The impact of quark many-body effects on exotic hadrons

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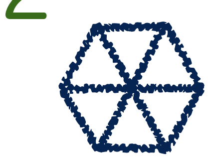
Today's menu

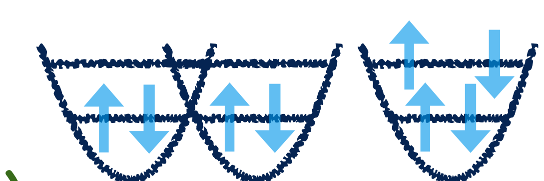
Q: Where and how can we see the quark degrees of freedom in the low-energy region?

A: By looking into the symmetry.

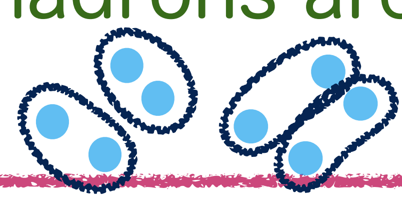
- There are 10 spin-3/2 baryons but the number of spin-1/2 baryons is 8.



- Some of the two-baryon systems have a large short-range repulsion caused by the Pauli-blocking. (Experiments, Quark models, the Lattice QCD)



- Two-hadron systems get a large short-range attraction because hadrons are composed of multiple quarks (many-body effect).



also in the exotic hadrons!

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- (a) Pauli-Blocking effects between quarks, and (b) Quark many-body effects

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T_{cc} $I(J^P) = 0(1^+)$

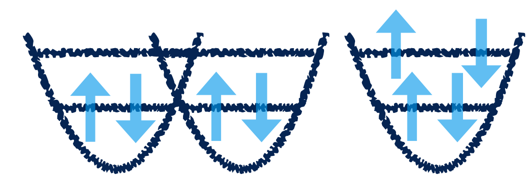
Dynamic calculation with the color-spin interaction

Summary and Outlook

Motivation

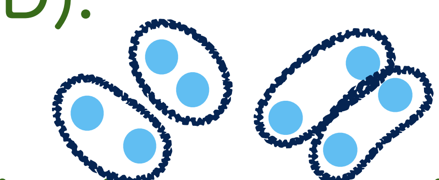
The interaction between the hadrons originated from the quark degrees of freedom consists of ...

(a) Pauli-Blocking effects between quarks



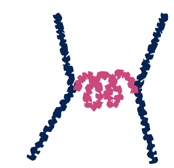
- gives strong repulsion to some of the two-baryon channels, e.g., (consistent with the experiments, LQCD).

(b) Quark many body effects



- gives attraction between the hadrons in the scattering states or shallow bound states. (spectroscopic factor, S-factor)

(c) color-spin interaction (color-magnetic interaction)

