Nuclear Structure Corrections to the Hyperfine Splitting in Muonic Deuterium

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JOHANNES GUTENBERG UNIVERSITÄT MAINZ JG|U

Cluster of Excellence PRISMA+ Precision Physics, Fundamental Interactions and Structure of Matter

In collaboration with Bijaya Acharya, Sonia Bacca, Jose Bonilla, Chen Ji, Simone Li Muli, Lucas Platter

Why Muonic Atoms?

- Muon 200 times heavier than electron—smaller Bohr radius

- Precision probe of nuclear electroweak structure

- Possible window into new physics beyond the Standard Model at the precision frontier

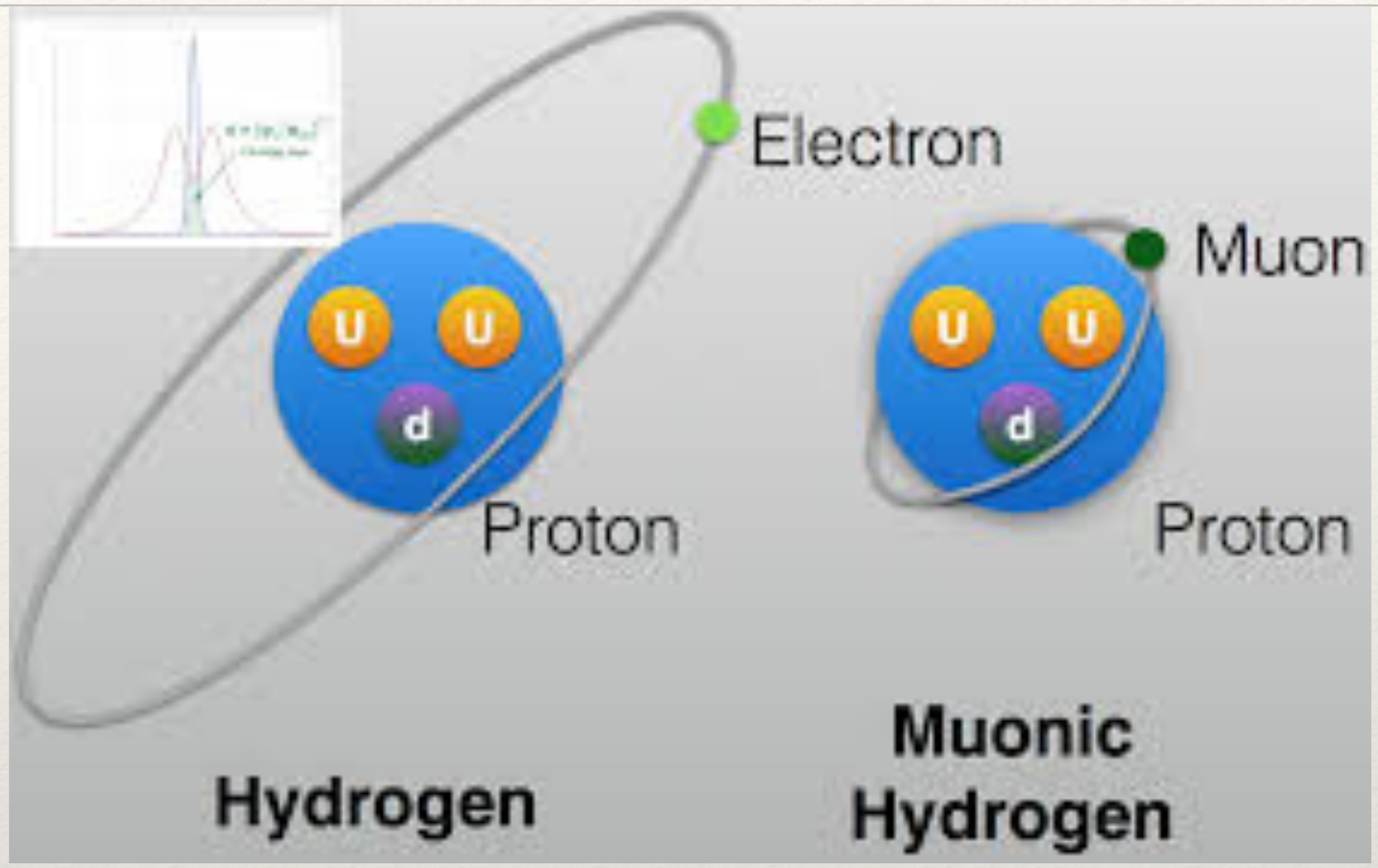


Fig: H. Gao

from PUZZLE
ASTI
to PRECISION

Experimental Effort—Hyperfine Splitting

- μH—CREMA, FAMU, J-PARC
- μD—CREMA (Pohl et al. Science 353)
- μ^3He^+—CREMA, J-PARC

