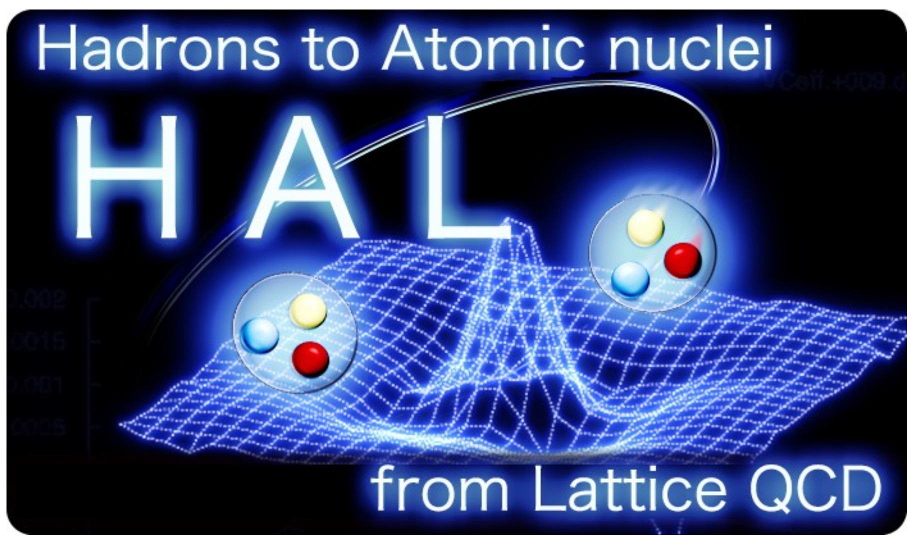
Influence of discretization error on the HALQCD baryon forces

Takashi Inoue @Nihon Univ.
for
HALQCD Collaboration



EFB25, Jul 30, 2023, Mainz

Plan

- Introduction
  - HALQCD method for multi-hadron systems in LQCD
  - Mainz group paper = Motivation of this study
  - Purpose: study "a" influence of HALQCD BB int.
- Current status
  - Gauge conf. generation at same M_π with 3 different "a"
  - Study flavor singlet BB interaction and H-dibaryon.
- Summary and outlook

Multi-hadron in LQCD

N. Ishii etal.
Phys. Rev. Lett . 99 022001 (2007)
Phys. Lett. B 712 437 (2012)

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$\psi(\vec{r},t)$: 4-point function contains NBS w.f.

  - and solve the Schrodinger eq.
  - Advantages
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    - Then, potential can be extracted.
    - Demand a minimal lattice volume. No need to extrapolate to V=∞.
    - Can output many observables.

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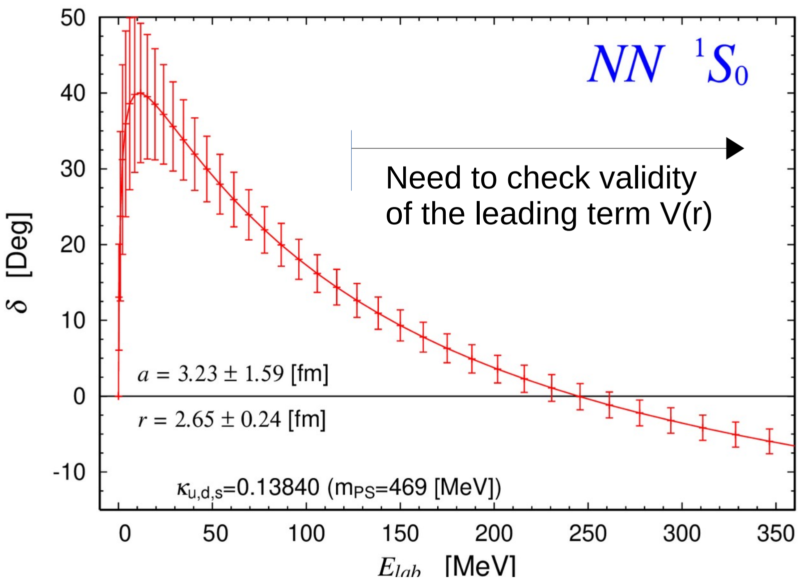
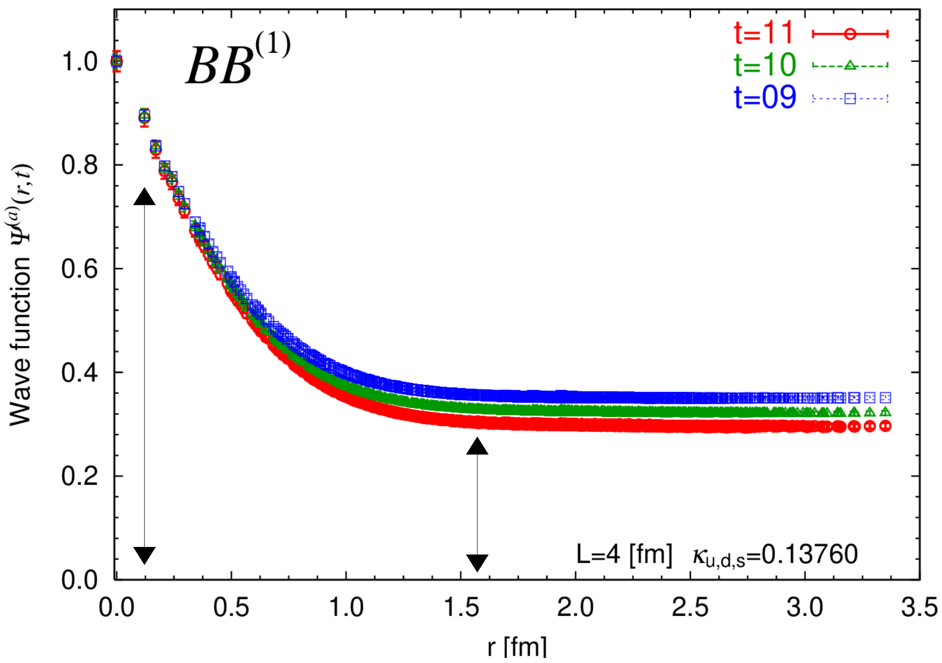
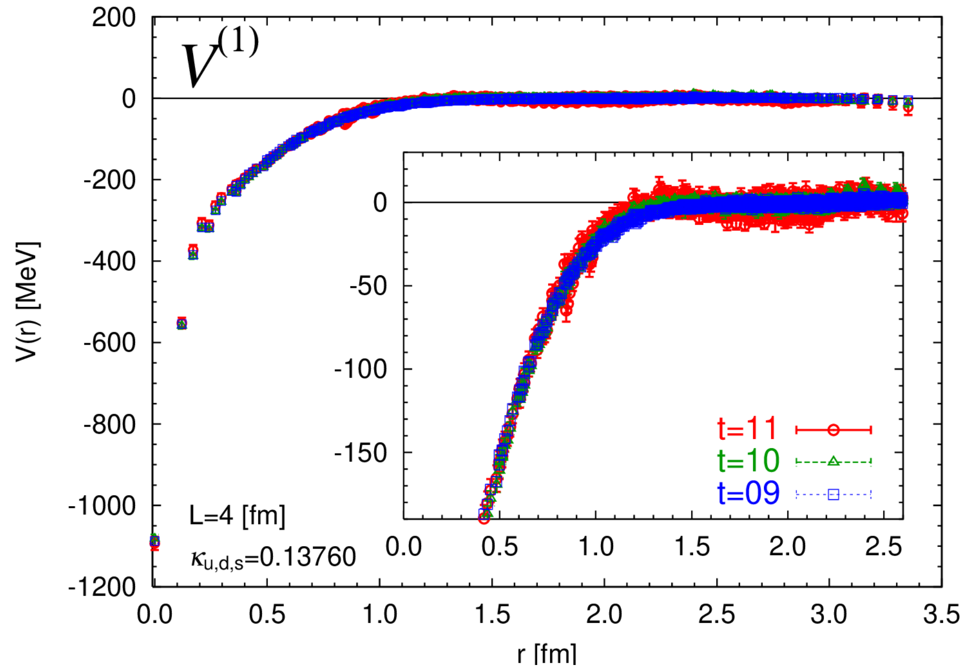


