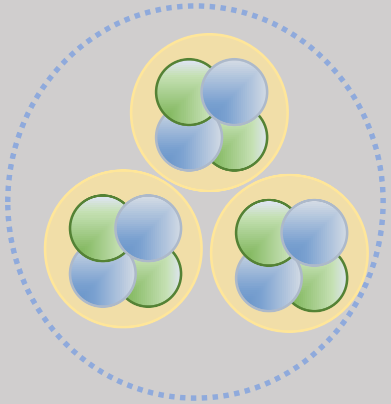
Structure of resonance states in three-alpha systems

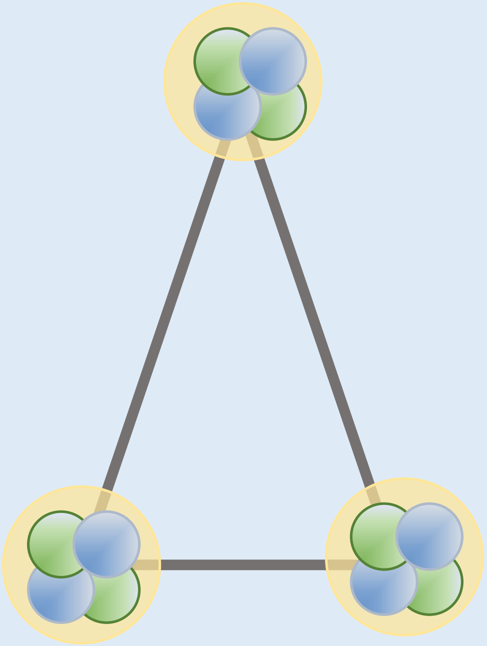


Souichi Ishikawa @Hosei University

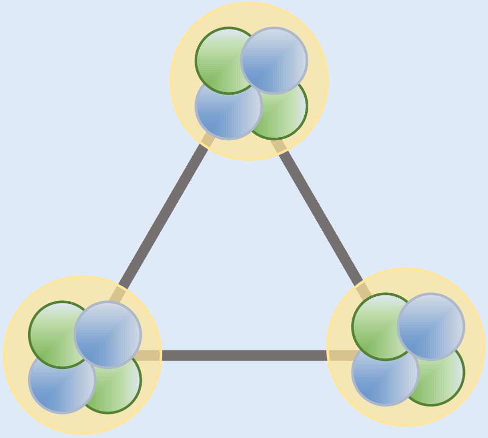
1. Introduction (1)

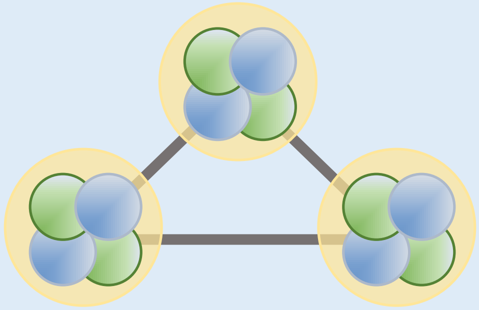
- Geometrical structure of low-energy excited states the ^{12}C nucleus in terms of 3-alpha configuration is interesting subject to study properties (transition strength, etc.) of these states.

Acute isosceles triangle



Equilateral triangle





Obtuse isosceles triangle (Bent arm)

1. Introduction (2)

Content of the talk:

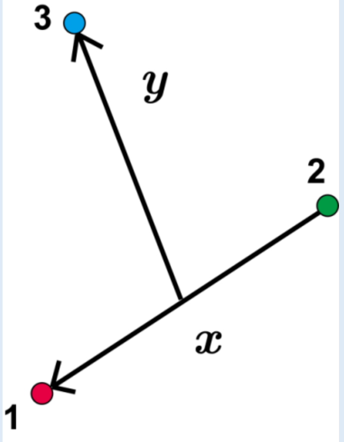
2. Density functions
3. 3α model (Hamiltonian)
4. Transition Strength Function $S_\lambda(E)$ for $^{12}\mathrm{C}(0_1^+) \to 3\alpha(\lambda)$
5. $^{12}\mathrm{C}(\alpha,\alpha')3\alpha$ reaction in $\mathbf{3}\alpha$-model
6. 3α density
7. Summary

2. Density functions

- 3α density function $\rho(x,y) = x^2 y^2 \int d\hat{x} d\hat{y} |\Psi(\vec{x},\vec{y})|^2$
- Bound states [$^{12}\mathrm{C}(0_1^+)$, $^{12}\mathrm{C}(2_1^+)$] $H_{3\alpha}|\Psi_b\rangle = E\,|\Psi_b\rangle$ (as Faddeev eq.)
- Continuum state function for to $^{12}\mathrm{C} \rightarrow 3\alpha$ by multipole operators:

$$|\Psi_\lambda\rangle = \frac{1}{E+i\varepsilon-H_{3\alpha}} \hat{O}_\lambda |\Psi(^{12}\mathrm{C})\rangle$$

$$\hat{O}_\lambda = \left[r^2,\ \ r^3 Y_1(\hat{r}),\ \ \ r^\lambda Y_\lambda(\hat{r})\right] \quad (\lambda = 0, \lambda = 1, \lambda \geq 2)$$



