Quantum-chaotic behavior of a particle interacting with independent scatterers

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Outline

Effects of additional interactions on chaotic systems

Independent interactions

Increase of the number of principal components

Numerical calculations for a harmonic waveguide with scatterers

Fluctuation decrease — eigenstate thermalization

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EFFECT OF AN ADDITIONAL INTERACTION

Integrable system: $\hat{H}_0$: $[\hat{H}_0, \hat{I}_j] = 0$, $[\hat{I}_j, \hat{I}_{j'}] = 0$

$\hat{H}_1 = \hat{H}_0 + \hat{V}_1$, $[\hat{V}_1, \hat{I}_j] \neq 0$ — non-integrable (chaotic) system

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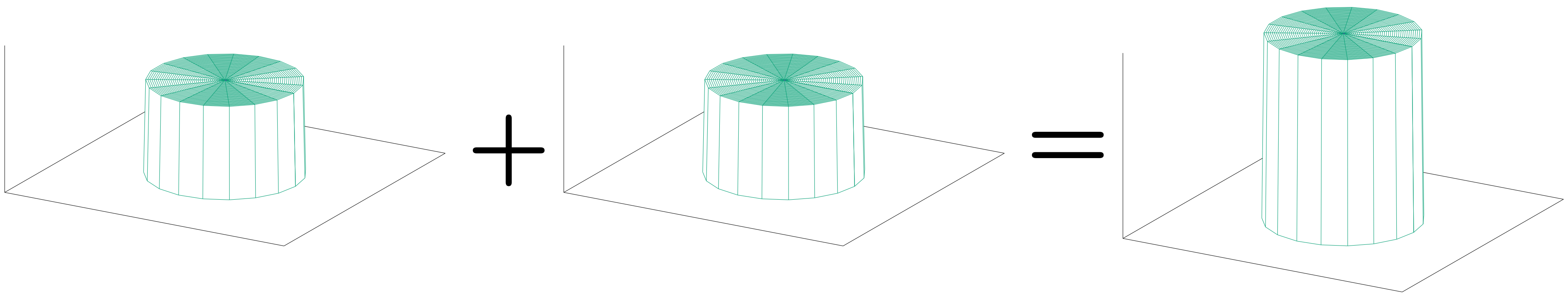

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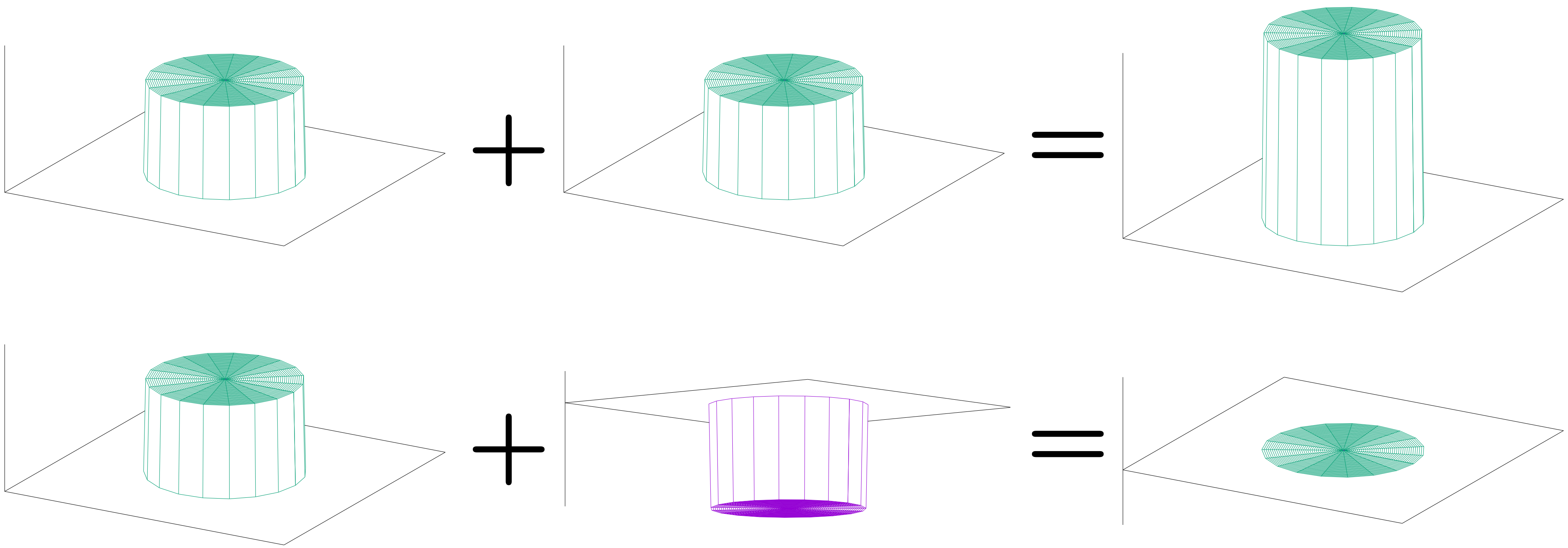

INCREASE OF "CHAOTICITY"

