Exotic pairing in few-body ultracold systems

Daniel Pęcak

3rd August 2023, Mainz

Pairing

BCS

Bardeen, Cooper, and Schrieffer

$$|\Psi_{\mathrm{BCS}}\rangle = \prod_k (u_k + v_k \hat{c}^\dagger_{\uparrow k} \hat{c}^\dagger_{\downarrow -k}) |F_0\rangle$$

Order parameter $\Delta \propto \langle \hat{c}_\uparrow \hat{c}_\downarrow \rangle$

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Fulde and Ferrell

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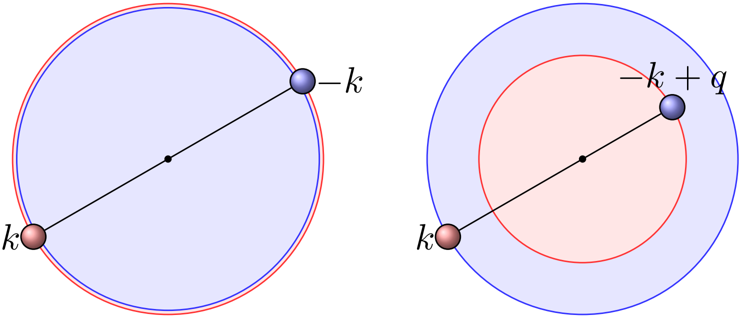
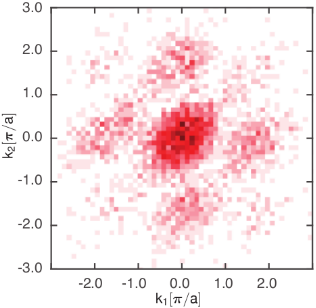


A. Bergschneider et al. Nature Physics **15**, 640–644 (2019)



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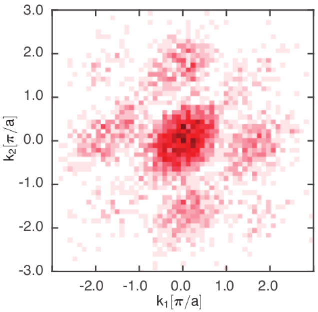
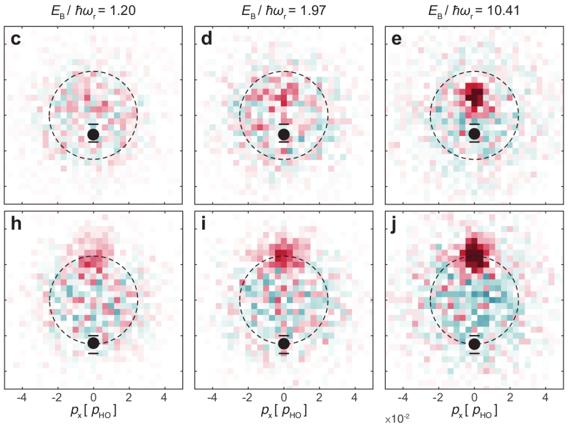
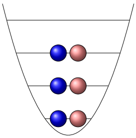
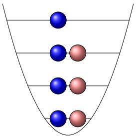
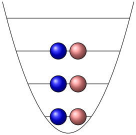


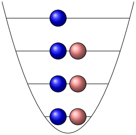


Holten et al. Nature 606.7913 (2022): 287-291



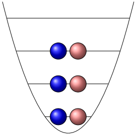


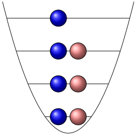




Experiment
$\omega \approx 2\pi \cdot 1.488\,\mathrm{kHz}$
$B = 1202\,\mathrm{G}$
$g = -1$

$$H = \sum_{i=1}^{N_\downarrow} \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_i^2} + \frac{m\omega^2}{2} x_i^2 \right] + \sum_{i=1}^{N_\uparrow} \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial y_i^2} + \frac{m\omega^2}{2} y_i^2 \right] + g \sum_{i,j=1}^{N_\downarrow, N_\uparrow} \delta(x_i - y_j)$$





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Input

- numbers of particles $N_\uparrow, N_\downarrow$
- interaction $g < 0$
- single-particle eigenstates $\phi_{n\sigma}(x)$

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- computation of all matrix elements of the many-body Hamiltonian $\hat{H}$
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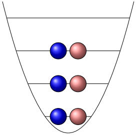