- Multipole strength functions: $S_\lambda(E) = -\frac{1}{\pi} \mathrm{Im} \left\langle \Psi_b \middle| \hat{O}_\lambda^\dagger \middle| \Psi_\lambda \right\rangle$
- For continuum state: $\Psi(\vec{x},\vec{y}) = \mathrm{Im}\left[|\Psi_\lambda\rangle\right] \propto \int dc\, |\Psi_c\rangle\langle\Psi_c|\hat{O}_\lambda|\Psi_b\rangle$

$$\frac{1}{E+i\varepsilon-H_{3\alpha}} = \mathrm{P}\frac{1}{E+i\varepsilon-H_{3\alpha}} - i\pi\delta(E-H_{3\alpha})$$

- Normalization: $1 = \int_0^{x_{\max}} dx \int_0^{y_{\max}} dy\, \rho(x,y)$, $x_{\max} = y_{\max} = 12$ fm
- References: Faddeev calculations for $3\alpha(0^+)$ systems:

S. I. : PRC **87** (2013) 055804, PRC **90** (2014) 061604, PRC **94** (2016) 061603

3. 3α model - Hamiltonian

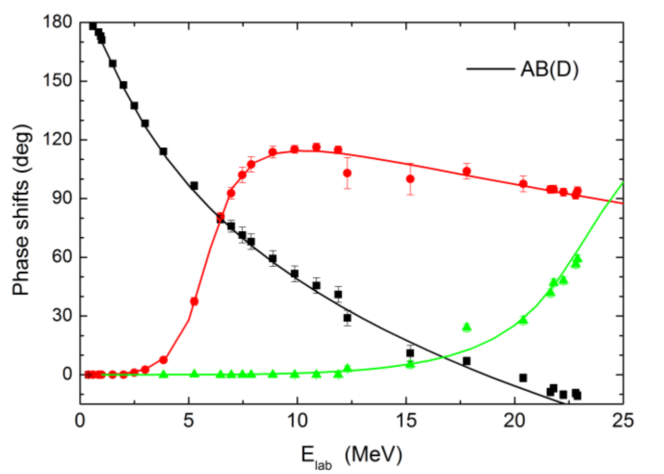
$L_{\alpha\alpha}$	E_r(MeV)	Γ (MeV)
0	0.0918	6.8e-6
2	3.132	1.5
4	11.49	3.5

- 3α Hamiltonian $H_{3\alpha} = K + V_{12} + V_{23} + V_{31} + V_{3\alpha}$
- Phenomenological $\alpha\alpha$ potential: Ali-Bodmer Model D (L=0,2,4)

  Ref.: NP**80** (1966) 99

$$V_{\alpha\alpha}(x) = \left(V_R^{(0)}\hat{P}_{L=0} + V_R^{(2)}\hat{P}_{L=20}\right)e^{-(x/a_R)^2} + V_A e^{-(x/a_A)^2}$$

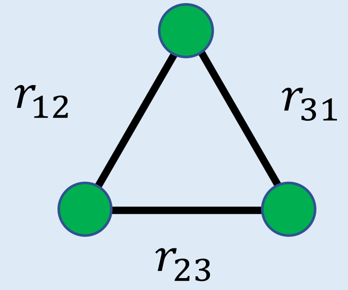
a_R (fm)	$V_R^{(0)}$ (MeV)	$V_R^{(R)}$ (MeV)	a_A (fm)	V_A (MeV)
1.40	500.0	320.0	2.11	-130.0



- Gaussian 3α potential $V_{3\alpha} = W_3 \exp\left[-\frac{{r_{12}}^2+{r_{23}}^2+{r_{31}}^2}{a^2}\right]$

