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JÜLICH FORSCHUNGSZENTRUM

# The Proton Radius and its Relatives A.D. 2023

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- Summary and outlook

# Introductory remarks

# Facets of the nucleon

- Nucleons (neutrons, protons) are the building blocks of the matter surrounding us
- Various probes (electromagnetic, weak, ...) see different facets/scales

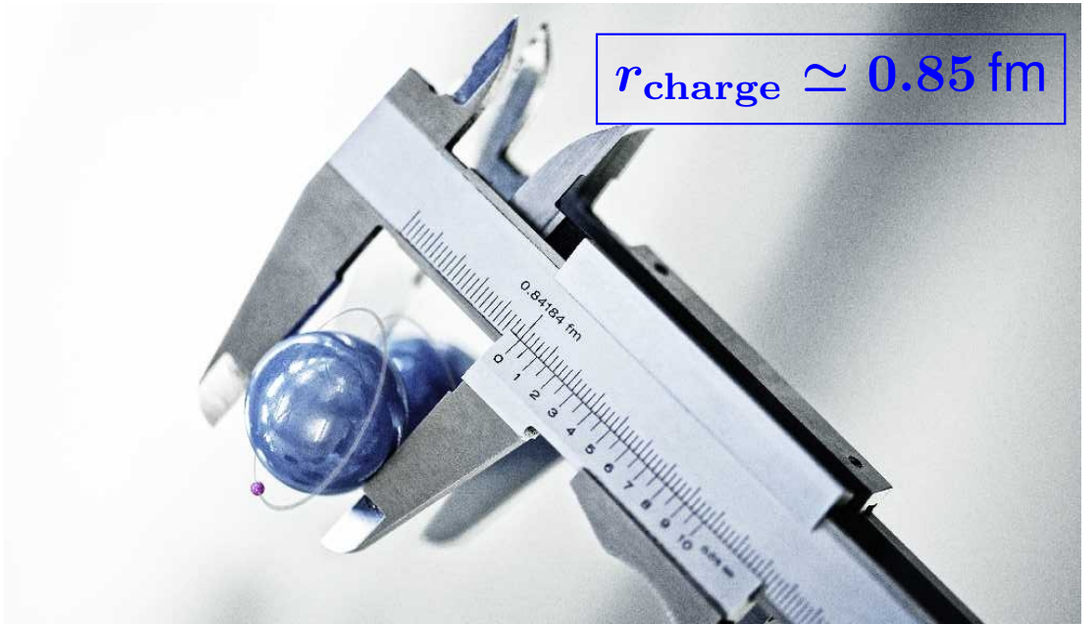



Fig. from Pohl et al., Nature 466 (2010) 213

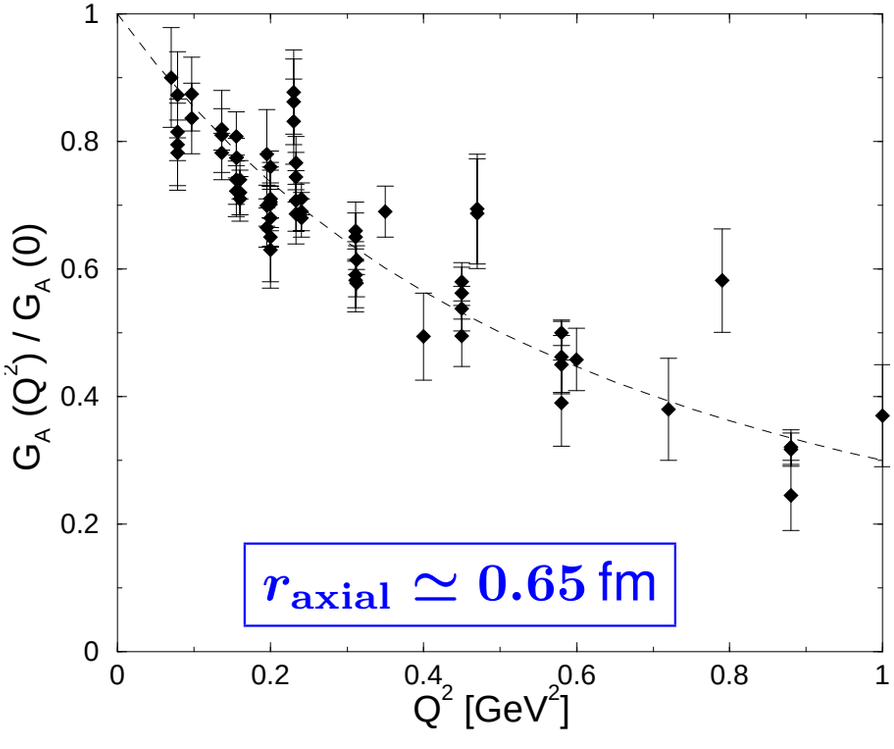


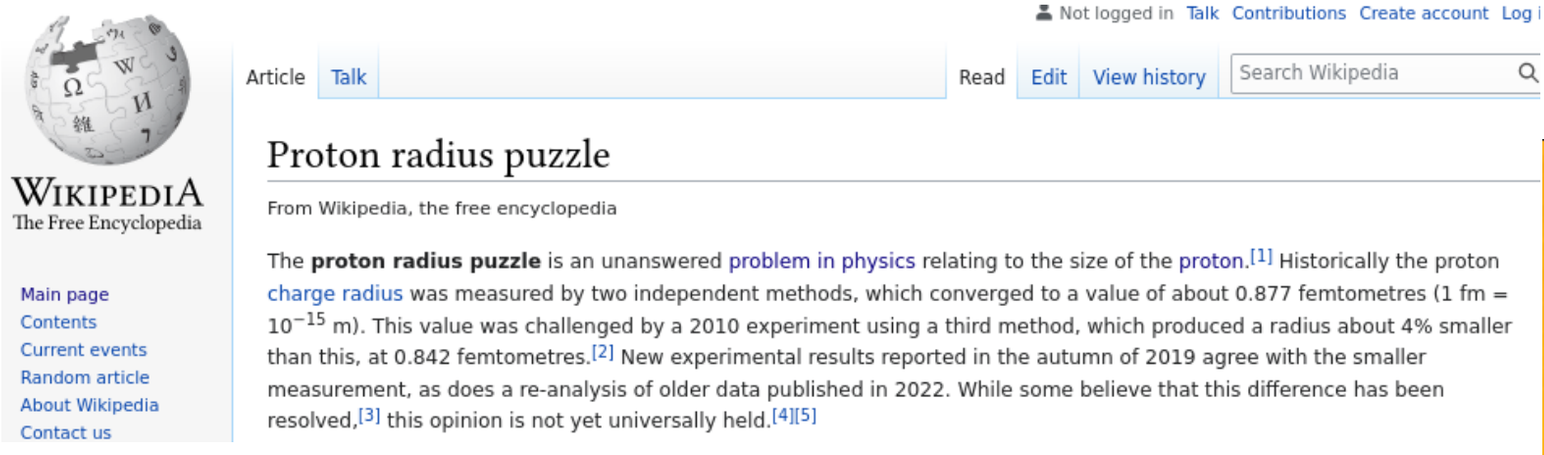

Fig. from Bernard, Elouadrhiri, UGM, J.Phys.G 28 (2002) R1

$\hookrightarrow$ learn more about the nucleon form factors & sizes

# The proton radius "puzzle"



- As I will show: Don't trust Wikipedia or popular media



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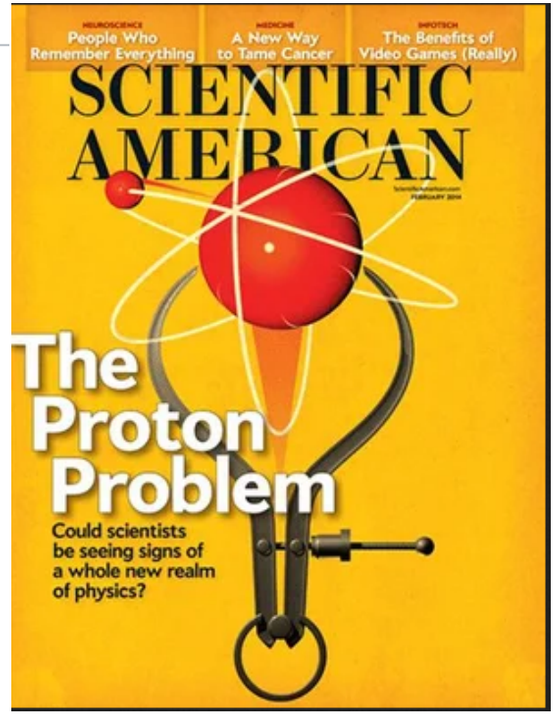
## Proton radius puzzle

From Wikipedia, the free encyclopedia

The **proton radius puzzle** is an unanswered problem in physics relating to the size of the proton.[1] Historically the proton charge radius was measured by two independent methods, which converged to a value of about 0.877 femtometres (1 fm = $10^{-15}$ m). This value was challenged by a 2010 experiment using a third method, which produced a radius about 4% smaller than this, at 0.842 femtometres.[2] New experimental results reported in the autumn of 2019 agree with the smaller measurement, as does a re-analysis of older data published in 2022. While some believe that this difference has been resolved,[3] this opinion is not yet universally held.[4][5]




- Or stated differently: It's all about precision

Science Bulletin 65 (2020) 257–258

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News & Views

The proton radius: from a puzzle to precision

Hans-Werner Hammer [a,b], Ulf-G. Meißner [c,d,e,*]

# Theoretical framework

Review: Lin, Hammer, UGM, Eur. Phys. J. **A 57** (2021) 255

# Basic definitions



- Nucleon matrix elements of the electromagnetic vector current $J_\mu = \bar{q}\mathcal{Q}\gamma_\mu q$

$$\boxed{\langle N(p')|J_\mu|N(p)\rangle = \bar{u}(p')\left[F_1(t)\,\gamma_\mu + i\,\frac{F_2(t)}{2m}\,\sigma_{\mu\nu}q^\nu\right]u(p)}$$

$\star$ $q = (u,d)^{\rm T}$ , $\quad \mathcal{Q} = e\,{\rm diag}(2/3,-1/3)$

$\star$ $F_1$ = Dirac form factor, $F_2$ = Pauli form factor

$\star$ four-momentum transfer $t \equiv q^2 = (p'-p)^2 \equiv -Q^2$

$\star$ isospin basis $F_i^S = (F_i^p + F_i^n)/2$ , $\quad F_i^V = (F_i^p - F_i^n)/2$

$\star$ Normalizations: $F_1^V(0) = F_1^S(0) = 1/2$, $F_2^{S,V}(0) = (\kappa_p \pm \kappa_n)/2$

$\star$ Sachs form factors: $G_E = F_1 + \frac{t}{4m^2}F_2$ , $\quad G_M = F_1 + F_2$

$\star$ Nucleon radii: $F(t) = F(0)\,\left[1 + t\langle r^2\rangle/6 + \ldots\right]$ [except for the neutron charge ff]

$\hookrightarrow$ analyze in a model-independent approach: Dispersion Relations

# Dispersion relations

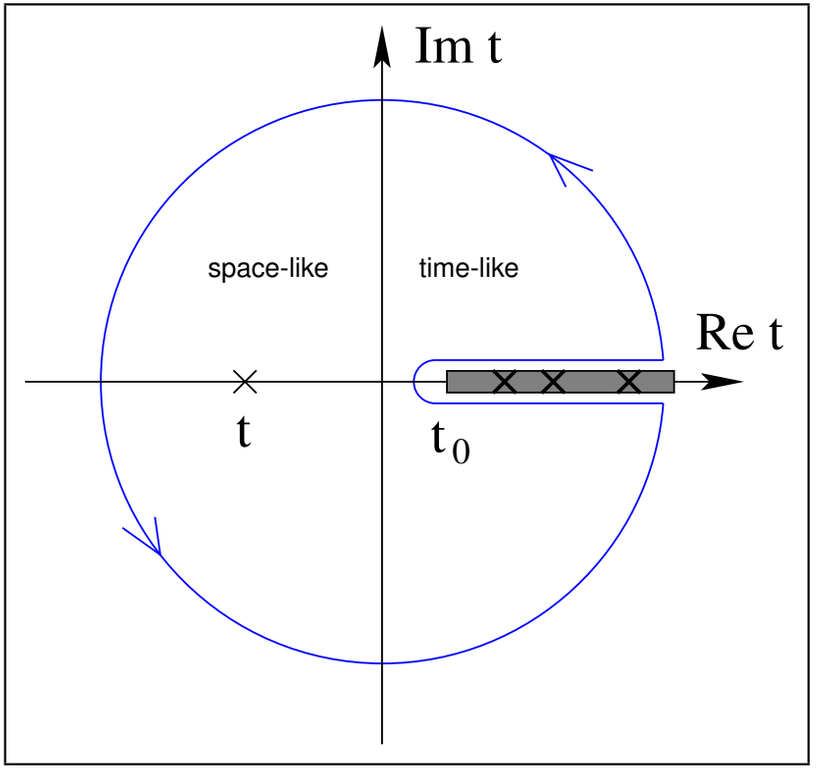


Federbush, Goldberger, Treiman, Drell, Zachariasen, Frazer, Fulco, Höhler, . . .

- The form factors have cuts in the interval $[t_n, \infty[$ $(n = 0, 1, 2, \ldots)$ and also poles

$\Rightarrow$ Dispersion relations for $F_i(t)$ $(i = 1, 2)$:

$$F_i(t) = \frac{1}{\pi} \int_{t_0}^{\infty} dt' \, \frac{\text{Im } F_i(t')}{t' - t}$$




- no subtractions
  [only proven in perturbation theory]
- suppression of higher mass states
- central objects: spectral functions

$$\boxed{\text{Im } F_i(t)}$$

– cuts $\triangleq$ multi-meson continua
– poles $\triangleq$ vector mesons

} sometimes intertwined!

# Isovector spectral function

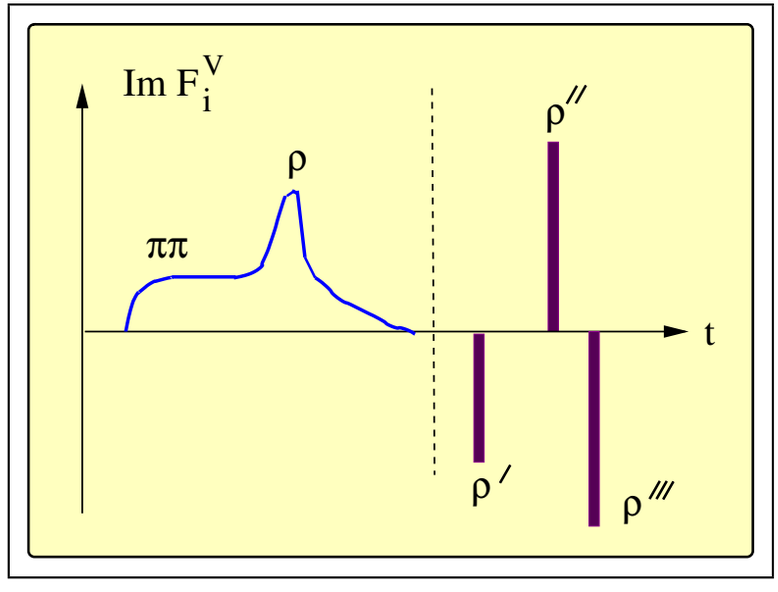


Frazer, Fulco, Höhler, Pietarinen, . . .

