New studies of the $K^{-}pp$ system

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Interest to antikaon-nucleon systems: quasi-bound state in the K^-pp system
$\rightarrow$ experimental and theoretical efforts with different results

Experimental evidences (FINUDA, DISTO); J-PARK E15 experiment:
clear observation of the K^-pp quasi-bound state with
$BE = 42 \pm 3$ MeV, $\Gamma = 100 \pm 7$ MeV.

The problem: big difference between theoretical and experimental widths

Our K^-pp calculations: Faddeev-type dynamically exact equations with coupled $\bar{K}NN - \pi\Sigma N$ channels, different $\bar{K}N - \pi\Sigma - \pi\Lambda$ and NN interactions

What could change the theoretical results:

- More accurate model of the $\Sigma N - \Lambda N$ interaction (previous calculations: "ΣN is important to include, but no strong effects")
- Inclusion of the πN interaction (was switched off till now),
- Direct inclusion of the $\pi\Lambda N$ channel

Faddeev-type three-body AGS equations, two channels

$$U_{ij}^{\alpha\beta} = \delta_{\alpha\beta}(1-\delta_{ij})(G_0^{\alpha})^{-1} + \sum_{k=1}^{3}\sum_{\gamma=1}^{3}(1-\delta_{ik})T_k^{\alpha\gamma}G_0{}^{\gamma}U_{kj}^{\gamma\beta}$$

$\bar{K}N$ interaction is stronlgly coupled to $\pi\Sigma$ via $\Lambda(1405)$ resonance $\rightarrow$ $\pi\Sigma$ channel was included directly. Particle channels:

$$\alpha = 1 : |\bar{K}_1 N_2 N_3\rangle, \qquad \alpha = 2 : |\pi_1 \Sigma_2 N_3\rangle \qquad \alpha = 3 : |\pi_1 N_2 \Sigma_3\rangle$$

Separable form of the potentials:

$$V_i^{\alpha\beta} = \lambda_i^{\alpha\beta}|g_i^{\alpha}\rangle\langle g_i^{\beta}| \qquad \rightarrow \qquad T_i^{\alpha\beta} = |g_i^{\alpha}\rangle\tau_i^{\alpha\beta}\langle g_i^{\beta}|$$

the three-body equations are rewritten as

$$X_{ij}^{\alpha\beta}(z) = \delta_{\alpha\beta}Z_{ij}^{\alpha} + \sum_{k=1}^{3}\sum_{\gamma=1}^{3} Z_{ik}^{\alpha}\,\tau_k^{\alpha\gamma}\,X_{kj}^{\gamma\beta}$$

with new four-body transition $X_{ij}^{\alpha\beta}$ and kernel Z_{ij}^{α} operators.

Input for the system: two-body (potentials) T-matrices.

Three antikaon-nucleon interaction models:

- phenomenological $\bar{K}N - \pi\Sigma$ with one-pole $\Lambda(1405)$ resonance
- phenomenological $\bar{K}N - \pi\Sigma$ with two-pole $\Lambda(1405)$ resonance
- chirally motivated $\bar{K}N - \pi\Sigma - \pi\Lambda$ potential, two-pole $\Lambda(1405)$

reproduce (with the same level of accuracy):

- $1s$ level shift and width of kaonic hydrogen *(SIDDHARTA)*
  direct inclusion of Coulomb interaction, no Deser-type formula used
- Cross-sections of $K^- p \to K^- p$ and $K^- p \to MB$ reactions
- Threshold branching ratios γ, R_c and R_n (or γ, $R_{\pi\Sigma}(R_c, R_n)$ for phen.)
- ◦ $\Lambda(1405)$ resonance (*one- or two-pole structure*)
  $M^{PDG}_{\Lambda(1405)} = 1405.1^{+1.3}_{-1.0}$ MeV, $\Gamma^{PDG}_{\Lambda(1405)} = 50.5 \pm 2.0$ MeV [*PDG (2016)*]