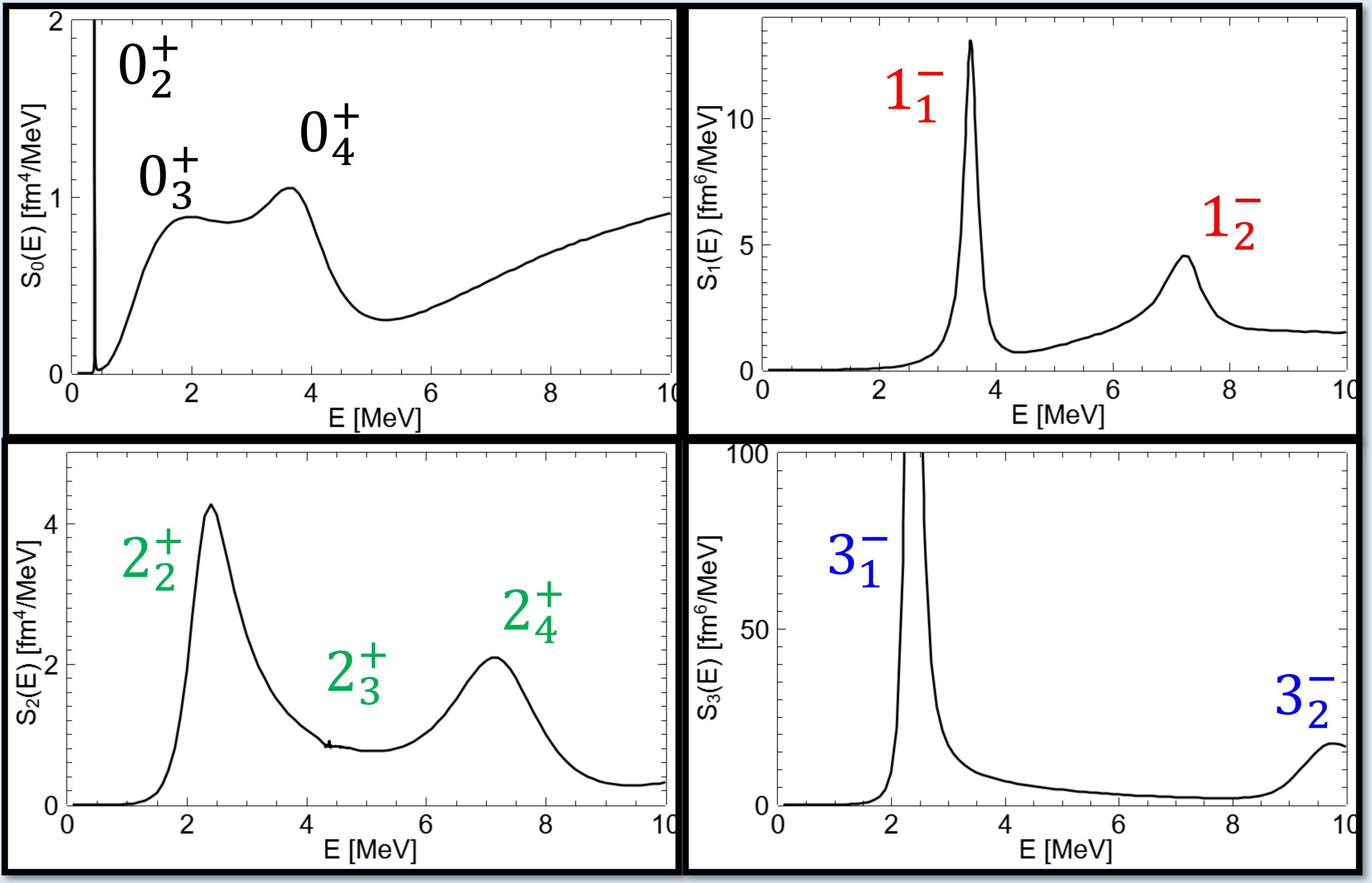
Fitted to $E[0_2^+] = 0.379$ MeV, $E[1_1^-] = 3.569$MeV

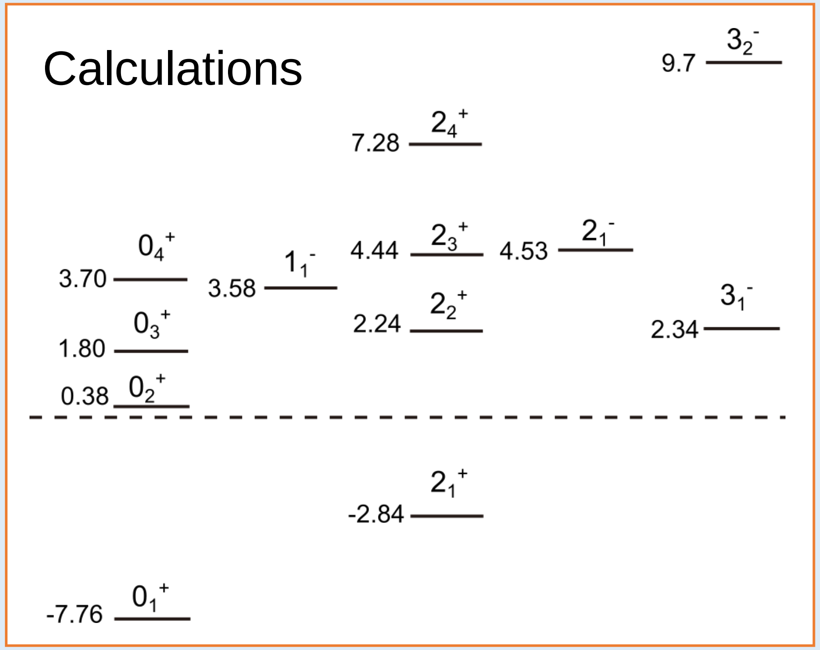
$E[2_1^+] = -2.836$ MeV, $E[3_1^-] = 2.336$ MeV