- exact $2\pi$ continuum is known from threshold $t_0 = 4M_\pi^2$ to $t \simeq 40\, M_\pi^2$

$$\boxed{\mathrm{Im}\, G^V_{E,M}(t) = \frac{q_t^3}{\sqrt{t}} F_\pi(t)^\star f^1_\pm(t)}$$

  - $\star$ $F_\pi(t)$ = pion vector form factor
  - $\star$ $f^1_\pm(t)$ = P-wave pion-nucleon partial waves in the t-channel




- Spectral functions inherit singularity on the second Riemann sheet in $\pi N \to \pi N$

$$\boxed{t_c = 4M_\pi^2 - M_\pi^4/m^2 \simeq 3.98\, M_\pi^2} \quad \to \text{strong shoulder} \to \text{isovector radii}$$

- For a recent determination of the $2\pi$ continuum, see HKRHM, EPJA **52** (2016) 331

  $\hookrightarrow$ based on a precise RS-equation analysis

  Hoferichter, de Elvira, Kubis, UGM, Phys. Rept. **625** (2016) 1

- Higher mass states represented by poles (not necessarily physical masses)

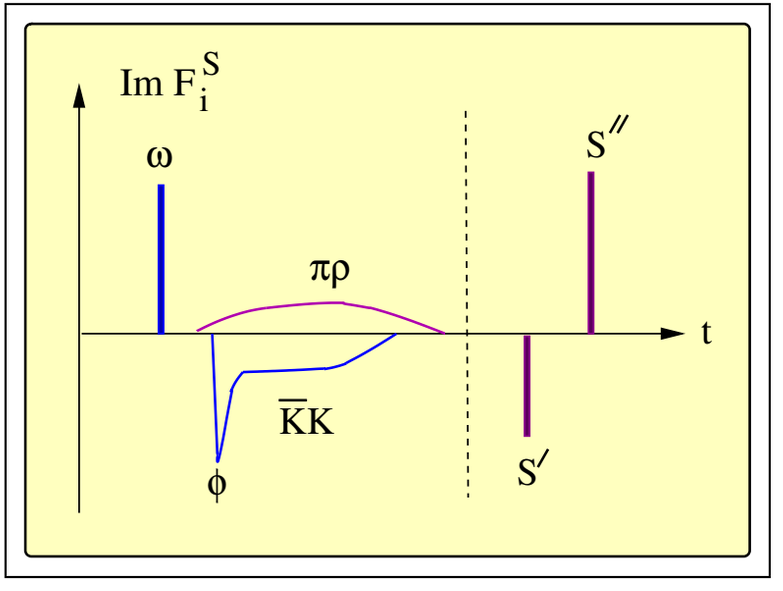
# Isoscalar spectral function

- $K\bar{K}$ continuum can be extracted from analytically cont. $KN$ scattering amplitudes

  $\rightarrow$ analytic continuation must be stabilized

  $\rightarrow$ generates most of the $\phi$ contribution

  Hammer, Ramsey-Musolf, Phys. Rev. C **60** (1999) 045204, 045205




- Further strength in the $\phi$-region generated by correlated $\pi\rho$ exchange

  $\rightarrow$ strong cancellations ($K\bar{K}, K^*K, \pi\rho$)

  $\rightarrow$ takes away sizeable strength from the $\phi$

  UGM, Mull, Speth, van Orden, Phys. Lett. B **408** (1997) 381

- Spectral functions exhibit anomalous threshold (analyzed in 2-loop CHPT)

$$t_c = M_\pi^2 \left(\sqrt{4 - M_\pi^2/m^2} + \sqrt{1 - M_\pi^2/m^2}\right)^2 \simeq 8.9\, M_\pi^2$$

$\rightarrow$ effectively masked

Bernard, Kaiser, M, Nucl. Phys. A **611** (1996) 429

- Higher mass states represented by poles (with a finite width)

# Constraints on the spectral functions

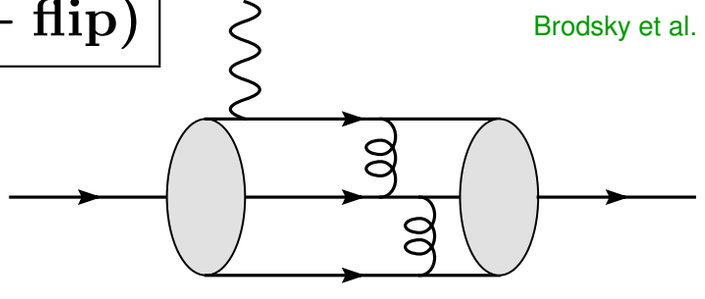
- Normalizations: electric charges, magnetic moments
- Radii *not* imposed [except for the neutron charge radius]
- Superconvergence relations $\cong$ leading pQCD behaviour

Filin et al., Phys. Rev. C **103** (2021) 024313

$$\langle r_n^2 \rangle = -0.105^{+0.005}_{-0.006}\ \mathrm{fm}^2$$

$$\boxed{F_1(t) \sim 1/t^2\ , F_2(t) \sim 1/t^3\ \ (\mathbf{helicity-flip})}$$

Brodsky et al.



$$\Rightarrow \boxed{\int_{t_0}^{\infty} \mathrm{Im}\, F_1(t)\, dt = 0\ , \quad \int_{t_0}^{\infty} \mathrm{Im}\, F_2(t)\, dt = \int_{t_0}^{\infty} \mathrm{Im}\, F_2(t)\, t\, dt = 0}$$

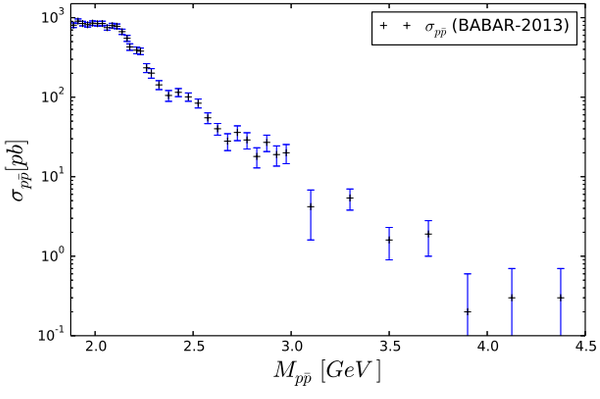
$\Rightarrow$ severely restricts the number of fit parameters

# Form factors in the time-like region

- Xsection for $e^+e^- \leftrightarrow \bar{p}p, \bar{n}n$ in the one-photon approximation

$$\sigma(s) = \frac{4\pi\alpha_{\rm EM}^2\beta}{3s}\, C(s)\, \left[|G_M(s)|^2 + \frac{2m_N^2}{s}\,|G_E(s)|^2\right], \quad \beta = \frac{k_p}{k_e}$$

$$= \frac{4\pi\alpha_{\rm EM}^2\beta}{3s}\, C(s)\, \left(1 + \frac{2m_N^2}{s}\right)\, |G_{\rm eff}|^2$$



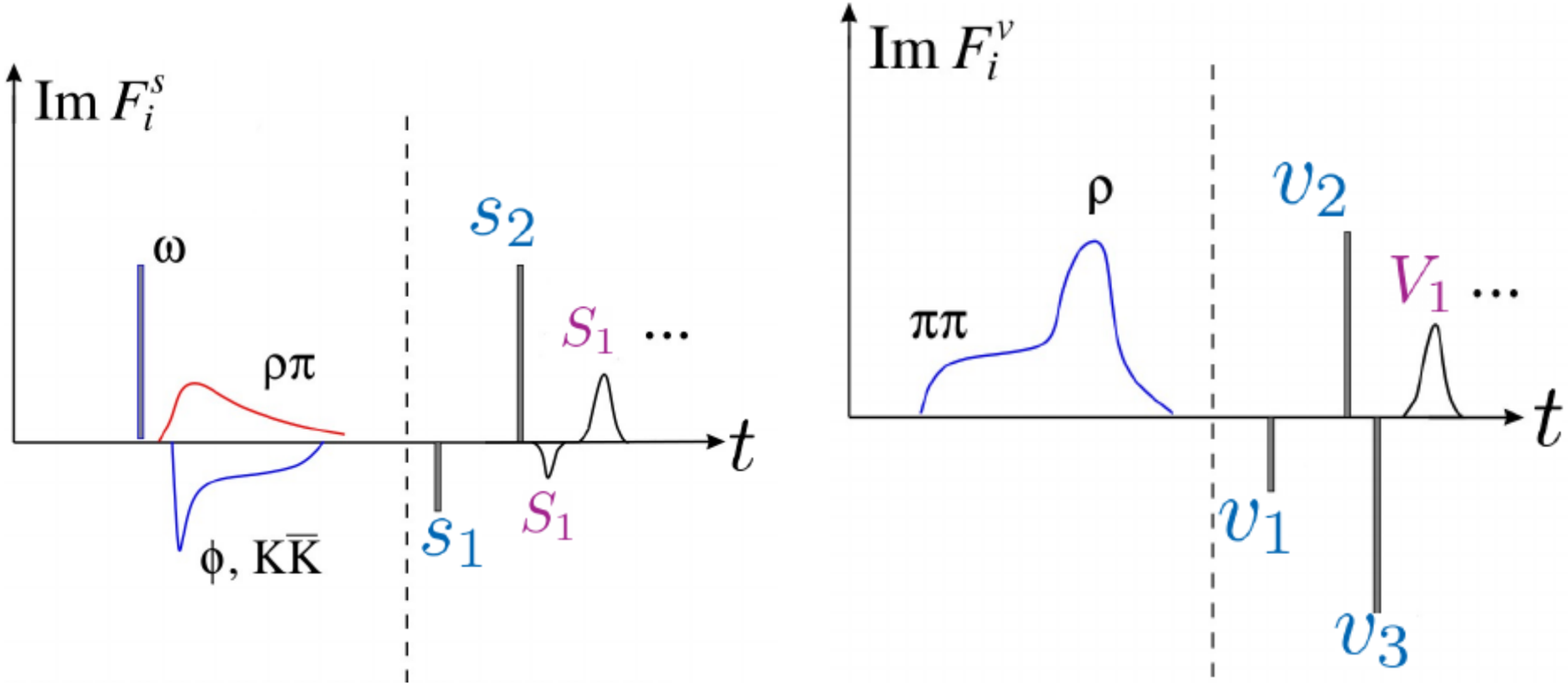

- $G_{E,M}(s)$ are complex for $s \geq 4m_N^2$
- Threshold constraint: $G_E(4m_N^2) = G_M(4m_N^2)$
- Gamov-Sommerfeld factor (only for the proton):

$$C = \frac{y}{1 - e^{-y}}, \quad y = \frac{\pi\alpha_{\rm EM} m_p}{k_p}, \quad \sqrt{s} = 2\sqrt{m_p^2 + k_p^2}$$

- Data from $e^+e^- \to \bar{N}N$ & $\bar{N}N \to e^+e^-$: strong threshold enhancement & oscillations

# Summary: spectral functions

- Cartoons of the isoscalar/isovector spectral functions:



# Fit procedure & theoretical uncertainties

Review: Lin, Hammer, UGM, Eur. Phys. J. **A 57** (2021) 255

# Data basis



- Data basis:

| Region | Observables | Source | $\lvert t\rvert$ GeV$^2$ | number |
|---|---|---|---|---|
| Spacelike($t<0$) | $d\sigma/d\Omega$ | MAMI | 0.00384–0.977 | 1422 |
| | | PRad | 0.000215–0.058 | 71 |
| | $\mu_p G_E^p/G_M^p$ | JLab | 1.18–8.49 | 16 |
| | $\mu_n G_E^n/G_M^n$ | world | 1.58–3.41 | 4 |
| | $G_E^n$ | world | 0.14–3.41 | 29 |
| | $G_M^n$ | world | 0.071–10.0 | 49 |
| Timelike($t>0$) | $\lvert G_{\rm eff}^p\rvert$ | world | 3.52–20.25 | 153 |
| | $\lvert G_{\rm eff}^n\rvert$ | world | 3.53–9.49 | 32 |
| | $\lvert G_E^p/G_M^p\rvert$ | BaBar | 3.52–9.0 | 6 |
| | $d\sigma/d\Omega$ | BESIII | 3.52–3.80 | 10 |

- Number of data/fit parameters:

$$\#_{\rm data} = 1792\,, \quad \#_{\rm fitpara.} = \underbrace{4}_{\omega+\phi} + 3\underbrace{(N_s+N_v)}_{best\ fit:\ 3+5} + 4\underbrace{(N_S+N_V)}_{best\ fit:\ 3+3} - 11 + \underbrace{31+2}_{\rm norm.}$$

# Fit procedure & uncertainty quantification



- Fit strategy (weigthed average): $\chi^2_{\rm total} = \sum_{i \text{ in data basis}} \frac{\chi^2_i}{\#\text{ data points}}$

  $\hookrightarrow$ use covariance matrix whenever available

- Systematic and statistical uncertainties, only recently fully incorporated

- Systematics: Vary the number of poles around the best solution
  so that $\delta\chi^2_{\rm total} \leq 1\%$ Höhler et al. (1976)

- Statistical errors, use

  – bootstrap method Efron, Tibshirani (1986)

  – Bayesian analysis Bayes (1763)

