Detailed studies of 12C structure and reactions

Lorenzo Fortunato
Univ. Padova & INFN
Italy

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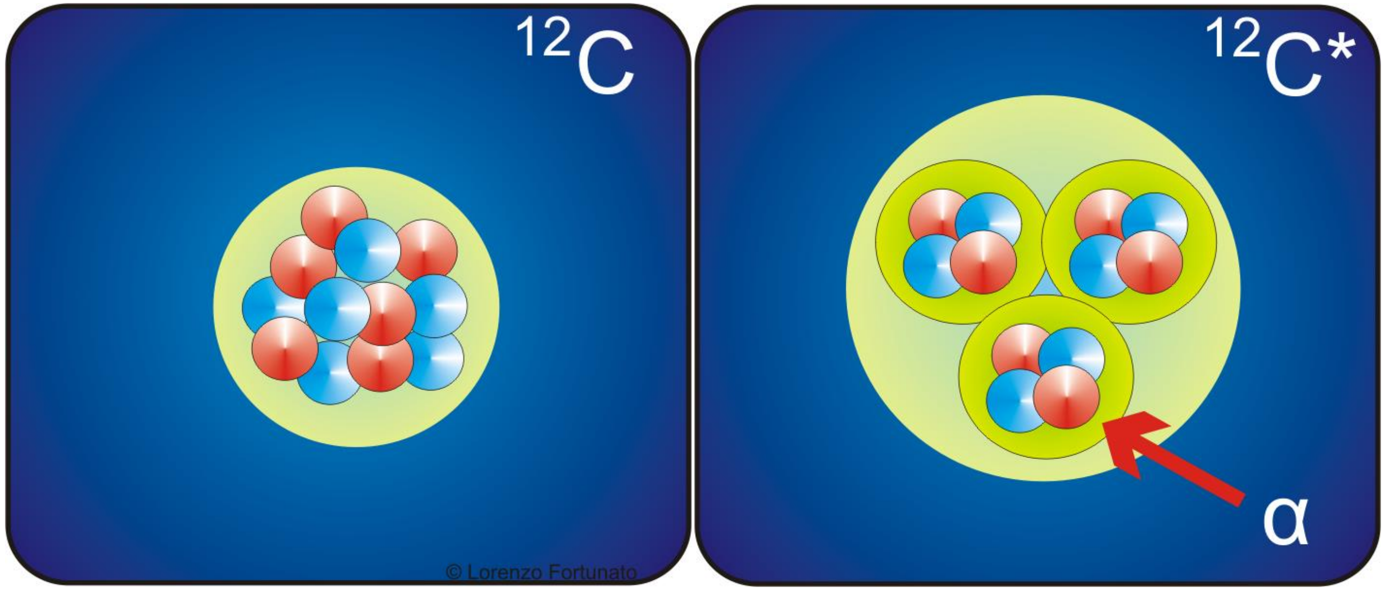


Our understanding of the life supporting 12C nucleus still unfolds multiple unsolved problems, conundra and impacts other branches of science:

- single particle vs. cluster
- role of discrete symmetries
- fermionic vs. bosonic, role of Pauli exc. pr.
- consistency of structure and reaction theories
- Bose-Einstein condensation
- abundance in the universe: importance in astrophysics and anthropogenesis

Università degli Studi di Padova 800 anni 1222·2022
Dipartimento di Fisica e Astronomia Galileo Galilei
INFN Istituto Nazionale di Fisica Nucleare
MOdeling Nuclear STructure and REactions

A little bit of history … and references

- Electromagnetic selection rules for 12C in a 3 alpha cluster model - Presented at **EFB23 Aarhus (DK) 2016**
  - G. Stellin, L. Fortunato and A. Vitturi, **J.Phys. G: Nucl. Part. Phys. 43(2016) 085104**
  - L. Fortunato, G. Stellin, A. Vitturi, **Few-Body Syst. (2017) 58:19**

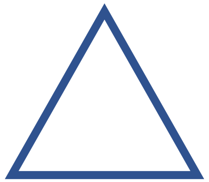
- How to determine the shape of nuclear molecules with polarized gamma-rays - Presented at **EFB24 Guildford (UK) 2019**
  - L. Fortunato, **Phys. Rev. C 99, 031302(R) (2019)**
  - L. Fortunato, **SciPost Phys. Proc. 3, 035 (2020)**

- **Detailed studies of 12C structure and reactions – Presented at EFB25 Mainz (DE) 2023**
  - A. Vitturi, J. Casal, L. Fortunato, E. G. Lanza, **Phys. Rev. C 101, 014315 (2020)**
  - J. Casal, L. Fortunato, E. G. Lanza, A. Vitturi, **Eur. Phys. J. A (2021) 57:33**
  - H. Moriya, W. Horiuchi, J. Casal, L. Fortunato, **Few-Body Syst (2021) 62:46**
  - H. Moriya, W. Horiuchi, J. Casal, L. Fortunato, **Eur. Phys. J. A (2023) 59:37**

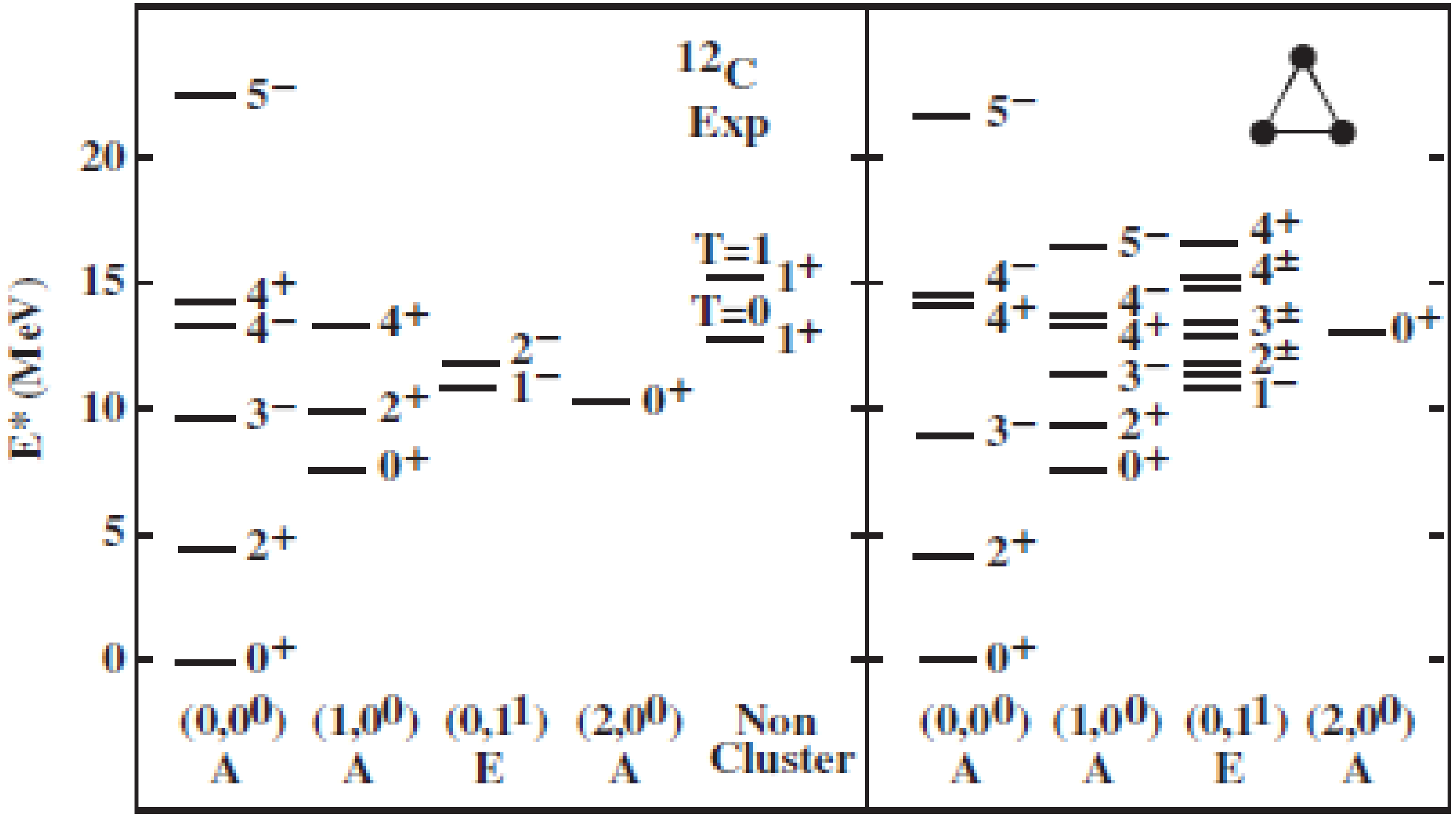
25 EFB

Algebraic cluster model for 3 alphas

Bijker and Iachello (Ann.Phys. 298, 2002) have clearly demonstrated the succesfull application of the ACM, or algebraic cluster model, to the vibrational-rotational spectrum of alpha-conjugate nuclei like 12C and 16O.



Note that rotational bands DO NOT conform to the usual quadrupole rotational bands that we are used to, but they have a different symmetry!
Rather, these are the L^{π} compatible with the transformation of an equil. triangle into itself.



What are the implication of the triangular structure in reactions?