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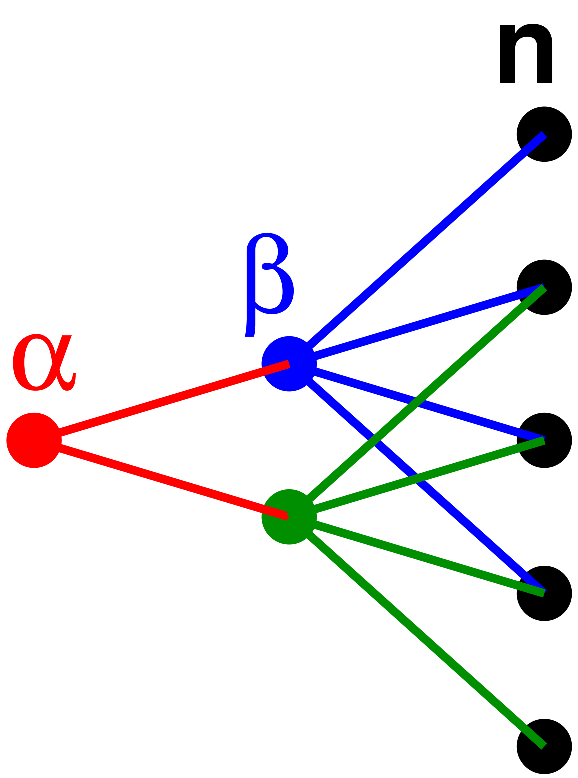

↑

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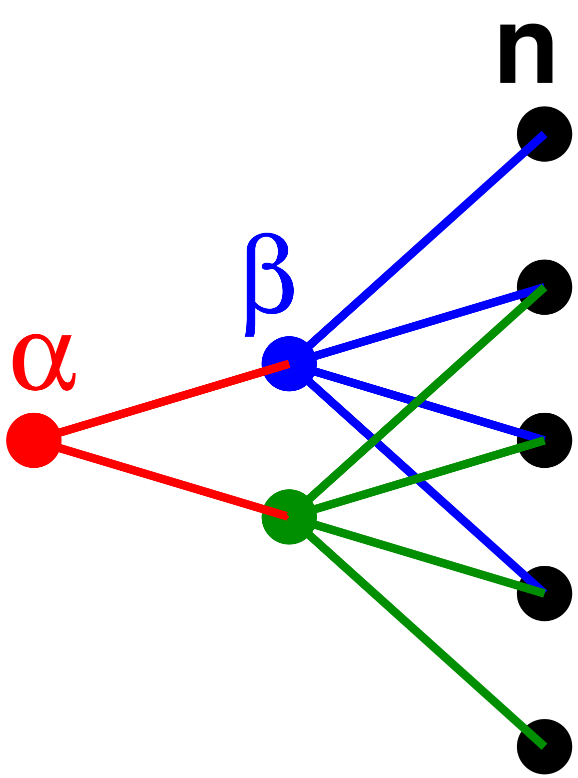

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When the interference terms can be neglected?

INDEPENDENT INTERACTIONS

$$\sum_{\mathbf{n},\mathbf{n}'} \left\langle \mathbf{n} | \hat{V}_1 | \mathbf{n}' \right\rangle \left\langle \mathbf{n}' | \hat{V}_2 | \mathbf{n} \right\rangle \ll \sum_{\mathbf{n},\mathbf{n}'} \left(\left| \left\langle \mathbf{n} | \hat{V}_1 | \mathbf{n}' \right\rangle \right|^2 + \left| \left\langle \mathbf{n} | \hat{V}_2 | \mathbf{n}' \right\rangle \right|^2 \right) \qquad (\hat{H}_0 \left| \mathbf{n} \right\rangle = E_{\mathbf{n}} \left| \mathbf{n} \right\rangle)$$

Summation over a microcanonical window around an energy E.

Length scale — the de Broglie wavelength $\hbar/\sqrt{mE}$

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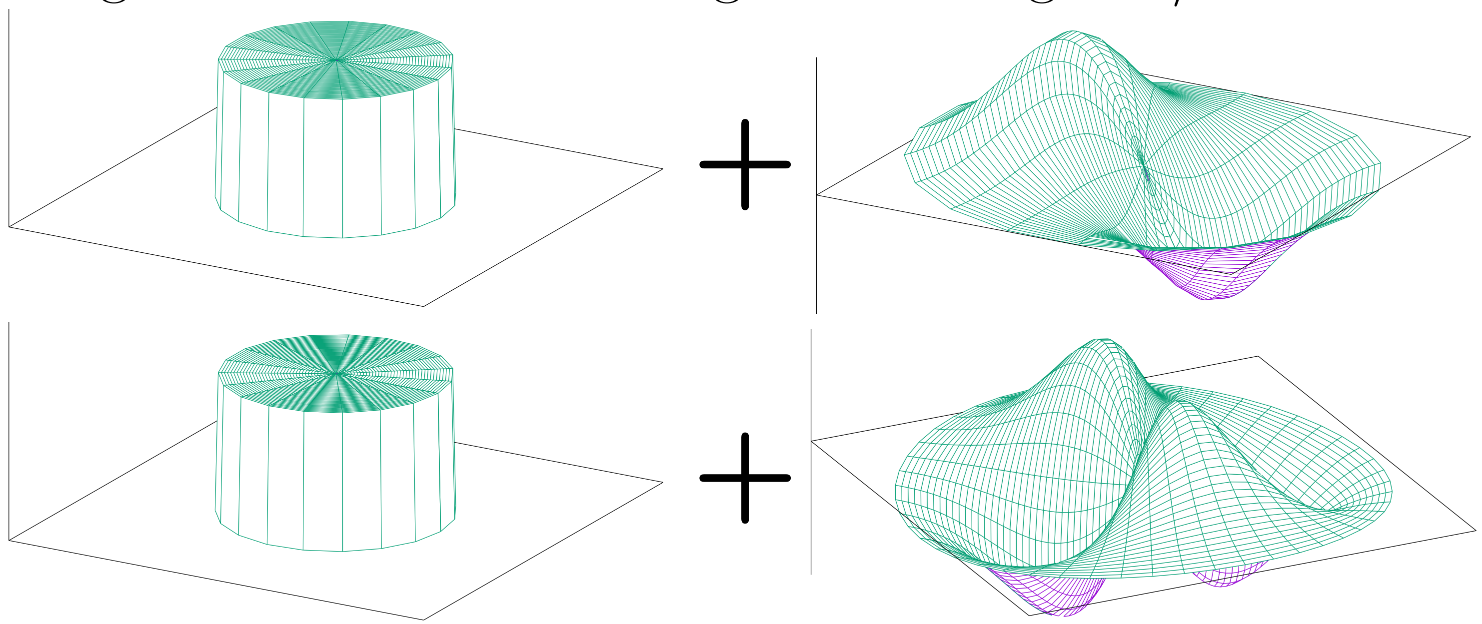