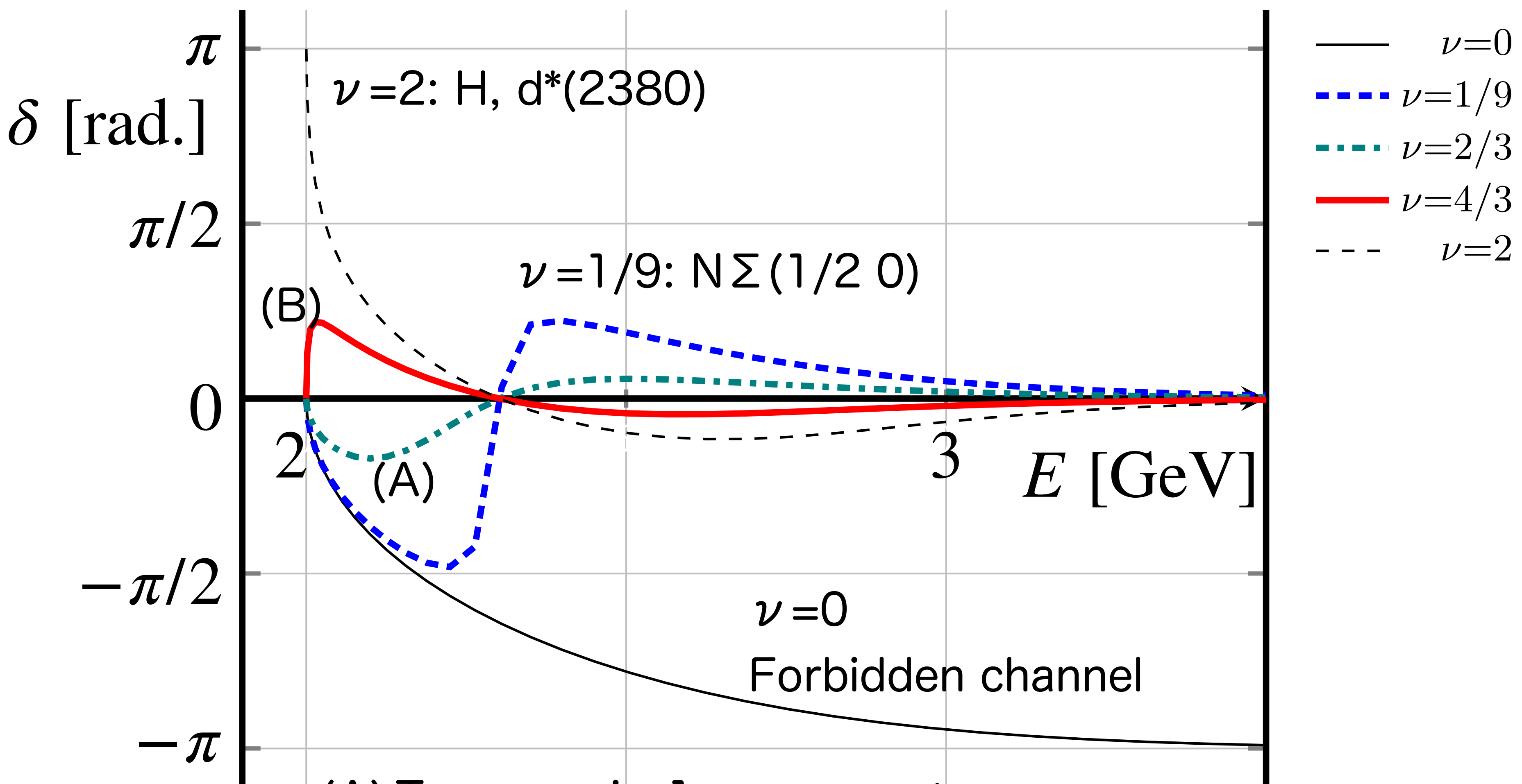


- The one-gluon exchange or instanton induced interactions that give the hyperfine splittings, say, N-Δ, or D-D* mass difference.

For compact states, (c) matters. But for scattering or loosely bound states, (a) $\simeq$ (b) > (c) in size in general. To investigate the mechanism to bind two hadrons one has to look also into (a) and (b).

Size of the effects (a)+(b)

- Both of the hadron masses are 1GeV, all quark masses are equal.



(A):Tcc, cc spin-1 component

(B):Tcc, cc spin-0 component

Hadron potential from the Quark antisymmetrization (How to derive (a) and (b))

Suppose quark antisymmetrization occurs only within the $(0s)^n$ states.

- Single hadrons
  - $|B\rangle = \mathscr{A}_q^{(3)}|q^3\rangle,\ |M\rangle = |q\bar{q}\rangle$, with $\langle B|B\rangle = 1,\ \ \langle M|M\rangle = 1$
- Wave function of two hadron states with the number of antiquarks is 0 or 1:
  - $|\Psi(hh')\rangle \propto \mathscr{A}_q^{(n)}|hh'\rangle \propto \left(1 - n_h n_{h'} P_{14}^{ex}|0s\rangle\langle 0s|\right)|hh'\rangle$

    with n_h the number of quarks in hadron h, and P_{14}^{ex} express the one-quark interchange between the two hadrons, and $|0s\rangle = |(0s)^n\rangle$.
  - Note that $P_{14}^{ex,orb}$ does not change the orbital $(0s)^n$ configuration,

    $P_{14}^{ex}|0s\rangle\langle 0s| = |0s\rangle\langle 0s|P_{14}^{ex} = P_{14}^{ex,sfc}|0s\rangle\langle 0s|$,

    so the wave function becomes:

    $|\Psi(hh')\rangle \propto \left(\overline{|0s\rangle\langle 0s|} + \nu_{hh'}^{sfc}|0s\rangle\langle 0s|\right)|hh'\rangle$

    with $\nu_{hh'}^{sfc} = \langle hh'0s|\mathscr{A}|hh'0s\rangle = \langle hh'|\left(1 - n_h n_{h'} P_{14}^{ex,sfc}\right)|hh'\rangle_{sfc}$

    Or, operator 1 for the quarks becomes operator N for the hadrons

    $1 = \left(\overline{|0s\rangle\langle 0s|} + |0s\rangle\langle 0s|\right) \quad \rightarrow \quad N = \left(\overline{|0s\rangle\langle 0s|} + \nu_{hh'}^{sfc}|0s\rangle\langle 0s|\right)$

Hadron potential from the Quark antisymmetrization (How to derive (a) and (b))

Suppose quark antisymmetrization occurs only within the $(0s)^n$ states.

