# Baryon-baryon interactions from lattice QCD

**Hartmut Wittig**

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Johannes Gutenberg-Universität Mainz

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***Johannes Gutenberg-Universität Mainz***
**2 August 2023**

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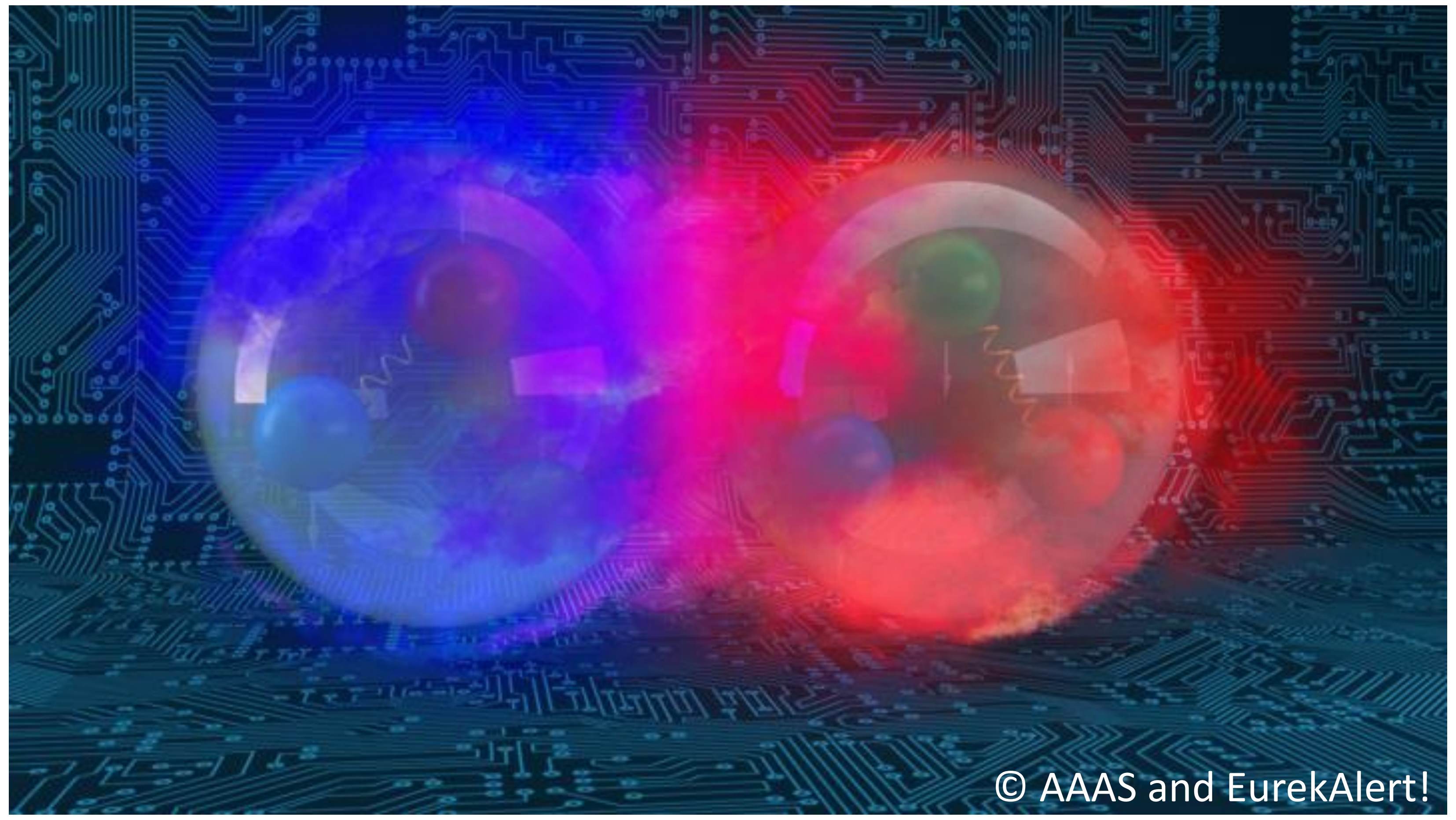
# Introduction

**Nucleon-nucleon interactions**

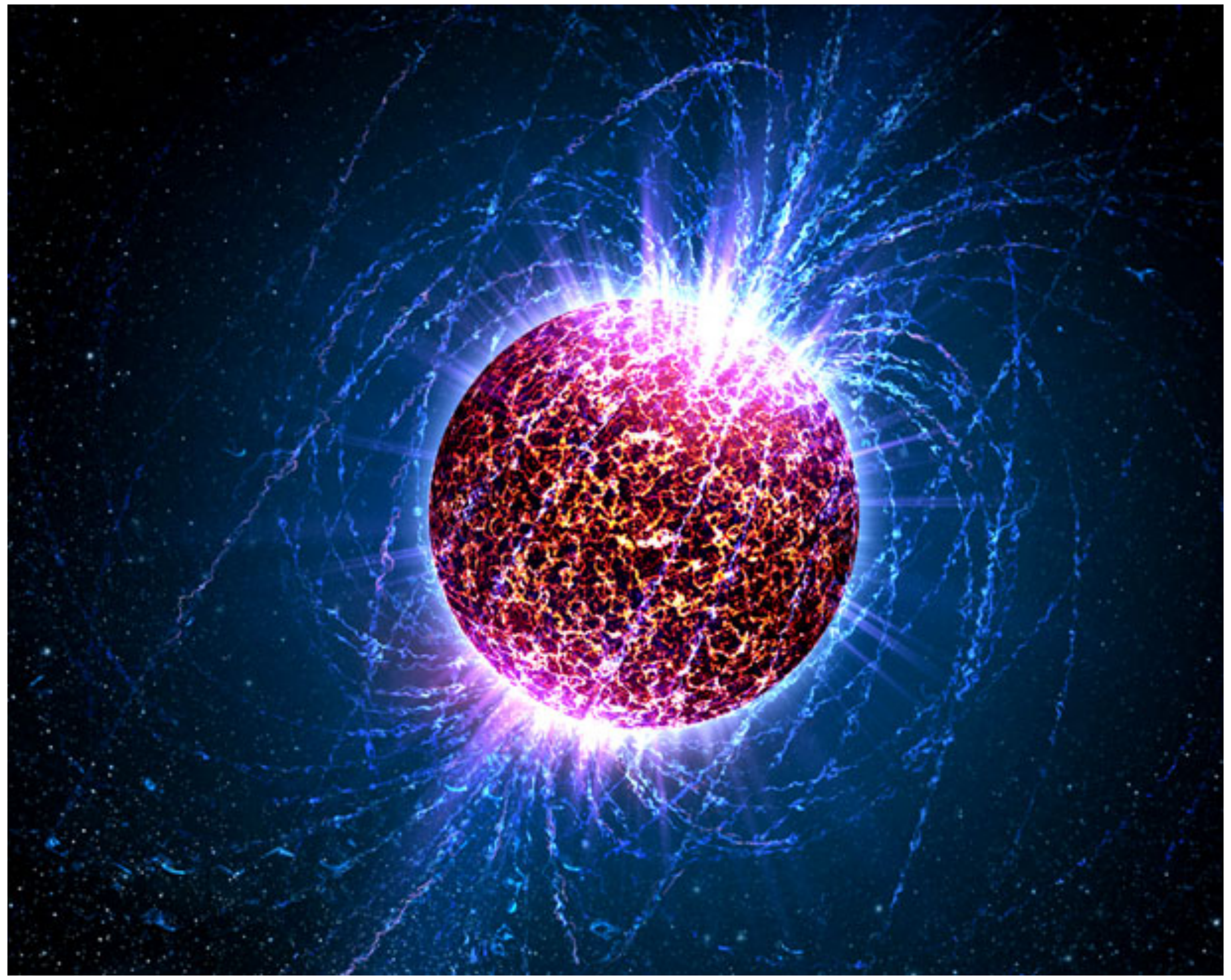
- Simplest systems to study formation of light nuclei from first principles
- How does the weak binding of the deuteron emerge from QCD?




**Hyperon-nucleon and hyperon-hyperon interactions**

- Relevant for physics of (double) hypernuclei, neutron-rich matter, neutron stars
- Does a bound *H* dibaryon exist?

[Emiko Hiyama, MON 11:15]



# The *H* Dibaryon

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**Perhaps a Stable Dihyperon***

R. L. Jaffe†
*Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305, and Department of Physics and Laboratory of Nuclear Science,‡ Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*
(Received 1 November 1976)

In the quark bag model, the same gluon-exchange forces which make the proton lighter than the $\Delta(1236)$ bind six quarks to form a stable, flavor-singlet (with strangeness of $-2$) $J^P=0^+$ dihyperon ($H$) at 2150 MeV. Another isosinglet dihyperon ($H^*$) with $J^P=1^+$ at 2335 MeV should appear as a bump in $\Lambda\Lambda$ invariant-mass plots. Production and decay systematics of the $H$ are discussed.

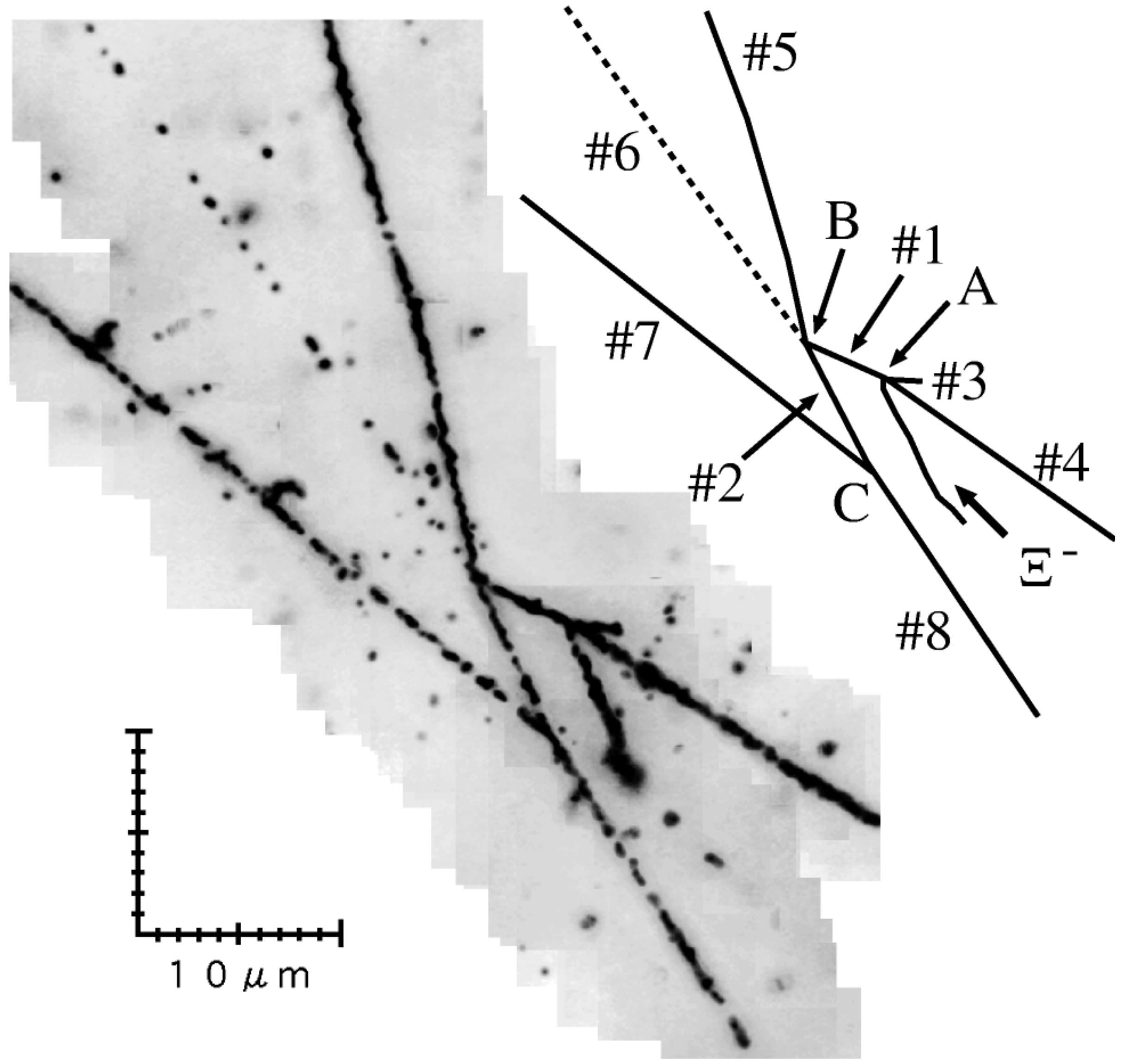
- "Nagara event": Observation of a ${}^{\ 6}_{\Lambda\Lambda}\mathrm{He}$ double-hypernucleus

  Binding energy: $B_{\Lambda\Lambda} = 7.25 \pm 0.19\,\left({}^{+0.18}_{-0.11}\right)\,\mathrm{MeV}$

  Interpreted as sequential weak decay of ${}^{\ 6}_{\Lambda\Lambda}\mathrm{He}$ :

  $m_H > 2m_\Lambda - B_{\Lambda\Lambda} = 2223.7\,\mathrm{MeV}$ @ 90% CL

  *[Takahashi et al., PRL 87 (2001) 212502]*



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Deeply bound *udsuds* state ("sexaquark") proposed / discussed as dark matter candidate:

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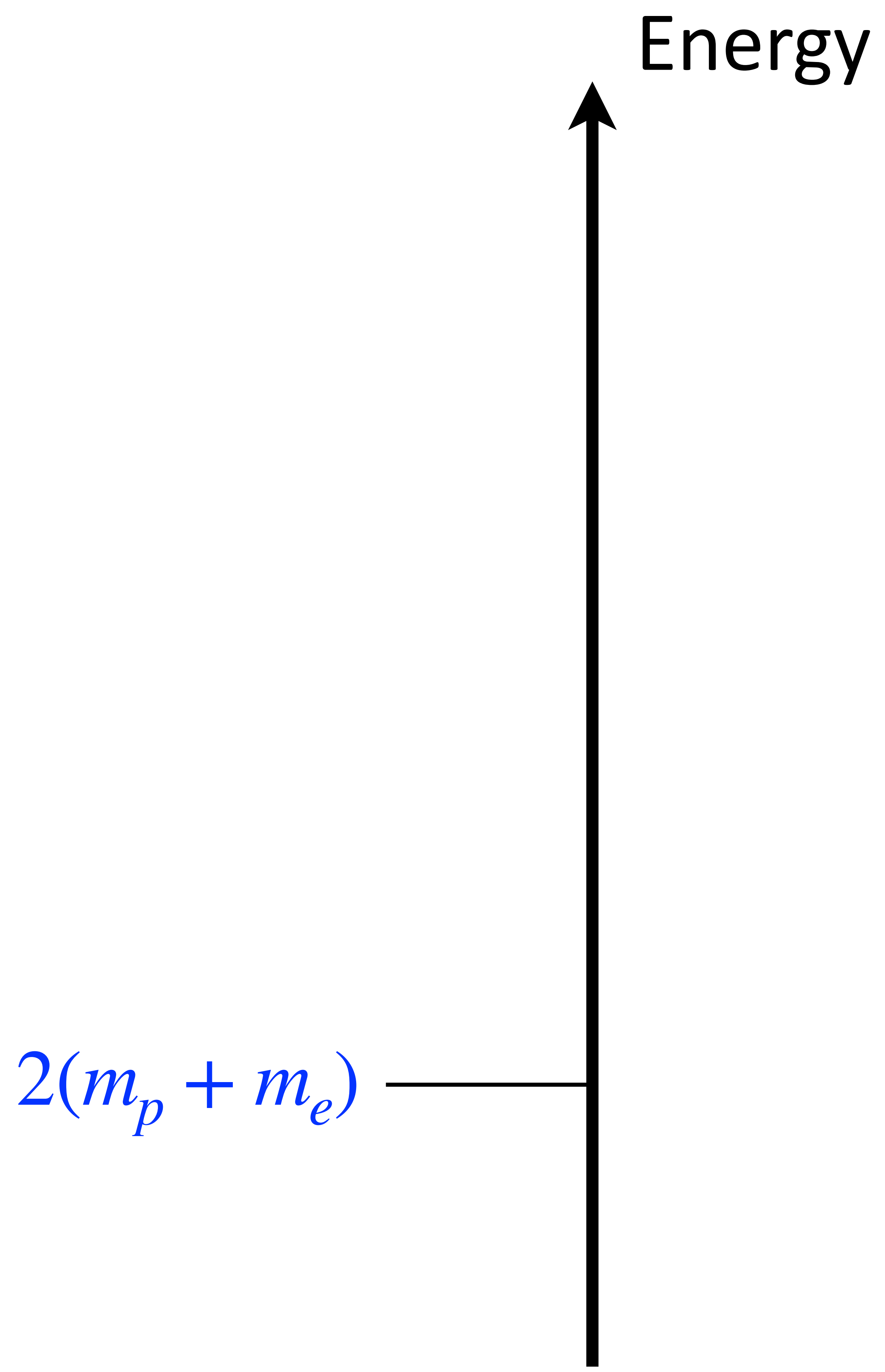
→ *H* dibaryon absolutely stable



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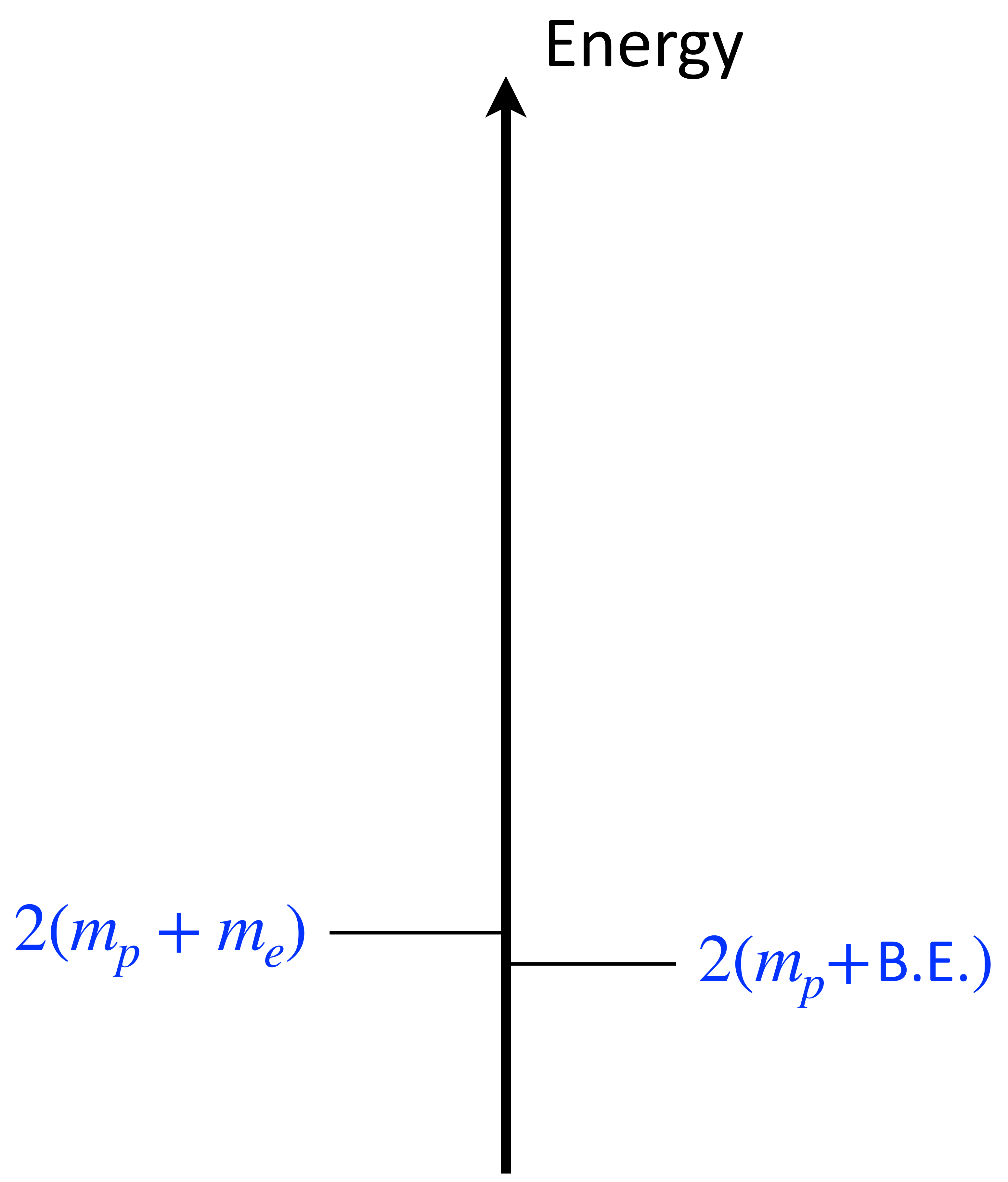

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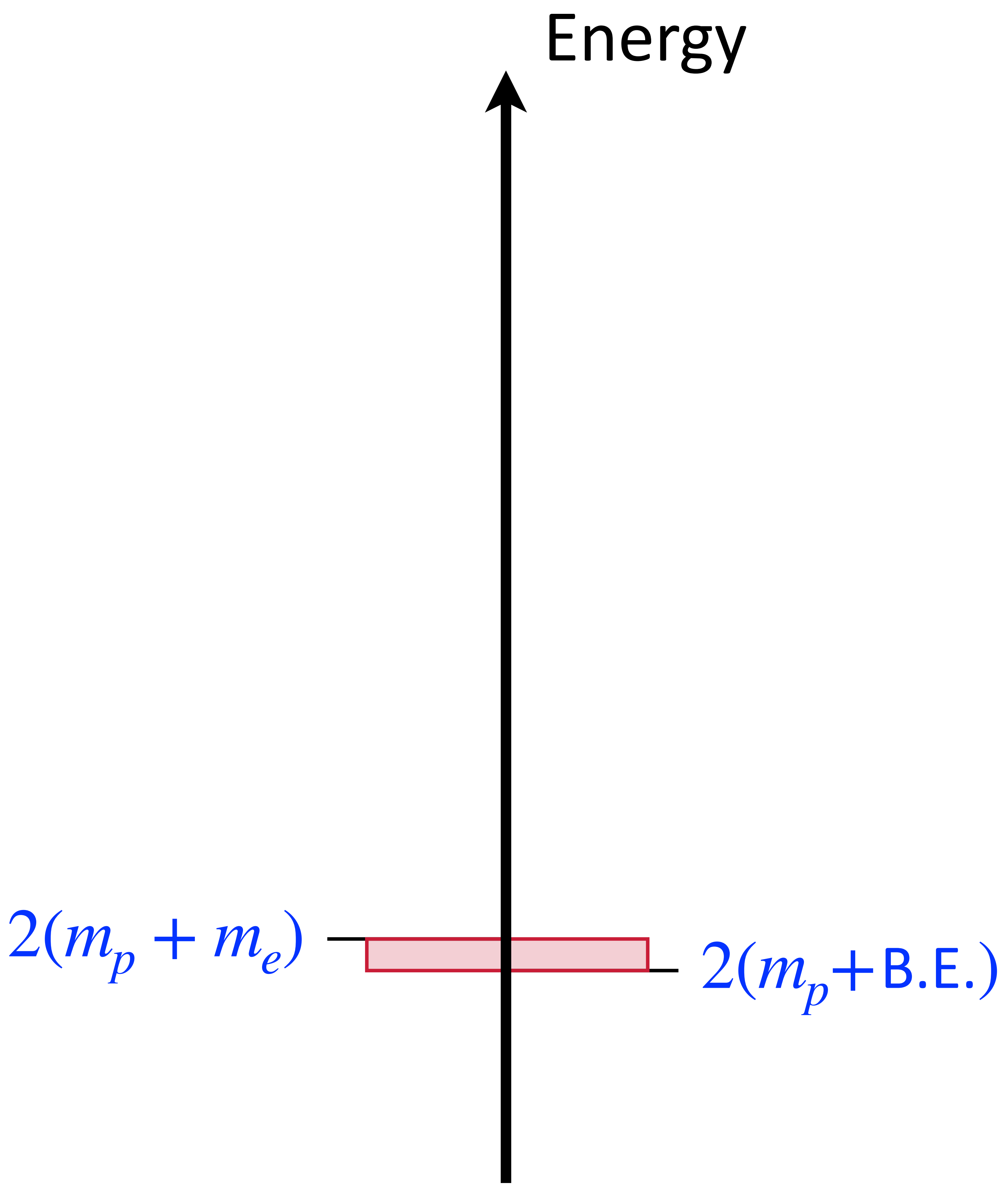

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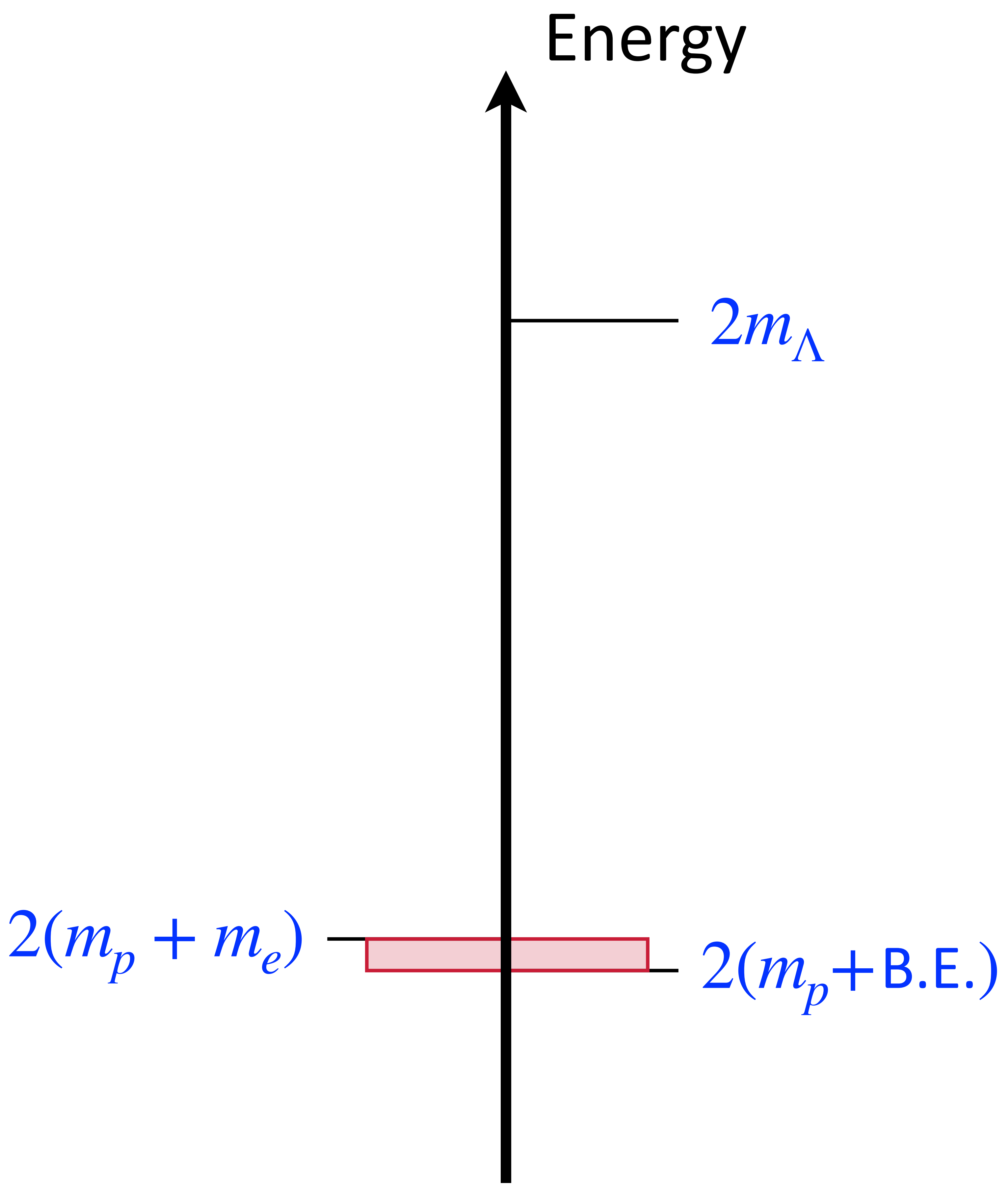

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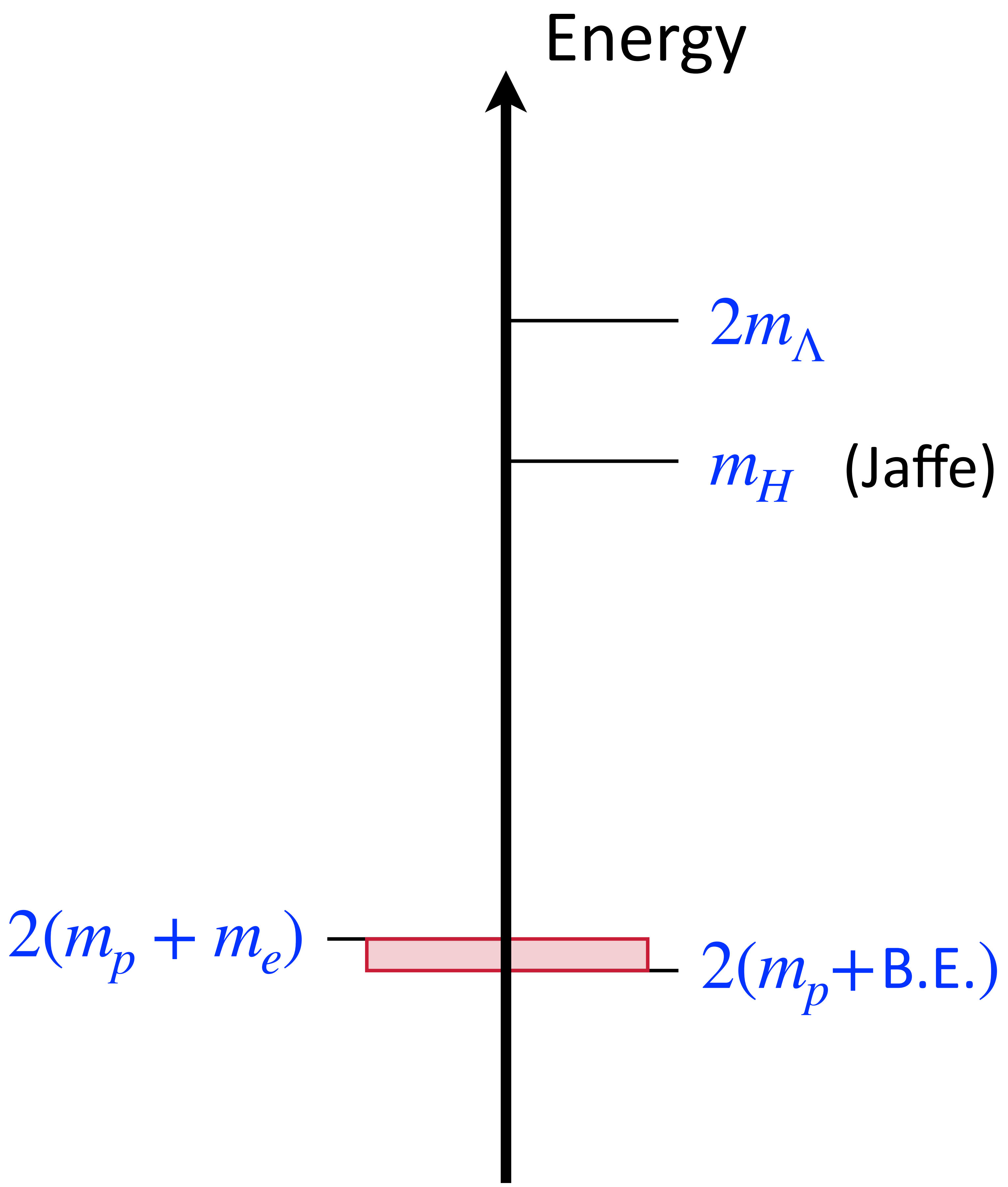