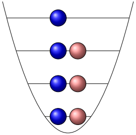

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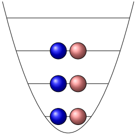

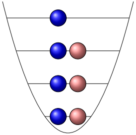

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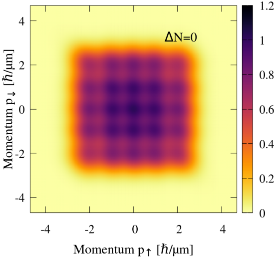
Observables

- single-particle density profile ρ_σ
- higher order correlations $\mathcal{G}_T(p_1, p_2)$

$$\mathcal{G}_T(p_1, p_2) = \langle \hat{n}_\uparrow(p_1)\hat{n}_\downarrow(p_2)\rangle_T - \langle \hat{n}_\uparrow(p_1)\rangle_T \langle \hat{n}_\downarrow(p_2)\rangle_T,$$



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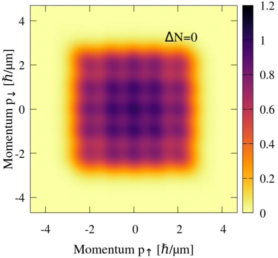

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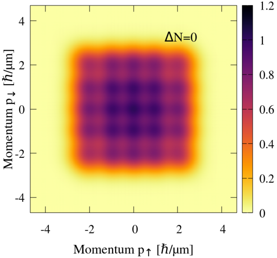

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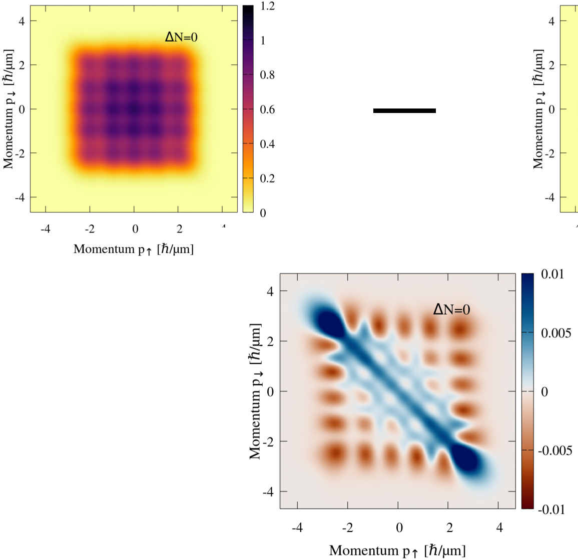
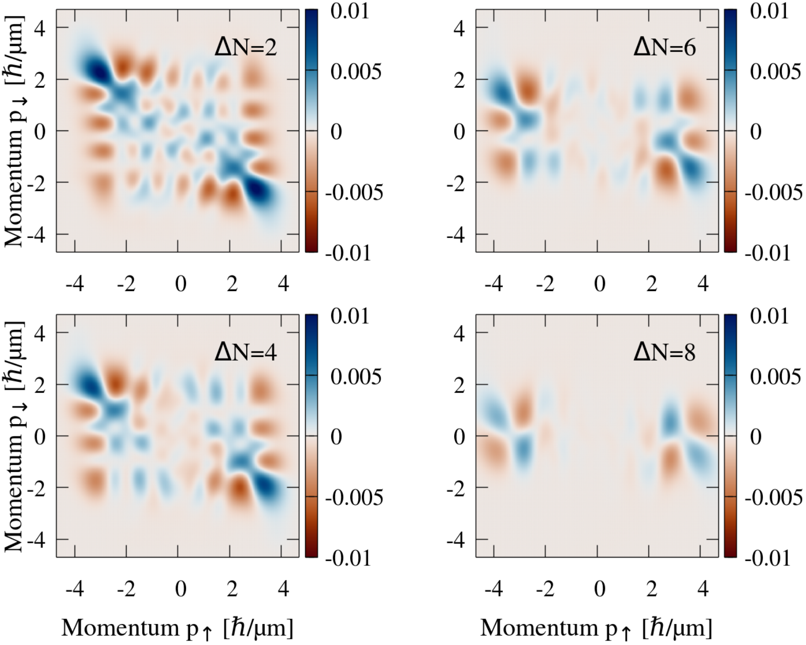
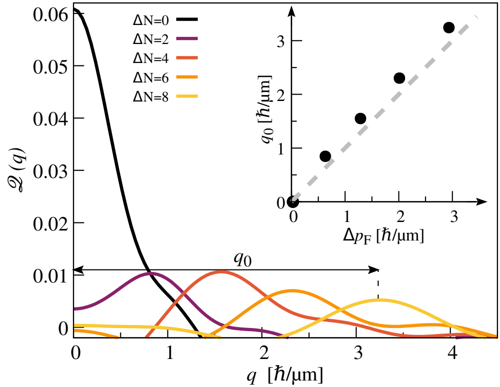