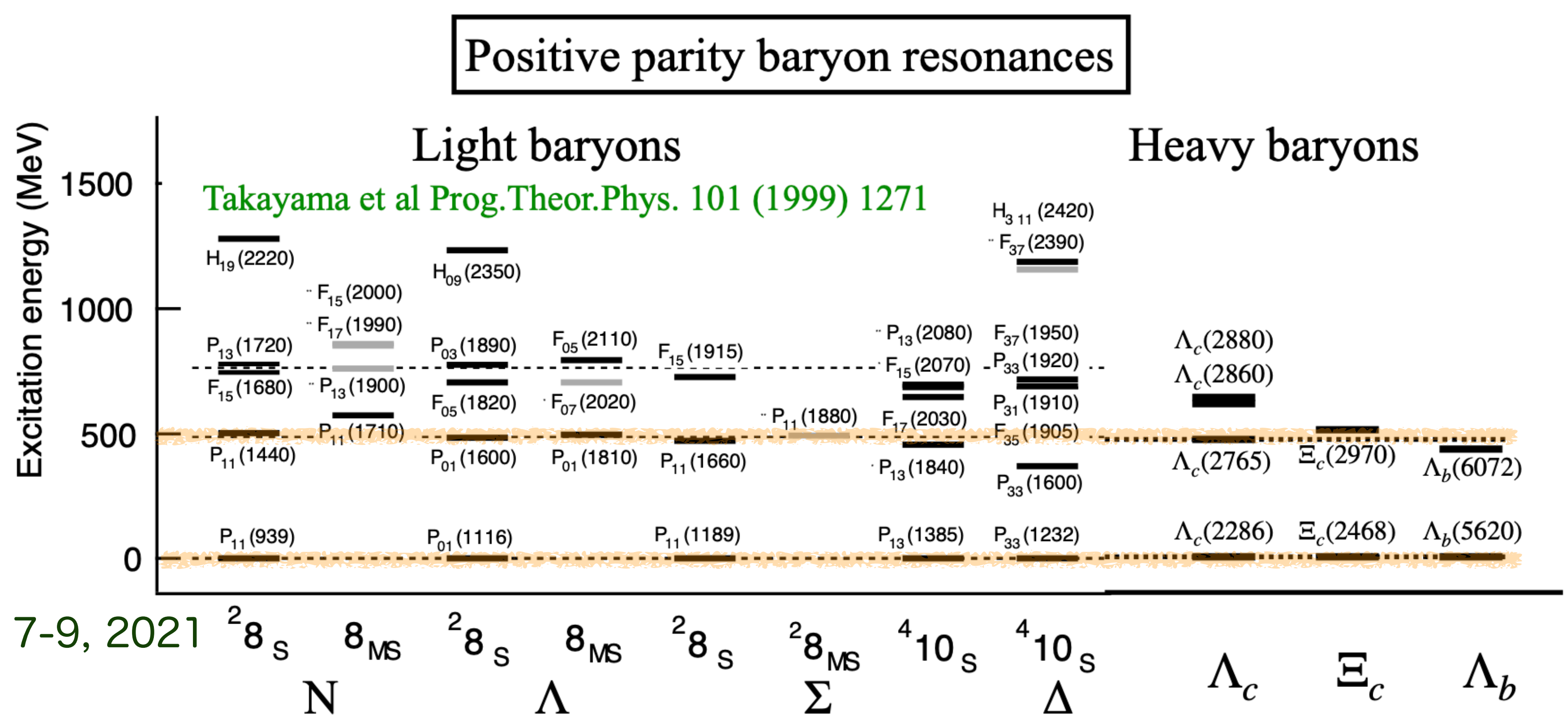
- Wave function of two hadron states with the number of antiquarks is 0 or 1:
  - Kinetic operator for quarks becomes

$$H_0 = \sum m_i + \sum_{i\in h} K_i + \sum_{i\in h'} K_i + K_{hh'}, \qquad \mathscr{A}_q H_0 = H_0 \mathscr{A}_q$$
$$\langle hh'0s|\mathscr{A}H_0 = \nu^{sfc}_{hh'}\langle hh'0s|H_0, \qquad H_0\mathscr{A}|hh'0s\rangle = \nu^{sfc}_{hh'} H_0|hh'0s\rangle$$

  - So, the kinetic operator for hadrons becomes

$$H_0 \to H_{h0} = (M_h + M'_h)N + |0s\rangle\nu^{sfc}_{hh'}\frac{3}{4}\hbar\omega\langle 0s| + \nu^{sfc}_{hh'}\frac{\sqrt{6}}{4}\hbar\omega(|1s\rangle\langle 0s| + |0s\rangle\langle 1s|) + \text{rest of } K_{hh'}$$

- We take $\hbar\omega$ not depend on the flavor because 1st excitation energies of hadrons are almost flavor independent.



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Hadron potential from the Quark antisymmetrization (How to derive (a) and (b))

Suppose quark antisymmetrization occurs only within the $(0s)^n$ states.