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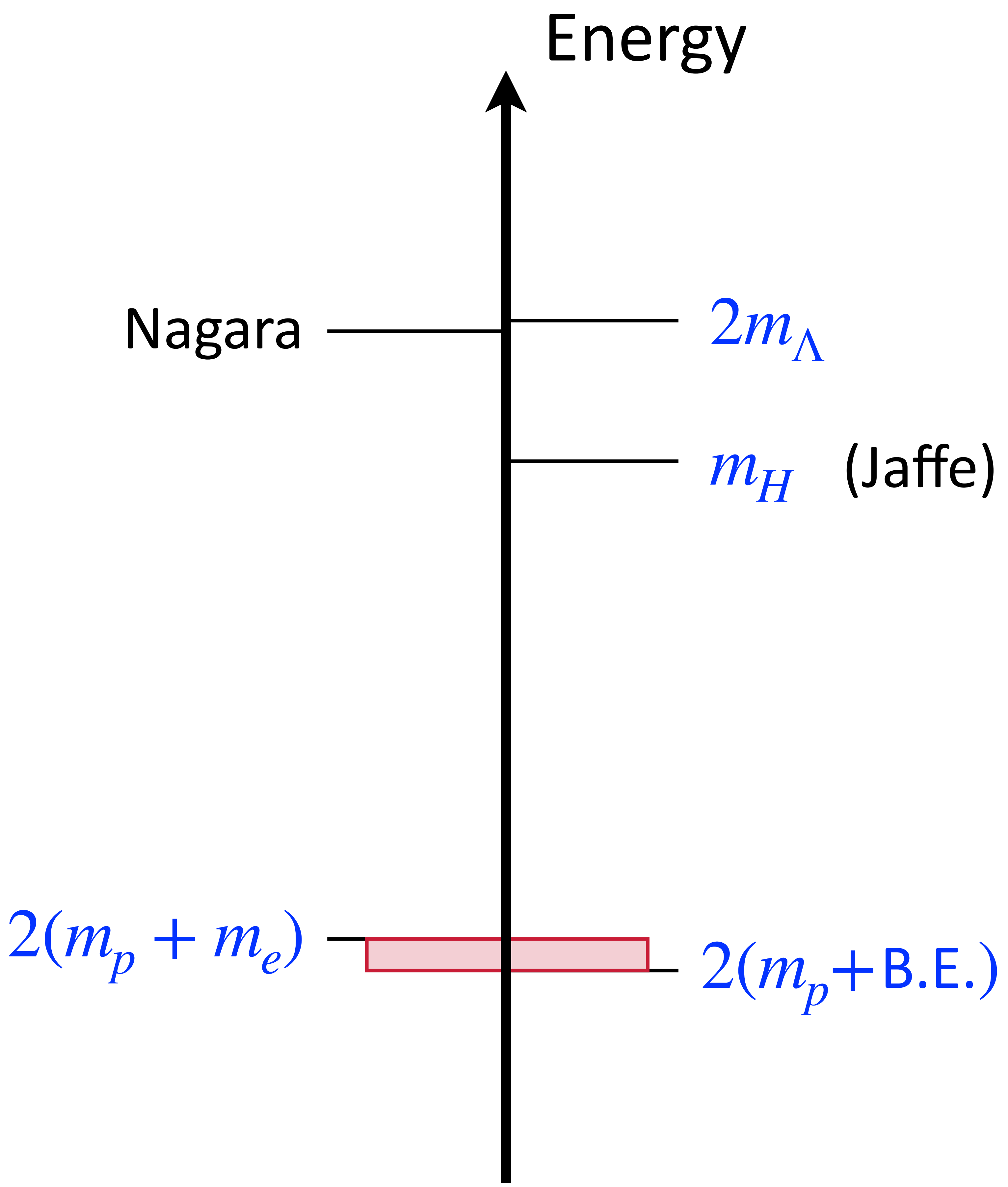

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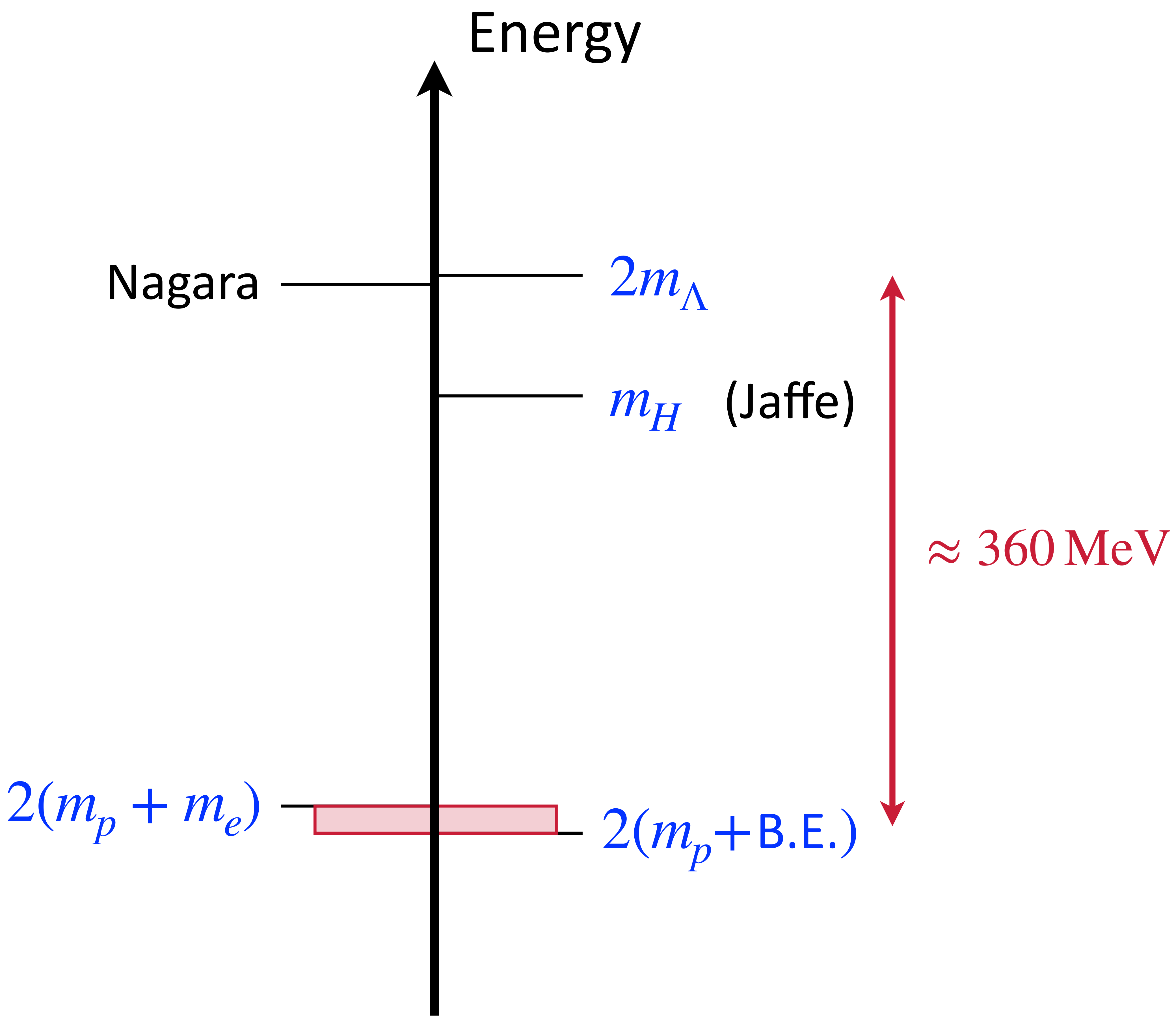


→ Scenario requires very large binding energy of $\approx 360\,\mathrm{MeV}$

# Outline

Dibaryons in Lattice QCD

The *H* dibaryon: Lattice results

Nucleon-nucleon scattering

Charmed tetraquarks

Summary — Conclusions — Outlook

# Dibaryons in Lattice QCD

## Computing the hadron spectrum in Lattice QCD

Spectral information encoded in correlation functions

$$\sum_{\boldsymbol{x},\boldsymbol{y}} \mathrm{e}^{i\boldsymbol{p}\cdot(\boldsymbol{y}-\boldsymbol{x})} \left\langle O_{\mathrm{had}}(y)\, O^{\dagger}_{\mathrm{had}}(x)\right\rangle = \sum_{n} w_n(\boldsymbol{p})\,\mathrm{e}^{-E_n(\boldsymbol{p})(y_0-x_0)} \quad \xrightarrow{(y_0-x_0)\to\infty} \quad w_1(\boldsymbol{p})\,\mathrm{e}^{-E_1(\boldsymbol{p})(y_0-x_0)}$$

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  e.g. Nucleon: $O_N = \epsilon_{abc} \left(u^a\, C\gamma_5\, d^b\right) u^c$

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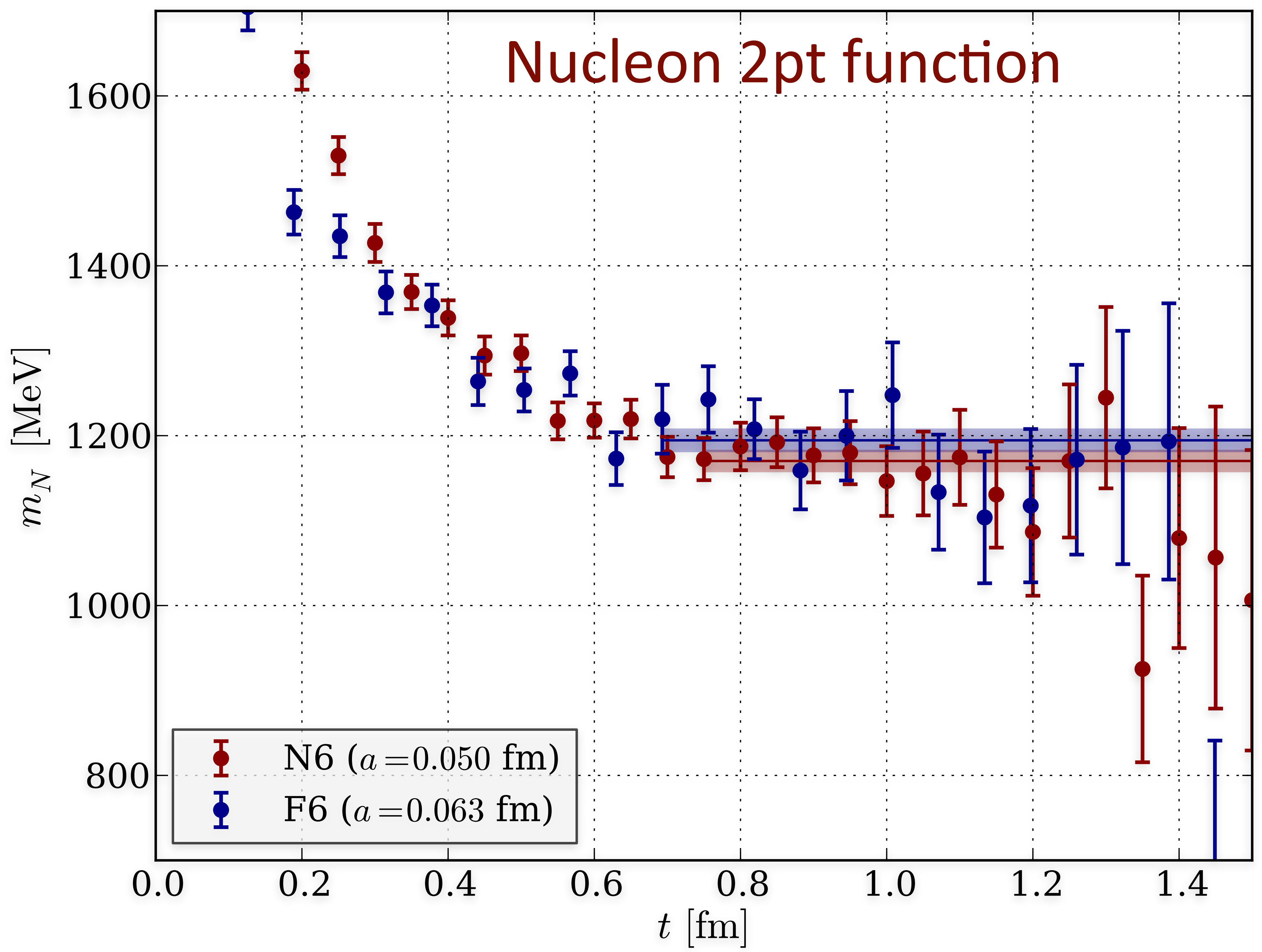

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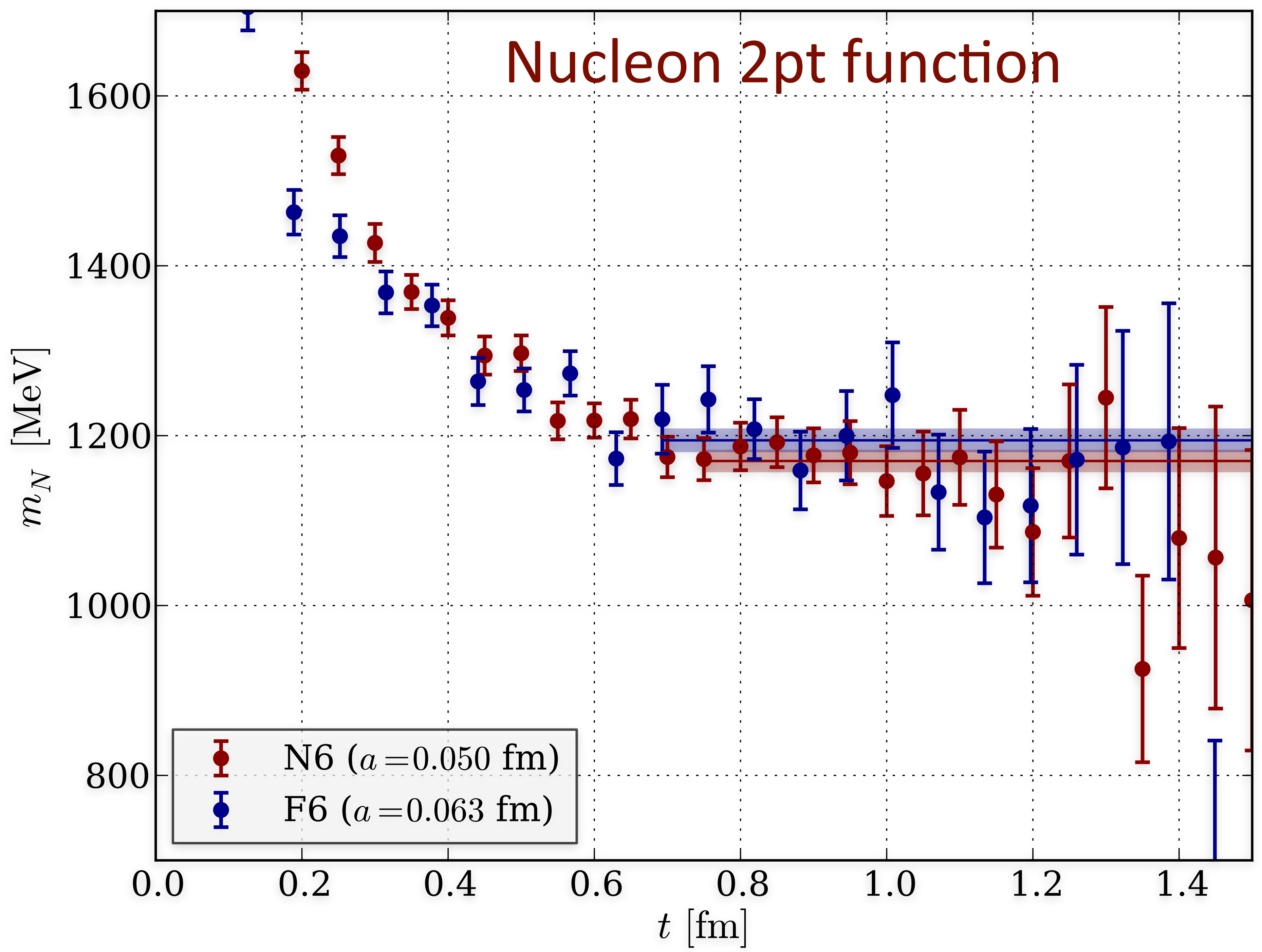
- Excited states are sub-leading contributions



# Dibaryons in Lattice QCD

## Flavour structure of two octet baryons

Irreducible representations

$$\mathbf{8} \otimes \mathbf{8} = (\mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27})_{\mathrm{S}} \oplus (\mathbf{8} \oplus \mathbf{10} \oplus \overline{\mathbf{10}})_{\mathrm{A}}$$

- *H* dibaryon lies in **1**-dimensional irrep of $\mathrm{SU}(3)_{\mathrm{flavour}}$
- Upon SU(3)-symmetry breaking, $\mathbf{8}$ and $\mathbf{27}$ mix with singlet
- Singlet, octet and $\mathbf{27}$plet operators constructed from linear combinations of $\Lambda\Lambda$, $\Sigma\Sigma$ and $N\Xi$ operators

e.g. $$[\mathbf{1}] = -\sqrt{\frac{1}{8}}[\Lambda\Lambda]^{I=0} + \sqrt{\frac{3}{8}}[\Sigma\Sigma]^{I=0} + \sqrt{\frac{4}{8}}[N\Xi_s]^{I=0}$$

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**Other dibaryons:**

- Dineutron lies in $\mathbf{27}$ irrep
- Deuteron lies in $\overline{\mathbf{10}}$ irrep with $J^P = 1^+$

# Dibaryons in Lattice QCD

## Interpolating operators for the *H* dibaryon

Hexaquark operators (inspired by Jaffe's original bag model calculation):

$$[rstuvw] = \epsilon_{ijk}\epsilon_{lmn}\left(s^i C\gamma_5 P_+ t^j\right)\left(v^l C\gamma_5 P_+ w^m\right)\left(r^k C\gamma_5 P_+ u^n\right)$$

$$H^{(1)} = \frac{1}{48}\left([sudsud] - [udusds] - [dudsus]\right)$$

$$H^{(27)} = \frac{1}{48\sqrt{3}}\left(3[sudsud] + [udusds] - [dudsus]\right)$$

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Momentum-projected two-baryon operators:

$$B_\alpha \equiv [rst]_\alpha = \epsilon_{ijk}\left(s^i C\gamma_5 P_+ t^j\right) r^k_\alpha$$

$$(BB)(\boldsymbol{P};t) = \sum_{\boldsymbol{x}} e^{-i\boldsymbol{p}_1\cdot\boldsymbol{x}} B_1(\boldsymbol{x},t)\,(C\gamma_5 P_+)\sum_{\boldsymbol{y}} e^{-i\boldsymbol{p}_2\cdot\boldsymbol{y}} B_2(\boldsymbol{y},t), \quad \boldsymbol{P} = \boldsymbol{p}_1 + \boldsymbol{p}_2$$

→ project onto $(BB)^{(1)},\ (BB)^{(8)},\ (BB)^{(27)}$

# Dibaryons in Lattice QCD

## Correlator matrices and GEVP

Consider set on $N_{\rm op}$ interpolating operators for a given hadron:

Correlation matrix: $C_{ij}(\boldsymbol{P},\tau) = \left\langle O_i(\boldsymbol{P},t) O_j(\boldsymbol{P},t')^\dagger \right\rangle, \quad \tau = t - t', \quad i,j = 1,\ldots,N_{\rm op}$

- Variational method: solve Generalised Eigenvalue Problem (GEVP)

$$\boldsymbol{C}(t_1)\, v_n(t_1,t_0) = \lambda_n(t_1,t_0)\, \boldsymbol{C}(t_0)\, v_n(t_1,t_0)$$

$$w_n^\dagger(t_1,t_0)\, \boldsymbol{C}(t_1) = \lambda_n(t_1,t_0)\, w_n^\dagger(t_1,t_0)\, \boldsymbol{C}(t_0), \quad n = 1,\ldots,N_{\rm op}$$

- Project onto approximately diagonal correlator: $\boldsymbol{\Lambda}_{mn}(t) = w_n^\dagger\, \boldsymbol{C}(t)\, v_m$

- Compute the effective $n^{\rm th}$ energy level: $E_n^{\rm eff}(t) = \dfrac{1}{\Delta t} \ln \dfrac{\Lambda_{nn}(t)}{\Lambda_{nn}(t+\Delta t)}$

# Dibaryons in Lattice QCD

## Distillation with Laplace-Heaviside (LapH) smearing

Timeslice-to-all quark propagator in the subspace spanned by eigenmodes $v^{(n,t)}$ of smearing kernel

"Perambulator": $\tau_{\alpha\beta}^{nn'}(t,\, t_0) = \sum_{i,\,j,\,\vec{x},\,\vec{x}'} v_i^{(n',t)*}(\vec{x}')\, D^{-1}(\vec{x}',t;\, \vec{x}, t_0)\, v_j^{(n,t_0)}(\vec{x}),$ $\quad n,\, n' = 1,\ldots, N_{\rm LapH}$

"Mode triplets": $T_{lnm}(t,\, \vec{p}\,) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \epsilon^{ijk}\, v_i^{(l,t)}(\vec{x}\,)\, v_j^{(n,t)}(\vec{x}\,)\, v_k^{(m,t)}(\vec{x}\,)$ (tensorial structure of baryon-baryon correlators)

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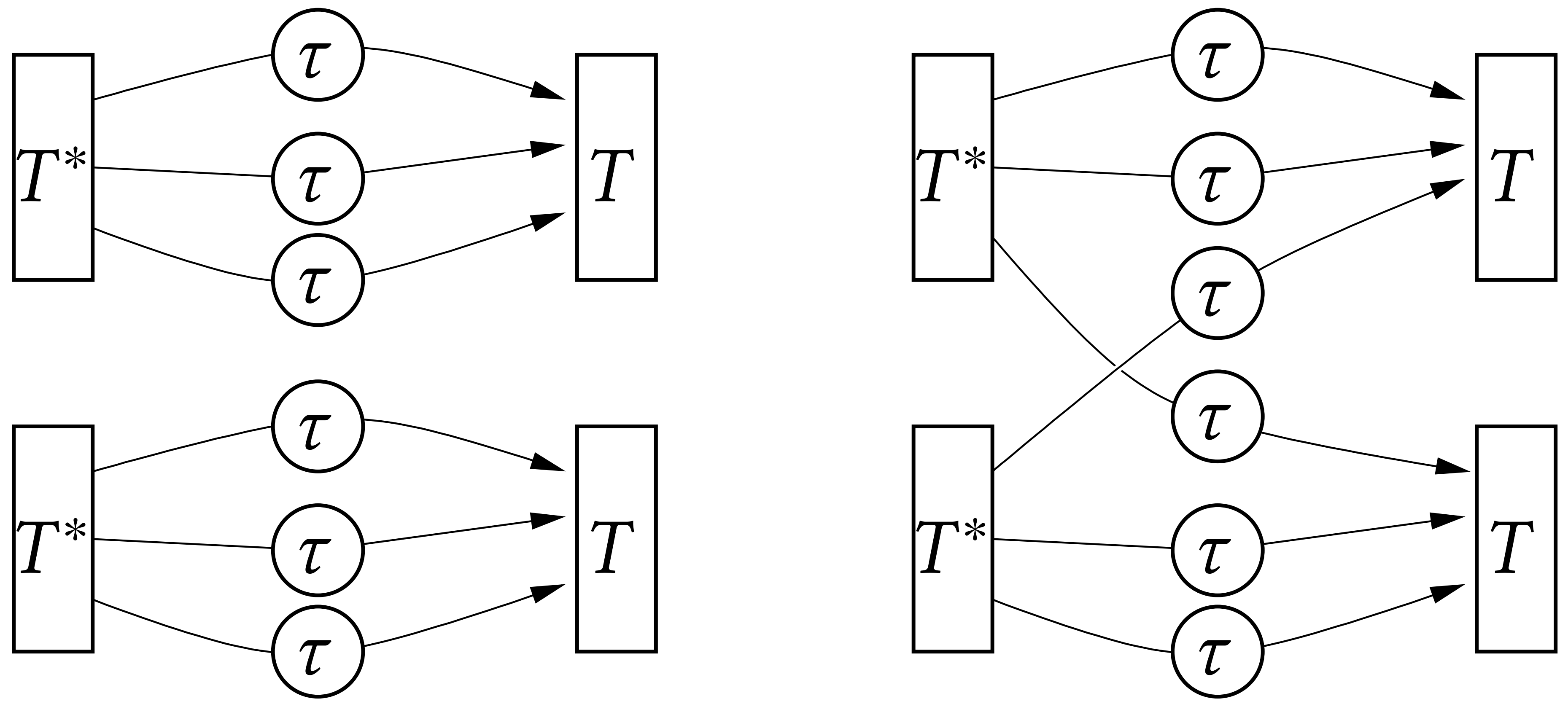
Two topologies of Wick contractions $\rightarrow$ computational cost scales naively like $N_{\mathrm{LapH}}^6$:



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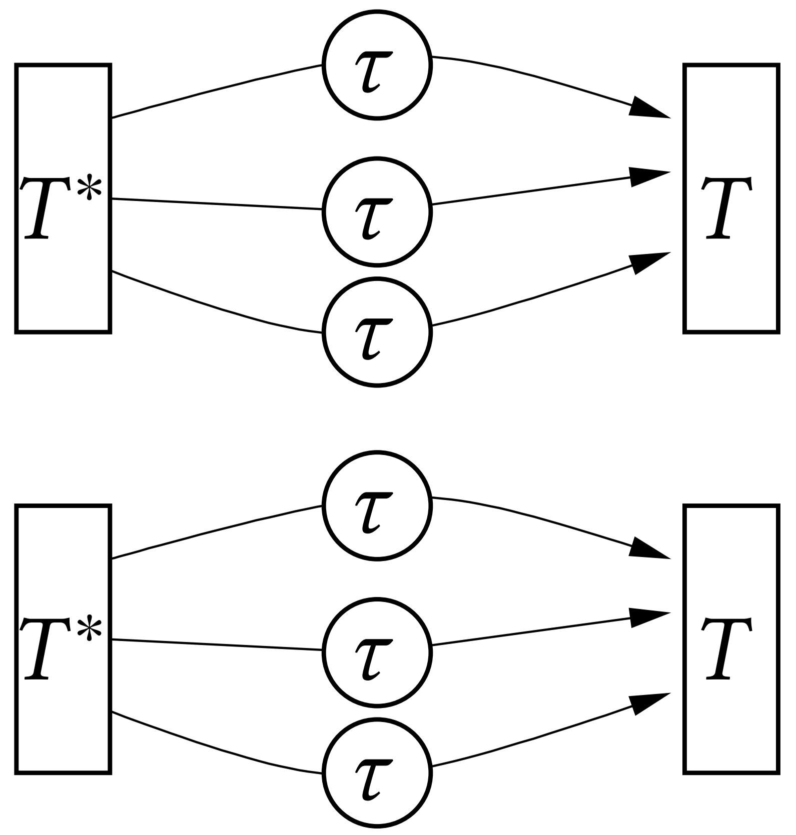
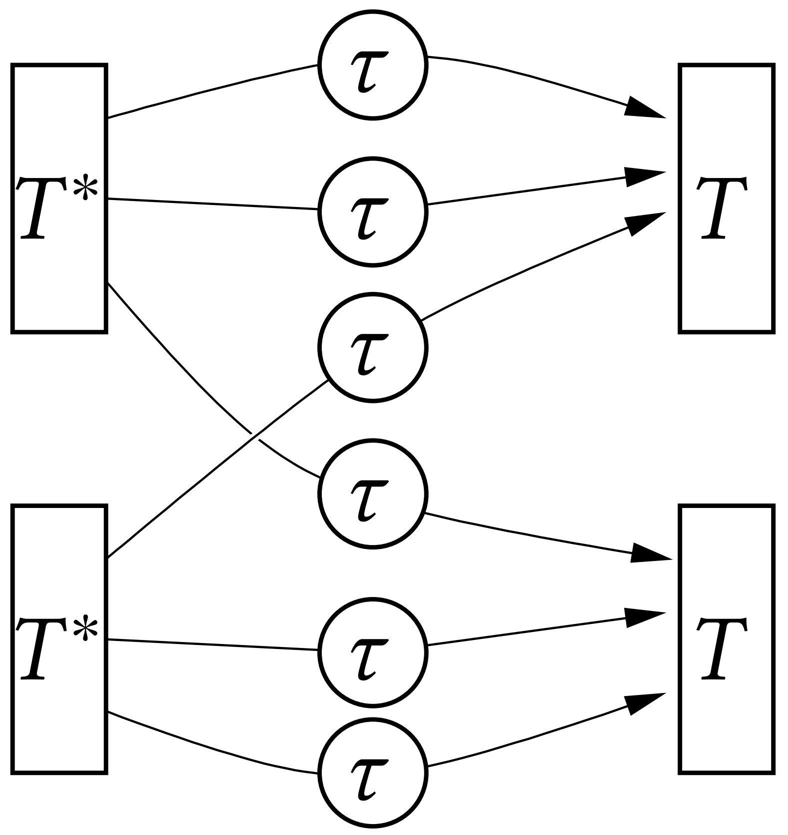
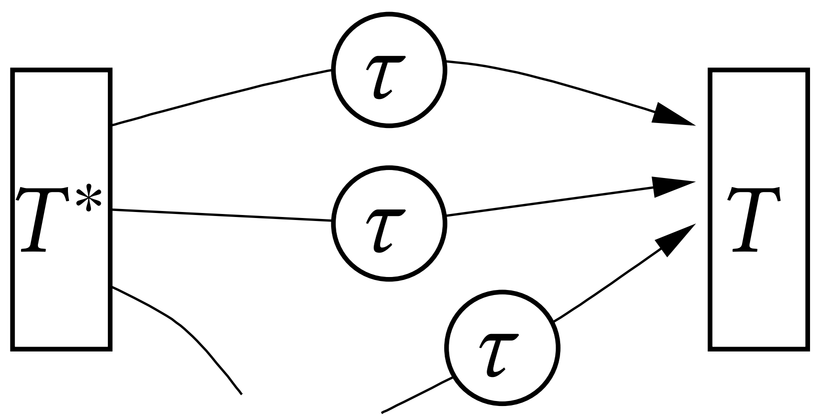




Building block:



Cost scaling $\sim N^4_{\rm LapH}$

# Dibaryons in Lattice QCD

## Finite-volume quantisation condition

$$\det\left(\tilde{\mathcal{K}}^{-1}(p^2) - B(p^2,\, L)\right) = 0$$

*[Lüscher 1990–91, Rummukainen & Gottlieb 1995,….]*

$\tilde{\mathcal{K}}(p^2)$ : $2 \to 2$ scattering amplitude

$B(p^2,\, L)$ : analytically known function

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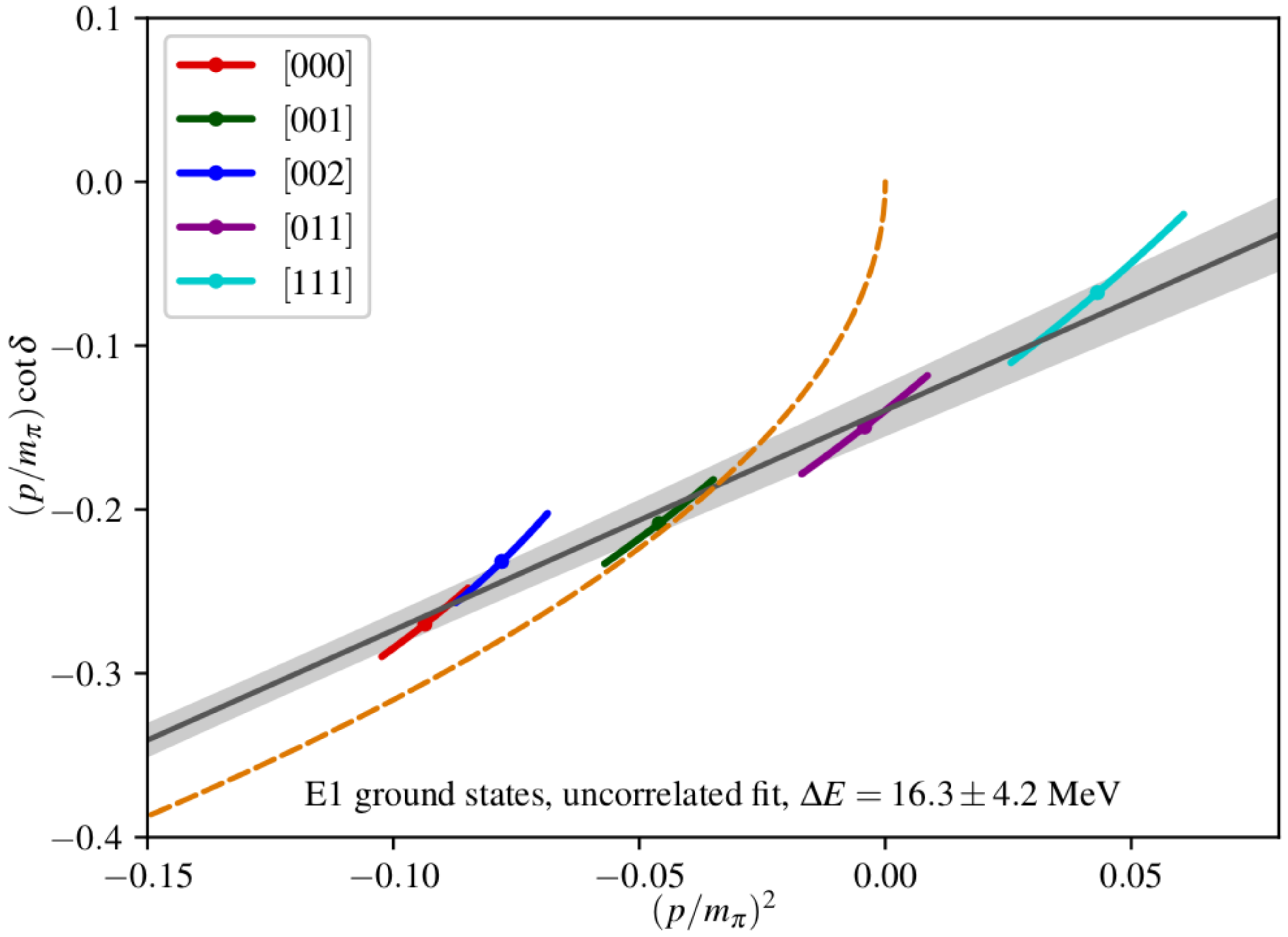
Pole of scattering amplitude: $\mathcal{A} \propto \dfrac{1}{p\cot\delta(p) - ip}$

