Momentum dependent nucleon-nucleon contact interactions and their effect on p-d scattering observables

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Summary

N3LO 2N Contact Interactions from Relativistic constraints

The general expression of the relativistic NN contact Lagrangian invariant under parity, charge conjugation and time reversal symmetry consists of products of fermion bilinears as

$$(\bar{\psi}\overleftrightarrow{\partial}_{\mu_1}\cdots\overleftrightarrow{\partial}_{\mu_i}\Gamma_A\psi)\partial_{\lambda_1}\cdots\partial_{\lambda_k}(\bar{\psi}\overleftrightarrow{\partial}_{\nu_1}\cdots\overleftrightarrow{\partial}_{\nu_j}\Gamma_B\psi)$$

ψ = relativistic nucleon field, $\overleftrightarrow{\partial} = \overrightarrow{\partial} - \overleftarrow{\partial}$ and $\Gamma_{A,B}$ = generic elements of the Clifford algebra

- Regarding the isospin degrees of freedom structures as $1 \otimes 1$ and $\tau^a \otimes \tau^a$ are allowed, however the latter can be disregarded since it can be eliminated by Fierz rearrangements
- To specify the chiral order of each building block, it is necessary to identify the powers of soft nucleon momenta p
  - The derivatives ∂ acting on the entire bilinear $\sim p$
  - $\overleftrightarrow{\partial} \sim p^0$ due to the presence of the heavy fermion mass scale
  - $\gamma_5 \sim p$ since it mixes the large and small components of the Dirac spinor
- Use of equations of motion to reduce the number of terms

These criteria lead to the complete (although non-minimal) set of relativistic contact operators up to $O(p^4)$

$\Gamma_A \otimes \Gamma_B$	Operators	Gradient structures
$1 \otimes 1$	$\tilde{O}_{1-6} = \bar{\psi}\psi\bar{\psi}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2, (\overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B)^2, (\overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B) d^2, d^4\}$
$1 \otimes \gamma$	$\tilde{O}_{7-9} = \frac{i}{2m}\bar{\psi}\overleftrightarrow{\partial}^{\mu}\psi\bar{\psi}\gamma_\mu\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$1 \otimes \gamma\gamma_5$	$\tilde{O}_{10-12} = \frac{-1}{8m^3}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\overleftrightarrow{\partial}_{\mu}\psi\partial_\nu\bar{\psi}\gamma_\alpha\gamma_5\overleftrightarrow{\partial}_{\beta}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$\gamma_5 \otimes \gamma_5$	$\tilde{O}_{13-15} = -\bar{\psi}\gamma_5\psi\bar{\psi}\gamma_5\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$\gamma_5 \otimes \sigma$	$\tilde{O}_{16} = \frac{1}{16m^4}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_5\overleftrightarrow{\partial}^{\gamma}\overleftrightarrow{\partial}_{\mu}\psi\partial_\nu\bar{\psi}\sigma_{\alpha\gamma}\overleftrightarrow{\partial}_{\beta}\psi$	1
$\gamma \otimes \gamma$	$\tilde{O}_{17-19} = \bar{\psi}\gamma_\mu\psi\bar{\psi}\gamma^\mu\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
	$\tilde{O}_{20-22} = \frac{-1}{4m^2}\bar{\psi}\gamma_\mu\overleftrightarrow{\partial}_{\nu}\psi\bar{\psi}\gamma^\nu\overleftrightarrow{\partial}^{\mu}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$\gamma \otimes \gamma\gamma_5$	$\tilde{O}_{23} = \frac{-i}{16m^4}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_\mu\overleftrightarrow{\partial}^{\gamma}\overleftrightarrow{\partial}_{\nu}\psi\partial_\alpha\bar{\psi}\gamma_\gamma\gamma_5\overleftrightarrow{\partial}_{\beta}\psi$	1
	$\tilde{O}_{24} = \frac{-i}{16m^4}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma^\gamma\overleftrightarrow{\partial}_{\mu}\psi\partial_\nu\bar{\psi}\gamma_\alpha\gamma_5\overleftrightarrow{\partial}_{\gamma}\overleftrightarrow{\partial}_{\beta}\psi$	1
	$\tilde{O}_{25-27} = \frac{i}{4m^2}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_\mu\overleftrightarrow{\partial}_{\nu}\psi\partial_\alpha\bar{\psi}\gamma_\beta\gamma_5\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
	$\tilde{O}_{28-30} = \frac{i}{4m^2}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_\mu\psi\partial_\nu\bar{\psi}\gamma_\alpha\gamma_5\overleftrightarrow{\partial}_{\beta}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$\gamma\gamma_5 \otimes \gamma\gamma_5$	$\tilde{O}_{31-36} = \bar{\psi}\gamma^\mu\gamma_5\psi\bar{\psi}\gamma_\mu\gamma_5\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2, (\overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B)^2, (\overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B) d^2, d^4\}$
	$\tilde{O}_{37-39} = \frac{-1}{4m^2}\bar{\psi}\gamma^\mu\gamma_5\overleftrightarrow{\partial}^{\nu}\psi\bar{\psi}\gamma_\nu\gamma_5\overleftrightarrow{\partial}_{\mu}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$\gamma\gamma_5 \otimes \sigma$	$\tilde{O}_{40-42} = \frac{-i}{8m^3}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_\mu\gamma_5\overleftrightarrow{\partial}_{\nu}\overleftrightarrow{\partial}^{\gamma}\psi\bar{\psi}\sigma_{\alpha\gamma}\overleftrightarrow{\partial}_{\beta}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
	$\tilde{O}_{43-45} = \frac{i}{2m}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_\mu\gamma_5\psi\bar{\psi}\sigma_{\nu\alpha}\overleftrightarrow{\partial}_{\beta}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
	$\tilde{O}_{46-48} = \frac{i}{2m}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_\mu\gamma_5\overleftrightarrow{\partial}_{\nu}\psi\bar{\psi}\sigma_{\alpha\beta}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$\sigma \otimes \sigma$	$\tilde{O}_{49-51} = \bar{\psi}\sigma_{\mu\nu}\psi\bar{\psi}\sigma^{\mu\nu}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
	$\tilde{O}_{52-54} = \frac{-1}{4m^2}\bar{\psi}\sigma^{\mu\alpha}\overleftrightarrow{\partial}^{\beta}\psi\bar{\psi}\sigma_{\mu\beta}\overleftrightarrow{\partial}_{\alpha}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$

A complete non-minimal set of relativistic contact operators.
Here $\overleftrightarrow{d}_X = \overleftrightarrow{\partial}_X/(2m)$ $(X = A, B)$ and $d = \partial/(2m)$.