PHYSICAL REVIEW C **101**, 014315 (2020)

Transition densities and form factors in the triangular α-cluster model of ^{12}C with application to $^{12}C + \alpha$ scattering

A. Vitturi,[1,2] J. Casal,[1,2] L. Fortunato,[1,2] and E. G. Lanza[3,4]

[1]*Dipartimento di Fisica e Astronomia "G. Galilei", Università di Padova*
[2]*I.N.F.N., Sez. di Padova, I-35131 Padova, Italy*
[3]*I.N.F.N., Sez. di Catania, I-95123 Catania, Italy*
[4]*Dipartimento di Fisica e di Astronomia "Ettore Majorana", Università Catania, Italy*

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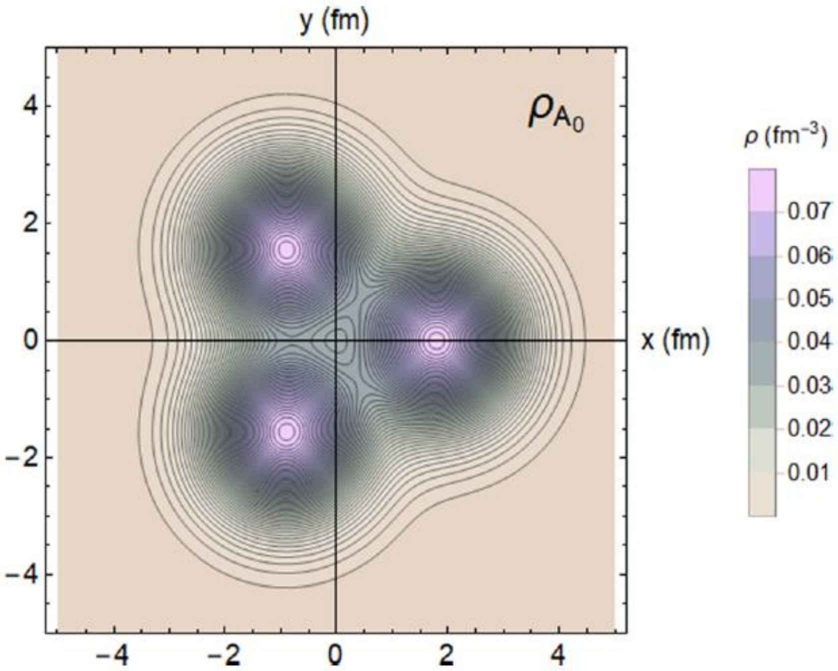
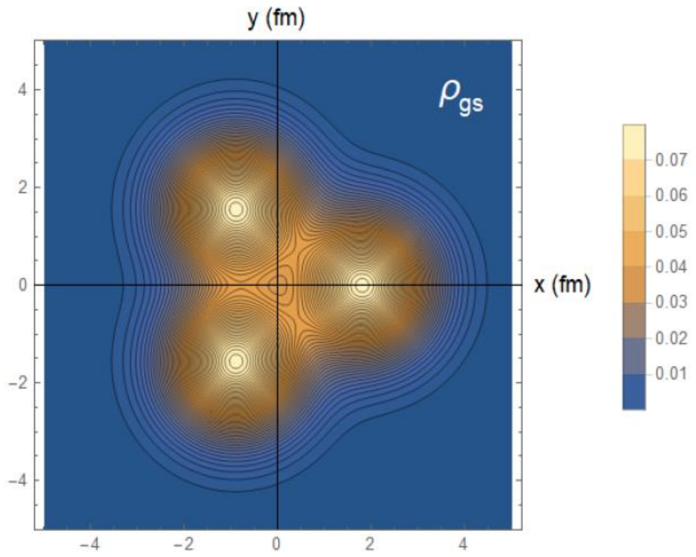
Densities and transition densities are computed in an equilateral triangular α-cluster model for ^{12}C, in which each α particle is taken as a Gaussian density distribution. The ground state, the symmetric vibration (Hoyle state), and the asymmetric bend vibration are analyzed in a molecular approach and dissected into their components in a series of harmonic functions, revealing their intrinsic structures. The transition densities in the laboratory frame are then used to construct form factors and to compute distorted-wave Born approximation inelastic cross sections for the $^{12}C(\alpha, \alpha')$ reaction. The comparison with experimental data indicates that the simple geometrical model with rotations and vibrations gives a reliable description of reactions where α-cluster degrees of freedom are involved.



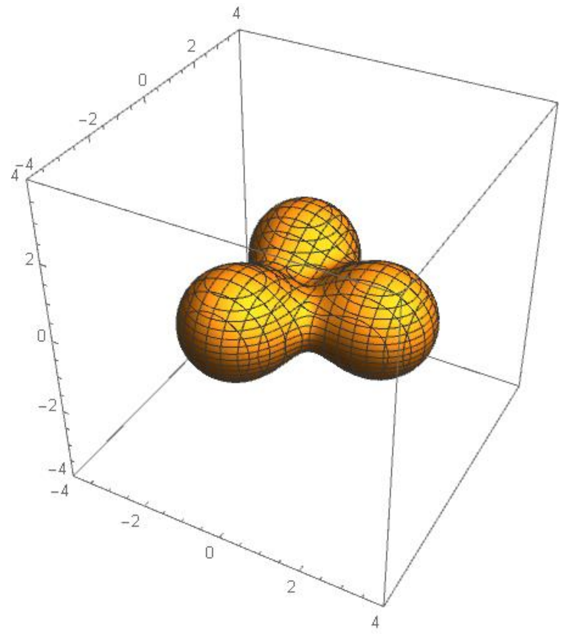
M. Kamimura, Nucl. Phys. A **351**, 456 (1981).
D. C. Cuong, D. T. Khoa, and Y. Kanada En'yo, Phys. Rev. C **88**, 064317 (2013).
M. Ito, Phys. Rev. C **97**, 044608 (2018).
Y. Kanada-En'yo and K. Ogata, Phys. Rev. C **99**, 064601 (2019).

Transition densities in 12C

In collaboration with A. Vitturi, E. Lanza and J. Casal

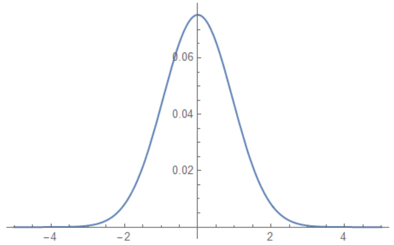


Ground state



This model assume guassian densities for each alpha particle

$$\rho_\alpha(\vec{r}) = \left(\frac{\alpha}{\pi}\right)^{3/2} e^{-\alpha r^2}$$



and a total density that is the sum of three displaced alpha's

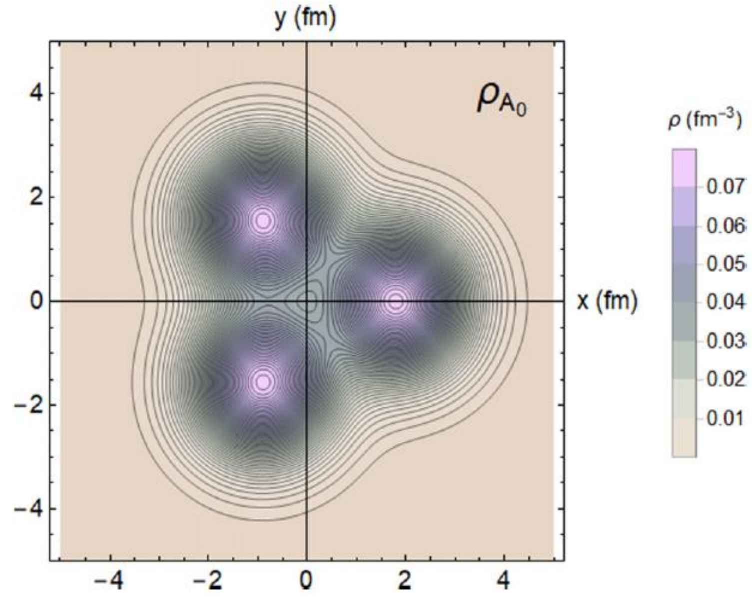
$$\rho_{gs}(\vec{r}, \{\vec{r}_k\}) = \sum_{k=1}^{3} \rho_\alpha(\vec{r} - \vec{r}_k),$$

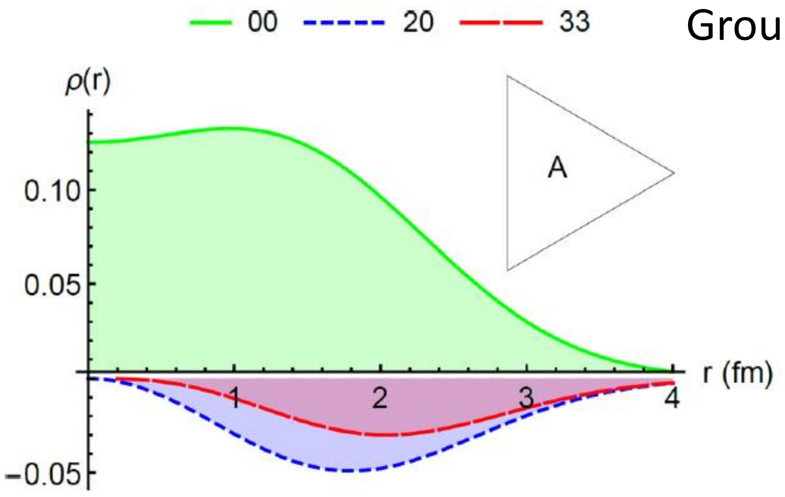
which is then expanded in spherical harmonics

$$\rho_{gs}(\vec{r}) = \sum_{\lambda\mu} \rho_{gs}^{\lambda,\mu}(r) Y_{\lambda,\mu}(\theta, \varphi),$$