- Sch-eq for the hadron states:
  - $(H_0 - E)|\Psi(hh')\rangle = 0 \quad \to \quad (K_h - EN)|hh'\rangle = 0$
    $K_h = \left(|0s\rangle \nu_{hh'}^{sfc}\frac{3}{4}\hbar\omega\langle 0s| + \nu_{hh'}^{sfc}\frac{\sqrt{6}}{4}\hbar\omega(|1s\rangle\langle 0s| + |0s\rangle\langle 1s|)\right) + \text{rest of } \mathrm{H}_0$
  - To remove the energy dependent exchange term, we rewrite the eq as$N^{1/2}(N^{-1/2}K_h N^{-1/2} - E)N^{1/2}|hh'\rangle = 0$
    $N^x = \overline{|0s\rangle\langle 0s|} + \left(\nu_{hh'}^{sfc}\right)^x |0s\rangle\langle 0s|$, We can consider $|hh'\rangle\rangle = N^{1/2}|hh'\rangle$ as the hadron wave function in stead of $|hh'\rangle$. Then the Sch-eq becomes
    $(N^{-1/2}K_h N^{-1/2} - E)|hh'\rangle\rangle = 0$

No effects on the compact states

  - The matrix element between 0s states becomes
    $\langle\langle hh'0s|N^{-1/2}K_h N^{-1/2}|hh'0s\rangle\rangle = \langle\langle hh'|\nu^{-1/2}\frac{3}{4}\hbar\omega\,\nu\nu^{-1/2}|hh'\rangle\rangle = \frac{3}{4}\hbar\omega$
  - Matrix element between 0s and 1s states becomes
    $\langle\langle hh'1s|N^{-1/2}K_h N^{-1/2}|hh'0s\rangle\rangle = \langle\langle hh'|1^{-1/2}\frac{\sqrt{6}}{4}\hbar\omega\,\nu\nu^{-1/2}|hh'\rangle\rangle = \frac{\sqrt{6}}{4}\hbar\omega\,\nu^{1/2}$

Effects on the scattering states or loosely bound states

Hadron potential from the Quark antisymmetrization (How to derive (a) and (b))

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    Effects on the scattering states or loosely bound states

Channel dependence of the effects

Size of the effects can be evaluated by $(\nu^{sfc}_{hh'} - 1)$

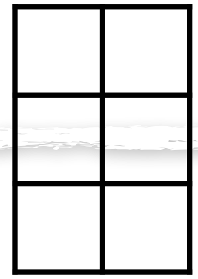
$K_h = \sum_{nm} |ns\rangle K_{nm} \langle ms|$

$$K_{nm} = \begin{pmatrix} \frac{3}{2} & \sqrt{\frac{3}{2}}\sqrt{\nu} & 0 & \cdots \\ \sqrt{\frac{3}{2}}\sqrt{\nu} & \frac{7}{2} & \cdots & \\ 0 & \cdots & \frac{11}{2} & \cdots \end{pmatrix} \frac{\hbar\omega}{2}$$

- $(\nu^{sfc}_{hh'} - 1) < 0$: (a) Pauli-blocking effect
  - 0s-1s mixing reduces → repulsion
- $(\nu^{sfc}_{hh'} - 1) > 0$: (b) Quark many body effects
  - 0s-1s mixing enhances → attraction
- $\nu^{sfc}_{hh'}$ is determined by the color-flavor-spin symmetry
  $\nu^{sfc}_{hh'} = \langle hh'0s | \mathscr{A} | hh'0s \rangle = \langle hh' | \left(1 - n_h n_{h'} P^{ex,sfc}_{14}\right) | hh' \rangle_{sfc}$
- Examples:
  - Two baryon systems (q^3-q^3) $0 \leq \nu^{sfc}_{hh'} \leq 2$
  - Pentaquarks (q^3-$q\bar{q}$) $0 \leq \nu^{sfc}_{hh'} \leq \frac{4}{3}$
  - Two meson systems ($q\bar{q}$-$q\bar{q}$) $0 \leq \nu^{sfc}_{hh'} \leq \frac{4}{3}$
- ‘taking one hadron out of 2 hadrons’ is different from ‘taking qqq out of $qqqq$ (+$\bar{q}$) or $q\bar{q}$ out of $q\bar{q}q\bar{q}$.’