Most demonstrating case

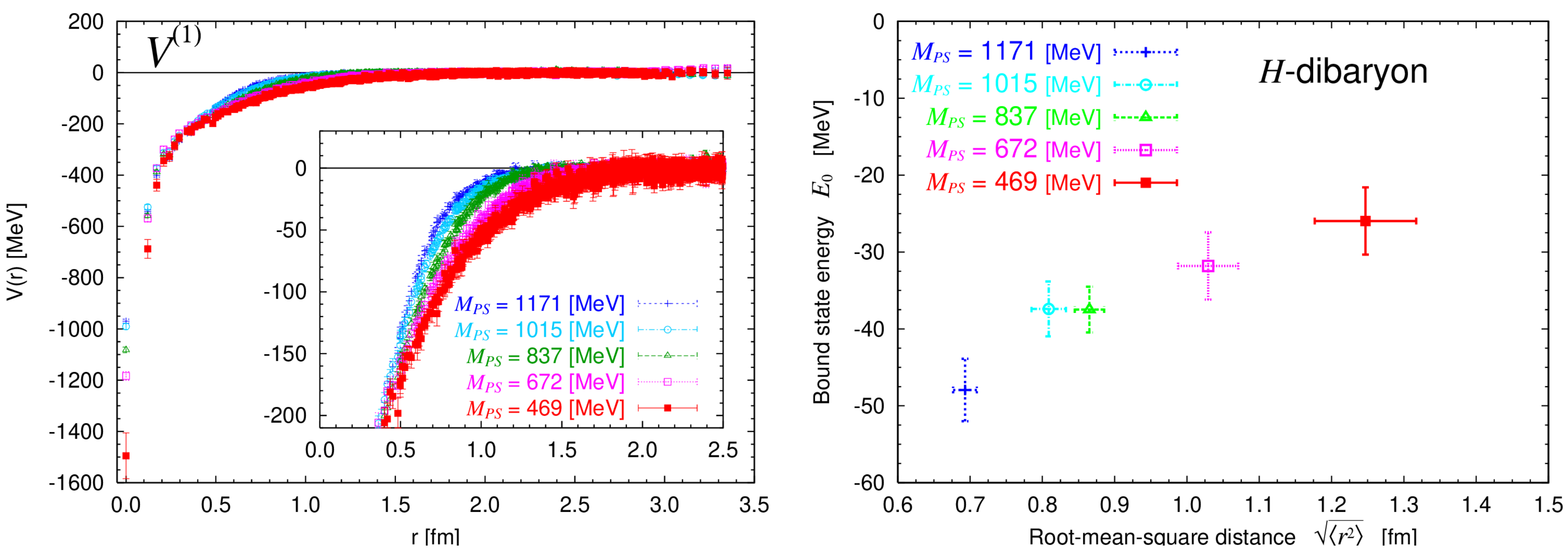




- Left: Measured 4pt of the flavor singlet BB $^1S_0$ channel
  - The peaking value at short distance is a bound state contribution
  - The constant value at large distance is a scattering state contr.
  - Difficult (impossible) to suppress the scattering contribution
- Right: Extracted potential of the channel
  - independent of the sink-time t after t_Hadron

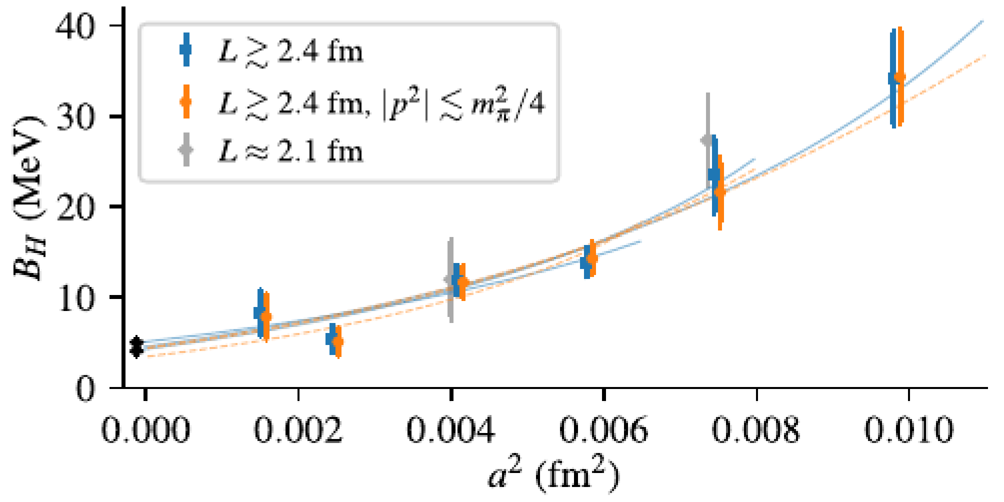
H-dibaryon

T. I. etal [HALQCD collaboration]
Phys. Rev. Lett. 106 162002 (2011)
Nucl. Phys. A881, 28 (2012)



- Left: Flavor singlet BB potential at five quark mass
- Right: the ground state from the potential
  - which is 20 - 50 [MeV] below the threshold
  - A stable(bound) H-dibaryon exists in these SU(3)F sym. world!

Mainz group paper 2021



J. R. Green, A. D. Hanlon, P. M. Junnarkar and H. Wittig,
Phys. Rev. Lett. 127, 242003 (2021)

- Flavor singlet BB $^1S_0$ sector in LQCD numerical calculation
- At a flavor SU(3) limit with $M_\pi = M_K \simeq$ 420 [MeV],
- Lüschers' finite volume method w/ energy levels extracted with a variational method
- Lattice w/ six $a = 0.10 - 0.05$ [fm] , $L = 3.1 - 2.1$ [fm]
- They clame strong "a" dependence of observables.

At the continum limit $B_H = 4.56 \pm 1.13 \pm 0.63$ [MeV]

Purpose

- We were surprised by the Mainz group result.
- We study influence of "a" on HALQCD BB interactions by performing comptation on finer lattice than before but with large box enough to accommodate BB.
- Use Fugaku supercomputer at R-CCS