Ground and Hoyle bands

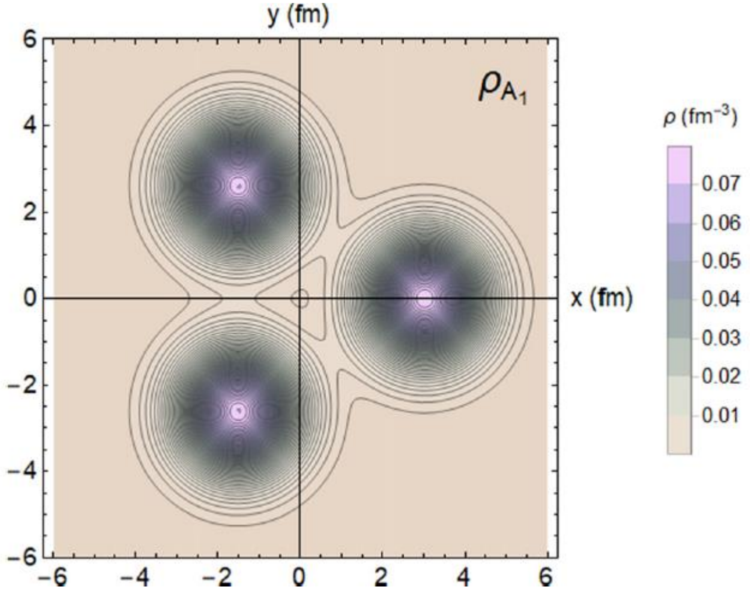
In collaboration with A. Vitturi, E. Lanza and J. Casal

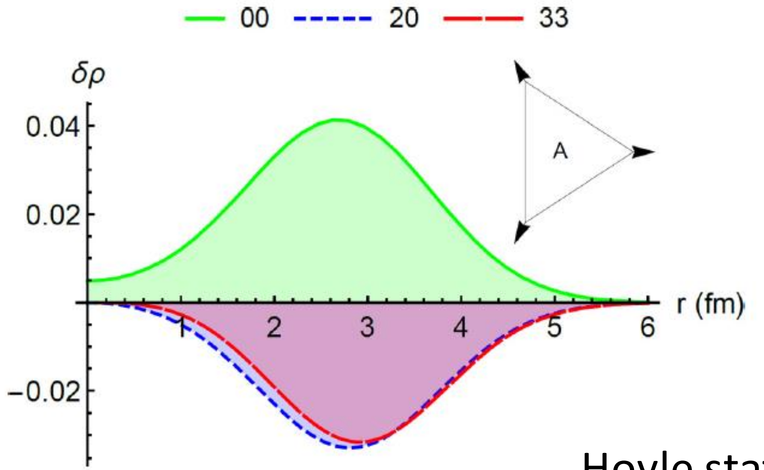




Ground state band

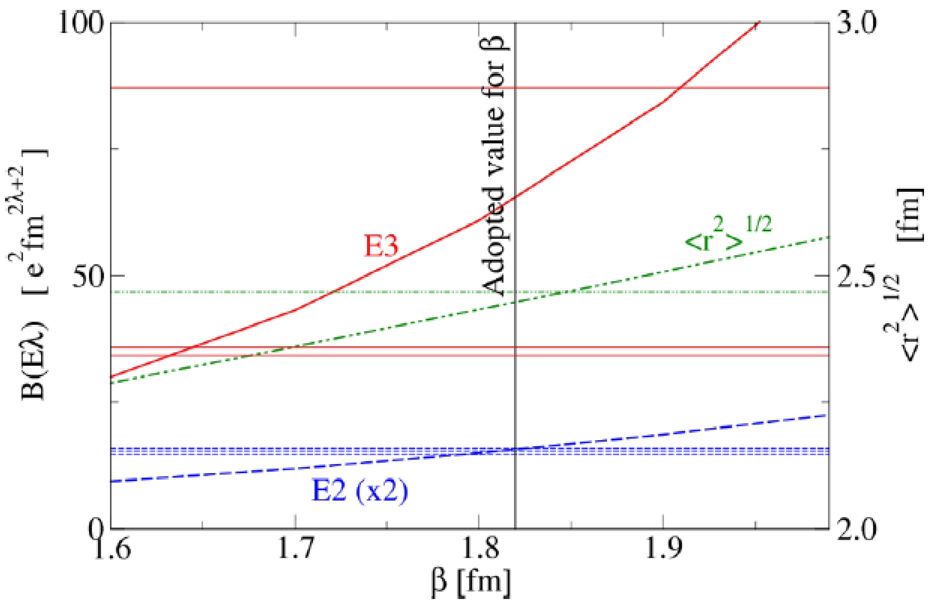
D_{3h}	$\mathbb{I}$	$2C_3$	$3C_2$	σ_h	$2S_3$	$3\sigma_V$
A_1'	1	1	1	1	1	1
A_2'	1	1	-1	1	1	-1
E'	2	-1	0	2	-1	0
A_1''	1	1	1	-1	-1	-1
A_2''	1	1	-1	-1	-1	1
E''	2	-1	0	-2	1	0





Hoyle state band

Parameters phenomenologically adjusted



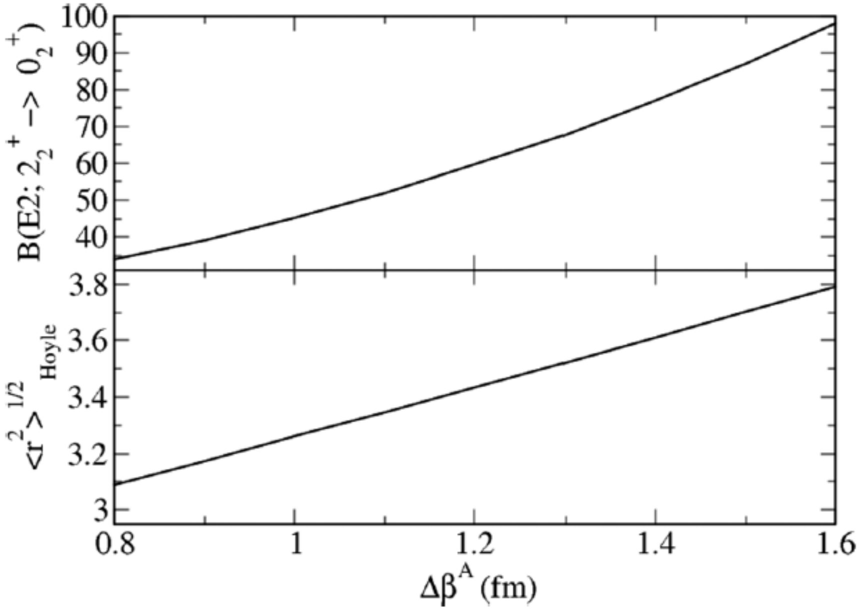


TABLE I. Calculated observables within the ground-state band.

$\langle r^2\rangle^{1/2}_{0_1^+}$	2.45 (fm)
$B(E2; 2_1^+ \to 0_1^+)$	7.86 (e^2 fm^4)
$B(E3; 3_1^- \to 0_1^+)$	65.07 (e^2 fm^6)
$B(E4; 4_1^+ \to 0_1^+)$	96.99 (e^2 fm^8)

TABLE II. Quantities calculated in the present work for the Hoyle band using the values of β and χ_1 given in the text.

$\langle r^2\rangle^{1/2}_{0_2^+}$	3.44 (fm)
$B(E2; 2_2^+ \to 0_1^+)$	0.58 (e^2 fm^4)
$B(E2; 0_2^+ \to 2_1^+)$	2.90 (e^2 fm^4)
$B(E3; 3_2^- \to 0_1^+)$	70.42 (e^2 fm^6)
$M(E0; 0_2^+ \to 0_1^+)$	5.4 (e fm^2)