PSI, Switzerland

Japan

RIKEN-RAL, UK

Hyperfine Splitting in Muonic Deuterium

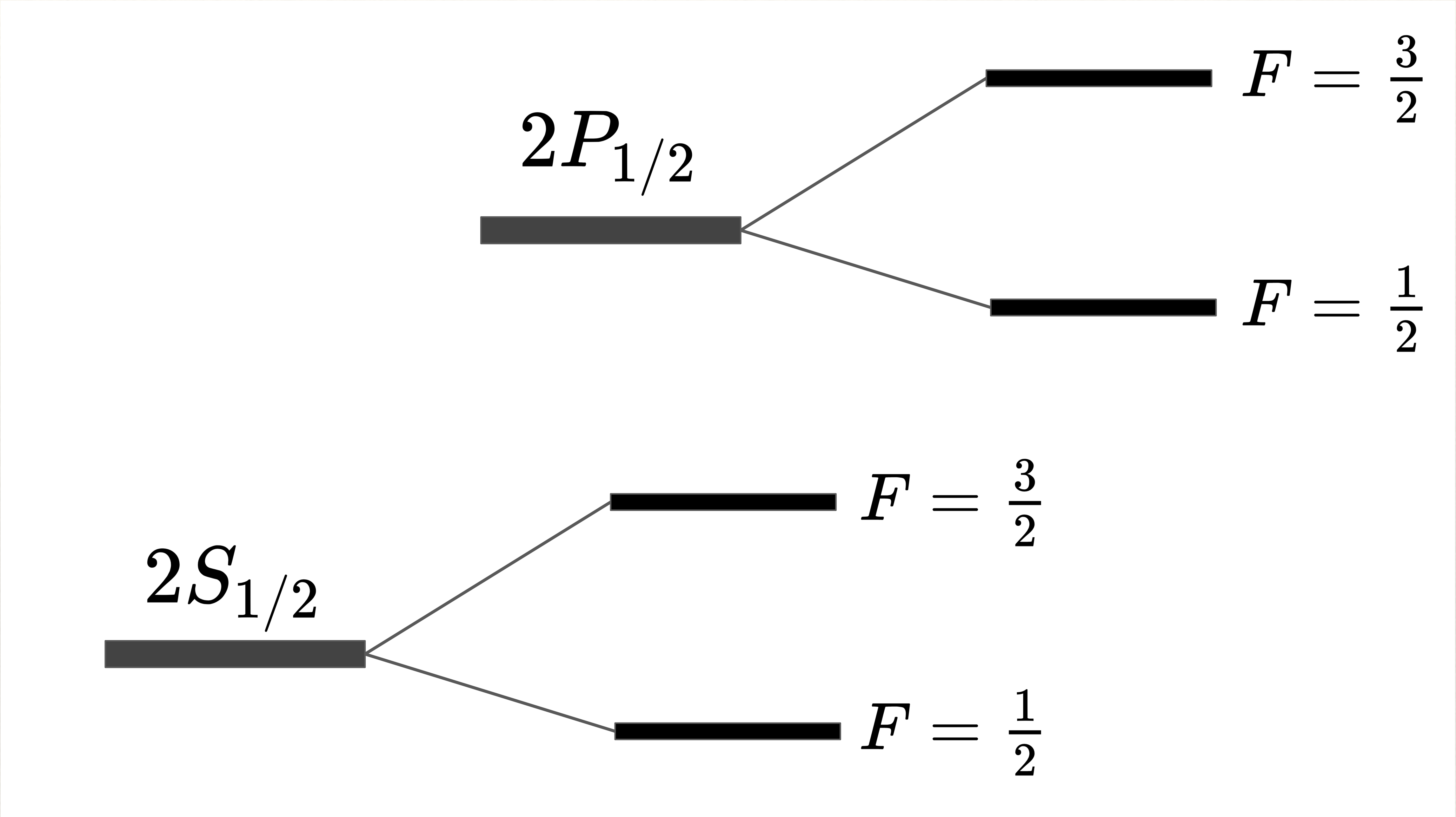
$$\Delta E_{\mathrm{HFS}} = E_F \left(1 + \delta_{\mathrm{QED}} + \delta_{\mathrm{FS}} + \delta_{\mathrm{pol}}\right)$$

$$E_F = \frac{4\pi\alpha\mu_N}{3m_\mu} |\phi_n(0)|^2 \frac{\sigma \cdot \mathbf{J}}{J} = 6.1359 \text{ meV}$$

$$\Delta E_{\mathrm{FS}} = -2m_r Z\alpha E_F R_z$$

$$R_Z = \int d^3r d^3r' r \rho_E(\mathbf{r} - \mathbf{r}')\rho_M(\mathbf{r}')$$

$$\delta_{\mathrm{pol}} = \delta^A_{\mathrm{pol}} + \delta^N_{\mathrm{pol}}$$



$$\Delta E_{\mathrm{HFS}}(2S)_{\mathrm{exp}} = 6.2747(70)_{\mathrm{stat}}(20)_{\mathrm{syst}} \text{ meV}$$

$$\Delta E_{\mathrm{HFS}}(2S)_{\mathrm{theory}} = 6.2791(50) \text{ meV}$$

Krauth et al., Annals 366; Pohl et al. Science 353

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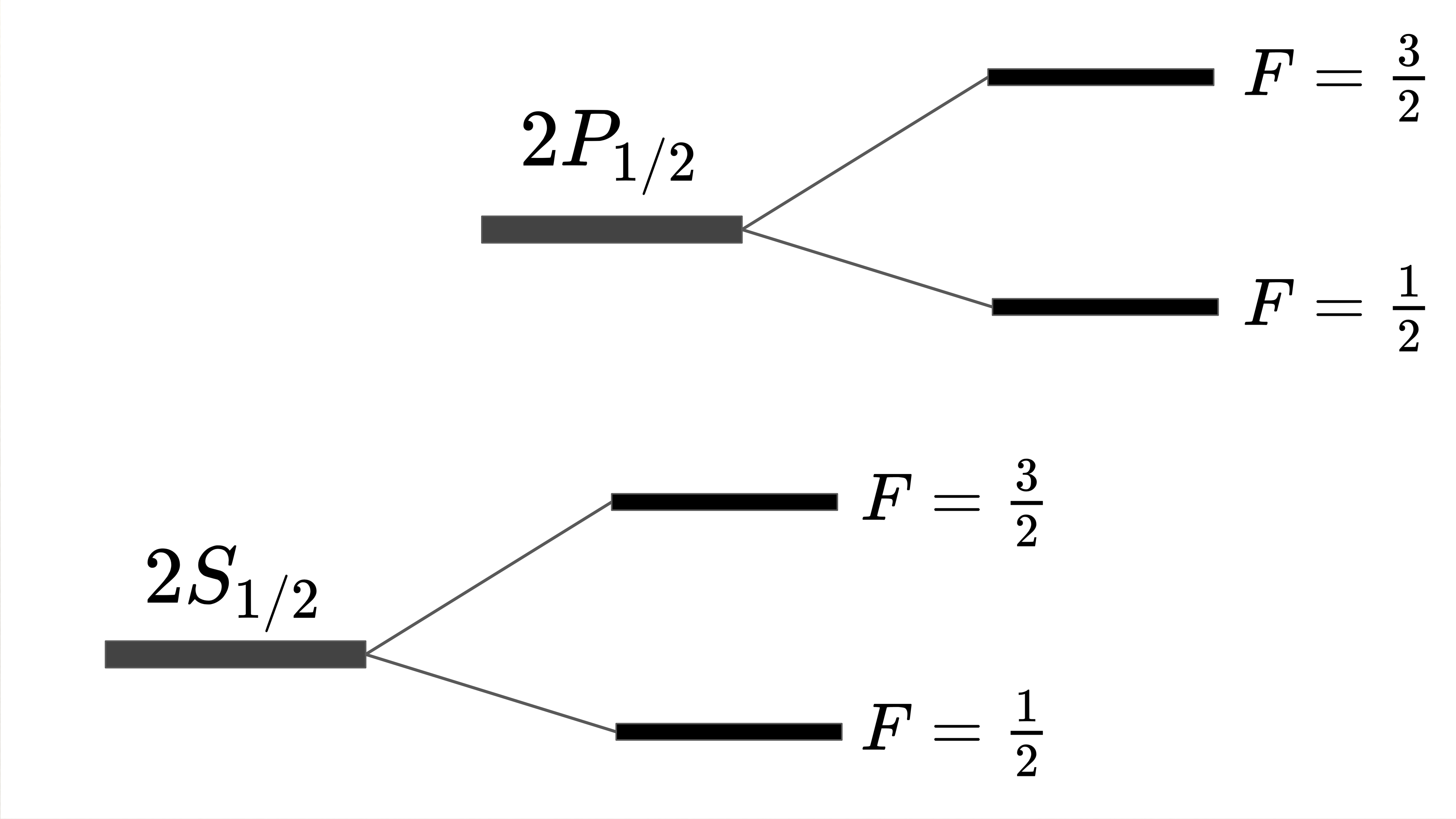

