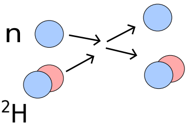
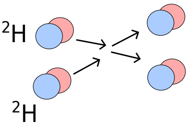




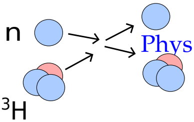
Few Nucleon Scattering in Pionless Effective Field Theory

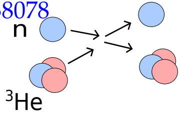
Betzalel Bazak
The Racah institute of physics
The Hebrew university of Jerusalem

THE HEBREW UNIVERSITY OF JERUSALEM

with M. Schäfer, M. Bagnarol, and N. Barnea

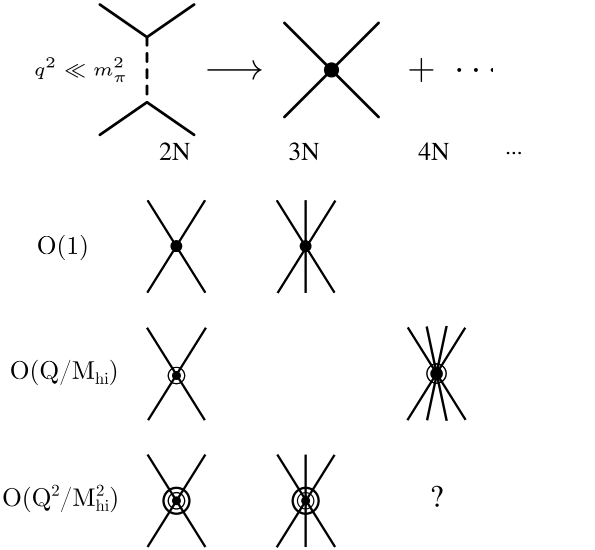
Phys Rev. C 107 (2023) 064001; Phys. Lett. B 844 (2023) 138078



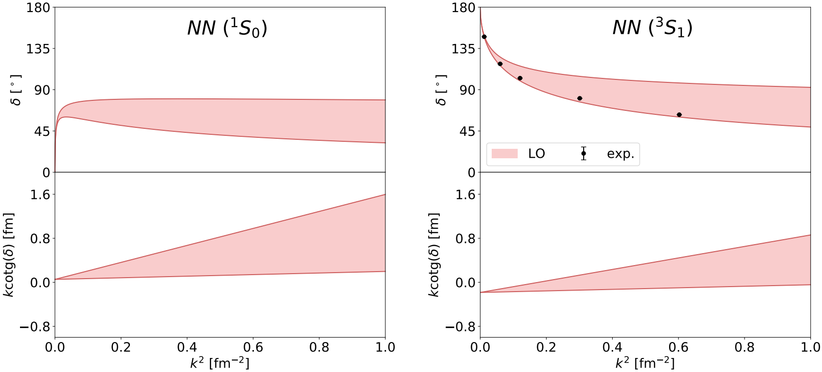


EFB25 @ Mainz
August 1st, 2023

πEFT



H.-W. Hammer, S. König, and U. van Kolck, Rev. Mod. Phys. 92, 025004 (2020)



Leading order (LO):

(exp. constraints)

$$a^0_{NN}\ (a^0_{nn}) = \ -18.95(40)\ \text{fm}$$

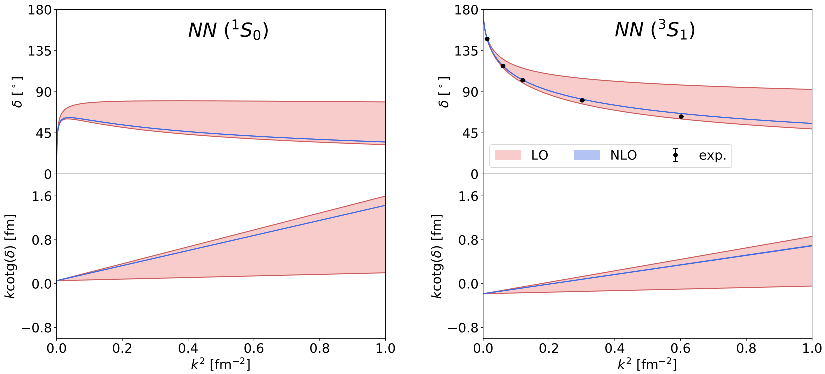
$$a^1_{NN}\ (a^1_{np}) = \ 5.419(7)\ \text{fm}$$

$$B(^3\text{H}) = 8.482\ \text{MeV}$$

Effective range expansion:

$$k\text{cotg}(\delta) = -\frac{1}{a} + \frac{1}{2} r k^2 + \ldots$$

NLO $\not\pi$EFT



Leading order (LO):