[E. Filandri and L. Girlanda, Phys. Lett. B (2023)]

$\Gamma_A \otimes \Gamma_B$	Operators	Gradient structures
$1 \otimes 1$	$\tilde{O}_{1-6} = \bar{\psi}\psi\bar{\psi}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2, (\overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B)^2, (\overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B)d^2, d^4\}$
$1 \otimes \gamma$	$\tilde{O}_{7-9} = \frac{i}{2m}\bar{\psi}\overleftrightarrow{\partial}^{\mu}\psi\bar{\psi}\gamma_{\mu}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$1 \otimes \gamma\gamma_5$	$\tilde{O}_{10-12} = \frac{-1}{8m^3}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\overleftrightarrow{\partial}_{\mu}\psi\partial_{\nu}\bar{\psi}\gamma_{\alpha}\gamma_5\overleftrightarrow{\partial}_{\beta}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$\gamma_5 \otimes \gamma_5$	$\tilde{O}_{13-15} = -\bar{\psi}\gamma_5\psi\bar{\psi}\gamma_5\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
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$\gamma \otimes \gamma$	$\tilde{O}_{17-19} = \bar{\psi}\gamma_{\mu}\psi\bar{\psi}\gamma^{\mu}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
	$\tilde{O}_{20-22} = \frac{-1}{4m^2}\bar{\psi}\gamma_{\mu}\overleftrightarrow{\partial}_{\nu}\psi\bar{\psi}\gamma^{\nu}\overleftrightarrow{\partial}^{\mu}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$\gamma \otimes \gamma\gamma_5$	$\tilde{O}_{23} = \frac{-i}{16m^4}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_{\mu}\overleftrightarrow{\partial}^{\gamma}\overleftrightarrow{\partial}_{\nu}\psi\partial_{\alpha}\bar{\psi}\gamma_{\gamma}\gamma_5\overleftrightarrow{\partial}_{\beta}\psi$	1
	$\tilde{O}_{24} = \frac{-i}{16m^4}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma^{\gamma}\overleftrightarrow{\partial}_{\mu}\psi\partial_{\nu}\bar{\psi}\gamma_{\alpha}\gamma_5\overleftrightarrow{\partial}_{\gamma}\overleftrightarrow{\partial}_{\beta}\psi$	1
	$\tilde{O}_{25-27} = \frac{i}{4m^2}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_{\mu}\overleftrightarrow{\partial}_{\nu}\psi\partial_{\alpha}\bar{\psi}\gamma_{\beta}\gamma_5\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
	$\tilde{O}_{28-30} = \frac{i}{4m^2}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_{\mu}\psi\partial_{\nu}\bar{\psi}\gamma_{\alpha}\gamma_5\overleftrightarrow{\partial}_{\beta}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$\gamma\gamma_5 \otimes \gamma\gamma_5$	$\tilde{O}_{31-36} = \bar{\psi}\gamma^{\mu}\gamma_5\psi\bar{\psi}\gamma_{\mu}\gamma_5\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2, (\overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B)^2, (\overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B)d^2, d^4\}$
	$\tilde{O}_{37-39} = \frac{-1}{4m^2}\bar{\psi}\gamma^{\mu}\gamma_5\overleftrightarrow{\partial}^{\nu}\psi\bar{\psi}\gamma_{\nu}\gamma_5\overleftrightarrow{\partial}_{\mu}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$\gamma\gamma_5 \otimes \sigma$	$\tilde{O}_{40-42} = \frac{-i}{8m^3}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_{\mu}\gamma_5\overleftrightarrow{\partial}_{\nu}\overleftrightarrow{\partial}^{\gamma}\psi\bar{\psi}\sigma_{\alpha\gamma}\overleftrightarrow{\partial}_{\beta}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
	$\tilde{O}_{43-45} = \frac{i}{2m}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_{\mu}\gamma_5\psi\bar{\psi}\sigma_{\nu\alpha}\overleftrightarrow{\partial}_{\beta}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
	$\tilde{O}_{46-48} = \frac{i}{2m}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_{\mu}\gamma_5\overleftrightarrow{\partial}_{\nu}\psi\bar{\psi}\sigma_{\alpha\beta}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$\sigma \otimes \sigma$	$\tilde{O}_{49-51} = \bar{\psi}\sigma_{\mu\nu}\psi\bar{\psi}\sigma^{\mu\nu}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
	$\tilde{O}_{52-54} = \frac{-1}{4m^2}\bar{\psi}\sigma^{\mu\alpha}\overleftrightarrow{\partial}^{\beta}\psi\bar{\psi}\sigma_{\mu\beta}\overleftrightarrow{\partial}_{\alpha}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$

A complete non-minimal set of relativistic contact operators. Here $\overleftrightarrow{d}_X = \overleftrightarrow{\partial}_X/(2m)$ $(X = A, B)$ and $d = \partial/(2m)$.

