

Relativistic current operators

W. N. Polyzou

Jacek Gloak

The University of Iowa

EFB 25 -

Johannes Gutenberg University



Probes of Hadronic structure

- Short-distance electromagnetic probes of strongly interacting systems require relativistic momentum transfers, a **relativistic model** of the strong interaction dynamics and a **consistent** strong electromagnetic current. ($\Delta p \sim \hbar/2\Delta x$, $\Delta x = .1 \text{ fm} \rightarrow \Delta p \sim 1 \text{ GeV}$)
- Model input:

$$H = H_s + H_{\gamma e} + \int (I_s^\mu(0, \mathbf{x}) + I_e^\mu(0, \mathbf{x})) A_\mu(0, \mathbf{x}) dx$$

Relativistic models of strong interaction dynamics
one-photon-exchange approximation

$$U(\Lambda, a) \rightarrow U_s(\Lambda, a) \otimes U_{QED}(\Lambda, a)$$

$$\begin{aligned} [P_s^\mu, P_s^\nu] &= 0, & [J_s^i, P_s^j] &= i\epsilon^{ijk} P_s^k, & [J_s^i, J_s^j] &= i\epsilon^{ijk} J_s^k, \\ [J_s^i, K_s^j] &= i\epsilon^{ijk} K_s^k, & [K_s^i, K_s^j] &= -i\epsilon^{ijk} J_s^k \\ [K_s^i, P_s^j] &= i\delta^{ij} H_s & [K_s^i, H_s] &= iP_s^i. \end{aligned}$$

Dynamical constraints on hadronic current
(current covariance and current conservation)

$$\begin{aligned} [K_s^i, I_s^j(0)] &= i\delta^{ij} I_s^0(0), & [K_s^i, I_s^0(0)] &= iI_s^i(0) \\ [H_s, I_s^0(0)] &= \sum_i [P_s^i, I_s^i(0)] \end{aligned}$$

Not satisfied by impulse (one-body) current.

S-matrix preserving change of representation

$$H'_s = WH_sW^\dagger \quad I_s'^{\mu}(0) = WI_s^\mu(0)W^\dagger$$

$$S_s = \Omega_{s+}^\dagger \Omega_{s-}$$

$$\Omega_{s\pm} = s - \lim_{t \rightarrow \pm\infty} e^{iH_s t} \Pi e^{-iH_0 t} \quad \Omega_{s\pm}' = s - \lim_{t \rightarrow \pm\infty} e^{iH'_s t} \Pi' e^{-iH_0 t}$$

$$S_s = S_s' \iff W^\dagger W = I$$

and short range condition

$$s - \lim_{t \rightarrow \pm\infty} (W\Pi - \Pi') e^{-iH_0 t} = 0 \quad \text{both time limits!}$$

Models and many-body strong currents are representation dependent.

For translationally and rotationally invariant W

$$[W, \mathbf{P}_s] = [W, \mathbf{J}_s] = 0$$

A representation independent calculation requires:

$$H'_s = WH_sW^\dagger$$

$$\mathbf{K}'_s = W\mathbf{K}_sW^\dagger$$

$$I_s^{\mu\nu}(0) = WI_s^\mu(0)W^\dagger$$

Problem: Construct a current operator, $I_s^\mu(0)$, consistent with the model dynamics?

Weyl representation

Irreducible set of operators $\{\hat{\mathbf{q}}_i, \hat{\mathbf{p}}_i\}$

Any operator, \hat{H} , (local or non-local) can be expressed in the form:

$$\hat{H} = \int d^{3N} \mathbf{a} d^{3N} \mathbf{b} h(\mathbf{a}, \mathbf{b}) e^{i\mathbf{a} \cdot \hat{\mathbf{q}}} e^{i\mathbf{b} \cdot \hat{\mathbf{p}}}$$

where

$$\hat{\mathbf{p}} = (\hat{p}_1, \dots, \hat{p}_N)$$

$$\hat{\mathbf{q}} = (\hat{q}_1, \dots, \hat{q}_N)$$

$$[\hat{q}_i, \hat{p}_j] = i\delta_{ij}.$$

Relativistic case - irreducible operators $\{\mathbf{q}_i, \mathbf{p}_i\}$

$\mathbf{q}_i =$ Newton-Wigner position operator

function of single-particle Poincaré generators

$$\mathbf{q}_i := -\frac{1}{2} \left\{ \frac{1}{H_i}, \mathbf{K}_i \right\} - \frac{\mathbf{P}_i \times (H_i \mathbf{J}_i - \mathbf{K}_i)}{M_i H_i (M_i + H_i)} = i \nabla_p$$

where the partial derivative is computed **holding the canonical spin constant** (recall Wigner rotations are momentum dependent).

