Meson exchange currents in the clothed-particle representation.

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Contents

Recollections

We are going on applications of a field theoretical approach based upon method of unitary clothing transformations (UCTs):

{GreSch58} Greenberg O., Schweber S.: Nuovo Cim. **8** (1958) 378;
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{SheShi00} Shebeko A., Shirokov M.: Prog. Part. Nucl. Phys. **44** (2000) 75;
doi: 10.1016/S0146-6410(00)00060-0

{DuShe10} Dubovyk I., Shebeko O.: Few Body Syst. **48** (2010) 109;
arXiv:1012.5406 [nucl-th]

{SheFro12} Shebeko A., Frolov P.: Few Body Syst. **52** (2012) 125;
arXiv:1107.5877 [hep-th]

{She13} Shebeko A.: Few Body Syst. **54** (2013) 2271;
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with its description of electromagnetic (e.m.) properties of deuteron. See our contribution to APFB20 (2021), Kanazawa

{KosShe21} Kostylenko, Y., Shebeko, A.: Few-Body Syst. **62** (2021) 41;
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Underlying formalism

Method in question is aimed at expressing total Hamiltonian $H(\alpha)$ for interacting fields through so-called clothed-particle creation (annihilation) operators α_c, e.g., $a_c^\dagger(a_c)$, $b_c^\dagger(b_c)$ and $d_c^\dagger(d_c)$ via UCTs $W(\alpha_c) = W(\alpha) = \exp R$, $R = -R^\dagger$ in similarity transformation

$$\alpha = W(\alpha_c)\alpha_c W^\dagger(\alpha_c)$$

that connects a primary set α in bare-particle representation (BPR) with new operators in clothed particle representation (CPR).

A key point of clothing procedure is to remove so-called bad terms from Hamiltonian

$$H \equiv H(\alpha) = H_F(\alpha) + H_I(\alpha) = W(\alpha_c)H(\alpha_c)W^\dagger(\alpha_c) \equiv K(\alpha_c),$$

By definition, such terms prevent physical vacuum $|\Omega\rangle$ (H lowest energy eigenstate) and one-clothed-particle states $|n\rangle_c = a_c^\dagger(n)|\Omega\rangle$ to be H eigenvectors for all quantum numbers n included. Bad terms occur every time when any normally ordered product

$$a^\dagger(1')a^\dagger(2')...a^\dagger(n_C')a(n_A)...a(2)a(1)$$

of class [C.A] embodies, at least, one substructure $\in [k.0]$ ($k = 1, 2...$) or/and $[k.1]$ ($k = 2, 3, ...$). In this context all primary Yukawa-type (trilinear) couplings should be eliminated.

It results in form

$$H(\alpha) = W\, H(\alpha_c) W^\dagger = K(\alpha_c) = K_F(\alpha_c) + K_I(\alpha_c),$$

where free part $K_F(\alpha_c) = H_F(\alpha_c)$ while operator $K_I(\alpha_c)$ contains interactions between clothed particles. By construction, latter has property

$$K_I(\alpha_c)|\Omega\rangle = 0.$$

Together with Hamiltonian Poincaré group generators: total linear momenta $\vec{P}$, total angular momenta $\vec{M}$ and Lorentz boosts $\vec{N}$ meet commutation relations of Poincaré algebra. Within instant form of relativistic dynamics used here generators $\vec{P}$, $\vec{M}$ do not contain interactions, whereas boosts have structure

$$\vec{N}(\alpha) = W\, \vec{N}(\alpha_c) W^\dagger = \vec{B}(\alpha_c) = \vec{B}_F(\alpha_c) + \vec{B}_I(\alpha_c).$$

See derivation of boost operators in CPR {SheFro12}.
Along with requirement to remove bad terms clothing transformations in Dirac (D) picture should meet

$$\lim_{t\to\pm\infty} W_D(t) = 1$$

that leads to equivalence between S-matrix in BPR and CPR, which is provided by isomorphism between $\{\alpha\}$ and $\{\alpha_c\}$ algebras and gives a constructive way for calculating S-matrix elements in CPR and BPR {She13}.