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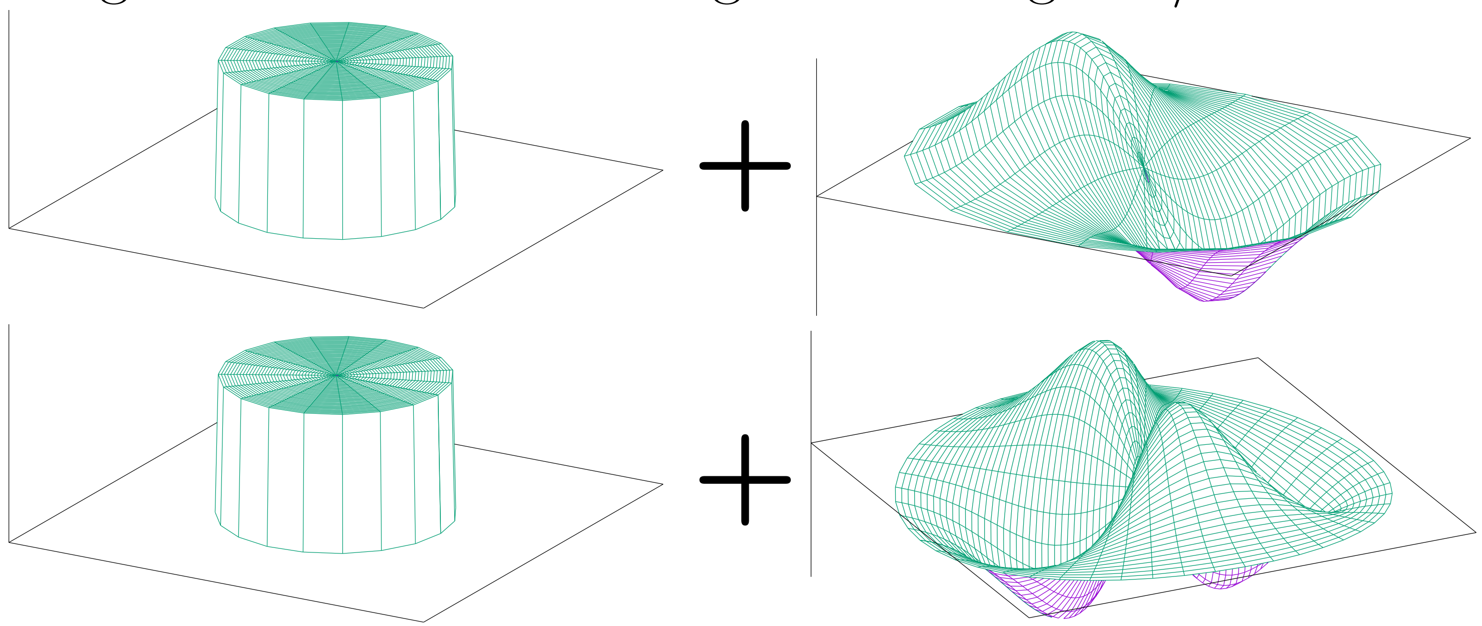