Operator	Operator
$O_S = 1$	$O_1^{*\prime\prime} = ik^2(\mathbf{k} \times \mathbf{P} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2))$
$O_T = \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$	$O_2^{*\prime\prime} = \mathbf{k} \cdot \mathbf{P}(\mathbf{k} \cdot \boldsymbol{\sigma}_1\,\mathbf{Q} \cdot \boldsymbol{\sigma}_2 - \mathbf{k} \cdot \boldsymbol{\sigma}_2\,\mathbf{Q} \cdot \boldsymbol{\sigma}_1)$
$O_1' = k^2$	$O_3^{*\prime\prime} = k^2(\mathbf{Q} \cdot \boldsymbol{\sigma}_1\,\mathbf{P} \cdot \boldsymbol{\sigma}_2 - \mathbf{Q} \cdot \boldsymbol{\sigma}_2\,\mathbf{P} \cdot \boldsymbol{\sigma}_1)$
$O_2' = Q^2$	$O_4^{*\prime\prime} = k^2P^2$
$O_3' = k^2\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$	$O_5^{*\prime\prime} = k^2P^2\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$
$O_4' = Q^2\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$	$O_6^{*\prime\prime} = (\mathbf{k} \cdot \mathbf{P})^2$
$O_5' = i\frac{\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2}{2} \cdot \mathbf{Q} \times \mathbf{k}$	$O_7^{*\prime\prime} = (\mathbf{k} \cdot \mathbf{P})^2\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$
$O_6' = \boldsymbol{\sigma}_1 \cdot \mathbf{k}\boldsymbol{\sigma}_2 \cdot \mathbf{k}$	$O_8^{*\prime\prime} = P^2\,\mathbf{k} \cdot \boldsymbol{\sigma}_1\,\mathbf{k} \cdot \boldsymbol{\sigma}_2$
$O_7' = \boldsymbol{\sigma}_1 \cdot \mathbf{Q}\boldsymbol{\sigma}_2 \cdot \mathbf{Q}$	$O_9^{*\prime\prime} = \mathbf{k} \cdot \mathbf{P}(\mathbf{k} \cdot \boldsymbol{\sigma}_1\,\mathbf{P} \cdot \boldsymbol{\sigma}_2 + \mathbf{k} \cdot \boldsymbol{\sigma}_2\,\mathbf{P} \cdot \boldsymbol{\sigma}_1)$
$O_1^{*\prime} = i\frac{\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2}{2} \cdot \mathbf{P} \times \mathbf{k}$	$O_{10}^{*\prime\prime} = k^2\,\mathbf{P} \cdot \boldsymbol{\sigma}_1\,\mathbf{P} \cdot \boldsymbol{\sigma}_2$
$O_2^{*\prime} = \boldsymbol{\sigma}_1 \cdot \mathbf{P}\boldsymbol{\sigma}_2 \cdot \mathbf{Q} - \boldsymbol{\sigma}_1 \cdot \mathbf{Q}\boldsymbol{\sigma}_2\mathbf{P}$	$O_{11}^{*\prime\prime} = iQ^2(\mathbf{k} \times \mathbf{P} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2))$
$O_3^{*\prime} = P^2$	$O_{12}^{*\prime\prime} = i\mathbf{Q} \cdot \mathbf{P}(\mathbf{Q} \times \mathbf{k} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2))$
$O_4^{*\prime} = \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2P^2$	$O_{13}^{*\prime\prime} = iP^2(\mathbf{k} \times \mathbf{Q} \cdot (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2))$
$O_5^{*\prime} = \boldsymbol{\sigma}_1 \cdot \mathbf{P}\boldsymbol{\sigma}_2 \cdot \mathbf{P}$	$O_{14}^{*\prime\prime} = i\mathbf{P} \cdot \mathbf{k}(\mathbf{Q} \times \mathbf{P} \cdot (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2))$
$O_1'' = k^4$	$O_{15}^{*\prime\prime} = i\mathbf{Q} \cdot \mathbf{P}(\mathbf{P} \times \mathbf{k} \cdot (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2))$
$O_2'' = Q^4$	$O_{16}^{*\prime\prime} = iP^2(\mathbf{k} \times \mathbf{P} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2))$
$O_3'' = k^2Q^2$	$O_{17}^{*\prime\prime} = Q^2(\mathbf{Q} \cdot \boldsymbol{\sigma}_1\,\mathbf{P} \cdot \boldsymbol{\sigma}_2 - \mathbf{Q} \cdot \boldsymbol{\sigma}_2\,\mathbf{P} \cdot \boldsymbol{\sigma}_1)$
$O_4'' = (\mathbf{k} \times \mathbf{Q})^2$	$O_{18}^{*\prime\prime} = P^2Q^2$
$O_5'' = k^4\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$	$O_{19}^{*\prime\prime} = (\mathbf{P} \cdot \mathbf{Q})^2$
$O_6'' = Q^4\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$	$O_{20}^{*\prime\prime} = P^2Q^2\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$
$O_7'' = k^2Q^2\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$	$O_{21}^{*\prime\prime} = (\mathbf{P} \cdot \mathbf{Q})^2\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$
$O_8'' = (\mathbf{k} \times \mathbf{Q})^2\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$	$O_{22}^{*\prime\prime} = Q^2\,\mathbf{P} \cdot \boldsymbol{\sigma}_1\,\mathbf{P} \cdot \boldsymbol{\sigma}_2$
$O_9'' = \frac{ik^2}{2}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\mathbf{Q} \times \mathbf{k})$	$O_{23}^{*\prime\prime} = \mathbf{P} \cdot \mathbf{Q}(\mathbf{P} \cdot \boldsymbol{\sigma}_1\,\mathbf{Q} \cdot \boldsymbol{\sigma}_2 + \mathbf{P} \cdot \boldsymbol{\sigma}_2\,\mathbf{Q} \cdot \boldsymbol{\sigma}_1)$
$O_{10}'' = \frac{iQ^2}{2}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\mathbf{Q} \times \mathbf{k})$	$O_{24}^{*\prime\prime} = P^2\,\mathbf{Q} \cdot \boldsymbol{\sigma}_1\,\mathbf{Q} \cdot \boldsymbol{\sigma}_2$
$O_{11}'' = k^2\boldsymbol{\sigma}_1 \cdot \mathbf{k}\boldsymbol{\sigma}_2 \cdot \mathbf{k}$	$O_{25}^{*\prime\prime} = P^2(\mathbf{P} \cdot \boldsymbol{\sigma}_1\,\mathbf{Q} \cdot \boldsymbol{\sigma}_2 - \mathbf{P} \cdot \boldsymbol{\sigma}_2\,\mathbf{Q} \cdot \boldsymbol{\sigma}_1)$
$O_{12}'' = Q^2\boldsymbol{\sigma}_1 \cdot \mathbf{k}\boldsymbol{\sigma}_2 \cdot \mathbf{k}$	$O_{26}^{*\prime\prime} = P^4$
$O_{13}'' = k^2\boldsymbol{\sigma}_1 \cdot \mathbf{Q}\boldsymbol{\sigma}_2 \cdot \mathbf{Q}$	$O_{27}^{*\prime\prime} = P^4\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$
$O_{14}'' = Q^2\boldsymbol{\sigma}_1 \cdot \mathbf{Q}\boldsymbol{\sigma}_2 \cdot \mathbf{Q}$	$O_{28}^{*\prime\prime} = P^2\,\mathbf{P} \cdot \boldsymbol{\sigma}_1\,\mathbf{P} \cdot \boldsymbol{\sigma}_2$
$O_{15}'' = \boldsymbol{\sigma}_1 \cdot (\mathbf{k} \times \mathbf{Q})\,\boldsymbol{\sigma}_2 \cdot (\mathbf{k} \times \mathbf{Q})$	
$O_{16}'' = i\mathbf{k} \cdot \mathbf{Q}\,\mathbf{Q} \times \mathbf{P} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)$	
$O_{17}'' = \mathbf{k} \cdot \mathbf{Q}\,(\mathbf{k} \times \mathbf{P}) \cdot (\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2)$	

Complete basis of non-relativistic operators computed between states of two nucleons with $\mathbf{p}_1(\mathbf{p}_1') = \mathbf{P}/2 + \mathbf{Q} \pm \mathbf{k}/2$ and $\mathbf{p}_2(\mathbf{p}_2') = \mathbf{P}/\mathbf{2} - \mathbf{Q} \pm \mathbf{k}/2$.