$$[q_i, p_j] = -i \delta_{ij}$$

Replace \mathbf{p} by a gauge covariant derivative

$$\hat{H}_S \rightarrow \hat{H}_s = \int d^{3N} \mathbf{a} d^{3N} \mathbf{b} h(\mathbf{a}, \mathbf{b}) e^{i\mathbf{a} \cdot \hat{\mathbf{q}}} e^{i(\hat{\mathbf{p}} - e\hat{\mathbf{A}}(\hat{\mathbf{q}})) \cdot \mathbf{b}}$$

The current is the coefficient of the term linear in $\hat{\mathbf{A}}(\mathbf{q})$ (use Trotter product formula to get)

$$e \frac{d\hat{H}_S}{de} \Big|_{e=0} = - \int_0^1 d\lambda \int d^{3n} \mathbf{a} d^{3n} \mathbf{b} h(\mathbf{a}, \mathbf{b}) e^{i\mathbf{a} \cdot \hat{\mathbf{q}}} e^{i\lambda \hat{\mathbf{p}} \cdot \mathbf{b}} \sum_j \hat{\mathbf{A}}(\hat{\mathbf{q}}_j) \cdot \mathbf{b}_j e^{i(1-\lambda)\hat{\mathbf{p}} \cdot \mathbf{b}}$$

Since \mathbf{p} generates translations $\hat{\mathbf{A}}(\hat{\mathbf{q}}_j)$ can be factored out of non-commuting operators

Expression for vector part of the current

$$\begin{aligned} & \langle \mathbf{p}'_1 \cdots \mathbf{p}'_n | \mathbf{I}(\mathbf{q}, 0) | \mathbf{p}_1 \cdots \mathbf{p}_n \rangle = \\ & -e \int_0^1 d\lambda \int \frac{d^{3N} \mathbf{a} d^{3N} \mathbf{b} d^{3N} \mathbf{q}'}{(2\pi)^{3N}} h(\mathbf{a}, \mathbf{b}) e^{-i\mathbf{q}' \cdot (\mathbf{p}' - \mathbf{p} - \mathbf{a})} e^{i\mathbf{p} \cdot \mathbf{b}} \times \\ & \quad \sum_j \delta(\mathbf{q}'_j + \lambda \mathbf{b}_j - \mathbf{q}) \mathbf{b}_j. \end{aligned}$$

Covariance gives the charge density

$$I^0(\mathbf{0}, 0) = i[\mathbf{K}^i, I^i(\mathbf{0}, 0)] \quad \text{no sum, any } i$$

Example

$$H = \sqrt{\mathbf{p}^2 + m^2}$$

The above method gives

$$\langle \mathbf{p}' | \mathbf{I}(\mathbf{x}, 0) | \mathbf{p} \rangle = -e \frac{1}{(2\pi)^3} \int_0^1 d\lambda \frac{(1-\lambda)\mathbf{p} + \lambda\mathbf{p}'}{\sqrt{((1-\lambda)\mathbf{p} + \lambda\mathbf{p}')^2 + m^2}} e^{i\mathbf{x} \cdot (\mathbf{p} - \mathbf{p}')}$$

Compared to non-relativistic result ($H = \frac{\mathbf{p}^2}{2m}$):

$$\langle \mathbf{p}' | \mathbf{I}(\mathbf{x}, 0) | \mathbf{p} \rangle = -e \frac{1}{(2\pi)^3} \frac{\mathbf{p}' + \mathbf{p}}{2m} e^{i\mathbf{x} \cdot (\mathbf{p} - \mathbf{p}')}$$

Light-front case

$$P^\pm = H \pm P^3, \quad \mathbf{E}_\perp = \mathbf{K}_\perp - \hat{\mathbf{z}} \times \mathbf{J}$$

Irreducible canonical algebra:

$$p_i^+, q_i^- = \frac{1}{2} \left\{ \frac{1}{p_i^+}, K_i^3 \right\} = i \frac{\partial}{\partial p_i^+}$$

$$\mathbf{p}_{i\perp}, \mathbf{q}_{i\perp} = i \frac{1}{p^+} \mathbf{E}_\perp = i \frac{\partial}{\partial \mathbf{p}_{\perp i}}$$

Weyl representation

$$\hat{P}^- = \int d^{3N} \mathbf{a} d^{3N} \mathbf{b} p^-(\mathbf{a}, \mathbf{b}) e^{i\mathbf{a} \cdot \hat{\mathbf{q}}} e^{i\mathbf{b} \cdot \hat{\mathbf{p}}}$$

Current covariance

$$I^-(0) = I^+(0) - 2[J^2 [J^2, I^+(0)]]$$

Remarks - conclusions

- The result is a current operator rather than a matrix element - it can be used in different reactions.
- Method is applicable to non-local interactions.
- Invariant under change of representation.
- Consistent with the dynamics - based on local gauge invariance.
- Light-front version available for hadronic models.
- Charge density operator requires a representation of the dynamical boost generator.
- Construction assumes point charges - needs to be generalized to treat extended charge distributions (particles with form factors).

Thanks - organizers!