Nucleon-nucleon interactions in the large-N_c expansion

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25th European Conference on Few-Body Problems in Physics
Mainz, July 31- August 4, 2023

Work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics

Large-N_c QCD

- QCD with number of colors $N_c \to \infty$
- Systematic expansion in $1/N_c$
- Phenomenologically successful
- Baryon sector: emergent SU(2F) symmetry
- Application to NN interactions:
  - Determine large-N_c scaling of contributions to potential

Combine with EFTs to obtain dual expansion

  - Derive $1/N_c$ hierarchy among low-energy coefficients (LECs)

't Hooft (1974); Witten (1979); Dashen, Jenkins, Manohar (1994,95); Kaplan, Savage (1996); Kaplan, Manohar (1997)

NN potential in large-N_c expansion

$$V(p_-, p_+) = \langle N_C(p_1'), N_D(p_2') \,|\, H \,|\, N_A(p_1), N_B(p_2) \rangle$$

- Effective Hamiltonian

$$H = N_c \sum_{s,t,u} v_{stu} \left(\frac{S}{N_c}\right)^s \left(\frac{I}{N_c}\right)^t \left(\frac{G}{N_c}\right)^u$$

- Building blocks

$$S^i = q^\dagger \frac{\sigma^i}{2} q, \quad I^a = q^\dagger \frac{\tau^a}{2} q, \quad G^{ia} = q^\dagger \frac{\sigma^i \tau^a}{4} q$$

- Coefficients v_{stu}
  - Momentum dependent
  - Constrained by symmetries

Dashen, Jenkins, Manohar (1994); Kaplan, Savage (1996); Kaplan, Manohar (1997); Cohen et al. (2002)

Large-N_c scaling

- Nucleon matrix elements

$$\langle N'|G^{ia}|N\rangle \sim \langle N'|1|N\rangle \sim O(N_c)$$
$$\langle N'|S^{i}|N\rangle \sim \langle N'|I^{a}|N\rangle \sim O(1)$$

- Momenta (in t-channel)

$$p_- = (p_1' - p_2') - (p_1 - p_2) \qquad \sim O(1)$$
$$p_+ = (p_1' - p_2') + (p_1 - p_2) \sim 1/M_N \sim O(1/N_c)$$

- Coefficients (excluding momenta)

$$\tilde{v}_{stu} \sim 1$$

Dashen, Jenkins, Manohar (1994,95); Kaplan, Savage (1996); Kaplan, Manohar (1997)

Caveats

- Nuclear matter forms classical crystal for $N_c \to \infty$?
  - Assume that symmetries of NN interactions do not change
- Nucleon and Δ degenerate in large-N_c limit
- Δ plays important role in meson-baryon interactions
  - Ignore intermediate Δ states (for now)

Kaplan, Manohar (1997); Banerjee, Cohen, Gelman (2002)

Pionless EFT

- Very low energies
- Only nucleons and external fields
- LO: *S*-wave interactions

$$\mathscr{L} = -\frac{1}{2}C_S(N^\dagger N)(N^\dagger N) - \frac{1}{2}C_T(N^\dagger \sigma^i N)(N^\dagger \sigma^i N)$$

- LECs C_S, C_T assumed “natural” in EFT
- Alternatively: partial-wave basis

$$C^{(^1S_0)} = (C_S - 3C_T), \quad C^{(^3S_1)} = (C_S + C_T)$$

Pionless EFT and the large-N_c expansion

- Spin-isospin structure of operators

$$(N^\dagger N)(N^\dagger N) \sim 1_1 \cdot 1_2 \qquad (N^\dagger \sigma^i N)(N^\dagger \sigma^i N) \sim \hat{S}_1 \cdot \hat{S}_2$$

- Large-N_c scaling of LECs

$$C_S \sim O(N_c) \qquad C_T \sim O(1/N_c)$$

- In large-N_c limit

$$C^{(^1S_0)} = C^{(^3S_1)}$$

- Wigner SU(4) symmetry (in S waves)