Transition densities in 12C

A-band

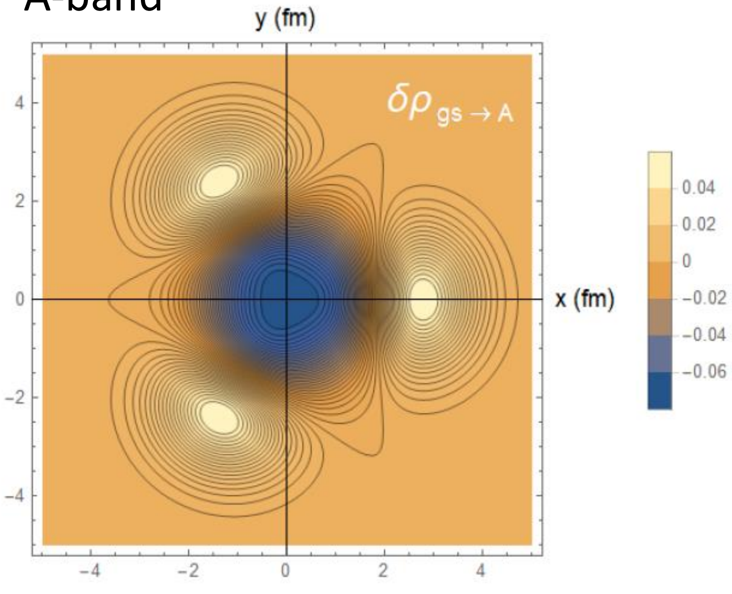
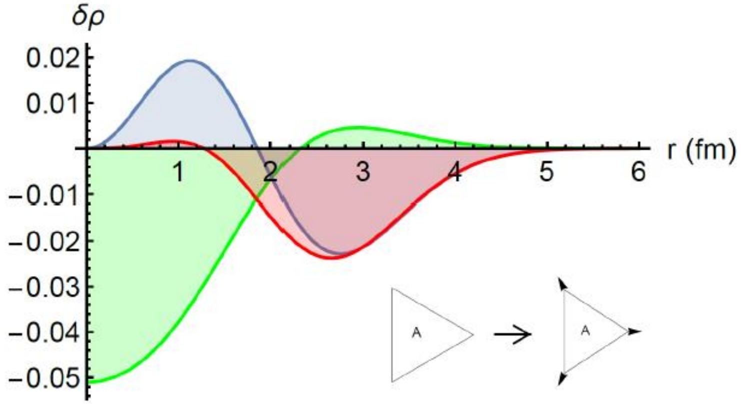
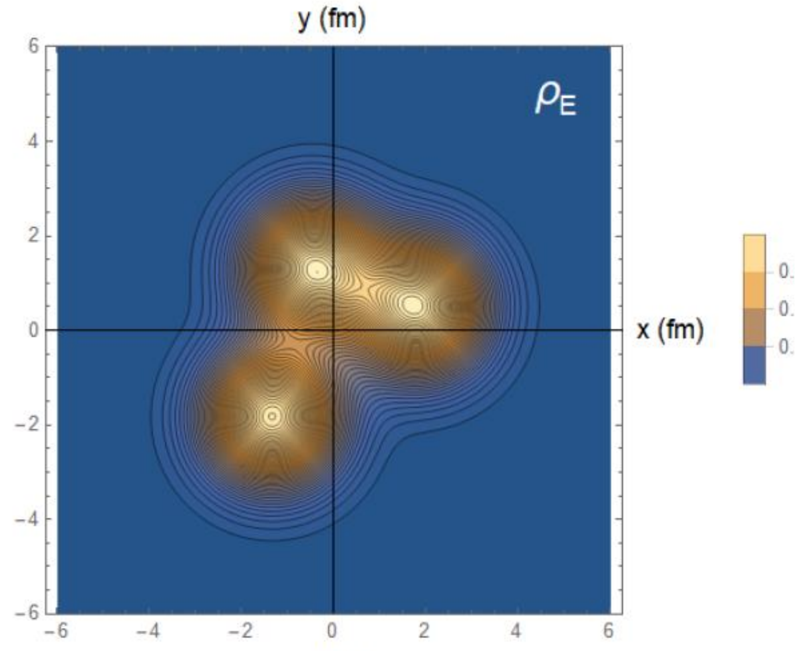
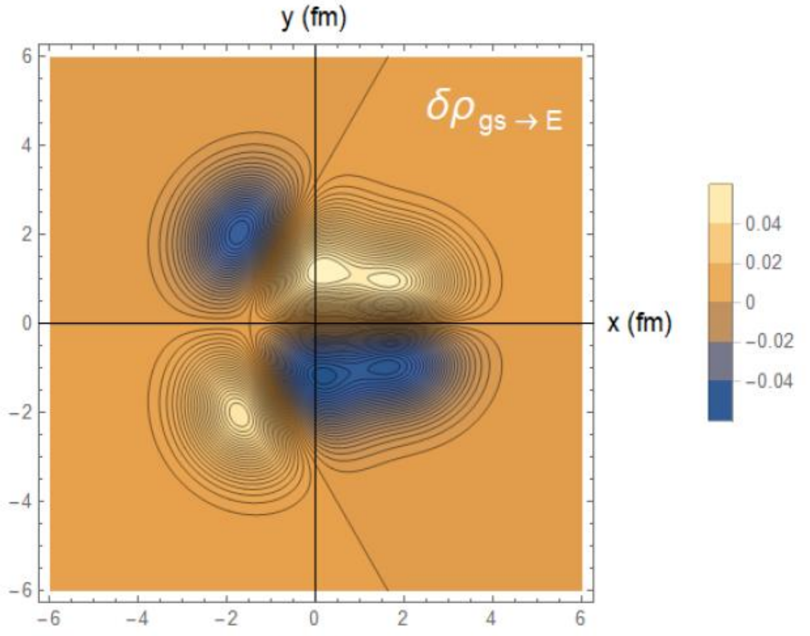
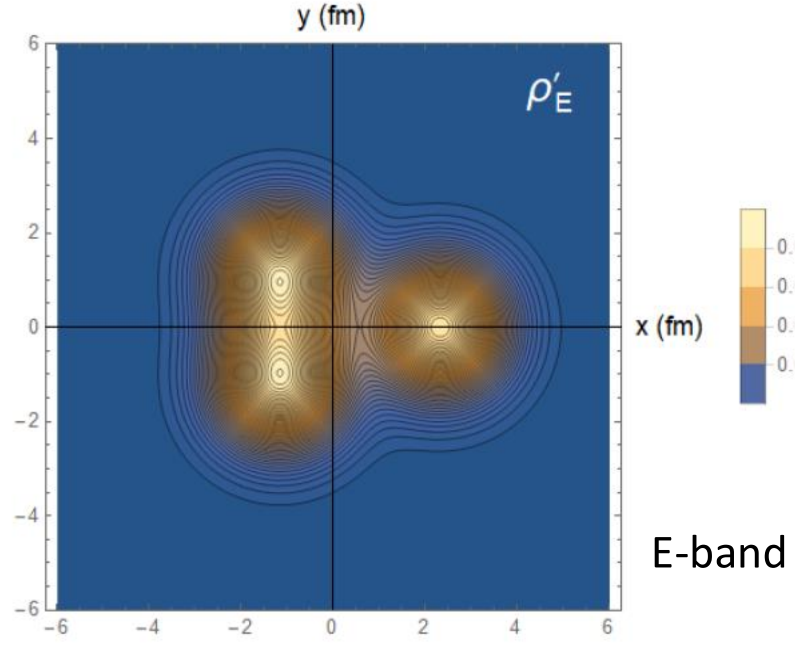


FIG. 7: Transition density for the first A-type vibration.

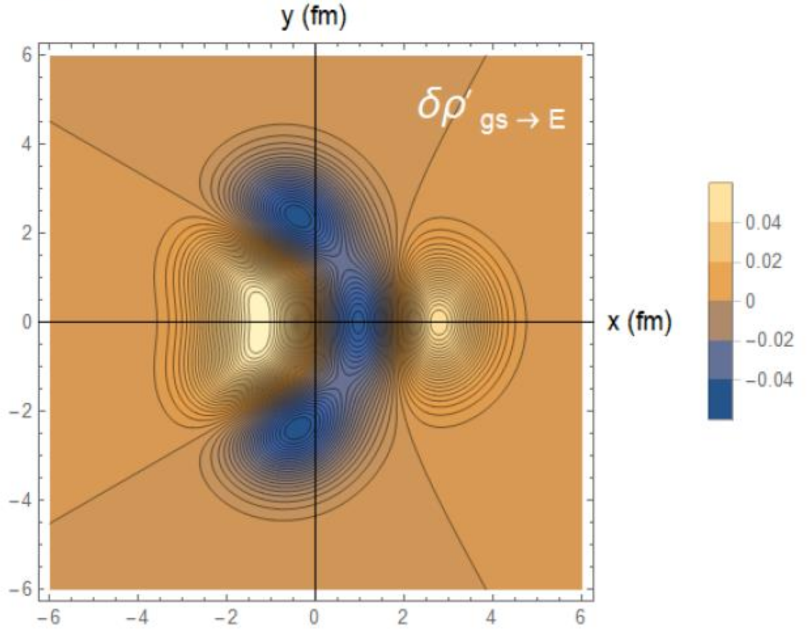








E-band



Transition densities -> Form Factors -> Coupled Channels -> Cross-sections

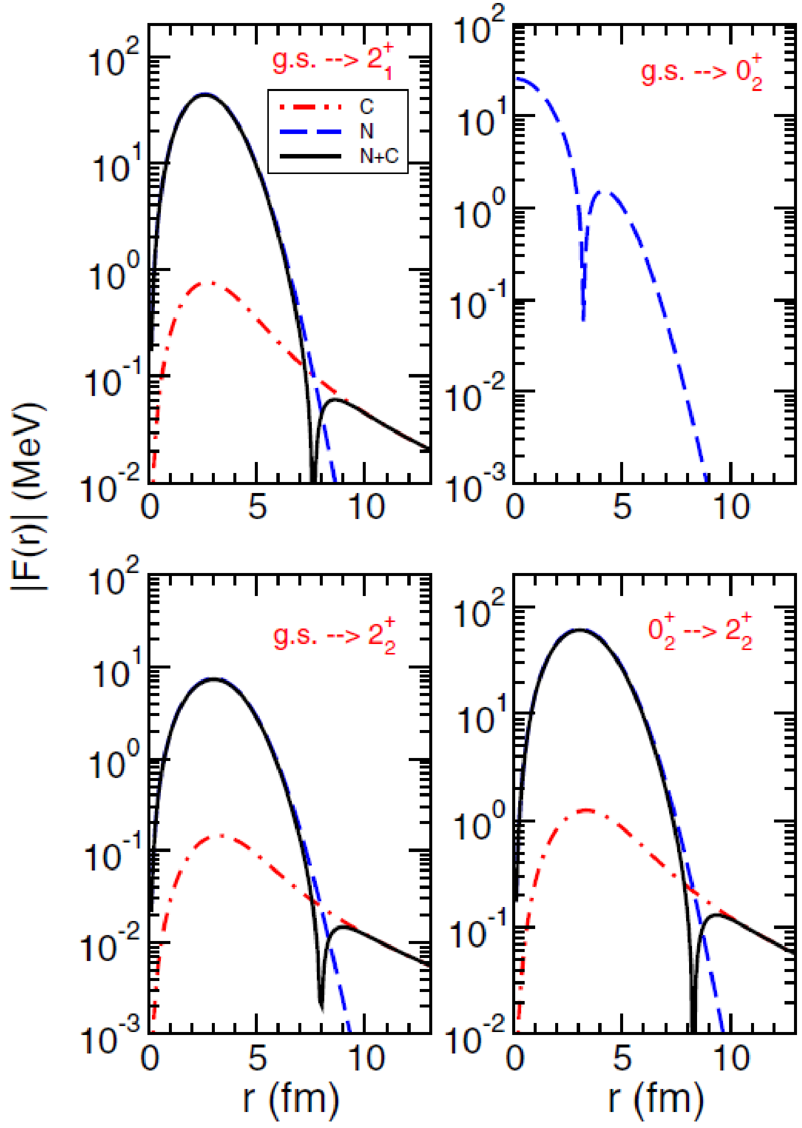


FIG. 15. Form factors in logarithmic scale for a few inelastic excitation processes of interest. We show the nuclear, Coulomb, and total form factors.

