The Three-Proton Correlation function

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Introduction

- Recently the nnn and ppp systems have attracted a particular interest
- In the two-body case, the nn system is much less known than the pp (or np) system
- In s-wave they interact in the $S = 0$, $T = 1$ channel.
- The associate scattering lengths are negative and large indicating the presence of a virtual state:
  $a_{nn} \approx -18.5\,\text{fm}$, $a^{sr}_{pp} \approx -17.5\,\text{fm}$, $a_{np} \approx -23.7\,\text{fm}$
- The effective ranges are all similar:
  $r_{nn} \approx r_{pp} \approx r_{np} \approx 2.8\,\text{fm}$
- The nnn and nnnn have been subject of intense investigations to determine if they have a low energy resonance state
- Here I discuss the $3 \rightarrow 3$ scattering: nnn and ppp

The pp correlation function

- Experimentally, the pp correlation function is defined as

$$C_{pp}(k) = \xi(k) \otimes \frac{N_{\mathrm{same}}(k)}{N_{\mathrm{mixed}}(k)}$$

- $\xi(k) \rightarrow$ corrections for experimental effects
- $N_{\mathrm{same}}(k) \rightarrow$ number of detected particle pairs from the same event
- $N_{\mathrm{mixed}}(k) \rightarrow$ number of uncorrelated pairs, the so-called mixed-event technique

- Theoretically the definition of the correlation function is

$$C_{pp}(k) = \int d^3y \; S_R(y) |\Psi|^2$$

The pp correlation function

- The theoretical ingredients

$$C_{pp}(k) = \int d^3y \, S_R(y) |\Psi|^2$$

with S_R the source function defined as

$$S_R(y) = \frac{1}{(4\pi R^2)^{3/2}} e^{-(y/2R)^2}$$

and Ψ the pp scattering wave function

$$\Psi = \sum_{[LSJ]} u_{LSJ}(y) [Y_L(\hat{y}) \chi_S]_J = \Psi^0 + \sum_{[LSJ]}^{\bar{J}} \Psi_{LSJ}$$

with Ψ^0 the free scattering wave function. In Ψ_{LSJ} the interaction has been considered up to $\bar{J}$.

The pp free wave function

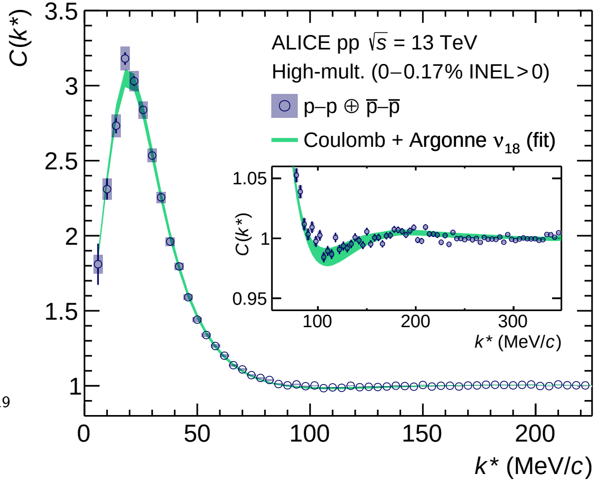
In the case of two protons, the scattering wave function has to be expanded in terms of Coulomb functions

$$\Psi_s^0 = 4\pi \sum_{[\ell S]} i^\ell (kr)^{-1} F_\ell(\eta, kr) \mathcal{Y}_{[\ell S]}(\hat{r}) \mathcal{Y}^*_{[\ell S]}(\hat{k})$$

with $F_\ell(\eta, kr)$ a regular Coulomb wave function and $\eta = e^2/(\hbar^2 k/m)$. The norm results

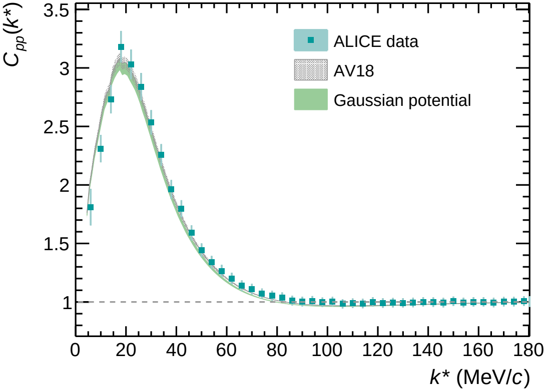
$$|\Psi_s^0|^2 = \frac{1}{2} \sum_{\ell \text{ even}} (kr)^{-2} F_\ell^2(\eta, kr)(2\ell + 1) + \frac{3}{2} \sum_{\ell \text{ odd}} (kr)^{-2} F_\ell^2(\eta, kr)(2\ell + 1)$$