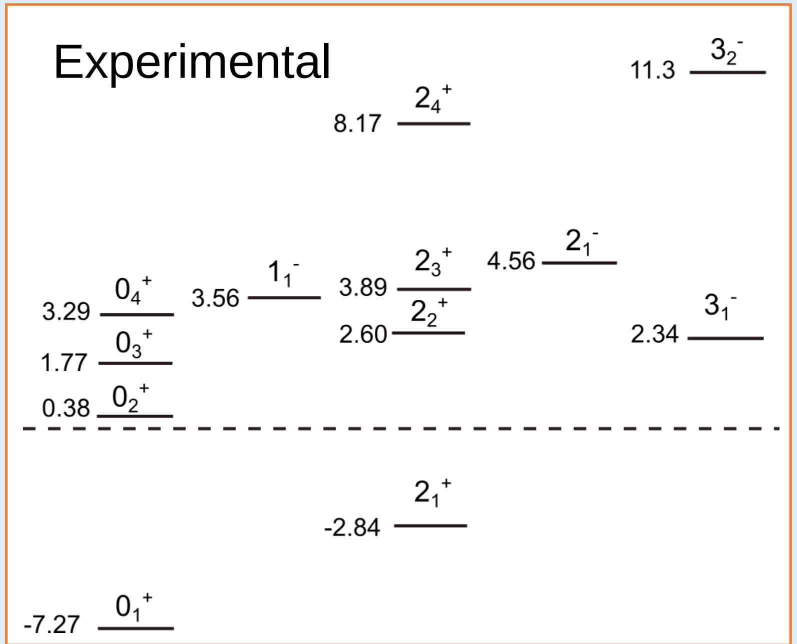


a [fm]	$W_3^{[0]}$[MeV]	$W_3^{[1]}$[MeV]	$W_3^{[2]}$[MeV]	$W_3^{[3]}$[MeV]
3.0	-156.9	-79	-78.3	25.0