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Uncertainties in the Polarizability Contribution

- (2016) Recommended value

$$\delta^A_{\rm pol} = 0.2121(42)\ {\rm meV}$$

Khriplovich and Milstein, JETP 98; Krauth et al., Annals 366

- Reassessment in 2018

TABLE II: Nuclear structure corrections for hyperfine splitting of the $1S$ and $2S$ states of muonic deuterium, in meV. Numerical results are obtained with the AV18 potential [12].

Correction	$1S$	$2S$	Source
$\delta E_{\rm pol1}$	-1.1007	-0.1376	Eq. (22)
$\delta E_{\rm pol2}$	-0.0823	-0.0103	Eq. (25)
$\delta E_{\rm pol3}$	0.1513	0.0189	Eq. (26)
$\delta E_{\rm pol4}$	-0.1979	-0.0283	Eq. (30)
$\delta E_{\rm pol5}$	-0.0327	-0.0041	Eq. (32)
$\delta E_{\rm pol}$	$-1.2623(631)$	$-0.1578(79)$	Eq. (33)
$\delta E_{\rm 1nucl}$	$-0.9450(224)$	$-0.1181(28)$	Eq. (15)
$\delta E_{\rm Low}$	2.566	0.3208	Eq. (14)
$\delta^{(1)} E_{\rm nucl}$	0.3587(670)	0.0448(84)	Eq. (12)
$\delta^{(2)} E_{\rm nucl}$	$-0.0547(137)$	$-0.0065(16)$	Eq. (77)
$\delta E_{\rm nucl,theo}$	0.304(68)	0.0383(86)	Eq. (8)
$\delta E_{\rm nucl,exp}$		0.0966(73)	Eq. (7)
difference		0.0583(113)	

Kalinowski et al. PRA 98

Polarizability from Two Photon Exchange

- HFS contained in muon spin-dependent (antisymmetric) terms

$$\Delta E_{2\gamma} = i(4\pi\alpha)^2 \, |\phi_n(0)| \int \frac{d^4 q}{(2\pi)^4} T^{\mu\nu}(q) L^{\rho\tau}(q) D_{\mu\rho}(q) D_{\nu\tau}(q)$$

$$T^{\mu\nu}(q) = \sum_{N \neq N_0} \langle N_0 | \frac{J^\mu(-\mathbf{q})|N\rangle\langle N|J^\nu(\mathbf{q})}{q_0 - \omega_N + i\epsilon} - \frac{J^\nu(\mathbf{q})|N\rangle\langle N|J^\mu(-\mathbf{q})}{q_0 + \omega_N - i\epsilon} | N_0 \rangle$$

$$L^{\mu\nu} = (ie)^2 \, \bar{u}_r(k) \gamma^\mu \frac{\not{q} + \not{k} + m_\mu}{(q+k)^2 - m_\mu^2 + i\epsilon} \gamma^\nu u_s(k)$$

Bernabéu and Jarlskog NPB 75; Rosenfelder, NPA 393; Friar and Payne, PLB 618, PRC 72

Effective Field Theory: From the Top Down

Λ

???