Examples

color [222]

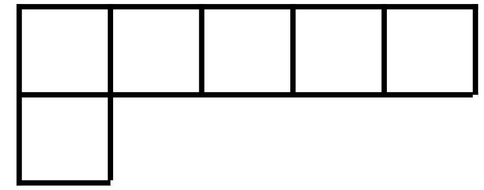


$P_{14} = \frac{1}{3}$

spin-flavor [33]

$P_{14} = -\frac{1}{3} \quad \to \nu = 1 - 9P_{14} = 2$

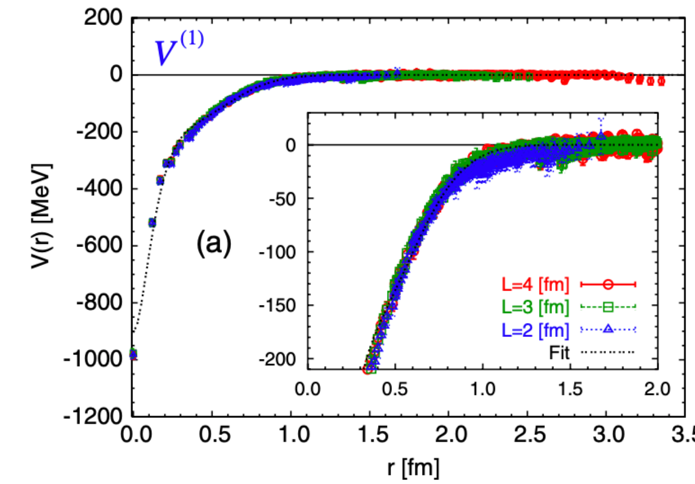
spin-flavor [51]

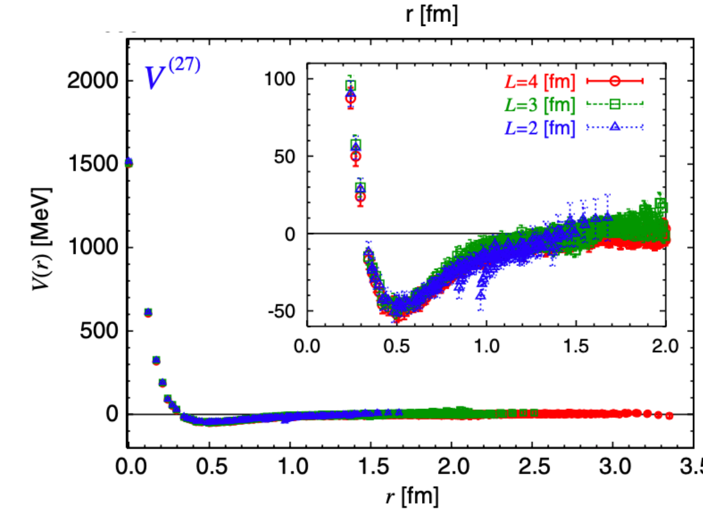


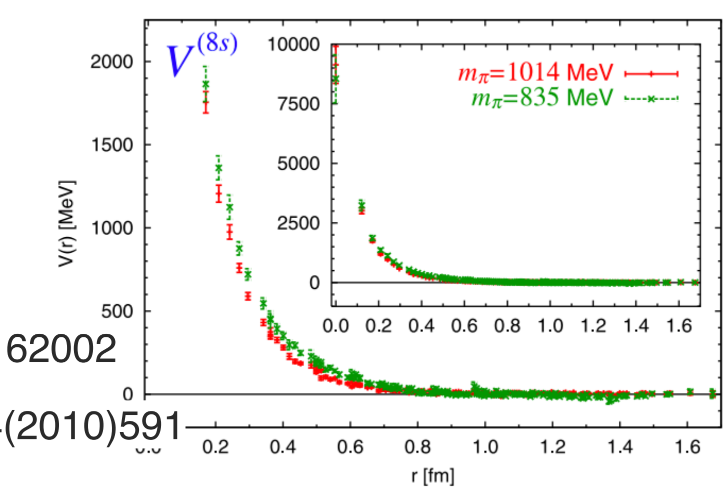
$P_{14} = \frac{1}{3} \quad \to \nu = 0$

Two baryon systems

- H dihyperon, $d^*(2380)\ I(J^P) = 0(3^+)$
  - $\nu = 2$, strongly attractive.
    the color-spin term is also strongly attractive.
- $NN(TS) = (10), (01)$
  - $\nu = \frac{10}{9}$, small effect.
    the color-spin term is repulsive.
- $N\Sigma(TS) = (\frac{1}{2}, 0)$
  - $\nu = \frac{1}{9}$, strongly repulsive.