- NR expansions of relativistic operators can be expressed via the basis of NR operators presented [E. Filandri and L. Girlanda, Phys. Lett. B (2023)]

$\Gamma_A \otimes \Gamma_B$	Operators	Gradient structures
$1 \otimes 1$	$\tilde{O}_{1-6} = \bar{\psi}\psi\bar{\psi}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2, (\overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B)^2, (\overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B)d^2, d^4\}$
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$\gamma \otimes \gamma$	$\tilde{O}_{17-19} = \bar{\psi}\gamma_\mu\psi\bar{\psi}\gamma^\mu\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
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	$\tilde{O}_{25-27} = \frac{i}{4m^2}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_\mu\overleftrightarrow{\partial}_{\nu}\psi\partial_\alpha\bar{\psi}\gamma_\beta\gamma_5\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
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$\gamma\gamma_5 \otimes \gamma\gamma_5$	$\tilde{O}_{31-36} = \bar{\psi}\gamma^\mu\gamma_5\psi\bar{\psi}\gamma_\mu\gamma_5\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2, (\overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B)^2, (\overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B)d^2, d^4\}$
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Operator	Operator
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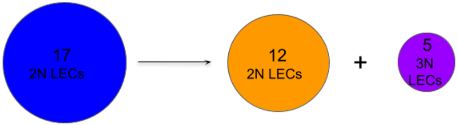
$O_1' = k^2$	$O_3^{*\prime\prime} = k^2(\mathbf{Q}\cdot\boldsymbol{\sigma}_1\,\mathbf{P}\cdot\boldsymbol{\sigma}_2-\mathbf{Q}\cdot\boldsymbol{\sigma}_2\,\mathbf{P}\cdot\boldsymbol{\sigma}_1)$
$O_2' = Q^2$	$O_4^{*\prime\prime} = k^2P^2$
$O_3' = k^2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$	$O_5^{*\prime\prime} = k^2P^2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$
$O_4' = Q^2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$	$O_6^{*\prime\prime} = (\mathbf{k}\cdot\mathbf{P})^2$
$O_5' = i\frac{\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2}{2}\cdot\mathbf{Q}\times\mathbf{k}$	$O_7^{*\prime\prime} = (\mathbf{k}\cdot\mathbf{P})^2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$
$O_6' = \boldsymbol{\sigma}_1\cdot\mathbf{k}\boldsymbol{\sigma}_2\cdot\mathbf{k}$	$O_8^{*\prime\prime} = P^2\,\mathbf{k}\cdot\boldsymbol{\sigma}_1\,\mathbf{k}\cdot\boldsymbol{\sigma}_2$
$O_7' = \boldsymbol{\sigma}_1\cdot\mathbf{Q}\boldsymbol{\sigma}_2\cdot\mathbf{Q}$	$O_9^{*\prime\prime} = \mathbf{k}\cdot\mathbf{P}(\mathbf{k}\cdot\boldsymbol{\sigma}_1\,\mathbf{P}\cdot\boldsymbol{\sigma}_2+\mathbf{k}\cdot\boldsymbol{\sigma}_2\,\mathbf{P}\cdot\boldsymbol{\sigma}_1)$
$O_1^{*\prime} = i\frac{\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2}{2}\cdot\mathbf{P}\times\mathbf{k}$	$O_{10}^{*\prime\prime} = k^2\,\mathbf{P}\cdot\boldsymbol{\sigma}_1\,\mathbf{P}\cdot\boldsymbol{\sigma}_2$
$O_2^{*\prime} = \boldsymbol{\sigma}_1\cdot\mathbf{P}\boldsymbol{\sigma}_2\cdot\mathbf{Q}-\boldsymbol{\sigma}_1\cdot\mathbf{Q}\boldsymbol{\sigma}_2\mathbf{P}$	$O_{11}^{*\prime\prime} = iQ^2(\mathbf{k}\times\mathbf{P}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2))$
$O_3^{*\prime} = P^2$	$O_{12}^{*\prime\prime} = i\mathbf{Q}\cdot\mathbf{P}(\mathbf{Q}\times\mathbf{k}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2))$
$O_4^{*\prime} = \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2P^2$	$O_{13}^{*\prime\prime} = iP^2(\mathbf{k}\times\mathbf{Q}\cdot(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2))$
$O_5^{*\prime} = \boldsymbol{\sigma}_1\cdot\mathbf{P}\boldsymbol{\sigma}_2\cdot\mathbf{P}$	$O_{14}^{*\prime\prime} = i\mathbf{P}\cdot\mathbf{k}(\mathbf{Q}\times\mathbf{P}\cdot(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2))$
$O_1'' = k^4$	$O_{15}^{*\prime\prime} = i\mathbf{Q}\cdot\mathbf{P}(\mathbf{P}\times\mathbf{k}\cdot(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2))$
$O_2'' = Q^4$	$O_{16}^{*\prime\prime} = iP^2(\mathbf{k}\times\mathbf{P}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2))$
$O_3'' = k^2Q^2$	$O_{17}^{*\prime\prime} = Q^2(\mathbf{Q}\cdot\boldsymbol{\sigma}_1\,\mathbf{P}\cdot\boldsymbol{\sigma}_2-\mathbf{Q}\cdot\boldsymbol{\sigma}_2\,\mathbf{P}\cdot\boldsymbol{\sigma}_1)$
$O_4'' = (\mathbf{k}\times\mathbf{Q})^2$	$O_{18}^{*\prime\prime} = P^2Q^2$
$O_5'' = k^4\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$	$O_{19}^{*\prime\prime} = (\mathbf{P}\cdot\mathbf{Q})^2$
$O_6'' = Q^4\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$	$O_{20}^{*\prime\prime} = P^2Q^2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$
$O_7'' = k^2Q^2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$	$O_{21}^{*\prime\prime} = (\mathbf{P}\cdot\mathbf{Q})^2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$
$O_8'' = (\mathbf{k}\times\mathbf{Q})^2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$	$O_{22}^{*\prime\prime} = Q^2\,\mathbf{P}\cdot\boldsymbol{\sigma}_1\,\mathbf{P}\cdot\boldsymbol{\sigma}_2$
$O_9'' = \frac{ik^2}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\mathbf{Q}\times\mathbf{k})$	$O_{23}^{*\prime\prime} = \mathbf{P}\cdot\mathbf{Q}(\mathbf{P}\cdot\boldsymbol{\sigma}_1\,\mathbf{Q}\cdot\boldsymbol{\sigma}_2+\mathbf{P}\cdot\boldsymbol{\sigma}_2\,\mathbf{Q}\cdot\boldsymbol{\sigma}_1)$
$O_{10}'' = \frac{iQ^2}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\mathbf{Q}\times\mathbf{k})$	$O_{24}^{*\prime\prime} = P^2\,\mathbf{Q}\cdot\boldsymbol{\sigma}_1\,\mathbf{Q}\cdot\boldsymbol{\sigma}_2$
$O_{11}'' = k^2\boldsymbol{\sigma}_1\cdot\mathbf{k}\boldsymbol{\sigma}_2\cdot\mathbf{k}$	$O_{25}^{*\prime\prime} = P^2(\mathbf{P}\cdot\boldsymbol{\sigma}_1\,\mathbf{Q}\cdot\boldsymbol{\sigma}_2-\mathbf{P}\cdot\boldsymbol{\sigma}_2\,\mathbf{Q}\cdot\boldsymbol{\sigma}_1)$
$O_{12}'' = Q^2\boldsymbol{\sigma}_1\cdot\mathbf{k}\boldsymbol{\sigma}_2\cdot\mathbf{k}$	$O_{26}^{*\prime\prime} = P^4$
$O_{13}'' = k^2\boldsymbol{\sigma}_1\cdot\mathbf{Q}\boldsymbol{\sigma}_2\cdot\mathbf{Q}$	$O_{27}^{*\prime\prime} = P^4\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$
$O_{14}'' = Q^2\boldsymbol{\sigma}_1\cdot\mathbf{Q}\boldsymbol{\sigma}_2\cdot\mathbf{Q}$	$O_{28}^{*\prime\prime} = P^2\,\mathbf{P}\cdot\boldsymbol{\sigma}_1\,\mathbf{P}\cdot\boldsymbol{\sigma}_2$
$O_{15}'' = \boldsymbol{\sigma}_1\cdot(\mathbf{k}\times\mathbf{Q})\,\boldsymbol{\sigma}_2\cdot(\mathbf{k}\times\mathbf{Q})$	
$O_{16}'' = i\mathbf{k}\cdot\mathbf{Q}\,\mathbf{Q}\times\mathbf{P}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)$	
$O_{17}'' = \mathbf{k}\cdot\mathbf{Q}\,(\mathbf{k}\times\mathbf{P})\cdot(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)$	