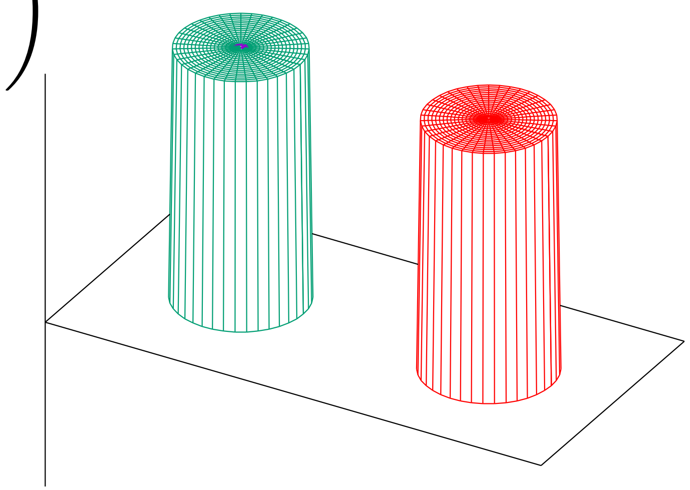

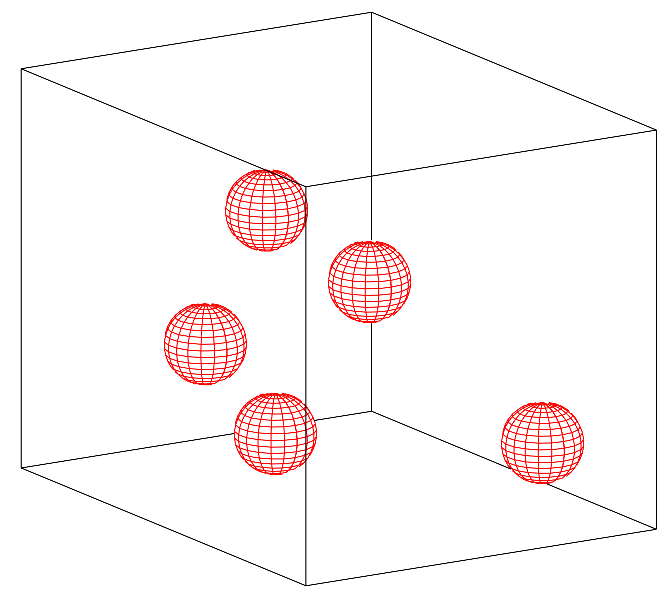

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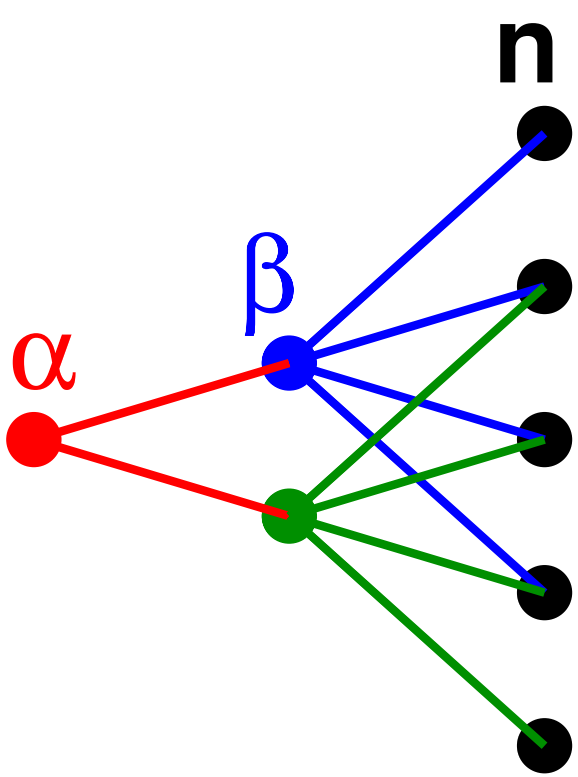

The interference terms are neglected $\rightarrow$

$|\alpha\rangle$ involves more $|\mathbf{n}\rangle$ than $|\beta\rangle$

Qualitative conclusion.

Quantitative?

NUMBER OF PRINCIPAL COMPONENTS (NPC)

Inverse Participation Ratio $\eta = \sum_{\mathbf{n}} |\langle \mathbf{n} | \alpha \rangle|^4$, η^{-1} — NPC

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— Lorentzian approximation (for β—Γ_1)

ΔE— average state energy difference

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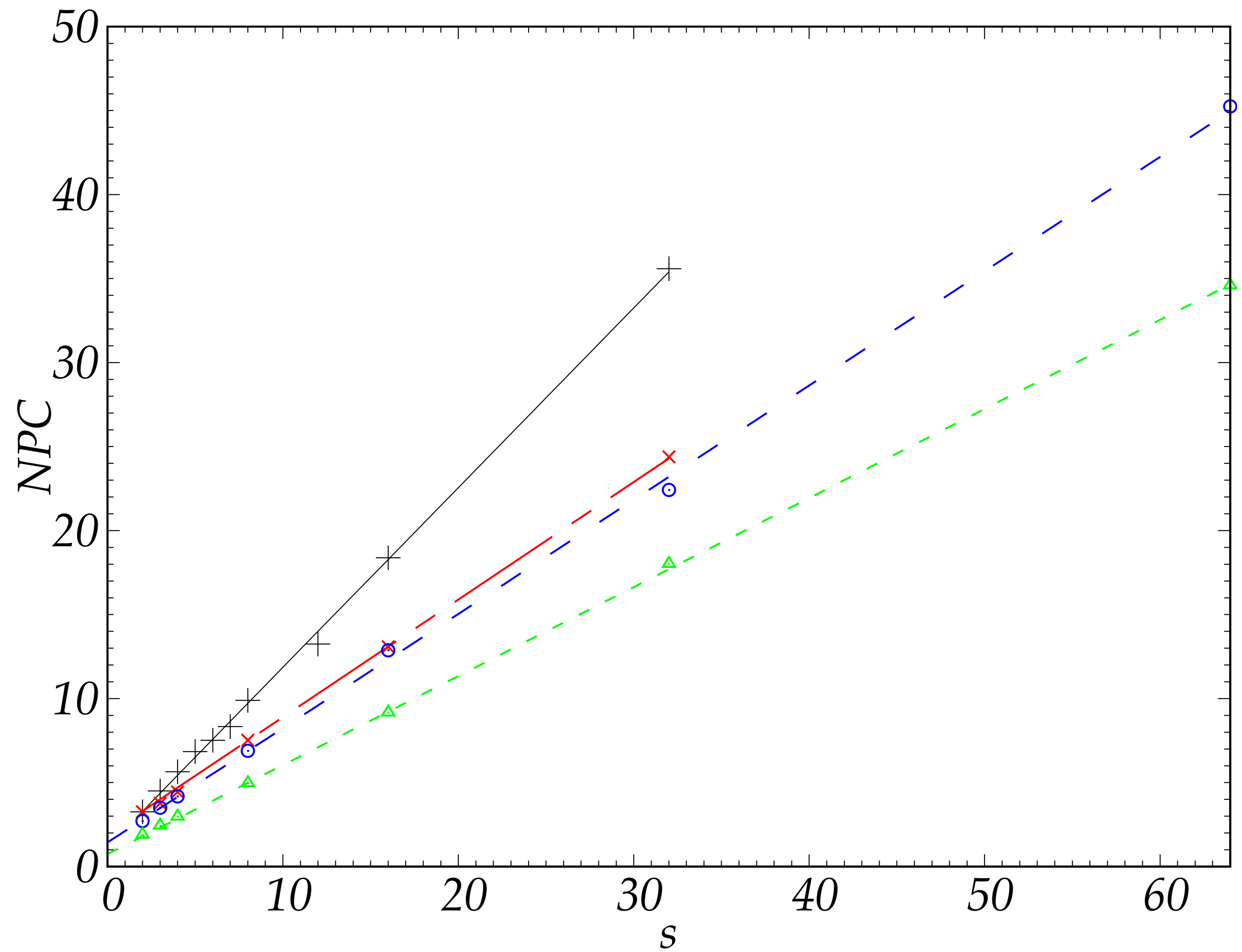
$\hat{H}_s = \hat{H}_0 + \sum_{s'=1}^{s} \hat{V}_{s}$: $\boxed{\eta_s^{-1} = \eta_{s-1}^{-1} + \nu}$.