T.Inoue etal, HAL QCD Collab, *PRL106*(2011)162002

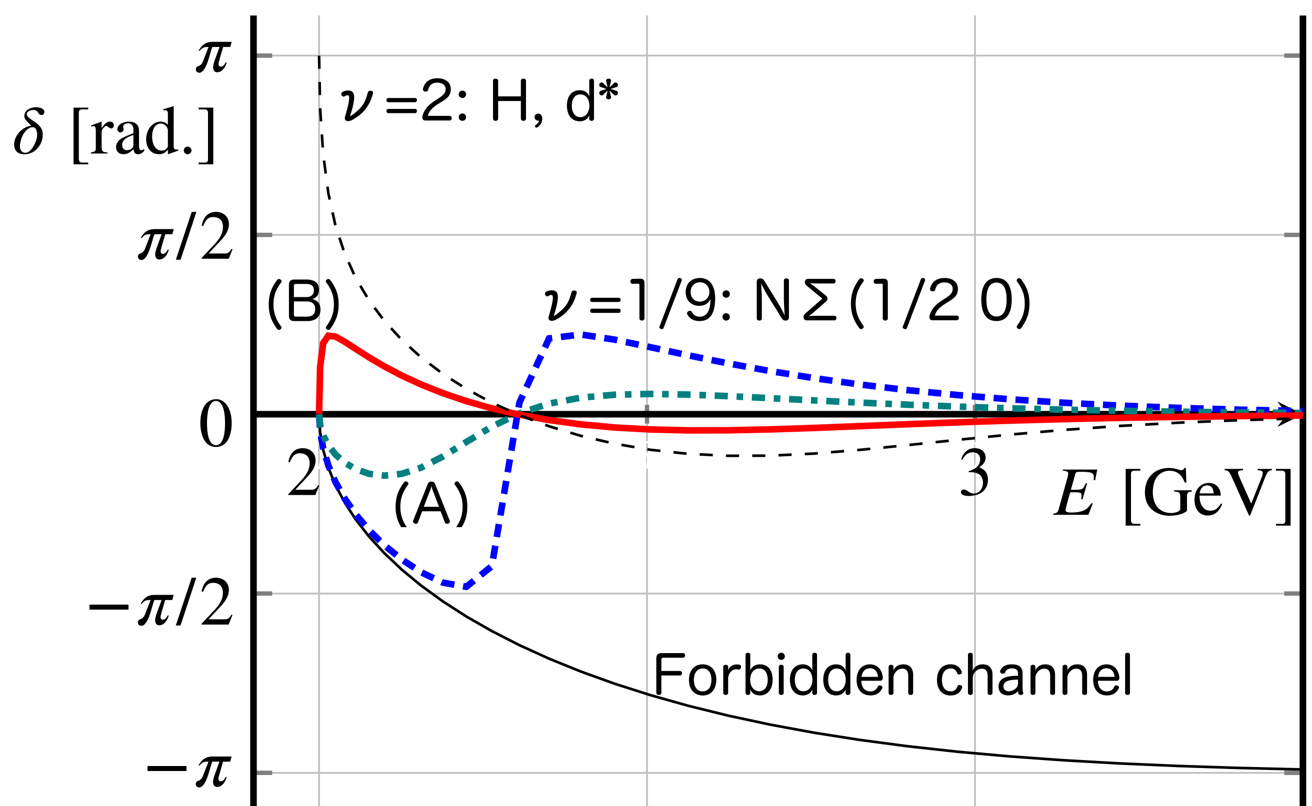
T.Inoue etal, HAL QCD Collaboration, *PTP* 124(2010)591

Potential, summary

Harmonic expansion of the nonrela kinetic term, $\frac{1}{2}k^2$:

- The 0s state is isolated if ν=0
- The mixing enhances for ν >1
- reduces for ν <1

$$K = \begin{pmatrix} \frac{3}{2} & \sqrt{\frac{3}{2}}\sqrt{\nu} & 0 & \cdots \\ \sqrt{\frac{3}{2}}\sqrt{\nu} & \frac{7}{2} & \cdots & \\ 0 & \cdots & \frac{11}{2} & \cdots \end{pmatrix} \hbar\omega$$



(A):Tcc, cc spin-1 component
(B):Tcc, cc spin-0 component

Potential that gives this effect has a node.
In the above K, the effects are written only between 0s-1s, but we performed a full calculation, which is necessary when mq=mqbar.

$T_{cc}\ I(J^P) = 0(1^+)$

LHCb,
Many theoretical references

2 meson system

- $(c\bar{u})_{color\ singlet}\ (c\bar{d})_{color\ singlet}$
  - $\nu = \langle (q\bar{q})(q\bar{q}) | (1-P_{13})(1-P_{24}) | (q\bar{q})(q\bar{q}) \rangle = \langle (q\bar{q})(q\bar{q}) | (1+P_{13}P_{24})(1-P_{24}) | (q\bar{q})(q\bar{q}) \rangle$
    $= \langle MM | (1-P_{24}) | MM \rangle$
    $= \langle \frac{1}{3} A^{c3}_{24}(1+P^{f\sigma}_{24}) + \frac{2}{3} S^{c6}_{24}(1-P^{f\sigma}_{24}) \rangle \le \frac{4}{3}$ for $q_1\bar{q}_2$ (and $q_3\bar{q}_4$) color-singlet systems
  - isospin 1, J=0: ν =2/3, 4/3
  - isospin 1, J=1: ν =2/3
  - isospin 1, J=2: ν =2/3
  - isospin 0, J=1: ν =2/3, 4/3 $\to T_{cc}$
- T_{cc} has two components:
  - (A) cc spin-1 $\bar{u}\bar{d}$ spin-0 (good diquark)
    ν repulsive CMI more attractive
  - (B) cc spin-0 $\bar{u}\bar{d}$ spin-1
    ν attractive CMI less attractive