4. Transition Strength Function $S_\lambda(E)$ for $^{12}C(0_1^+) \rightarrow 3\alpha(\lambda)$

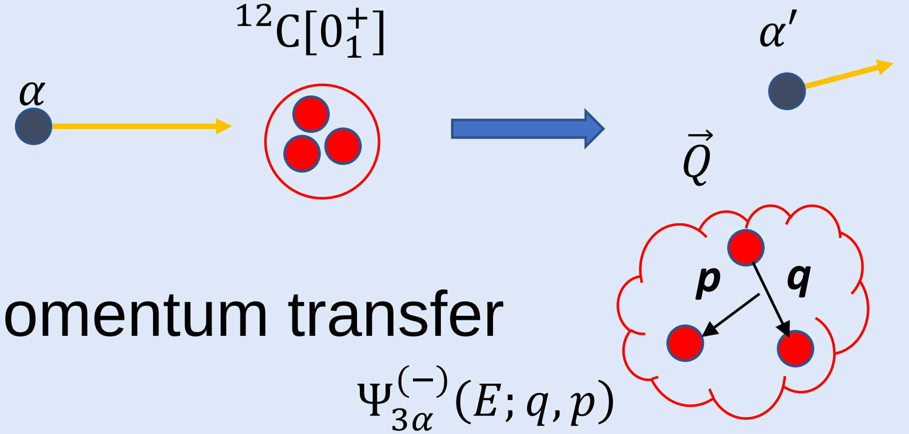






5. $^{12}\mathrm{C}(\alpha,\alpha')3\alpha$ reaction in 3α-model

- Spectrum of inelastic scattering $^{12}\mathrm{C}(\alpha,\alpha')3\alpha$

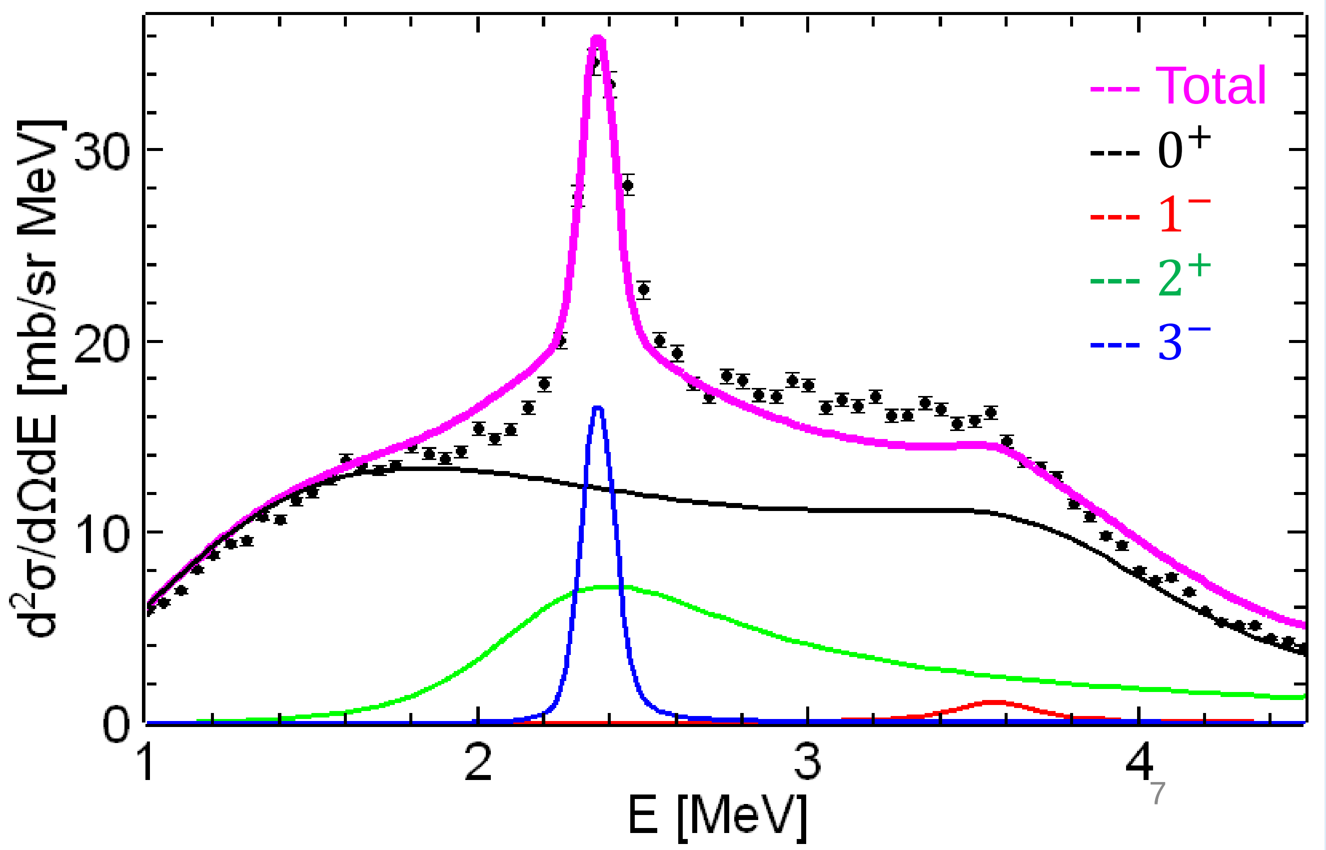


$$\frac{d^2\sigma}{d\Omega dE} = \sum_{\lambda=0}^{3} a_\lambda \frac{c_\lambda(Q)}{Q^4} S_\lambda(E)$$

Q : Momentum transfer

- Gauss convolution FWHM=100keV
- Fitting parameters (a_0, a_1, a_2, a_3)

$$E_\alpha = 386\ \mathrm{MeV} \qquad \theta_{\mathrm{Lab}} = 0°$$

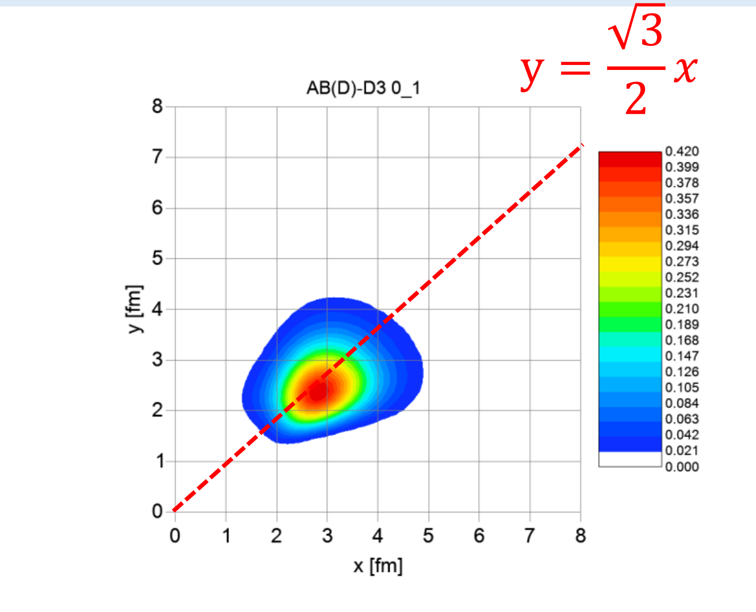


S.I.,paper in preparation

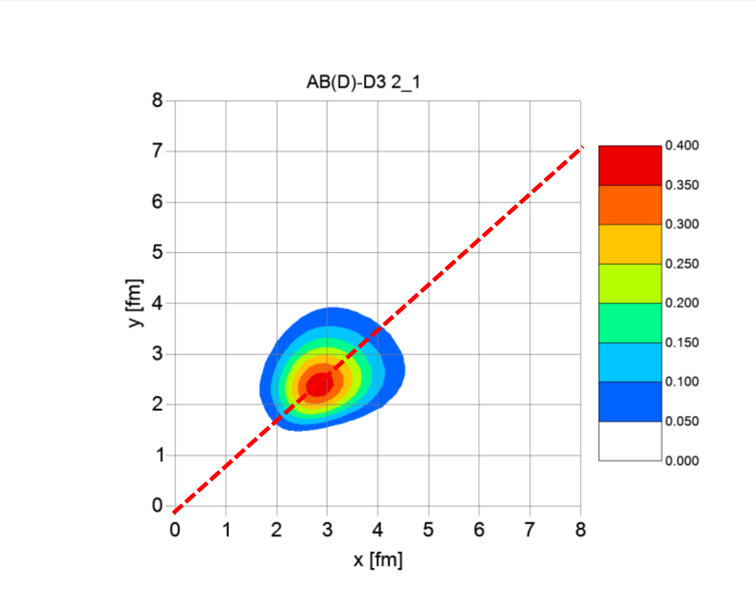
6. 3α density (1)