The pp Correlation Function



ALICE collaboration
Phys. Lett. B 805, (2020) 135419

The pp Correlation Function



Using a gaussian potential: $V_{pp}(r) = V_0 e^{-(r/r_0)^2} \mathcal{P}_0 + \frac{e^2}{r}$

The pd Correlation Function

- We now consider the pd correlation function:

$$A_d C_{pd}(k) = \frac{1}{6} \sum_{m_2, m_1} \int d^3 r_1 d^3 r_2 d^3 r_3 \; S_1(r_1) S_1(r_2) S_1(r_3) |\Psi_{m_2, m_1}|^2$$

- the probability of deuteron formation

$$A_d = \frac{1}{3} \sum_{m_2} \int d^3 r_1 d^3 r_2 \; S_1(r_1) S_1(r_2) |\phi_{m_2}|^2$$

- the single particle source function

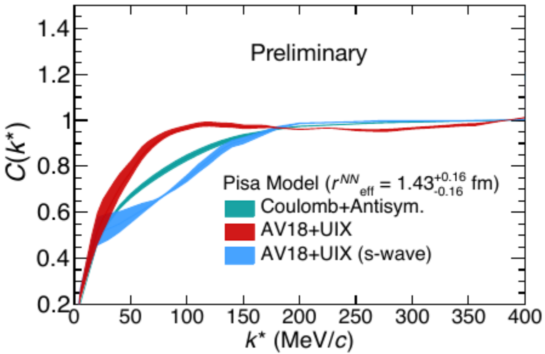
$$S_1(r) = \frac{1}{(2\pi R_{\mathrm{M}}^2)^{\frac{3}{2}}} e^{-r^2/2R_{\mathrm{M}}^2}$$

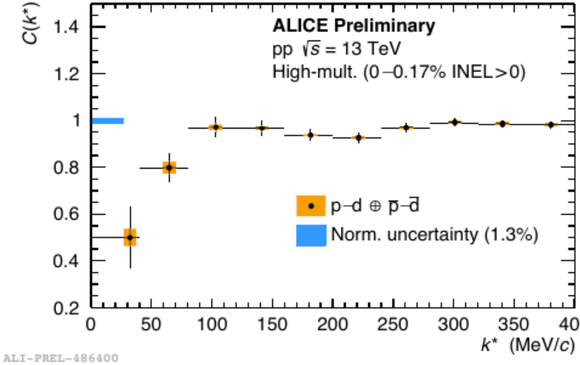
The pd Correlation Function

- the pd correlation function results

$$A_d C_{pd}(k) = \frac{1}{6} \sum_{m_2,m_1} \int \rho^5 d\rho d\Omega \, \frac{e^{-\rho^2/4R_M^2}}{(4\pi R_M^2)^3} |\Psi^{pd}_{m_2,m_1\vec{k}}|^2$$

$$\Psi^{pd}_{m_2,m_1,\vec{k}} = \sum_{LSJ} \sqrt{4\pi} i^L \sqrt{2L+1} e^{i\sigma_L} (1 m_2 \frac{1}{2} m_1 \mid S J_z)(L 0 S J_z \mid J J_z) \Psi_{LSJJ_z} \;\;,$$





M. Viviani, S. König, A. Kievsky, L.E. Marcucci, B. Singh, O. Vázquez Doce, arXiv:2306.02478 [nucl-th]
ALICE collaboration+theoreticians, in preparation

The ppp correlation function

- Now we consider the ppp correlation function:

$$C_{ppp}(Q) = \int \rho^5 d\rho d\Omega \; S_{\rho_0}(\rho)|\Psi_{ppp}|^2$$

with Q the hyper-momentum, S_{ρ_0} the source function defined as

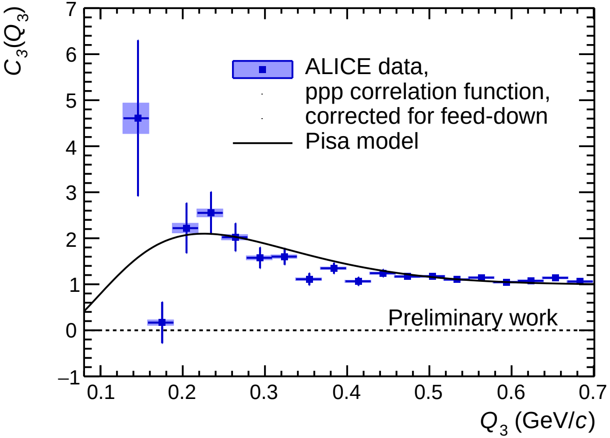
$$S_{\rho_0}(\rho) = \frac{1}{\pi^3 \rho_0^6} e^{-(\rho/\rho_0)^2}$$