	(A) ν repulsive	(B) ν attractive
	cc, qq: color $\bar{3}$ ν =2/3 $\lambda\lambda$=-8/3	cc, qq: color 6 ν =4/3 $\lambda\lambda$=4/3
cc	spin 1	spin 0
ud isospin 0	spin 0 cmi=-8	spin 1 cmi=-4/3
ud isospin 1	spin 1 cmi=8/3	spin 0 cmi=4

- if the flavor of $q_1 \neq q_3$, e.g. $b\bar{u}c\bar{d}$, more ν =4/3 attractive states

T_{cc} $I(J^P) = 0(1^+)$

LHCb,
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2 meson system

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    $= \langle MM | (1-P_{24}) | MM \rangle$
    $= \langle \frac{1}{3} A^{c3}_{24}(1+P^{f\sigma}_{24}) + \frac{2}{3} S^{c6}_{24}(1-P^{f\sigma}_{24}) \rangle \le \frac{4}{3}$ for $q_1\bar{q}_2$ (and $q_3\bar{q}_4$) color-singlet systems
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  - isospin 0, J=1: ν =2/3, 4/3 $\rightarrow T_{cc}$
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	(A) ν repulsive	(B) ν attractive
	cc, qq: color $\bar{3}$ ν=2/3 $\lambda\lambda$=-8/3	cc, qq: color 6 ν=4/3 $\lambda\lambda$=4/3
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ud isospin 1	spin 1 cmi=8/3	spin 0 cmi=4

- if the flavor of $q_1 \neq q_3$, e.g. $b\bar{u}c\bar{d}$, more ν =4/3 attractive states

(c) $q\bar{c}q\bar{c}$ interaction

$$V_{ij}^{q\alpha}(r,r') = (\lambda_i \,.\, \lambda_j)(\sigma_i \,.\, \sigma_j)\; c_{0s}^{\alpha}\; \langle r_{ij}\,|\,0s\rangle\langle 0s\,|\,r'_{ij}\rangle$$

Assumptions:

- potential between two quarks with relative $(0s)$.
- proportional to $\lambda\cdot\lambda\sigma\cdot\sigma$
- $c_{0\ell}^{\alpha}$'s are obtained from each $u\bar{u}, c\bar{c}, u\bar{c}$ meson mass diff.
  - $c_{0s}^{\sigma\sigma}$ from $\eta_c, J/\psi, D, D^*$ mass diff.
  - $c_{0s}^{\sigma\sigma}(ud)$ is from ΔN mass diff.

Two free parameters:

- quark mass ratio $m_q/m_{\bar{q}}$.
- scaled size of mesons, $x_0^2 = mb^2 = \omega_0^{-1}$
  - $x_0 \sim 0.63 - 0.76\mathrm{fm}^{1/2}$
    for nucleon charge rms $\sqrt{\langle r^2\rangle} \sim b = 0.5 - 0.6\mathrm{fm}$
    and the excitation energy $\omega_0 \sim 350 - 500\mathrm{MeV}$

Dynamical calculation

Potential to give the S-factor effects has a node, so the dynamical calculation is necessary (especially those with the short range attraction).

- $H_h = H_0 + V_K + V_{CS}$
  $V_K = (\sqrt{\nu} - 1)(|0s\rangle\langle 1s| + |1s\rangle\langle 0s|)\frac{\sqrt{6}}{2}\hbar\omega$ ・・・(a)Pauli (b) Many-body
  $V_{CS} = c_{0s}(\lambda \cdot \lambda)(\sigma \cdot \sigma)|0s\rangle\langle 0s|$ ・・・(c) CMI
- 3-channel calculation : $D^0 D^{*+}$, $D^+ D^{*0}$, $D^{*+} D^{*0}$ (all S-wave)