SMEFT — Standard Model + higher dimension operators

100 GeV

LEFT — $\mathrm{SU}(3)_c \times \mathrm{U}(1)_{\mathrm{EM}}$

RG

1 GeV

Lattice QCD — H. Wittig's talk

Chiral EFT

100 MeV

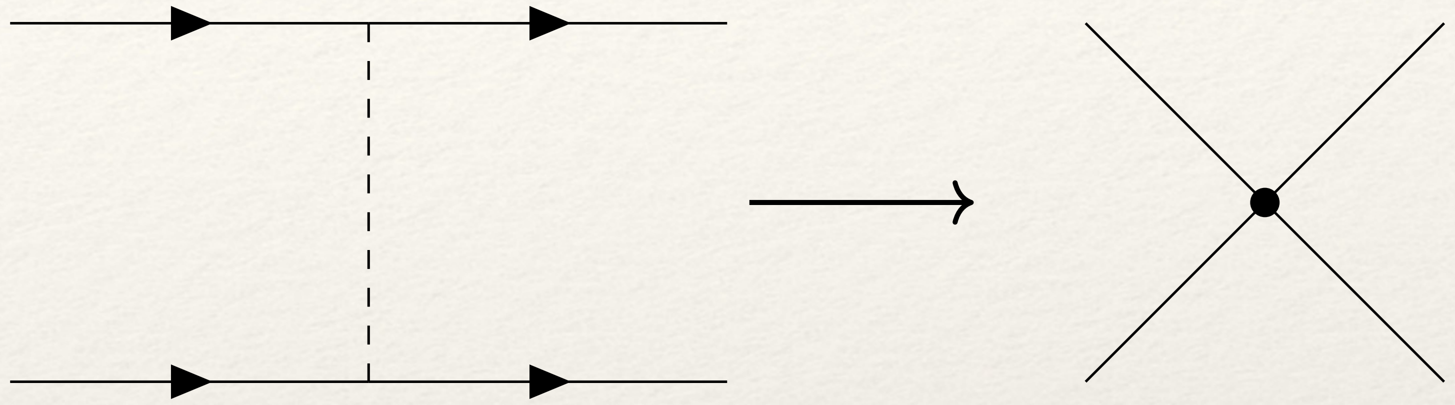
Hadrons

Pionless EFT

Few/many-body methods

S. Li Muli's poster

W. Dekens' talk



$$\frac{g^2}{p^2 - M^2} \qquad \frac{g^2}{M^2}$$

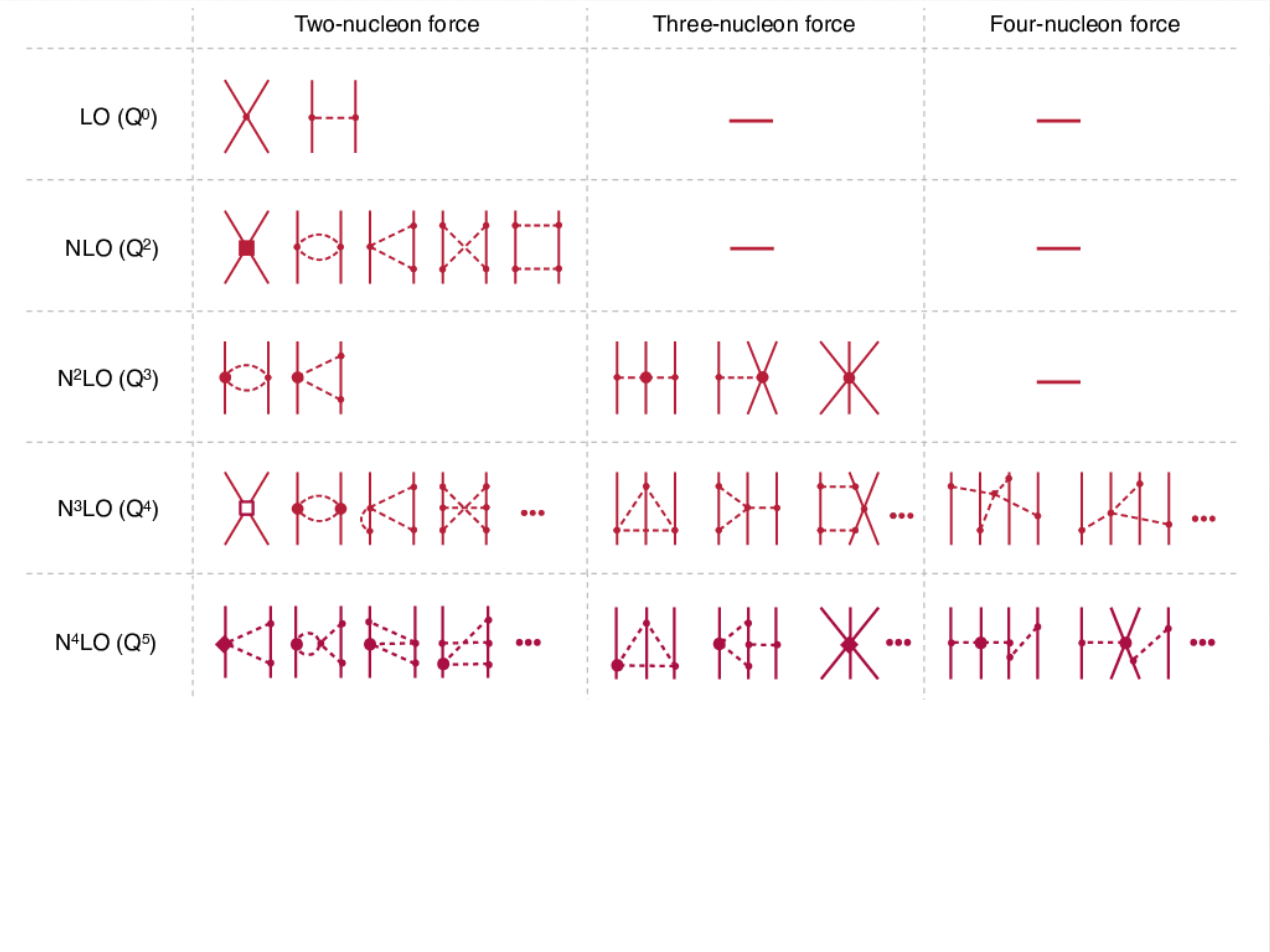
Chiral Effective Field Theory

- Extend ChPT to few-nucleon systems

$$\mathcal{L}_{\text{ChEFT}} = \mathcal{L}_{\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \cdots$$