Electron-Deuteron Scattering Amplitude: Gauge Invariance and Gauge Indepedence

Let us remind that in one-photon-exchange approximation (OPEA) elastic *e-d* scattering amplitude is proportional to contraction

$$T(ed \to e'd') = \varepsilon_\mu(e,e')\langle \vec{q}\,M'|J^\mu(0)|\vec{0}\,M\rangle$$

with polarization vector of virtual photon $\varepsilon_\mu(e,e') = \bar{u}_{e'}(k')\gamma_\mu u_e(k)$, and deuteron projection of total angular momentum on quantization axis M.

Current density operator $J^\mu(x) = \exp(iPx)\,J^\mu(0)\exp(-iPx)$ taken at the space-time point $x = (t,\vec{x}) = 0$ has the property to be 4-vector

$$U(\Lambda)J^\mu(0)U^\dagger(\Lambda) = J^\nu(0)\Lambda_\nu{}^\mu, \quad \forall\Lambda \in \text{full Lorenz group.}$$

In particular, for moving system with momentum $\vec{q}$

$$U(\Lambda = L(q)) = \exp(i\vec{\beta}\vec{B}), \quad \vec{\beta} = \beta\frac{\vec{q}}{|\vec{q}|}, \;\; \tan(\beta) = \frac{|\vec{q}|}{m_d}.$$

Of course, current density operator should satisfy continuity equation (CE)

$$\partial_\mu J^\mu(x) = 0 \quad \text{or} \quad [P_\mu, J^\mu(0)] = 0,$$

this requirement of gauge invariance principle in the first order in charge e (cf. our lecture for Erice school (1999)).
Along with it, we emphasize its consequence

$$(P_f - P_i)_\mu \langle f|J^\mu(0)|i\rangle = 0 \qquad (*)$$

that is valid for current density operator being sandwiched between the exact $P = (H, \vec{P})$ eigenvectors. Following {Kaz82} we distinguish gauge invariance and gauge independence (GI) conditions. Latter is presented by Eq. $(*)$.
In this context, we calculate amplitude of interest using an original way of ensuring GI {LevShe93}.

$$T(ed \to e'd') = \varepsilon_\mu(e,e') J^\mu_{M'M}(\vec{q}) - \vec{\varepsilon}(e,e')\vec{J}^G_{M'M}(\vec{q}),$$

$$J^\mu_{M'M}(\vec{q}) = \langle \vec{q}\, M'|J^\mu(0)|\vec{0}\, M\rangle,$$

$$\vec{J}^G_{M'M}(\vec{q}) = \int\limits_0^1 \frac{d\lambda}{\lambda} \nabla_{\vec{q}} \left\{ \left[\sqrt{\lambda^2 \vec{q}^2 + m_d^2} - m_d \right] \rho_{M'M}(\lambda \vec{q}) - \lambda \vec{q}\, \vec{J}_{M'M}(\lambda \vec{q}) \right\}.$$

(cf., {FriFal86})

Property of current density to be 4-vector and GI leads to relations:

$$G_M(Q^2) = \sqrt{2}(\eta(\eta+1))^{-\frac{1}{2}}(\vec{J}_{10}(\vec{q}) + \vec{J}^G_{10}(\vec{q}))_x,$$
$$G_C(Q^2) = \frac{1}{3}(\eta+1)^{-1} \sum_{M=0,\pm1} \rho_{MM}(\vec{q}),$$
$$G_Q(Q^2) = \frac{1}{2}(\eta(\eta+1))^{-1}\left(\rho_{00}(\vec{q}) - \rho_{11}(\vec{q})\right),$$

where charge monopole (G_C), magnetic dipole (G_M) and electric quadrupole (G_Q) form factors (FFs) of deuteron are determined in lab. system with Z-axis directed along momentum transfer $\vec{q}$ (cf. {GarOrd02}). Argument of FFs $Q^2 \equiv -\omega^2 + \vec{q}^2$, where ω is energy transfer so $\eta = \dfrac{Q^2}{4m_d^2} > 0$. In our case

$$\omega = \sqrt{\vec{q}^2 + m_d^2} - m_d \;\text{ and }\; Q^2 = 2m_d^2\left(\sqrt{1 + \frac{\vec{q}^2}{m_d^2}} - 1\right).$$