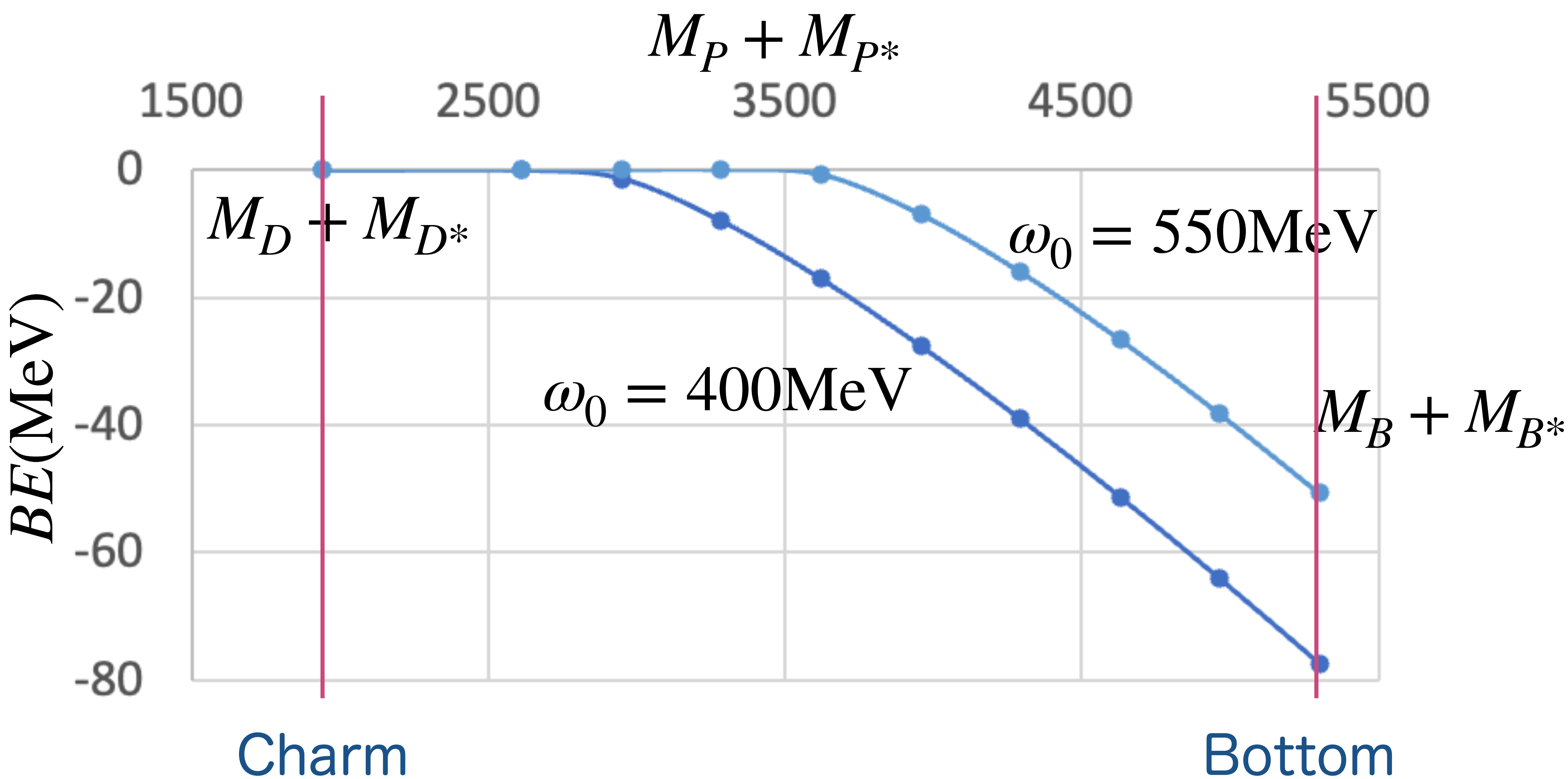
OPEP is attractive, but ambiguous results.
No bound : S.Ohkoda etal PRD 86, 034019 (2012), and corrected one (in preparation)
Bound: Ning Li etal, PRD 88, 114008 (2013)

Results

- no bound state T_{cc} $I(J^P) = 0(1^+)$
  needs OPEP? …
- a bound state for T_{bb} $I(J^P) = 0(1^+)$ with a BE of 51-77 MeV($x_0 = 0.6 - 0.7$), with ~0.2 of the B^*B^* component.

charm to bottom

As the mass of charm goes to the mass of bottom…



The quark effects are attractive.

Summary and Outlook

- We study the effects of the quark Pauli-blocking effects and the quark many-body effects in the multiquark systems.
- They can be expressed by the non-local potential with 0s-1s mixing. For compact states, the effects are small, but scattering states, the effects can be very large.
- For example, they explain the attraction of H, d*.
- In some of the multiquark systems, like T_{QQ}, there are color-spin-attractive, and S-factor repulsive configuration, and vice versa.
- T_{QQ}'s are interesting. By introducing OPEP, and studying the decay or production, we can construct comprehensive picture of the exotic states ⋯