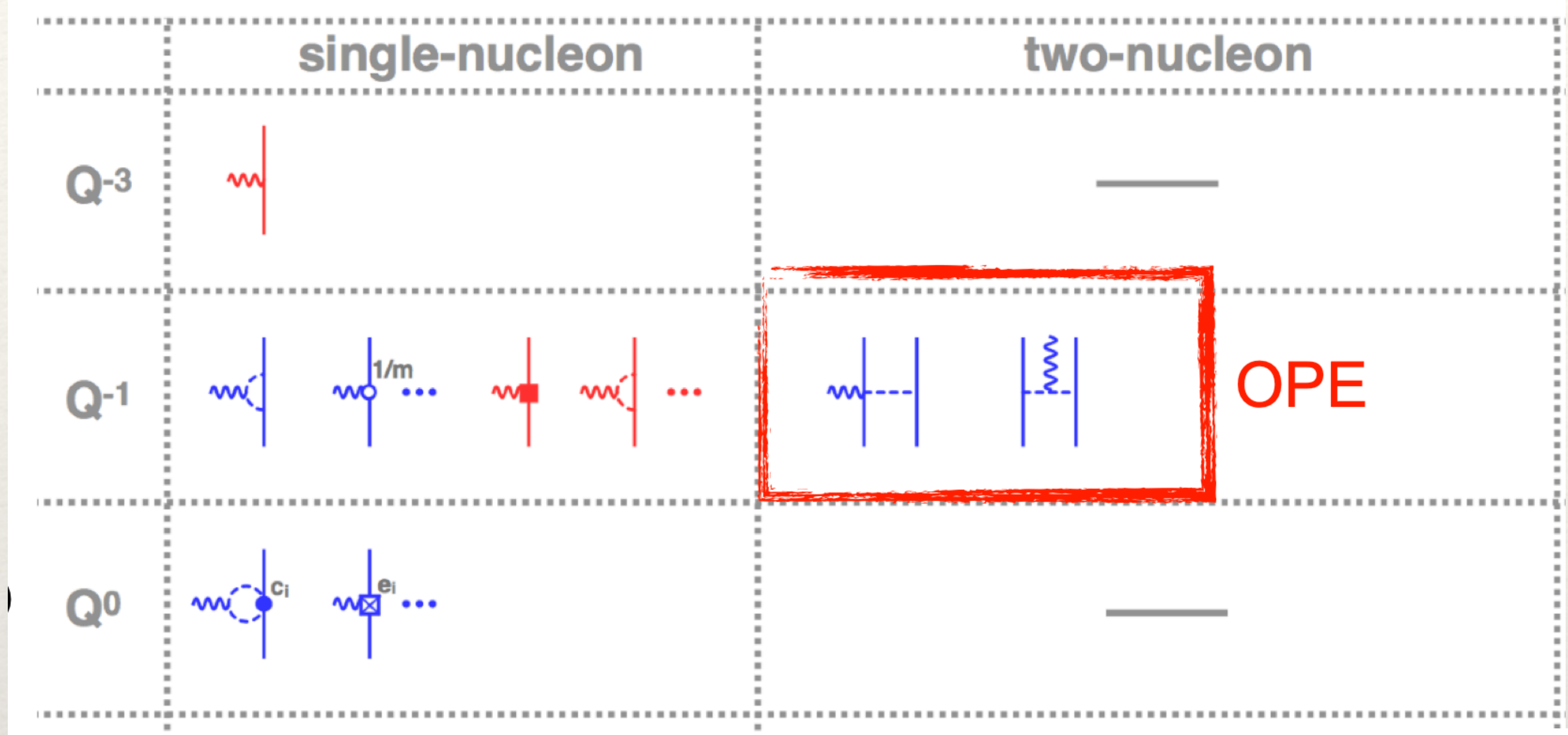
- NN potential = sum of “irreducible” diagrams

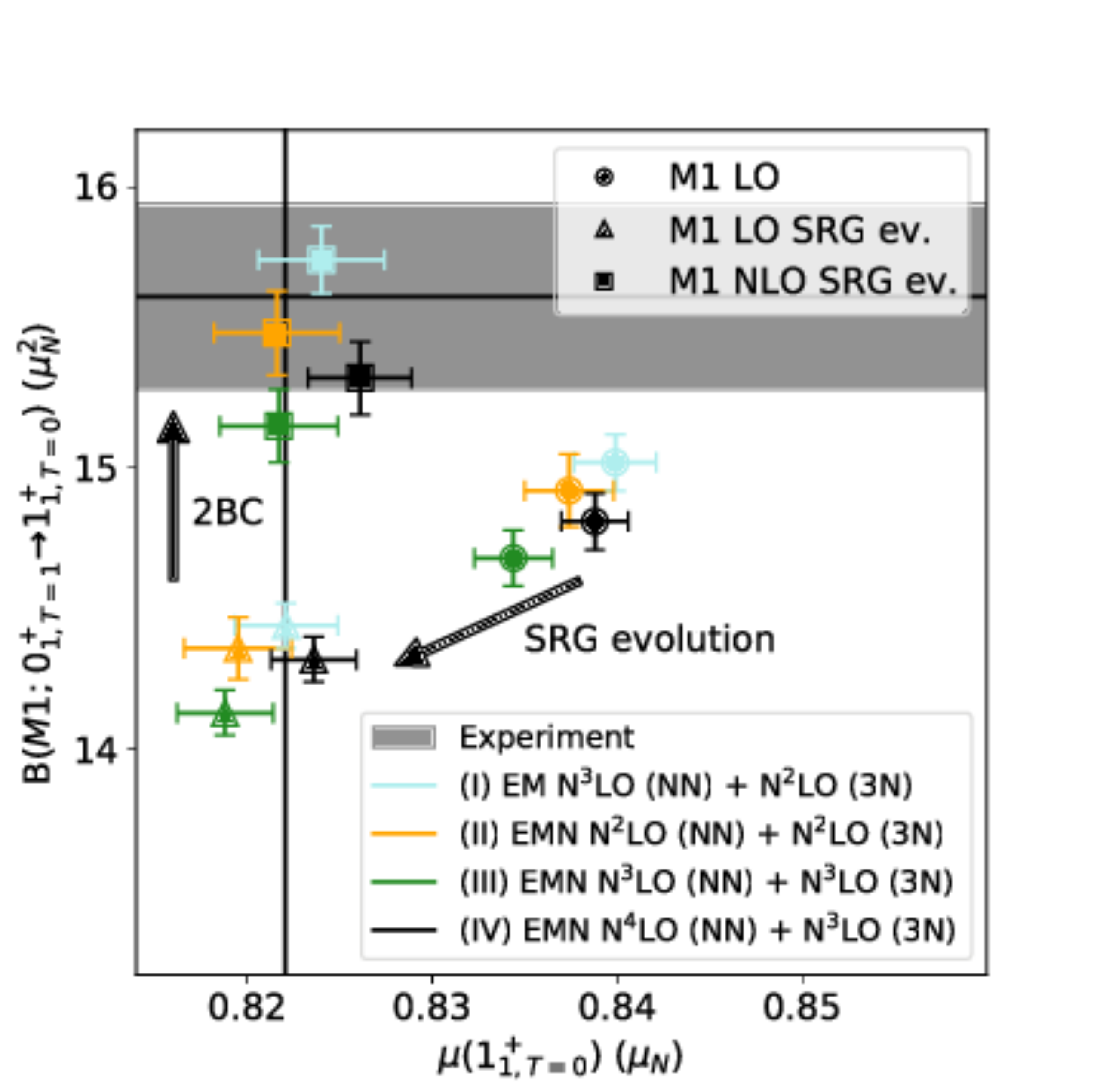
- *Ab initio* program: Construct chiral potentials and solve many-body Schrödinger equation