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Gauge conf. generation

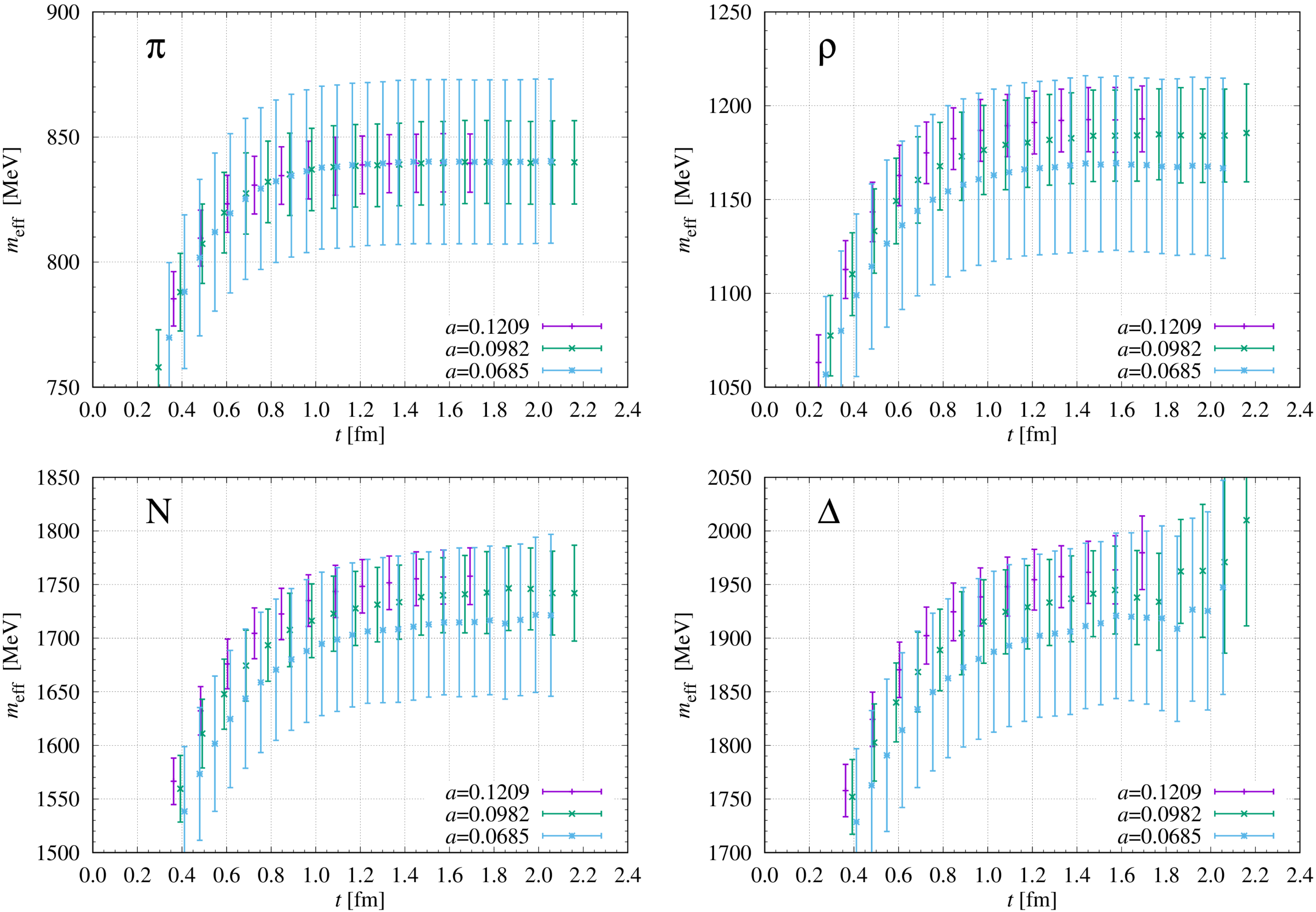
- Actions
  · renormalization-group improved gauge action
  · non-perturbatively O(a) improved clover quark action
  - from www.jldg.org/ildg-data/CPPACS+JLQCDconfig.html
  - β=1.83, Kuds=0.13760, Csw=1.761 (a=0.1209(16) fm) 1.3%
  - β=1.90, Kuds=0.13640, Csw=1.715 (a=0.0982(19) fm) 1.9%
  - β=2.05, Kuds=0.13540, Csw=1.628 (a=0.0685(26) fm) 3.8%

  K-input

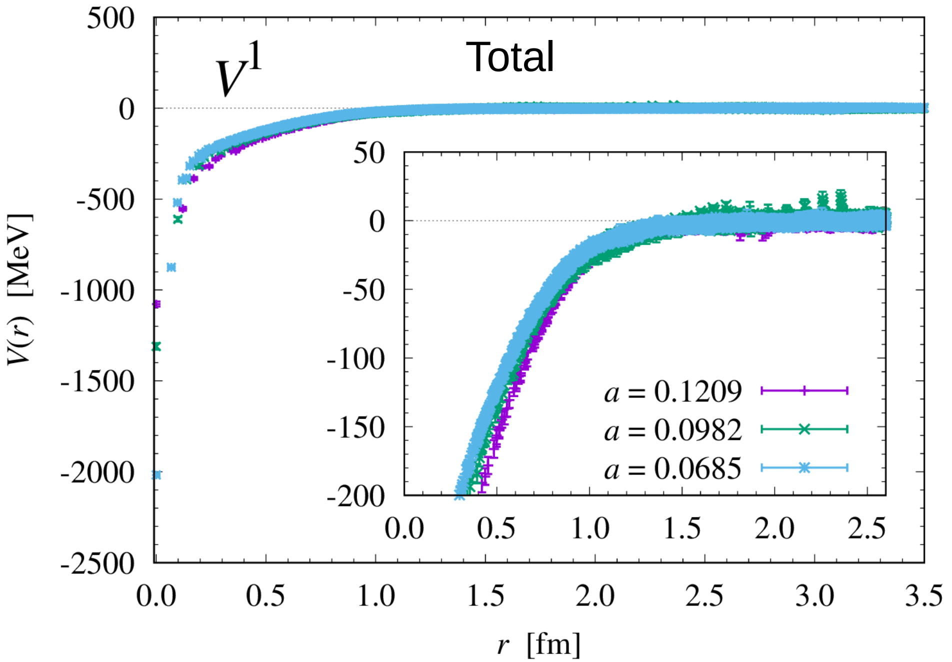
- Lattice
  - β=1.83 : $32^3 \times 32$ Lattice, 4×4×4×1 MPI ← same as 10 years ago
  - β=1.90 : $48^3 \times 48$ Lattice, 8×6×4×1 MPI
  - β=2.05 : $64^3 \times 64$ Lattice, 8×8×8×1 MPI

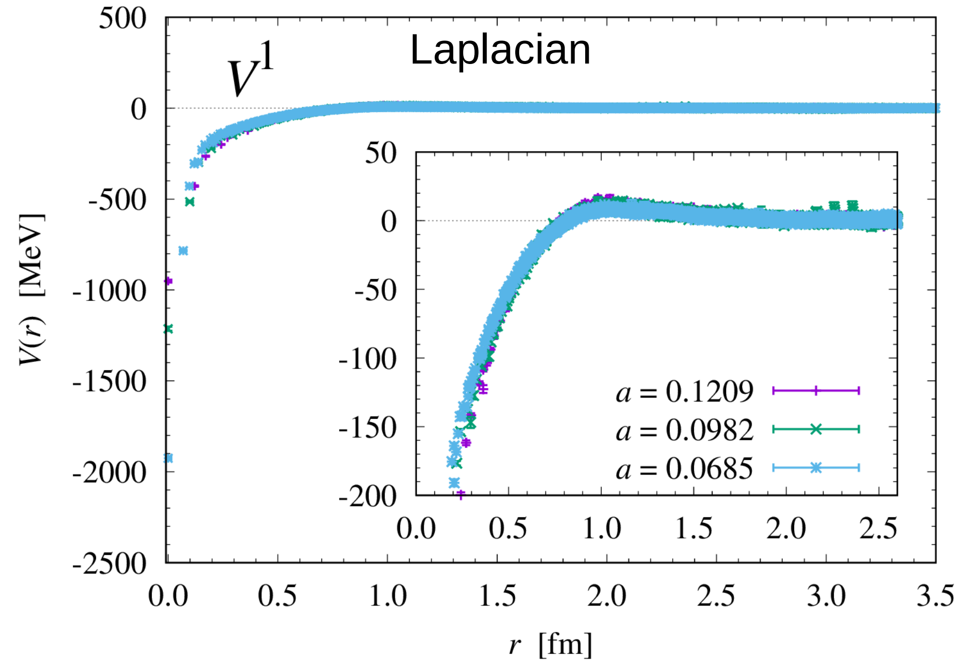
  (β=1.90, β=2.05) ← New
  - Bigger than their lattice chosen so that $L \simeq 4$ [fm]
- Quark mass
  - Tune parameter κ so that three M_π agree.