  $\Rightarrow$ both give the same result, bootstrap simpler to implement

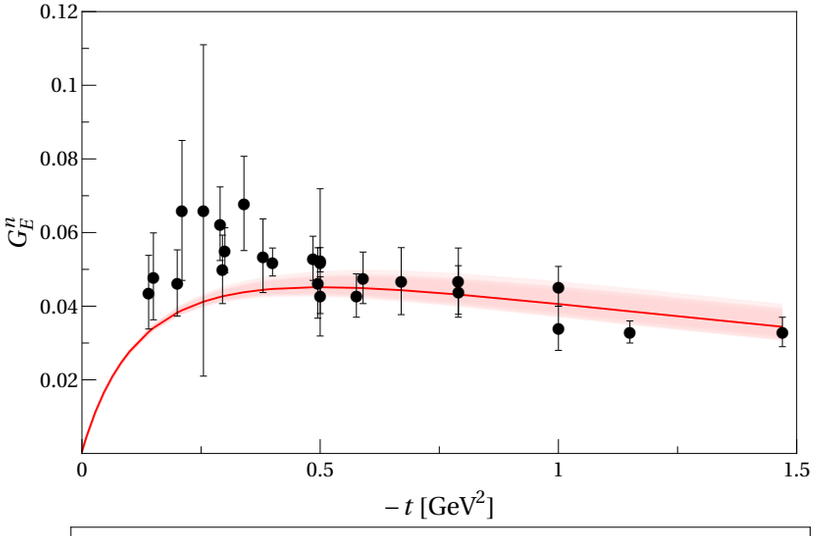
# Results

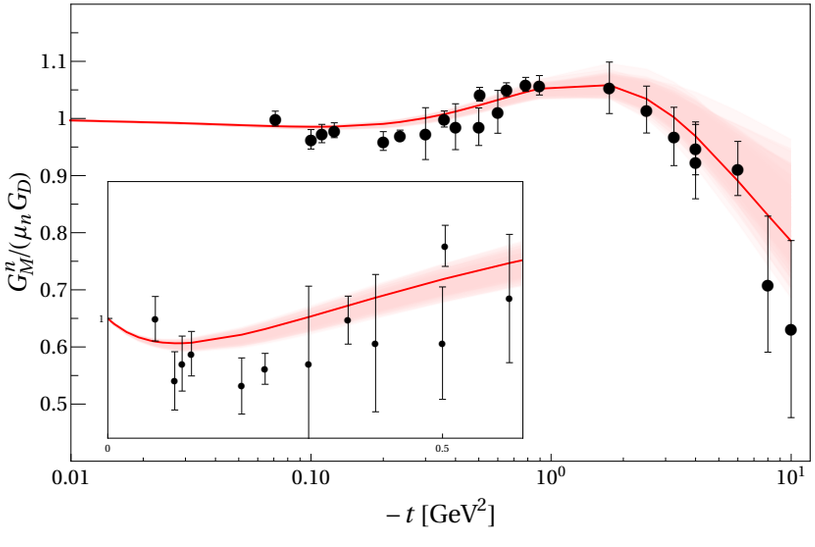
Lin, Hammer, UGM, Phys. Rev. Lett. **128** (2022) 5, 052002

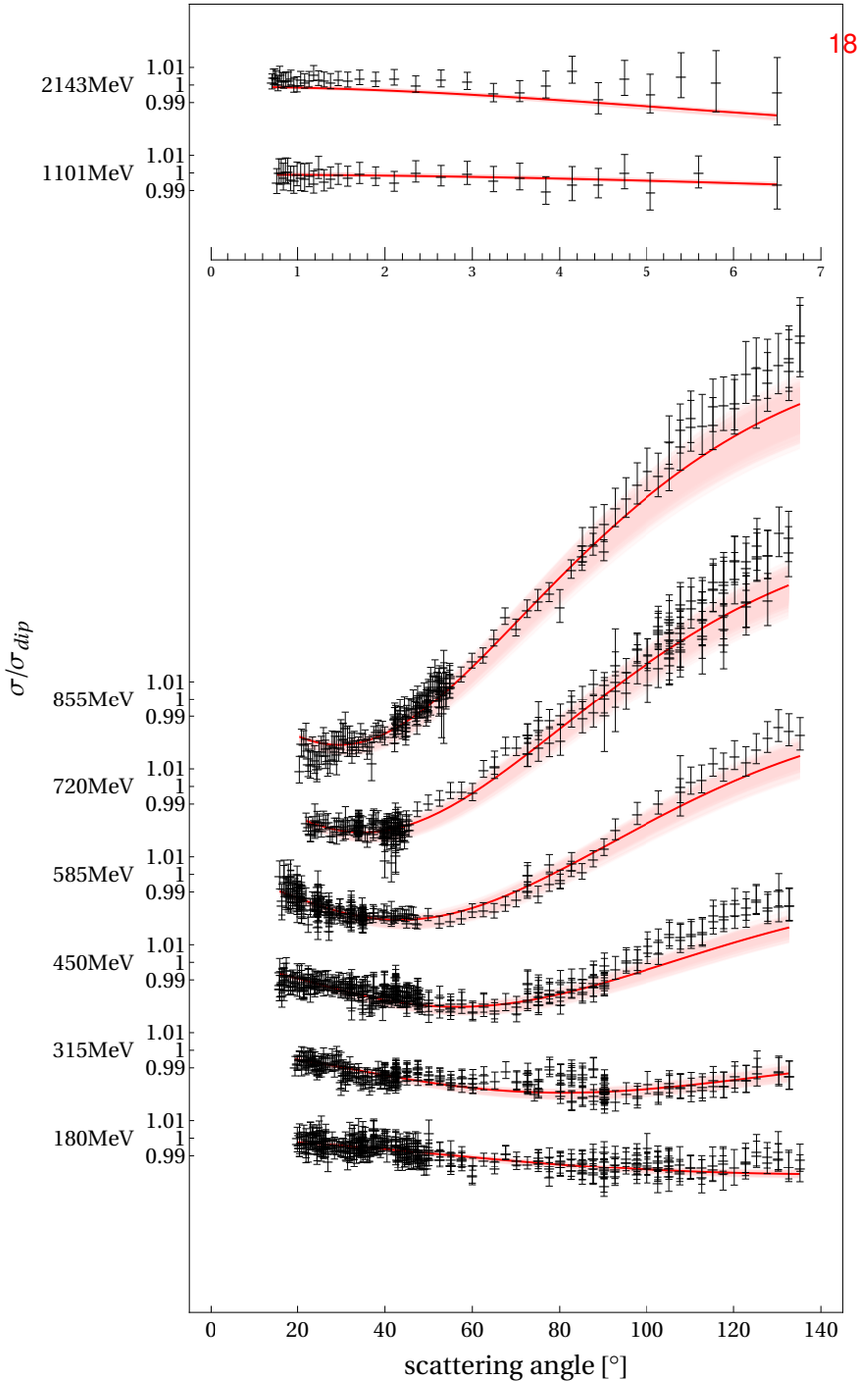
# Space-like results I



- $ep$ scattering data and neutron ffs:
  [error bands from bootstrap, $\chi^2$/dof=1.223]



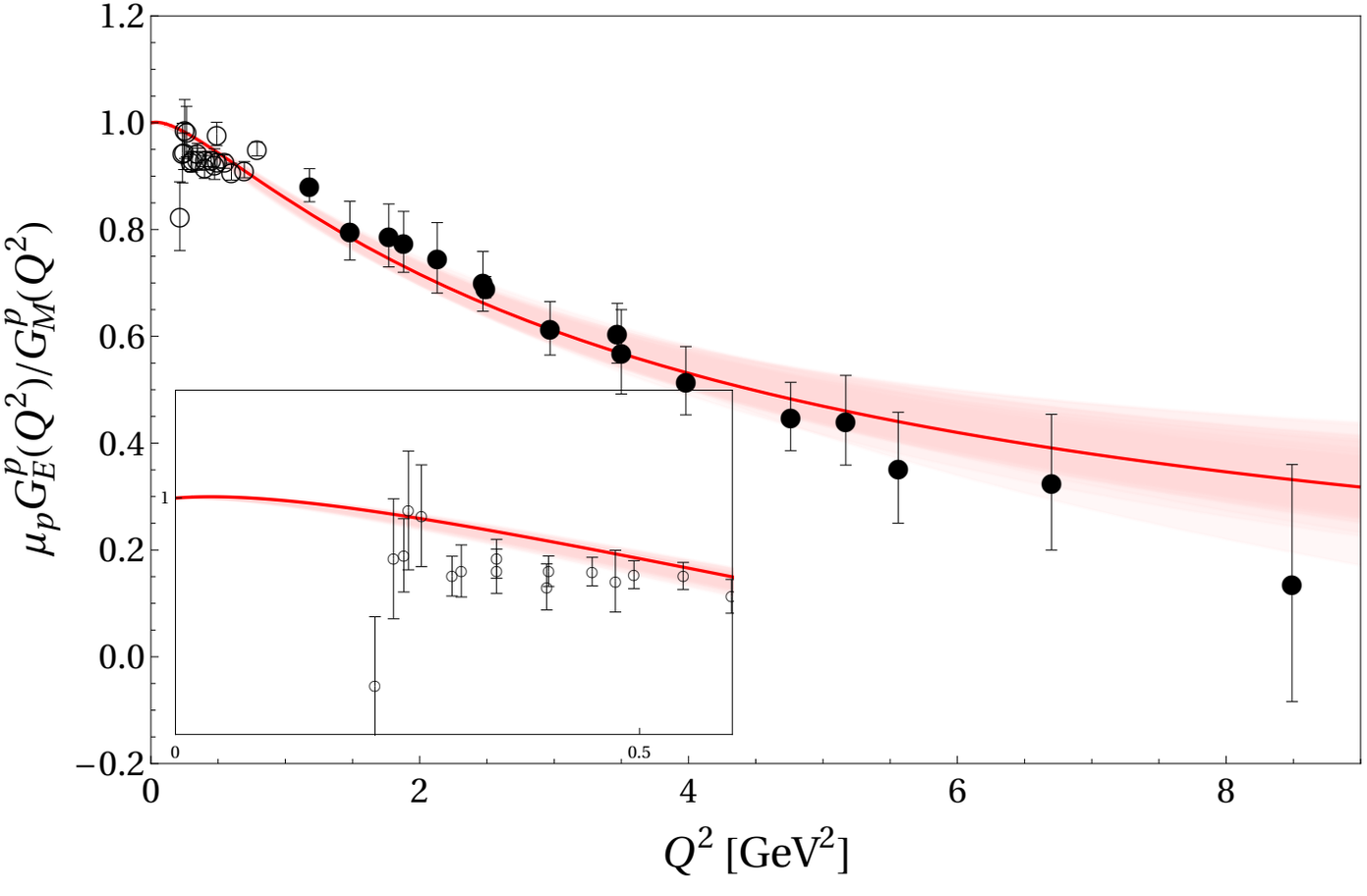



# Space-like results II



- Proton form factor ratio [Jlab data]:

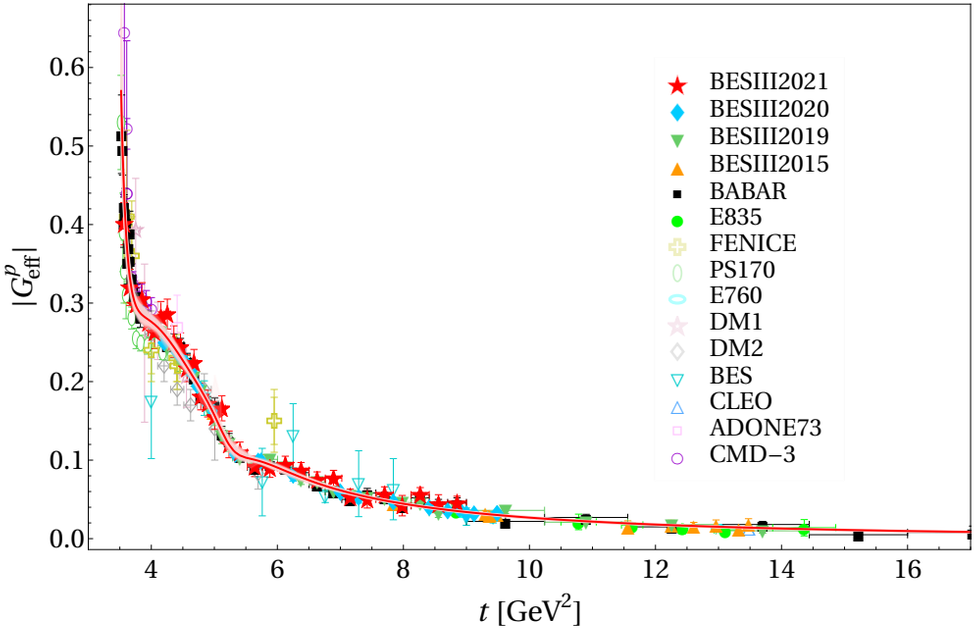


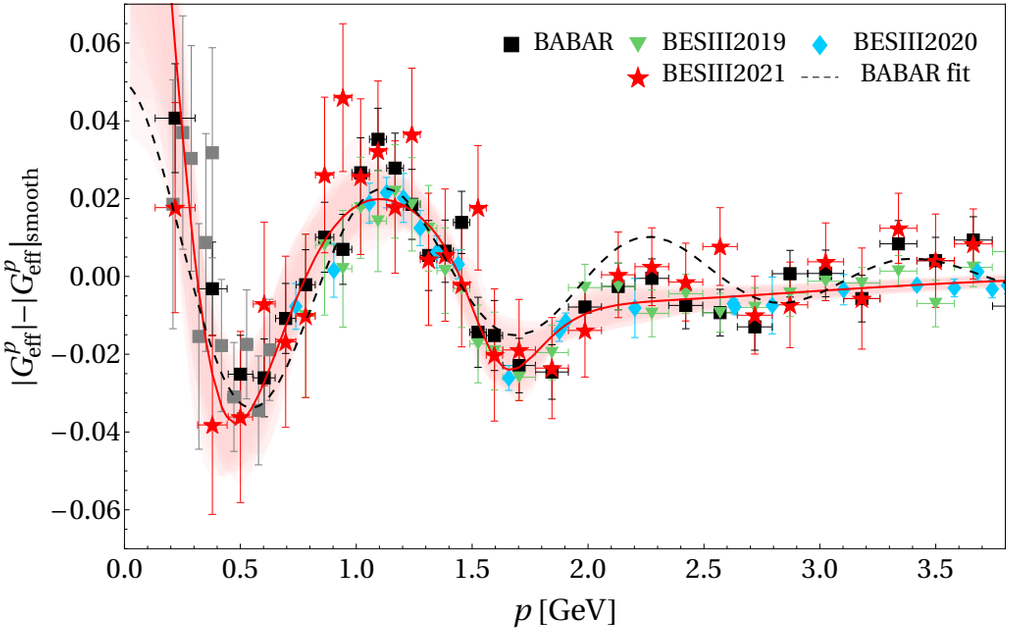

  - empty symbols: not fitted
  - zero crossing disfavored

# Time-like results I



- Proton effective form factor:




open symbols not fitted

$$|G^p_{\rm eff}|_{\rm smooth} = \frac{7.7}{(1+t/14.8)(1-t/0.71)^2}$$

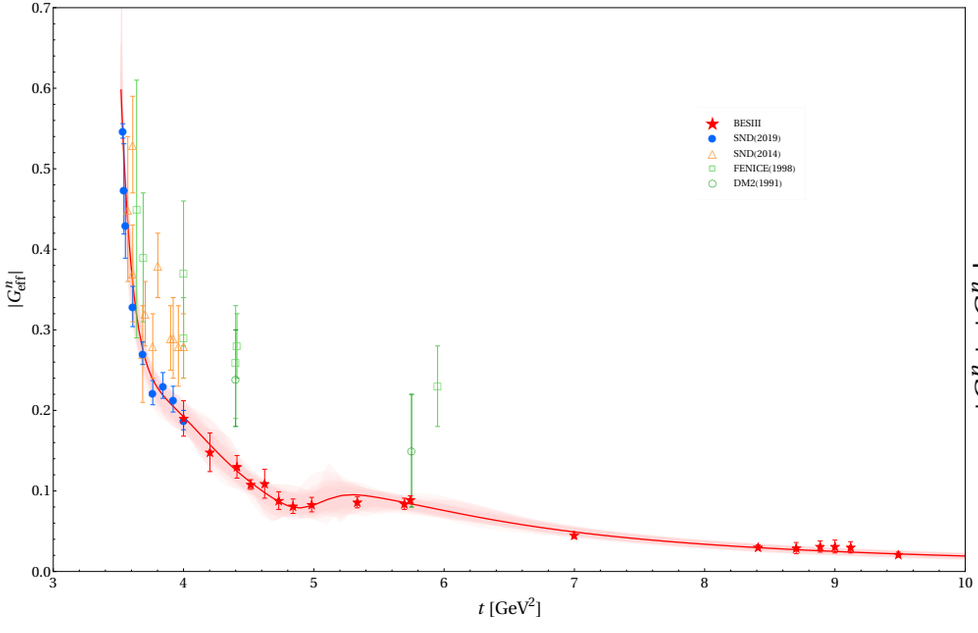
BaBar fit formula:

