phys. Rev. D 102, 114030 (2020),



- Universe 7 (2021) 4, 94:

"there are **significant disagreements** between different theoretical approaches. Indeed, Refs. [...] predict heavy tetraquark masses **below or slightly above the thresholds** of the decays to two quarkonia and, thus, stable or significantly suppressed against fall-apart decays with a very narrow decay width. ... other approaches predict such tetraquark masses **significantly above** these thresholds and, thus, they can be observed only as broad resonances."

Therefore it is important to compare theoretical basis of these approaches.

- A universal set of equations is derived which produces all of these approaches in different approximations. Exposing three body  $Mq\bar{q}, \bar{D}qq, D\bar{q}\bar{q}$  states is inevitable to compare them on the theoretical foundation level.

- Giessen Group eq, in the approximation of only double scattering,  $T_3 \sim T_{12}^{(2)} T_{34}^{(2)}$ ,  
 $\phi = V_G \phi$
- If we take into account the single scatterings  $T_{12}^{(2)} + T_{34}^{(2)}$  in  $T_3 = T_{12}^{(2)} T_{34}^{(2)} + T_{12}^{(2)} + T_{34}^{(2)}$ , the Giessen Group eq modifies to the **unified** eq,  
 $\phi = (V_G + V_M) \phi$ ,  
where  $V_M$  is the Moscow Group kernel.

- The structure of tetraquarks can be studied by calculating tetraquark **currents**?
- The complexity of the tetraquark eqs would make the construction of the **gauge invariant currents** impossible if not for the "**gauging equations method**" [KB, PRC60, 044004 (1999)]. It makes the construction **extremely** simple just as, for example, in the case of the quark-diquark model for baryons.

THANK YOU!