(1) Equilateral triangle configuration

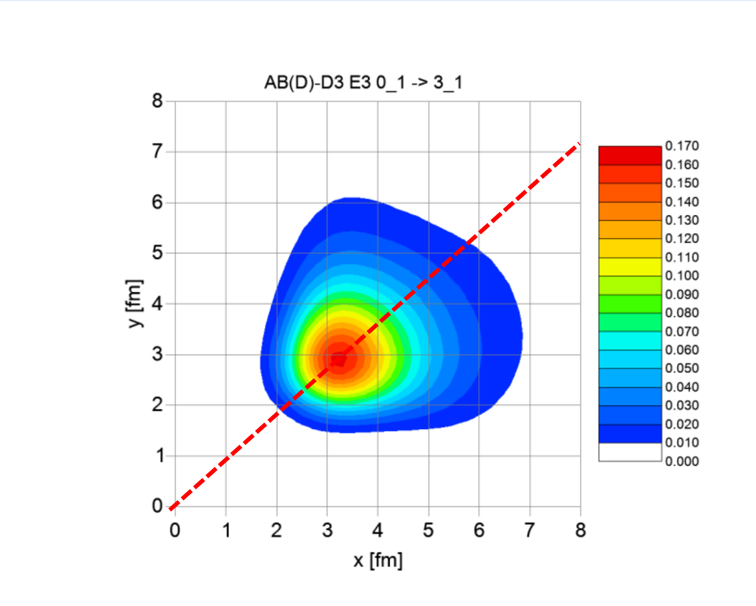
$^{12}\mathrm{C}(0_1^+)$ ground state

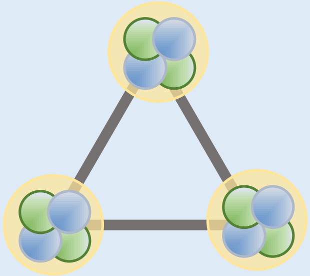


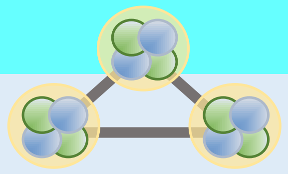
$^{12}\mathrm{C}(2_1^+)$ 1st excited state