M_{eff} with systematic error of a



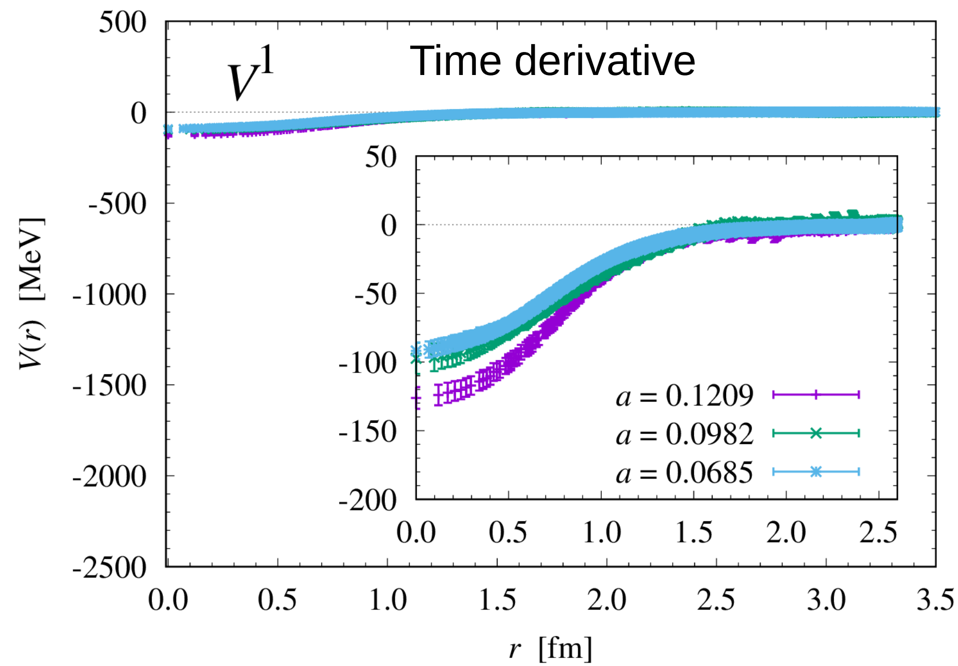
Flavor singlet BB potential





Laplacian | Time derivative

$$V(\vec{r}) = \boxed{\frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)}} \boxed{-\frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B}$$



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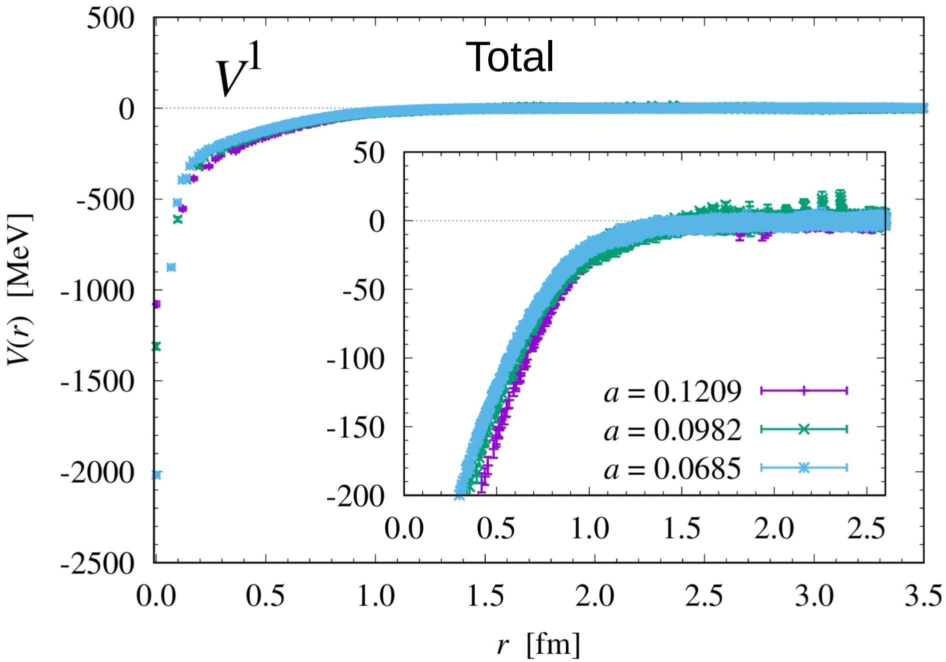

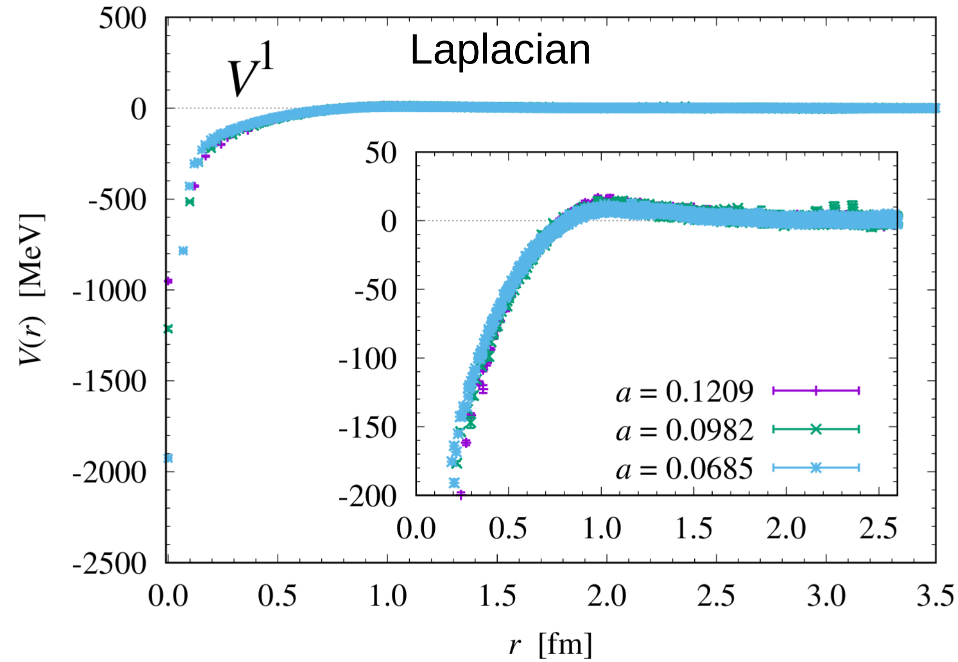

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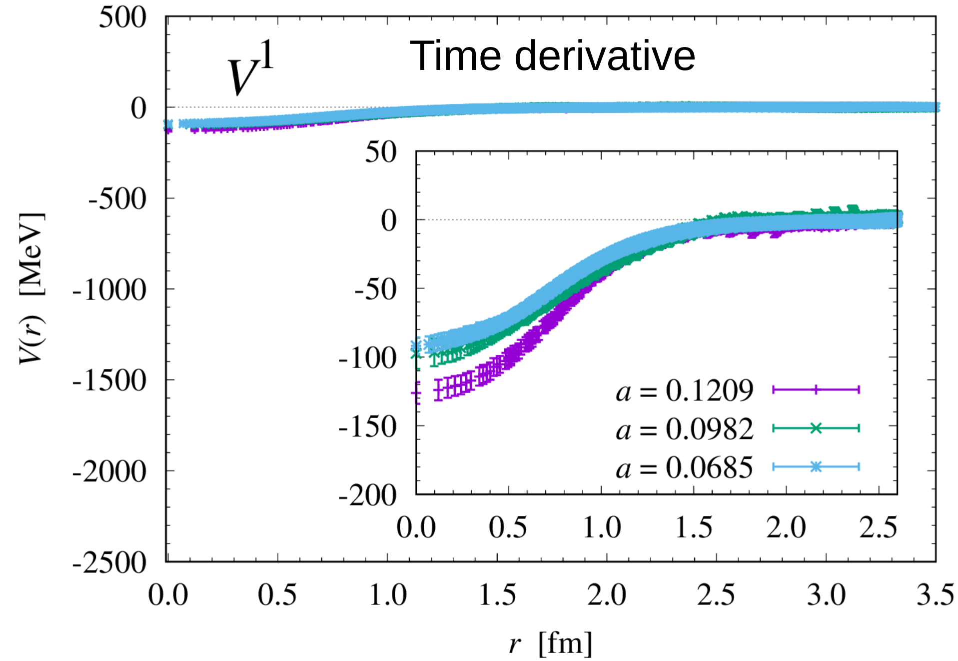

H-dibaryon

- Binding energy of H-dibaryon from the potential
  - $a = 0.1209(16) : B_{\mathrm{H}} = 40.354 \pm 4.045 \pm 2.236 \pm 0.579 \ \ [\mathrm{MeV}]$
  - $a = 0.0982(19) : B_{\mathrm{H}} = 22.893 \pm 3.632 \pm 1.537 \pm 0.805 \ \ [\mathrm{MeV}]$
  - $a = 0.0685(26) : B_{\mathrm{H}} = 14.350 \pm 1.676 \pm 1.445 \pm 1.351 \ \ [\mathrm{MeV}]$