Complete basis of non-relativistic operators computed between states of two nucleons with $\mathbf{p}_1(\mathbf{p}_1') = \mathbf{P}/2 + \mathbf{Q} \pm \mathbf{k}/2$ and $\mathbf{p}_2(\mathbf{p}_2') = \mathbf{P}/2 - \mathbf{Q} \pm \mathbf{k}/2$.

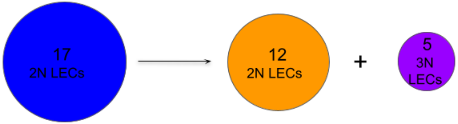
- NR expansions of relativistic operators can be expressed via the basis of NR operators presented
  - 26 independent combinations
  - Existence of 2 free LECs which parameterize an interaction that depends on P
  - All the remaining P-dependent interactions are uniquely determined as relativistic corrections

[E. Filandri and L. Girlanda, Phys. Lett. B (2023)]

Redundancy at N3LO



Redundancy at N3LO



$$U = e^{\alpha_i T_i}$$

$$T_1 = \int d^3x N^\dagger \overleftrightarrow{\nabla}^i N \nabla^i \left(N^\dagger N\right)$$

$$T_2 = \int d^3\mathbf{x} N^\dagger \overleftrightarrow{\nabla}^i \sigma^j N \nabla^i \left(N^\dagger \sigma^j N\right)$$

$$T_3 = \int d^3x \left[N^\dagger \overleftrightarrow{\nabla}^i \sigma^i N \nabla^j \left(N^\dagger \sigma^j N\right) + N^\dagger \overleftrightarrow{\nabla}^i \sigma^j N \nabla^j \left(N^\dagger \sigma^i N\right)\right]$$

[P. Reinert et al., Eur. Phys. J. A (2018)]

$$T_4 = i\epsilon^{ijk} \int d^3x N^\dagger \overleftrightarrow{\nabla}^i N N^\dagger \overleftrightarrow{\nabla}^j \sigma^k N$$

$$T_5 = \int d^3x \left[N^\dagger \overleftrightarrow{\nabla}^i \sigma^i N \nabla^j \left(N^\dagger \sigma^j N\right) - N^\dagger \overleftrightarrow{\nabla}^i \sigma^j N \nabla^j \left(N^\dagger \sigma^i N\right)\right]$$

[L. Girlanda et al., Phys. Rev. C (2020)]

$$U^\dagger H_0 U = H_0 + \alpha_i [H_0, T_i] + \cdots \equiv H_0 + \alpha_i \delta_i H_0 + \cdots \Rightarrow$$

$$U^\dagger H_{C_S/C_T}\Big($$  $$\Big)U = H_{C_S/C_T} + \alpha_i [H_{C_S/C_T}, T_i] + \cdots \equiv H_{C_S/C_T} + \alpha_i \delta_i H_{C_S/C_T} + \cdots \Rightarrow$$