ΔN=2
ΔN=6
ΔN=4
ΔN=8
0.01
0.005
0
-0.005
-0.01
Momentum p↓ [ħ/μm]
Momentum p↑ [ħ/μm]
-4
-2
0
2
4

$$\mathcal{Q}(q) = \int dp_1 dp_2\, \mathcal{G}_T(p_1, p_2)\mathcal{F}(p_1 + p_2 + q)$$



$$\Delta p_\mathrm{F} \approx p_{F\uparrow} - p_{F\downarrow} = \sqrt{2\left(N_\uparrow - \frac{1}{2}\right)} - \sqrt{2\left(N_\downarrow - \frac{1}{2}\right)}$$

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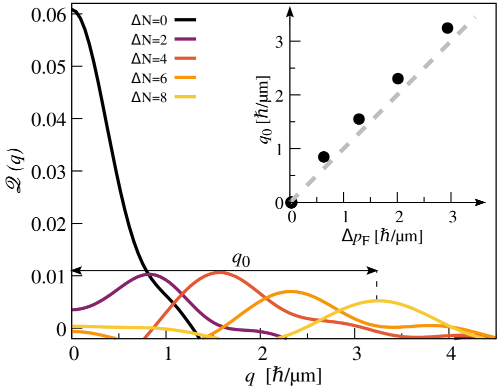

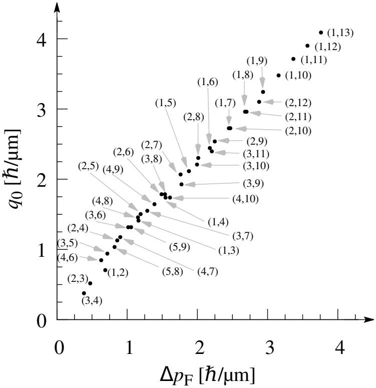

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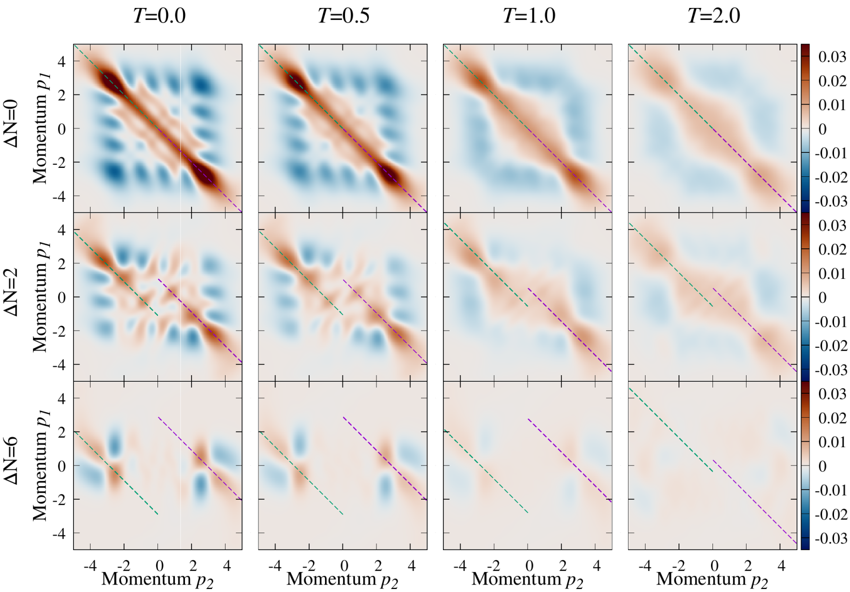
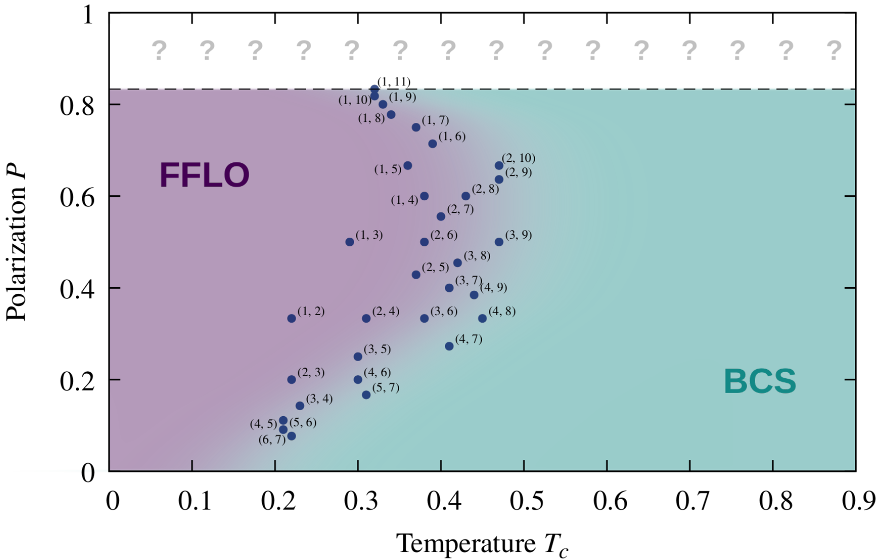