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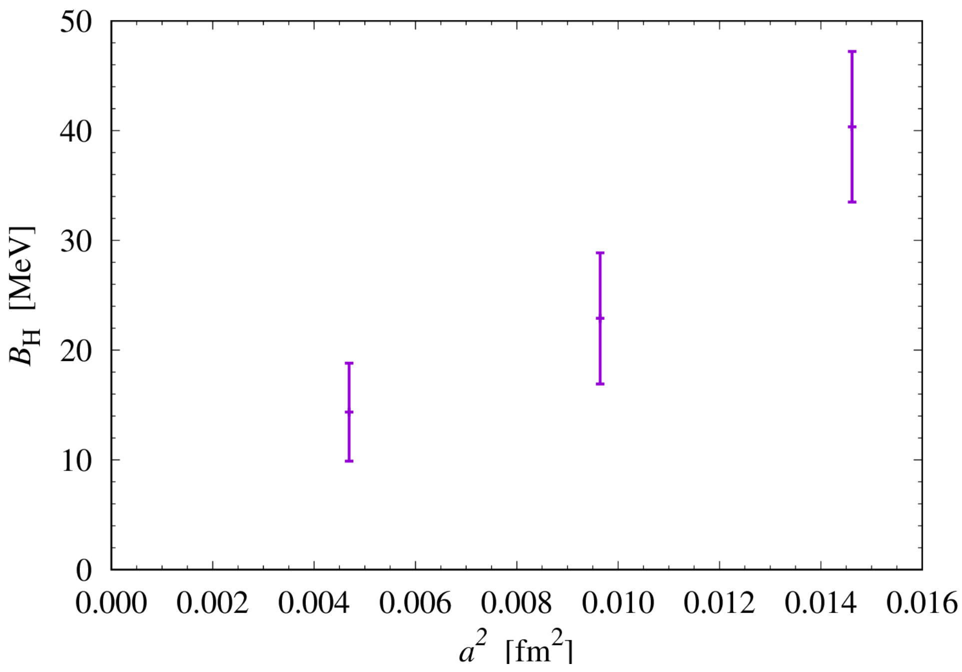

- B_H become smaller as "a" decrease...
- The trend is consistent with the Mainz result.
- The cause is not understood well at this moment.
- Probably, the action we use is not enough for the present purpose.

Summary and outlook

- Background, motivation and pupose
  - Motivation: Mainz group paper
  - Purpose: Study "a" influence of HALQCD BB int.
- Current status
  - Generate gauge conf. at the same M_π with 3 different "a".
  - Measure BB 4pt functions and extract BB interaction potentials.
  - Through $V^1(r)$, it seems that B_H decrease as "a" decrease.
  - This trend agrees with the Mainz paper result.
  - (may suggest an alert common to LQCD studies on multi-hadron)
- Need to study more.
  - Is this real ot not? How about in other sectors?
  - What is the cause? Any prescriptions?
  - Same quark mass point? We'll tune K_{uds} again respecting M_π/M_ρ .
  - We need more accuate "a". We may determine it by ourselves.
  - We may need to develop better action.

$$8 \times 8 = 27 + 8s + 1 + 10^* + 10 + 8a$$

Thank you for your attention

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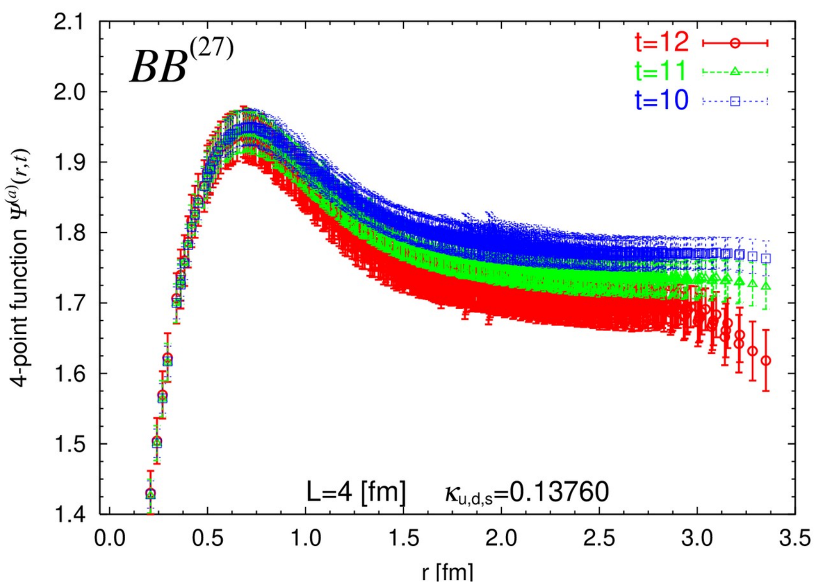

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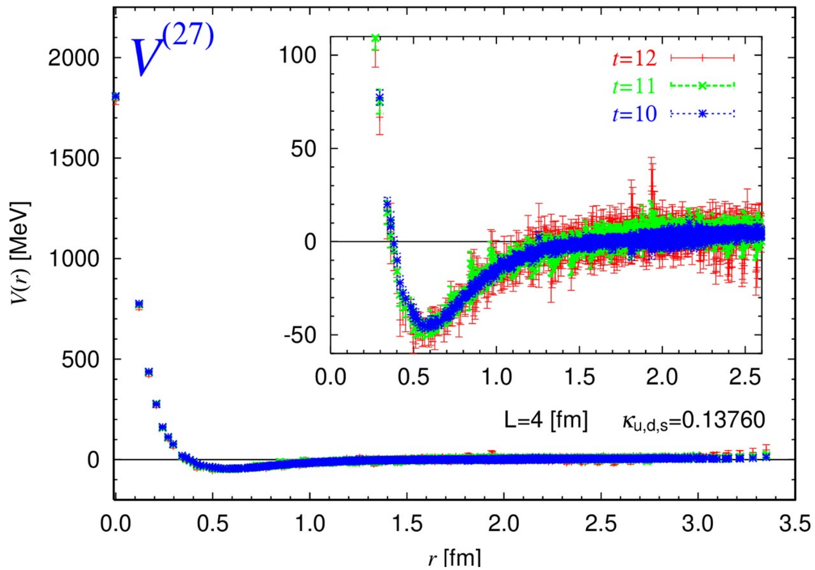

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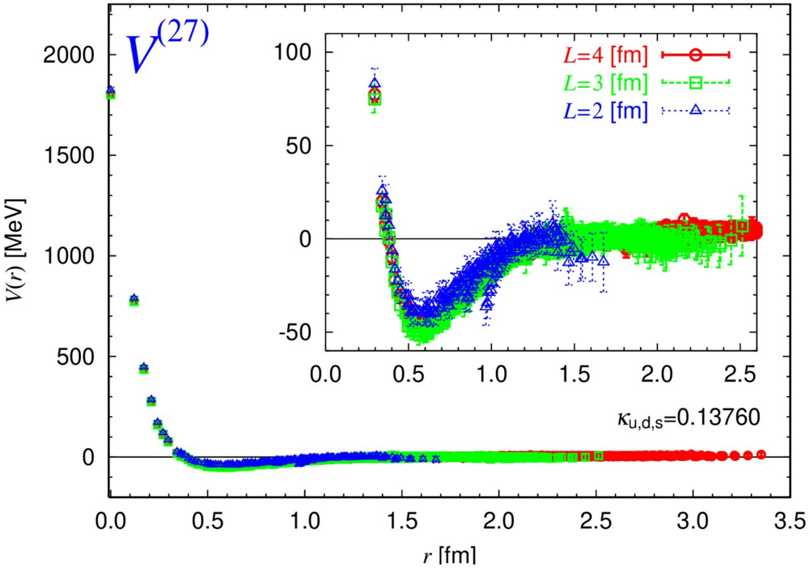