α_i are related with the D_i and E_i LECs

$$\delta E_1 = \alpha_1 (C_S + C_T) + \alpha_2 (C_S - 2C_T)$$
$$\delta E_2 = 3\alpha_2 C_T + 2\alpha_3 C_T - 8\alpha_4 C_T + 2\alpha_5 C_T$$
$$\delta E_3 = 2\alpha_1 C_T + \alpha_2 (2C_S - C_T) + \frac{2}{3}\alpha_3 (2C_S - C_T) + 8\alpha_4 C_T - 2\alpha_5 C_T$$
$$\delta E_4 = \frac{2}{3}\alpha_1 C_T + \frac{1}{3}\alpha_2 (2C_S - 7C_T) - \frac{2}{3}\alpha_3 C_T + \frac{8}{3}\alpha_4 C_T - \frac{2}{3}\alpha_5 C_T$$
$$\delta E_5 = 2\alpha_1 C_T + 2\alpha_2 (C_S - 2C_T) + \frac{2}{3}\alpha_3 (2C_S - C_T) + 8\alpha_4 C_T - 2\alpha_5 C_T$$
$$\delta E_6 = \frac{2}{3}\alpha_1 C_T + \frac{2}{3}\alpha_2 (C_S - 2C_T) - \frac{2}{3}\alpha_3 C_T + \frac{8}{3}\alpha_4 C_T - \frac{2}{3}\alpha_5 C_T,$$
$$\delta E_7 = 24\alpha_4 C_T$$
$$\delta E_8 = \frac{1}{3}\delta E_7$$
$$\delta E_9 = 3\alpha_1 C_T + 3\alpha_2 (C_S - 2C_T) + 2\alpha_3 (C_S - 2C_T) - \alpha_4 (C_S - 11C_T) + 2\alpha_5 (C_S - 2C_T)$$
$$\delta E_{10} = \alpha_1 C_T + \alpha_2 (C_S - 2C_T) - \frac{1}{3}\alpha_4 (3C_S - 15C_T)$$
$$\delta E_{11} = 3\alpha_1 C_T + 3\alpha_2 (C_S - 2C_T) + 2\alpha_3 (C_S - 2C_T) + \alpha_4 (C_S - 11C_T) - 2\alpha_5 (C_S - 2C_T)$$
$$\delta E_{12} = \alpha_1 C_T + \alpha_2 (C_S - 2C_T) + \frac{1}{3}\alpha_4 (3C_S - 15C_T)$$
$$\delta E_{13} = -16\alpha_4 C_T + 4\alpha_5 C_T$$

$$\alpha_1 = \frac{m}{16}(16D_1 + D_2 + 4D_3)$$
$$\alpha_2 = \frac{m}{16}(16D_5 + D_6 + 4D_7)$$
$$\alpha_3 = \frac{m}{32}(D_{14} + 16D_{11} + 4D_{12} + 4D_{13})$$
$$\alpha_4 = \frac{m}{2}D_{16}$$
$$\alpha_5 = \frac{m}{16}(8D_{17} - D_{14} - 16D_{11} - 4D_{12} - 4D_{13})$$

Therefore:

- It is possible to remove 5 D_i LECs from the NN contact potential as long as we consider the shifts δE_i as an effect at N3LO
- The five LECs parametrizing the N3LO NN off-shell interaction can be fitted to observables of the 3N system and interpreted as a 3N interaction

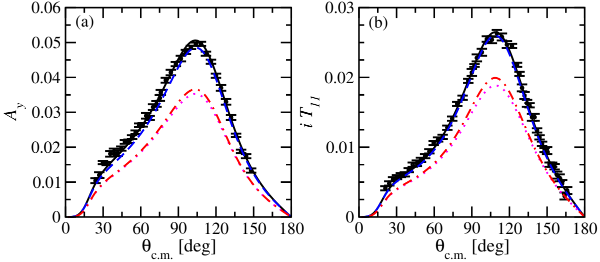
$$\Downarrow$$

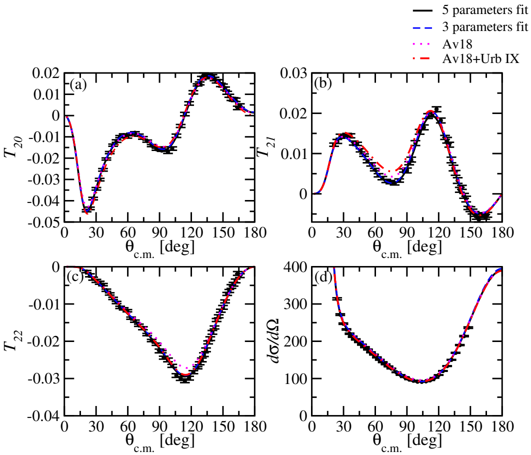
Using the relations between the δE_i and α_i, we tried to fit the α parameters to the p-d scattering observables

As a first step, we used and hybrid model with a phenomenological two body potential (the Av18), the E_0 term and the δE_i N3LO terms

Hybrid Fit on p-d observables

$\Lambda = 500$ MeV
$\chi^2/d.o.f. = 1.7$
$B(^3H) = 8.482$ MeV
$a_2 = 0.651$ fm





[L. Girlanda, E. Filandri, A. Kievsky, L. E. Marcucci, M. Viviani, Phys. Rev. C (2023)]

Results of the 5-parameters [3-parameters fits, setting $\alpha_4 = \alpha_5 = 0$]. Here $e_0 = E_0 F_\pi^4 \Lambda$, $\tilde{\alpha}_i = \alpha_i F_\pi^4 \Lambda^3$.

Fit	
χ^2/d.o.f.	1.7 [2.3]
e_0	0.685 [-1.570]
$\tilde{\alpha}_1 C_S$	1.410 [-3.611]
$\tilde{\alpha}_2 C_S$	0.211 [-0.483]
$\tilde{\alpha}_3 C_S$	-0.370 [0.209]
$\tilde{\alpha}_4 C_S$	1.735 [0]
$\tilde{\alpha}_5 C_S$	2.266 [0]
$^2a_{nd}$ [fm]	0.648 [0.647]
$^4a_{nd}$ [fm]	6.31 [6.32]

Estimation of some N3LO LECs combinations from fit. The D_i are in units of 10^4 GeV^{-4} and $\tilde{D}_{13} = 16D_1 + D_2 + 4D_3$, $\tilde{D}_{14} = 16D_5 + D_6 + 4D_7$, and $\tilde{D}_{15} = D_{14} + 16D_{11} + 4D_{12} + 4D_{13}$. In the last column, the values obtained in [R. Machleidt and D. R. Entem, Phys. Rept. (2011)] and used for the Idaho N3LO 2N potential with $\Lambda = 500$ MeV are shown.