(exp. constraints)

$a^0_{NN}\ (a^0_{nn}) = \ -18.95(40)$ fm

$a^1_{NN}\ (a^1_{np}) = \ 5.419(7)$ fm

$B(^3\text{H}) = 8.842$ MeV

Next-to-leading order (NLO):

(exp. constraints)

$r^0_{NN}\ (r^0_{nn}) = \ 2.75(11)$ fm

$r^1_{NN}\ (r^1_{np}) = \ 1.753(8)$ fm

$B(^4\text{He}) = 28.296$ MeV

$\pi\!\!\!/$EFT potential at LO and NLO

Leading order potential (3 LECs):

$$V_\lambda^{(\mathrm{LO})} = \sum_{i<j}\left[C_0^{(0)}(\lambda)P_{ij}^{I=1,S=0} + C_1^{(0)}(\lambda)P_{ij}^{I=0,S=1}\right]e^{-\frac{\lambda^2}{4}\mathbf{r}_{ij}^2}$$

$$+ D_0^{(0)}(\lambda)\sum_{i<j<k}\mathcal{Q}_{ijk}^{I=1/2,S=1/2}\sum_{\mathrm{cyc}}e^{-\frac{\lambda^2}{4}(\mathbf{r}_{ij}^2+\mathbf{r}_{jk}^2)}$$

Next-to-leading order potential (6 LECs):

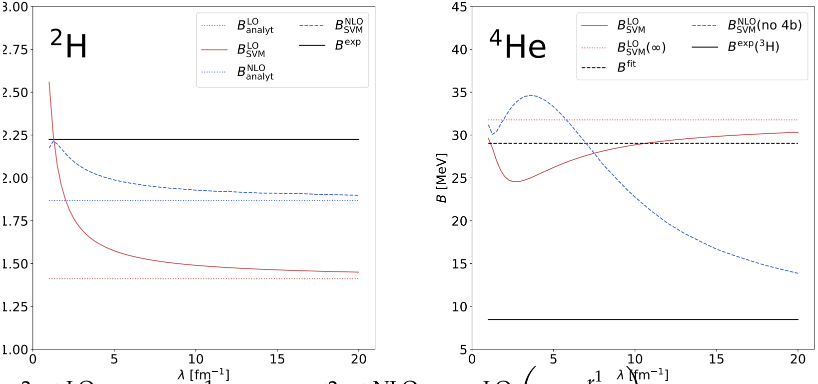
$$V_\lambda^{(\mathrm{NLO})} = \sum_{i<j}\left[C_0^{(1)}(\lambda)P_{ij}^{T=1,S=0} + C_1^{(1)}(\lambda)P_{ij}^{I=0,S=1}\right]e^{-\frac{\lambda^2}{4}\mathbf{r}_{ij}^2}$$

$$+ \sum_{i<j}\left[C_2^{(1)}(\lambda)P_{ij}^{T=1,S=0} + C_3^{(1)}(\lambda)P_{ij}^{T=0,S=1}\right](\mathbf{k}^2+\mathbf{q}^2)e^{-\frac{\lambda^2}{4}\mathbf{r}_{ij}^2}$$

$$+ D_0^{(1)}(\lambda)\sum_{i<j<k}\mathcal{Q}_{ijk}^{T=1/2,S=1/2}\sum_{\mathrm{cyc}}e^{-\frac{\lambda^2}{4}(\mathbf{r}_{ij}^2+\mathbf{r}_{jk}^2)}$$

$$+ E_0^{(1)}(\lambda)\sum_{i<j<k<l}\mathcal{Q}_{ijkl}^{T=0,S=0}e^{-\frac{\lambda^2}{4}(\mathbf{r}_{ij}^2+\mathbf{r}_{ik}^2+\mathbf{r}_{il}^2+\mathbf{r}_{jk}^2+\mathbf{r}_{jl}^2+\mathbf{r}_{kl}^2)}$$