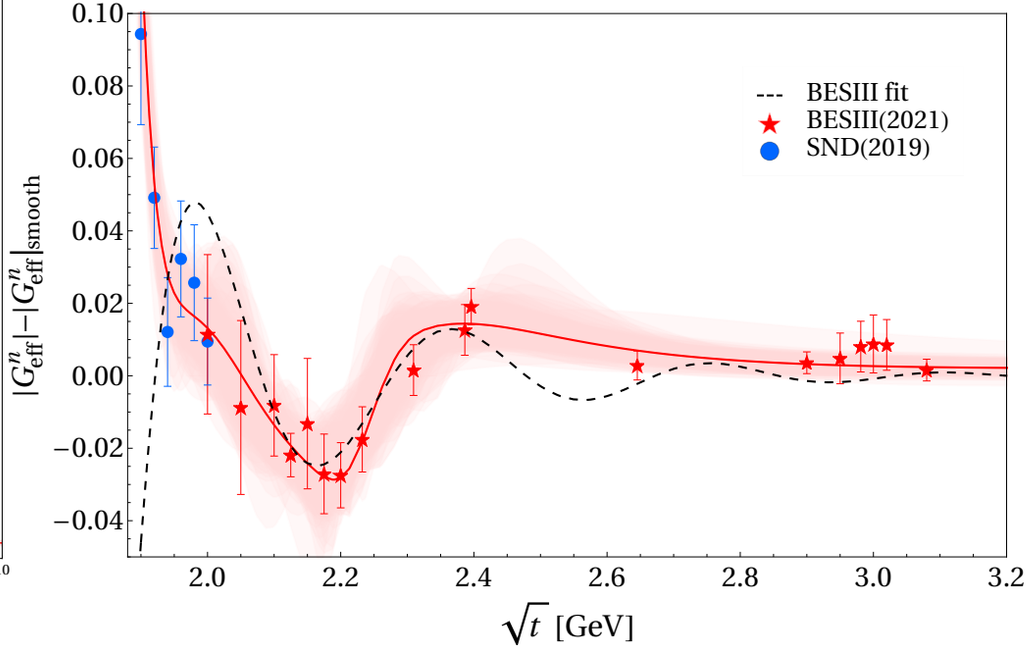
$$F_p = A^{\rm osc}\exp(-B^{\rm osc}p) \times \cos(C^{\rm osc}p + D^{\rm osc})$$

# Time-like results II



- Neutron effective form factor:




open symbols not fitted

$$|G_{\rm eff}^n|_{\rm smooth} = \frac{4.87}{(1+t/14.8)(1-t/0.71)^2}$$

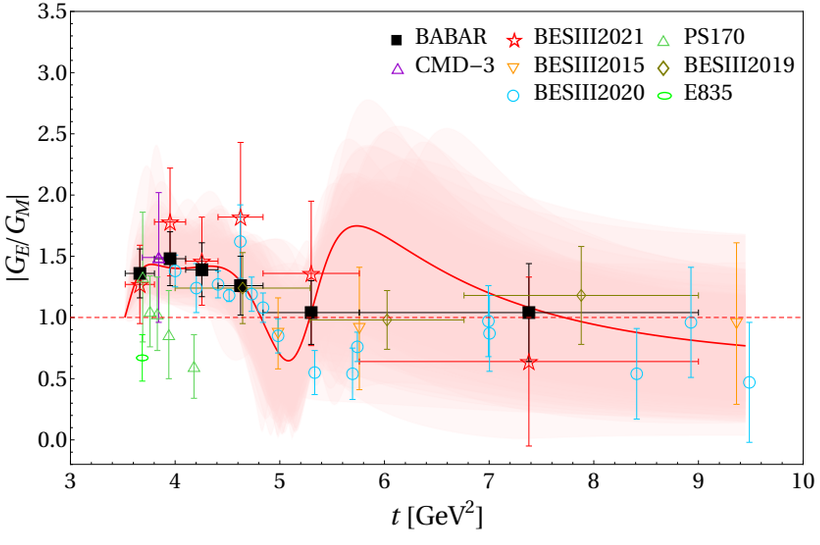
BESIII fit formula:

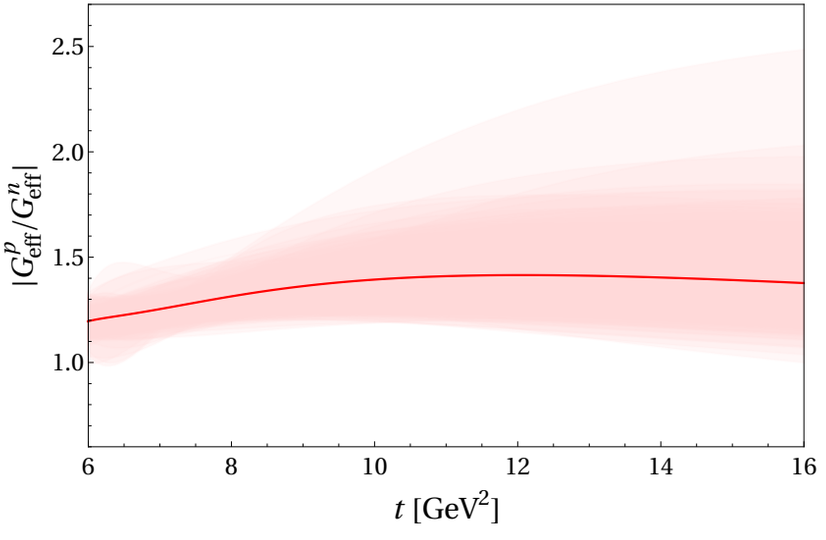
$$F_p = A^{\rm osc} \exp(-B^{\rm osc} p) \times \cos(C^{\rm osc} p + D^{\rm osc})$$

# Time-like results III

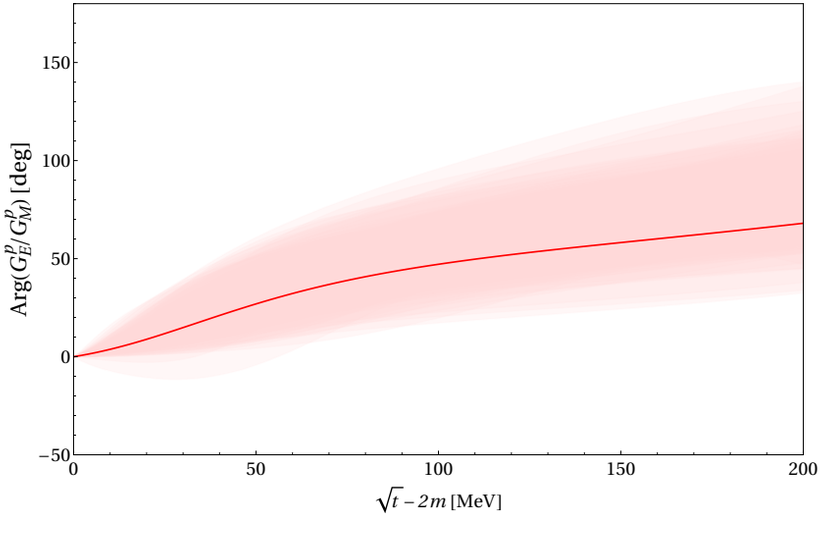


- More fits and predictions [pQCD & phases]




open symbols not fitted



# The proton radius and its relatives

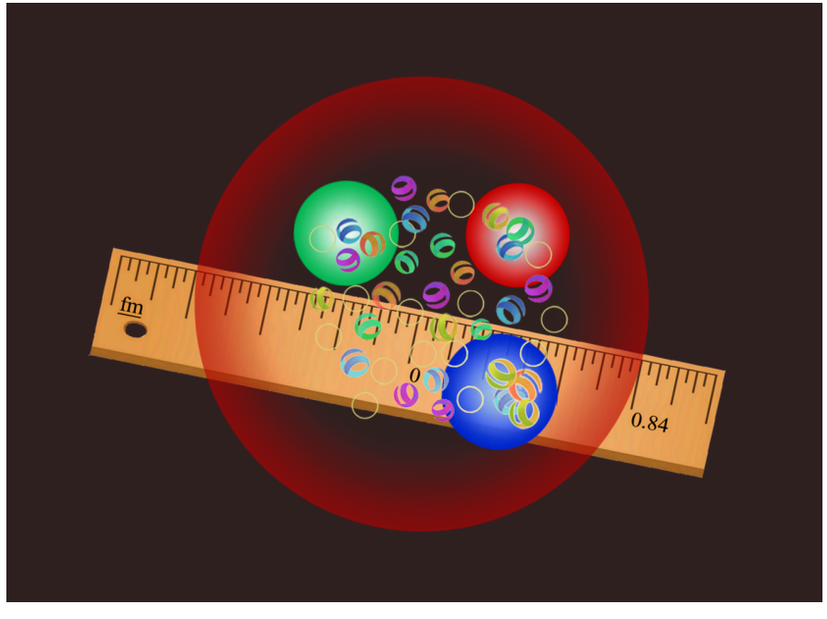



Fig. courtesy Yong-Hui Lin

# Proton charge radius



- Definition: $\boxed{r_p^2 \equiv -6\, G_E'(0)}$ [not discussing charge distribution here!]

- Measurements:

  – Leptonic hydrogen Lamb shift (LS) [in principle 2 numbers: $r_p$ & $R_\infty$]

$$\Delta E_{LS} = \Delta E_1 + \Delta E_2 C(r_p^2) + \mathcal{O}(m_{\rm red}\alpha_{\rm EM}^2)$$

$$C(r_p^2) = c_1 + c_2 r_p^2 + \mathcal{O}(m_{\rm red}\alpha_{\rm EM}^2)$$

  – Lepton-proton scattering (Rosenbluth separation)

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_{\rm Mott}}{d\Omega}\frac{1}{1+\tau}\left(G_E^2 + \frac{\tau}{\varepsilon}G_M^2\right)(1+\delta_{\rm rad.}) + \mathcal{O}(m_{\rm red}\alpha_{\rm EM}^2)$$

- The neglected sibling, the proton magnetic radius:

$$(r_p^M)^2 \equiv -(6/\mu_p)\, G_M'(0)$$

# Proton charge & magnetic radius

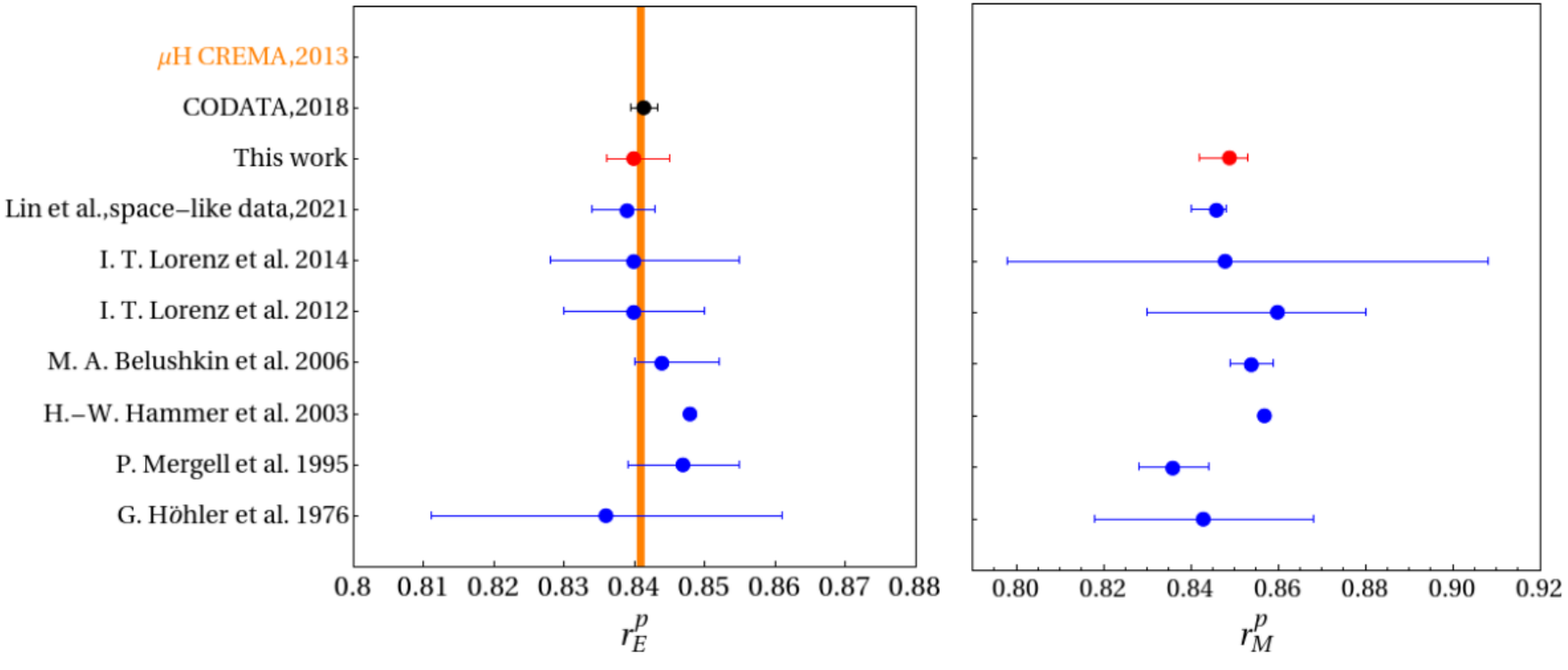


Lin, Hammer, UGM, Phys. Lett. B **816** (2021) 136254 [2102.11642 [hep-ph]]
Phys. Rev. Lett. **128** (2022) 052002 [2109.12961 [hep-ph]]