Two-term Separable New potential (TSN) of nucleon-nucleon interaction

[N.V.S. Few-body syst 61, 27 (2020)]

$$V_{NN}^{\mathrm{TSN}}(k,k') = \sum_{m=1}^{2} g_m(k)\,\lambda_m\, g_m(k')\,,$$

$$g_m(k) = \sum_{n=1}^{3} \frac{\gamma_{mn}}{(\beta_{mn})^2 + k^2}, \quad \text{for } m = 1,2$$

fitted to Argonne $V18$ potential *[R.B. Wiringa, V.G.J. Stoks, R. Schiavilla, Phys. Rev. C 51, 38 (1995)]* phase shifts

Triplet and singlet scattering lengths a and effective ranges r_{eff}

$$a_{np}^{\mathrm{TSN}} = -5.400\,\mathrm{fm}, \qquad r_{\mathrm{eff},np}^{\mathrm{TSN}} = 1.744\,\mathrm{fm}$$
$$a_{pp}^{\mathrm{TSN}} = 16.325\,\mathrm{fm}, \qquad r_{\mathrm{eff},pp}^{\mathrm{TSN}} = 2.792\,\mathrm{fm},$$

deuteron binding energy $E_{\mathrm{deu}} = 2.2246$ MeV.

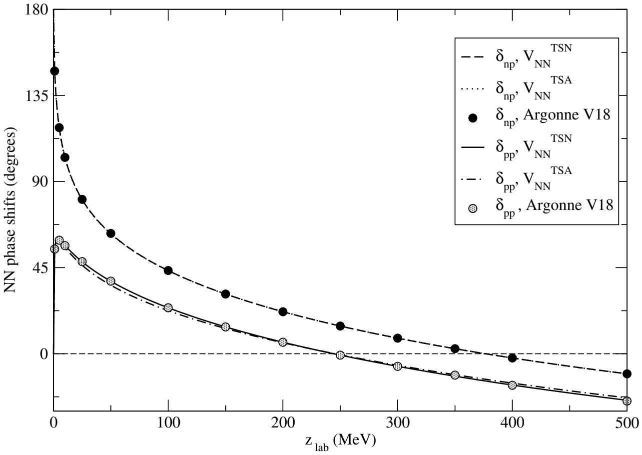

Figure: Phase shifts of np and pp scattering calculated using the new V_{NN}^{TSN} and $V_{NN}^{\mathrm{TSA-B}}$ potentials plus phase shifts of Argonne $V18$

Previously used YN potential: parameters of

$$V_{I,S}^{\Sigma N}(k,k') = \lambda_{I,S}^{\Sigma N}\, g_{I,S}^{\Sigma N}(k)\, g_{I,S}^{\Sigma N}(k'), \quad g_{I,S}^{\Sigma N}(k) = \frac{1}{(k^2 + \beta_{I,S}^{\Sigma N})^2}$$

were fitted to experimental cross-sections

- $I = 3/2$: Real parameters, one-channel case
- $I = 1/2$:
  1. Two-channel $\Sigma N - \Lambda N$ potential, real parameters,
  2. One-channel (exact) optical $\Sigma N(-\Lambda N)$ potential, complex energy-dependent strength

To study dependence of the $K^- pp$ pole position on the $\Sigma N - \Lambda N$ interaction – new fits: spin-dependent (SDep) or spin-independent (SInd) potentials fitted to experimental cross-sections with/without scattering lengths (ScL/noScL) from an "advanced" potential [*J. Haidenbauer et al., nucl-th/2301.00722*]

Dependence on $\Sigma N - \Lambda N$

Dependence of the binding energy B (MeV) and width Γ (MeV) of the quasi-bound state in the $K^- pp$ system on $\Sigma N - \Lambda N$ interaction models. Previous results are from [*N.V.S. Few-body syst 61, 27 (2020)*]