Requirement of $(S=0, T=0)$ four-body force



$\rightarrow B(^2\mathrm{H})^{\mathrm{LO}}_{\mathrm{analyt}} = \frac{1}{m_N (a^1_{NN})^2}, \quad B(^2\mathrm{H})^{\mathrm{NLO}}_{\mathrm{analyt}} = B^{\mathrm{LO}}_2 \left(1 + \frac{r^1_{NN}}{a^1_{NN}}\right)$

$\rightarrow$ divergence of $B(^4\mathrm{He})$ without 4-body force

$\rightarrow H_0(\lambda)$ 4-body LEC fitted to $B(^4\mathrm{He}) = 28.296$ MeV

Similar results for 4 bosons: BB, Kirscher, König, Pavón Valderrama, Barnea, and van Kolck, Phys. Rev. Lett. **122** (2019) 143001.

Bush formula

→ assumption of short range potential with range $R \ll b_{\text{HO}} = \sqrt{\frac{2}{m\omega}}$

Bush formula

$$-\sqrt{4\mu\omega}\,\frac{\Gamma\left(3/4 - \epsilon_n/2\omega\right)}{\Gamma\left(1/4 - \epsilon_n/2\omega\right)} = k\,\text{cotg}(\delta), \quad k = \sqrt{2\mu\epsilon_n}$$

(A. Suzuki, Phys. Rev. A 80 (2009) 033601, T. Bush Found. of Phys. 28 (1998) 4)

LO $\not\pi$EFT calculations:

$$H(\omega) = T_k + V_\lambda^{(\text{LO})} + V_{\text{HO}}(\omega) \quad \longrightarrow \quad H(\omega)\psi_n = \epsilon_n\psi_n \quad \overset{\text{BUSH}}{\overbrace{\longrightarrow}} \quad k\,\text{cotg}(\delta)$$

NLO $\not\pi$EFT calculations:

$$\epsilon_n^{(\text{NLO})} = \epsilon_n + \langle\psi_n|\, V_\lambda^{(\text{NLO})}\,|\psi_n\rangle \quad \overset{\text{BUSH}}{\overbrace{\longrightarrow}} \quad k\,\text{cotg}(\delta^{(\text{NLO})})$$

Stochastic Variational Method

$$H\psi = E\psi, \qquad \psi = \sum_{i=0}^{N} c_i \varphi^i$$

Basis states

- antisymmetrized correlated Gaussians (assuming L=0)

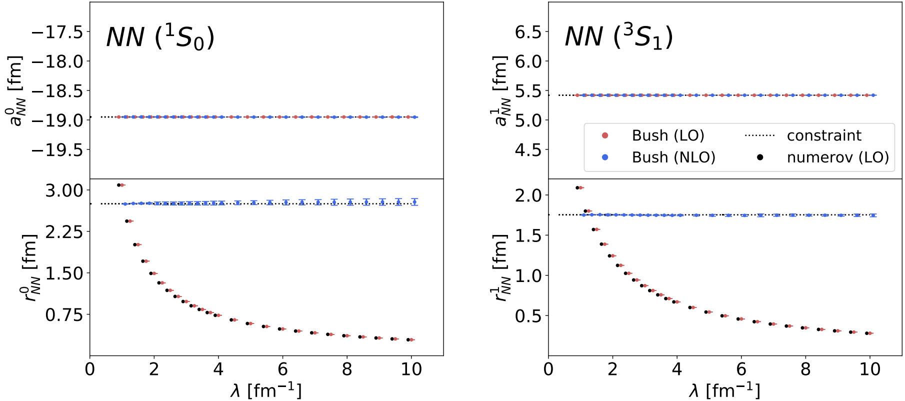
$$\varphi^i_{SM_STM_T}(\mathbf{x}, A_i) = \hat{\mathcal{A}}\{G_{A_i}(\mathbf{x})\chi_{SM_S}\eta_{TM_T}\}, \;\; G_{A_i}(\mathbf{x}) = e^{-\frac{1}{2}\mathbf{x}A_i\mathbf{x}}$$

- Jacobi coordinates $\mathbf{x}$, A_i symmetric positive definite matrix of $\frac{N(N-1)}{2}$ real parameters, spin χ_{SM_S} and isospin η_{TM_T} parts