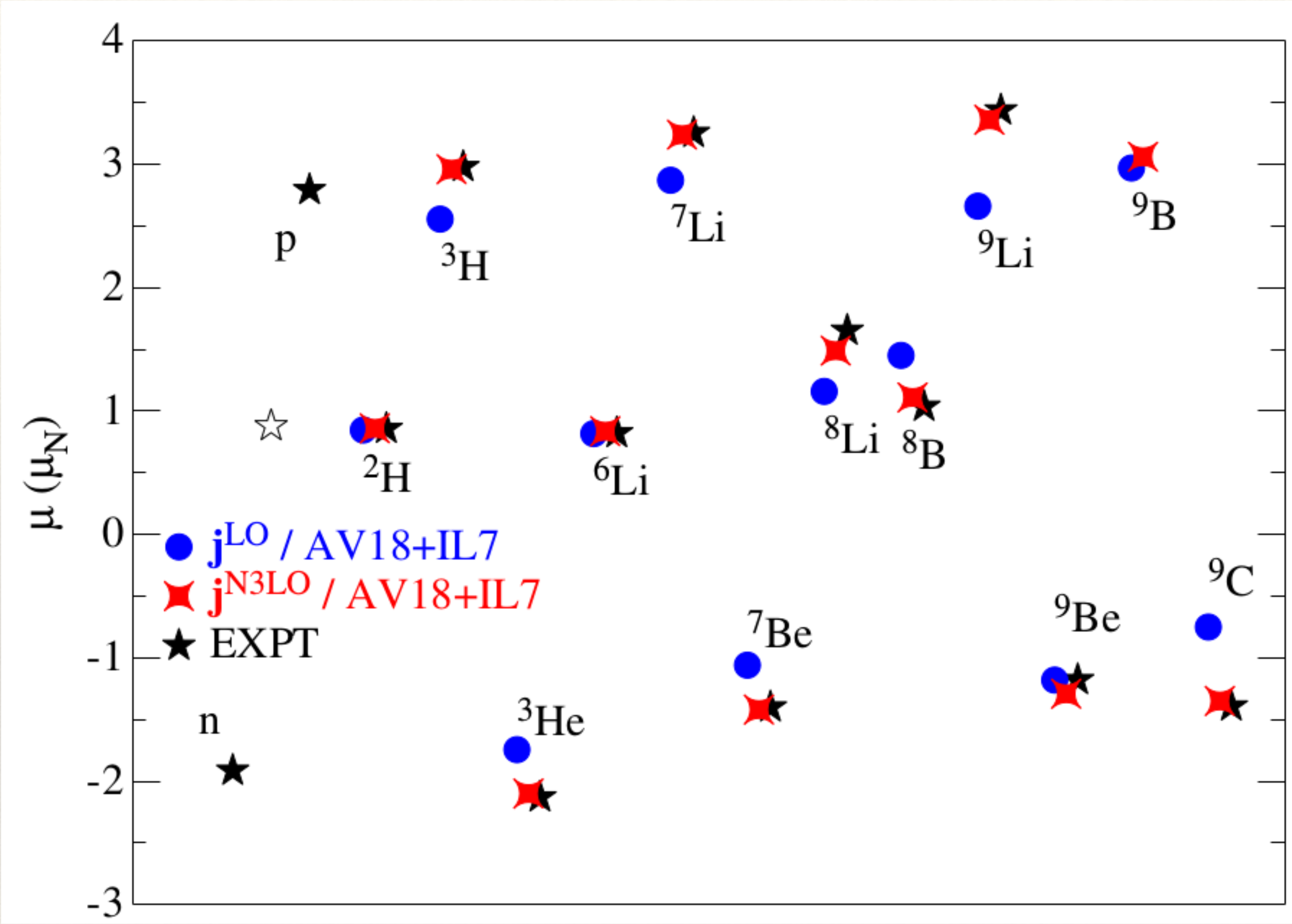
Weinberg, Entem, Machleidt, Epelbaum, Meißner…