LECs	Fit	Ref.[Machleidt 2011]
$\tilde{D}_{13}$	-3.96 [10.15]	6.41
$\tilde{D}_{14}$	-0.59 [1.36]	4.05
D_{15}	2.08 [-1.17]	-3.04
D_{16}	-0.61 [0]	-
D_{17}	-0.54 [-0.15]	-

Conclusions

- There are 5 free LECs in the 3N force at N3LO to improve the description of scattering data
- Preliminary investigations show that the N-d Ay problem could be solved in this way

.. and Outlook

- Use of unitary transformations in the N3LO Chiral potential
  - Add the δE_i N3LO terms
  - Remove the redundant D_i terms
  - Unitarity transformation of the term:

$$H_{\pi N} = \frac{g_A}{2F_\pi} \int d\mathbf{x} N^\dagger \boldsymbol{\nabla}\pi^a \cdot \boldsymbol{\sigma}\tau^a N \Rightarrow V_{3NF} = -\frac{g_A}{F_\pi} \sum_{i \neq j \neq k} \frac{\mathbf{k}_k \cdot \boldsymbol{\sigma}_k \tau_i \cdot \tau_k}{k_k^2 + m_\pi^2} \{\alpha_1 \mathbf{k}_j \cdot \mathbf{k}_k \mathbf{k}_k \cdot \boldsymbol{\sigma}_i$$

$$+ \alpha_2 \left[\mathbf{k}_j \cdot \mathbf{k}_k \mathbf{k}_k \cdot \boldsymbol{\sigma}_j + 2i\mathbf{k}_j \cdot \left(\mathbf{Q}_i - \mathbf{Q}_j\right) \mathbf{k}_k \cdot \boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j\right]$$

$$+ (\alpha_3 + \alpha_5) \left[k_k^2 \mathbf{k}_j \cdot \boldsymbol{\sigma}_j - 2i\mathbf{k}_j \cdot \boldsymbol{\sigma}_j \mathbf{k}_k \cdot \mathbf{Q}_i \times \boldsymbol{\sigma}_i + 2i\mathbf{Q}_j \cdot \boldsymbol{\sigma}_j \mathbf{k}_k \cdot \mathbf{k}_j \times \boldsymbol{\sigma}_i\right]$$

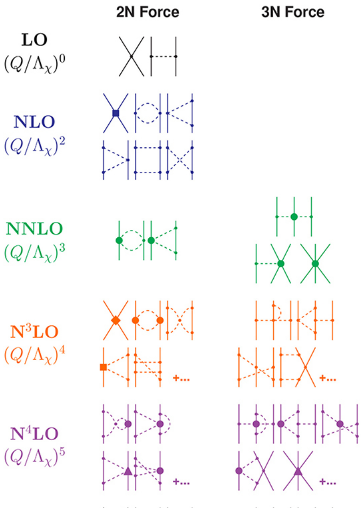
$$+ (\alpha_3 - \alpha_5) \left[\mathbf{k}_j \cdot \mathbf{k}_k \mathbf{k}_k \cdot \boldsymbol{\sigma}_j + 2i\mathbf{k}_j \cdot \boldsymbol{\sigma}_j \mathbf{k}_k \cdot \mathbf{Q}_j \times \boldsymbol{\sigma}_i - 2i\mathbf{Q}_i \cdot \boldsymbol{\sigma}_j \mathbf{k}_k \cdot \mathbf{k}_j \times \boldsymbol{\sigma}_i\right]$$

$$-2\alpha_4 \left[\mathbf{k}_k \cdot \boldsymbol{\sigma}_i \mathbf{k}_k \cdot \mathbf{Q}_j \times \boldsymbol{\sigma}_j - 2i\mathbf{k}_k \cdot \mathbf{Q}_i \left(\mathbf{k}_k \cdot \mathbf{Q}_i \mathbf{Q}_j \cdot \boldsymbol{\sigma}_i - \mathbf{k}_k \cdot \mathbf{Q}_j \mathbf{Q}_i \cdot \sigma_i\right)\right]\}$$

- Calculation of scattering observables also exploring the energy dependence and quantitative error estimation

Backup slides

Chiral EFT potentials

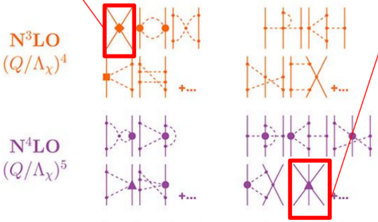


Chiral EFT potentials

$$V^{(4)} = D_1 k^4 + D_2 Q^4 + D_3 k^2 Q^2 + D_4 (\mathbf{k}\times\mathbf{Q})^2 + \left(D_5 k^4 + D_6 Q^4 + D_7 k^2 Q^2 + D_8 (\mathbf{k}\times\mathbf{Q})^2\right)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$$
$$+ \frac{i}{2}\left(D_9 k^2 + D_{10} Q^2\right)(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\mathbf{Q}\times\mathbf{k}) + \left(D_{11}k^2 + D_{12}Q^2\right)(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k}) + \left(D_{13}k^2 + D_{14}Q^2\right)(\boldsymbol{\sigma}_1\cdot\mathbf{Q})(\boldsymbol{\sigma}_2\cdot\mathbf{Q})$$
$$+ D_{15}\boldsymbol{\sigma}_1\cdot(\mathbf{k}\times\mathbf{Q})\boldsymbol{\sigma}_2\cdot(\mathbf{k}\times\mathbf{Q}) + iD_{16}\mathbf{k}\cdot\mathbf{Q}\mathbf{Q}\times\mathbf{P}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2) + D_{17}\mathbf{k}\cdot\mathbf{Q}(\mathbf{k}\times\mathbf{P})\cdot(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)$$

$$V = \sum_{i\neq j\neq k}\Big[-E_1\mathbf{k}_i^2 - E_2\mathbf{k}_i^2\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j - E_3\mathbf{k}_i^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j - E_4\mathbf{k}_i^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j - E_5\left(3\mathbf{k}_i\cdot\boldsymbol{\sigma}_i\mathbf{k}_i\cdot\boldsymbol{\sigma}_j - \mathbf{k}_i^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\right)$$
$$- E_6\left(3\mathbf{k}_i\cdot\boldsymbol{\sigma}_i\mathbf{k}_i\cdot\boldsymbol{\sigma}_j - \mathbf{k}_i^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\right)\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j - \frac{i}{2}E_7\mathbf{k}_i\times(\mathbf{Q}_i-\mathbf{Q}_j)\cdot(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)$$
$$- \frac{i}{2}E_8\mathbf{k}_i\times(\mathbf{Q}_i-\mathbf{Q}_j)\cdot(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k - E_9\mathbf{k}_i\cdot\boldsymbol{\sigma}_i\mathbf{k}_j\cdot\boldsymbol{\sigma}_j - E_{10}\mathbf{k}_i\cdot\boldsymbol{\sigma}_i\mathbf{k}_j\cdot\boldsymbol{\sigma}_j\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$$
$$- E_{11}\mathbf{k}_i\cdot\boldsymbol{\sigma}_j\mathbf{k}_j\cdot\boldsymbol{\sigma}_i - E_{12}\mathbf{k}_i\cdot\boldsymbol{\sigma}_j\mathbf{k}_j\cdot\boldsymbol{\sigma}_i\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j - E_{13}\mathbf{k}_i\cdot\boldsymbol{\sigma}_j\mathbf{k}_j\cdot\boldsymbol{\sigma}_i\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_k\Big]$$