Optimization of variational basis in a **stochastic trial and error procedure**

K. Varga et al., NPA571 (1994) 447, K. Varga, Y. Suzuki, PRC52 (1995) 2885.

NN scattering - Bush vs. Numerov



$n + d \; (S = 1/2, T = 1/2)$ scattering

1) Near-threshold $^3\mathrm{H}^*$ virtual state

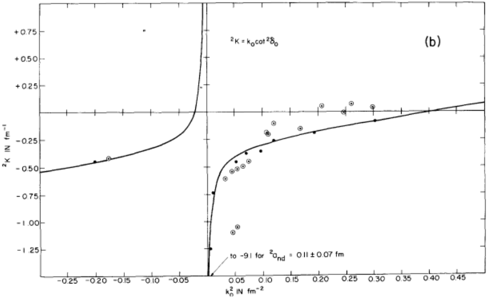
$\rightarrow$ pole of S-matrix

2) Near-threshold zero in S-matrix

$$\frac{1}{k\mathrm{cotg}(\delta) - \mathrm{i}k} = 0$$

$$\lim_{k \to k_0} k\mathrm{cotg}(\delta) = \pm\infty$$

$\rightarrow$ modified ERE



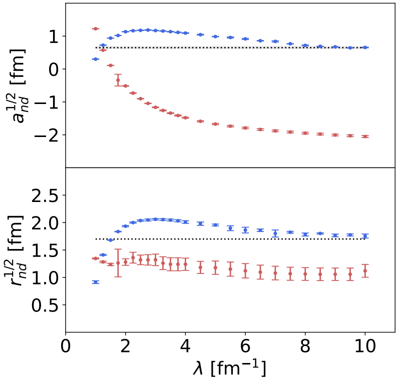
(Oers and Seagrave, Phys. Lett B 24 (1967) 11)

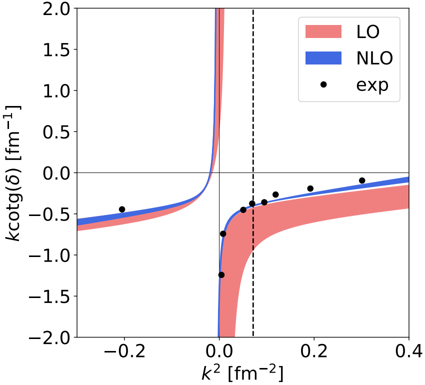
$$k\mathrm{cotg}(\delta) = A + B\,k^2 + \frac{C}{(1 + D\,k^2)}$$

$a = -\frac{1}{A+C}$ and $r = 2\,B$

$$a^{1/2}_{n^2\mathrm{H}} = 0.29 \text{ fm}$$
$$r^{1/2}_{n^2\mathrm{H}} = 1.70 \text{ fm}$$

$n + d \ (S = 1/2, T = 1/2)$ scattering

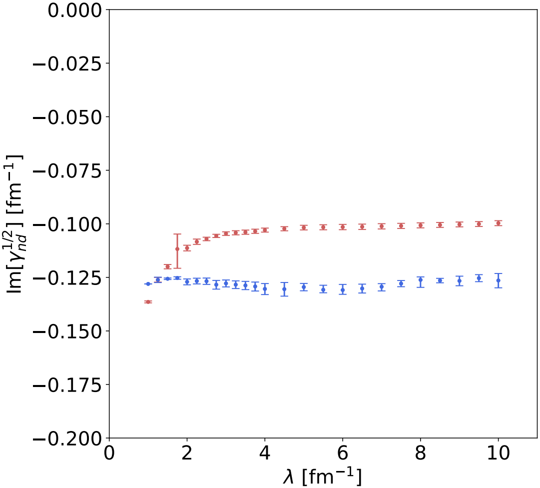




$n + d \ (S = 1/2, T = 1/2)$ scattering - $^3\mathrm{H}^*$ virtual state

Pole of S-matrix:

$$A + B\,k^2 + \frac{C}{(1 + D\,k^2)} - \mathrm{i}k = 0$$



LO :

$\mathrm{Im}[\gamma_{nd}^{1/2}] = -0.117(19)\ \mathrm{fm}^{-1}$

$E_{nd}^{1/2} = -0.43(14)$ MeV

NLO :