Charge e_d of deuteron and its dipole magnetic μ_d and electric quadrupole Q_d moments are given by normalization conditions

$$e_d = eG_C(0), \;\; \mu_d = G_M(0)\frac{e}{2m_d}, \;\; Q_d = G_Q(0)\frac{e}{m_d^2}$$

Deuteron eigenvalue problem in motion

Eigenvalue problem for deuteron in rest frame

$$[K_N + K_{NN}]\,|\vec{0}\,M\rangle = m_d|\vec{0}\,M\rangle$$

has been considered in {She13}. The deuteron state has been built in subspace $\mathcal{H}_{2N}$ spanned onto two-nucleon basis $b_c^\dagger b_c^\dagger|\Omega\rangle$ with $K_{NN} \sim b_c^\dagger b_c^\dagger b_c b_c$ and $K_N \sim b_c^\dagger b_c$

$$|\vec{0}\,M\rangle = \sqrt{2}\int \frac{d\vec{p}}{E_{\vec{p}}}\psi_M(p\mu_1\mu_2)b_c^\dagger(p\mu_1)b_c^\dagger(p\mu_2)|\Omega\rangle.$$

The deuteron state in moving frame is given by

$$|\vec{q}\,M\rangle = U(L(q))|\vec{0}\,M\rangle,$$

In its turn operator $U(L) = \exp(i\vec{\beta}(\vec{B}_F(\alpha_c) + \vec{B}_I(\alpha_c)))$ can be presented in form

$$U(L) = X(e^{i\vec{\beta}\vec{B}_F}B_I e^{-i\vec{\beta}\vec{B}_F})e^{i\vec{\beta}\vec{B}_F},$$

where operator X is measure of noncommutativity of operators B_F, and B_I.
A distinctive feature of CPR is that all generators of Poincaré group simultaneously get one and the same sparse structure.

For this consideration we confine ourselves to approximation $X \approx 1$.
Our derivation of such function depending on similarity transformation is underway.

After this, with the help of transformation property

$$e^{i\vec{\beta}\vec{B}_F} b_c^\dagger(p\mu) e^{-i\vec{\beta}\vec{B}_F} = D^{[1/2]}_{\bar{\mu}\mu}(W(L,p)) b_c^\dagger(Lp\bar{\mu}),$$

the matrix elements of interest $J^\mu_{M'M}(\vec{q}) = \langle \vec{q}\, M' | J^\mu(0) | \vec{0}\, M \rangle$ are reduced to expectation values over the physical vacuum state $|\Omega\rangle$.

Current density operator in the CPR

In CPR we have expansion in R-commutators {SheShi00} of original Nöther current

$$J(0) = WJ_c(0)W^\dagger = J_c(0) + [R, J_c(0)] + \frac{1}{2!}[R,[R,J_c(0)]] + \cdots ,$$

Here $J_c(0)$ is Nöther current in which bare operators $\{\alpha\}$ are replaced by clothed ones $\{\alpha_c\}$. It consists of nucleon and meson currents $J_c(0) = J_N(0) + J_M(0)$. This decomposition involves one-body, two-body and more complicated interaction currents. Since deuteron states are spanned onto two-clothed-nucleon states $|2N\rangle$ then only terms of $b^\dagger b$- and $b^\dagger b^\dagger bb$-type remain

$$\langle 2N'|J(0)|2N\rangle = \langle 2N'|J_{\text{one-body}}|2N\rangle + \langle 2N'|J_{\text{two-body}}|2N\rangle .$$