- Our determination incl. statistical and systematic errors:

$$r_E^p = 0.840^{+0.003+0.002}_{-0.002-0.002}\ \text{fm}\ , \quad r_M^p = 0.849^{+0.003+0.001}_{-0.003-0.004}\ \text{fm}$$

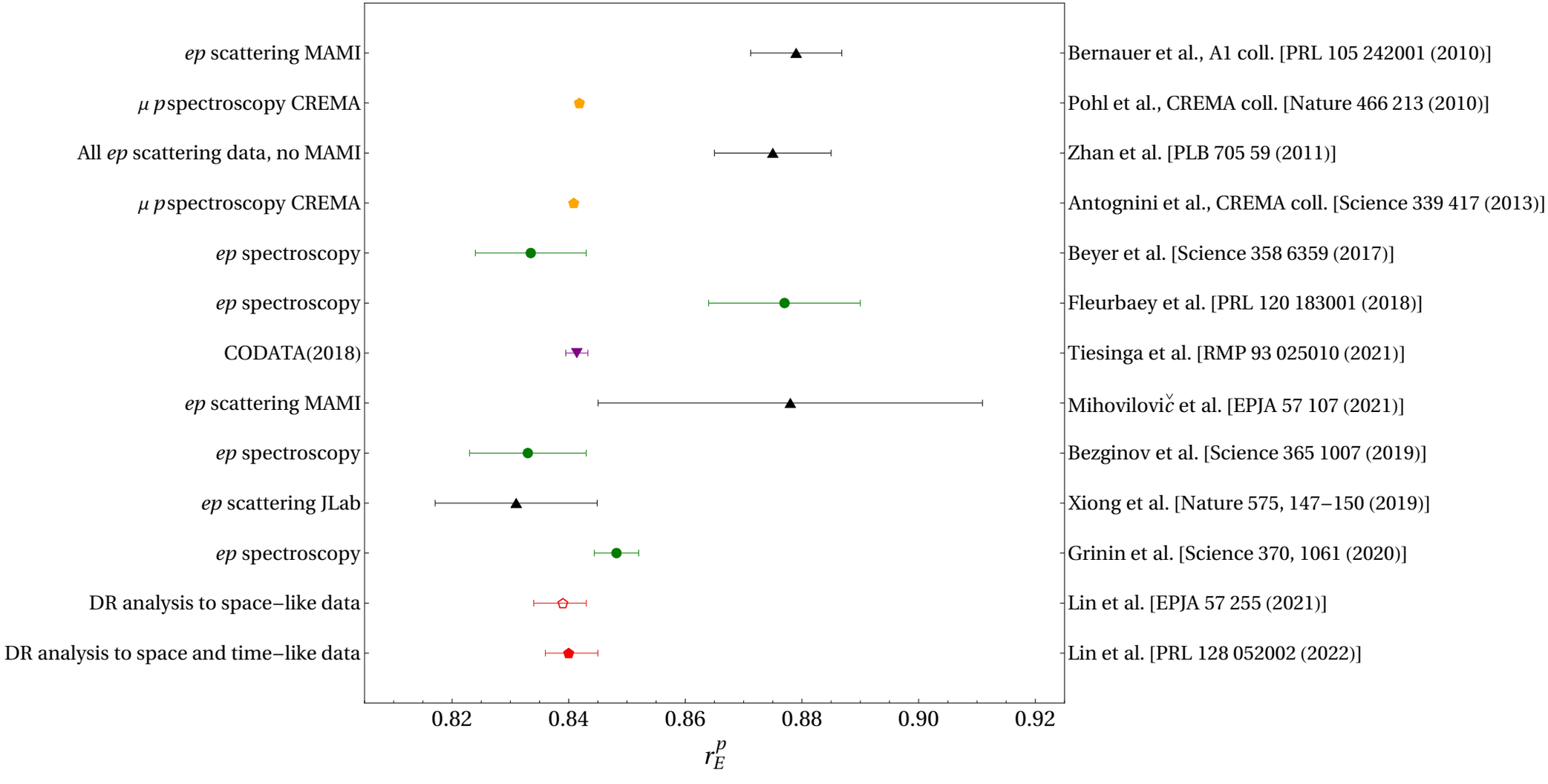
- Comparison to earlier DR determinations (and some data)



# Proton charge radius cont'd



- Comparison to recent measurements:



# Neutron radii

- The charge squared neutron $(r_E^n)^2$ radius was mostly input in DR analyses, but not the magnetic one

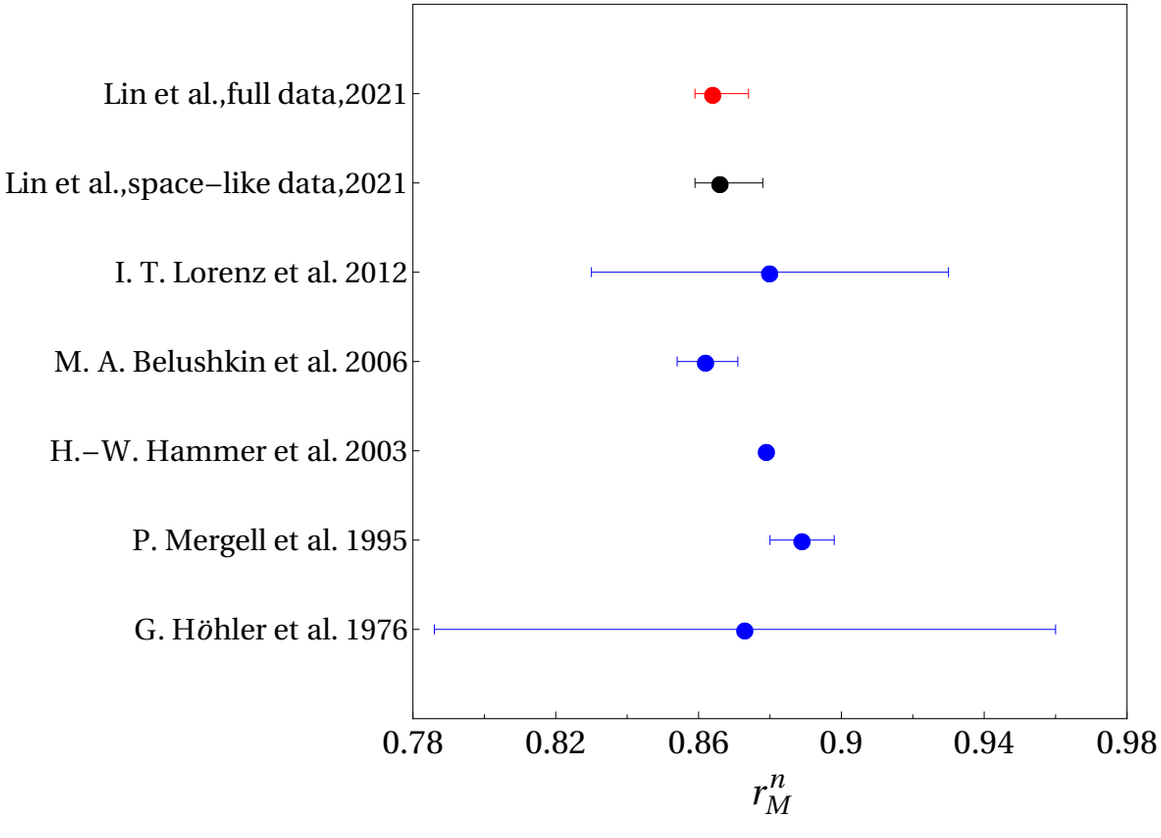
$$r_M^n = 0.864^{+0.004+0.006}_{-0.004-0.001}\ \text{fm}$$

↪ rather stable over time

↪ but larger variation

↪ always the largest em radius!

↪ lattice QCD gives rather comparable isovector radii (p& n)



# Comparison with lattice QCD



- Compare isovector radii, these are free of disconnect diagrams
- Show only calculations at the physical pion mass

| | $r_E^V$ [fm] | $r_M^V$ [fm] |
|---|---|---|
| Disp. rel. | 0.900(2)(2) | 0.854(1)(3) |
| Lattice/Mainz (new) [0] | 0.882(12)(15) | 0.814(7)(5) |
| Lattice/Cyprus [1] | 0.920(19)(–) | 0.742(27)(–) |
| Lattice/Mainz [2] | 0.894(14)(12) | 0.813(18)(7) |
| Lattice/ETMC [3] | 0.827(47)(5) | —- |
| Lattice/PACS [4] | 0.785(17)(21) | 0.758(33)(286) |
| Lattice/MIT [5] | 0.787(87) | —- |

[0] D. Djukanovic et al., (values PRELIMINARY)

[1] C. Alexandrou et al., in preparation (values PRELIMINARY)

[2] D. Djukanovic et al., Phys. Rev. D **103** (2021) 094522 [2102.07460 [hep-lat]].

[3] C. Alexandrou et al., Phys. Rev. D **101** (2020) 114504 [2002.06984 [hep-lat]]

[4] E. Shintani et al., Phys. Rev. D **99** (2019) 014510 [E] Phys. Rev. D **102** (2020) 019902

[5] N. Hasan et al., Phys. Rev. D **97** (2018) 034504 [1711.11385 [hep-lat]]

# A closer look at the hyperfine splitting



Antognini, Hagelstein, Pascalutsa, Ann.Rev.Nucl.Part.Sci. **72** (2022) 389

- Hyperfine splitting in electronic and muonic hydrogen sensitive to:
  - ⋆ the Zemach correction $\Delta_Z$

$$\Delta_Z = -2Z\alpha m_r\, r_Z\,, \quad r_Z = -\frac{4}{\pi}\int_0^\infty \frac{\mathrm{d}Q}{Q^2}\left[\frac{G_E(Q^2)G_M(Q^2)}{1+\kappa} - 1\right]$$

  - ⋆ the recoil correction $\Delta_R$

$$\Delta_R = \frac{Z\alpha}{\pi(1+\kappa_p)}\int_0^\infty \frac{\mathrm{d}Q}{Q}\left\{\frac{G_M(Q^2)}{Q^2}\frac{8m_\mu M_p}{v_l+v}\left(2F_1(Q^2) + \frac{F_1(Q^2)+3F_2(Q^2)}{(v_l+1)(v+1)}\right)\right.$$

$$\left. - \frac{8m_r G_M(Q^2)G_E(Q^2)}{Q} - \frac{m_\mu F_2^2(Q^2)}{M}\frac{5+4v_l}{(1+v_l)^2}\right\}$$

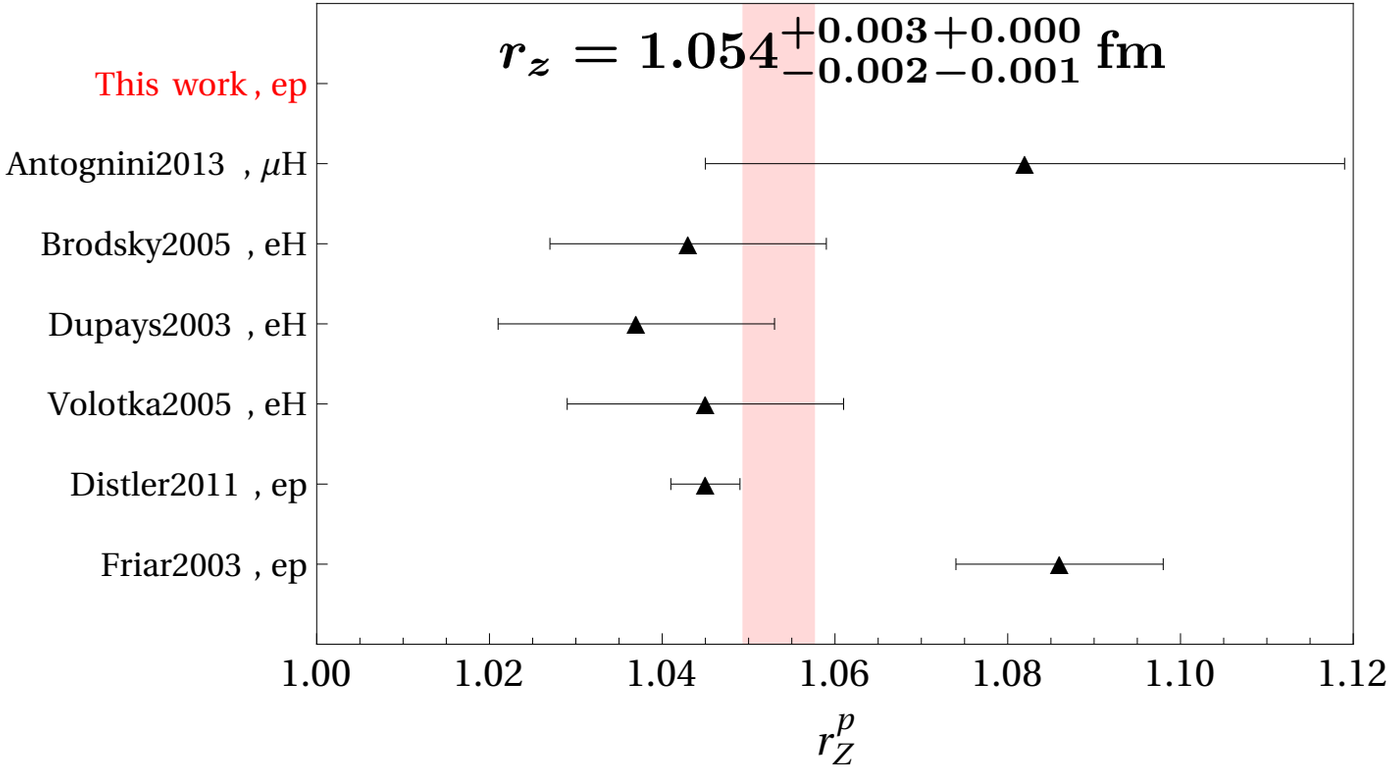
$$v = \sqrt{1+4M_p^2/Q^2}\,, \qquad v_l = \sqrt{1+4m_\mu^2/Q^2}$$

# Zemach radius & recoil correction



- Our determination of $r_Z$ incl. statistical and systematic errors:

Antognini, Lin, UGM. Phys. Lett. B **835** (2022) 137575 [2208.04025 [nucl-th]]




- and the results for $\Delta_R$ (slightly improved compard to Tomalak 2017)

$$\Delta_R^{\mu\mathrm{p}} = (837.6^{+1.7+2.2}_{-1.0-0.1}) \times 10^{-6} = (837.6^{+2.8}_{-1.0}) \times 10^{-6}$$