$\mathrm{Im}[\gamma_{nd}^{1/2}] = -0.1271(39)\ \mathrm{fm}^{-1}$

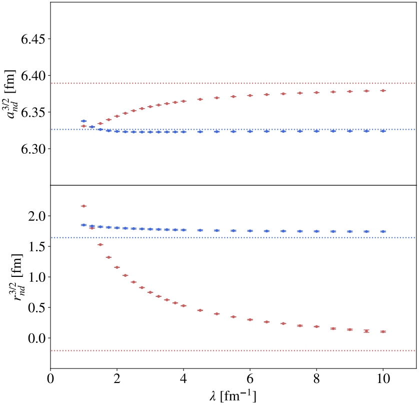
$E_{nd}^{1/2} = -0.503(31)$ MeV

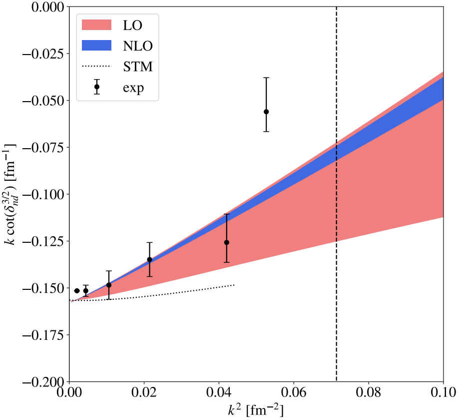
→ **Efimov virtual state**

(Phys. Rev. C 26 (1982) 77)

(Phys. Lett. B 791 (2019) 414)

$n + {}^2\mathrm{H}\ (S = 3/2, T = 1/2)$ scattering





Fermionic atom-dimer universality:

$\rightarrow \frac{a_{ad}}{a_{aa}} = 1.179066 - 0.03595\frac{r_{aa}}{a_{aa}}$

$\rightarrow \frac{r_{ad}}{a_{aa}} = -0.0383 + 1.0558\frac{r_{aa}}{a_{aa}}$

(G. V. Skorniakov et al., Zh. Eksp. Teor. Fiz. 31, 775 (1956); B. E. Grinyuk et al. Yad. Fiz. 39, 402 (1984))

Zero energy $n + {}^2\mathrm{H}$ scattering experiments

Coherent scattering length : $a^c_{n^2\mathrm{H}} = \frac{1}{3} a^{1/2}_{n^2\mathrm{H}} + \frac{2}{3} a^{3/2}_{n^2\mathrm{H}}$

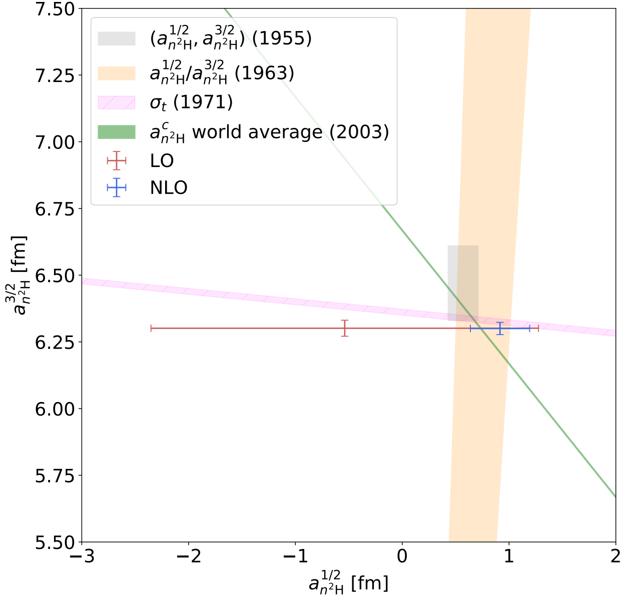
Incoherent scattering length : $\left(a^{inc}_{n^2\mathrm{H}}\right)^2 = \frac{1}{3}\left(a^{1/2}_{n^2\mathrm{H}}\right)^2 + \frac{2}{3}\left(a^{3/2}_{n^2\mathrm{H}}\right)^2$

Total cross section ($k = 0$) : $\sigma_t = 4\pi\left[\left(a^c_{n^2\mathrm{H}}\right)^2 + \left(a^{inc}_{n^2\mathrm{H}}\right)^2\right]$

$n + {}^2\mathrm{H}$

- **S. J. Nikitin et al.** (First Geneva Conf. 2 (1955) 81)
  $a^{1/2}_{n^2\mathrm{H}}$ and $a^{3/2}_{n^2\mathrm{H}}$ limits (ortho/para hydrogen)
- **W. Gissler** (Z. Kristallographie 118 (1963) 149)
  $a^{1/2}_{n^2\mathrm{H}} / a^{3/2}_{n^2\mathrm{H}}$ ratio
- **W. Dilg et al.** (Phys. Lett. B 36 (1971) 208)
  total crosssection
- **K. Schoen et al.** (Phys. Rev. C 67 (2003) 044005)
  coherent scattering length (the most recent world average)