One-body current is defined via usual on-mass-shell expression

$$J^\mu_{\text{one-body}} = \int \frac{d\vec{p'}}{E_{\vec{p}'}}\frac{d\vec{p}}{E_{\vec{p}}} F^\mu_{\text{one-body}}(p'\mu', p\mu) b_c^\dagger(p'\mu') b_c(p\mu),$$

in terms of Dirac and Pauli nucleon FFs

$$F^\mu_{\text{one-body}}(p'\mu', p\mu) = e\, m\, \bar{u}(p'\mu') \left\{ F_1[(p'-p)^2]\gamma^\mu + \frac{i}{2m}\sigma^{\mu\nu}(p'-p)_\nu F_2[(p'-p)^2] \right\} u(p\mu).$$

This expression with $p = (m, \vec{0})$ and $p' = (\omega, \vec{q})$ coincides with that used in Relativistic Impulse Approximation (RIA).

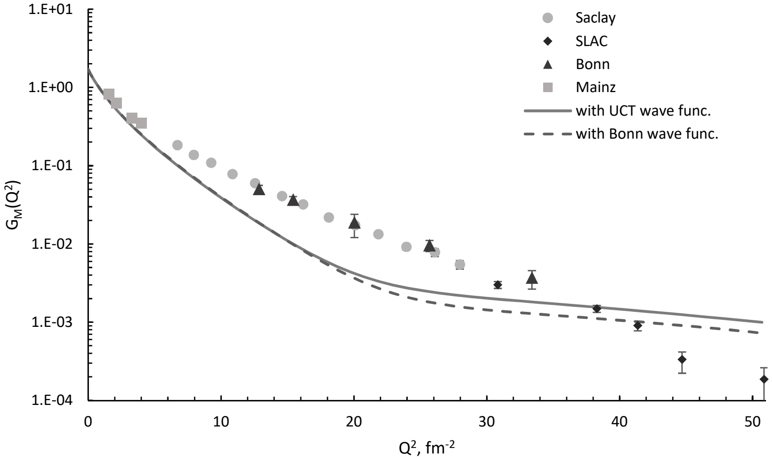

Figure 1: Our first approximation {KosShe21} (without two-body currents) to the deuteron magnetic form factor $G_M(Q^2)$ compared to the experimental data (see {JLab00} and refs. therein). We have used the Gari-Krümpelmann nucleon FFs {GarKru92} for this calculation.

Of course, RIA results should be corrected including more complex mechanisms of e-d scattering, that can be provided by including two-body or meson exchange currents (MECs)

$$J^{\mu}_{two-body} = \int \frac{d\vec{p}_1'}{E_{\vec{p}_1'}} \frac{d\vec{p}_2'}{E_{\vec{p}_2'}} \frac{d\vec{p}_1}{E_{\vec{p}_1}} \frac{d\vec{p}_2}{E_{\vec{p}_2}} F^{\mu}_{MEC}(p_1', p_2', p_1, p_2) b_c^{\dagger}(p_1') b_c^{\dagger}(p_1') b_c(p_1) b_c(p_2).$$

To determine it we have found $b^{\dagger}b^{\dagger}bb$-type terms that stem from commutator $\frac{1}{2!}[R,[R,J_c(0)]]$. The current density J_c composed of the meson J_M and nucleon J_N currents, so MEC also composed of two parts

$$F^{\mu}_{MEC} = F^{\mu}_{\text{mesonic}} + F^{\mu}_{\text{seagull}}\,.$$

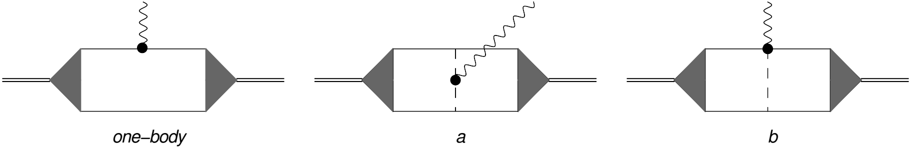

Figure 2: Various contributions to the deuteron e.m. current: (a) mesonic current, (b) seagull current.