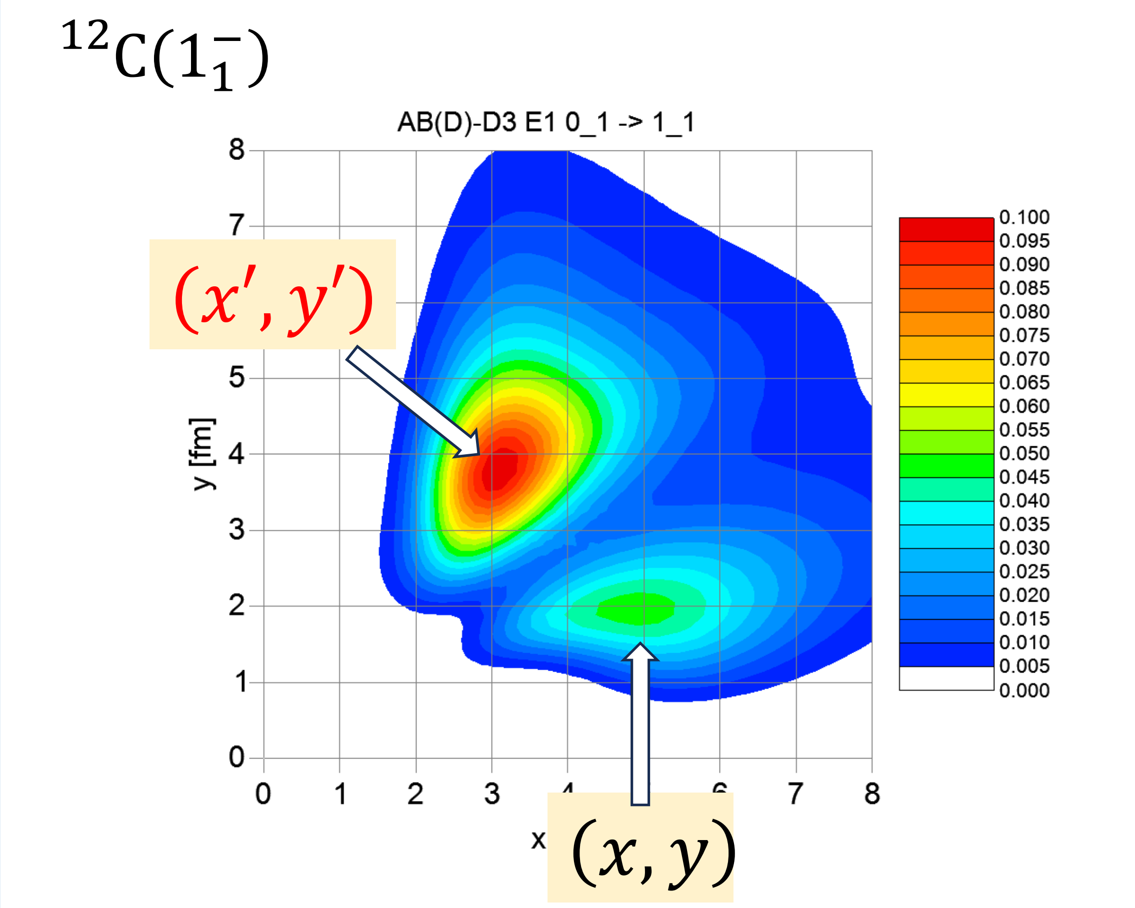
$^{12}\mathrm{C}(3_1^-)$ resonance

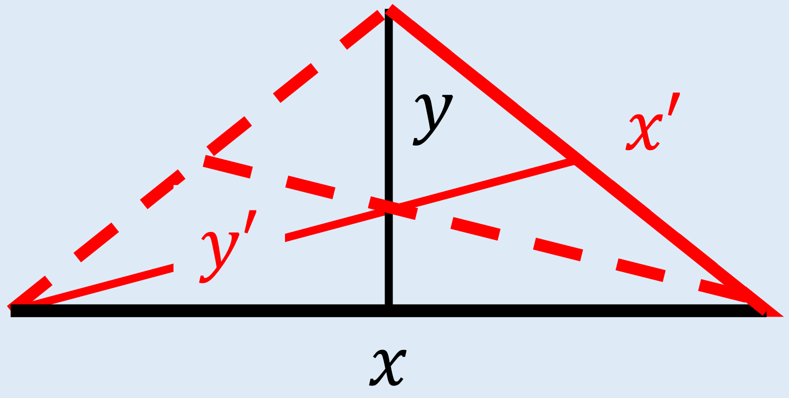






(2) Obtuse isosceles triangle configuration





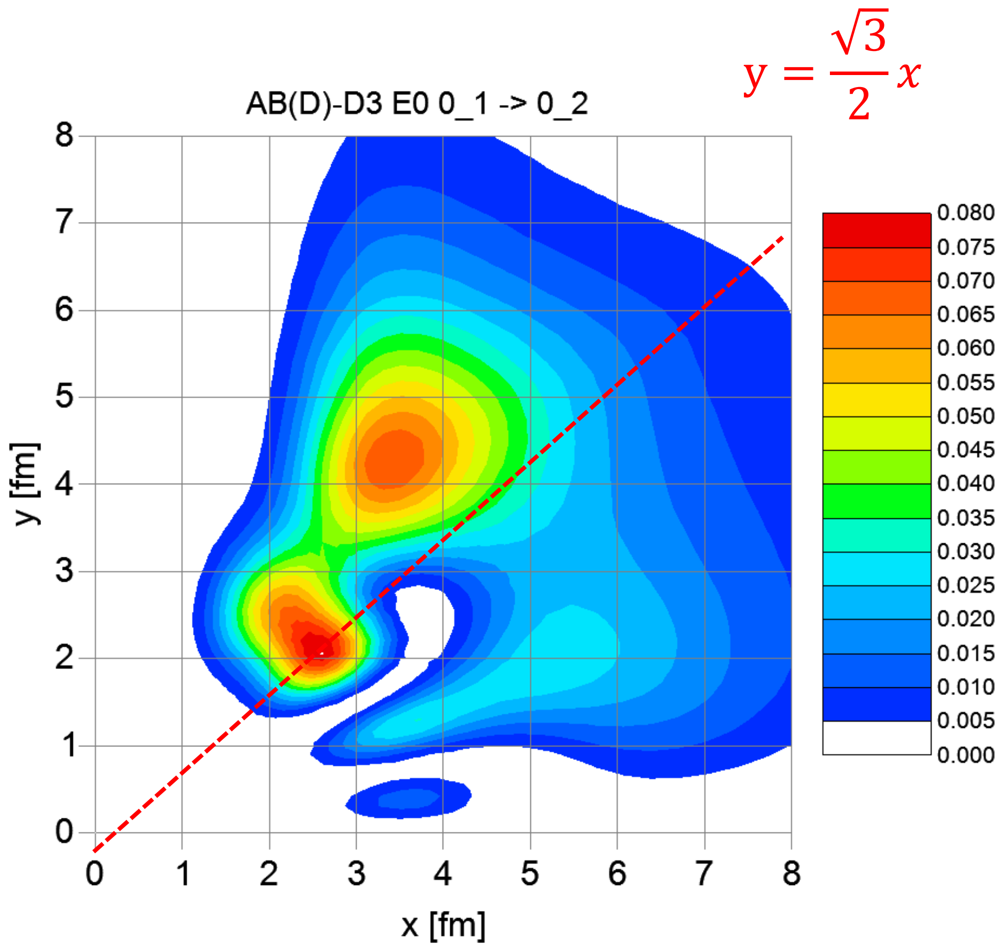
A peak (x, y) corresponding to an obtuse isosceles triangle

→

Associated peak at (x', y') in double strength

6. 3α density (3)

(3)Hoyle state 0_2^+



In addition to the equilateral triangle configuration, some peaks exist.