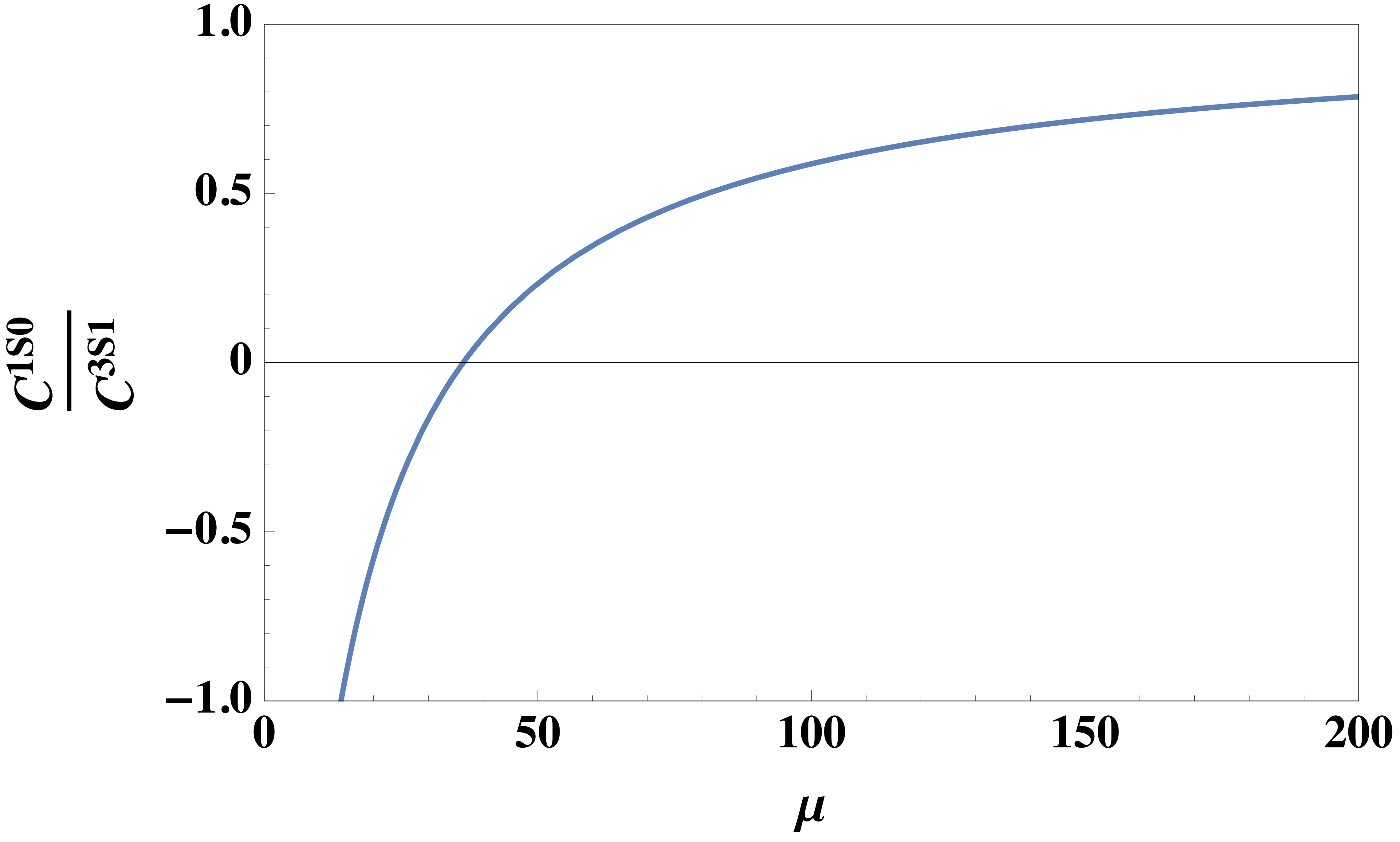
Kaplan, Savage (1996)

Renormalization-point dependence

- In PDS renormalization

$$\frac{C^{^1S_0}}{C^{^3S_1}} = \frac{1/a^{^3S_1} - \mu}{1/a^{^1S_0} - \mu} \xrightarrow{\mu\to 0} \frac{a^{^1S_0}}{a^{^3S_1}} \approx -4.4$$

- Agreement with large-N_c expected errors for $\mu \gtrsim m_\pi$



Kaplan, Savage (1996)