$n + {}^2\mathrm{H}$ scattering lengths



$d + d$ scattering $(S = 2, T = 0)$

- missing Coulomb interaction

  $a_{dd}^{2\ c} = 7.8 \pm 0.3$ fm

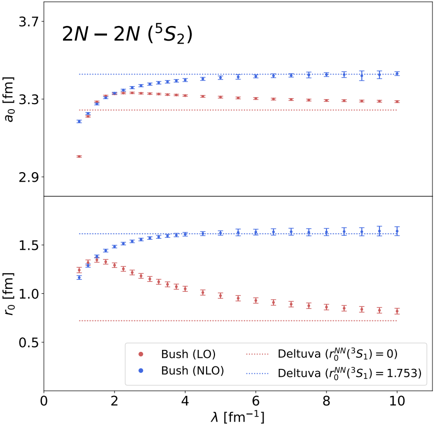
(Carew, Phys.Rev. C 103 (2021) 014002)

Only $NN(^3S_1)$ 2-body interaction

$\rightarrow$universal fermionic dimer-dimer scattering

$$\frac{a_{dd}}{a_{aa}} = 0.5986 + 0.105 \frac{r_{aa}}{a_{aa}} \pm 0.0005$$

$$\frac{r_{dd}}{a_{aa}} = 0.133 + 0.51 \frac{r_{aa}}{a_{aa}} \pm 0.002$$



Petrov, Salomon, and Shlyapnikov, Phys. Rev. Lett. 93 (2004) 090404;

Deltuva, Phys. Rev. A 96 (2017) 022701

Zero energy $n+{}^{3}\mathrm{H}$ and $n+{}^{3}\mathrm{He}$ scattering experiments

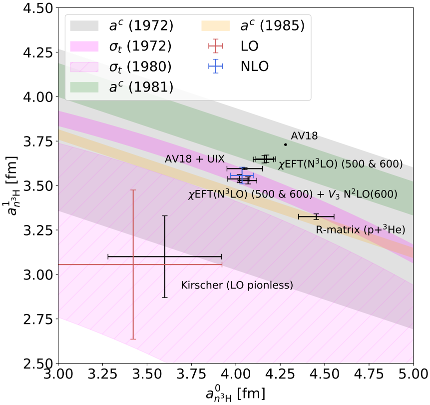
$n+{}^{3}\mathrm{H}$

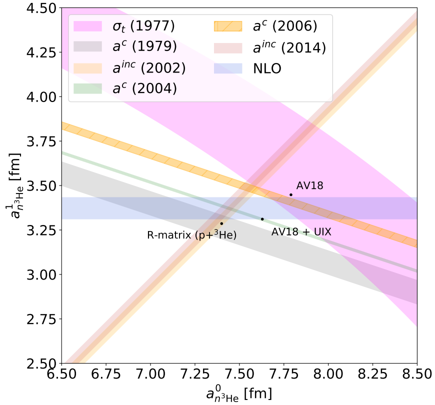
- (Phys. Rev. C (1972) 1952)
  coherent scatetring length,
  total crosssection
- **T. W. Phillips et al.**
  (Phys. Rev. C 22 (1980) 384)
  total crosssection
- **S. Hammerschmied et al.**
  (Z. Phys. A 302 (1981) 323)
  coherent scattering length
- **H. Rauch et al.**
  (Phys. Lett. B 165 (1985) 39)
  coherent scattering length

$n+{}^{3}\mathrm{He}$

- **V. P. Alfimenkov**
  (Sov. J. Nucl. Phys 25 (1977) 607)
  total crosssection
- **H. Kaiser et al.**
  (Z. Phys. A 291 (1979) 231)
  cohherent scattering length
- **O. Zimmer et al.**
  (Eur. Phys. J. Direct A 1 (2002) 1)
  incoherent scattering length
- **P.R. Huffman et al.**
  (Phys. Rev. C 70 (2004) 014004)
  coherent scattering length
- **W. Ketter et al.**
  (Eur. Phys. J. A 27 (2006) 243)
  coherent scattering length
- **M.G. Huber et al.**
  (Phys. Rev. C 90 (2014) 064004)
  incoherent scattering length

$n + {}^3\mathrm{H}$ and $n + {}^3\mathrm{He}$ scattering lengths





(Phys. Rev. C 42 (1990) 438; Phys. Rev. C 102 (2020) 034007; Few-Body Syst. 34 (2004) 105; Phys. Lett B 721 (2013) 355; Phys. Rev. C 68(R) (2003) 021002)

Experimental $n + {}^4\mathrm{He}$ elastic scattering

Only one s-wave channel (spin doublet)!