Fit to effective range expansion:

$$\Rightarrow \quad p\cot\delta_0(p) = A + Bp^2 + \ldots \overset{!}{=} -\sqrt{-p^2}$$



# Dibaryons in Lattice QCD: The HAL QCD Method

[Takashi Inoue, MON 17:30]

Baryon-baryon potential from Nambu-Bethe-Salpeter amplitude computed on the lattice

$$G_4(\boldsymbol{r}, t-t_0) = \left\langle 0 \left| (BB)^{(\alpha)}(\boldsymbol{r}, t)\, (\overline{BB})^{(\alpha)}(\boldsymbol{r}, t_0) \right| 0 \right\rangle = \phi(\boldsymbol{r}, t)\, \mathrm{e}^{-2M(t-t_0)}$$

$(BB)^{(\alpha)}(\boldsymbol{r}, t)$ : 2-baryon interpolating operator; flavour irrep. $\alpha$

$\phi(\boldsymbol{r}, t)$ : NBS wave function

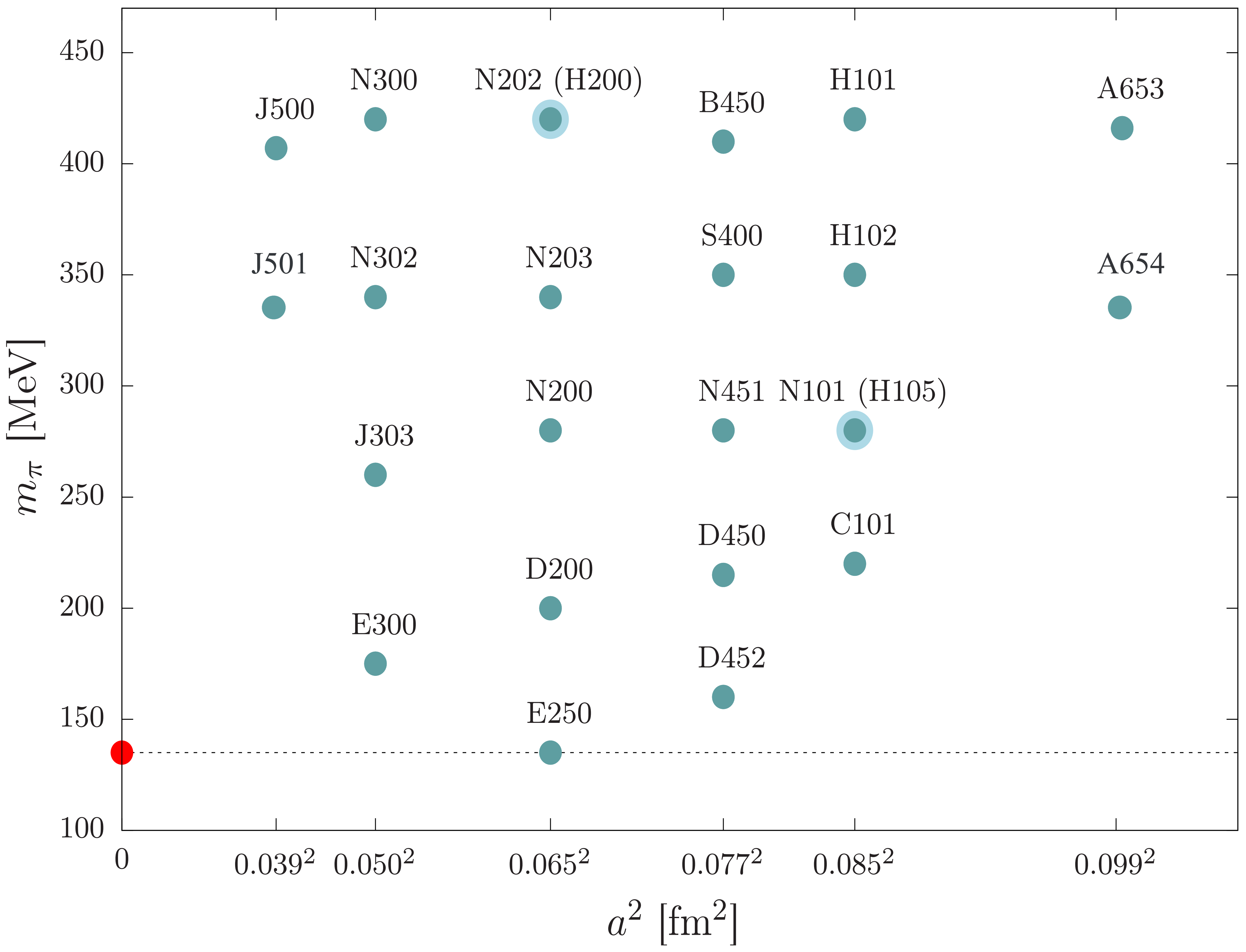
$M$ : single baryon mass

- No determination of energy levels from asymptotic exponential fall-off of correlator
- Determine potential via $V(r) = \dfrac{[-H_0 - (\partial/\partial t)]\,\phi(\boldsymbol{r}, t)}{\phi(\boldsymbol{r}, t)}$
- Solve Schrödinger equation $\rightarrow$ determine binding energies and scattering phase shifts

# Mainz Dibaryon Project with $3$ and $2+1$ flavours

Use CLS ensembles with $N_f = 2+1$ flavours of $\mathrm{O}(a)$ improved Wilson quarks

- Six lattice spacings: $a = 0.099 - 0.039\,\mathrm{fm}$; pion masses $m_\pi = 130 - 420\,\mathrm{MeV}$
- Timeslice-to-all propagators; chiral trajectory: $\mathrm{Tr}\,M_q = \mathrm{const.} \quad \Leftrightarrow \quad \frac{1}{2}m_\pi^2 + m_K^2 \approx \mathrm{const.}$



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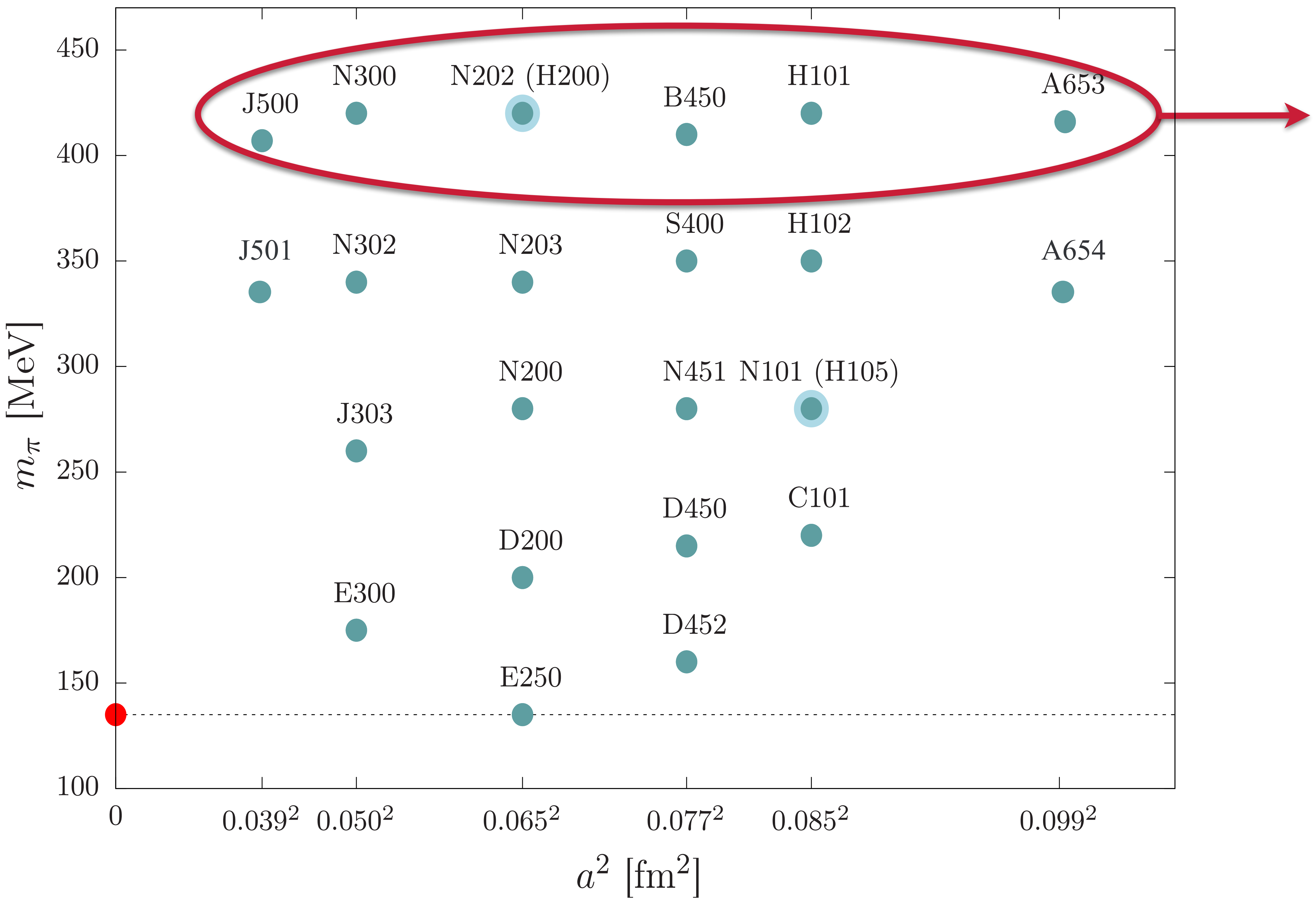


Scaling test at $m_\pi = m_K \approx 420\,\mathrm{MeV}$, SU(3)-symmetric point with $m_u + m_d + m_s$ at the physical value (published)

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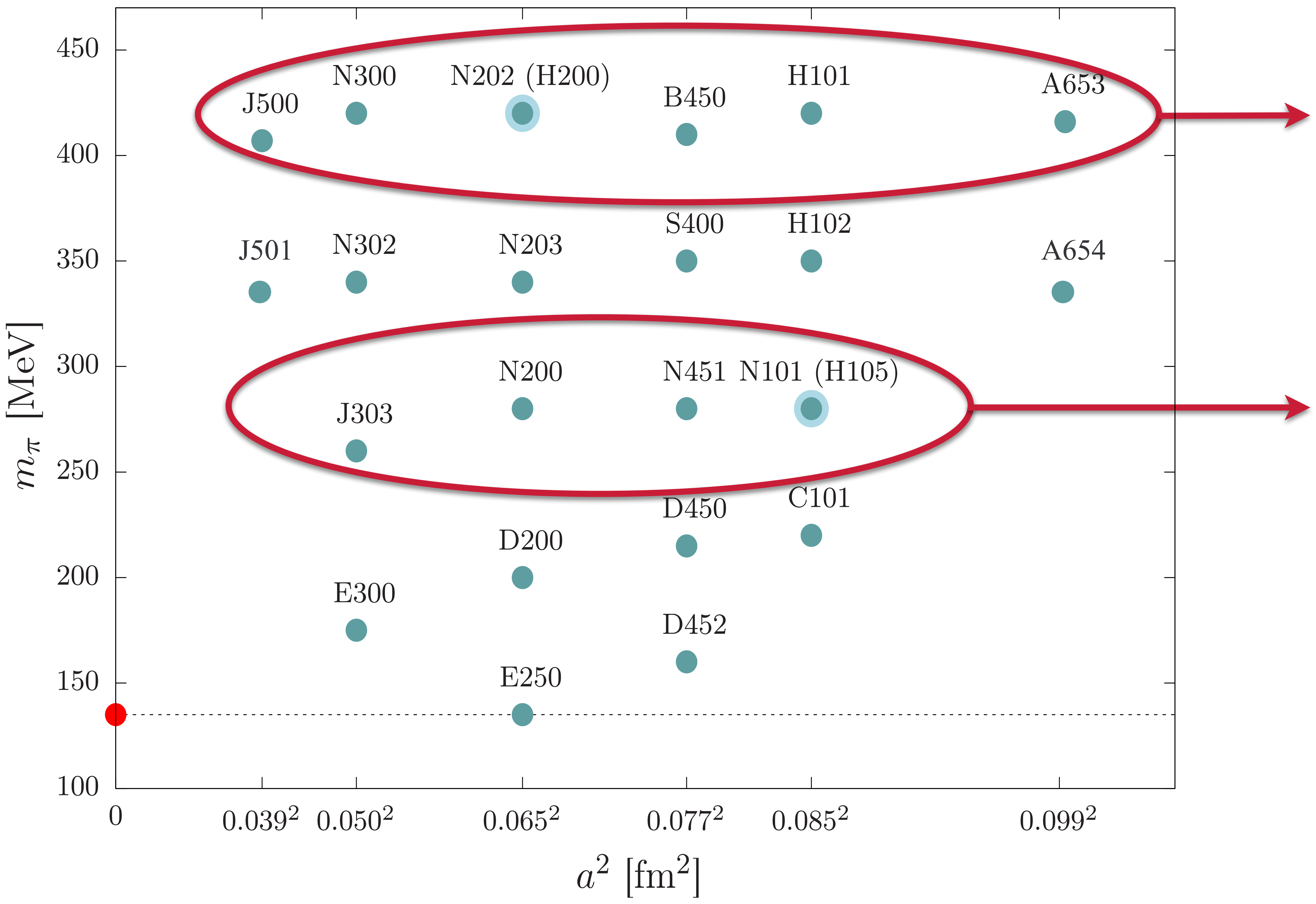


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Scaling test at $m_\pi \approx 280\,\mathrm{MeV}$, broken SU(3) (ongoing)

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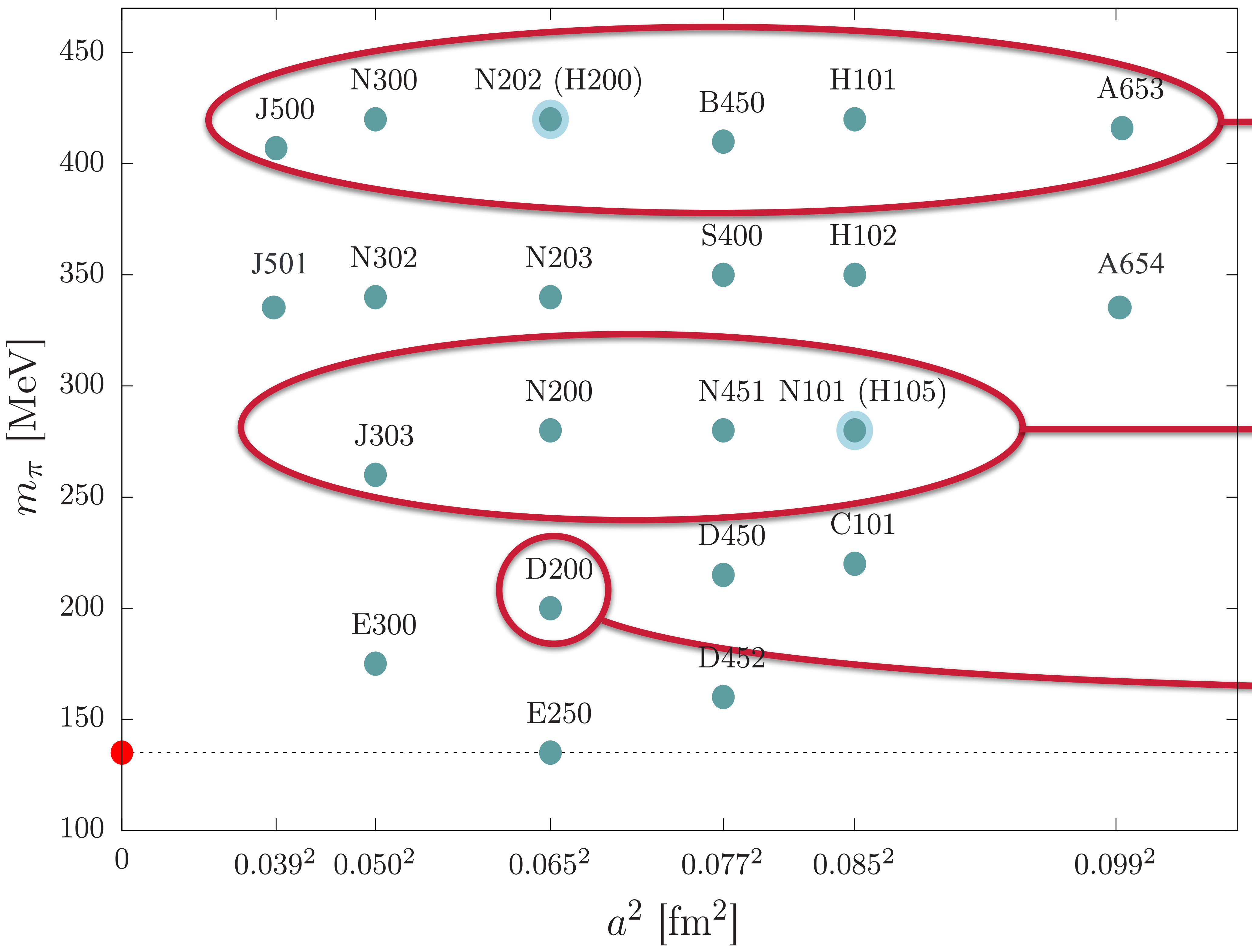


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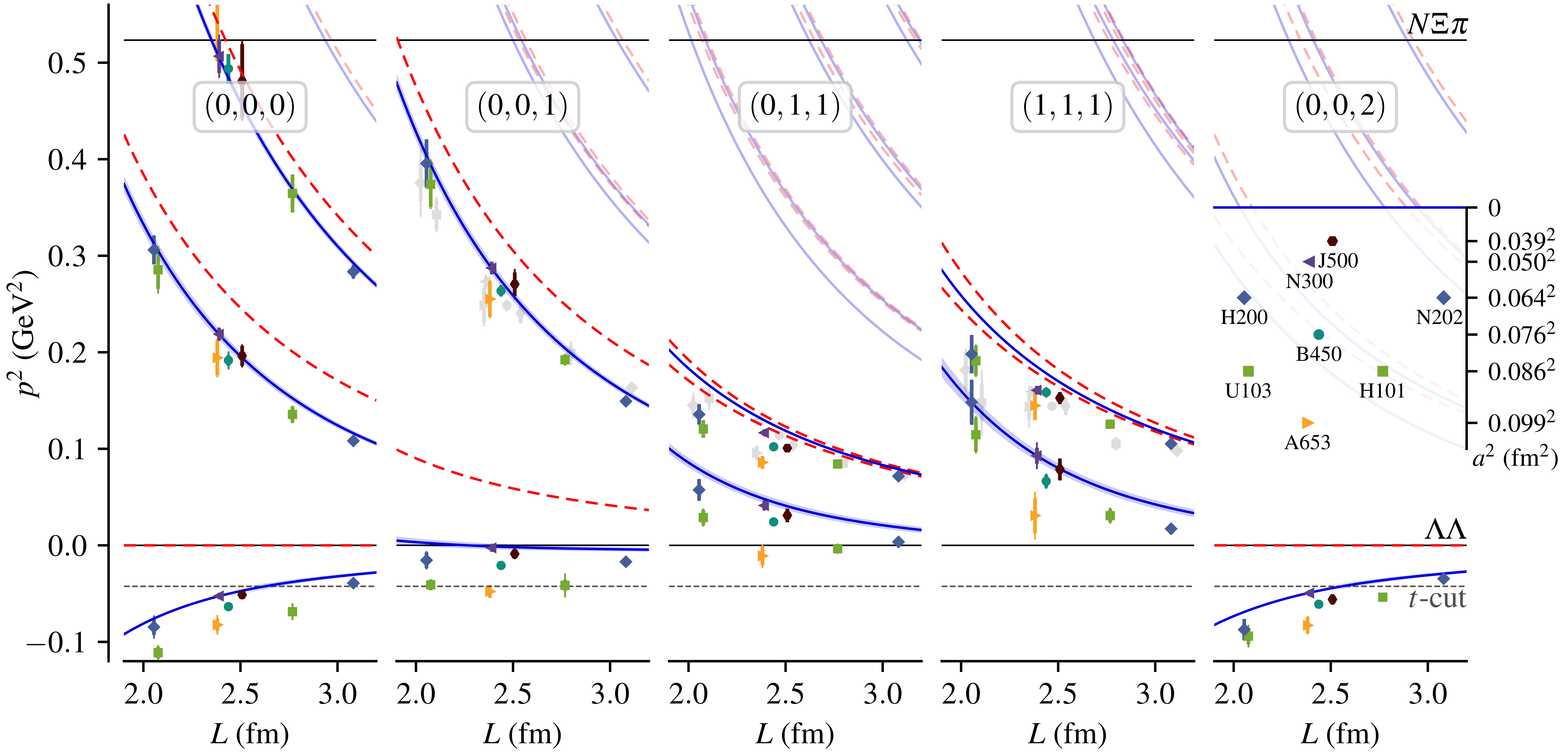
Scaling test at $m_\pi \approx 280\,\mathrm{MeV}$, broken SU(3) (ongoing)

Stochastic LapH at $m_\pi \approx 200\,\mathrm{MeV}$, — in collab. with BaSc

# *H* Dibaryon at the SU(3)-symmetric point

Scattering momenta in finite volume in different frames: $p^2 = \frac{1}{4}(E_L^2 - \boldsymbol{P}\cdot\boldsymbol{P}) - m_\Lambda^2$




: interacting spectrum in continuum limit

: non-interacting levels

*[Green, Hanlon, Junnarkar, HW, Phys Rev Lett 127 (2021) 242003]*

# *H* Dibaryon at the SU(3)-symmetric point

## Continuum extrapolation

Finite-volume quantisation condition only valid in continuum limit

Perform combined fit of $p\cot\delta(p)$ in both $p^2$ and $a$ :

$$p\cot\delta(p) = \sum_{i=0}^{N-1} c_i\, p^{2i} \overset{!}{=} -\sqrt{-p^2}\,, \qquad c_i = c_{i0} + c_{i1}a^2$$

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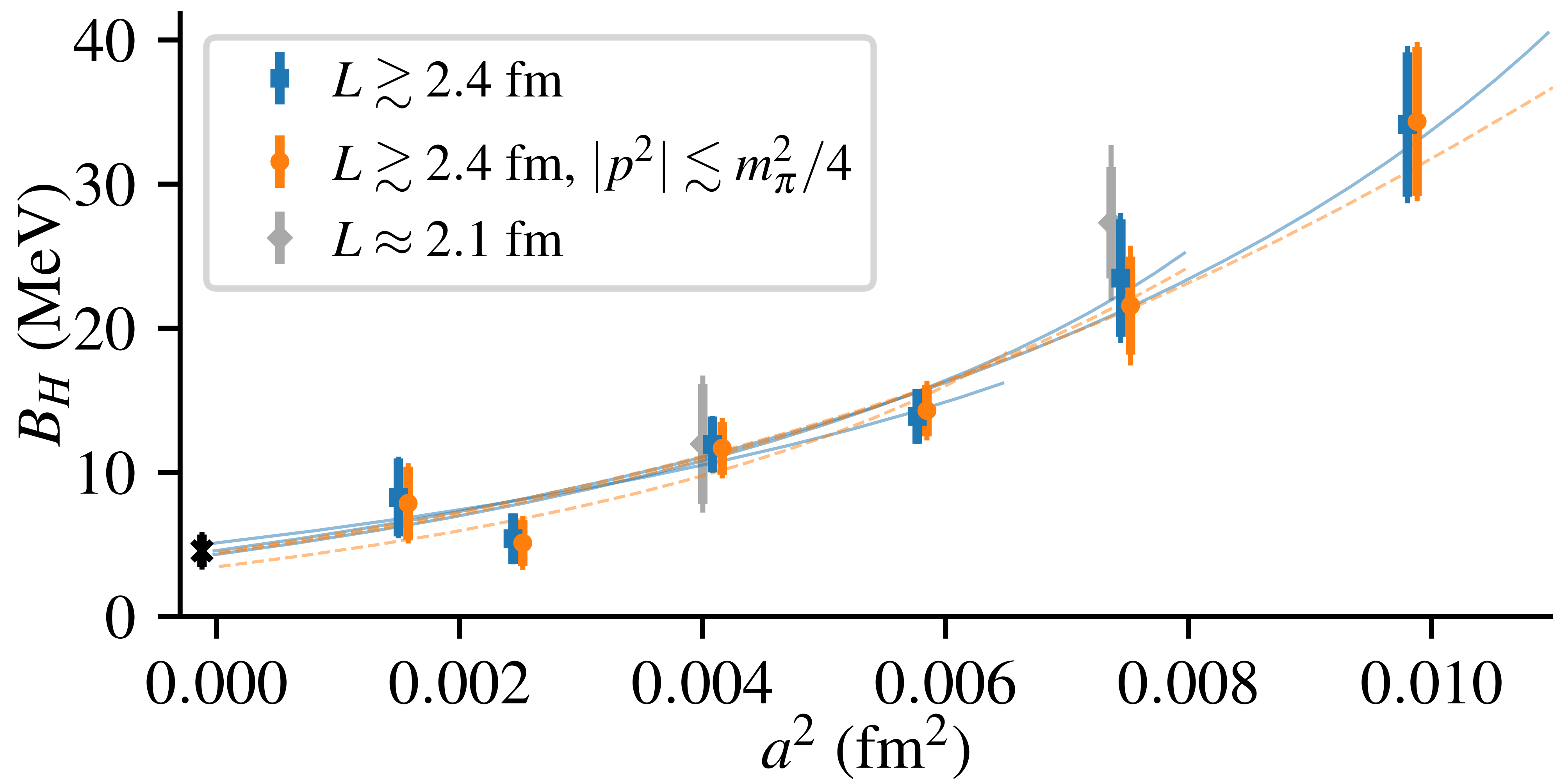


$\Rightarrow \quad B_H^{N_f=3} = 4.56 \pm 1.13 \pm 0.63\,\mathrm{MeV}$

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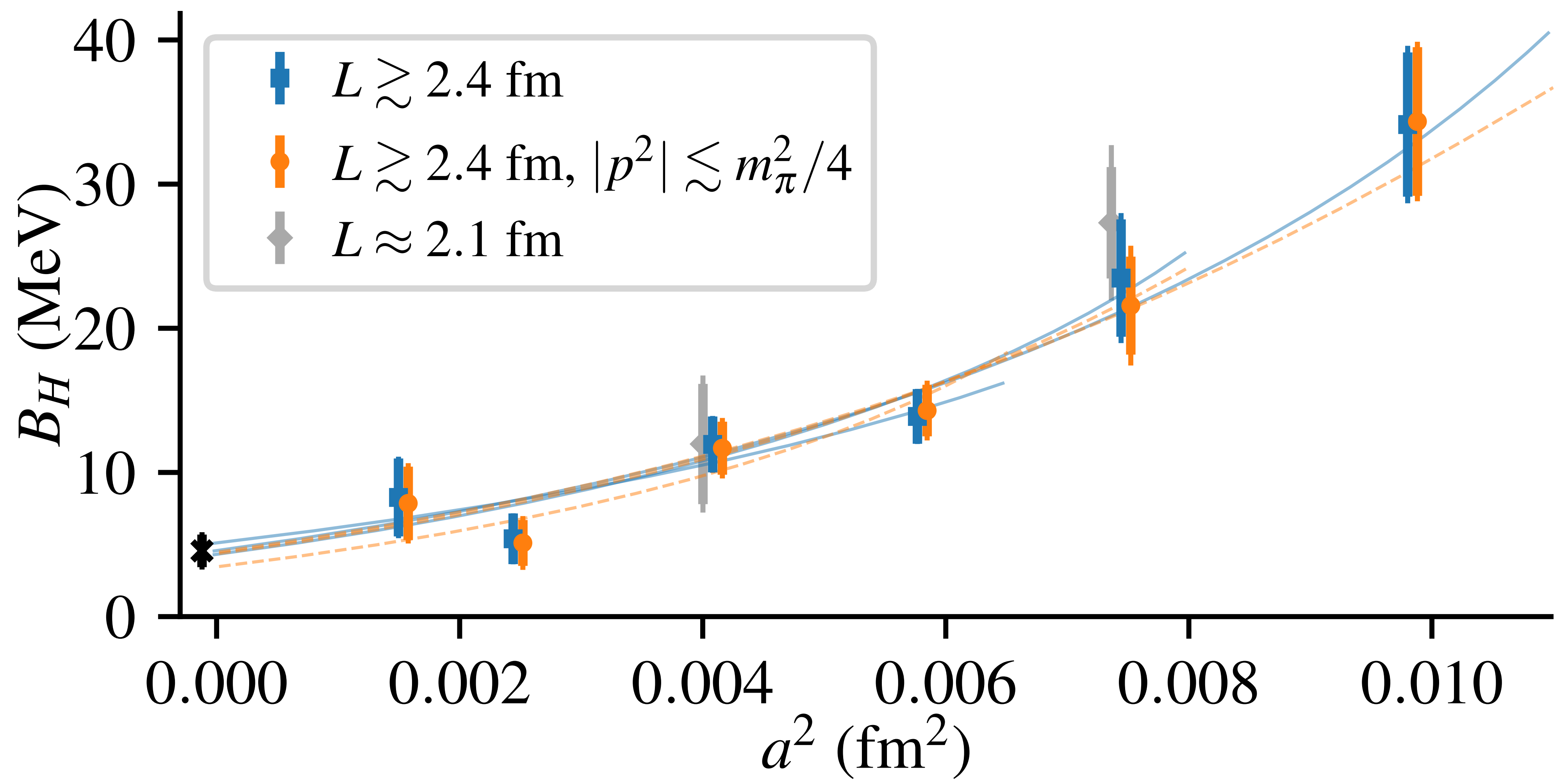