	$V^{1,\mathrm{SIDD}}_{\bar{K}N}$		$V^{2,\mathrm{SIDD}}_{\bar{K}N}$		$V^{\mathrm{Chiral}}_{\bar{K}N}$	
	B_{K^-pp}	Γ_{K^-pp}	B_{K^-pp}	Γ_{K^-pp}	B_{K^-pp}	Γ_{K^-pp}
$V^{\mathrm{Prev}}_{\Sigma N-\Lambda N}$	52.2	67.1	46.6	51.2	29.4	46.4
$V^{\mathrm{noScL,SInd}}_{\Sigma N-\Lambda N}$	63.5	96.5	52.2	61.0	28.3	49.5
$V^{\mathrm{noScL,SDep}}_{\Sigma N-\Lambda N}$	52.5	67.0	46.1	49.8	29.6	46.8
$V^{\mathrm{ScL,SInd}}_{\Sigma N-\Lambda N}$	38.1	48.9	35.4	41.1	29.5	39.3
$V^{\mathrm{ScL,SDep}}_{\Sigma N-\Lambda N}$	34.3	65.6	34.7	53.4	27.9	42.8

Strong dependence (phenomenological $\bar{K}N - \pi\Sigma$ potentials)

Previous calculations: "πN interaction is weak" – $V^{\pi N}$ was set to zero.

Now: parameters of

$$V_I^{\pi N}(k,k') = \lambda_I^{\pi N}\; g_I^{\pi N}(k)\, g_I^{\pi N}(k'), \quad g_I^{\pi N}(k) = \frac{1}{(k^2 + \beta_I^{\pi N})^2}$$

were arbitrary (no bound states) chosen in order to see at the dependences:

- $\beta_{I=1/2}^{\pi N} = \beta_{3/2}^{\pi N} = 3 \text{ fm}^{-1}; \quad \lambda_I^{\pi N} = \lambda_I^{\pi N} = -0.1$ (1a), 0.1 (1b);
- $\beta_{I=1/2}^{\pi N} = \beta_{3/2}^{\pi N} = 1.5 \text{ fm}^{-1}; \quad \lambda_I^{\pi N} = \lambda_I^{\pi N} = -0.03$ (2a), 0.03 (2b)

or fitted to experimental data ("ScL-1" and "ScL-2"):

- $a_{\pi N, I=1/2}^{\text{Exp}} = 0.25$ fm; $a_{\pi N, I=3/2}^{\text{Exp}} = -0.12$ fm; $\quad \beta_{I=1/2}^{\pi N} = \beta_{3/2}^{\pi N} = 3 \text{ fm}^{-1}$
- $a_{\pi N, I=1/2}^{\text{Exp}} = 0.25$ fm; $a_{\pi N, I=3/2}^{\text{Exp}} = -0.12$ fm; $\quad \beta_{I=1/2}^{\pi N} = \beta_{3/2}^{\pi N} = 1.5 \text{ fm}^{-1}$

Dependence on πN

Dependence of the binding energy B (MeV) and width Γ (MeV) of the quasi-bound state in the $K^- pp$ on πN interaction models (with $V^{\text{ScL,SDep}}_{\Sigma N-\Lambda N}$).

	$V^{1,\text{SIDD}}_{\bar{K}N}$		$V^{2,\text{SIDD}}_{\bar{K}N}$		$V^{\text{Chiral}}_{\bar{K}N}$	
	B_{K^-pp}	Γ_{K^-pp}	B_{K^-pp}	Γ_{K^-pp}	B_{K^-pp}	Γ_{K^-pp}
$V^{\text{Prev}}_{\pi N}$	34.3	65.6	34.7	53.4	27.9	42.8
$V^{(1a)}_{\pi N}$	34.6	66.4	34.9	54.2	27.7	43.3
$V^{(1b)}_{\pi N}$	34.1	64.8	34.4	52.7	28.0	42.4
$V^{(2a)}_{\pi N}$	34.6	69.5	35.1	56.6	27.1	43.9
$V^{(2b)}_{\pi N}$	33.9	62.3	34.1	50.8	28.4	41.6
$V^{\text{ScL}-1}_{\pi N}$	37.6	71.4	37.7	58.2	27.2	46.6
$V^{\text{ScL}-2}_{\pi N}$	35.7	70.5	35.9	57.4	27.0	44.6