Mesonic currents

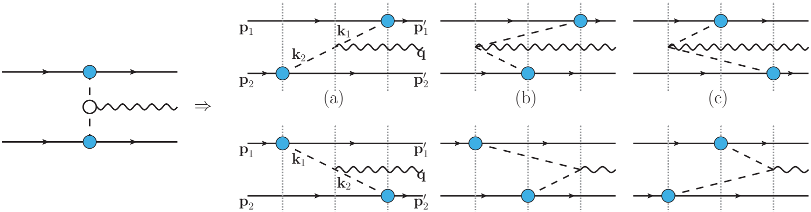

Figure 3: Illustration of the mechanisms that contribute to the mesonic current in the g^2-order. Blue dot corresponds to the g_{11}^{M} cutoff.

$$F^{\mu}_{\text{pionic}}(p_1', p_2', p_1, p_2) = i\frac{eg_\pi^2 m^2}{2(2\pi)^6}[\vec{\tau}_{\eta_1'\eta_1} \times \vec{\tau}_{\eta_2'\eta_2}]_z\, g_{11}^{\pi}(p_1' p_1) g_{11}^{\pi}(p_2' p_2)$$

$$\times \frac{\bar{u}(p_1')\gamma_5 u(p_1)}{(p_1' - p_1)^2 - m_\pi^2} \frac{\bar{u}(p_2')\gamma_5 u(p_2)}{(p_2' - p_2)^2 - m_\pi^2} (p_2' - p_1' + p_1 - p_2)^{\mu}.$$

Seagull currents

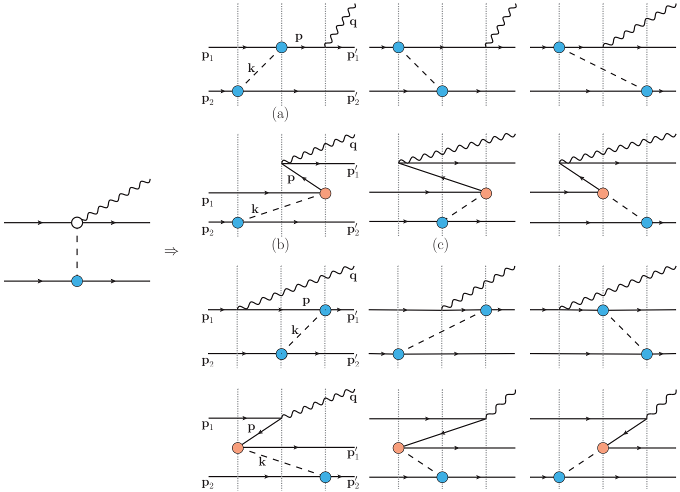

Figure 4: Illustration of the mechanisms that contribute to the seagull current in the g^2-order. Blue and orange dots correspond to the g_{11}^M and g_{12}^M cutoffs respectively.

$$F^{\mu}_{\text{seagull}}(p_1',p_2',p_1,p_2) = \frac{eg_\pi^2 m^2}{2(2\pi)^6} g_{11}^{\pi}(p_2'p_2)\frac{\bar{u}(p_2')\gamma_5 u(p_2)}{(p_2'-p_2)^2-m_\pi^2}$$
$$\times\Big[(\tau^z_{\eta_1'\eta_1}\tau^z_{\eta_2'\eta_2}+I_{\eta_1'\eta_1}\tau^z_{\eta_2'\eta_2}+\tau^+_{\eta_1'\eta_1}\tau^-_{\eta_2'\eta_2})g_{11}^{\pi}(p_1 s)\,\bar{u}(p_1')\gamma^\mu G(p_1',p_2',p_1,p_2)\gamma_5 u(p_1)$$
$$+(\tau^z_{\eta_1'\eta_1}\tau^z_{\eta_2'\eta_2}+I_{\eta_1'\eta_1}\tau^z_{\eta_2'\eta_2}+\tau^-_{\eta_1'\eta_1}\tau^+_{\eta_2'\eta_2})g_{11}^{\pi}(p_1's')\,\bar{u}(p_1')\gamma_5 G(p_1,p_2,p_1',p_2')\gamma^\mu u(p_1)\Big],$$