with $\mathbf{k}_i = \mathbf{p}_i' - \mathbf{p}_i$, $\mathbf{Q}_i = \frac{\mathbf{p}_i + \mathbf{p}_i'}{2}$



Choosing [Reinert, et al., Eur. Phys. J. A 54, 86 (2018)]

$$\alpha_1 = m(D_1+D_2+4D_3),\ \alpha_2 = m(16D_5+D_6+4D_7),\ \alpha_3 = m(8D_{11}+2D_{12}+2D_{13}+D_{14}/2),\ \alpha_4 = -\frac{m}{2}D_{16},\ \alpha_5 = -\frac{m}{2}D_{17}$$

it is possible to transform the N3LO two body contact potential

$$\begin{aligned} V^{(4)} =& D_1 k^4 + D_2 Q^4 + D_3 k^2 Q^2 + D_4(\mathbf{k}\times\mathbf{Q})^2 + \left(D_5 k^4 + D_6 Q^4 + D_7 k^2 Q^2 + D_8(\mathbf{k}\times\mathbf{Q})^2\right)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) \\ &+ \frac{i}{2}\left(D_9 k^2 + D_{10} Q^2\right)(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\mathbf{Q}\times\mathbf{k}) + \left(D_{11}k^2 + D_{12}Q^2\right)(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k}) + \left(D_{13}k^2 + D_{14}Q^2\right)(\boldsymbol{\sigma}_1\cdot\mathbf{Q})(\boldsymbol{\sigma}_2\cdot\mathbf{Q}) \\ &+ D_{15}\boldsymbol{\sigma}_1\cdot(\mathbf{k}\times\mathbf{Q})\boldsymbol{\sigma}_2\cdot(\mathbf{k}\times\mathbf{Q}) + iD_{16}\mathbf{k}\cdot\mathbf{Q}\mathbf{Q}\times\mathbf{P}\cdot(\sigma_1-\sigma_2) + D_{17}\mathbf{k}\cdot\mathbf{Q}(\mathbf{k}\times\mathbf{P})\cdot(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2) \end{aligned}$$

$$\Downarrow$$

$$\begin{aligned} V^{(4)} =& D'_1\left(k^4 - 4(\mathbf{Q}\cdot\mathbf{k})^2\right) + D'_2\left(Q^4 - \frac{1}{4}(\mathbf{Q}\cdot\mathbf{k})^2\right) + D'_3(\mathbf{k}\times\mathbf{Q})^2 \\ &+ \left(D'_4\left(k^4 - 4(\mathbf{Q}\cdot\mathbf{k})^2\right) + D'_5\left(Q^4 - \frac{1}{4}(\mathbf{Q}\cdot\mathbf{k})^2\right) + D'_6(\mathbf{k}\times\mathbf{Q})^2\right)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + \frac{i}{2}\left(D'_7k^2 + D'_8Q^2\right)(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\mathbf{Q}\times\mathbf{k}) \\ &+ D'_9\left(-\frac{1}{4}k^2\boldsymbol{\sigma}_1\cdot\mathbf{k}\boldsymbol{\sigma}_2\cdot\mathbf{k} + 4Q^2\boldsymbol{\sigma}_1\cdot\mathbf{Q}\boldsymbol{\sigma}_2\cdot\mathbf{Q}\right) + D'_{10}Q^2\left(\boldsymbol{\sigma}_1\cdot\mathbf{k}\boldsymbol{\sigma}_2\cdot\mathbf{k} - 4\boldsymbol{\sigma}_1\cdot\mathbf{Q}\boldsymbol{\sigma}_2\cdot\mathbf{Q}\right) + D'_{11}\left(k^2 - 4Q^2\right)\boldsymbol{\sigma}_1\cdot\mathbf{Q}\boldsymbol{\sigma}_2\cdot\mathbf{Q} \\ &+ D'_{12}\boldsymbol{\sigma}_1\cdot(\mathbf{k}\times\mathbf{Q})\boldsymbol{\sigma}_2\cdot(\mathbf{k}\times\mathbf{Q}) \end{aligned}$$

$$\text{with}\ \ k = p' - p,\quad Q = \frac{p'+p}{2},\quad P = p_1 + p_2$$

As long as we promote 5 terms of the N4LO 3B potential to N3LO

Bound and scattering wave functions

The ^3H wave function is written as an expansion over spin-isospin-Hyperspherical Harmonics (HH) states times hyperradial functions, which are themselves expanded on a basis of Laguerre polynomials,

$$\Psi = \sum_{\mu} c_{\mu} \phi_{\mu} \quad (1)$$

where μ denotes collectively the quantum numbers specifying the combination ϕ_{μ} of spin-isospin-HH states.

The Rayleigh-Ritz variational principle,

$$\delta \langle \Psi | H - E | \Psi \rangle = 0$$

is used to determine the expansion coefficients c_{μ} and bound state energy E

To describe $\mathrm{N} - \mathrm{d}$ scattering states below the deuteron breakup threshold the w.f. is taken as

$$\Psi = \Psi_C + \Omega^R + \sum_{\mu} \mathcal{R}_{\mu} \Omega^I_{\mu}$$

Ψ_C describes configurations in which all the particles of the system are close to each other and is decomposed as Eq (1), $\Omega^{\lambda = I,R}$ are functions describing the asymptotic region. $\mathcal{R}_{\mu}$ are the $\mathcal{R}$-matrix elements.

The Ψ_C coefficients c_{μ} and $\mathcal{R}_{\mu}$ are determined by using the Kohn variational principle which guarantees that the $\mathcal{R}$-matrix elements, considered as functionals of the w.f., are stationary with respect to variations of all the trial parameters