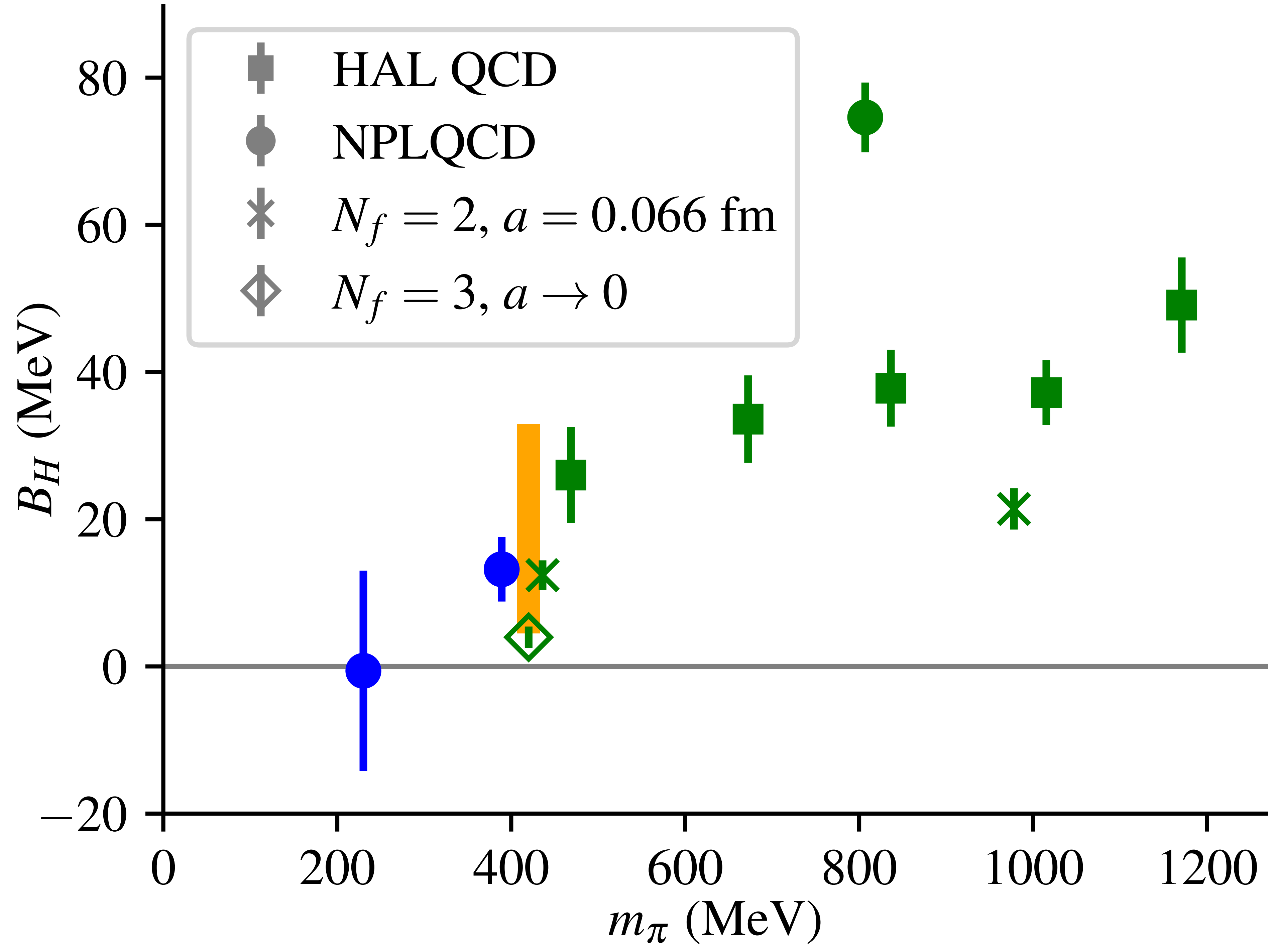


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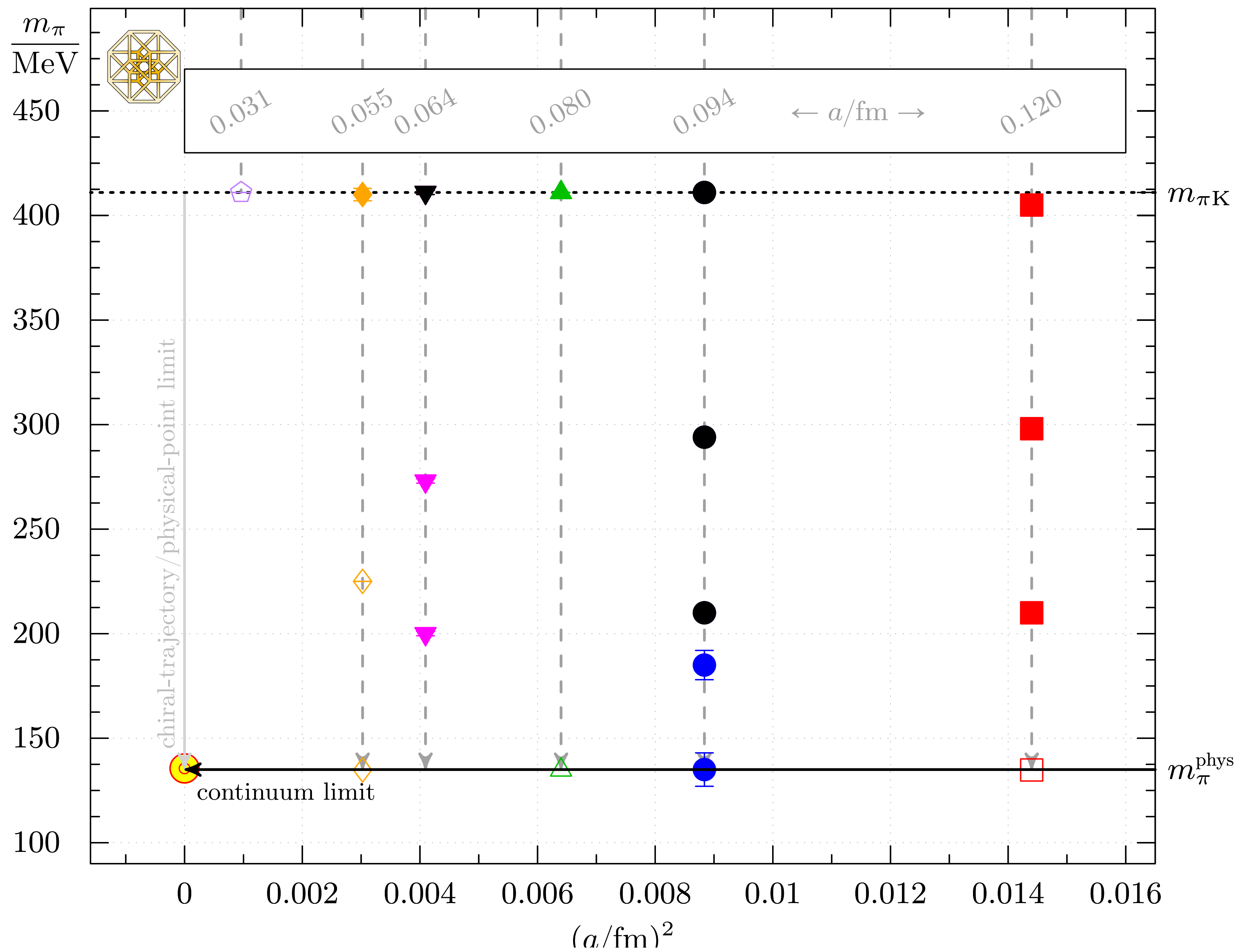
# $H$ Dibaryon at the SU(3)-symmetric point

## Cross-check using "Stabilised Wilson Fermions" (OpenLat)




$$D_{\text{wilson}} = \begin{pmatrix} D_{\text{ee}} & D_{\text{eo}} \\ D_{\text{oe}} & D_{\text{oo}} \end{pmatrix},$$

$$D_{\text{ee}} + D_{\text{oo}} = (4 + m_0)\,\exp\left\{\frac{c_{\text{sw}}}{4+m_0}\,\frac{i}{4}\,\sigma_{\mu\nu}\widehat{F}_{\mu\nu}\right\}$$

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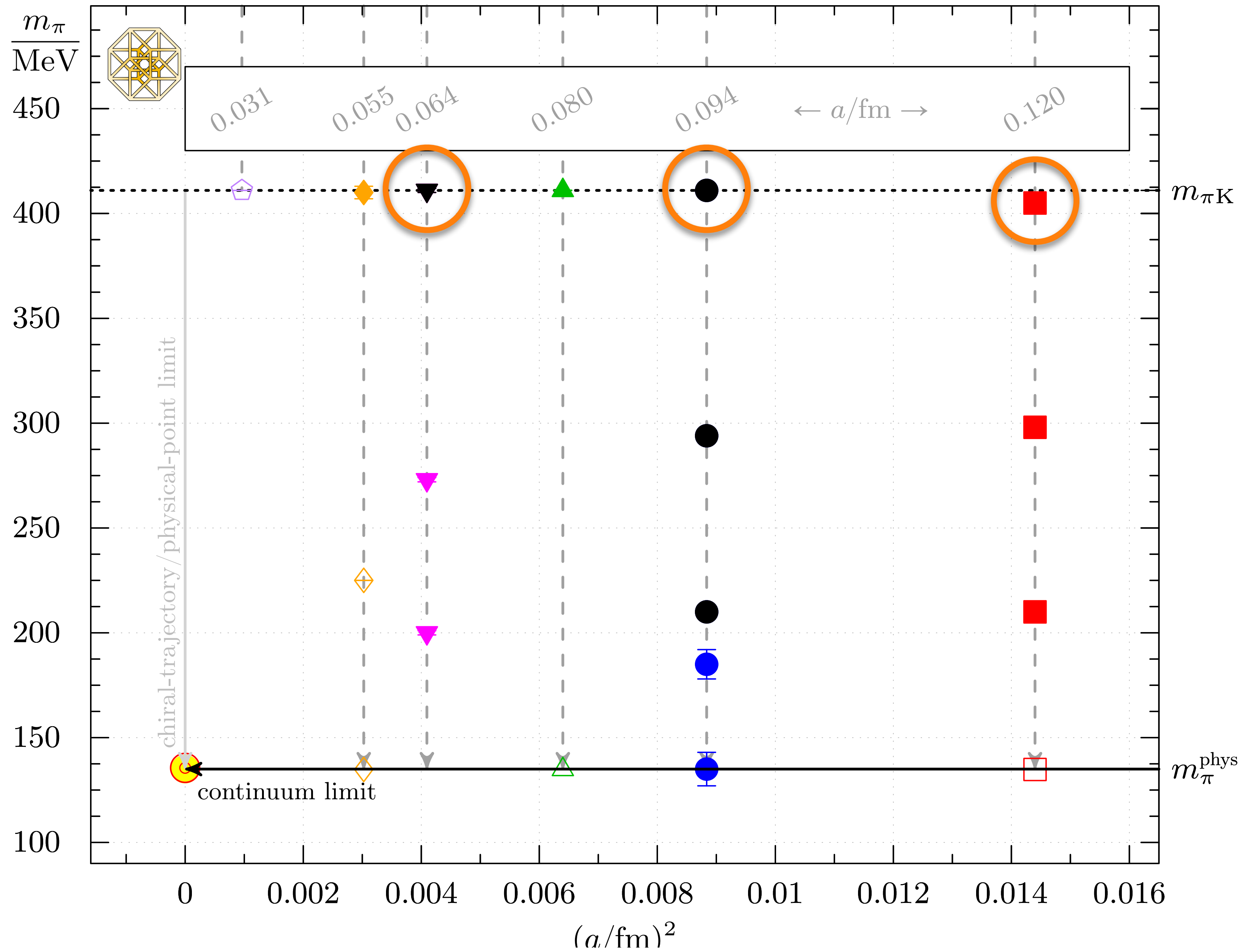


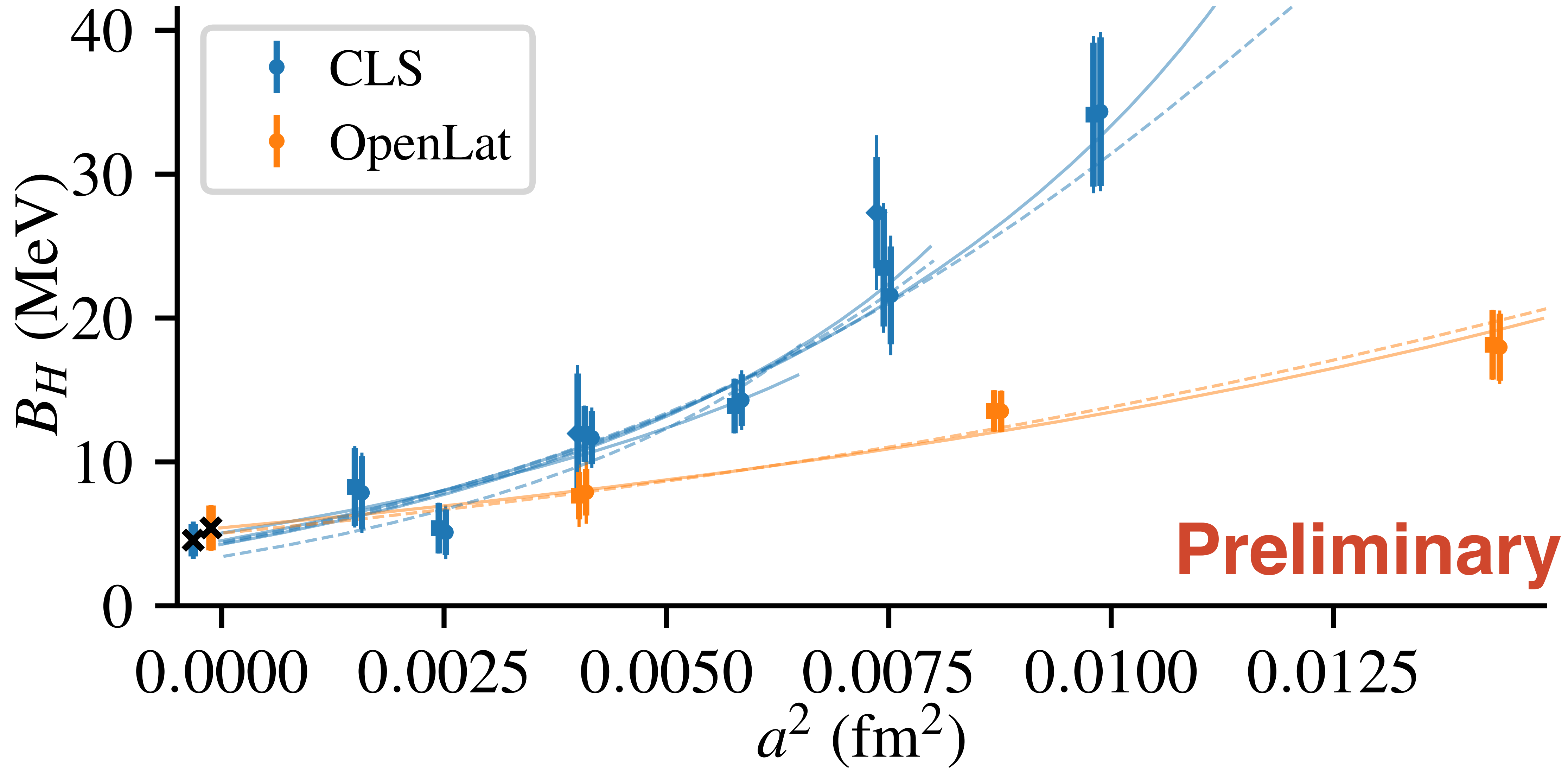


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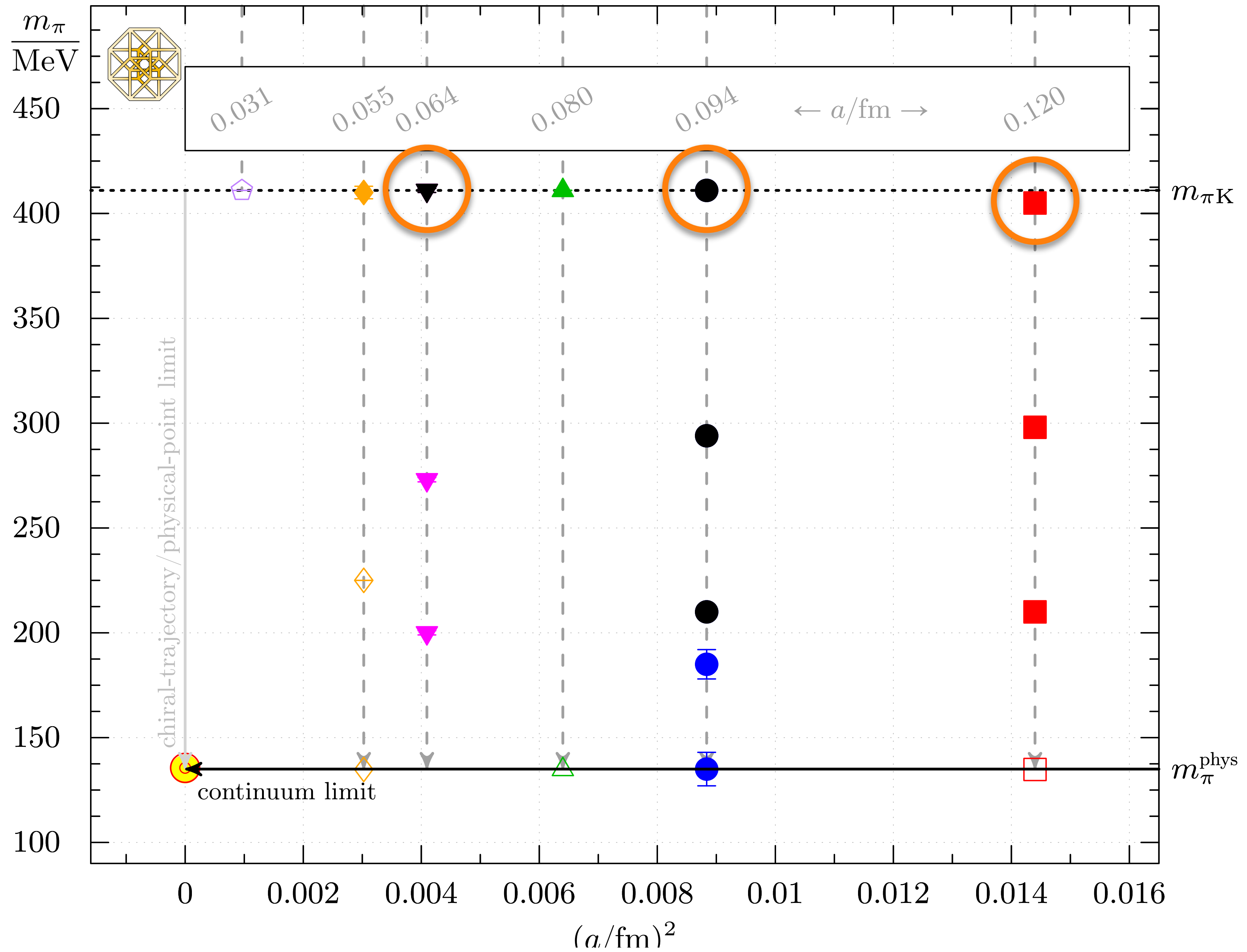


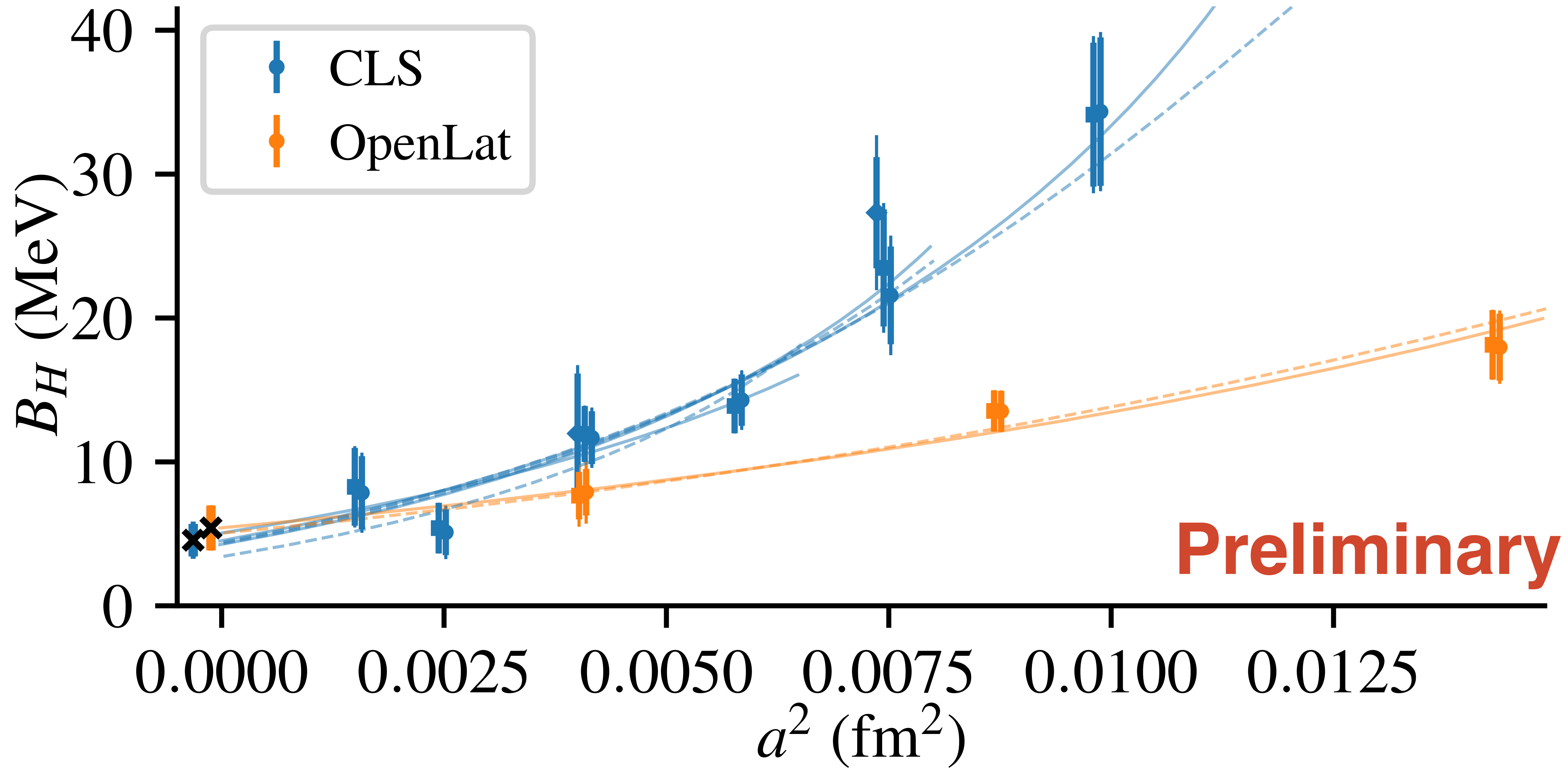


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$$B_H^{N_f=3} = \begin{cases} 4.56 \pm 1.13 \pm 0.63\,\mathrm{MeV} & \text{CLS} \\ 5.41 \pm 1.56 \pm 0.24\,\mathrm{MeV} & \text{OpenLat} \end{cases}$$

(CLS: systematic error includes fit error, plus cut in $a$, $L$ and $p^2$
OpenLat: systematic error from fit uncertainty only)

# $H$ Dibaryon at the SU(3)-symmetric point: HAL QCD

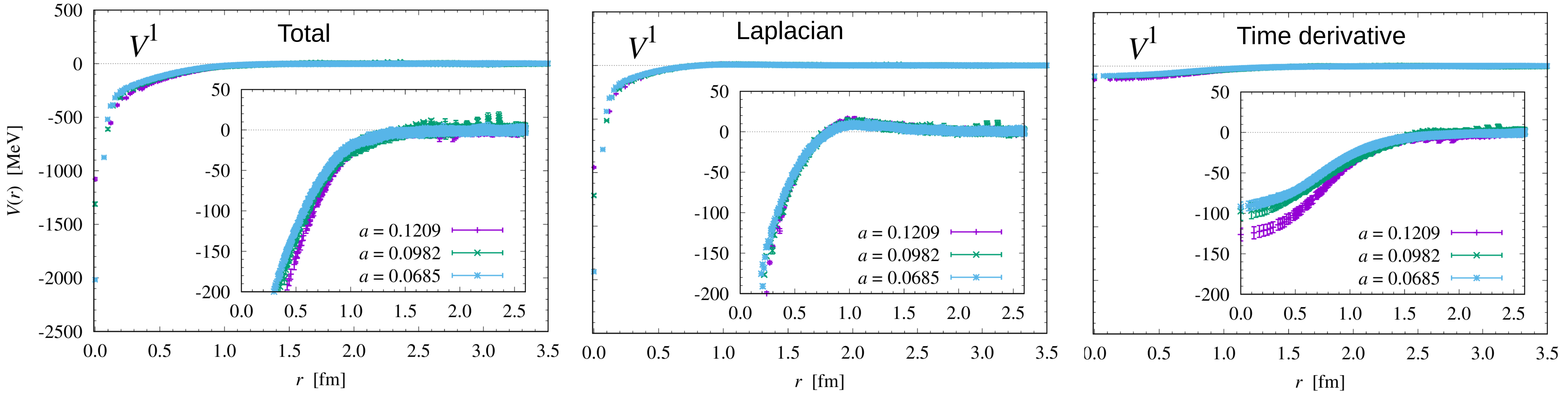
[Takashi Inoue, MON 17:30]

Ensembles with $N_f = 3$ flavours of $\mathrm{O}(a)$ RG-improved Wilson quarks

- Three lattice spacings: $a = 0.121, 0.098, 0.069\,\mathrm{fm}$; Volume: $L \gtrsim 4\,\mathrm{fm}$; Pion mass $m_\pi = 420\,\mathrm{MeV}$

Compute flavour-singlet $BB$ potential:

$$V(r) = \frac{[-H_0 - (\partial/\partial t)]\,\phi(\boldsymbol{r},t)}{\phi(\boldsymbol{r},t)}$$



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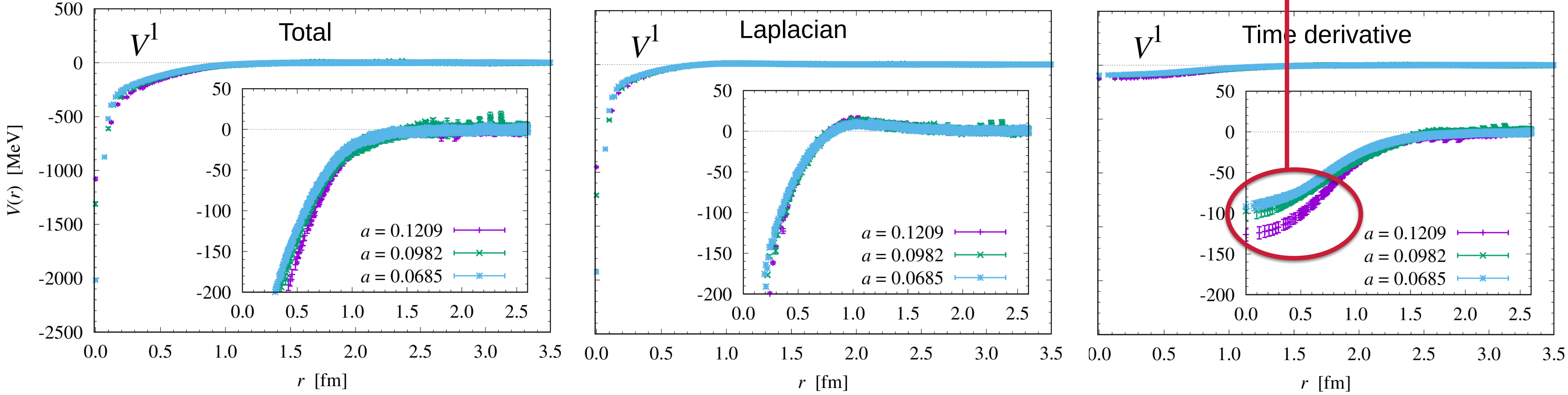
Significant lattice artefacts in short-distance potential



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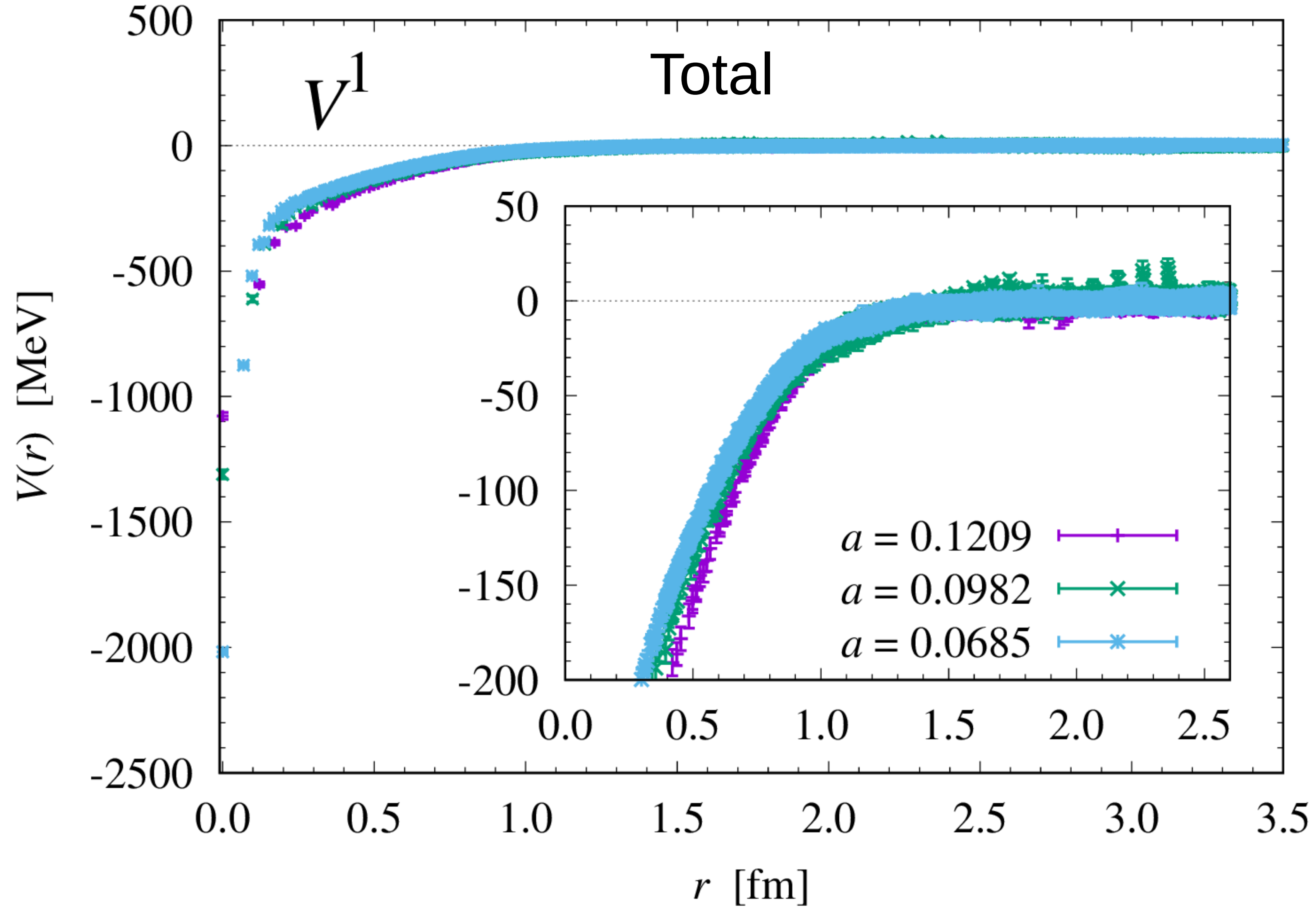
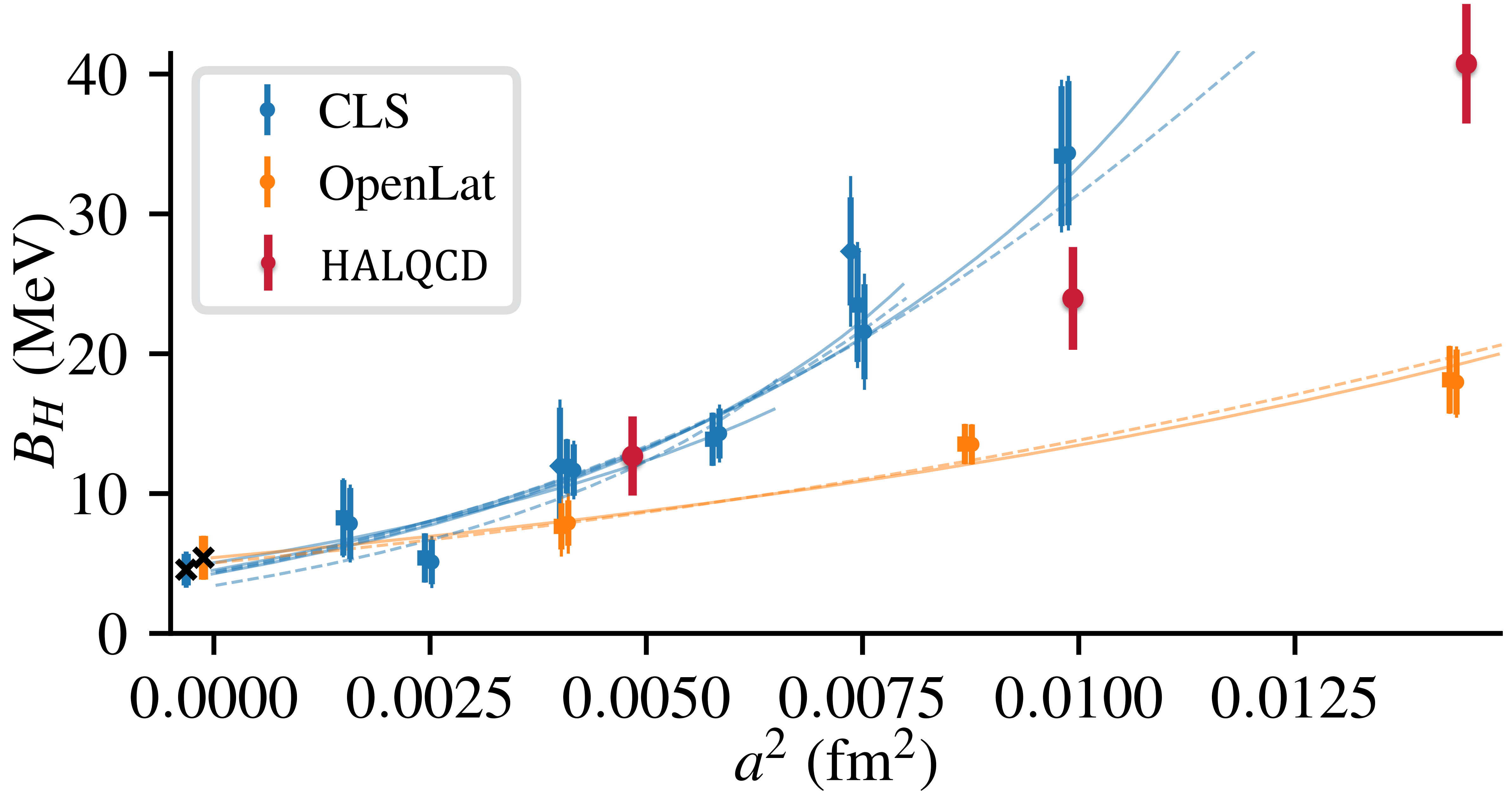
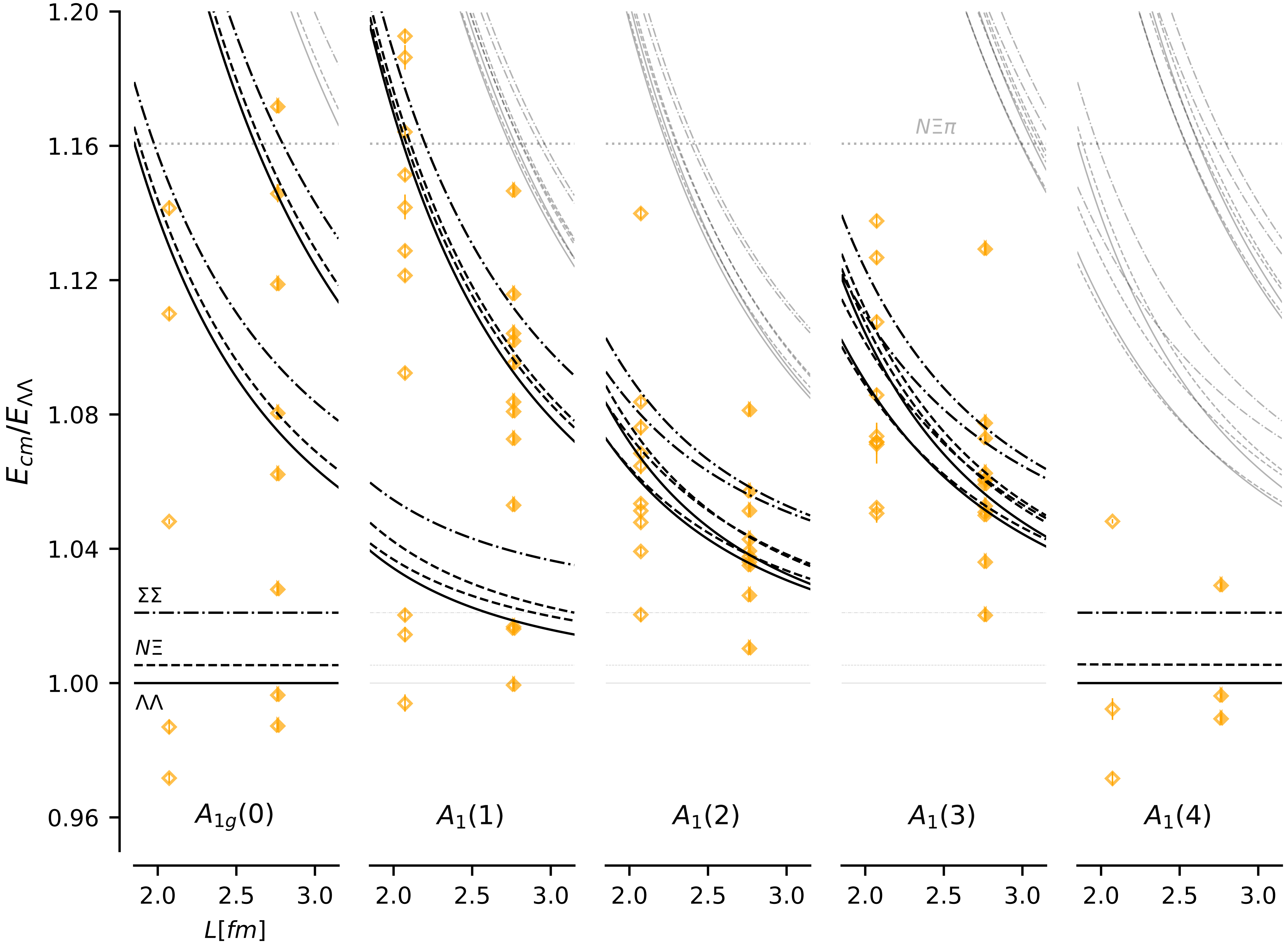