Faddeev-type three-body AGS equations, three channels

$$U_{ij}^{\alpha\beta} = \delta_{\alpha\beta}(1-\delta_{ij})(G_0^\alpha)^{-1} + \sum_{k=1}^{3}\sum_{\gamma=1}^{5}(1-\delta_{ik})T_k^{\alpha\gamma}G_0{}^\gamma U_{kj}^{\gamma\beta}$$

$\bar{K}N$ interaction is stronlgly coupled to $\pi\Sigma$ via $\Lambda(1405)$ resonance $\rightarrow$ $\pi\Sigma$ and $\pi\Lambda$ channel was included directly. Particle channels:

$$\alpha = 1 : |\bar{K}_1 N_2 N_3\rangle, \qquad \alpha = 2 : |\pi_1 \Sigma_2 N_3\rangle \qquad \alpha = 3 : |\pi_1 N_2 \Sigma_3\rangle$$
$$\alpha = 4 : |\pi_1 \Lambda_2 N_3\rangle \qquad \alpha = 5 : |\pi_1 N_2 \Lambda_3\rangle$$

The three-body equations are rewritten as

$$X_{ij}^{\alpha\beta}(z) = \delta_{\alpha\beta} Z_{ij}^\alpha + \sum_{k=1}^{3}\sum_{\gamma=1}^{5} Z_{ik}^\alpha \, \tau_k^{\alpha\gamma} \, X_{kj}^{\gamma\beta}$$

Two-body $\bar{K}N - \pi\Sigma - \pi\Lambda$ T-matrices are necessary. The chirally motivated potential was refitted, new phenomenological ones were constructed (before: $\pi\Lambda$ channel was taken in $V^{1,\mathrm{SIDD}}$, $V^{2,\mathrm{SIDD}}$ into account indirectly through complex $\lambda_{I=1}^{11}$).

Three antikaon-nucleon interaction models:

- phenomenological $\bar{K}N - \pi\Sigma - \pi\mathbf{\Lambda}$ with one-pole $\Lambda(1405)$ resonance
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reproduce (with the same level of accuracy):

- $1s$ level shift and width of kaonic hydrogen *(SIDDHARTA)*
  direct inclusion of Coulomb interaction, no Deser-type formula used
- Cross-sections of $K^- p \to K^- p$ and $K^- p \to MB$ reactions
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- ◦ $\Lambda(1405)$ resonance (*one- or two-pole structure*)
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New $\bar{K}N$ potentials, K^-p scattering

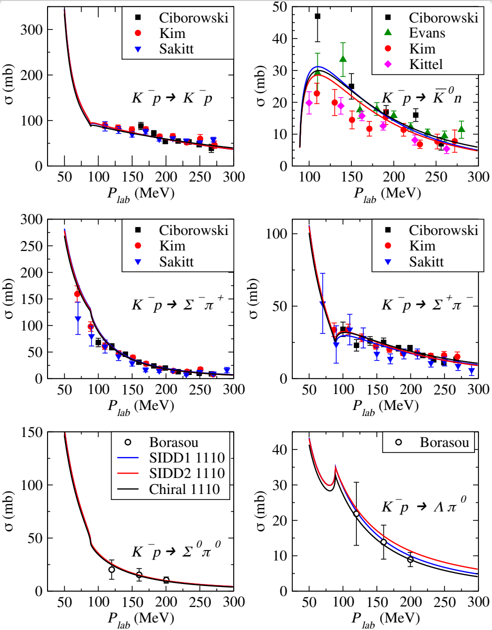

Figure: New $V_{\bar{K}N}$ potentials: one-pole, two-pole phenomenological and chirally motivated

Physical characteristics of the three new $\bar{K}N - \pi\Sigma - \pi\Lambda$ potentials

Physical characteristics of the three $\bar{K}N - \pi\Sigma - \pi\Lambda$ potentials: $1s$ level shift and width, strong pole(s), γ, R_c, R_n threshold branching ratios, a_{K^-p} scattering length (physical masses in all channels, Coulomb in K^-p).