T=0.0
T=0.5
T=1.0
T=2.0
ΔN=0
ΔN=2
ΔN=6
Momentum p1
Momentum p2
0.03
0.02
0.01
0
-0.01
-0.02
-0.03


FFLO
BCS
Polarization P
Temperature T_c
(1, 11)
(1, 10)
(1, 9)
(1, 8)
(1, 7)
(1, 6)
(1, 5)
(1, 4)
(1, 3)
(1, 2)
(2, 10)
(2, 9)
(2, 8)
(2, 7)
(2, 6)
(2, 5)
(2, 4)
(2, 3)
(3, 9)
(3, 8)
(3, 7)
(3, 6)
(3, 5)
(3, 4)
(4, 9)
(4, 8)
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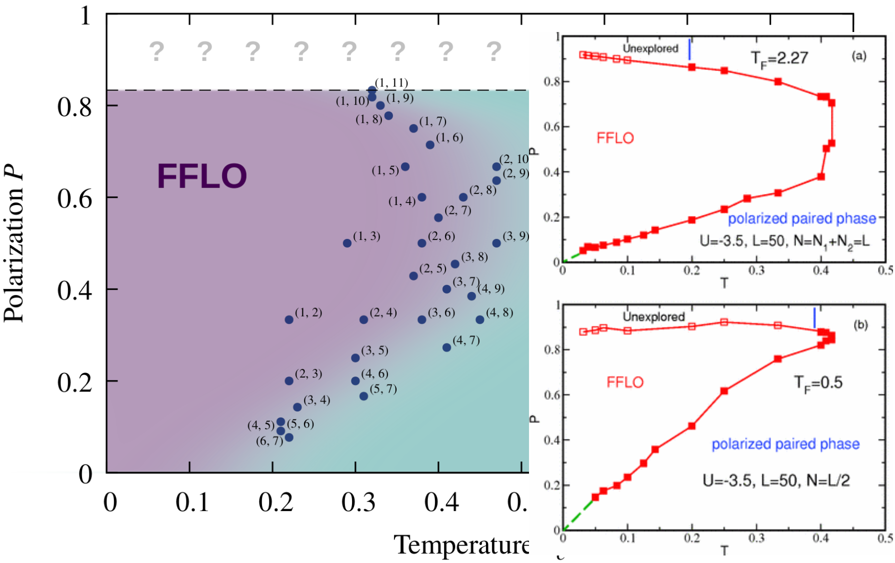
(4, 6)
(4, 5)
(5, 7)
(5, 6)
(6, 7)



Wolak et al. Physical Review A 82, 013614 (2010)

Take-home message

- correlation noise captures **pair's momentum**
- correlation found for **few-body, finite, non-uniform** system
- **linear** realtion beween mismatch and pair's momentum
- within state-of-the-art **experimental** capabilities
- robust for **small temperatures** due to the energy gap
- **crossover** character