# The *H* Dibaryon away from the SU(3)-symmetric point

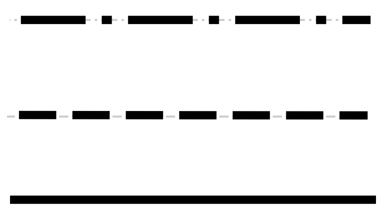
Finite-volume energy levels at decreasing pion mass:

*[M. Padmanath et al., arXiv:2111.11541]*




$m_\pi \sim 350\,\text{MeV},\ L = 2.1,\, 2.8\,\text{fm}$

$a = 0.0865\,\text{fm}$



: non-interacting levels

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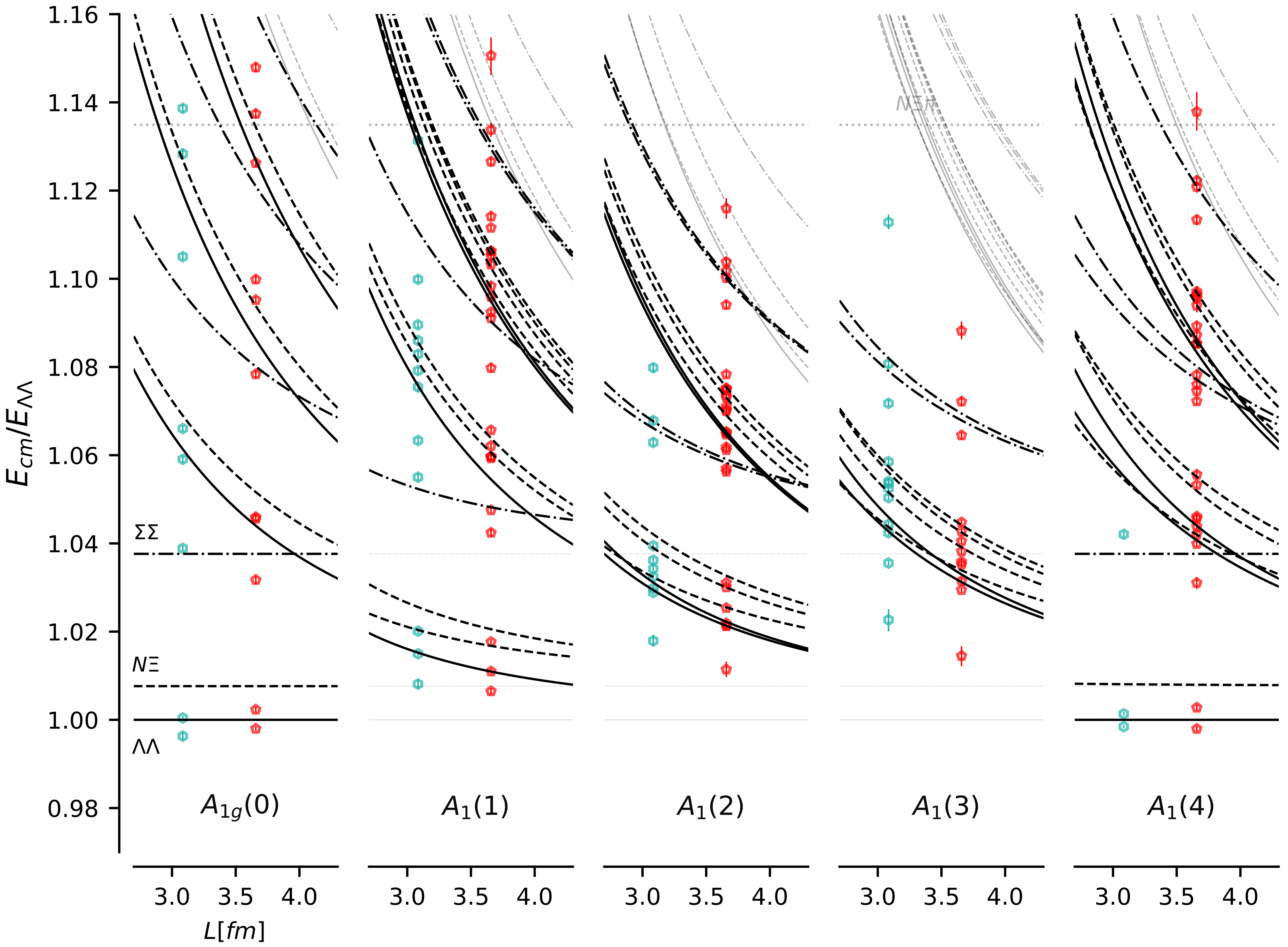
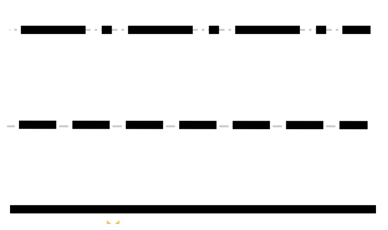



$m_\pi \sim 280\,\text{MeV},\ L = 3.1,\ 3.6\,\text{fm}$

$a = 0.0642,\ 0.0762\,\text{fm}$



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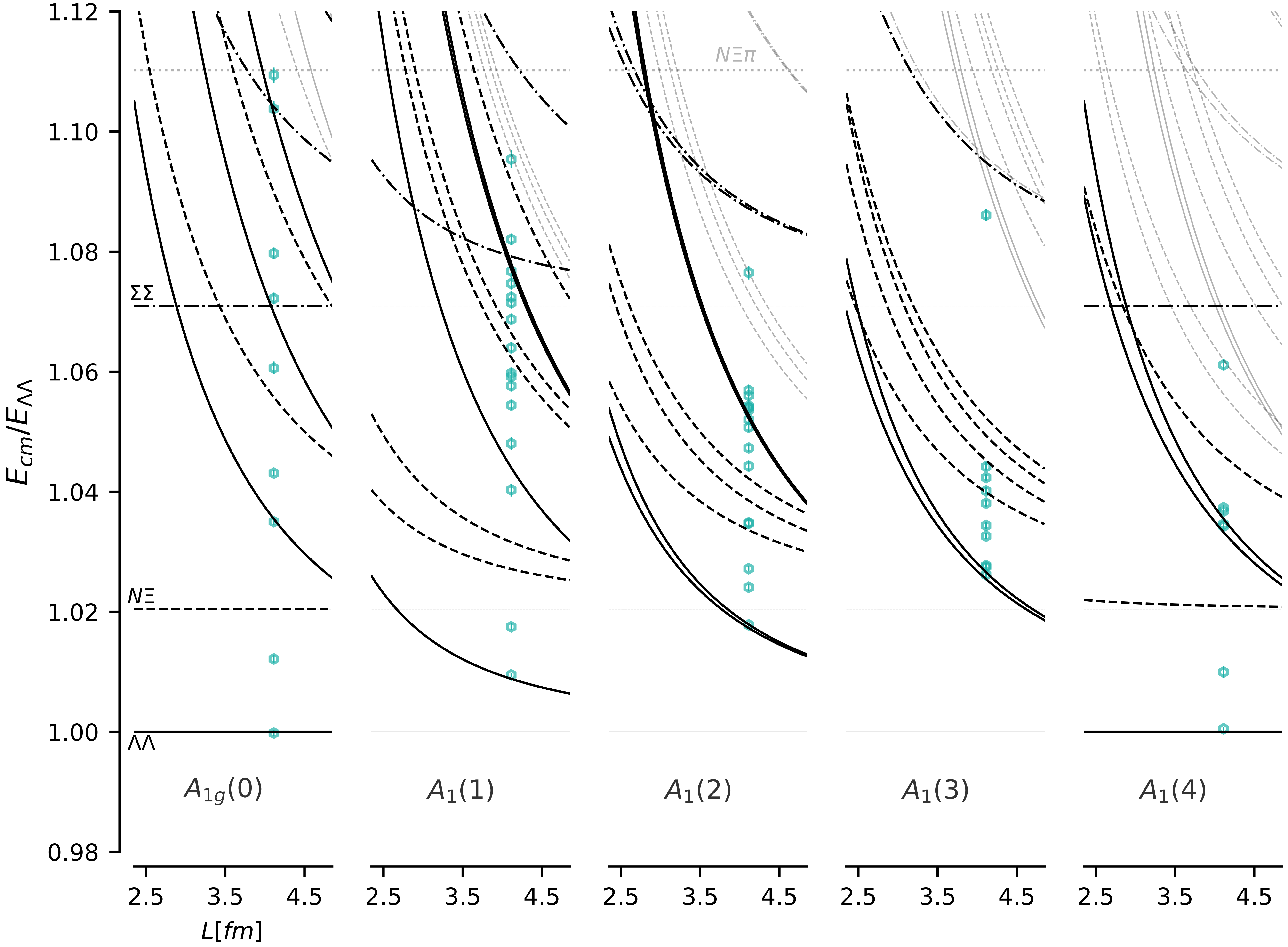


$m_\pi \sim 200\,\mathrm{MeV},\ L = 4.2\,\mathrm{fm}$

$a = 0.0642\,\mathrm{fm}$

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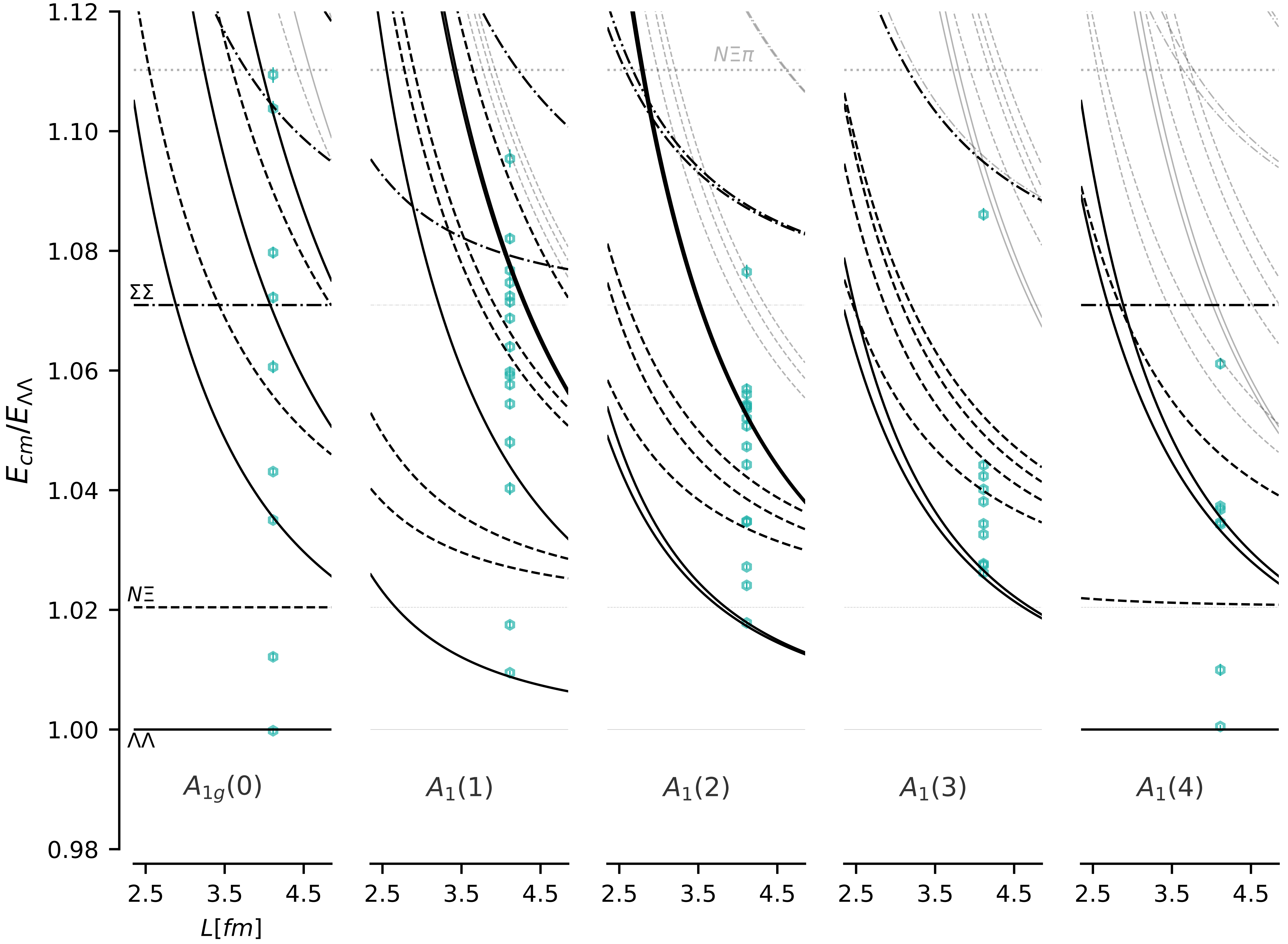



$m_\pi \sim 200\,\mathrm{MeV},\ L = 4.2\,\mathrm{fm}$

$a = 0.0642\,\mathrm{fm}$

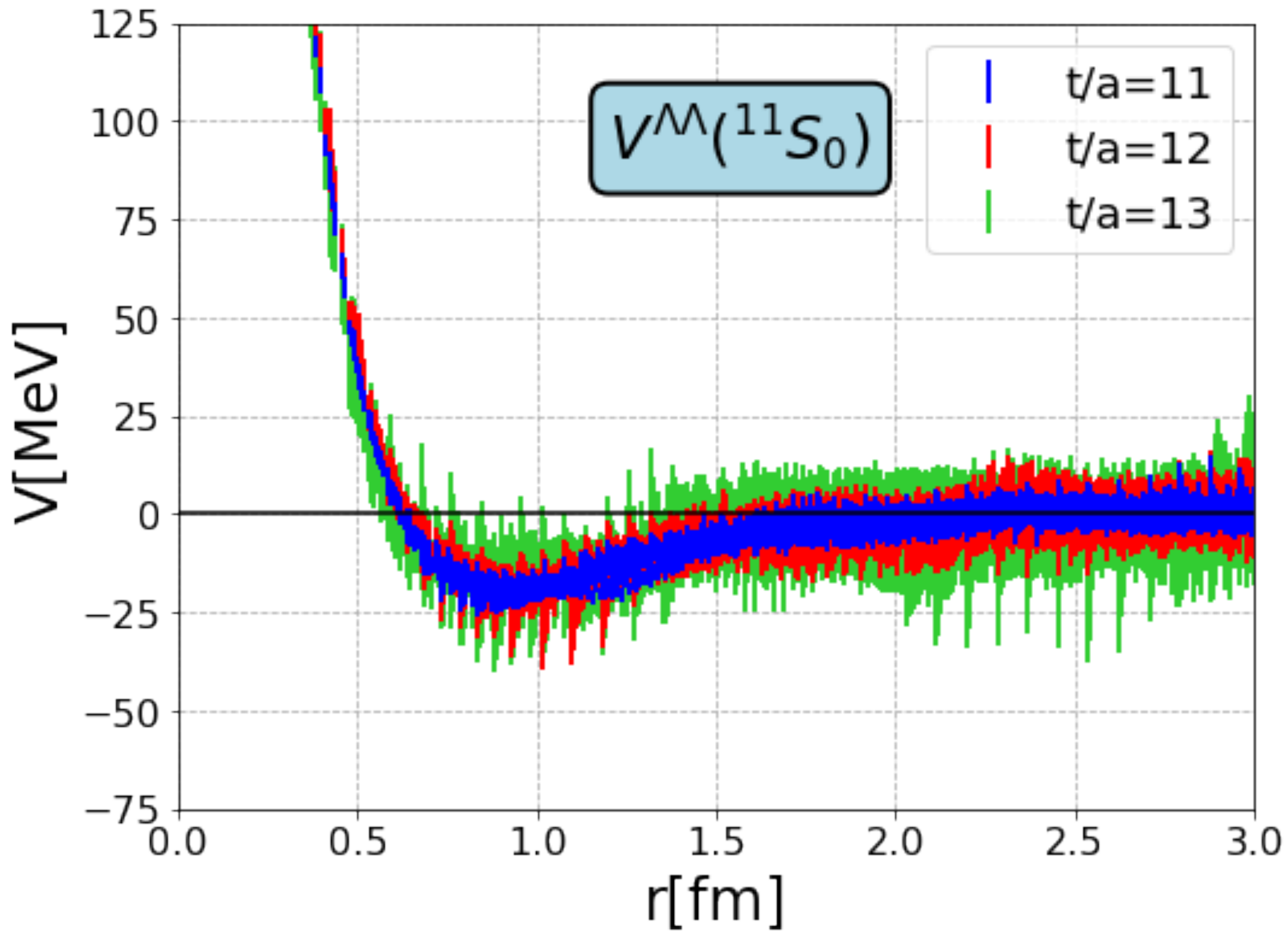
: non-interacting levels

- Resolve a dense spectrum of energy levels at several pion masses
- To be done: amplitude analysis

# The *H* Dibaryon at the physical pion mass: HAL QCD

Ensemble with $N_f = 2 + 1$ flavours of $\mathrm{O}(a)$ RG-improved Wilson quarks

- Single lattice spacing: $a = 0.0846\,\mathrm{fm}$; Volume: $L \approx 8.1\,\mathrm{fm}$;
- Near-physical pion mass: $m_\pi = 146\,\mathrm{MeV}$, $m_K = 525\,\mathrm{MeV}$



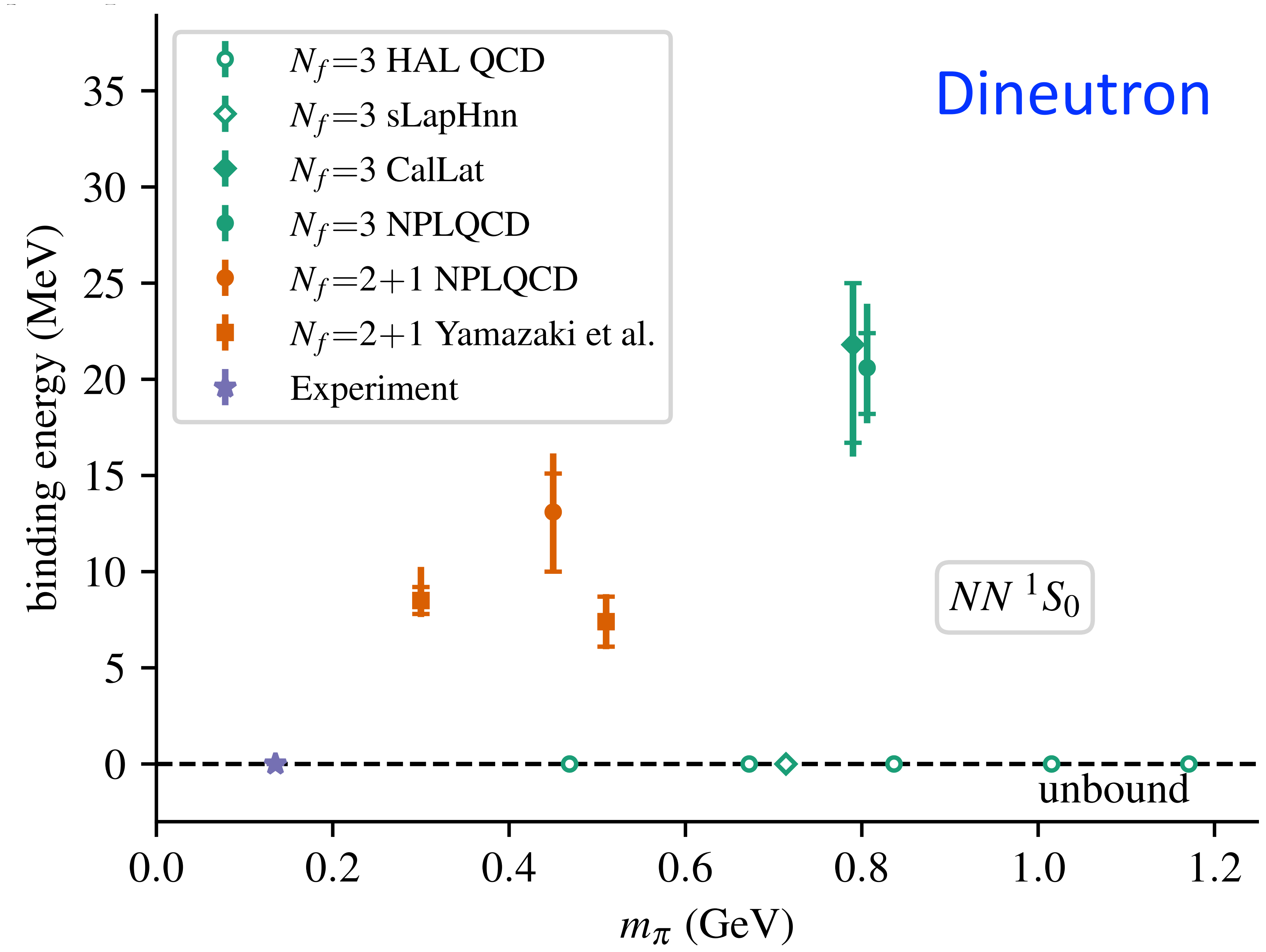

- $\Lambda\Lambda$ interaction weakly attractive
- No bound or resonant dihyperon near $\Lambda\Lambda$ threshold observed at the physical point

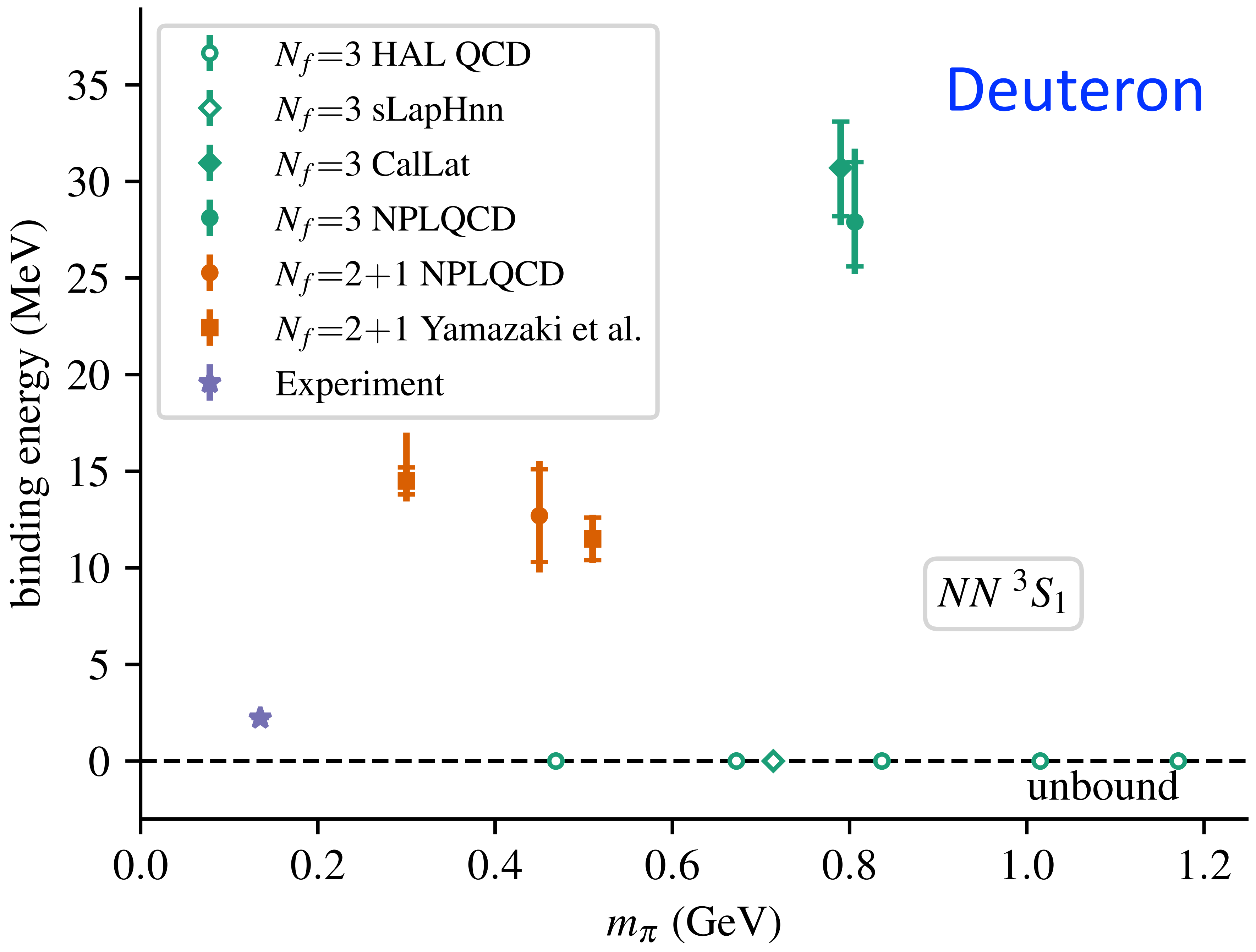
*[Sasaki et al., Nucl Phys A998 (2020) 121737, arXiv:1912.08630]*

# Nucleon-nucleon interactions

[Green, Hanlon, Junnarkar, HW (BaSc), arXiv:2212.09587]

Inconclusive results on existence of bound states at unphysical pion masses:

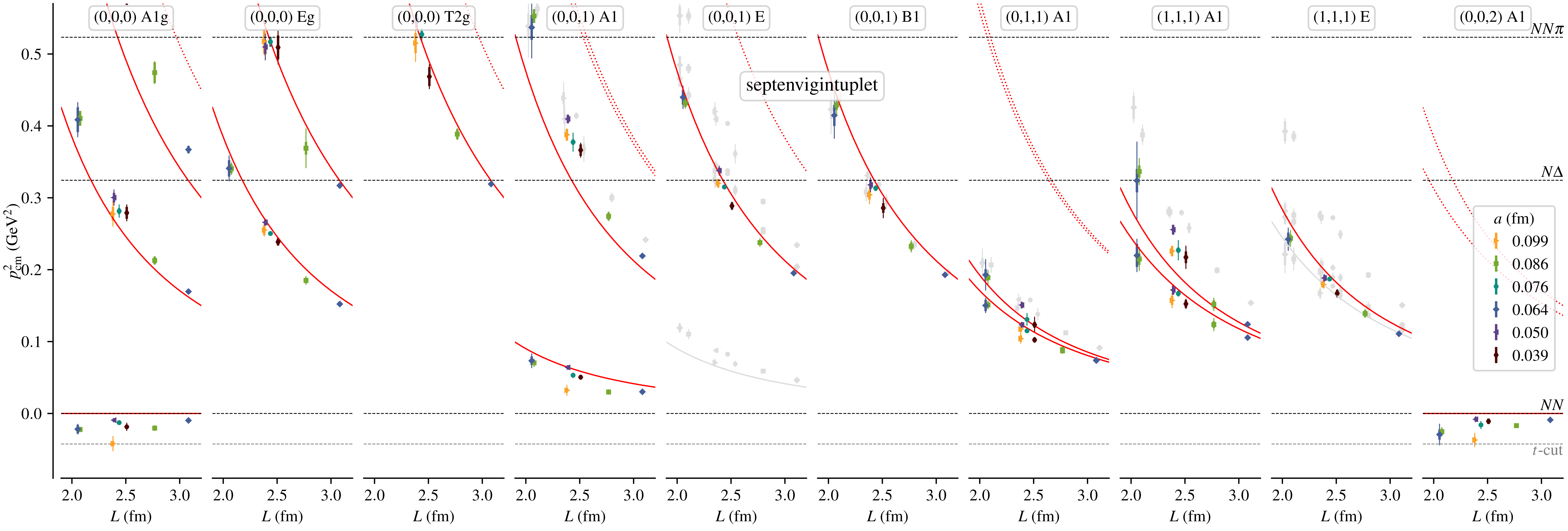



Study the dineutron and deuteron channels at SU(3)-symmetric point

- Employ distillation and symmetric GEVP
- Study dependence on lattice spacing

# 27-plet ($NN$, $I = 1$): spin-0 spectrum

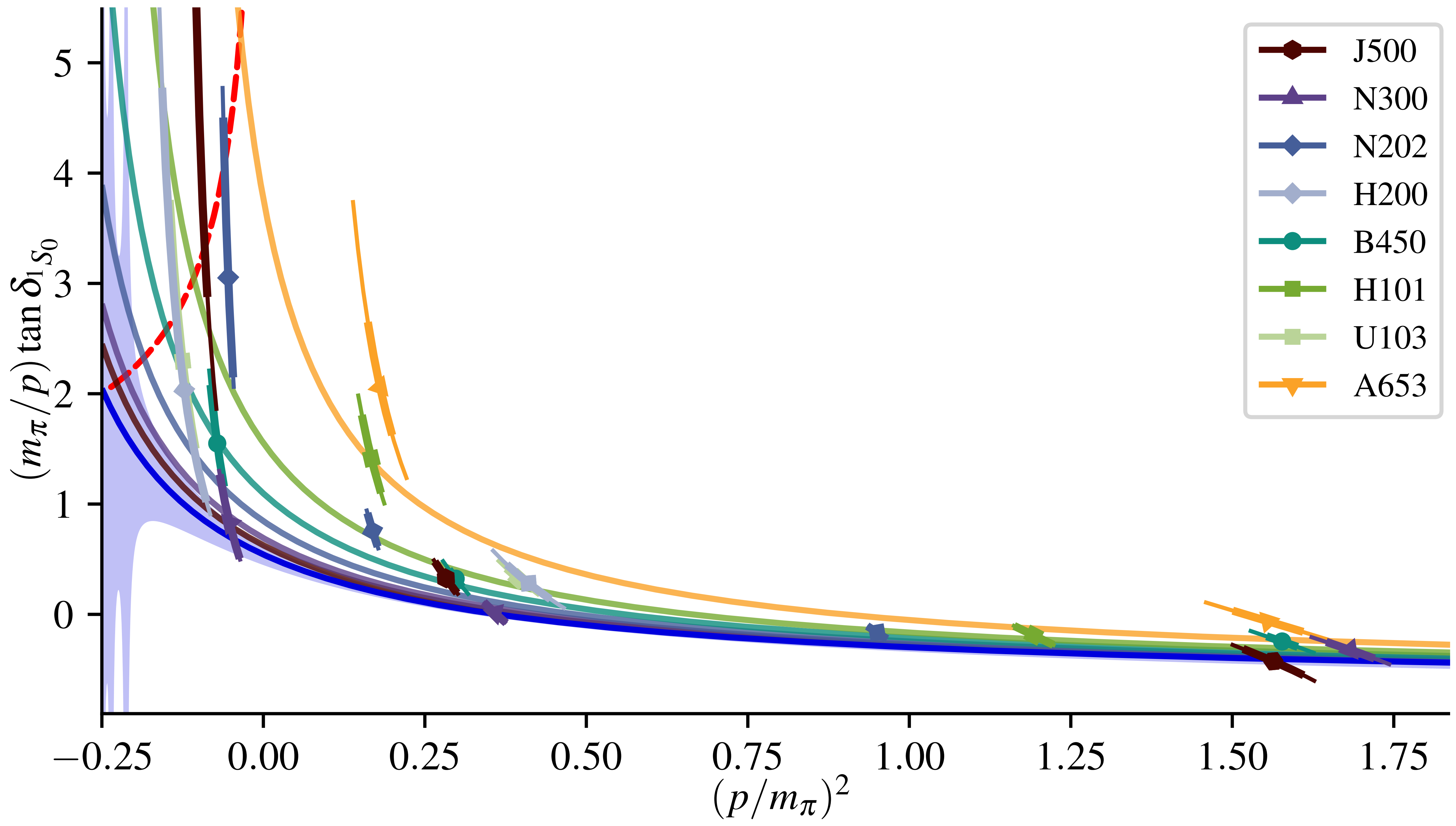



- Spin-1 states (grey) identified by overlaps
- Quantisation condition factorises in spin; $^1S_0$ and $^1D_2$ are relevant