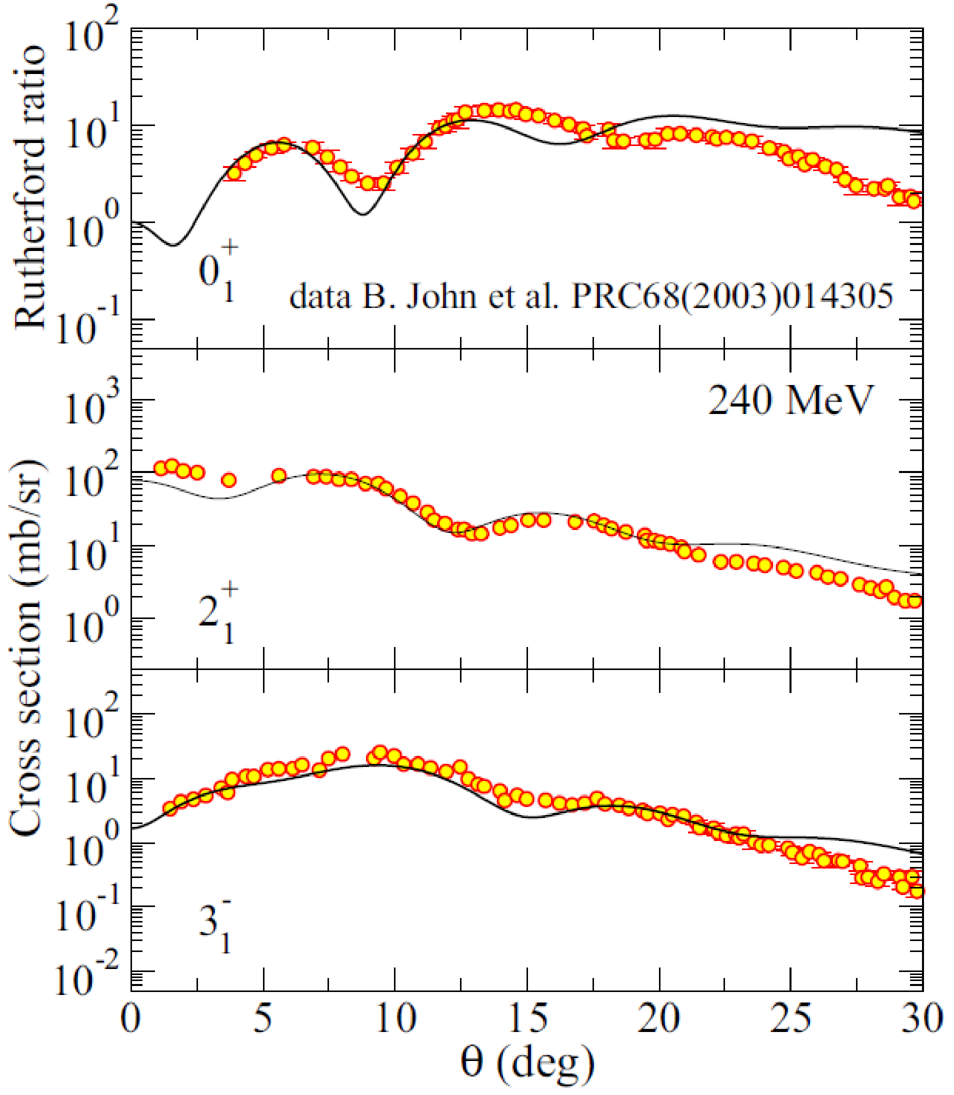


FIG. 16. Differential cross section for the elastic scattering and the transitions $0_1^+ \rightarrow 2_1^+$ and $0_1^+ \rightarrow 3_1^+$ at 240-MeV bombarding energy. Data are from Ref. [41] (retrieved through EXFOR).

Lots of results that I dont'have time to discuss in details.
They confirm that with just a simple triangular model one catches all the gross features, not only of the nuclear structure, but also of reaction dynamics of 12C.

Importance of the imaginary part of the ion-ion potential

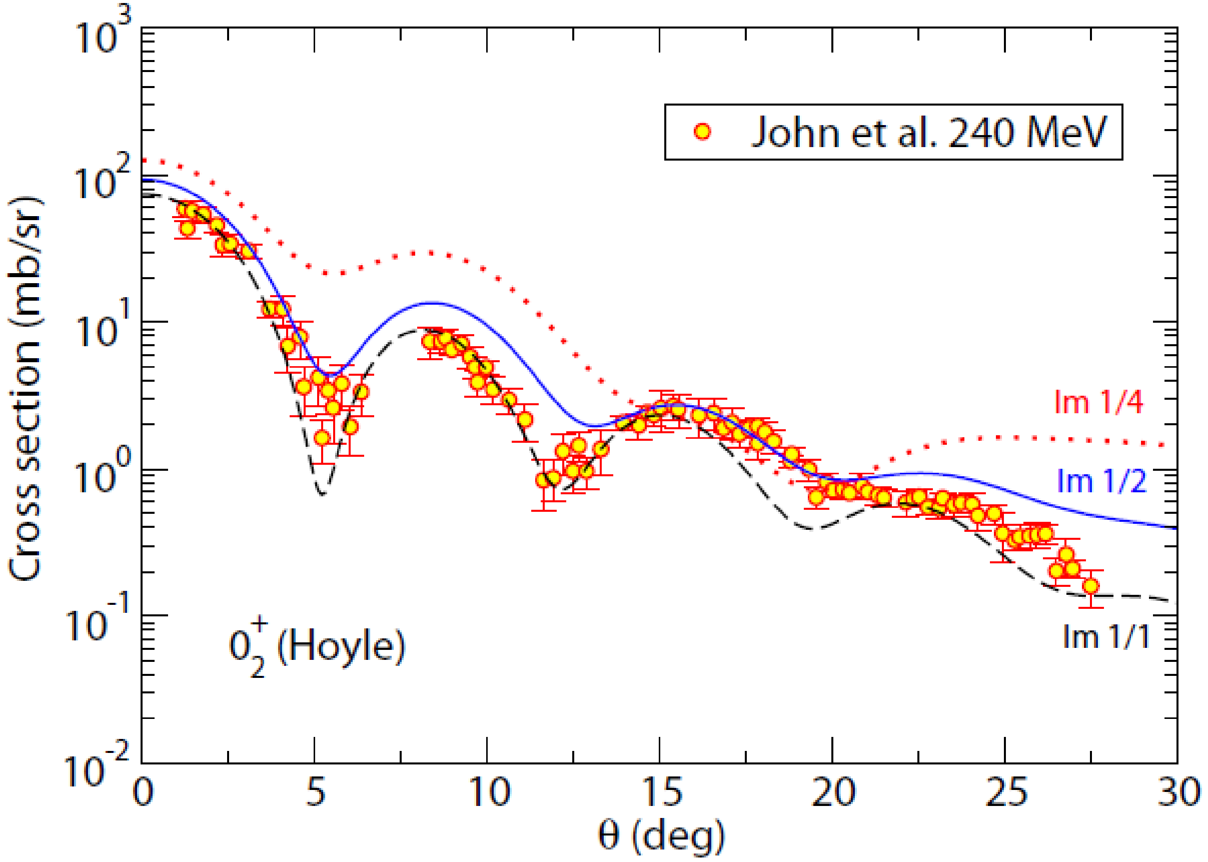


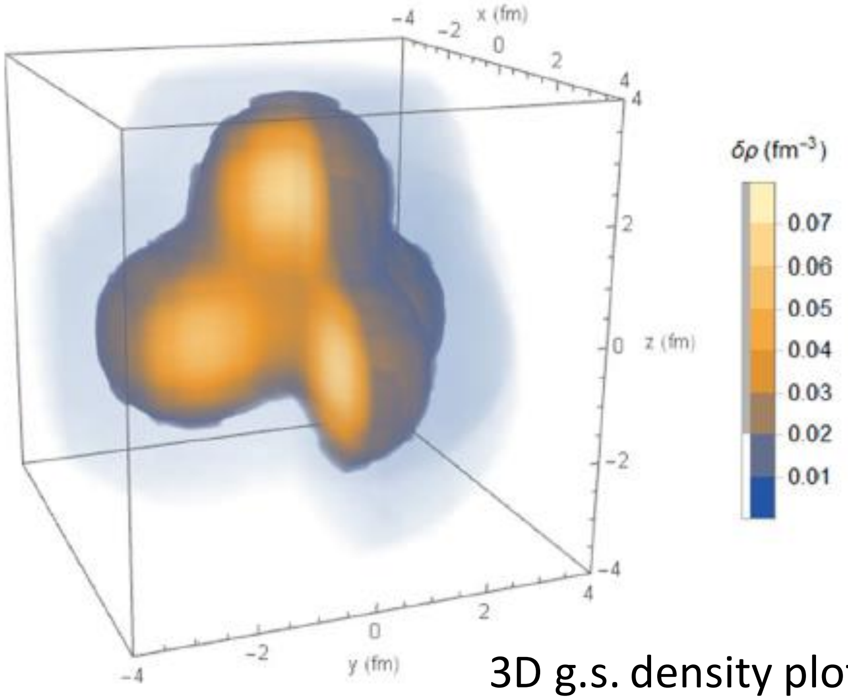
FIG. 17. Differential cross section for the transition $0_1^+ \rightarrow 0_2^+$ at 240-MeV bombarding energy. Data are from Ref. [41] (retrieved through EXFOR) and the three curves have different factors for the depth of the imaginary part as indicated in the figure.