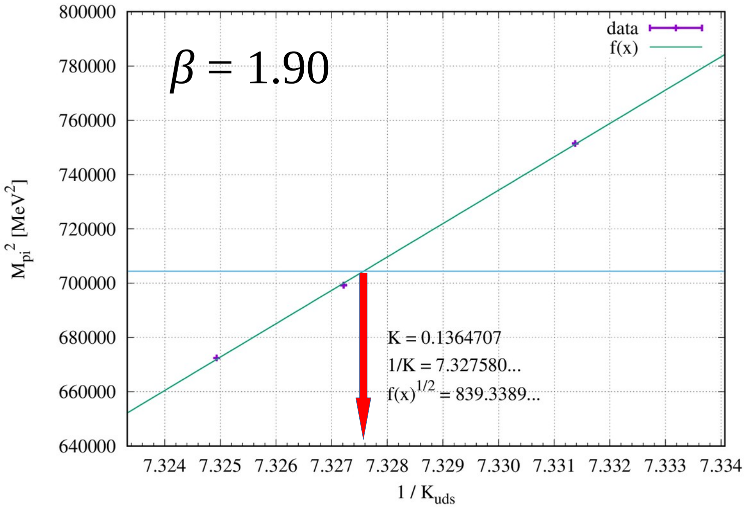
Tuning the paramete K_{uds}

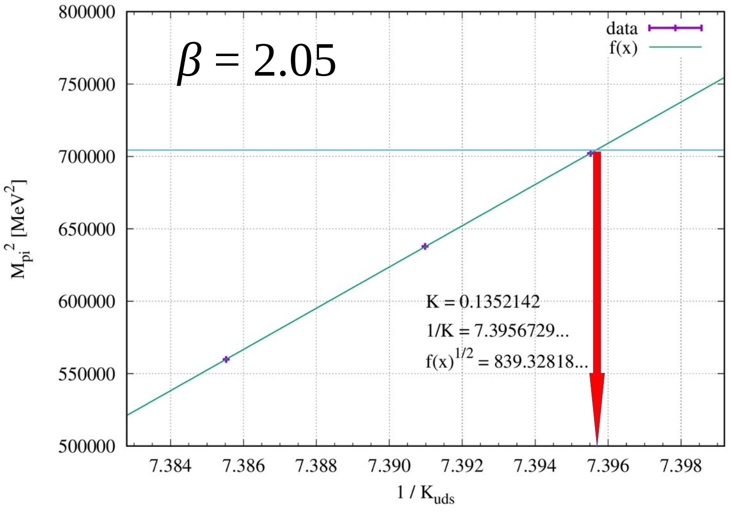
- Target is the most coarse one β=1.83, K_{uds}=0.13760, C_{sw}=1.761

pion	10 - 14	839.337 ± 0.476 ± 11.108
rho	10 - 14	1192.055 ± 1.089 ± 15.776
proton	11 - 14	1754.503 ± 1.752 ± 23.219
delta	11 - 14	1960.525 ± 3.087 ± 25.946

$$M_\pi / M_\rho = 0.704109 \pm 0.000390$$
$$M_\pi / M_N = 0.478390 \pm 0.000395$$

- Interpolate/Extrapolate $1/K_{uds}$ v.s. M_{pi}^2

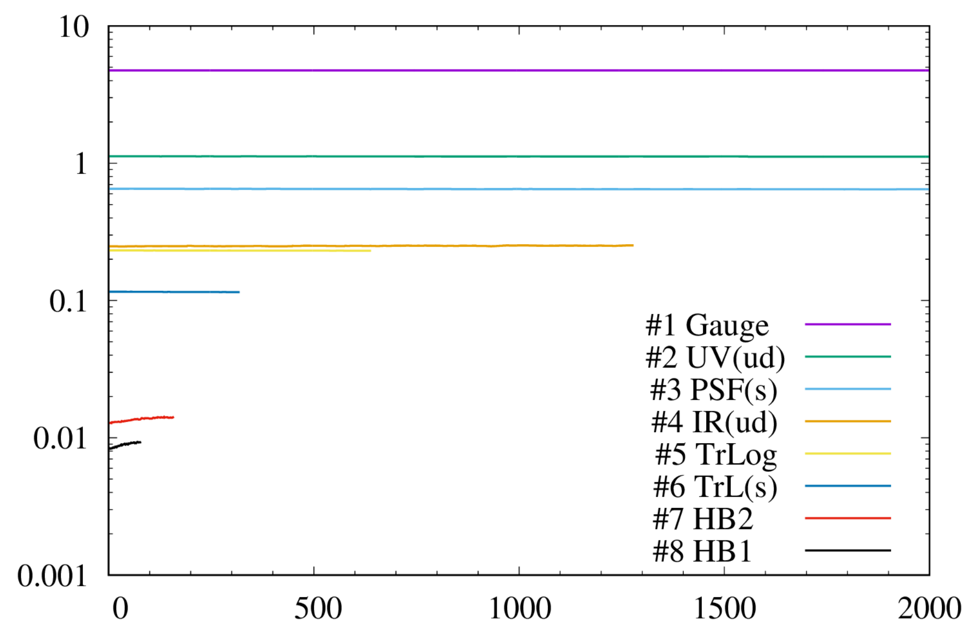




HMC parameters

Thanks to Prof. Ishikawa and Prof. Kanamori for providing and teaching the DDHMC code.

- Leapfrog algorithm, $dt = 1.0$
- NF2 HBHB: ρ_1=0.997, ρ_2=0.995 (Hasenbush, 2 stage)
- NF1 Rational Approximation: $[8.5\times10^{-6}, 5.0]$, order 20
- 4 MD depths:
  - To handle wide range of strengh of forces.
  - Gauge = 1st, UV(ud) = 2nd,
  - PSF(s), IR(ud), TrLog(ud), and TrLog(s) = 3rd
  - HB2 and HB1 = 4th
- Division of dt
  - 20 2 2 4 for β=1.83
  - 25 2 2 4 for β=1.90
  - 30 2 2 4 for β=2.05
- Tuned so that HMC Metropolis acceptance become 70 - 80 %.



Hadron mass

- β=1.83, Kuds=0.13760

systematic from a

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- β=1.90, Kuds=0.1364707

pion	17 - 22	839.878 ± 0.303 ± 16.250
rho	17 - 22	1184.289 ± 1.753 ± 22.914
proton	18 - 22	1744.635 ± 4.529 ± 33.756
delta	18 - 22	1955.090 ± 12.795 ± 37.828

$$M_\pi / M_\rho = 0.709183 \pm 0.000869$$
$$M_\pi / M_N = 0.481406 \pm 0.001137$$

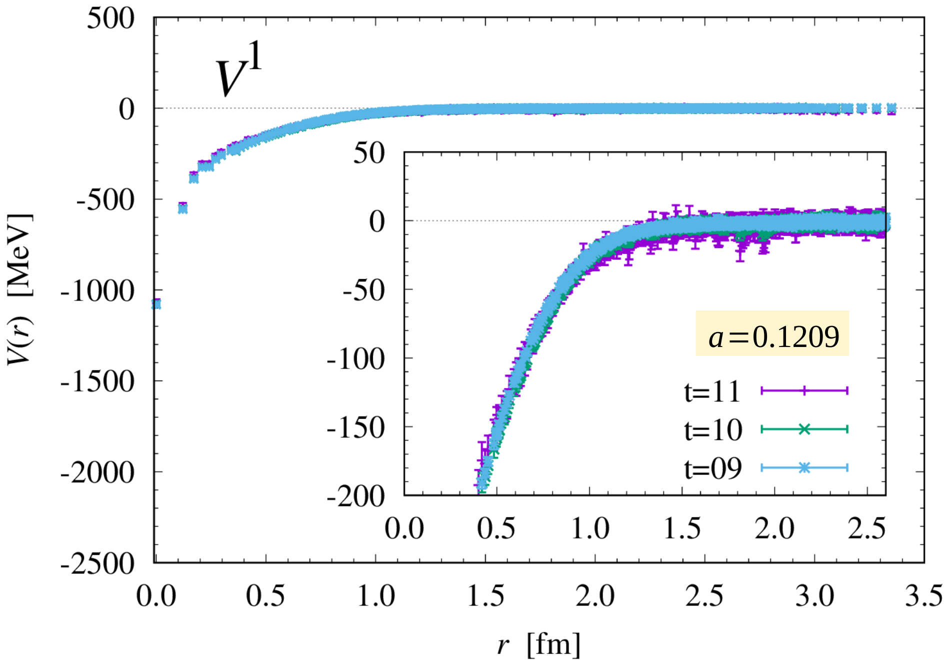
M_ρ, M_N, and M_Δ agree to β=1.83 with the large systematic error