ν is independent of s if strong $\hat{V}_s$ have the same shape

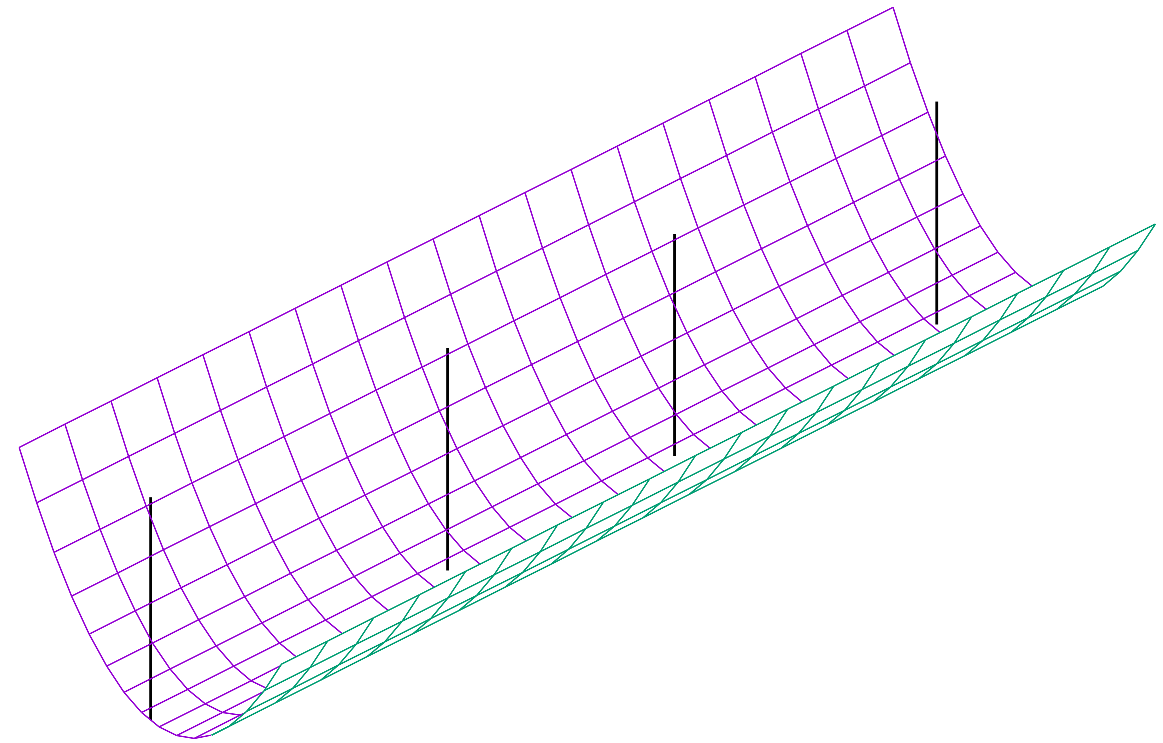


HARMONIC WAVEGUIDE WITH SCATTERERS

$$\hat{H}_0 = \frac{\hbar^2}{2m}\left[\left(\frac{1}{i}\frac{\partial}{\partial z} - A\right)^2 - \triangle_\rho\right] + \frac{m\omega_\perp^2\rho^2}{2}$$

$$\hat{V}_s = V_0\delta_{\rm reg}(\mathbf{r} - \mathbf{R}_s) \quad (\mathbf{R}_s = (0, 0, z_s))$$

In the sector of axially-symmetric states



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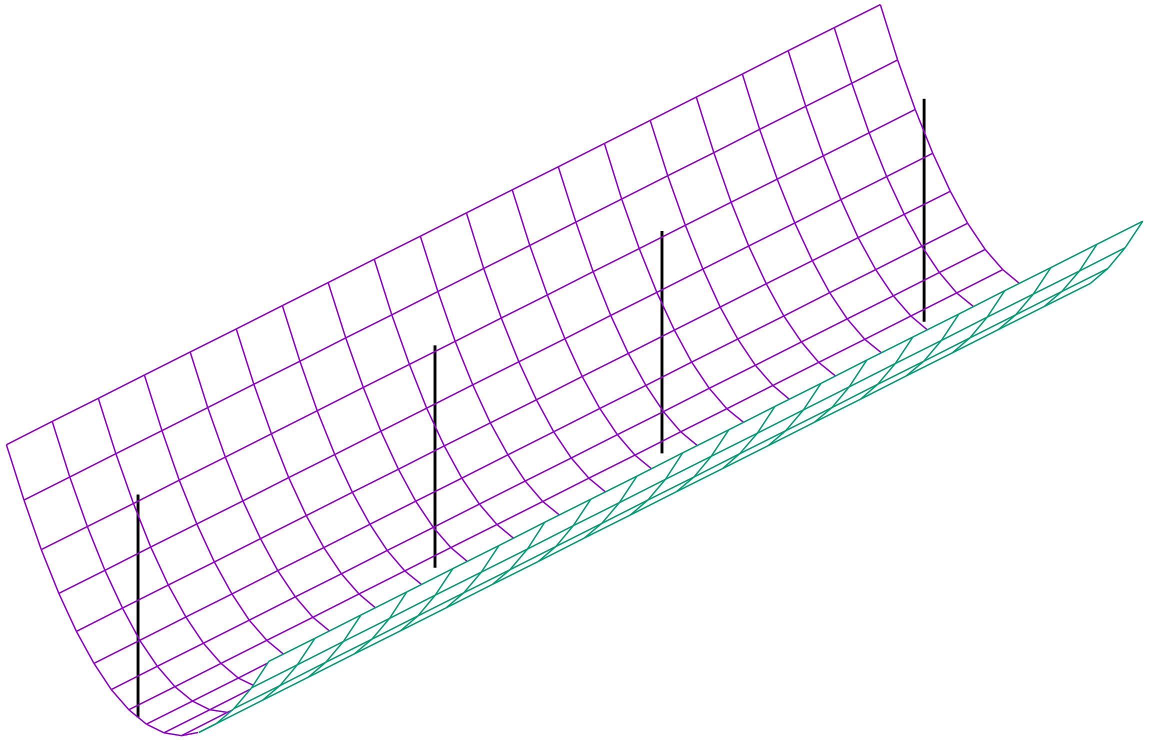
In the sector of axially-symmetric states