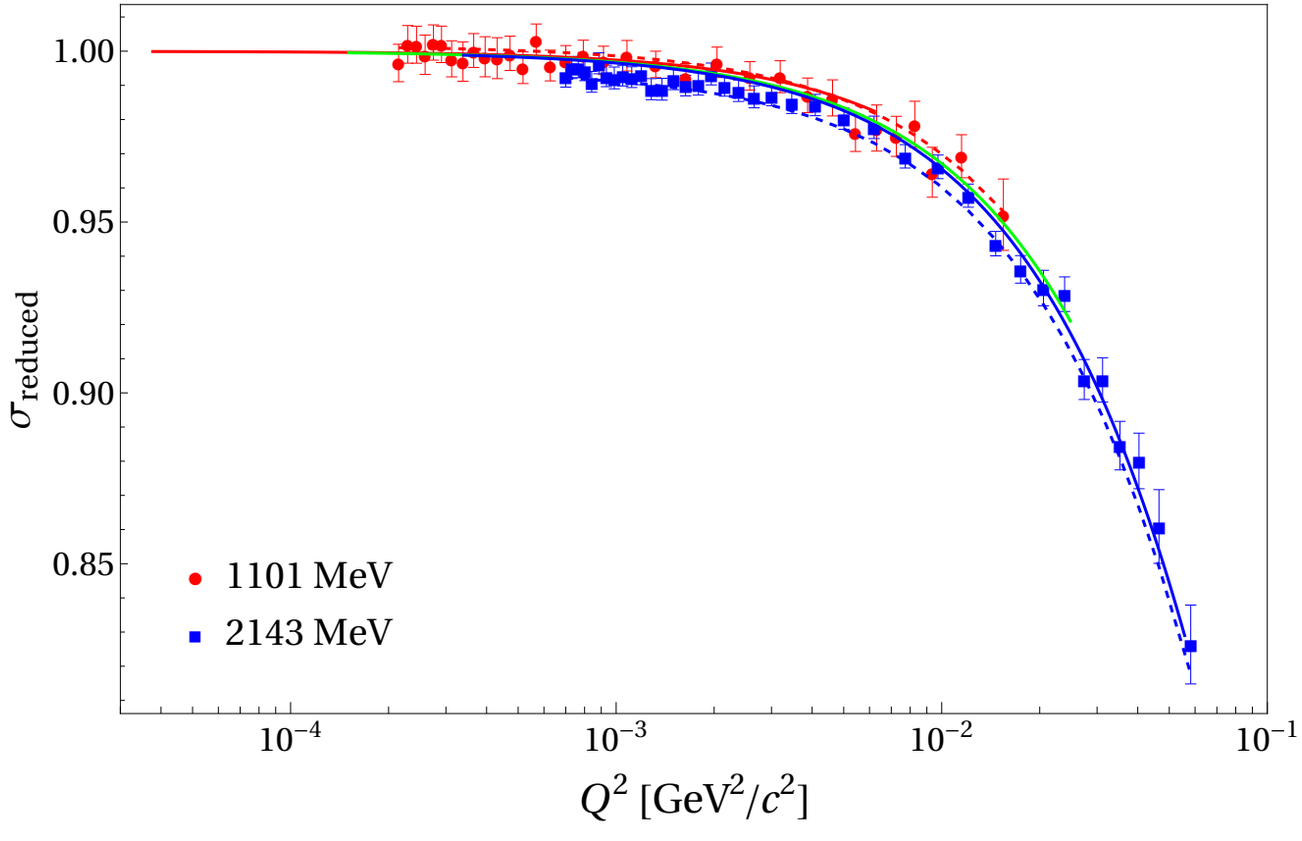
$$\Delta_R^{\mathrm{H}} = (526.9^{+1.1+1.3}_{-0.3-0.2}) \times 10^{-8} = (526.9^{+1.7}_{-0.4}) \times 10^{-8}$$

# Predictions for PRAD-II



Lin, Hammer, UGM, Phys. Lett. B **827** (2022) 136981 [2111.09619 [hep-ph]]

- Predictions for the upcoming PRad-II and $e^+p$ scattering




$\hookrightarrow$ Predictions for $E_\gamma = 0.7, 1.4, 2.1$ GeV

# The proton radius from $J/\psi$ decays

Y.-H. Lin, F.-K. Guo, UGM, *in preparation*

related work: J. Guttmann, M. Vanderhaeghen, Phys. Lett. B **719** (2013) 136

# The proton charge radius from $J/\psi$ decays



Lin, Guo, UGM, *in preparation*

- BESIII has a tremendous sample ($\sim 10^{10}$) of $J/\psi$ decays:

  $\hookrightarrow$ study the sensitivity of $J/\psi \to p\bar{p}e^+e^-$ to the nucleon em form factors

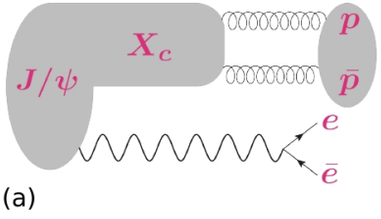
  $\hookrightarrow$ $e^+e^-$ threshold at $1.05 \times 10^{-6}$ GeV$^2$

- X-type: the same for $p\bar{p}$ and $n\bar{n}$

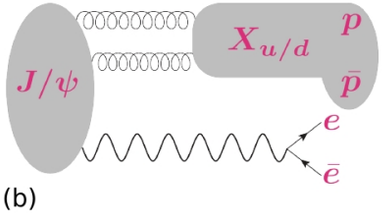
- Y- and Z-type:

  $\hookrightarrow$ proton $\to$ EMFFs

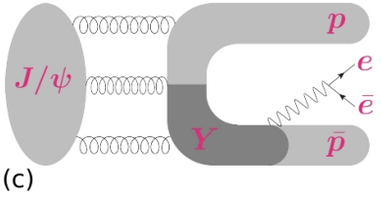
  $\hookrightarrow$ Delta $\to$ model the $N^*$ background



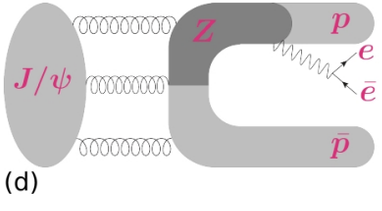
(a)



(b)



(c)



(d)

so that

$$|\mathcal{M}|^2 = |\mathcal{M}_{Y+Z}|^2 + \underbrace{\left(\mathcal{M}_{Y+Z}\mathcal{M}_X^* + \mathcal{M}_{Y+Z}^*\mathcal{M}_X\right)}_{\mathcal{M}_{\rm mix}} + |\mathcal{M}_X|^2$$

$\hookrightarrow$ subtracting the $J/\psi \to n\bar{n}e^+e^-$ data

# The proton charge radius from $J/\psi$ decays II



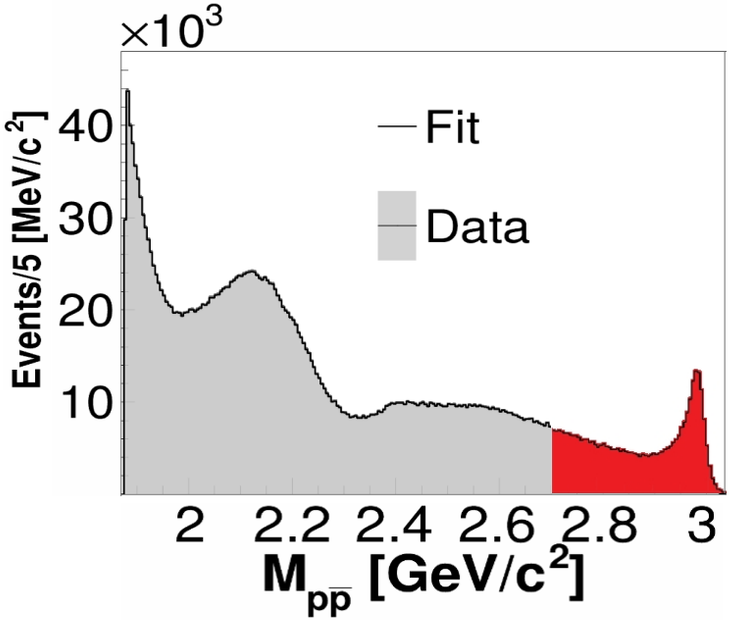
Lin, Guo, UGM, *in preparation*

- Selection of the $m_{p\bar{p}}$ region: search for best control of the backgrround

- $p\bar{p}$ invariant mass distribution in $J/\psi \to p\bar{p}\gamma$ from data taken in 2009-2018

| Region of $m_{p\bar{p}}$ [GeV] | $X$ resonances |
|---|---|
| $\sim 1.95$ | $X(1385)$ |
| $2.0 \sim 2.2$ | $f_0(2020), f_0(2100), f_0(220)$ $a_1(1930)$ $f_2(2010)$ |
| $2.7 \sim 3.05$ | $\eta_c$ |

R. Kappert, PhD thesis, U. Groningen (2022)




- Signal:

$$\frac{d\Gamma_{\rm signal}}{dm_{e^+e^-}} = \int_{2.7\ {\rm GeV}}^{M_{J/\psi}-m_{e^+e^-}} dm_{p\bar{p}} \int d\cos\theta_p^* dcos\theta_e' d\phi\, d\Gamma_{\rm signal}$$

$$d\Gamma_{\rm signal} \sim |\mathcal{M}|^2_{\rm signal} = |\mathcal{M}^N_{Y+Z}|^2 + \mathcal{M}^{N+\eta_c}_{\rm mix}$$

# The proton charge radius from $J/\psi$ decays III

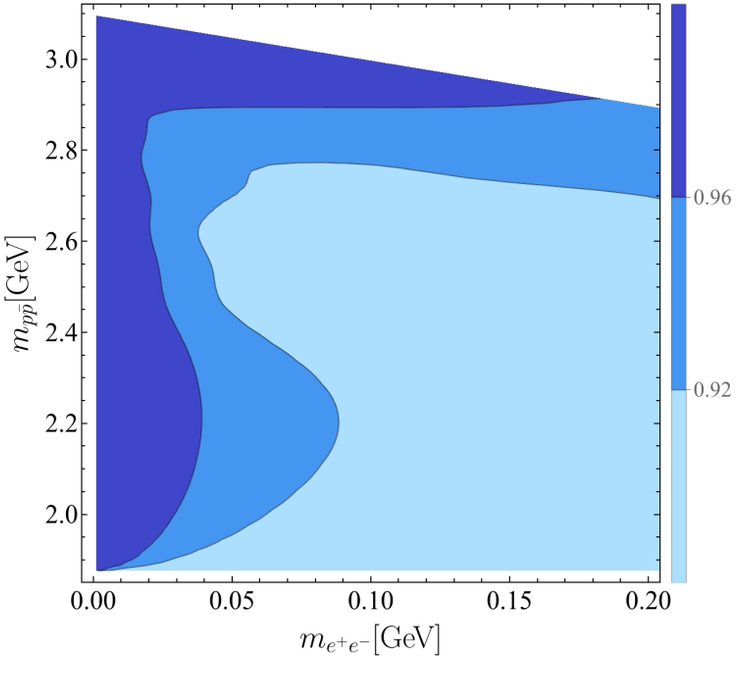


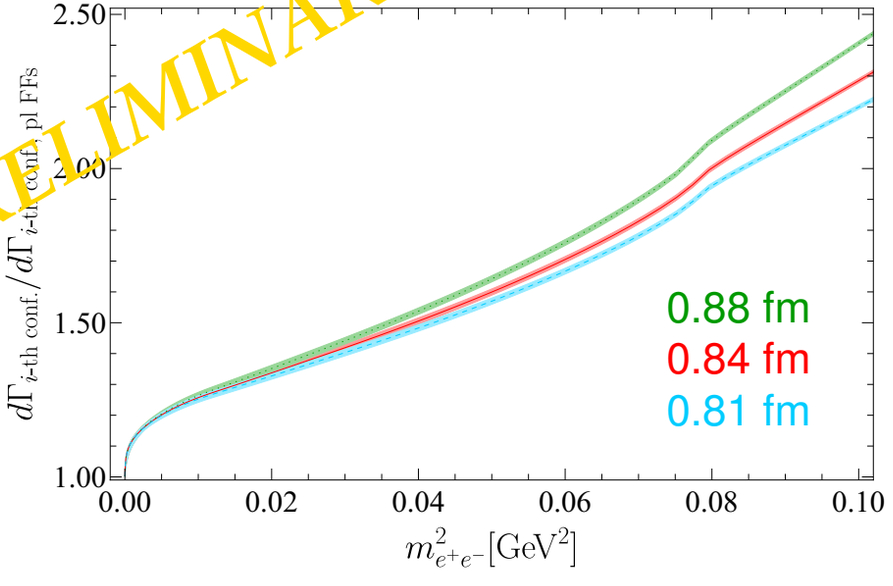
Lin, Guo, UGM, *in preparation*

- Results: Delta contribution and sensitivity to $r_p$

$$\frac{dR}{dm_{e^+e^-}dm_{p\bar{p}}} \sim \int \ldots \frac{d\Gamma^{N}_{Y+Z}}{d\Gamma^{N+\Delta}_{Y+Z}}$$

$$\frac{d\Gamma_{\text{signal}}}{dm^2_{e^+e^-}} \Big/ \frac{d\Gamma^{\text{pl}}_{\text{signal}}}{dm^2_{e^+e^-}} \rightarrow \textbf{little mod. dep.}$$




| $10^{10}\,J/\psi$ | $N^{N+\eta_c}_{\text{mix}}$ | $N^{N}_{Y+Z}$ |
|---|---|---|
| Events | 80 - 150 | 3000 - 25000 |

$\rightarrow$ determination of the possible accuracy of the $r_p$ extraction in the works

# Summary & outlook

- Dispersion theory is the best tool to analyze the nucleon em FFs
- Description of all data, time- and space-like
- Always a small proton charge radius (0.84 fm), magnetic one bigger (0.85 fm)
- Most recent experiments tend to the small radius
- From a “puzzle” to precision Hammer, UGM, Sci.Bull. **65** (2020) 257
- Predictions for PRad-II & positron-proton scattering (also MESA/Mainz)
- Magnetic radius of the neutron is the largest one
- Theory challenge I: Get the neutron ffs from fits to few-nucleon systems
- Theory challenge II: better understanding of the oscillations in $|G_{\rm eff}^{p,n}|$
- Experimental challenges
  - $\hookrightarrow$ proton form factor ratio at $Q^2 \simeq 10\,\text{GeV}^2$
  - $\hookrightarrow$ more resolved form factor measurements in the time-like region
  - $\hookrightarrow$ $\mu p$ scattering - testing lepton flavor universality (MUSE, AMBER)

# The last words from the press

- The story of the proton radius has left some aftertaste

48 FORSCHUNG UND TECHNIK Neue Zürcher Zeitung Samstag, 9. Januar 2021

Die Vermessung des Protons ist eine Geschichte der Irrungen und Wirrungen

Physiker sind bestrebt, die Natur unvoreingenommen zu beschreiben. Das Hin und Her um den Radius des Protons zeigt jedoch, dass sie diesem Anspruch nicht immer gerecht werden.
VON CHRISTIAN SPEICHER

**The measurements of the proton size is a story of trials and tribulations**

Physicists strive to describe nature in an unbiased way. However, the back and forth about the radius of the proton shows that they do not always live up to this claim.