Higher orders in pionless EFT

- Two-derivative operators: *S* waves, *P* waves, *S*-*D* mixing
- Fit to phase shifts and mixing parameter (μ=120 MeV)

$$\left.C_{1\cdot 1} = (-0.58 \pm 0.11)\ \mathrm{fm}^4\,, \quad C_{G\cdot G} = (0.40 \pm 0.05)\ \mathrm{fm}^4\,, \quad C'_{G\cdot G} = (0.84 \pm 0.05)\ \mathrm{fm}^4 \right\}\ \mathrm{LO}$$

$$\left.\begin{array}{l} C_{\tau\cdot\tau} = (0.15 \pm 0.07)\ \mathrm{fm}^4\,, \quad C_{\sigma\cdot\sigma} = (-0.39 \pm 0.07)\ \mathrm{fm}^4\,, \quad C'_{\sigma\cdot\sigma} = (0.78 \pm 0.1)\ \mathrm{fm}^4 \\ \overset{\leftrightarrow}{C}_{1\cdot\sigma} = (-0.17 \pm 0.12)\ \mathrm{fm}^4 \end{array}\right\}\ \mathrm{NLO}$$

- Fit to phase shifts and increase mixing parameter by factor of 3

$$\left.C_{1\cdot 1} = (-0.59 \pm 0.10)\ \mathrm{fm}^4\,, \quad C_{G\cdot G} = (0.11 \pm 0.06)\ \mathrm{fm}^4\,, \quad C'_{G\cdot G} = (1.72 \pm 0.13)\ \mathrm{fm}^4 \right\}\ \mathrm{LO}$$

$$\left.\begin{array}{l} C_{\tau\cdot\tau} = (0.16 \pm 0.07)\ \mathrm{fm}^4\,, \quad C_{\sigma\cdot\sigma} = (-0.10 \pm 0.08)\ \mathrm{fm}^4\,, \quad C'_{\sigma\cdot\sigma} = (-0.10 \pm 0.16)\ \mathrm{fm}^4 \\ \overset{\leftrightarrow}{C}_{1\cdot\sigma} = (-0.17 \pm 0.12)\ \mathrm{fm}^4 \end{array}\right\}\ \mathrm{NLO}$$

- Other physics can impact size of LECs

MRS, Singh, Springer (2018)

Magnetic couplings in pionless EFT

- NN contact terms coupled to magnetic field B

$$\mathcal{L} = eB_i \left[{}^{\not\pi}L_1 \left(N^T P_i N\right)^\dagger \left(N^T \bar{P}_3 N\right) - i\epsilon^{ijk}\, {}^{\not\pi}L_2 \left(N^T P_j N\right)^\dagger \left(N^T P_k N\right) \right] + \text{h.c.}$$

$P_i/\bar{P}_a$: projection onto ${}^3S_1/{}^1S_0$ partial waves

- Contributions to

    - ${}^{\not\pi}L_1$: radiative neutron capture $np \to d\gamma$, ${}^{\not\pi}L_2$: deuteron magnetic moment

- Extracted values at $\mu = m_\pi$: ${}^{\not\pi}L_1 = 7.24\,\text{fm}^4, \quad {}^{\not\pi}L_2 = -0.149\,\text{fm}^4$

- Simultaneously natural?

Chen, Rupak, Savage (1999); Beane et al. (2000)

Large-N$_c$ scaling of magnetic LECs

- Scaling not manifest in partial-wave basis
- "Large-N$_c$" basis

$$\mathcal{L} = eB^i \left[C_s^{(M)} \left(N^\dagger \sigma^i N\right) \left(N^\dagger N\right) + C_v^{(M)} \epsilon^{ijk} \epsilon^{3ab} \left(N^\dagger \sigma^j \tau^a N\right) \left(N^\dagger \sigma^k \tau^b N\right)\right]$$

- Large-N$_c$ scaling of LECs: $C_s^{(M)} \sim O(N_c^0), \quad C_v^{(M)} \sim O(N_c)$
- Numerical values at $\mu = m_\pi$:

$$C_s^{(M)} = 0.149\,\mathrm{fm}^4 \quad C_v^{(M)} = 0.905\,\mathrm{fm}^4$$

- Consistent to consider both $C_v^{(M)}$ and $C_s^{(M)}$ natural with large-N_c suppression