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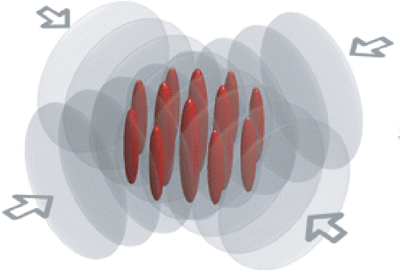
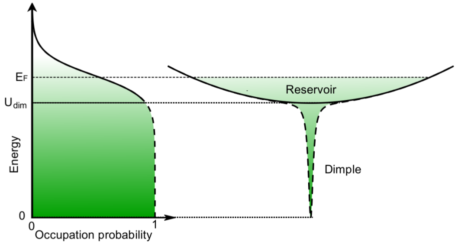
More details:

- DP & T. Sowiński, Phys. Rev. Res. 2, 012077(R) (2020)
  „Signatures of unconventional pairing in spin-imbalanced one-dimensional few-fermion systems"
- DP & T. Sowiński, Scientific Reports 12, 17476 (2022)
  „Unconventional pairing in few-fermion systems at finite temperature"

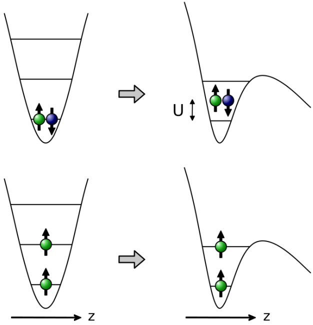
Elmar et al., Science **325**, 1224 (2009)



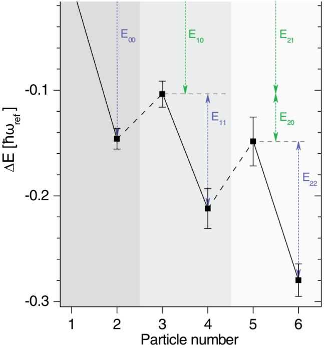
Gerhard Zürn's PhD thesis

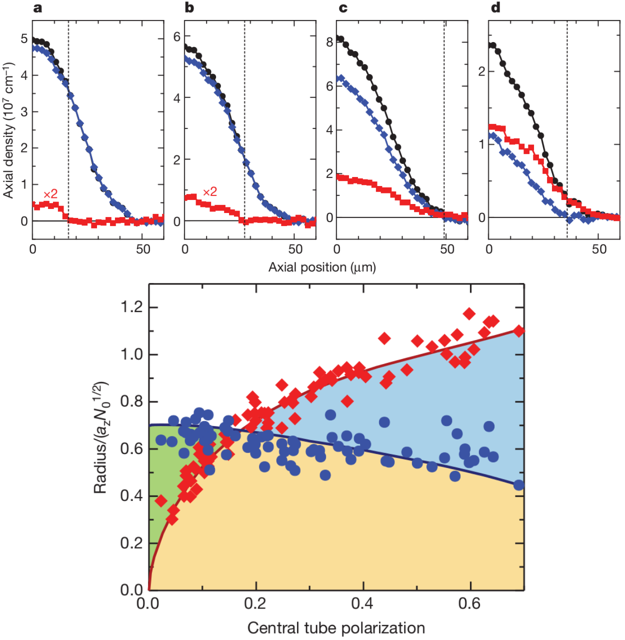


Zürn et al., Phys. Rev. Lett. **108**, 075303 (2012)



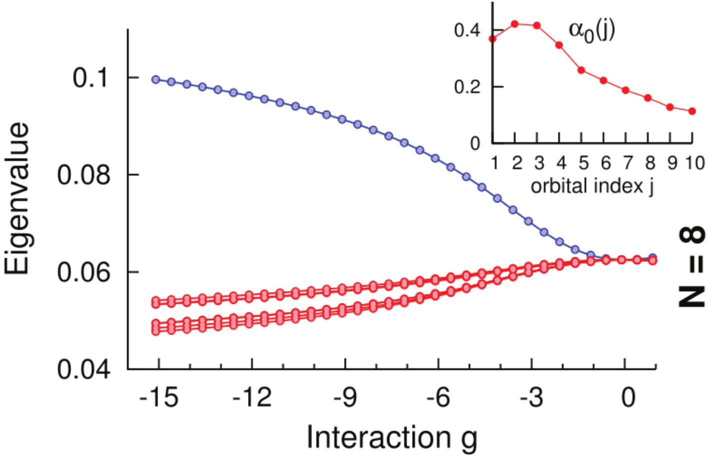
G. Zürn et al. PRL **111**, 175302 (2013)