: non-interacting levels

# 27-plet ($NN$, $I = 1$): spin-0

Phase shift analysis: $^1S_0$

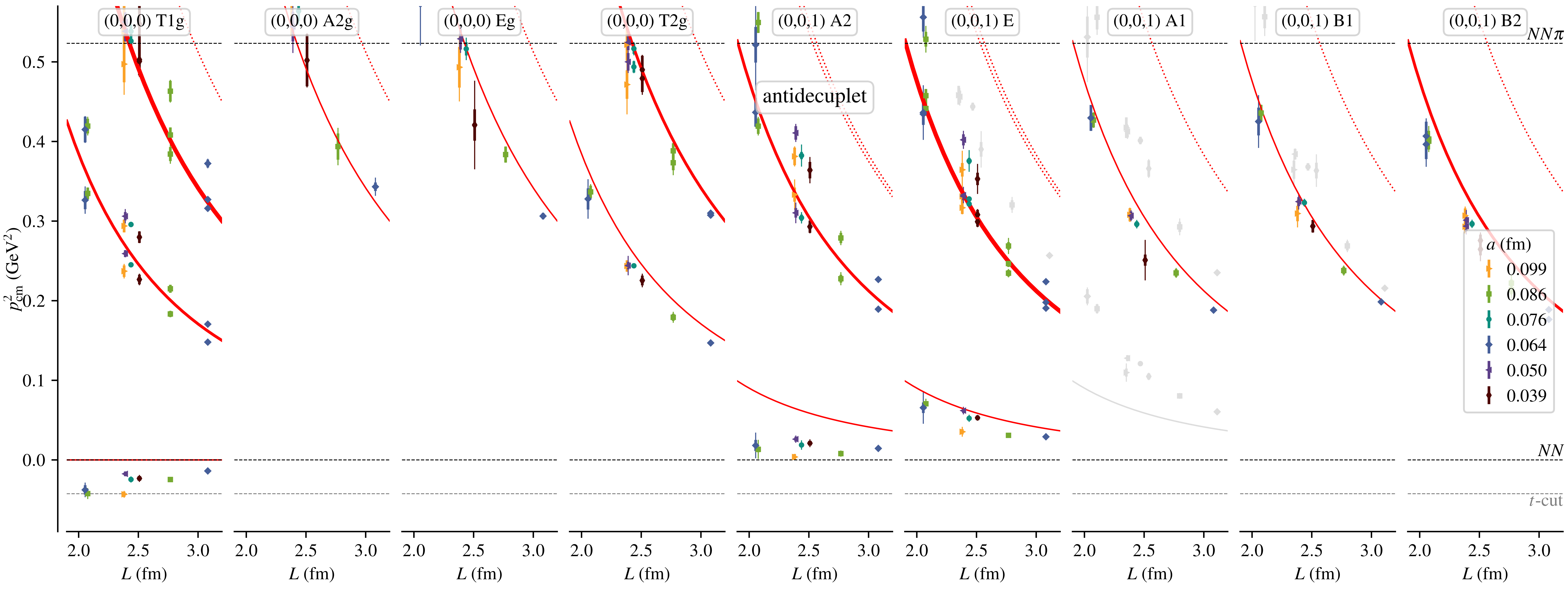



- Levels from rest frame and first moving frame
- Fit to rational function:

$$p \cot \delta(p) = \frac{c_0 + c_1 p^2}{1 + c_2 p^2}$$

- Observe virtual bound state
- Phase shift decreases towards continuum limit
  → Discretisation effects enhance baryon-baryon interactions

# Anti-decuplet ($NN$, $I = 0$): spin-1 spectrum




- Resolve ≈ 300 energy levels
- $^3S_1$, $^3D_1$, $^3D_2$ and $^3D_3$ can be relevant

: non-interacting levels
(thickness proportional to degeneracy)

(Spin-0 states (grey) identified by overlaps)

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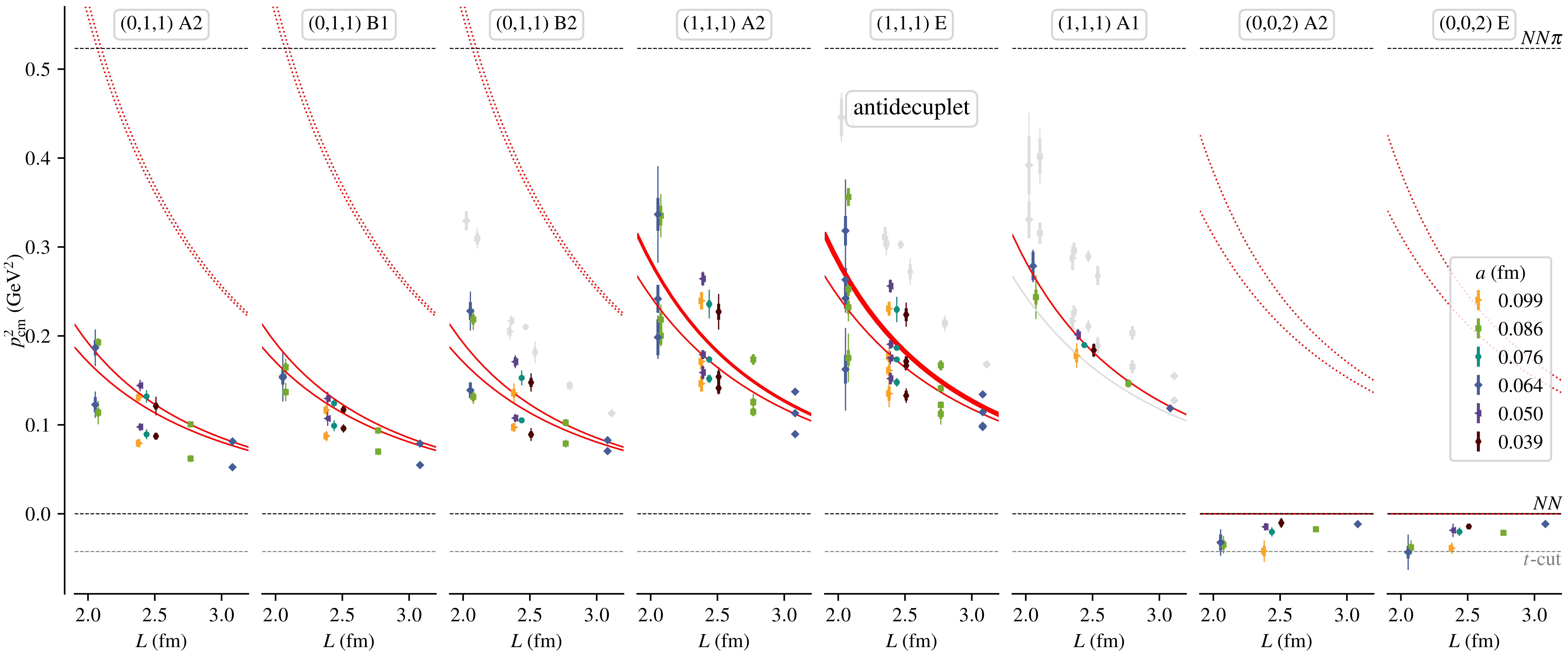


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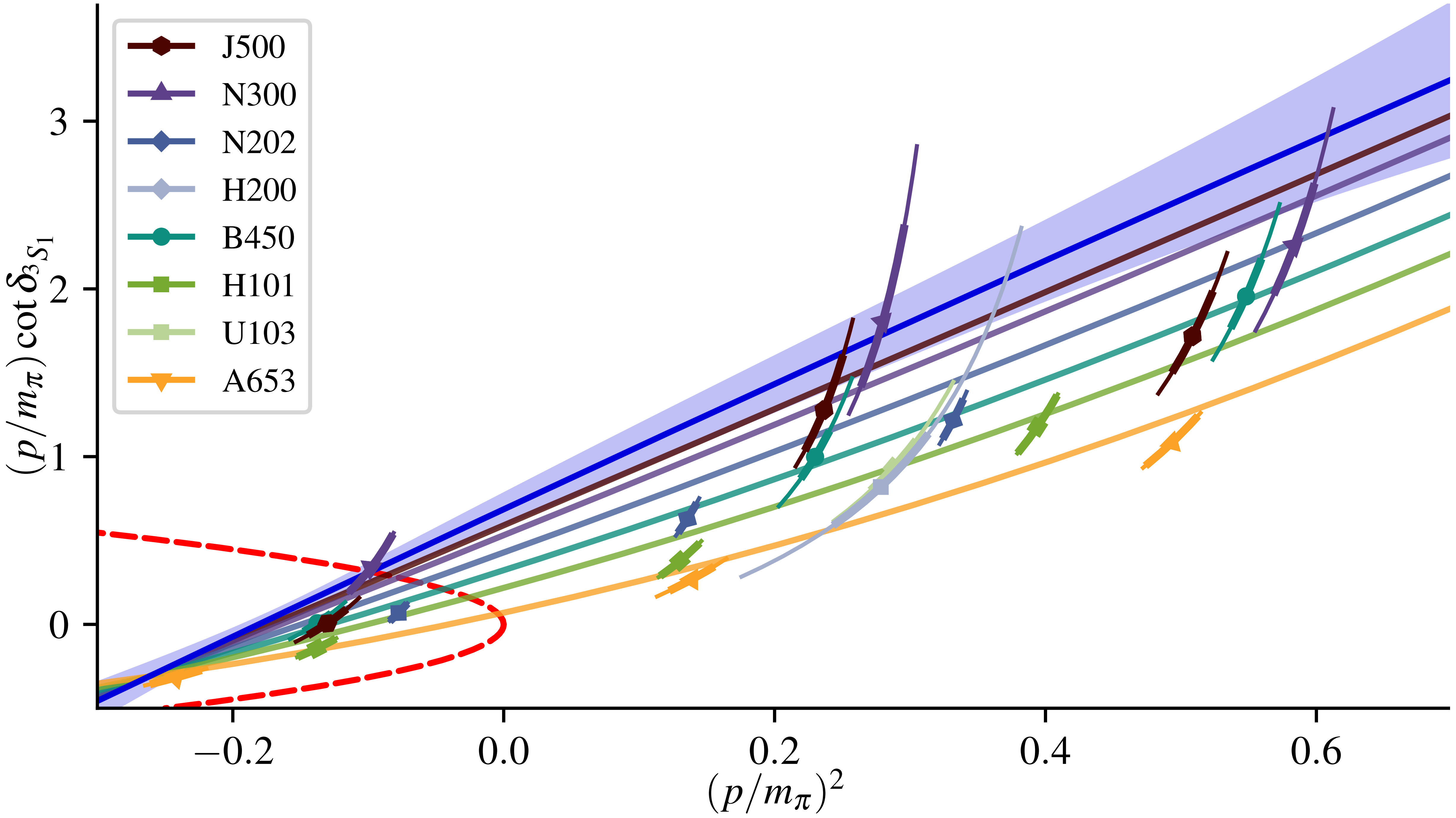
Phase shift analysis: $^3S_1$

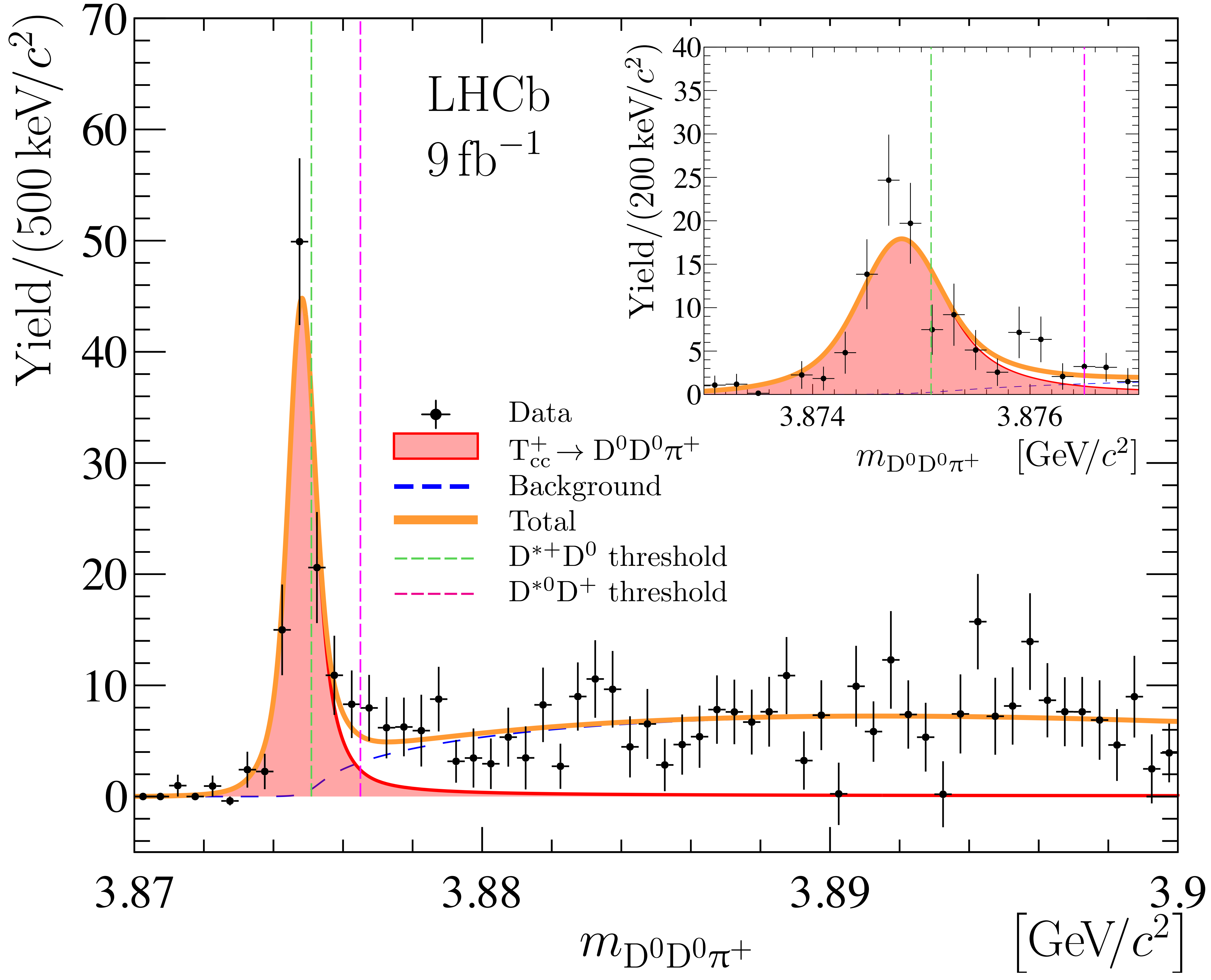



- Helicity-averaged levels from first two moving frames
- Neglect mixing with $^3D_1$

- Observe virtual bound state → Deuteron not bound at $m_\pi = m_K \sim 420\,\mathrm{MeV}$
- Phase shift decreases towards continuum limit → $NN$ interaction enhanced by lattice artefacts

# Charmed tetraquarks

LHCb: observation of doubly charmed tetraquark $T_{cc}^{+}$ with $I = 0,\ J^P = 1^+$




*[LHCb, Nature Phys. 18 (2022) 751]*

$\delta m_{BW} = -273 \pm 61\,\text{keV}$ below $D^{*+}D^0$ threshold

$\Gamma_{BW} = 410 \pm 165\,\text{keV}$

- Lattice QCD: discretisation effects may be large for heavy quark systems
- Perform scaling test for different lattice spacings

# Charmed tetraquarks: Lattice setup

[J. Green @ Lattice 2023 — Work in Progress]

Use same set of CLS ensembles at the SU(3)-symmetric point with $m_u + m_d + m_s$ at the physical value

$\rightarrow$ $D, D^*$ and $D_0^*$ all stable

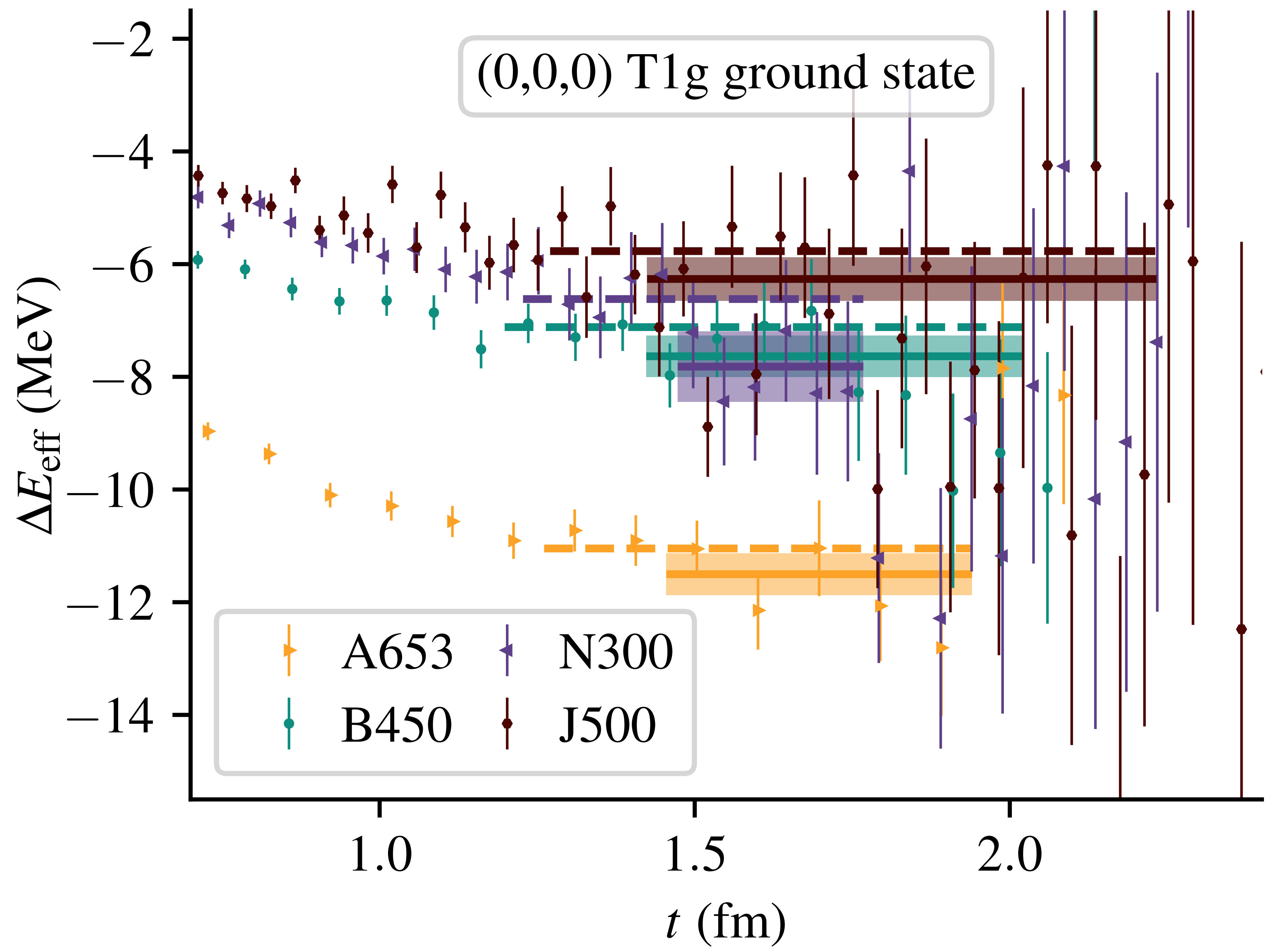
Apply distillation approach to momentum-projected meson-meson operators:

$$O(\boldsymbol{P};t) = \sum_{\boldsymbol{x}} e^{-i\boldsymbol{p}_1\cdot\boldsymbol{x}}(\bar{q}\Gamma_1 q)(\boldsymbol{x},t) \sum_{\boldsymbol{y}} e^{-i\boldsymbol{p}_2\cdot\boldsymbol{y}}(\bar{q}\Gamma_1 q)(\boldsymbol{y},t), \quad \boldsymbol{P} = \boldsymbol{p}_1 + \boldsymbol{p}_2$$

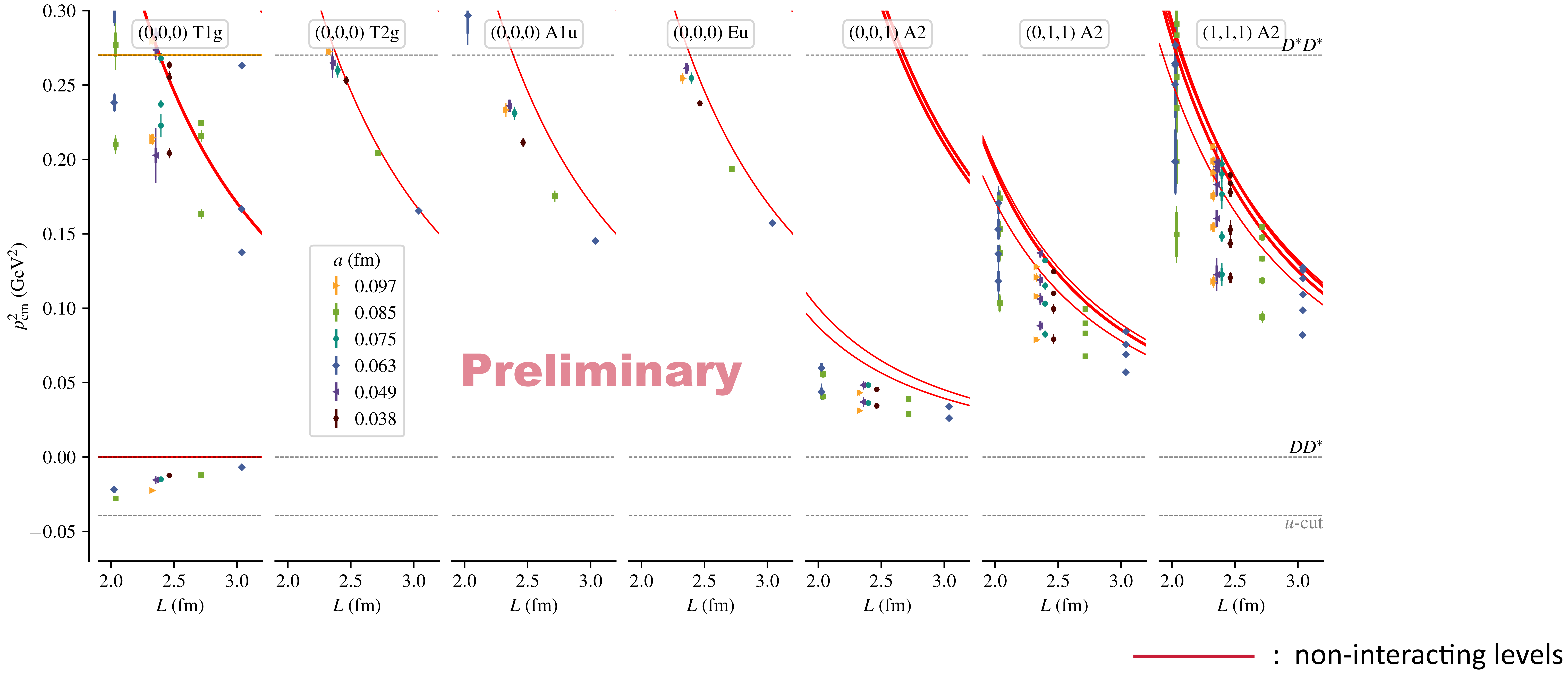
Compute $\quad C_{ij}(\boldsymbol{P},\tau) = \left\langle O_i(\boldsymbol{P},t) O_j(\boldsymbol{P},t')^\dagger \right\rangle, \quad \tau = t - t'$

Solve GEVP and fit to ratio of diagonalised correlator

$$R_n(t) \equiv \frac{\Lambda_{nn}(t)}{C_D^{\boldsymbol{p}_1}(t)\, C_{D^*}^{\boldsymbol{p}_2}(t)} \sim e^{-\Delta E t}$$



# Results: Energy levels and amplitude analysis



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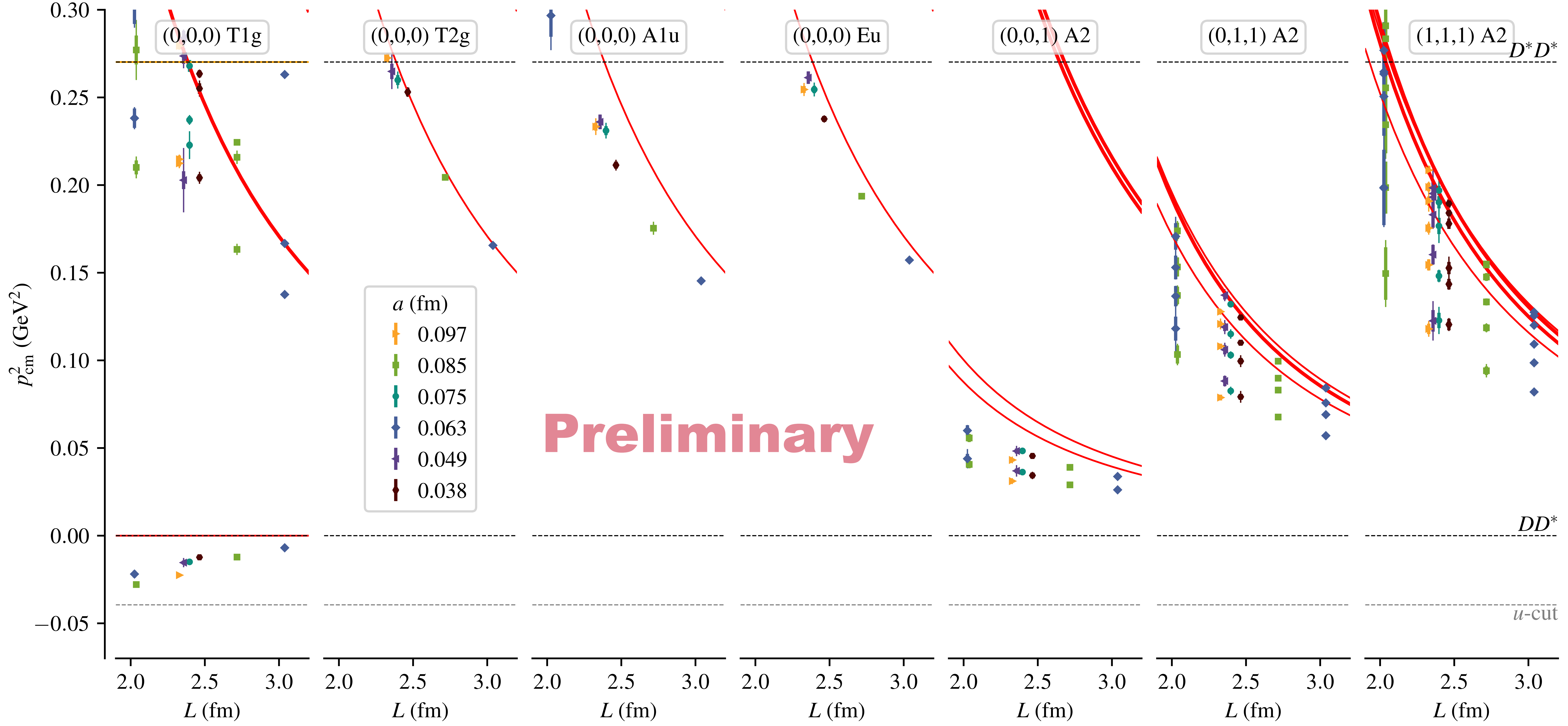
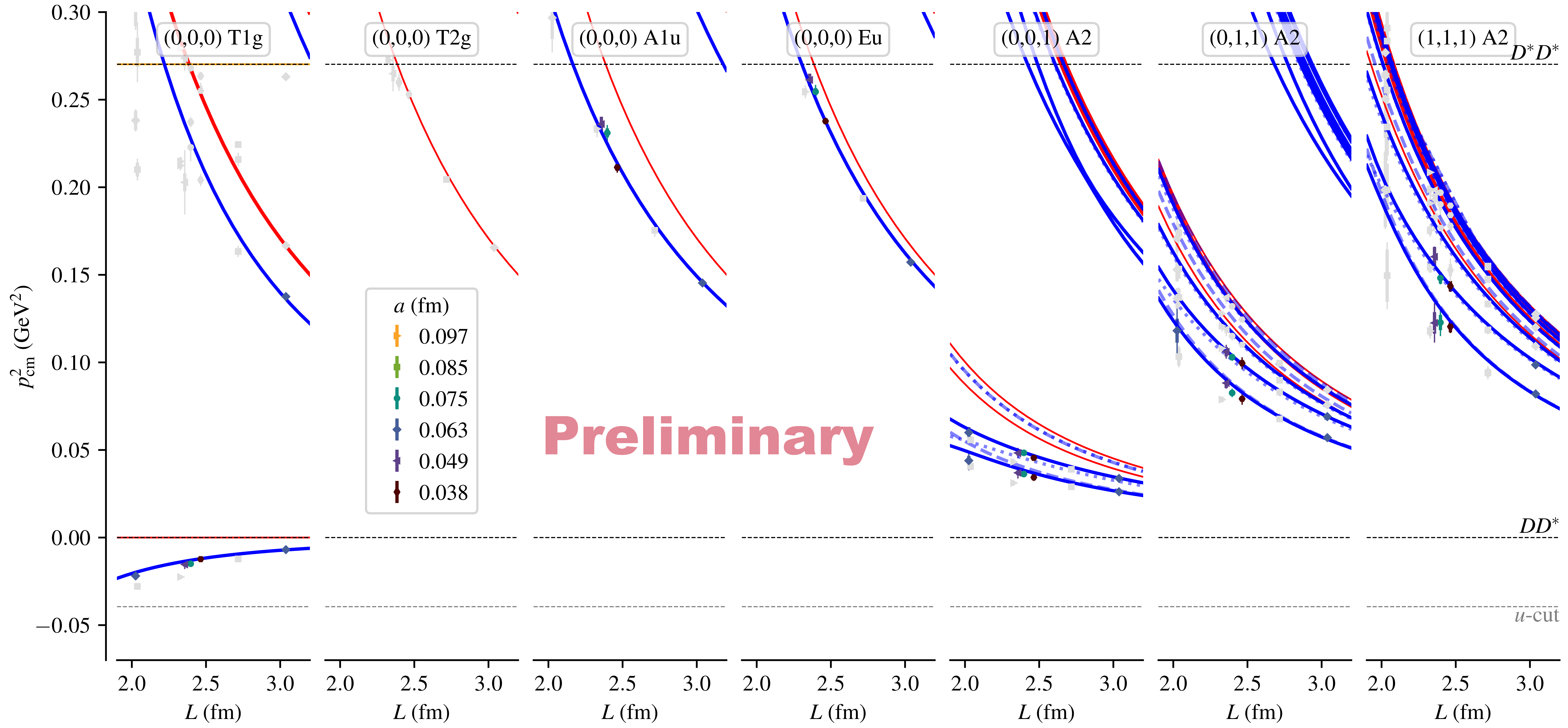