$E_{nl} = 2\hbar\omega_\perp n + \hbar^2(2\pi l/L - A)^2/(2m)$

— degeneracy is lifted by the vector potential A

$\Delta E \propto E^{-1/2}$— as in the 3D flat potential.



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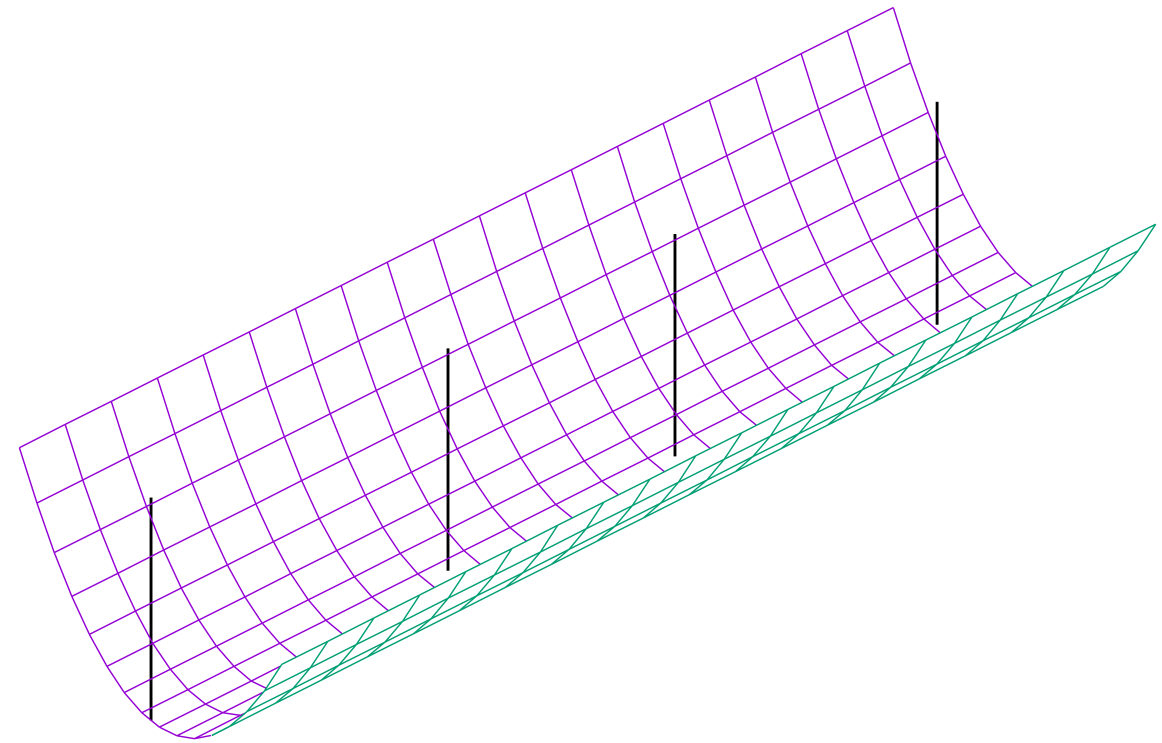
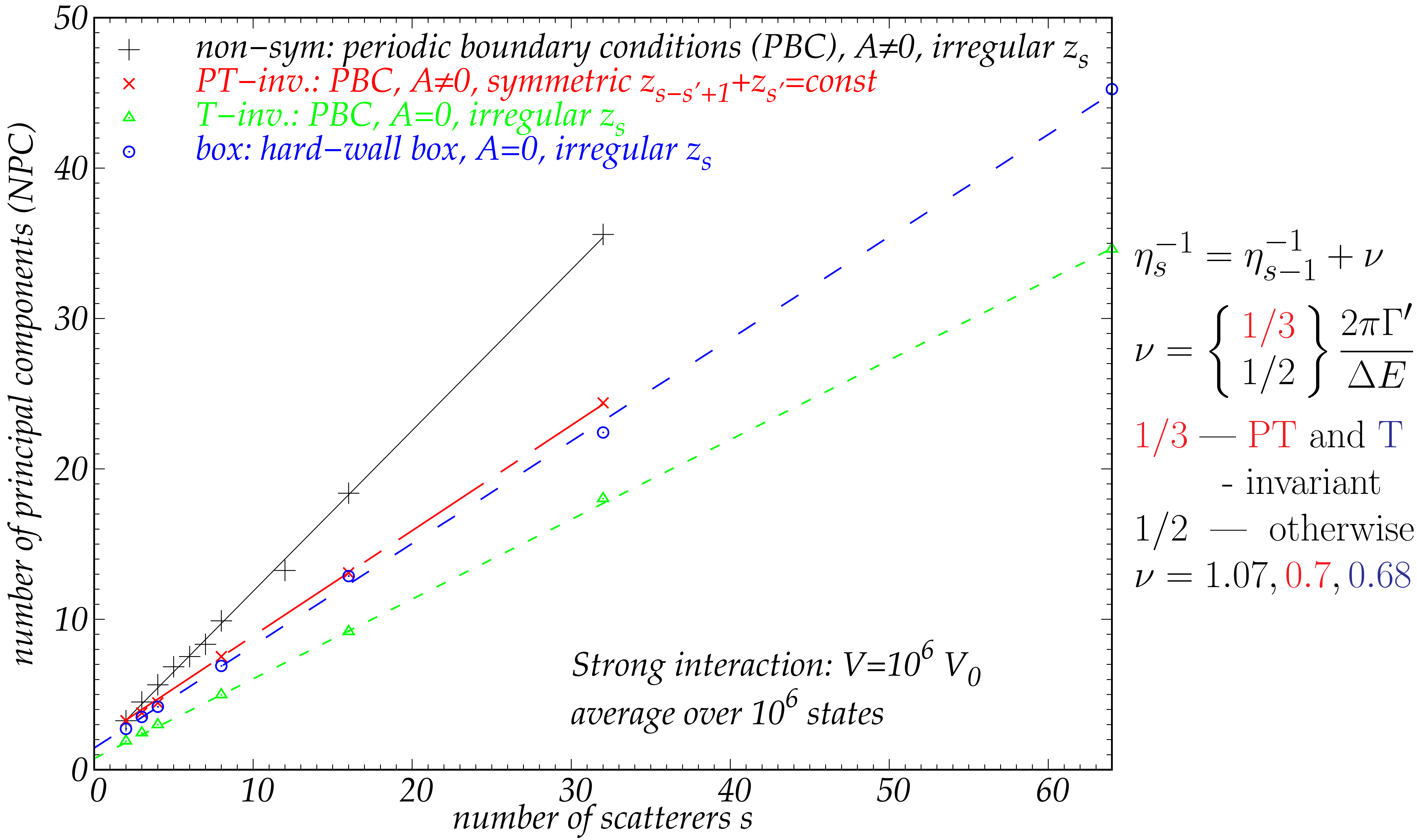