# Spares

# Best fit incl. time-like data

Lin, Hammer UGM, Phys. Rev. Lett. **128** (2022) 052002 [2109.12961 [hep-ph]]

- Masses [GeV], widths [GeV] and residua [GeV$^2$]

⋆ isoscalar poles

| $V_s$ | $M_V$ | $\Gamma_V$ | $a_1^V$ | $a_2^V$ |
|---|---|---|---|---|
| $\omega$ | 0.783 | 0 | 0.701 | 0.338 |
| $\phi$ | 1.019 | 0 | −0.526 | −0.997 |
| $s_1$ | 1.031 | 0 | 0.422 | −2.827 |
| $s_2$ | 1.120 | 0 | 0.122 | 3.655 |
| $s_3$ | 1.827 | 0 | 0.955 | −1.122 |
| $S_1$ | 1.903 | 0.973 | −2.653 | −1.753 |
| $S_2$ | 1.914 | 0.541 | −3.069 | 2.017 |
| $S_3$ | 1.879 | 0.895 | 4.953 | 0.501 |

⋆ isovector poles

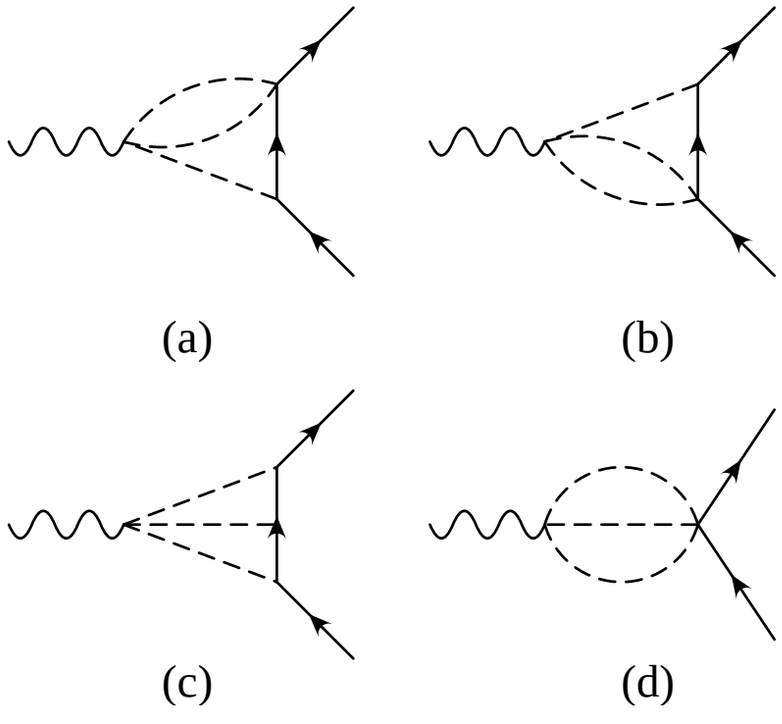
| $V_v$ | $M_V$ | $\Gamma_V$ | $a_1^V$ | $a_2^V$ |
|---|---|---|---|---|
| $v_1$ | 1.050 | 0 | 0.782 | −0.132 |
| $v_2$ | 1.323 | 0 | −4.873 | −0.645 |
| $v_3$ | 1.368 | 0 | 3.518 | −0.987 |
| $v_4$ | 1.462 | 0 | 2.243 | −3.813 |
| $v_5$ | 1.532 | 0 | −1.422 | 3.668 |
| $V_1$ | 2.256 | 0.239 | 2.552 | −1.217 |
| $V_2$ | 2.253 | 0.245 | −1.947 | 0.551 |
| $V_3$ | 2.220 | 0.362 | −0.985 | 1.061 |

# Isoscalar spectral function

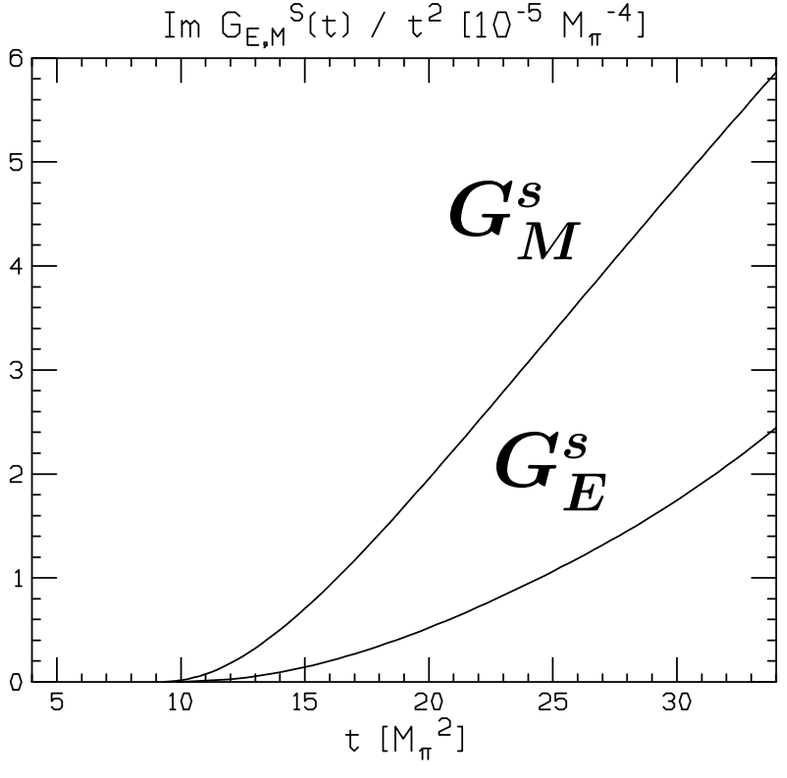


Bernard, Kaiser, UGM, Nucl. Phys. A **611** (1996) 429 [hep-ph/9607428]

- Two-loop CHPT calculation




- Electric/magnetic spectral fcts




⋆ **no** shoulder on the left wing

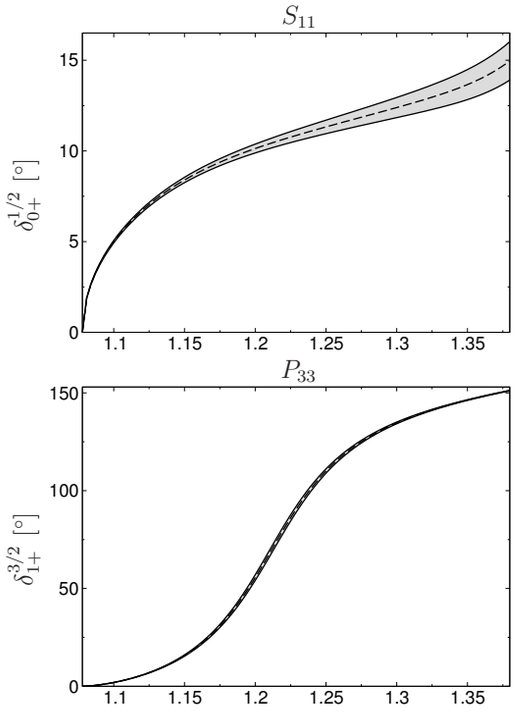
⋆ **clean** omega-pol dominance

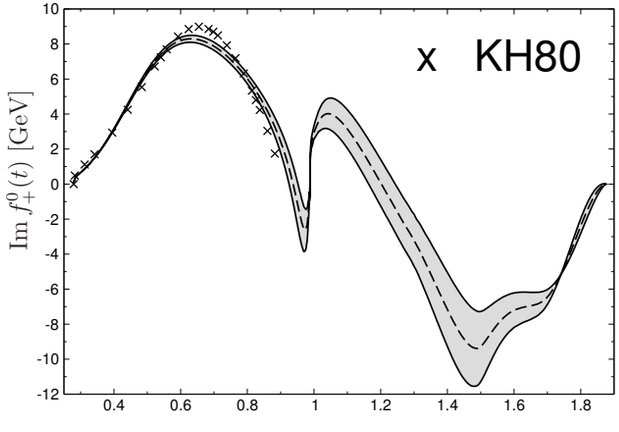
# Once more on the isovector spectral functions

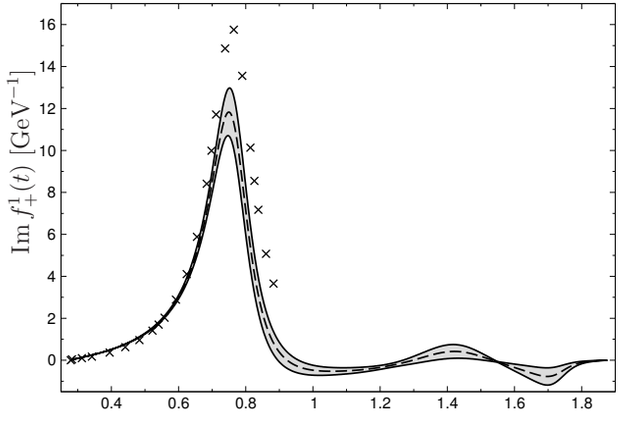
Hoferichter, Kubis, Ruiz de Elvira, Hammer, UGM, Eur. Phys. J. A **52** (2016)331
[arXiv:1609.06722 [nucl-th]]

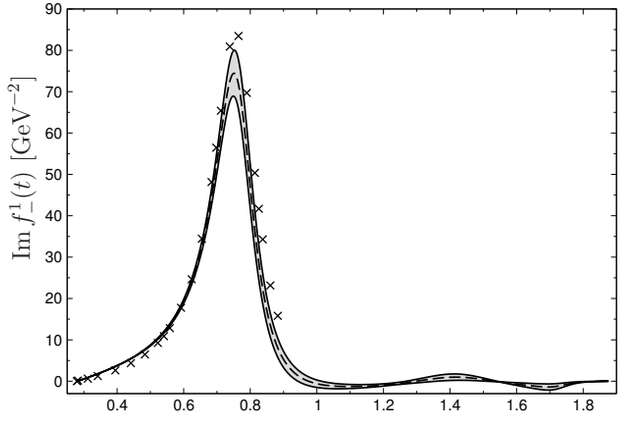
# Roy-Steiner equation analysis

- improve the isovector spectral functions by
  - ↪ updated $\pi N$ amplitudes from Roy-Steiner equations
  - ↪ include modern data (esp. pionic hydrogen & deuterium)
  - ↪ better treatment of isospin-violating effects
  - ↪ construct the pion FF from precise knowledge of $\delta_1^1(s)$
  - ↪ perform systematic error analysis



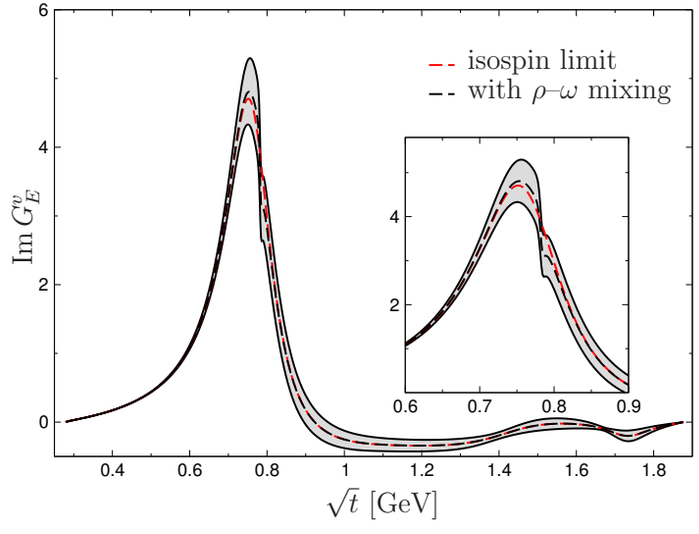



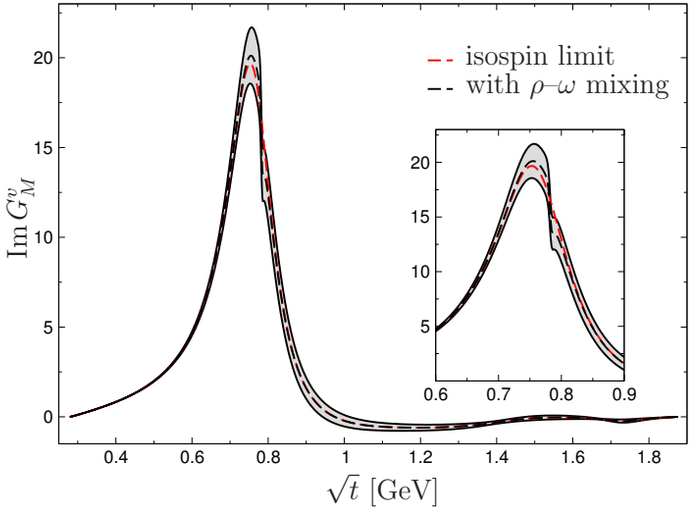

Hoferichter, Ruiz de Elvira, Kubis, UGM, Phys. Rev. Lett. **115** (2015) 092301; Phys. Rev. Lett. **115** (2015) 192301; Phys. Rept. **625** (2016) 1; J.Phys. G**45** (2018) 024001