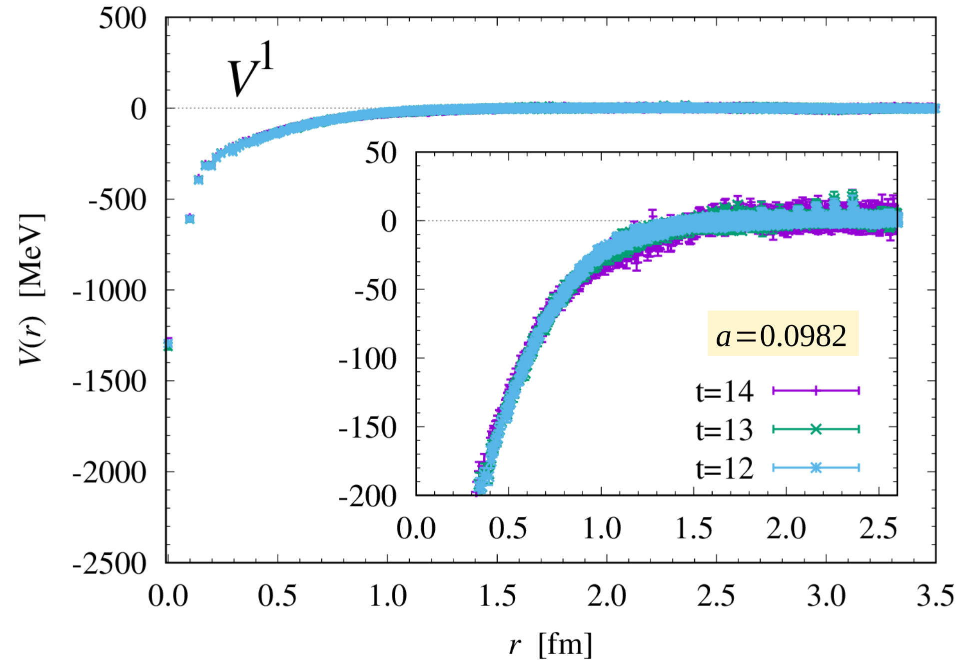
- β=2.05, Kuds=0.1352142

pion	21 - 30	840.083 ± 0.911 ± 31.886
rho	21 - 30	1168.348 ± 2.187 ± 44.346
proton	23 - 30	1715.704 ± 4.120 ± 65.122
delta	23 - 30	1918.915 ± 9.335 ± 72.835

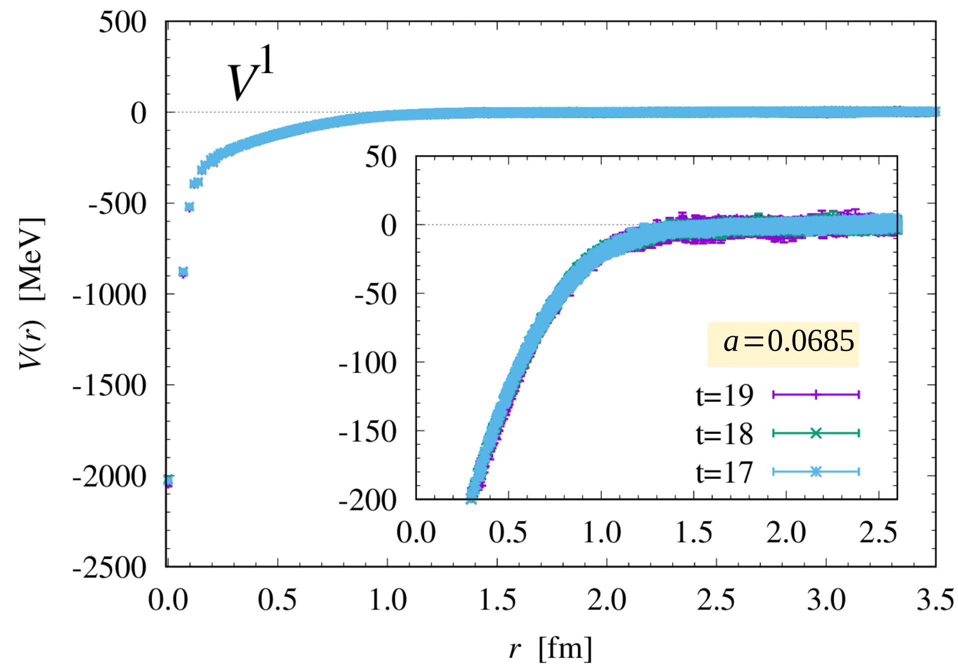
$$M_\pi / M_\rho = 0.719035 \pm 0.000702$$
$$M_\pi / M_N = 0.489643 \pm 0.000857$$