: non-interacting levels

**Amplitude analysis:** Neglect *D*-wave and higher

Fit $1^+$ *S*-wave, $0^-$ and $2^-$ *P*-wave phase shifts

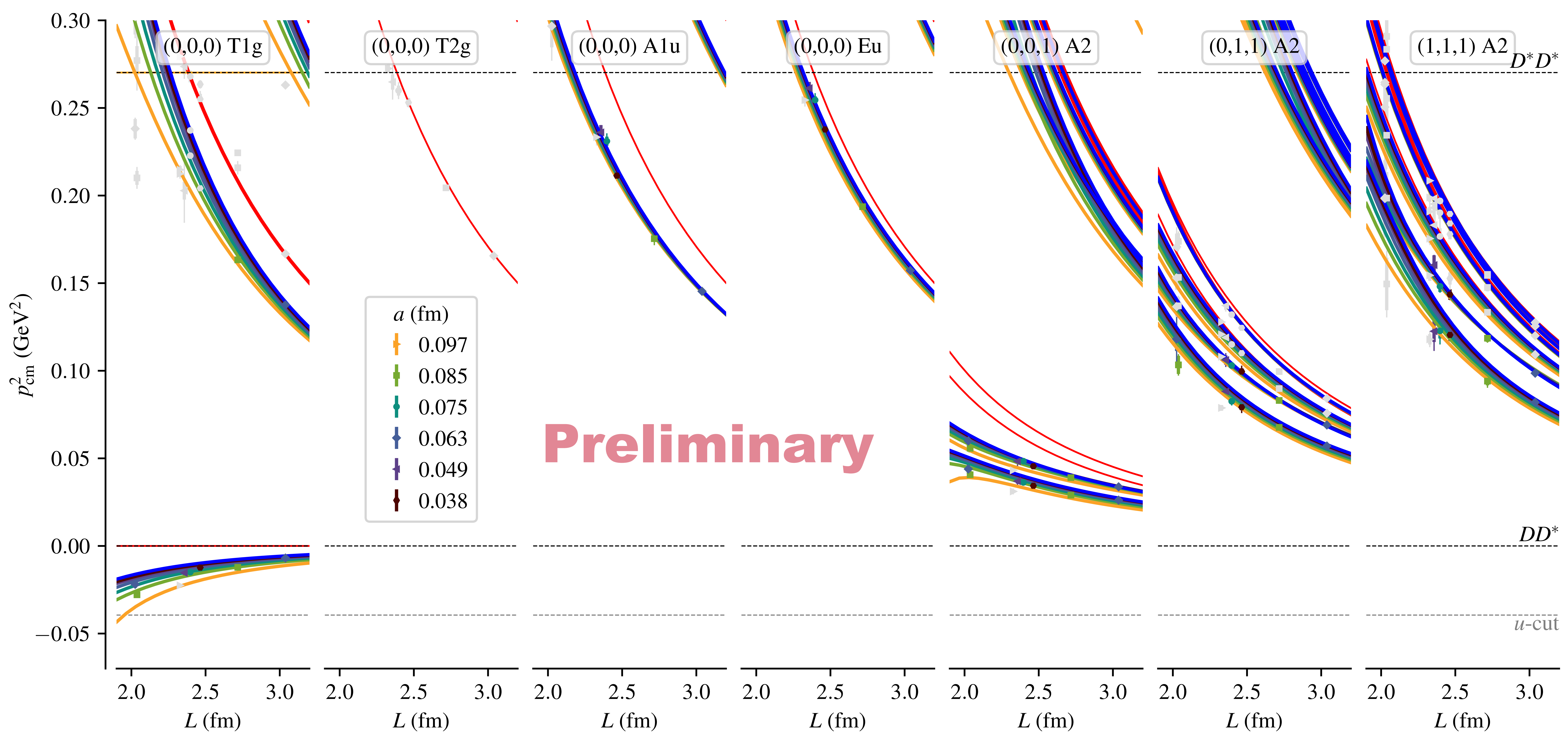
# Results: Energy levels and amplitude analysis




**Fit 1:** $a < 0.08$ fm, neglect $a^2$ lattice artefacts : non-interacting levels

(dashed: no $P$ waves; dotted: no $S$ wave; grey points not fitted)

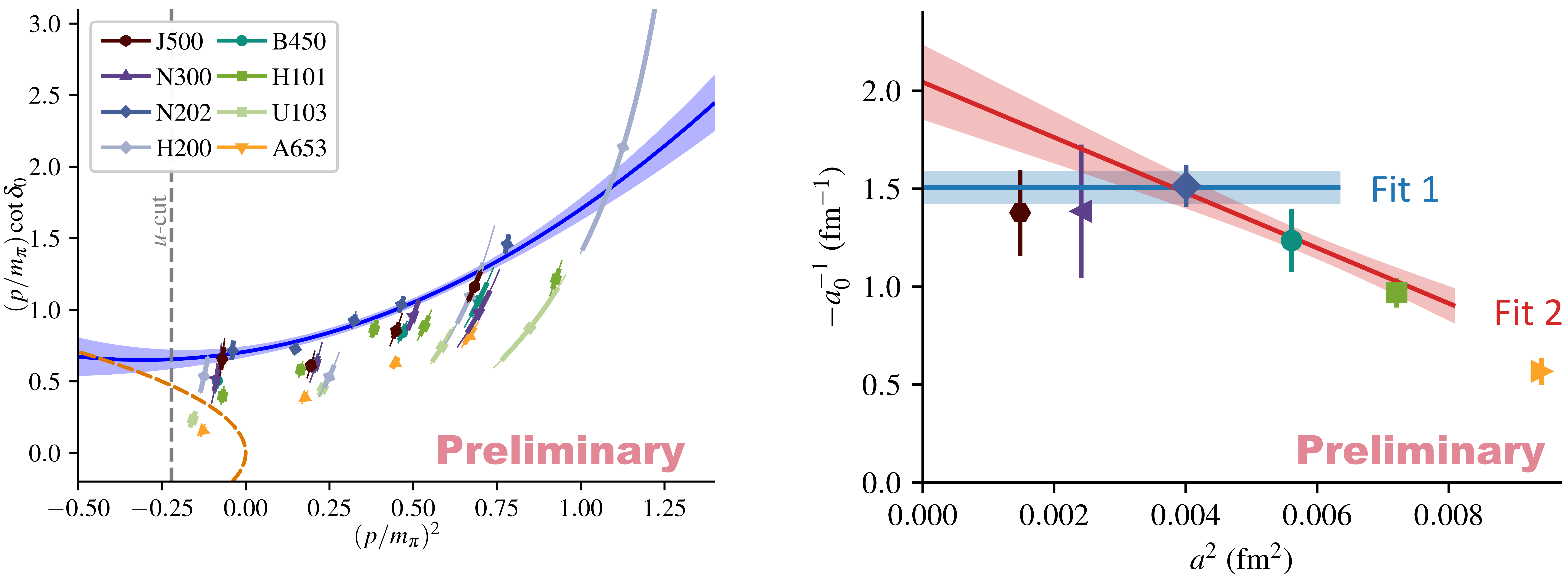
# Results: Energy levels and amplitude analysis




**Fit 2:** Continuum limit: $a < 0.09$ fm, including $a^2$ lattice artefacts : non-interacting levels

(other colours: non-zero lattice spacing; grey points not fitted)

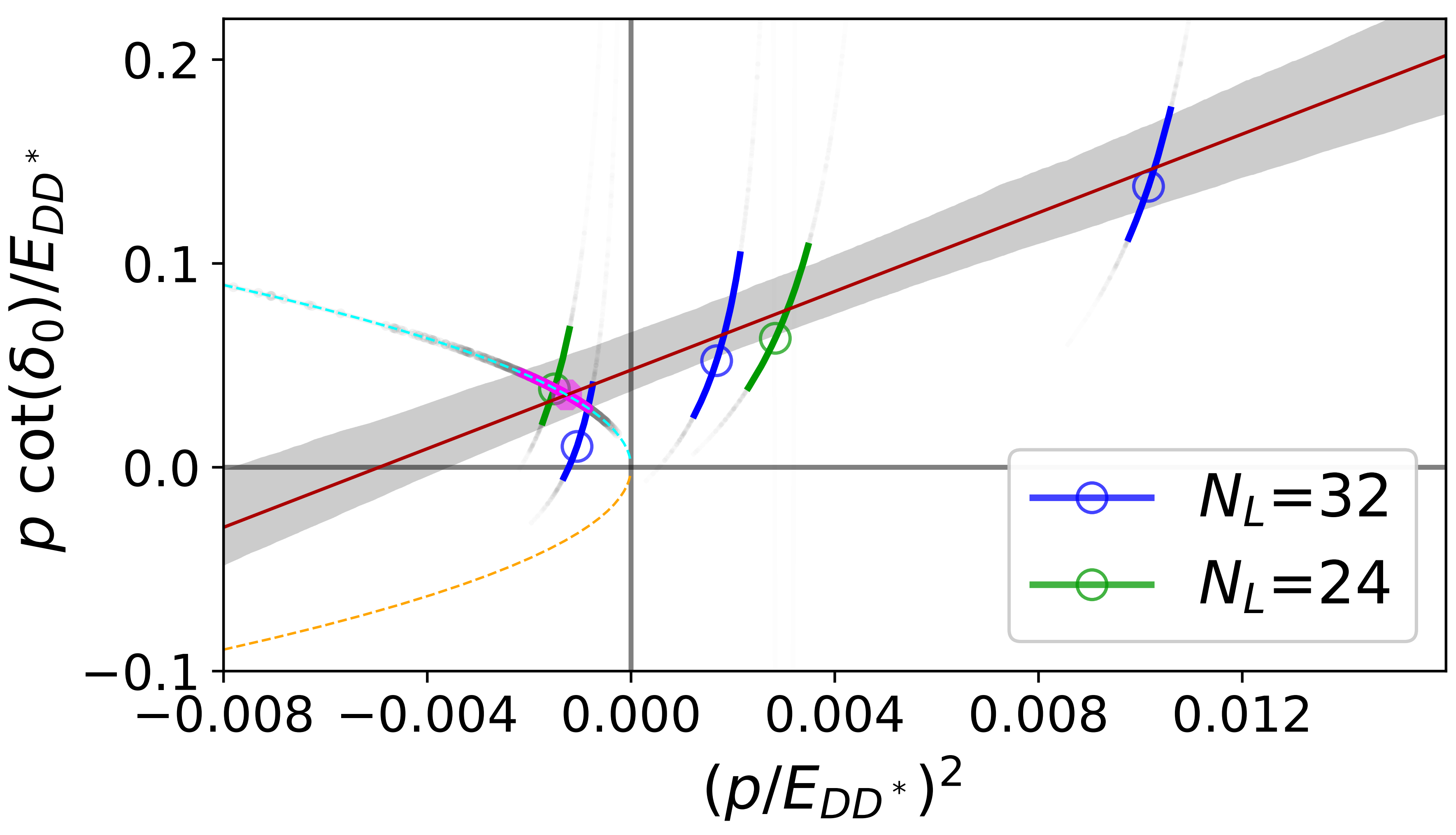
# Charmed tetraquarks: Amplitude analysis for $I = 0$ $S$-wave

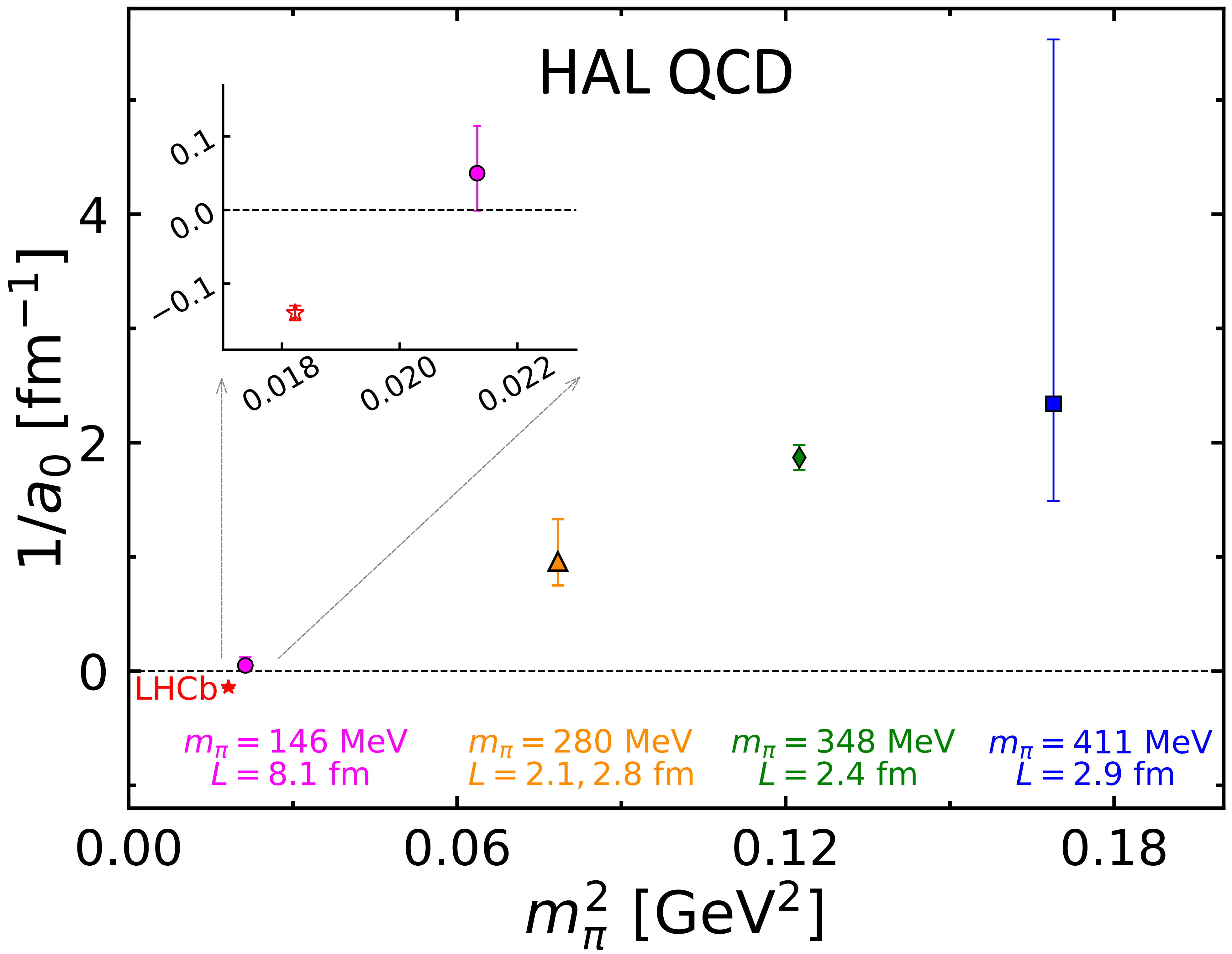



- Attractive $DD^*$ interaction but no bound state at SU(3)-symmetric point
- Very small discretisation effects for $a < 0.08$ fm but significant artefacts for coarser lattice spacings

# Charmed tetraquarks away from the SU(3)-symmetric point

Finite-volume quantisation




CLS $N_f = 2 + 1$ ensemble with $m_\pi \sim 280\,\mathrm{MeV}$

- Single lattice spacing: $a = 0.0864\,\mathrm{fm}$
- Virtual bound state observed

*[Padmanath & Prelovšek, PRL 129 (2022) 032002]*

- $m_\pi \sim 146\,\mathrm{MeV},\ a = 0.0846\,\mathrm{fm}$
- "Loosely bound state" observed

*[Lyu et al., arXiv:2302.04505]*

# Summary — Conclusions — Outlook

* Distillation, GEVP and Finite-volume Quantisation:
  Detailed and precise studies of two-particle interactions

* Discretisation effects in binding energies and scattering lengths can be sizeable:
  - Binding energy of *H* dibaryon much smaller in continuum limit: $\mathrm{O}(5\,\mathrm{MeV})$
  - Lattice artefacts enhance strength of hadron-hadron interactions
  - Confirmed using different lattice actions and formalisms

* No bound states in dineutron and deuteron channels observed at $m_\pi = m_K \sim 420\,\mathrm{MeV}$

* SU(3)-flavour breaking makes amplitude analysis much more complicated

* Mixing with higher partial waves under study

* Charmed tetraquarks: Virtual bound state observed at non-zero lattice spacings

# Backup

# Dibaryons in Lattice QCD

## Interpolating operators for other dibaryon channels

Building block: $B_\alpha \equiv [rst]_\alpha = \epsilon_{ijk}\left(s^i C\gamma_5 P_+ t^j\right) r^k_\alpha$

Spin-1 interpolator:

$$(BB)_i(\boldsymbol{p}_1, \boldsymbol{p}_2) = \sum_{\boldsymbol{x}} e^{-i\boldsymbol{p}_1\cdot\boldsymbol{x}} B_1(\boldsymbol{x},t)\,(C\gamma_i P_+) \sum_{\boldsymbol{y}} e^{-i\boldsymbol{p}_2\cdot\boldsymbol{y}} B_2(\boldsymbol{y},t)$$

Deuteron:

$$(BB)^{(n)}_{i;\,T_1^+} = \frac{1}{N} \sum_{\boldsymbol{p};\, p^2=n} (BB)_i(-\boldsymbol{p}, \boldsymbol{p})$$

# The Mainz dibaryon project

**Past and present members:**

Anthony Francis, Jeremy Green, Andrew Hanlon, Parikshit Junnarkar, M. Padmanath, Chuan Miao, Srijit Paul, Tom Rae, H.W.

Based on CLS ensembles with $N_f = 2$ and $N_f = 2 + 1$ flavours of $\mathrm{O}(a)$ improved Wilson quarks

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**Methodology:**

- Variational method: point-to-all ($N_f = 2$) and timeslice-to-all propagators
- Exact distillation: timeslice-to-all propagators *[Peardon et al., PRD 80 (2009) 054506]*
- Finite-volume quantisation
- Various dibaryon channels — extension to charmed tetraquarks

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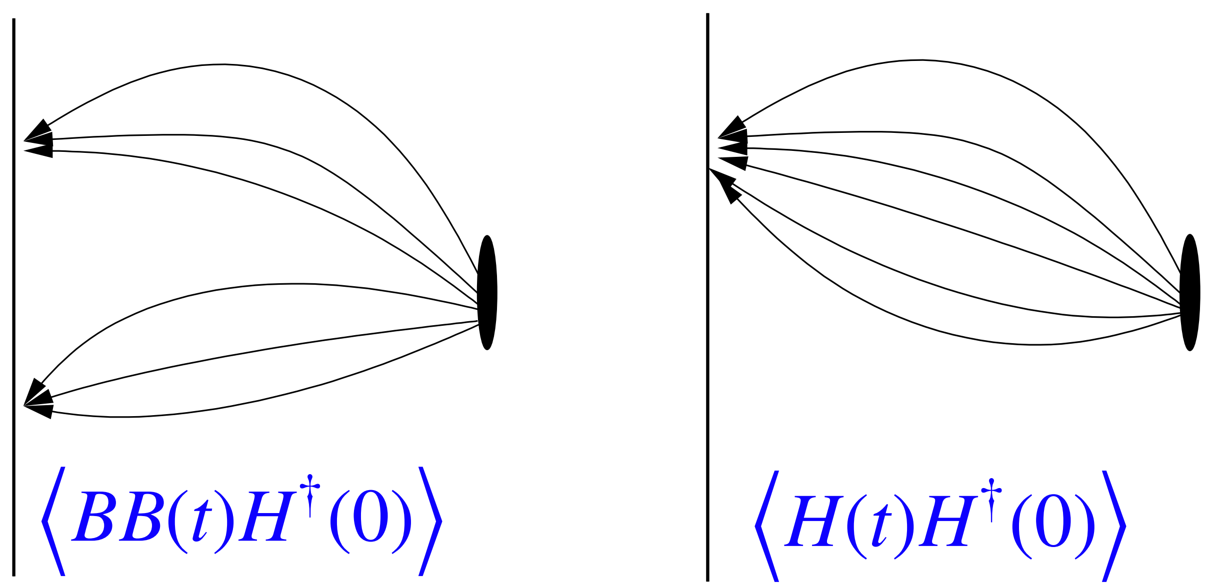
**Collaboration** within "Baryon Scattering" (BaSc) Collaboration

- Stochastic LapH on large physical volumes *[Morningstar et al., PRD 83 (2011) 114505]*
- Alternative discretisations: exponentiated Clover, domain wall

# Mainz Dibaryon project: Pilot study in 2-flavour QCD

- Pion masses match earlier calculations by NPLQCD and HALQCD
- Point-to-all propagators: asymmetric GEVP
- Hexaquark operators have poor overlap onto ground state
- Distillation: much better signal
- Finite-volume quantisation yields smaller binding energy

*[Francis et al., PRD 99 (2019) 074505]*



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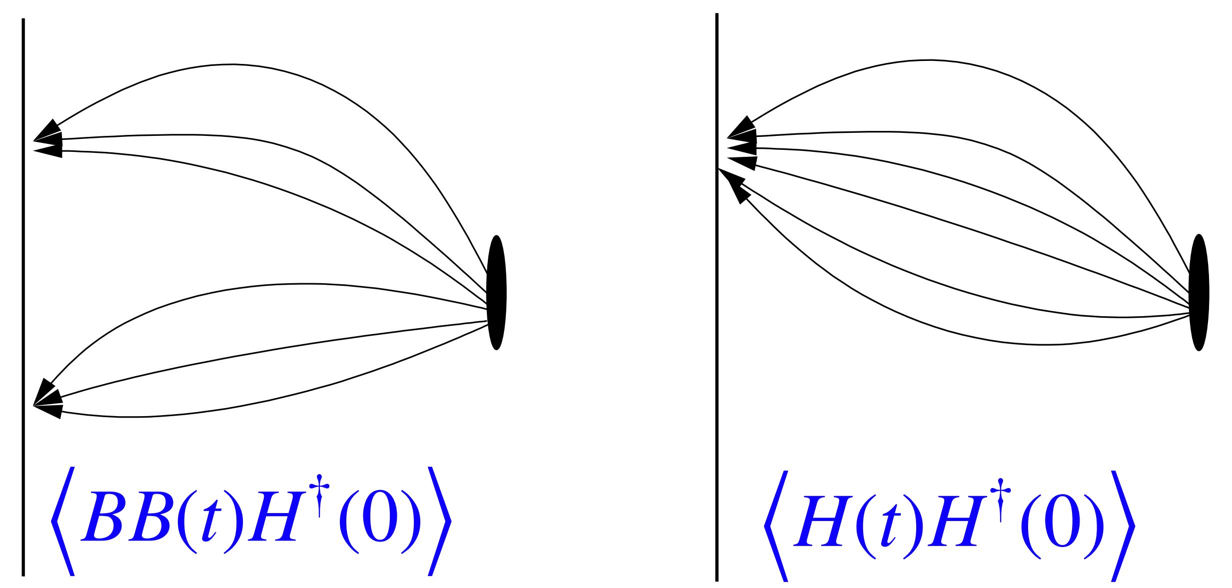


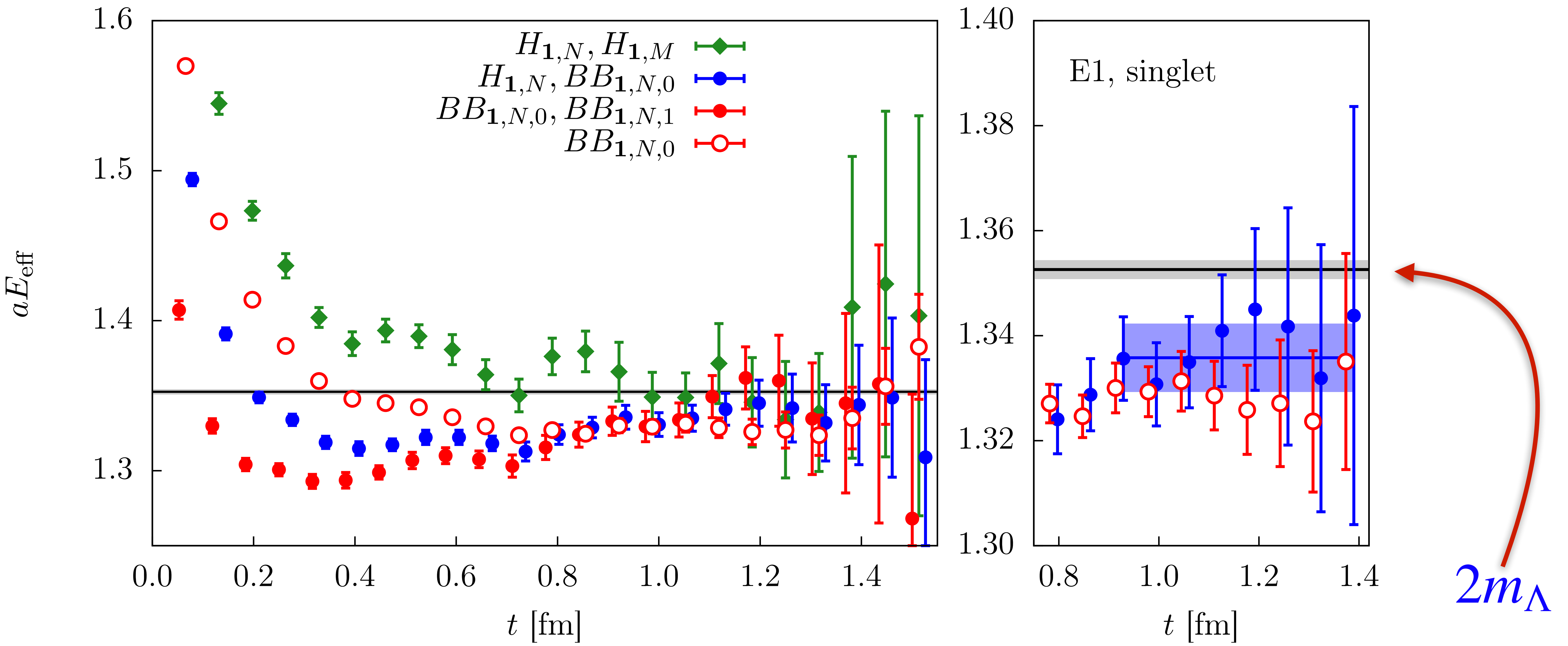

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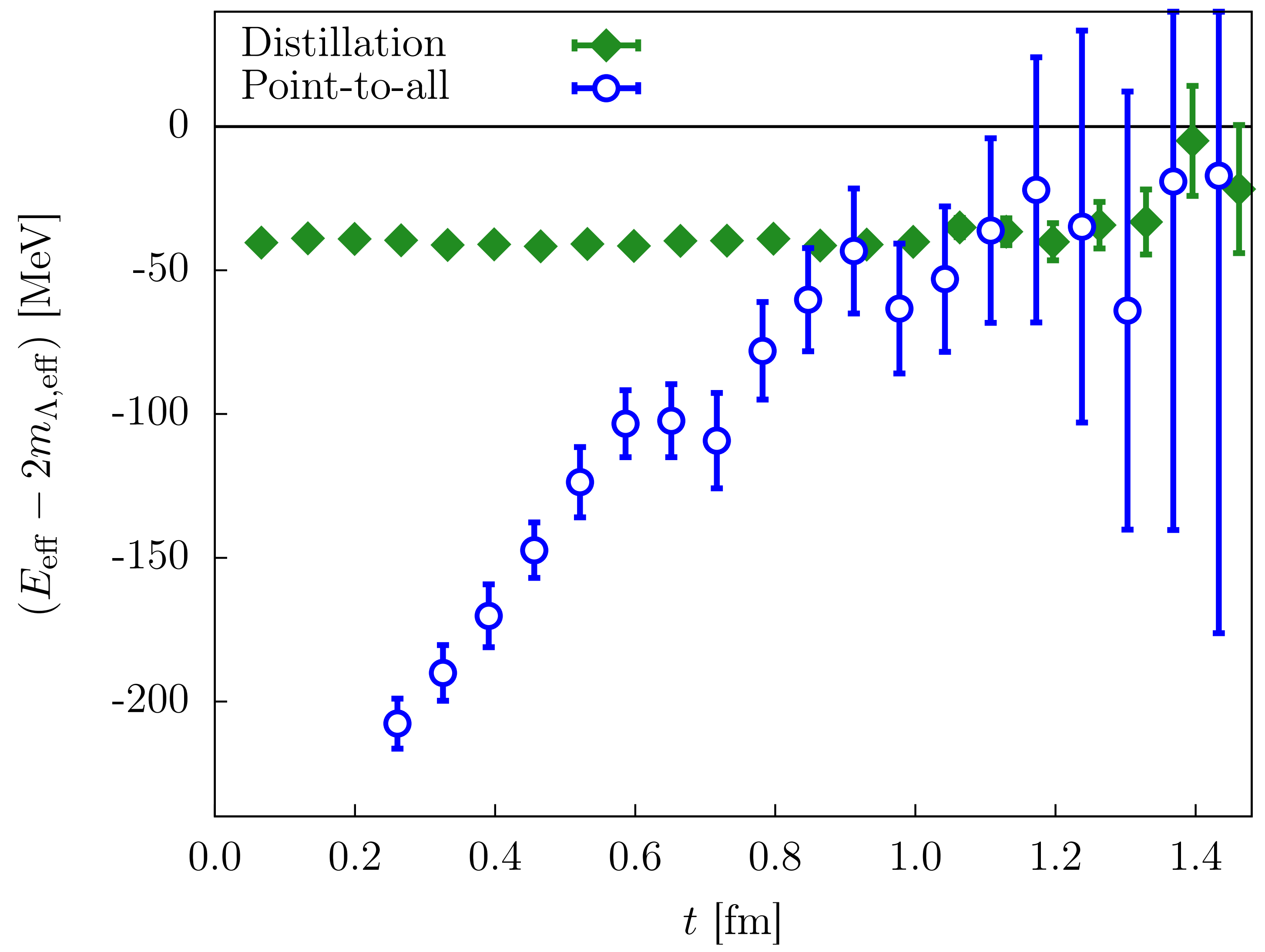