Ψ_{ppp} is the ppp scattering wave function

$$\Psi_{ppp} = \sum_{[K]} u_{[K]}(\rho)\mathcal{B}_{[K]}(\Omega) = \Psi^0 + \sum_{J,[K]}^{\overline{J},\overline{K}} \Psi^J_{[K]}$$

To be noticed that Ψ^0 is not well known. In $\Psi^J_{[K]}$ the interaction has been considered up to $\overline{J}$ and $\overline{K}$

Comparison to data (preliminary)



Some remarks

- In the pp correlation function the main ingredient is the Ψ_{pp} scattering function. Accordingly $C_{pp}(k)$ is sensitive to the NN interaction.
- In the pd and ppp correlation functions the main ingredients are the Ψ_{pd} and Ψ_{ppp} scattering functions. Accordingly $C_{pd}(k)$ and $C_{ppp}(Q)$ should be sensitive to the NN interaction (and NNN interaction).
- The ppp wave function is an open problem due to long-range coulomb interaction

The ppp Wave Function

The total wave function is

$$\Psi(\vec{x},\vec{y}) = \sum_i \psi(\vec{x}_i,\vec{y}_i) = \rho^{-5/2} \sum_{[K]} u_{[K]}(\rho) \mathcal{B}^{J\pi}_{[K]}(\Omega)$$

with $\mathcal{B}^{J\pi}_{[K]}$ antisymmetric HH-spin functions

The ppp wave is completely determined from the hyperradial functions $u_{[K]}(\rho)$. And they are determined from the boundary conditions as $\rho \to \infty$.

For a given energy, $E = \hbar^2 Q^2/m$, and in the nnn case

$$u_{[K]}(\rho \to \infty) \to \sqrt{Q\rho}\,[J_{K+2}(Q\rho) + \tan\delta_K\, Y_{K+2}(Q\rho)]$$

In the ppp case the asymptotic equations are coupled not allowing this simple picture

ppp Correlation Analysis

Using the property of the HH functions

$$\Psi_s^0 = e^{i\vec{Q}\cdot\vec{\rho}} = \frac{(2\pi)^3}{(Q\rho)^2}\sum_{[K]} i^K J_{K+2}(Q\rho)\mathcal{Y}_{[K]}(\Omega)\mathcal{Y}^*_{[K]}(\hat{Q})$$

where $\vec{Q}\cdot\vec{\rho} = \vec{k}_1\cdot\vec{x} + \vec{k}_2\cdot\vec{y}$ and J_{K+2} a Bessel function.

- For the case of three nucleons we have to include the correct symmetrization.
- For the case of three protons we have to include the correct asymptotics

The nnn case:

$$\Psi_s^0 = \frac{(2\pi)^3}{(Q\rho)^2}\sum_{[K]} i^K J_{K+2}(Q\rho)\mathcal{B}_{[K]}(\Omega)\mathcal{B}^*_{[K]}(\hat{Q})$$

with $\mathcal{B}_{[K]}(\Omega)$ antisymmetric in the hyperangle-spin space.

ppp Correlation Analysis

For three protons the asymptotic form changes (and it is not known in a close form)
The Coulomb interaction coupled the asymptotic equations through the term

$$\sum_{ij} \frac{e^2}{r_{ij}}$$

In this preliminary study we perform an average of the Coulomb interaction on the hyperangles

$$V_c(\rho) = \int d\Omega \sum_{ij} \frac{e^2}{r_{ij}} |\mathcal{Y}_0(\Omega)|^2 = \frac{16}{\pi}\frac{e^2}{\rho}$$

and the plane wave takes the form

$$e^{i\vec{Q}\cdot\vec{\rho}} \rightarrow \Psi_s^0 = \frac{1}{C_{3/2}(0)} \frac{(\pi)^3}{(Q\rho)^{5/2}} \sum_{[K]} i^K F_{K+3/2}(\eta, Q\rho) \mathcal{B}_{[K]}(\Omega) \mathcal{B}^*_{[K]}(\hat{Q})$$

ppp Correlation Analysis

The ppp wave function is

$$\Psi_{ppp} = \sum_{[K]} u_{[K]}(\rho)\mathcal{B}_{[K]}(\Omega) = \Psi^0 + \sum_{J,K}^{\overline{J},\overline{K}} \Psi_K^J$$