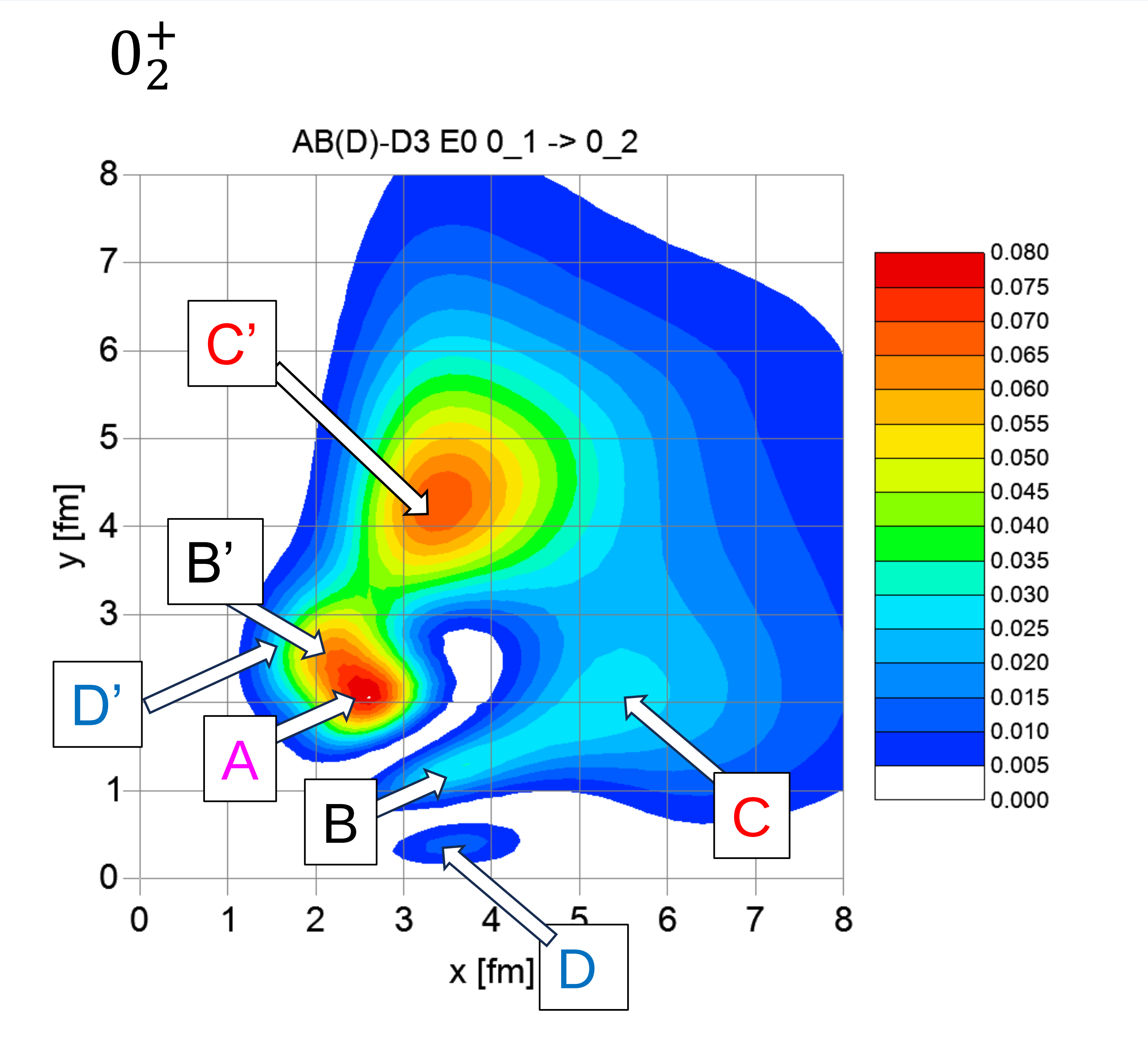
Refs.

[1] S.I. PRC **90** (2014) 061604(R).

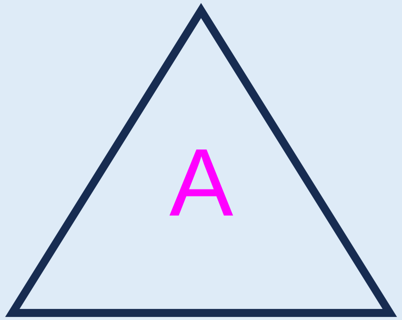
[2] N.B. Nguyen, et al., PRC **87** (2013) 054615.
https://doi.org/10.1103/PhysRevC.87.054615

[3] H. Moriya, et al. FBS **62** (2021) 46; EPJA **59** (2023) 1.

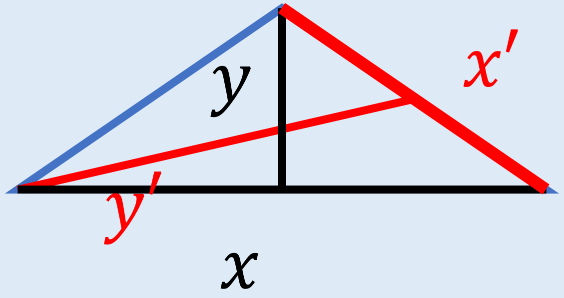
Density of Hoyle state



A:
Equilateral triangle



B and B', C and C', D and D'
Obtuse isosceles triangles



7. Summary

- Geometric structure of bound- and low energy resonance states of $^{12}\mathrm{C}$ nucleus is studied in 3α model.

- Equilateral triangle: $^{12}\mathrm{C}(0_1^+)$, $^{12}\mathrm{C}(2_1^+)$, $^{12}\mathrm{C}(3_1^-)$

- Obtuse isosceles triangle: $^{12}\mathrm{C}(1_1^-)$

- Mixture of equilateral- and obtuse isosceles triangles: $^{12}\mathrm{C}(0_2^+)$