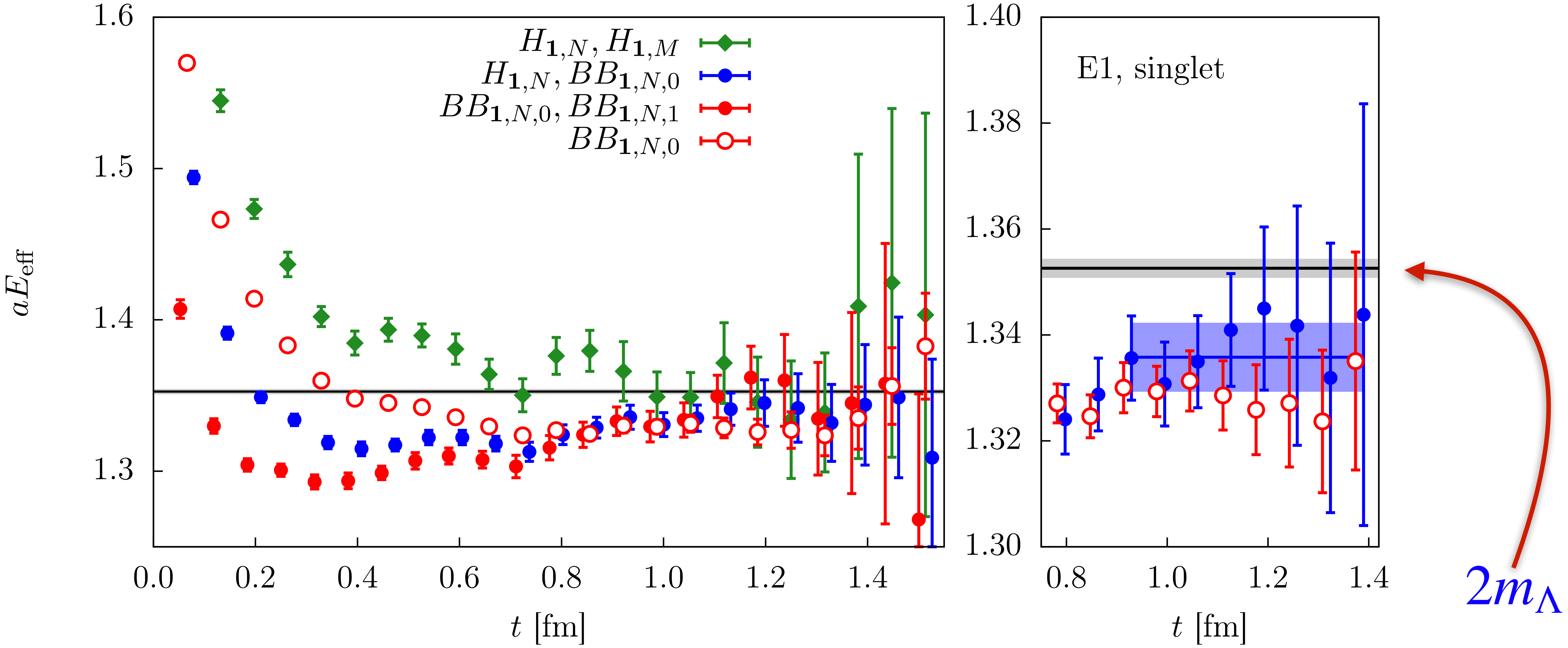


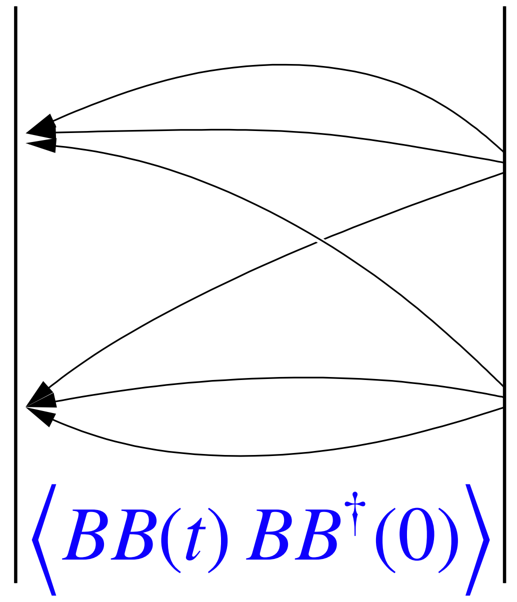

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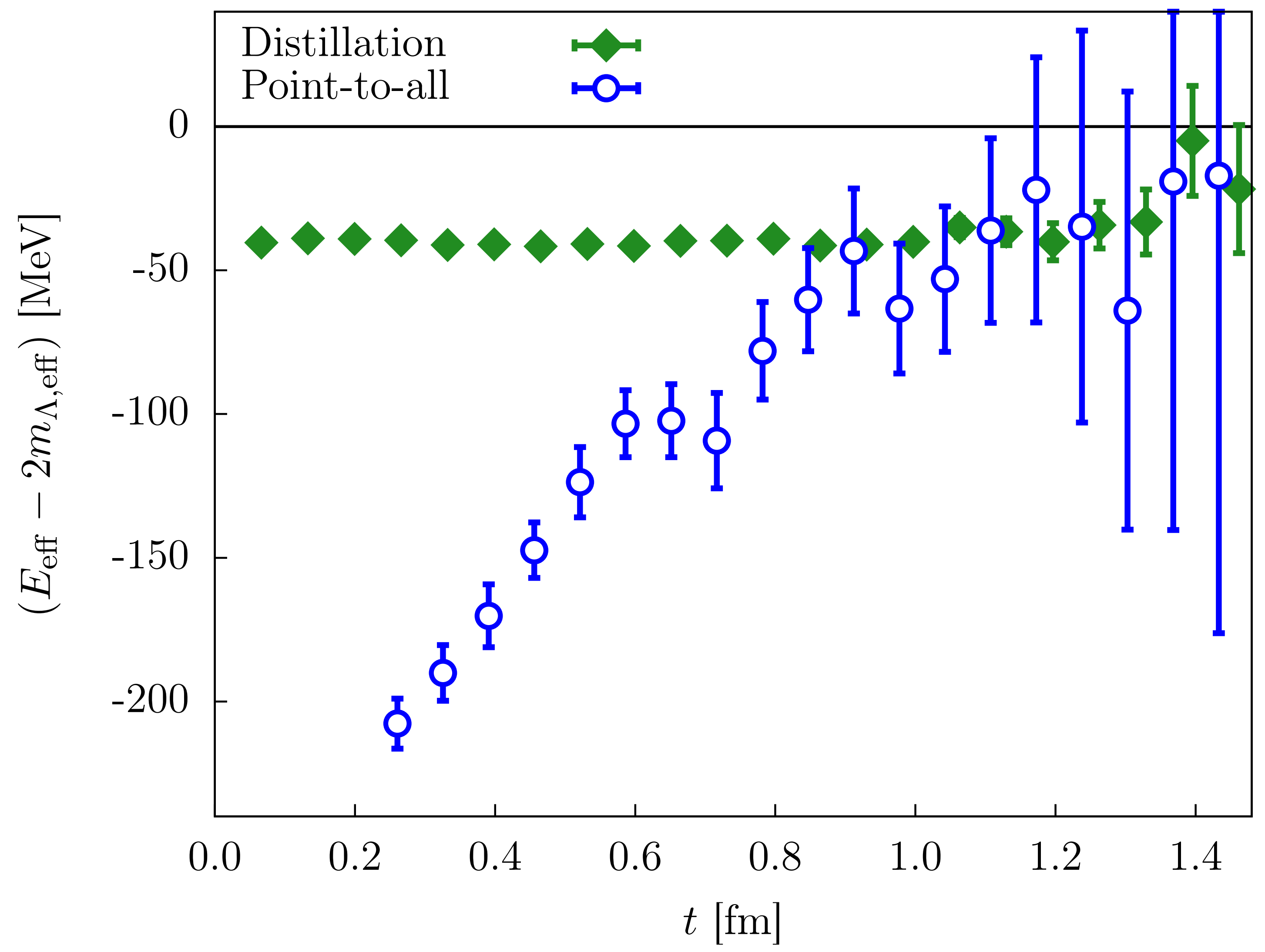
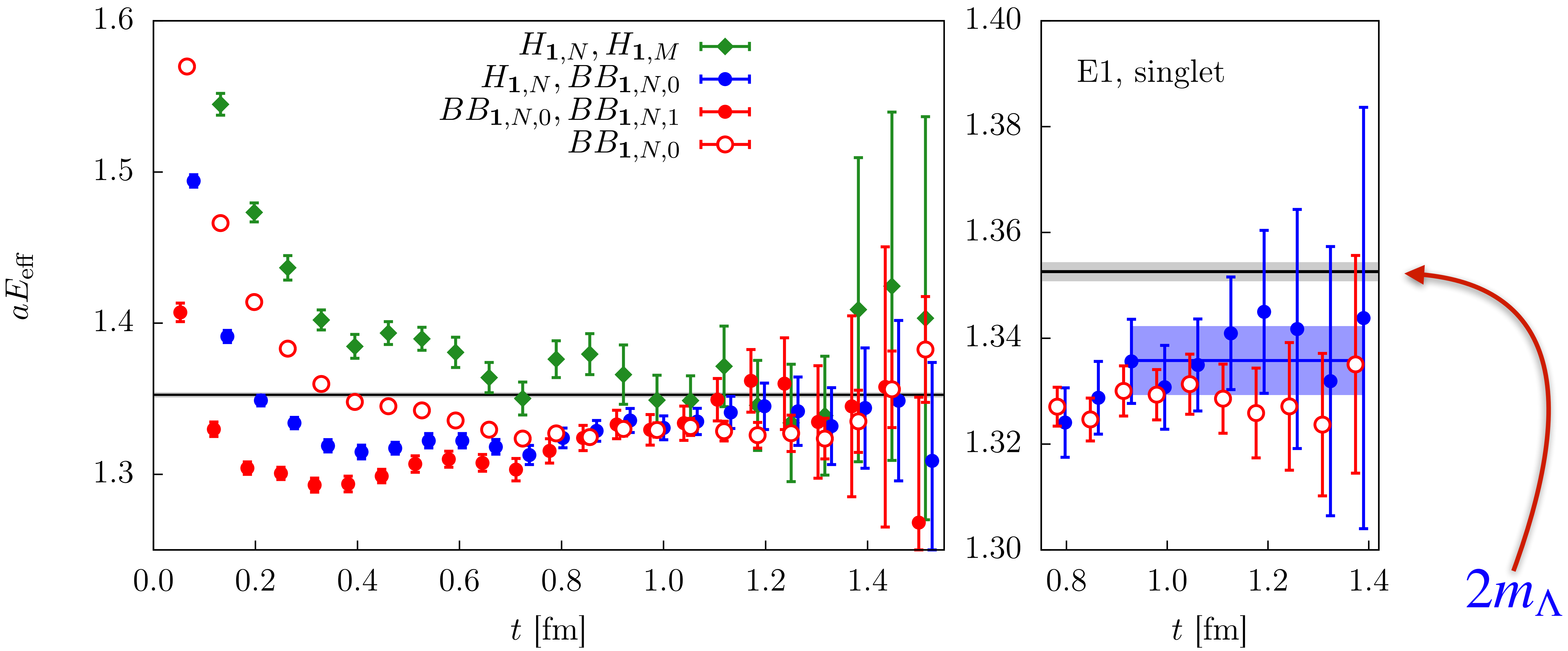
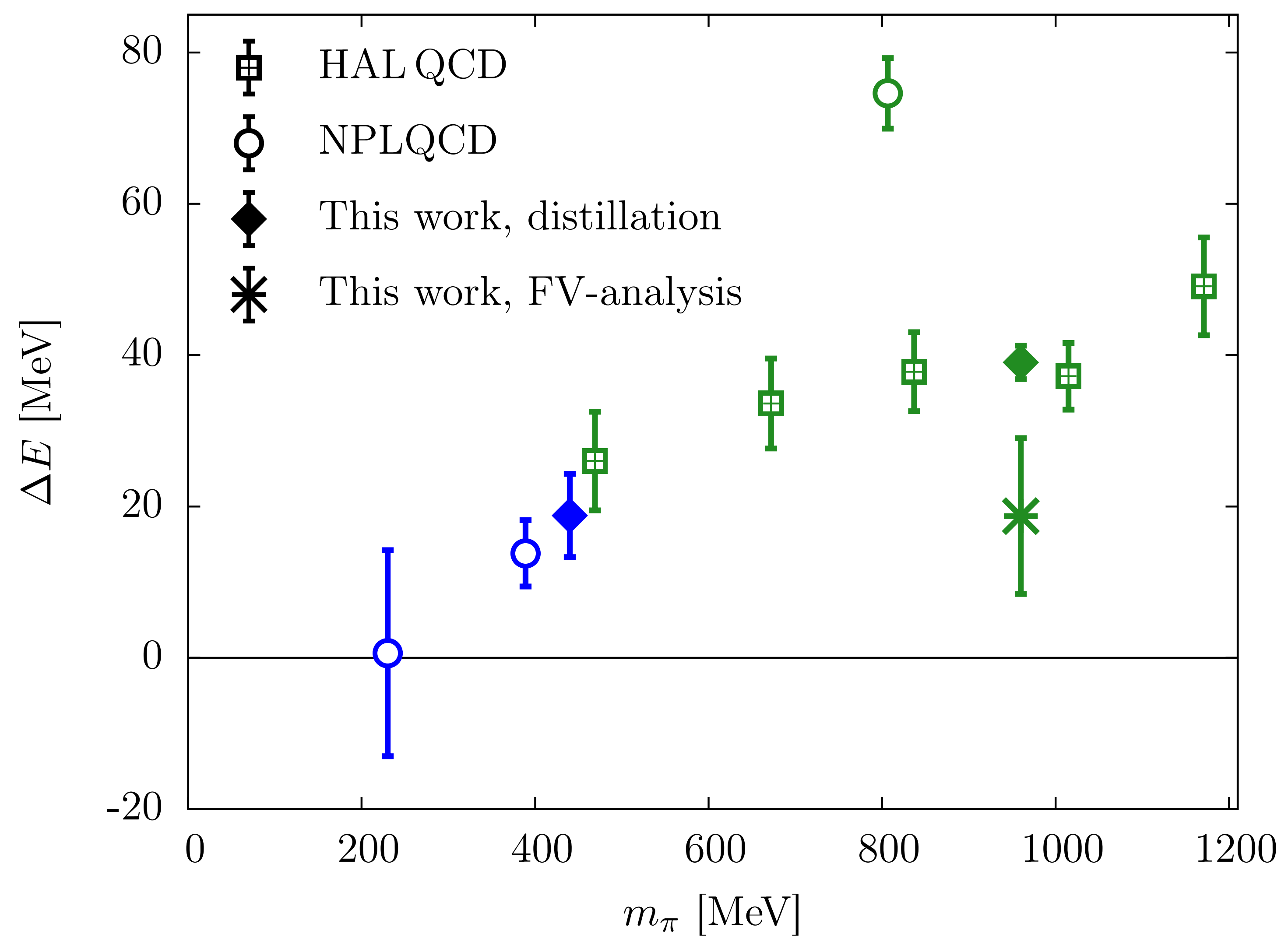
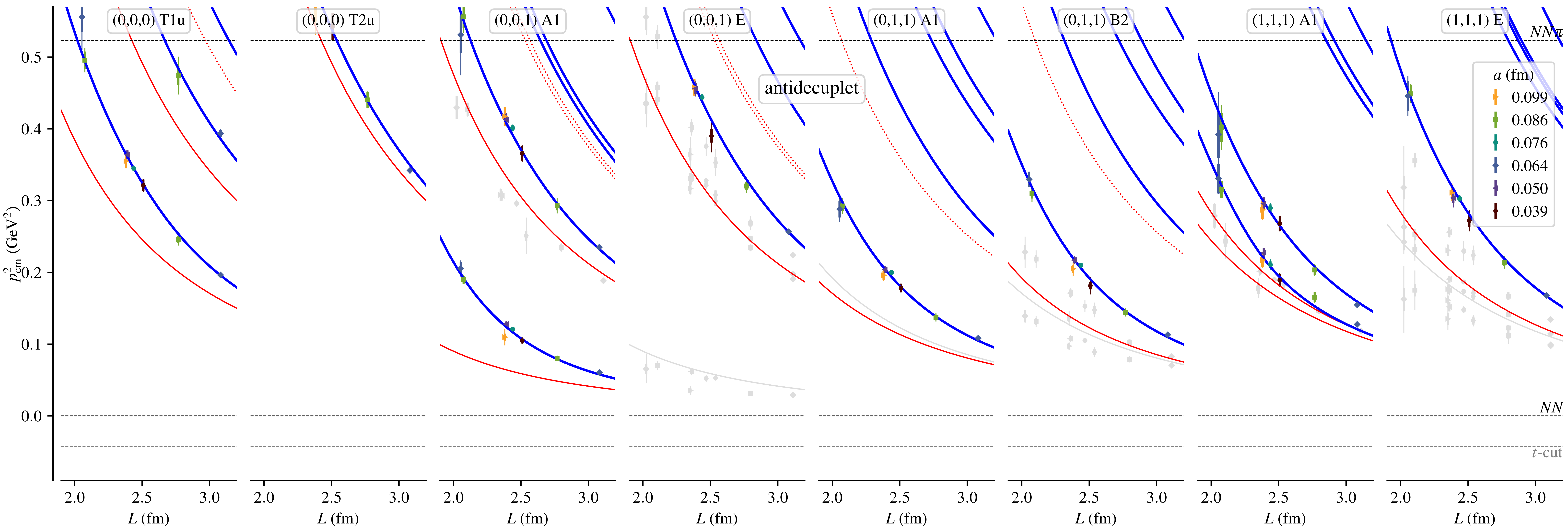


# Anti-decuplet ($NN$, $I = 0$): spin-0 spectrum




: 4-parameter fit to 70 energy levels

: non-interacting levels

$$p^3 \cot \delta_{^1P_1} = c_0 + c_1 p^2, \quad p^3 \cot \delta_{^1F_3} = c_2 + c_3 p^8$$

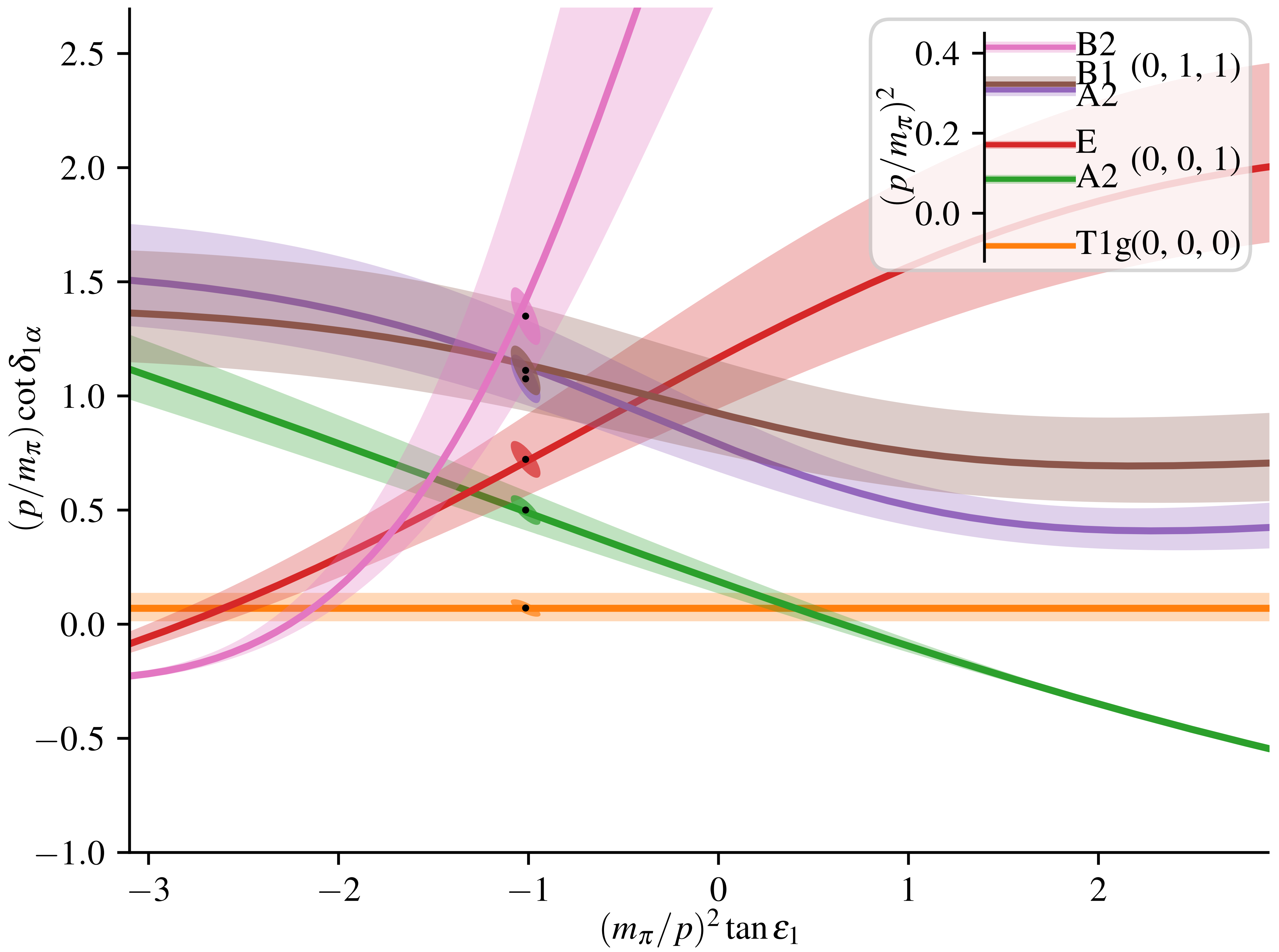
(good fit quality without terms describing lattice artefacts)

# Coupled partial waves

[Green, Hanlon, Junnarkar, HW (BaSc), arXiv:2212.09587]

$^3S_1 - {}^3D_1$ : Ansatz for $K$-matrix: Blatt-Biedenharn parameterisation

$$\tilde{K}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & p^2 \end{pmatrix} \begin{pmatrix} \cos\epsilon_1 & -\sin\epsilon_1 \\ \sin\epsilon_1 & \cos\epsilon_1 \end{pmatrix} \begin{pmatrix} p\cot\delta_{1\alpha} & 0 \\ 0 & p\cot\delta_{1\beta} \end{pmatrix} \begin{pmatrix} \cos\epsilon_1 & \sin\epsilon_1 \\ -\sin\epsilon_1 & \cos\epsilon_1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & p^2 \end{pmatrix}$$




$\epsilon_1 = 0 \quad \Leftrightarrow \quad \alpha \sim {}^3S_1 , \quad \beta \sim {}^3D_1$

$\delta_{1\beta} = 0$ : energy levels impose constraints

$$p\cot\delta_{1\alpha} = \frac{B_{00} + (B_{01} + B_{10})x + B_{11}x^2}{1 + p^4x^2}, \quad x = p^{-2}\tan\epsilon_1$$

Fit spectrum on N202 to

$$p\cot\delta_{1\alpha} = c_1 + c_2p^2, \quad p^{-2}\tan\epsilon_1 = c_3$$

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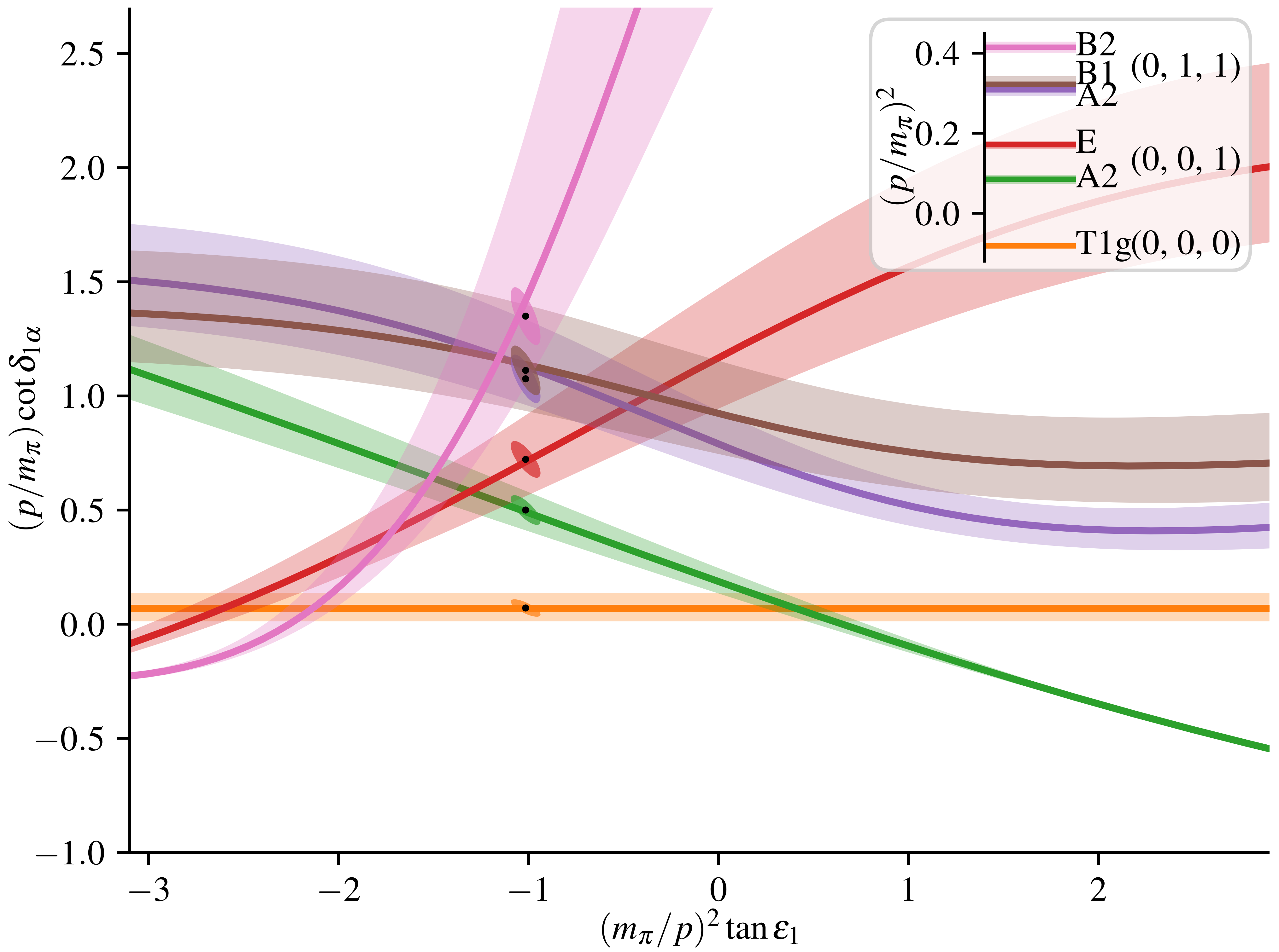


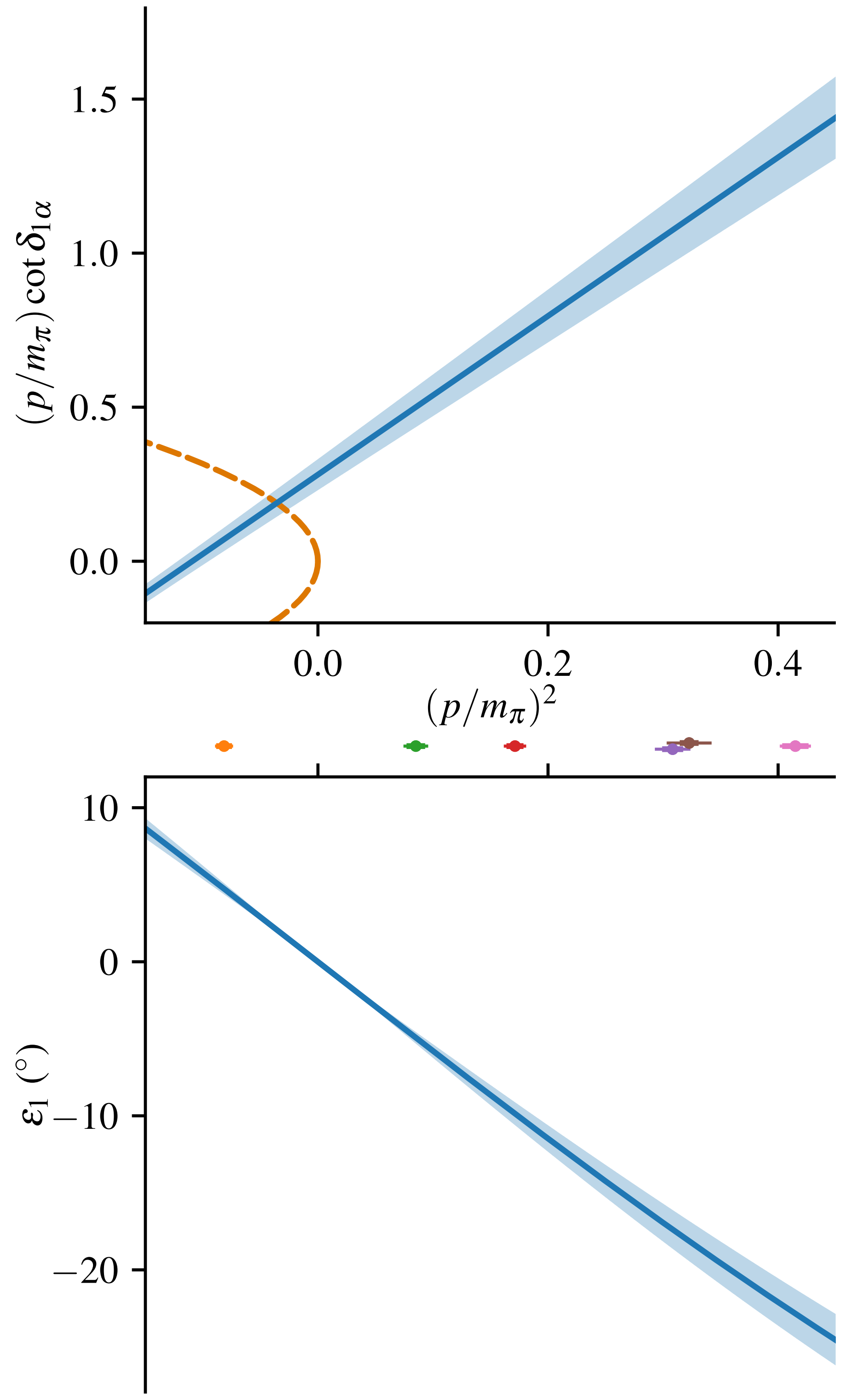


Fitted $p\cot\delta_{1\alpha}$ and $\epsilon_1$ versus $p^2$ :

Sign of $\epsilon_1$ opposite to experiment

# The *H* Dibaryon in 3-flavour QCD

Source positions on ensembles with SU(3) symmetry

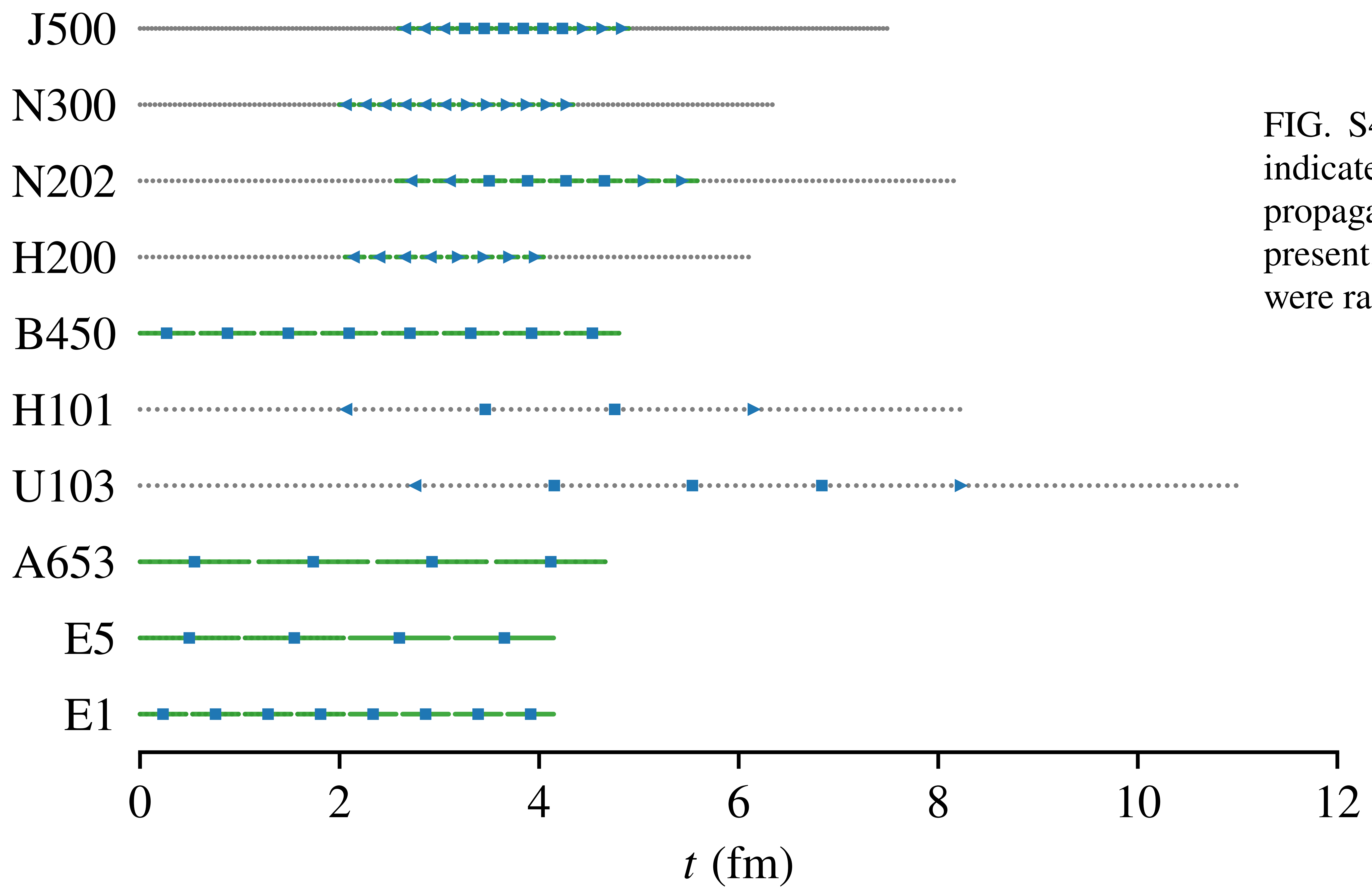


FIG. S4. Location of source times on all ensembles. Triangles indicate sources used for only forward-propgating or backward-propagating states, and squares indicate sources used for both. When present, green line segments indicate the range over which sources were randomly shifted on each gauge configuration.