To determine Ψ^J we use the Adiabatic Hyperspherical Harmonic basis:

$$-\frac{\hbar^2}{m}\frac{\Lambda^2(\Omega)}{\rho^2} = H_\Omega \phi_\nu(\rho,\Omega) = U_\nu(\rho)\phi_\nu(\rho,\Omega)$$

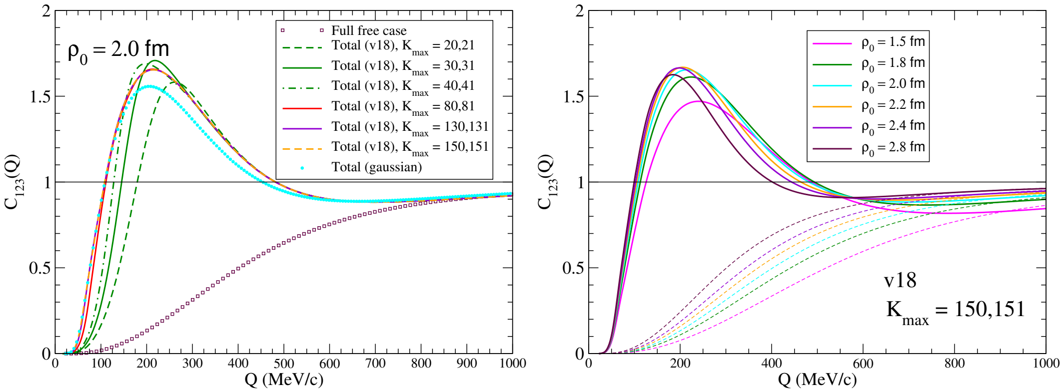
$$\Psi_K^J = \rho^{-5/2} \sum_\nu w_\nu^{J,K}(\rho)\phi_\nu(\rho,\Omega)$$

with the adiabatic functions $\phi_\nu(\rho \to \infty, \Omega) \to \mathcal{B}_{[K]}(\Omega)$

and the hyperradial functions $w_\nu^{J,K}(\rho) \to \sqrt{Q\rho}[\delta_{KK'} F_{K+3/2}(Q\rho) + T_{KK'}\mathcal{O}_{K'+3/2}(Q\rho)]$

The ppp correlation function $C_{123}(Q) = \int \rho^5 d\rho \, d\Omega \, S_{123} |\Psi_{ppp}|^2$

Convergence and preliminary results with different size sources



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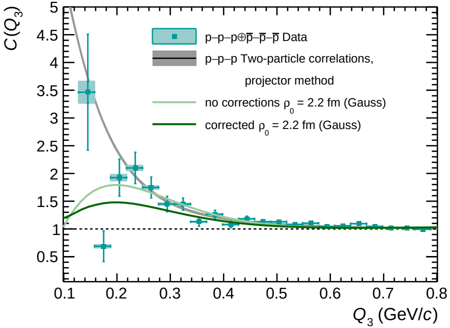

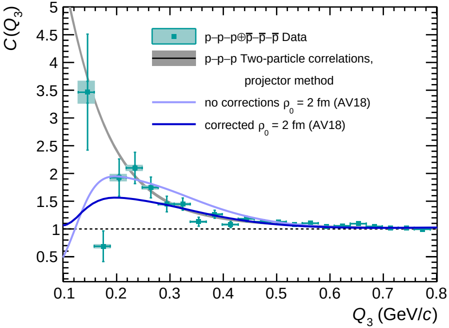

Summary

- Although its apparent simplicity, the three-nucleon problem is of great complexity
- Measurements of the correlation function allow for new tests of the NN and NNN interactions
- In the ppp case the Coulomb interaction couples the asymptotic dynamics increasing the difficulties of the numerical treatment
- In this preliminary study the Coulomb interaction was averaged on the hyperangles
- The Adiabatic expansion was used in the lowest channels

- The next work is to include more channels and to relax the average of the Coulomb force
- Studies on the ppΛ correlation function has been started