s zero-range scatterers — rank-s separble interaction
[Cheon & Shigehara, PRE (1996); Legrand, Mortessagne & Weaver, PRE (1997); Kanjilal, Bohn & Blume, PRA(2007); Yesha, J. Spectr. Theory (2018)]

α eigenstates are calculated with $\sim s^2\alpha^{5/3}$ operations (cf. with $\sim \alpha^3$ for direct diagonalization)


non−sym: periodic boundary conditions (PBC), A≠0, irregular z_s
PT−inv.: PBC, A≠0, symmetric z_{s−s′+1}+z_{s′}=const
T−inv.: PBC, A=0, irregular z_s
box: hard−wall box, A=0, irregular z_s
Strong interaction: V=10^6 V_0
average over 10^6 states
number of principal components (NPC)
number of scatterers s
η_s^{−1} = η_{s−1}^{−1} + ν
ν = {1/3, 1/2} 2πΓ′/ΔE
1/3 — PT and T
- invariant
1/2 — otherwise
ν = 1.07, 0.7, 0.68

FLUCTUATIONS

Variance of the expectation value fluctuations $\mathrm{Var}_\alpha(\hat{A}) = \overline{\left\langle \alpha \left| \hat{A} \right| \alpha \right\rangle^2} - \overline{\left\langle \alpha \left| \hat{A} \right| \alpha \right\rangle}^2$

$\mathrm{Var}_\alpha(\hat{A}) = \eta \mathrm{Var}_{\mathbf{n}}(\hat{A})$ [Neuenhahn & Marquardt, PRE (2012)]