with $s=(E_{\vec{s}},\vec{s})$, $s'=(E_{\vec{s}'},\vec{s}')$ and $\vec{s}=\vec{p}_1+\vec{p}_2-\vec{p}_2'$, $\vec{s}'=\vec{p}_1'+\vec{p}_2'-\vec{p}_2$. We have introduced the function G that can be divided into the Feynman-like part and rest

$$G = G_{\text{Feynman-like}}+G_{\text{rest}}$$
$$G_{\text{Feynman-like}}(p_1',p_2',p_1,p_2)=\frac{1}{\not p_1+\not p_2-\not p_2'-m},$$
$$G_{\text{rest}}(p_1',p_2',p_1,p_2)=\frac{1}{2E_{\vec{s}}}\Big[\frac{\not s+m}{E_{\vec{p}_2'}-E_{\vec{p}_1}-E_{\vec{p}_2}+E_{\vec{s}}}\frac{1}{(p_1-s)^2-m_\pi^2}$$
$$+\frac{\not s_- -m}{E_{\vec{p}_2'}-E_{\vec{p}_1}-E_{\vec{p}_2}-E_{\vec{s}}}\frac{1}{(p_1+s_-)^2-m_\pi^2}\Big][(p_2'-p_2)^2-m_\pi^2].$$

Under assumptions mentioned above our calculations are reduced to matrix elements

$$J_{M'M}(\vec{q}) = J^{[1]}_{M'M}(\vec{q}) + J^{[2]}_{M'M}(\vec{q}),$$

$$J^{[1]}_{M'M}(\vec{q}) = 2\int \frac{d\vec{p}}{E_{\vec{p}}}\psi^{*}_{M'}(p\,\bar{\mu}_1\bar{\mu}_2)\psi_M(L^{-1}p\,\mu'_1\mu_2)D^{[1/2]*}_{\bar{\mu}_1\mu_1}(W(L(q),p))D^{[1/2]*}_{\bar{\mu}_2\mu_2}(W(L(q),p_-)) \\ \times F_{\text{one-body}}(Lp\,\mu_1, L^{-1}p\,\mu'_1),$$

$$J^{[2]}_{M'M}(\vec{q}) = 2\int \frac{d\vec{p}'\,d\vec{p}}{E_{\vec{p}'}E_{\vec{p}}}\psi^{*}_{M'}(p'\,\bar{\mu}_1\bar{\mu}_2)\psi_M(p\,\mu_1\mu_2)D^{[1/2]*}_{\bar{\mu}_1\mu'_1}(W(L(q),p'))D^{[1/2]*}_{\bar{\mu}_2\mu'_2}(W(L(q),p'_-)) \\ \times \tau^{y}_{\eta'_1\eta'_2}\tau^{y}_{\eta_1\eta_2}F_{MEC}(Lp'\,\mu'_1\eta'_1, Lp'_-\,\mu'_2\eta'_2, p\,\mu_1\eta_1, p_-\,\mu_2\eta_2).$$

Summary

- For this exposition, we have shown a constructive way of describing the deuteron e.m. properties beyond the contemporary non-relativistic approach.
- We propose an original way of ensuring GI relayed upon generalization of Siegert's theorem.
- UCT method allows us to build a new family of MECs.
- Our consideration will be corrected taking into account the interaction part of the boost operator.
- In the course of our current work we are trying to understand to what extent deuteron form factors and structure functions that determine deuteron e.m. properties are sensitive to off-shell effects included in the new family of interactions and currents for the clothed particles.

Thank you very much for your attention!

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