$V(r) + iW(r)$

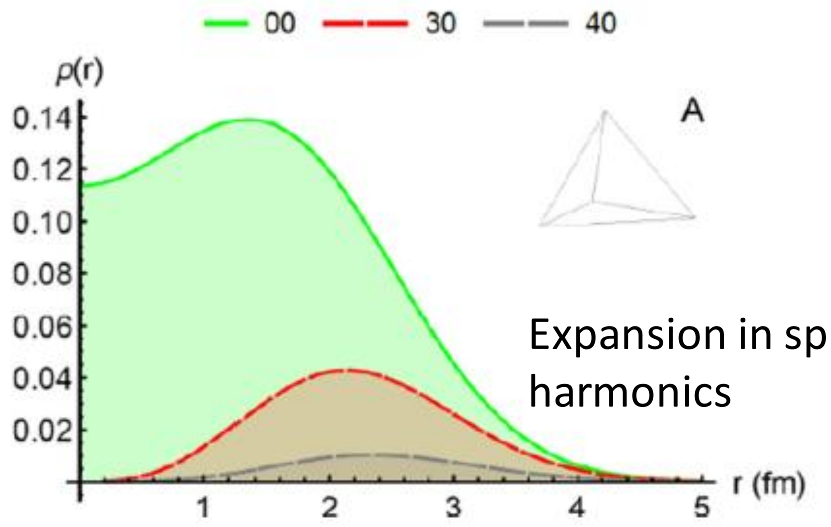
We have found that the «conventional wisdom» of taking the imaginary part ½ of the real part with the same geometry does not work here!
We obtain better results when the imaginary part is increased (= more absorption).

Extended to 16O in a tetrahedral arrangement

Eur. Phys. J. A (2021) 57:33



3D g.s. density plot



Expansion in spherical harmonics

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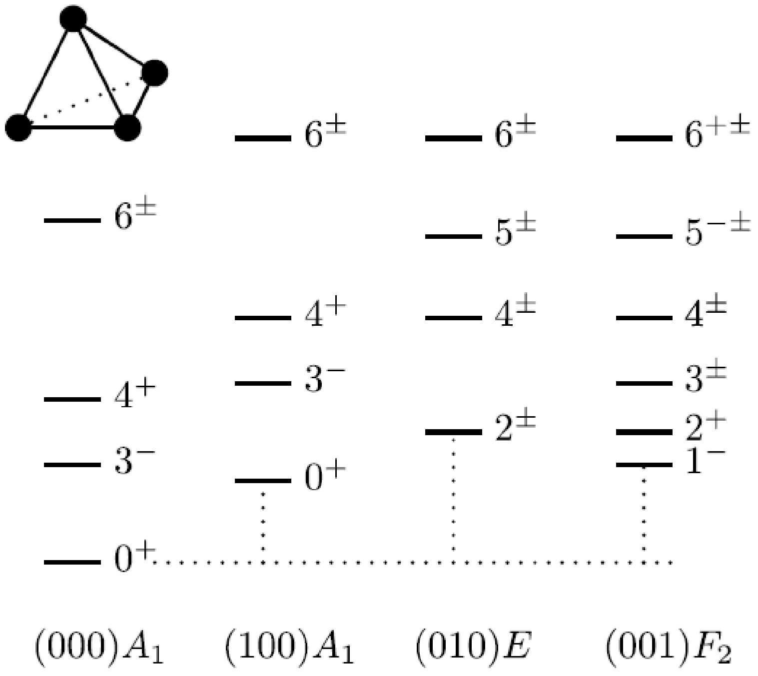
https://doi.org/10.1140/epja/s10050-021-00347-5
THE EUROPEAN PHYSICAL JOURNAL A
Regular Article - Theoretical Physics

Alpha-induced inelastic scattering and alpha-transfer reactions in ^{12}C and ^{16}O within the Algebraic Cluster Model

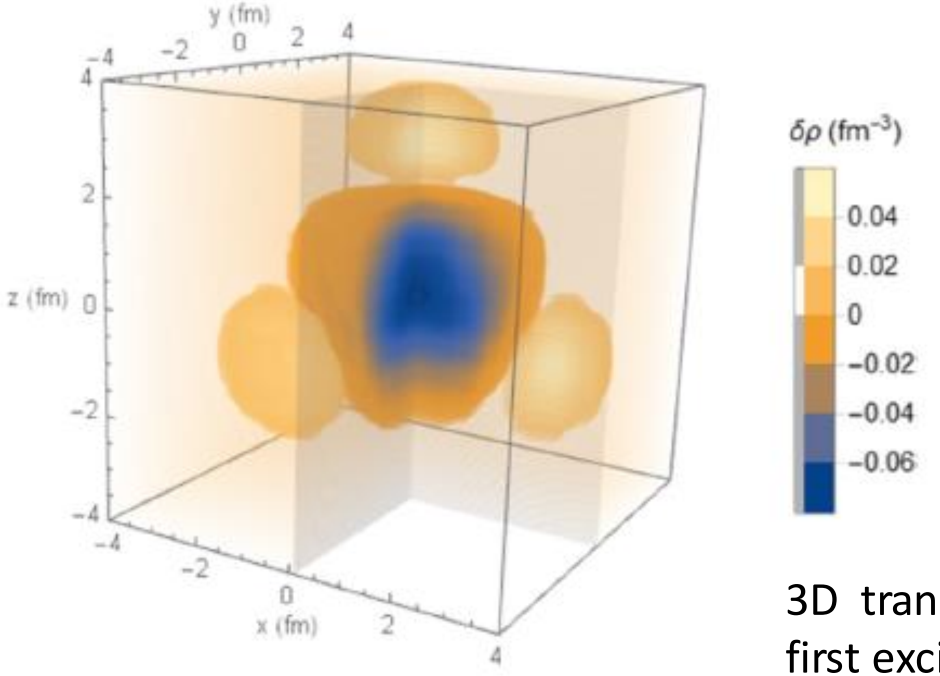
Jesus Casal[1,2], Lorenzo Fortunato[1,2], Edoardo G. Lanza[3,4], Andrea Vitturi[1,2,a]

The tetrahedral group Td allows for singly- , doubly- and triply-degenerate representations

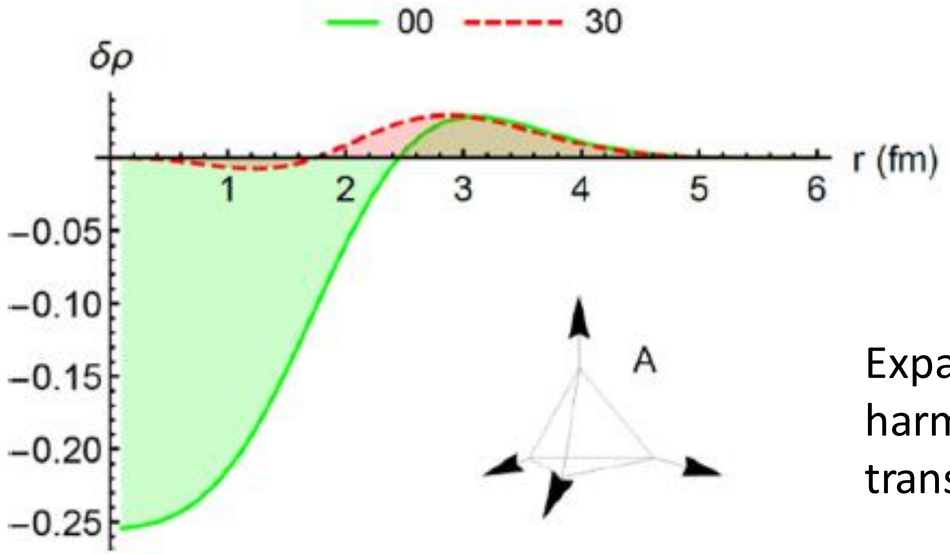
→ one can see all of these excitation modes in the spectrum of 16O !



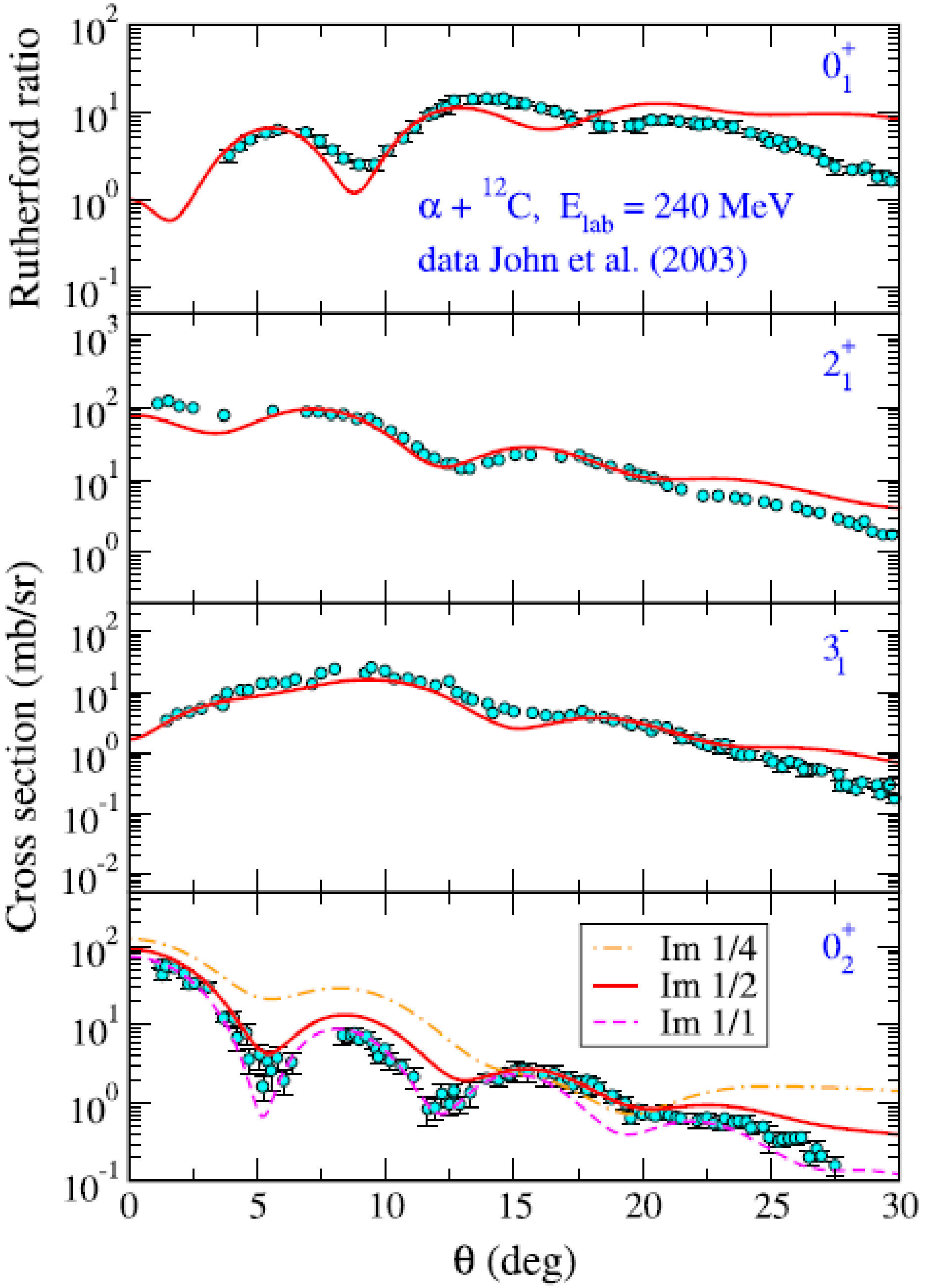
Extended to 16O in a tetrahedral arrangement -> 12C(α,γ)16O