Electromagnetic Few-body currents





Bacca et al. PRL 126



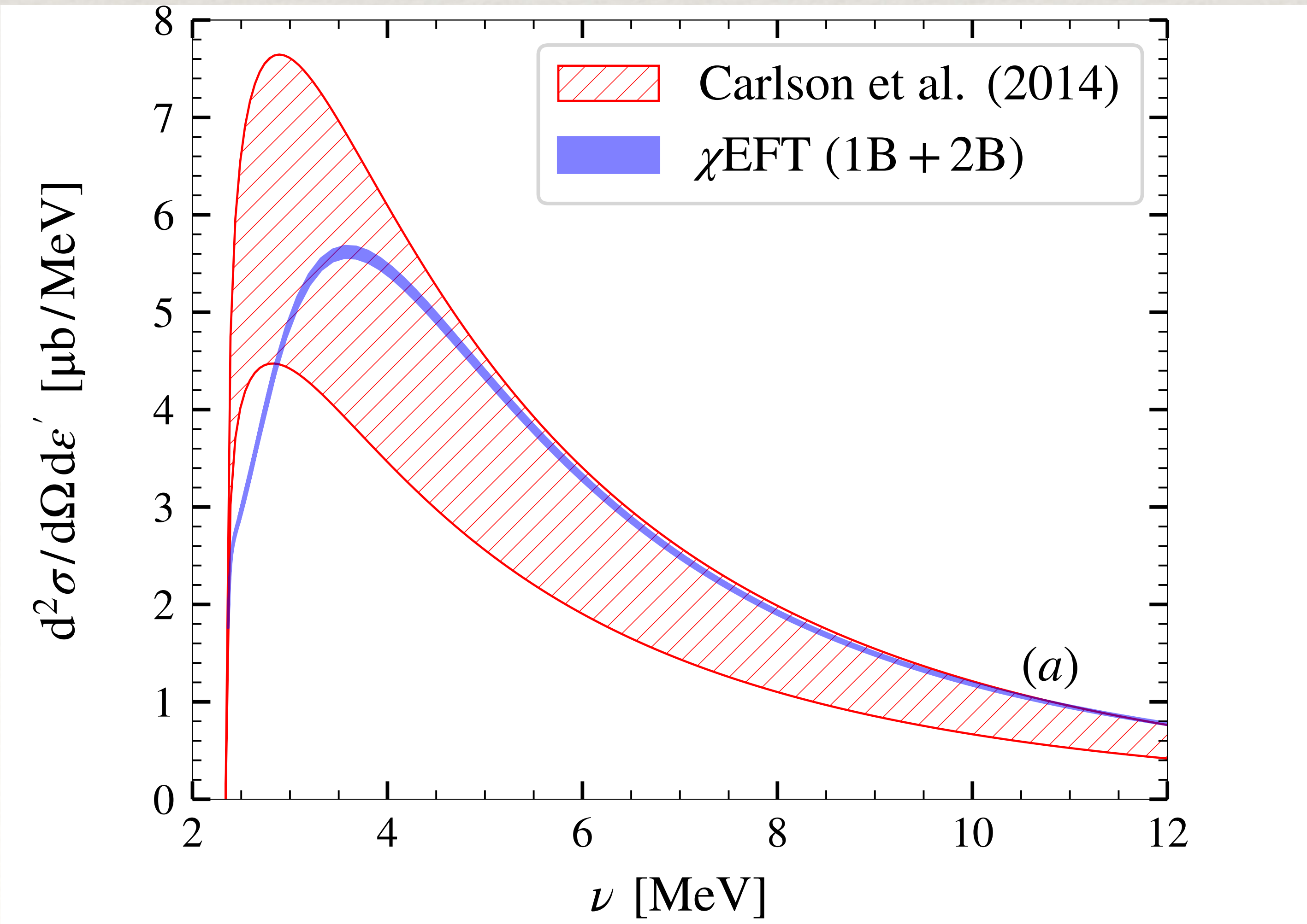
Pastore et al. PRC 90

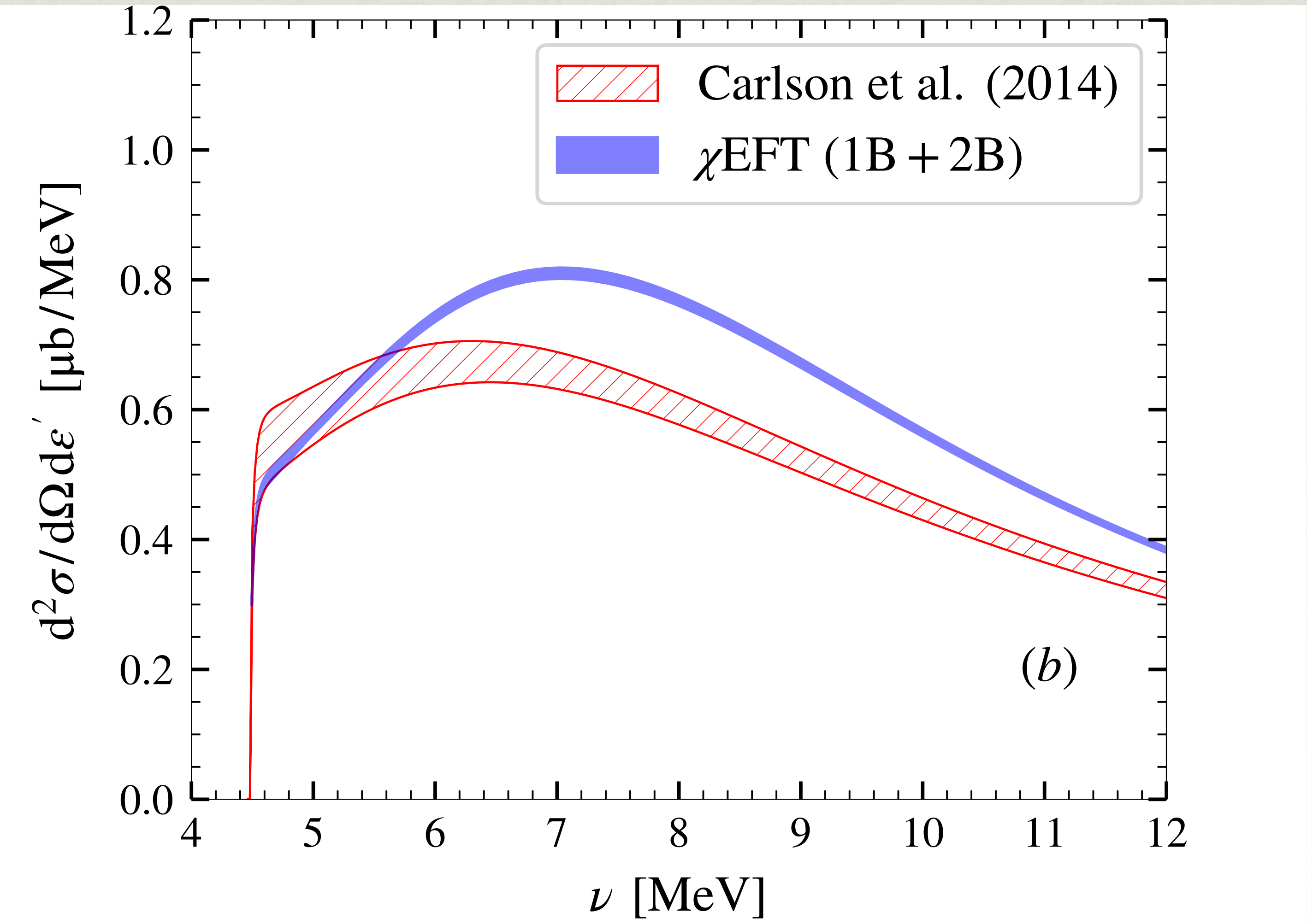
Deuterium Response Functions: Lamb Shift