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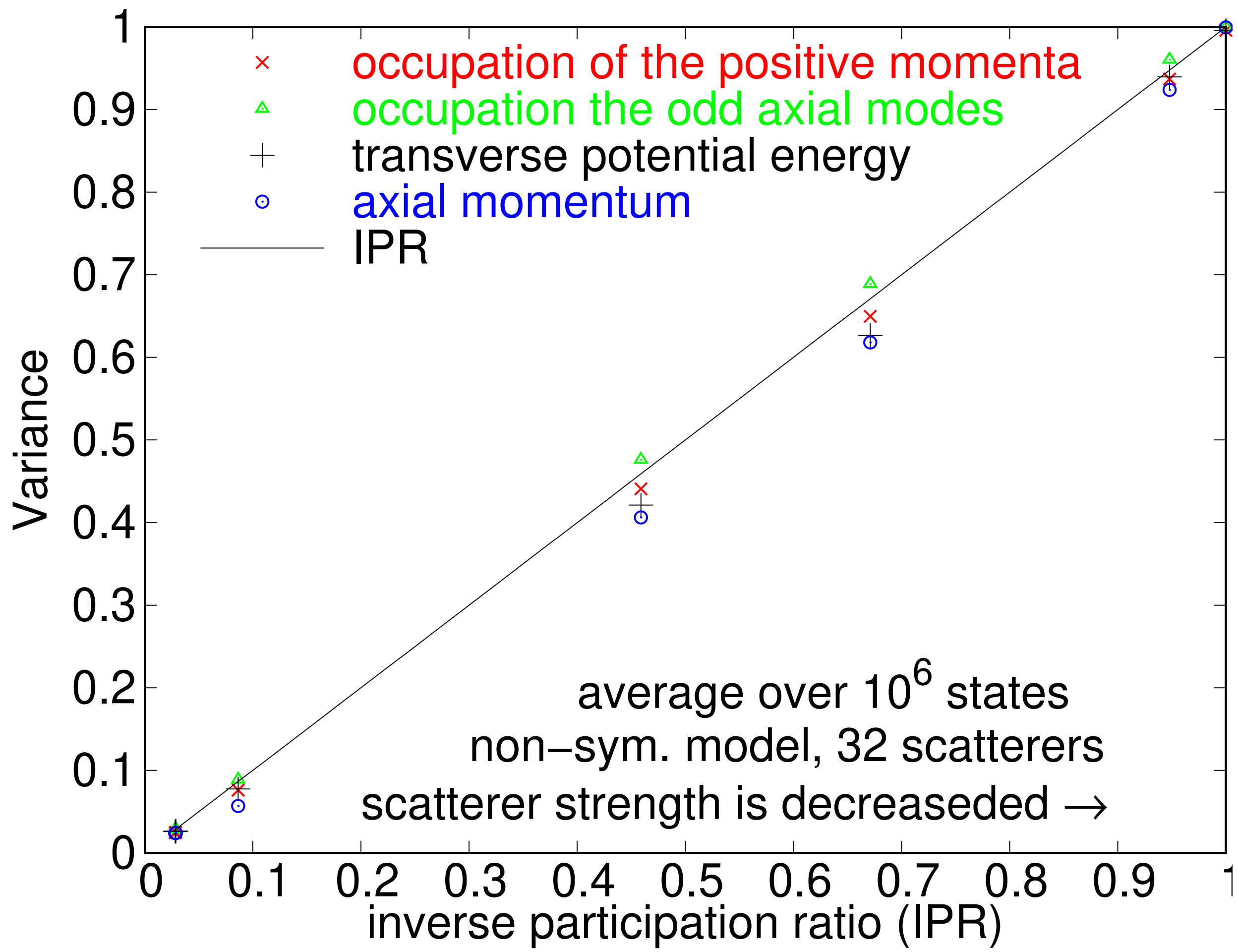

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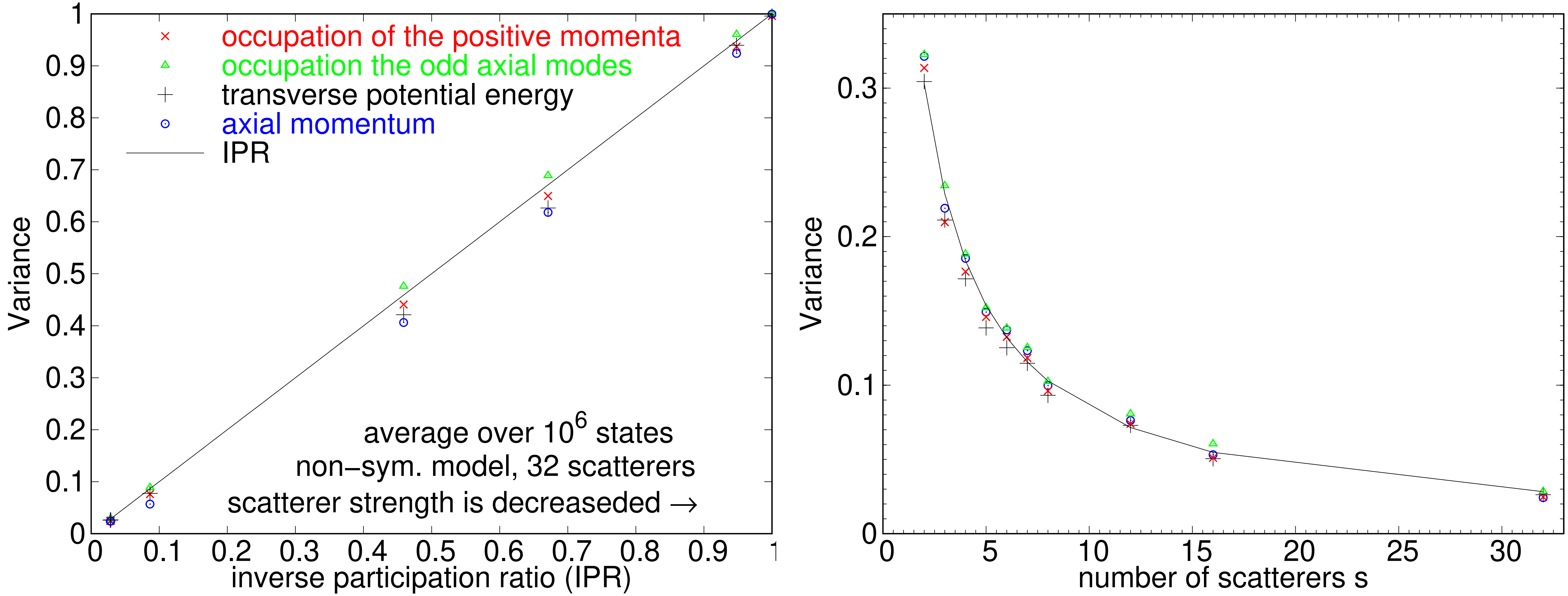

CONCLUSIONS

The addition of an independent interaction increases the system chaotisity.

The number of principal components — a characteristic of the system chaotisity — increases linearly with the number of independent interactions of the same shape.

This dependence is confirmed by numerical calculations for a harmonic waveguide with scatterers.

The variance of expectation value fluctuations between eigenstates decreases inversely proportionally to the number of scatterers, demonstrating approaching the eigenstate thermalization.