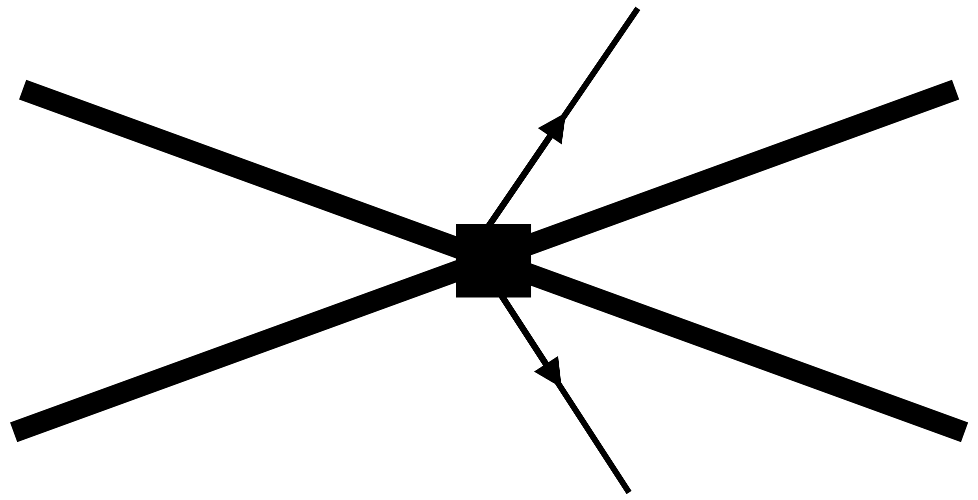
Richardson, MRS (2020); Detmold, Savage (2004)

Contact term for neutrinoless double beta decay

- LO light Majorana exchange transition operator for S-wave $nn \to ppe^-e^-$

$$V_{\nu,L}(\mathbf{q}) = \frac{\tau_1^+ \tau_2^+}{\mathbf{q}^2}\left[1 + 2g_A^2 + \frac{g_A^2 m_\pi^4}{(\mathbf{q}^2 + m_\pi^2)^2}\right]$$

- Evaluation between ${}^1S_0$ nn and pp states $\Rightarrow$ regulator dependence
- New short-range contribution $V_{\nu,S}$ at LO



- New undetermined LEC g_ν^{NN}
  - Related to CIB NN interaction with LEC C_1 by chiral symmetry
  - Data?
  - Lattice QCD?

Cirigliano et al. (2018, 2019)

g_ν^{NN} and large N$_c$

- Only linear combination $C_1 + C_2$ determined, where C_2 is independent CIB LEC
- Numerical impact estimated by assumption $g_\nu^{NN} \approx (C_1 + C_2)/2$
- Large-N$_c$ analysis

$$\mathcal{C}_1 = -3\bar{\mathcal{C}}_3 - 3\bar{\mathcal{C}}_6 = -3\bar{\mathcal{C}}_3\left[1 + O(1/N_c)\right]$$
$$\mathcal{C}_2 = -3\bar{\mathcal{C}}_3 + 3\bar{\mathcal{C}}_6 = -3\bar{\mathcal{C}}_3\left[1 + O(1/N_c)\right]$$

Supports assumption $g_\nu^{NN} \approx (C_1 + C_2)/2$

- Agrees with Cottingham-like analysis

Cirigliano et al. (2018, 2019); Richardson, MRS, Pastore, Springer (2021); Cirigliano et al. (2021)

Conclusions

- Large-N_c analysis
  - Captures nonperturbative QCD effects
  - Based on symmetry in baryon sector
  - Constraints in absence of data
  - Trends, not predictions
  - Only upper limits on size
  - Other scales can impact relative sizes

Conclusions

- Other applications
  - Three-nucleon interactions
  - Parity-violating NN interactions
  - Time-reversal-invariance-violating NN interactions
  - Two-nucleon EM and axial current operators
  - Dark matter couplings

Phillips, Schat (2013); Phillips, Samart, Schat (2015); Phillips, Samart, Schat, MRS (2016); MRS, Springer, Vanasse (2016); Nguyen, MRS, Springer (2021); Richardson, MRS (2020); Richardson, Lin, Nguyen (2022)

ANNUAL REVIEWS

Annual Review of Nuclear and Particle Science

Implications of Large-N_c QCD for the NN Interaction

Thomas R. Richardson,[1,2] Matthias R. Schindler,[3] and Roxanne P. Springer[2]