3D transition density to first excited A band (0+)



Expansion in spherical harmonics of the transition densities



Microscopic approaches lead to different routes (with Moriya, Horiuchi, Casal)

Three-body approach:
- stochastic variational method with correlated Gaussian basis functions

two methods are used:
1) orthogonality condition model (OCM)
2) shallow potential model (modified Ali-Bodmer alpha-alpha potential)

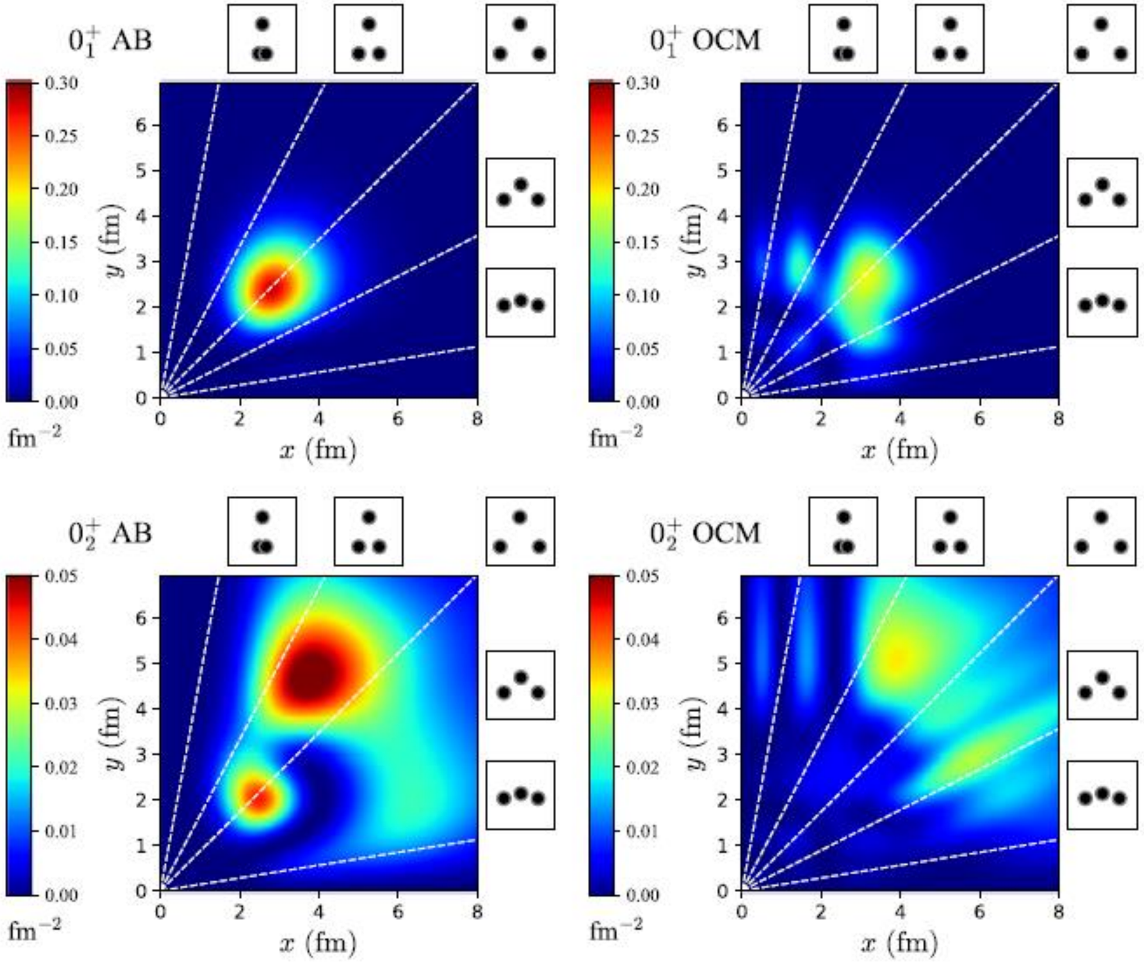


Fig. 1 Two-body density distributions for 0_1^+ and 0_2^+ with the AB and OCM. The white dashed lines are plotted as guide for some specific geometric configurations that are illustrated in the boxes at the end of the lines, e.g., the diagonal line corresponds to the equilateral triangle configuration

Includes the Pauli exclusion principle

(with Moriya, Horiuchi, Casal)

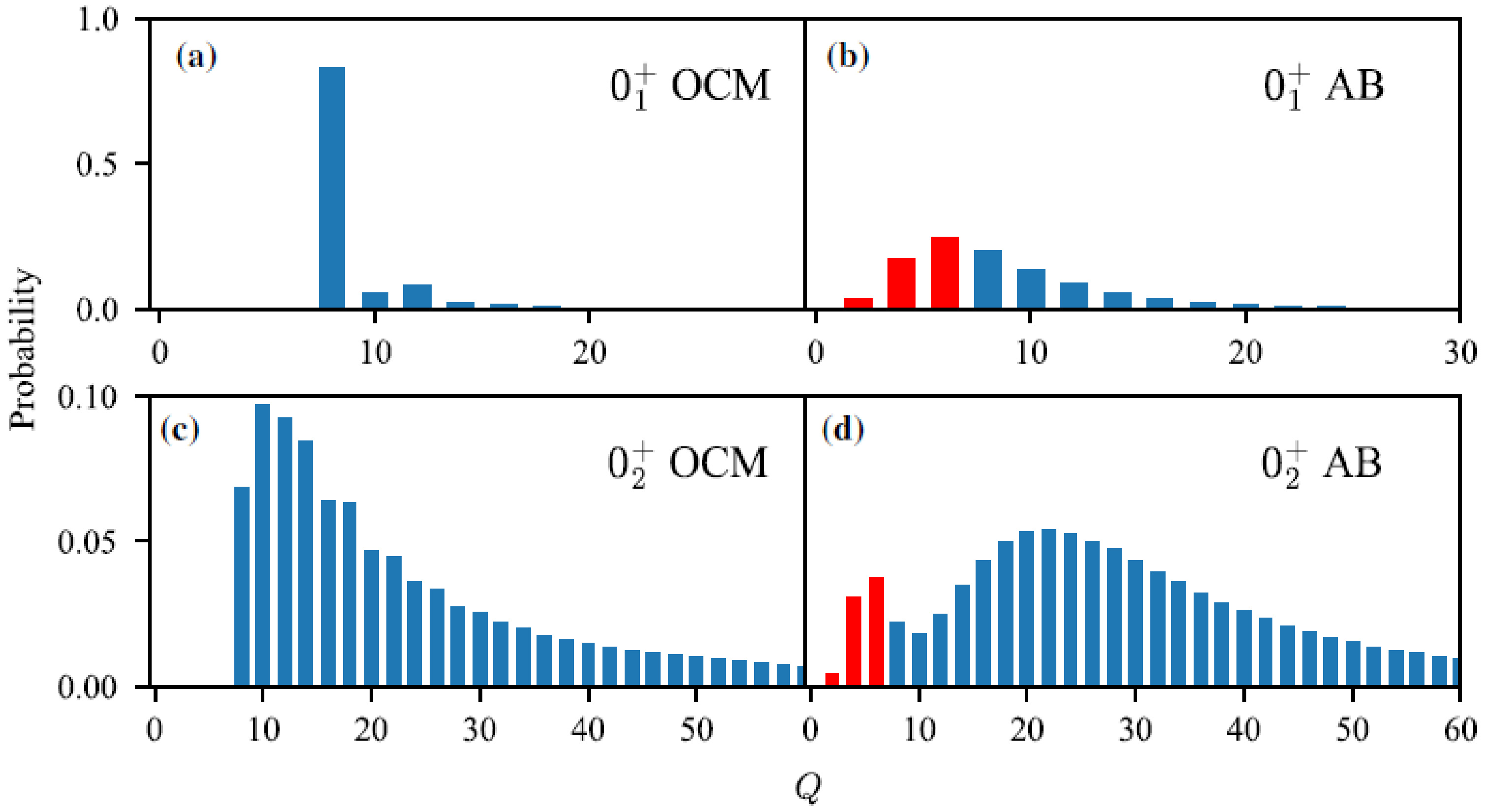


Fig. 2 Occupation probabilities of the harmonic oscillator quanta $Q\hbar\omega$ for **a**, **b** 0_1^+ and **c**, **d** 0_2^+. The oscillator energy $\hbar\omega$ is set to be 22 MeV, consistently with the size parameter of the forbidden states used in the OCM. The states that are forbidden in the OCM ($Q = 0$–6) are colored in red

The ACM «forgets» the Pauli Exclusion principle, in the sense that, if the three alphas are all (1s)^4 configurations, then the missing quanta of the Wildermuth condition must be attributed to the relative motion (two Jacobi vectors).