	$V^{1,\mathrm{SIDD}}_{\bar{K}N-\pi\Sigma-\pi\Lambda}$	$V^{2,\mathrm{SIDD}}_{\bar{K}N-\pi\Sigma-\pi\Lambda}$	$V^{\mathrm{Chiral}}_{\bar{K}N-\pi\Sigma-\pi\Lambda}$	Exp
ΔE_{1s}	-322.6	-323.5	-311.6	$-283 \pm 36 \pm 6$
Γ_{1s}	645.4	633.8	605.8	$541 \pm 89 \pm 22$
E_1	$1429.5 - i\,35.0$	$1430.9 - i\,41.6$	$1429.6 - i\,33.2$	
E_2	–	$1380.4 - i\,79.9$	$1367.8 - i\,66.5$	
γ	2.35	2.36	2.36	2.36 ± 0.04
R_c	0.666	0.664	0.664	0.664 ± 0.011
R_n	0.190	0.189	0.190	0.189 ± 0.015
a_{K^-p}	$-0.77 + i\,0.97$	$-0.78 + i\,0.95$	$-0.75 + i\,0.90$	

Three-channel $\bar{K}NN - \pi\Sigma N - \pi\Sigma\Lambda$ calculations: dependence of the binding energy B (MeV) and width Γ (MeV) of the quasi-bound state in the K^-pp on πN interaction models (with $V^{\mathrm{ScL,SDep}}_{\Sigma N-\Lambda N}$).

	$V^{1,\mathrm{SIDD}}_{\bar{K}N}$		$V^{2,\mathrm{SIDD}}_{\bar{K}N}$		$V^{\mathrm{Chiral}}_{\bar{K}N}$	
	B_{K^-pp}	Γ_{K^-pp}	B_{K^-pp}	Γ_{K^-pp}	B_{K^-pp}	Γ_{K^-pp}
$V^{\mathrm{ScL-1,Prev}}_{\pi N}$	37.6	71.4	37.7	58.2	27.2	46.6
$V^{\mathrm{ScL-1,3ch}}_{\pi N}$	34.5	52.0	42.9	60.4	26.8	59.6
$V^{\mathrm{ScL-2,Prev}}_{\pi N}$	35.7	70.5	35.9	57.4	27.0	44.6
$V^{\mathrm{ScL-2,3ch}}_{\pi N}$	34.3	52.1	40.2	57.7	27.2	56.3

Influence of $\Sigma N - \Lambda N$ and πN potentials and the $\pi\Lambda N$ channel on the K^-pp quasi-bound state:

- different models of $\Sigma N - \Lambda N$ interaction could change the three-body K^-pp pole position quite strongly (two-channel calculations); $V^{\mathrm{ScL,SDep}}_{\Sigma N-\Lambda N} \to$ smaller B_{K^-pp}
- variation of the πN interaction parameters lead to smaller differences (two-channel calculations); $V^{\mathrm{ScL}-1}_{\pi N}$, $V^{\mathrm{ScL}-2}_{\pi N} \to$ larger Γ_{K^-pp}
- the results evaluated with chirally motivated model of antikaon-nucleon interaction is less sensitive to the $\Sigma N - \Lambda N$ and πN interactions than those obtained with the phenomenological potentials (two-channel calc.)
- direct inclusion of the third channel in the $\bar{K}NN - \pi\Sigma N - \pi\Lambda N$ system: approximately equal K^-pp widths were obtained with the tree-channel two-pole phenomenological and chiral potentials ~ 58 MeV, and smaller width with one-channel phenomenological ~ 52 MeV. Binding energies: $B^{\mathrm{Chiral}}_{K^-pp}(\sim 27\,\mathrm{MeV}) < B^{1,\mathrm{SIDD}}_{K^-pp}(\sim 34\,\mathrm{MeV}) < B^{2,\mathrm{SIDD}}_{K^-pp}(\sim 41\,\mathrm{MeV})$