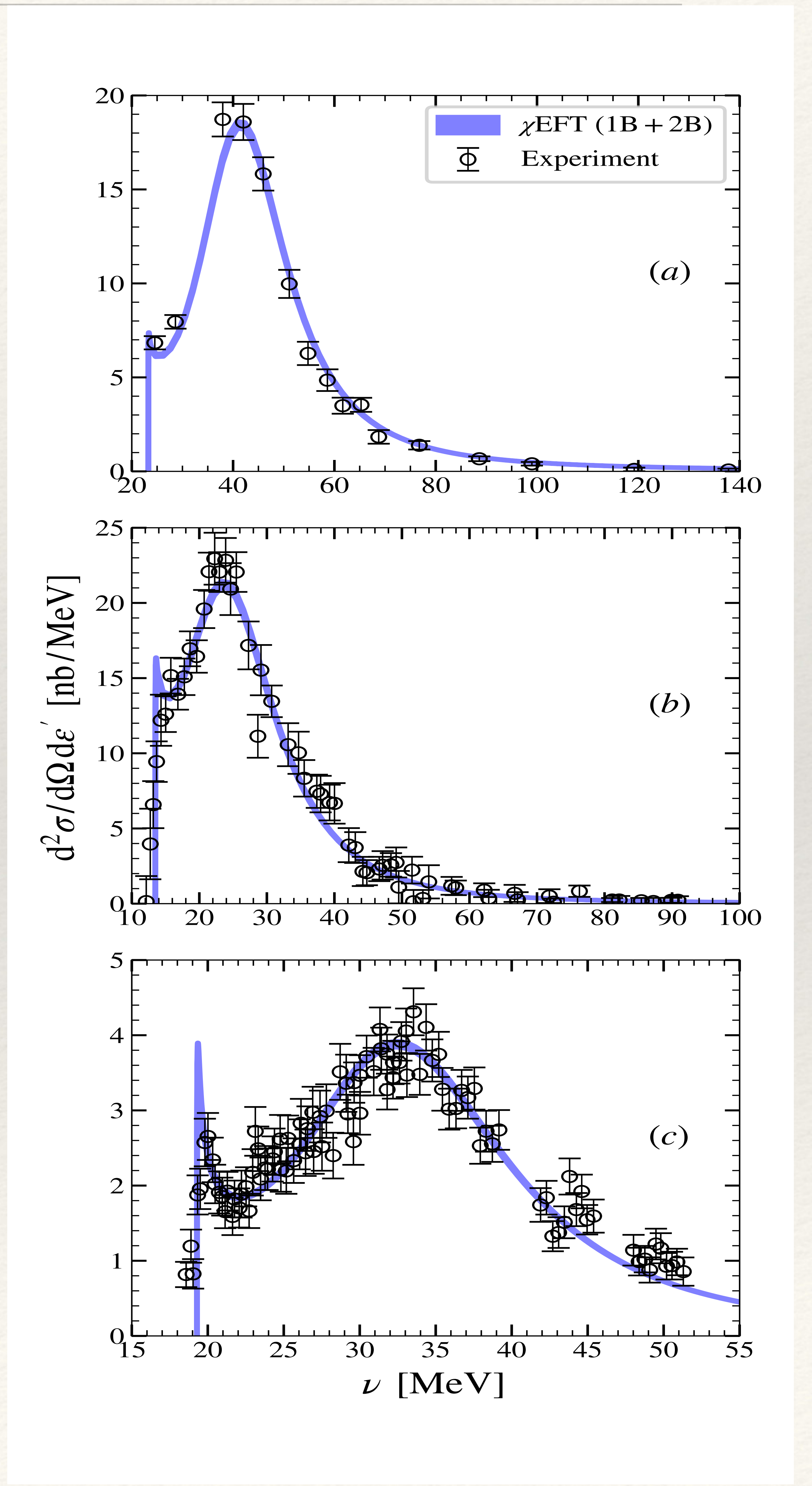
- Input from chiral EFT for dispersion relations

$$R_L(\nu,q) = \frac{1}{3}\sum_{m_d}\sum_{s,m_s}\sum_{t}\int \frac{d^3p}{(2\pi)^3}\frac{1}{2}\delta(\nu + M_d - E_+ - E_-)\,|\langle \mathbf{p}, sm_s, t0|\rho|\psi_d m_d\rangle|^2$$

$$R_T(\nu,q) = \frac{1}{3}\sum_{m_d}\sum_{s,m_s}\sum_{t}\int \frac{d^3p}{(2\pi)^3}\frac{1}{2}\delta(\nu + M_d - E_+ - E_-)\sum_{\lambda=\pm1}|\langle \mathbf{p}, sm_s, t0|J_\lambda|\psi_d m_d\rangle|^2$$







Carlson, et al., PRA 89; Acharya, PRC 103

Two Photon Exchange Contribution to the Lamb Shift

- Polarizability contribution to the Lamb shift

$$\Delta E_{2S}^{\rm 2PE} = \Delta E_{2S}^{\rm inel+subt} + \Delta E_{2S}^{\rm el} + \Delta E_{2S}^{\rm hadr} + \Delta E_{2S}^{\rm Coul}$$

$$\Delta E_{2S}^{\rm inel+subt} = -1.511(12)\ \rm meV$$

$$\Delta E_{2S}^{\rm el} = -0.417(2)\ \rm meV$$

$$\Delta E_{2S}^{\rm hadr} = -0.028(2)\ \rm meV$$

$$\Delta E_{2S}^{\rm Coul} = 0.262(2)\ \rm meV$$

$$\Delta E_{2S}^{2PE} = -1.695(13)\ \rm meV$$

	$\Delta E_{2S}^{\rm TPE}$ [meV]
This work	
— 1B+2B	-1.695(13)
— Siegert	-1.703(15)
Ref. [8]	-1.680(16)
Ref. [9]	-1.717(20)
Ref. [11]	-1.690(20)
Ref. [12]	-1.712(21)
Ref. [13]	-1.703
Ref. [14]	-2.011(740)

Acharya, PRC 103

Towards Two Photon Exchange for the Hyperfine Splitting

- Lamb shift requires

$$\left|\langle \mathbf{p}, sm_s, t0|\rho|\psi_d m_d\rangle\right|^2 \qquad\qquad \sum_\lambda \left|\langle \mathbf{p}, sm_s, t0|J_\lambda|\psi_d m_d\rangle\right|^2$$

- Need *polarized* response functions and different matrix element combinations

$$T^{0i} \to \langle N_0|\rho(-\mathbf{q})|N\rangle\langle N|\mathbf{q}\times\mathbf{J}(\mathbf{q})|N_0\rangle$$

Ordinary atoms + closure approximation (Low term)

Friar and Payne

$$T^{ij} \to \epsilon^{ijk}\langle N_0|J^j(-\mathbf{q})|N\rangle\langle N|J^k(\mathbf{q})|N_0\rangle$$

Meson exchange currents, suppressed?

Kalinowski et al.

- Technology is *mostly* the same as that used in the Lamb shift

Summary

- Disagreement in predictions of polarizability contribution to HFS—resolve with modern EFT techniques

- Effective field theory connects (beyond) Standard Model to low-energy observables

  ➡ Systematically improvable predictions for Standard Model process in light nuclei

- Progress in chiral EFT for the Lamb shift in muonic deuterium is encouraging

  ➡ Apply the same toolbox to HFS