See P.O.Hess, Symmetry 2023, 15, 1197

The present approach dooes not suffer from this limitation.

One-body density distributions

(with Moriya, Horiuchi, Casal)

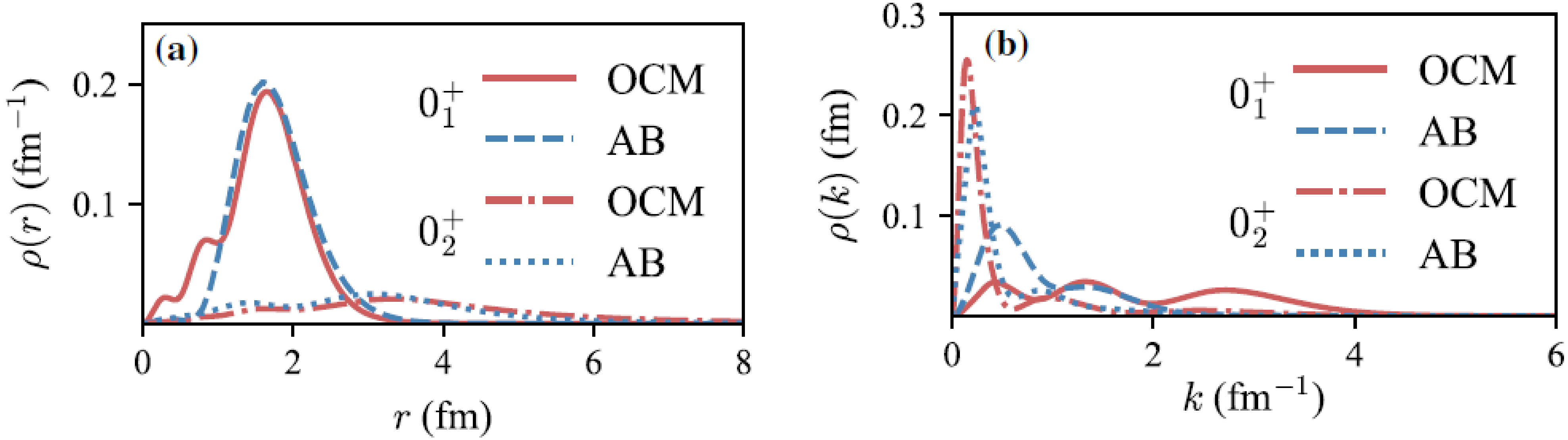


Fig. 3 One-body density distributions in **a** coordinate and **b** momentum space for 0_1^+ and 0_2^+ with the OCM and AB

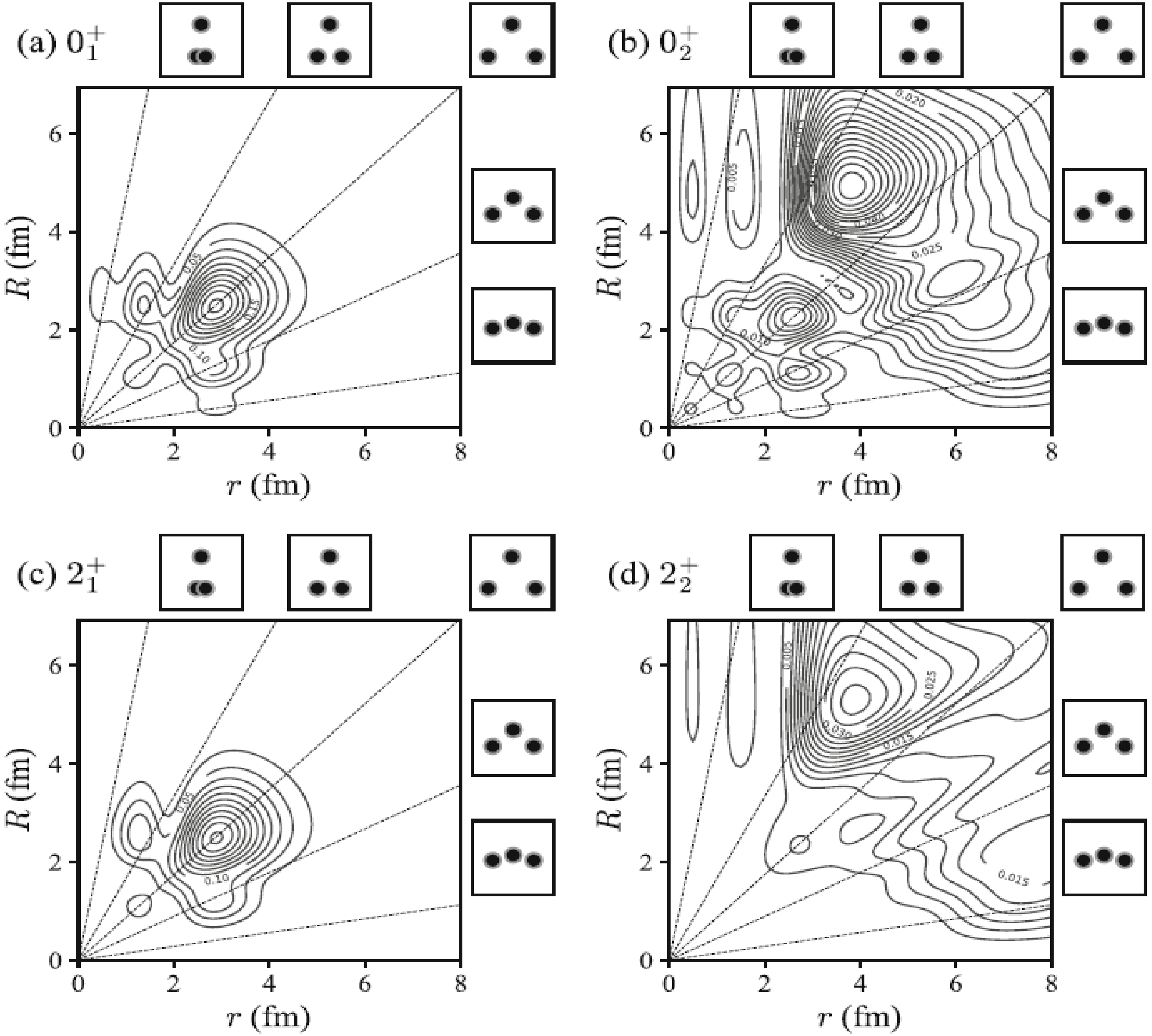
The 0_2^+ state is clearly more external, compatible with a first breathing osciillation of the clusters in and out.

This is the one-body density, if you multiply by r^2 you get lines of about the same height.

Two-body densities shows correlations

(with Moriya, Horiuchi, Casal)

Fig. 2 Two-body density distributions $\rho(r, R)$ of the **a** $J^\pi = 0_1^+$, **b** 0_2^+, **c** 2_1^+, and **d** 2_2^+ states. Contour intervals are 0.025 fm^{-2} for 0_1^+ and 2_1^+ and 0.0025 fm^{-2} for 0_2^+ and 2_2^+. Specific r/R ratios are indicated by dashed lines and their geometric configurations are illustrated in small panels, e.g., the diagonal dashed line indicates the equilateral triangle configurations



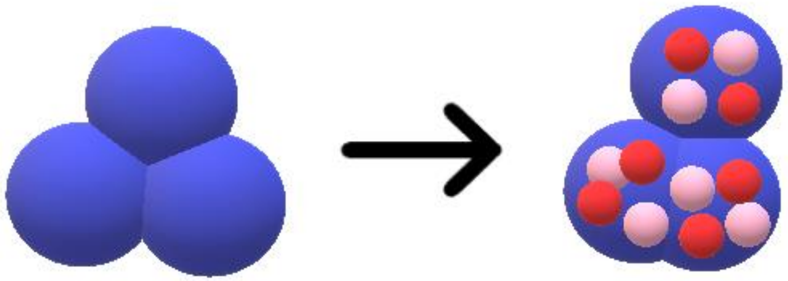
The two-body density distributions indicate that the main configurations of both the second 0_2^+ and 2_2^+ states have acute iscosceles triangle shapes coming mostly from 8Be(0^+) + α configurations and find some hints that the second 2_2^+ state is not an ideal rigid rotational band member of the Hoyle state band.

…things get distorted with raising energy…

Summary

- ✓ The discrete geometrical symmetry group of the equilateral triangle D_{3h} (containing also the permutations of the three alphas) shows its importance not only in the energy spectrum of 12C, but also in reaction properties, like elastic and inelastic scattering.
- ✓ The same can be said of the tetrahedral configurations of four alphas in 16O.

- ✓ Things are not so simple, though, due to the composite nature of the alpha particles (four correlated fermions, subject to the Pauli principle, instead of just a spin 0 boson). When you incorporate these effects, you get some distortion of the geometry in favour of an acute triangle, showing some 8Be+alpha correlations.



3 spin-0 bosons   12 spin-1/2 fermions