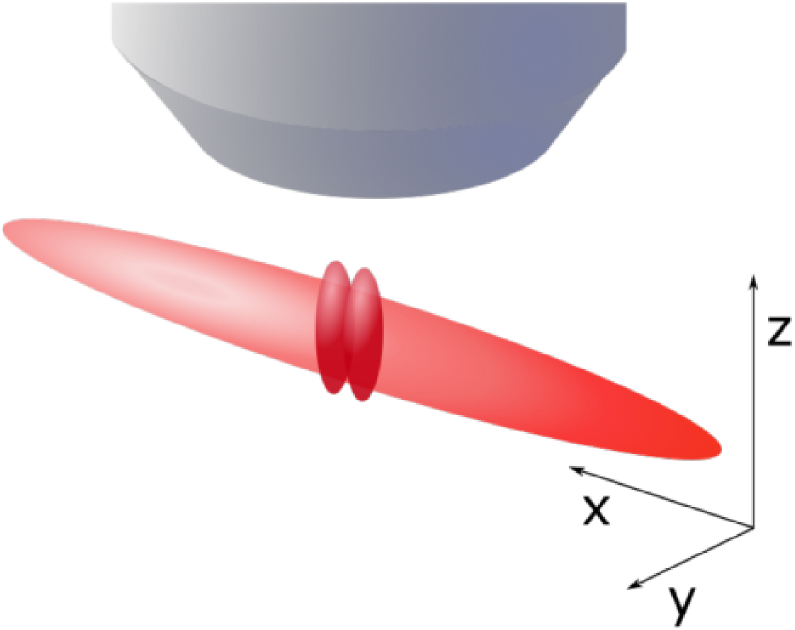


Liao et al. Nature **467**, 567–569 (2010)

$$\rho^{(2)}(x_1, x_2; y_1, y_2) = \langle \psi_i | \hat{\Psi}^\dagger_\uparrow(x_1) \hat{\Psi}^\dagger_\downarrow(y_1) \hat{\Psi}_\downarrow(y_2) \hat{\Psi}_\uparrow(x_2) | \psi_i \rangle$$



T. Sowiński, M. Gajda, and K. Rzążewski Europhys. Lett. **109**, 26005 (2015)



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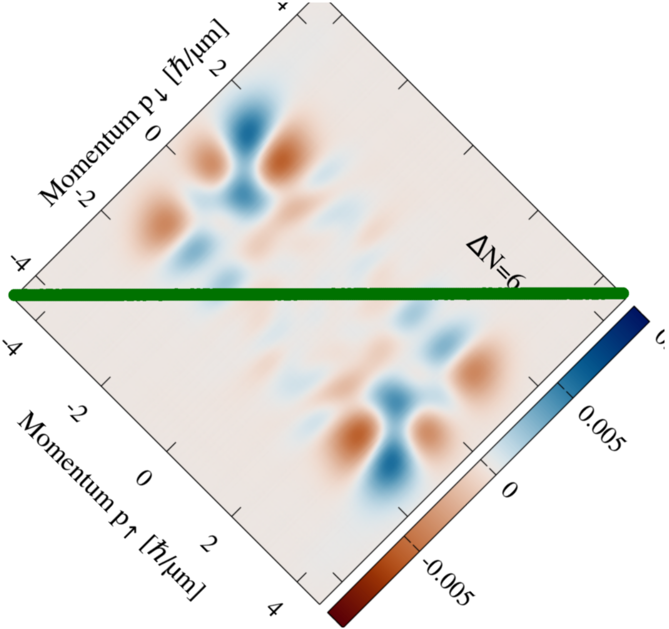

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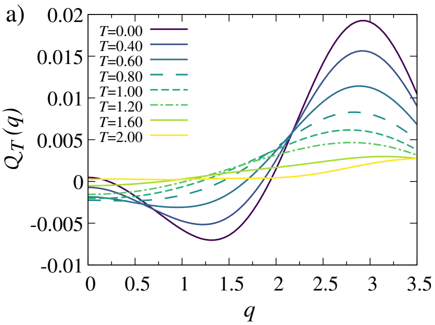

$$\mathcal{Q}_T(q) = \int dp_1 dp_2\, \mathcal{G}_T(p_1, p_2)\mathcal{F}(p_1 + p_2 + q).$$

$$\mathcal{F}(p) = \frac{1}{\sqrt{\pi\kappa}} \exp\left(-\frac{p^2}{2\kappa^2}\right).$$

$$\xi(T) = \mathcal{Q}_T(q_0) - \mathcal{Q}_T(0)$$