Zero-energy experiments

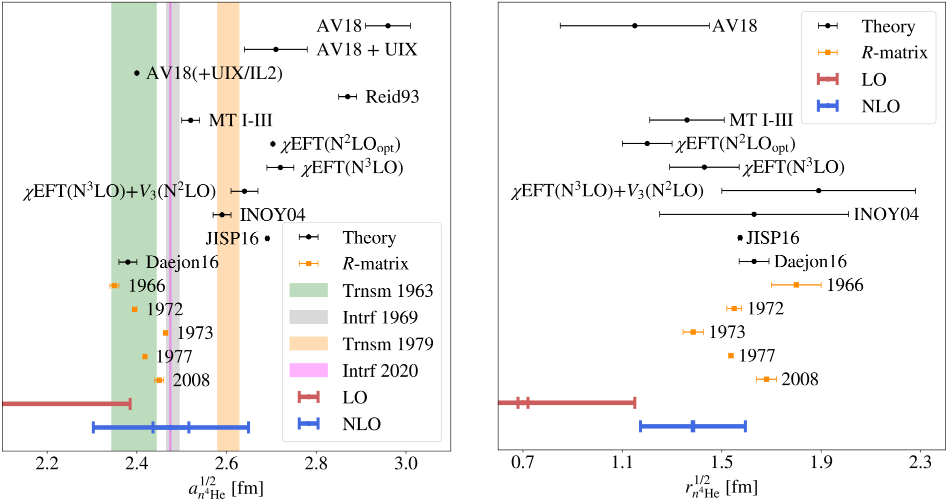
- **A. W. McRaynolds**
  (Phys. Rev. 84 (1951) 969)
  transmission measurement
- **R. Genin et al.**
  (J. Phys. Rodium 24 (1963) 21)
  transmission measurement
- **D. C. Rover et al.**
  (Nucl. Phys. A 133 (1969) 410)
  neutron interferometry
- **H. Kaiser at al.**
  (Zeit. Phys. A 291 (1979) 231)
  transmission measurement
- **R. Haun et al.**
  (Phys. Rev. Lett. 124 (2020) 012501)
  neutron interferometry

$$a^{1/2}_{\mathrm{n}^4\mathrm{He}} = 2.4746 \begin{array}{l}\pm 0.0017(\mathrm{stat}) \\ \pm 0.0011(\mathrm{syst})\end{array} \;\mathrm{fm}$$

Phaseshifts (R-matrix)

- **Hoop, Barschall**
  (Nucl. Phys. 83 (1966) 65)
- **Th. Stammbad, R. L. Walter**
  (Nucl. Phys. A 180 (1972) 225)
- **R. A. Arndt, D. L. Roper**
  (Nucl. Phys. A 209 (1973) 447)
- **Bond, Firch**
  (Nucl. Phys. A 287 (1977) 317)
- **G. M. Hale**
  (Private communication (2008))

$n + {}^4\mathrm{He}$ scattering



Phys. Lett. B 844 (2023) 138078; See M. Bagnarol's poster!

Summary & Outlook

- We study perturbative NLO $\not{\pi}$EFT for $A \leq 5$ nuclear systems using HO trap.
- Results for all 2-, 3-, 4-, and 5-body $L = 0$ channels converge with cutoff; four-body force is needed in the $S = 0$, $T = 0$ channel.
- Precise calculations of low-energy n^2H, n^3H, n^3He, and n^4He elastic scattering.
- NLO terms significantly increase accuracy of our predictions.

M. Schäfer and BB, Phys Rev. C 107 (2023) 064001;
M. Bagnarol, M. Schäfer, BB, and N. Barnea, Phys. Lett. B 844 (2023) 138078

Next steps:

- Coulomb interaction (p^2H, p^3H, p^3He, n^3He, $^4\text{He}(0_2^+)$, dd)
- Higher-body p-shell nuclear systems and p-wave scattering
- Nuclear reactions
- ...