Flavor-Singlet BB potential

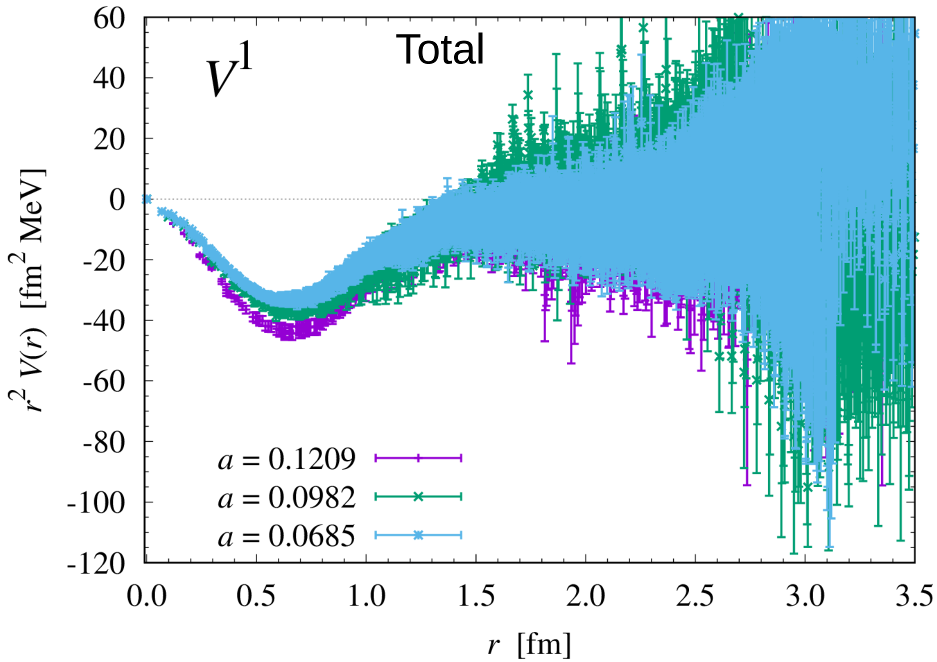


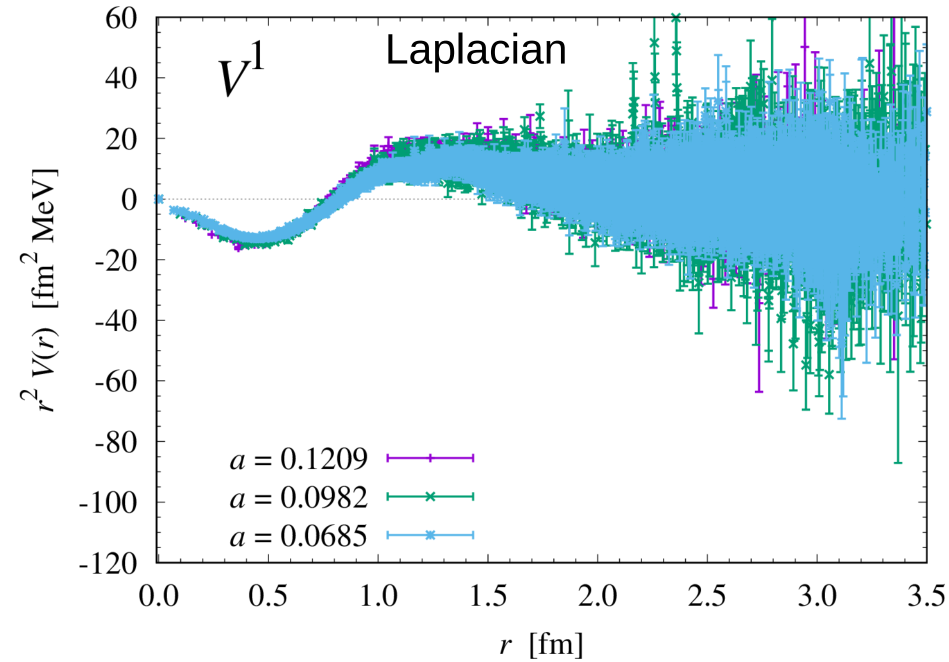


- Statistic error only.
- $V^1(r)$ become singular as "a" decrease i.e.
  - deeper at the origing
  - and narrower.



Potential comparison by parts





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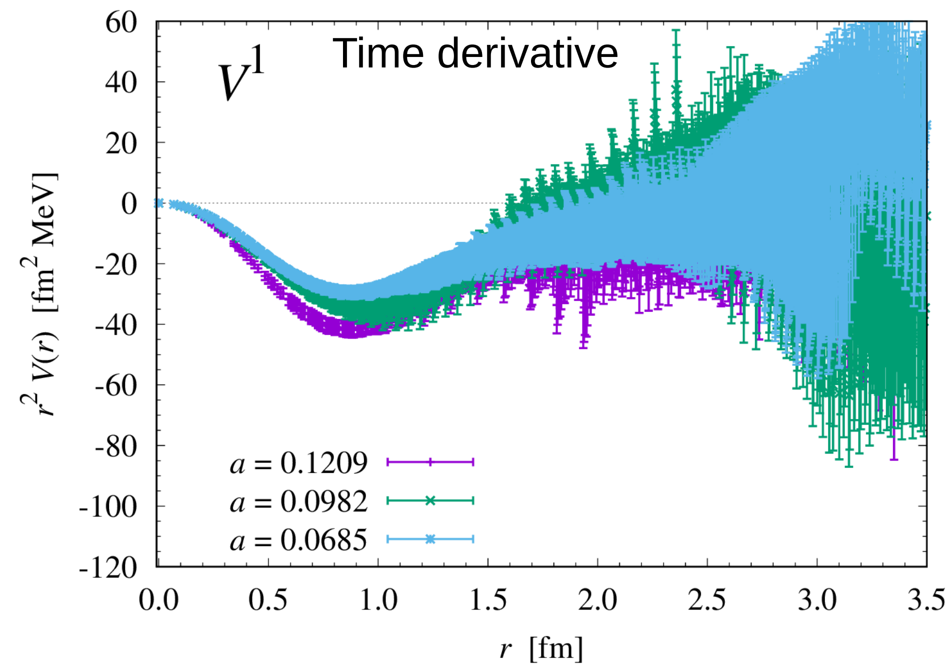
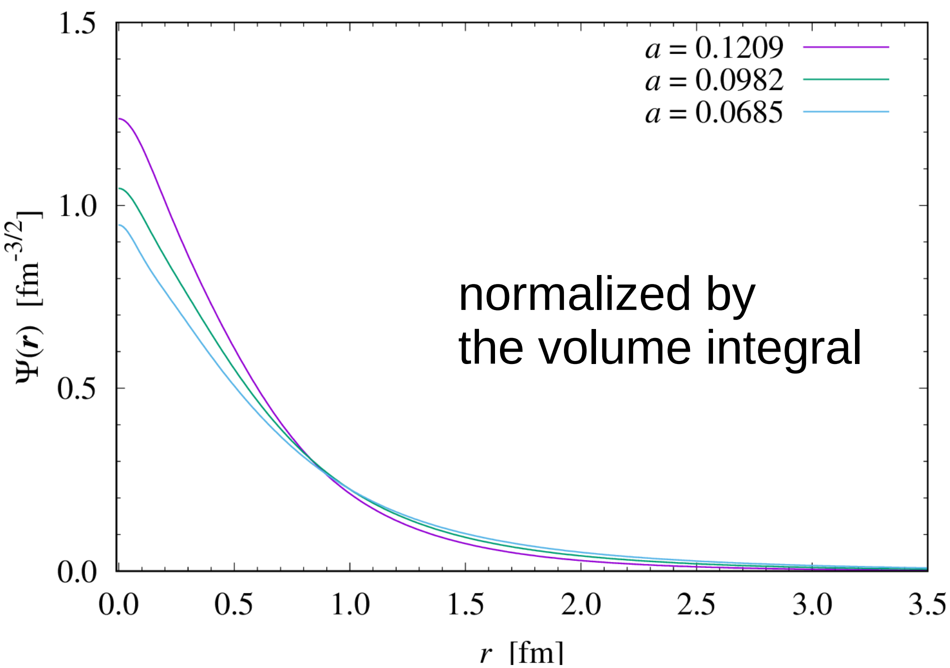
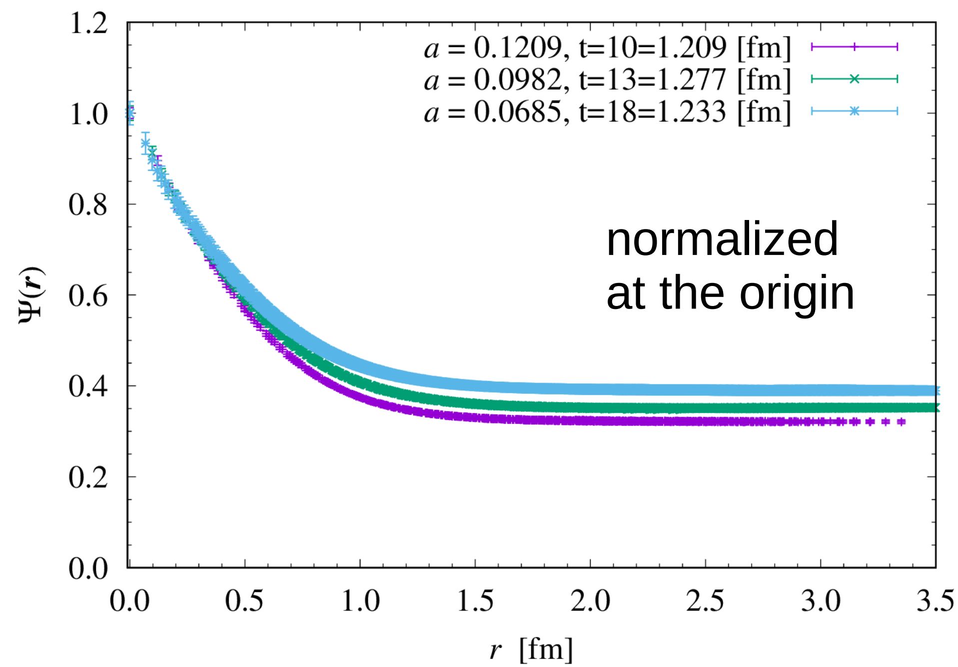


Wave functions





- Left: Resulting H-dibaryon w.f. from the potential $V^1(r)$.
  - We see that the w.f. spreads as "a" decrease.
- Right: Measured 4-point fouction in flavor singlet BB.
  - (Is the above feature seen in the original LQCD data too?)
  - (If so, the failure in the analysis is not the couse. But, it is not clear...)