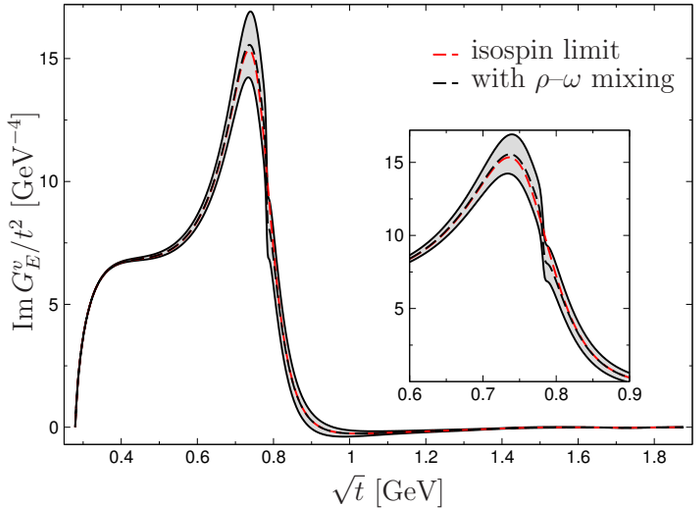
# New isovector spectral functions

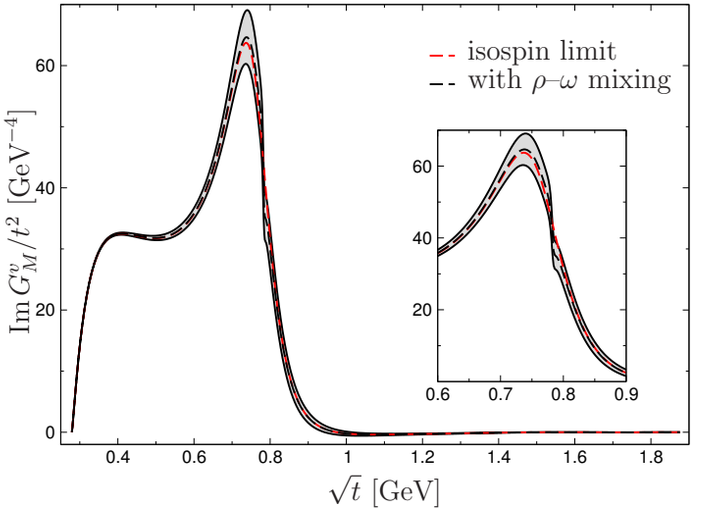


- Precise determinations of the isovector spectral functions



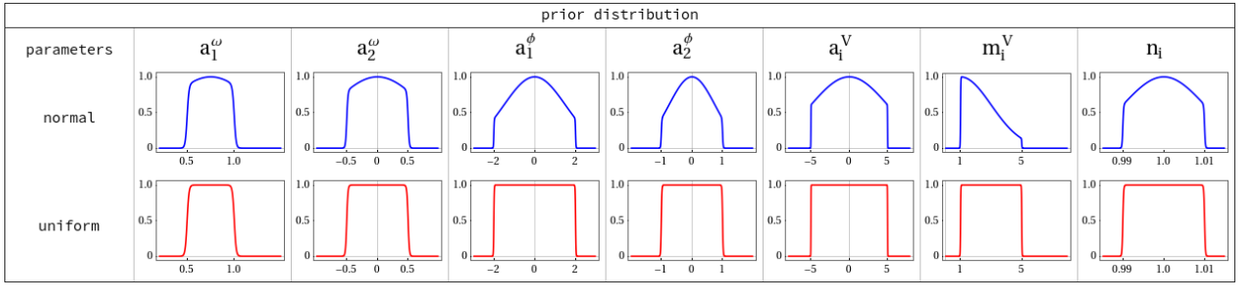



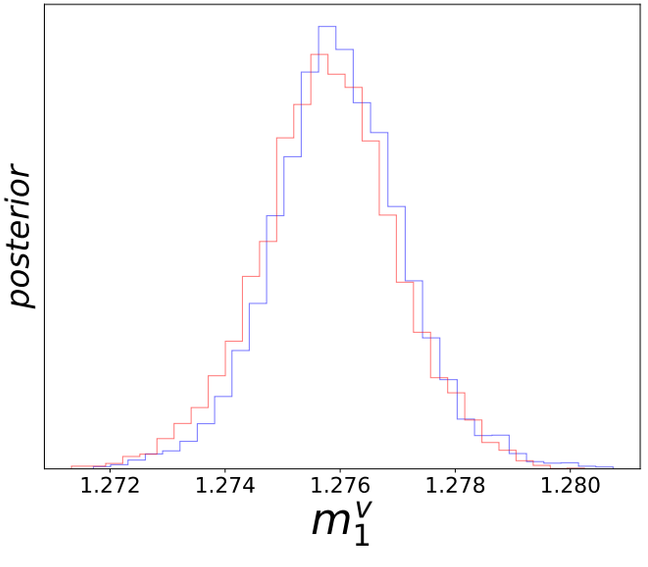
# BAYESIAN ANALYSIS

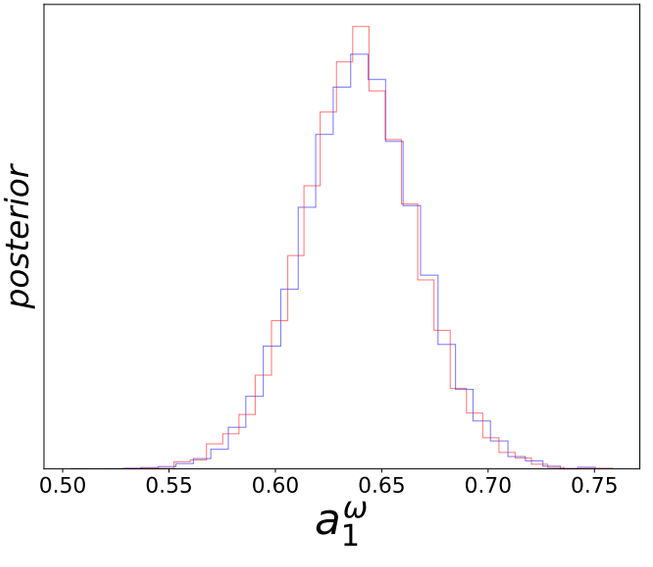
- Bayes theorem: $P(\text{parameters}|\text{data}) = \dfrac{P(\text{parameters})P(\text{data}|\text{parameters})}{P(\text{data})}$

posterior $\sim$ prior $\times$ likelihood

- Bayesian analysis of the PRad data (71 data pts)



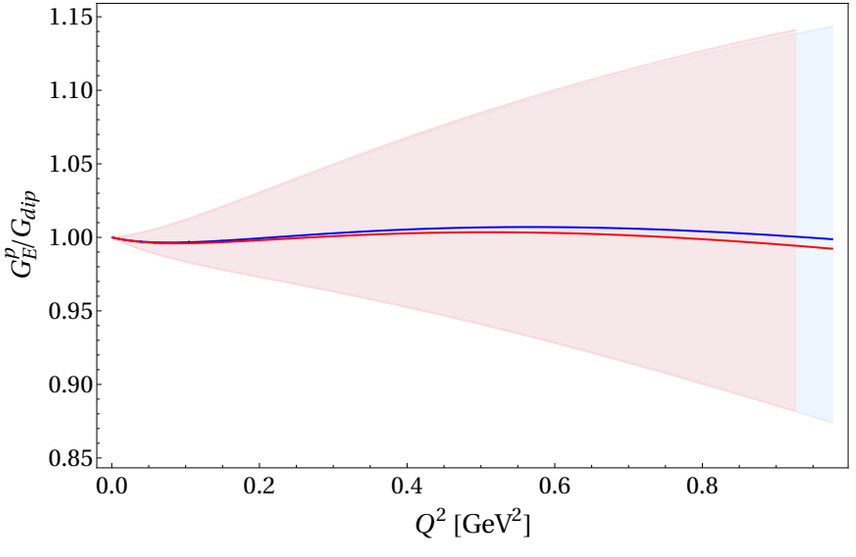



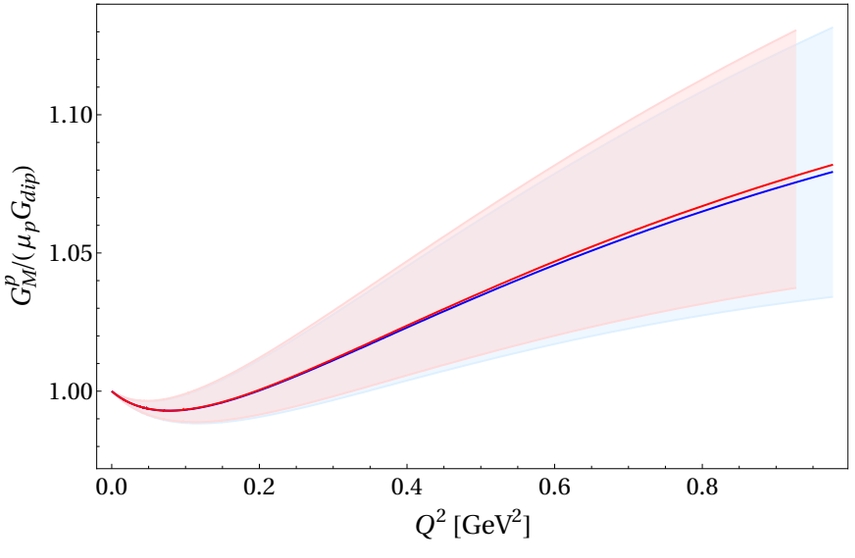
# BAYESIAN versus BOOTSTRAP



- Bootstrap sampling in comparison to the Bayesian analysis [PRad data]

| Method | $r_E^p$ [fm] | $r_M^p$ [fm] |
|---|---|---|
| Bayes normal | $\mathbf{0.828 \pm 0.011}$ | $\mathbf{0.843 \pm 0.004}$ |
| Bayes uniform | $\mathbf{0.828 \pm 0.011}$ | $\mathbf{0.843 \pm 0.004}$ |
| Bootstrap | $\mathbf{0.828 \pm 0.012}$ | $\mathbf{0.843 \pm 0.005}$ |




$\hookrightarrow$ identical results, but bootstrap much faster

$\hookrightarrow$ will use the bootstrap method to determine the statistical error

# PROTON CHARGE RADIUS - LEPTONIC LAMB SHIFT



- Energy levels in hydrogen:

$$E_{n\ell j} = R_\infty \left( -\frac{1}{n^2} + f_{n\ell j}\left(\alpha, \frac{m_e}{m_p}, \ldots\right) + \delta_{\ell 0} \frac{C_{\rm NS}}{n^3} r_p^2 \right)$$

$$f_{n\ell j}\left(\alpha, \tfrac{m_e}{m_p}, \ldots\right) = X_{20}\alpha^2 + X_{30}\alpha^3 + X_{31}\alpha^3 \ln\alpha + X_{40}\alpha^4 + \ldots$$

  - $n, \ell, j$ - principal, orbital, total ang. momentum quantum numbers
  - $f_{n\ell j}$ - relativistic corr's, vacuum effects, other QED corrections
  - $m_e/m_p$ enters through the coefficients $X_{20}$, $X_{30}$, ... (recoil)
  - $C_{\rm NS}$ calculable leading order correction due to the finite $r_p$
  - higher order charge distributions are included in $f_{n\ell j}$

  $\Rightarrow$ must measure at least 2 transitions to pin down the two unknowns

  $\Rightarrow$ this is done in recent measurements, but not before! [inconsistency]

# COMPARISON TO SPECTROSCOPY OBSERVABLES

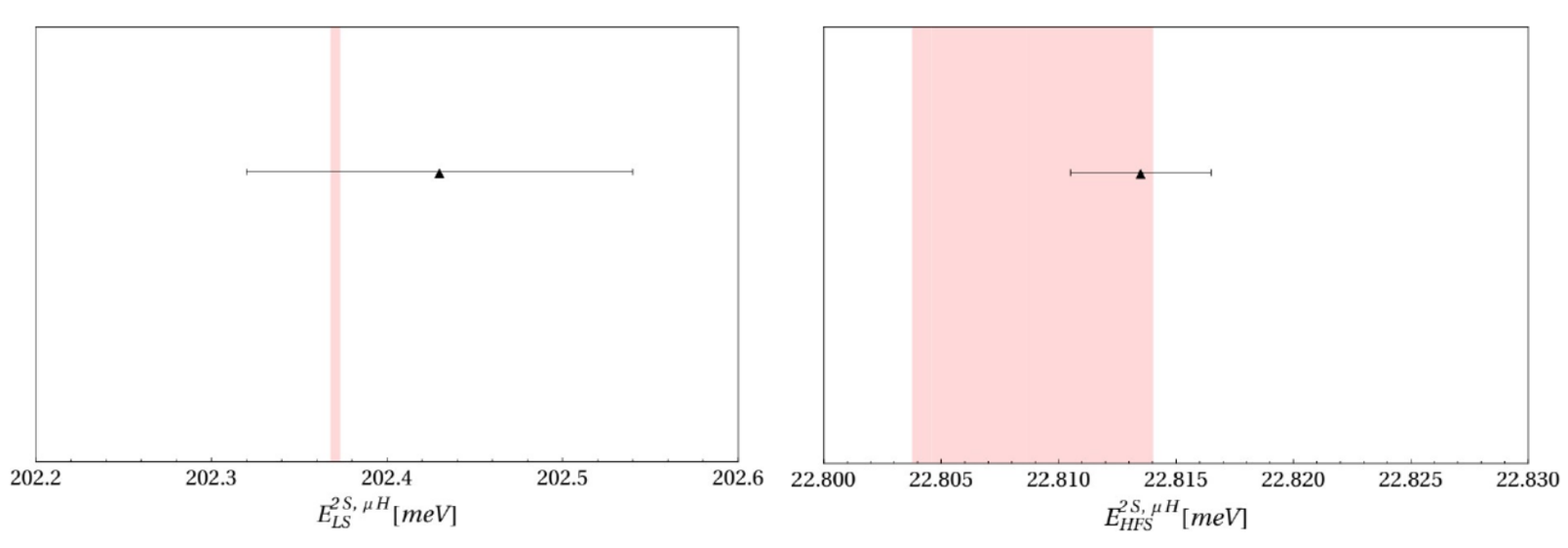


- Relevant formulas:

$$\Delta E_{LS} = 206.0336(15) - 5.2275(10)\langle r_p^2\rangle + 0.0347\langle r^3\rangle_{(2)}$$

$$\Delta E_{HFS} = 22.9843(30) - 0.1621(10)\, r_Z$$

Antognini et al., Ann. Phys. **331** (2013) 127




black triangles = our results

red bands from Antognini et al., Science **339** (2013) 417

# The last words from the press

LE PROTON EST PETIT... ET NORMAL

## LA TAILLE DU PROTON NE FAIT PLUS DÉBAT

0,84 femtomètre (fm), soit 0,00000000000084 mm : c'est ce que mesure le rayon d'un proton ! La taille de cette particule était sujette à un vif débat depuis les années 2010, que nous vous racontions en décembre 2019 *(S&V n°1227, p. 26)*. Jusqu'en 2010, le rayon officiel du proton était de 0,88 fm. Mais cette année-là, une mesure indirecte basée sur l'interaction de muons avec des électrons avait semé le doute : elle l'estimait à 0,84 fm seulement, soit une différence de tout de même 5 % ! Pour en avoir le cœur net, une autre équipe internationale avait alors lancé une expérience et, huit ans après, elle enfonçait le clou : oui, le proton semble mesurer 0,84 fm seulement. Aujourd'hui, *"le doute n'est plus permis, cette valeur basse est la seule correcte"*, tranche Ulf Meissner. Avec son équipe de l'université de Bonn, le chercheur a analysé les données de toutes les expériences de physique des particules effectuées dans le monde consistant à bombarder des électrons sur des protons, ou de les faire s'annihiler avec des positrons pour former des protons, comme BaBar ou BESIII. De quoi permettre aux physiciens de remonter par de savants calculs au rayon du proton : 0,84 fm. *"Le débat est clos,* conclut Ulf Meissner. *Il ne s'agit plus de discuter si la valeur correcte est la valeur haute ou la basse, mais d'augmenter la précision sur cette dernière. Ce à quoi nous allons désormais nous consacrer."*

**Benoît Rey**

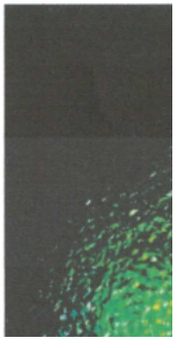




## Die Vermessung des Protons ist eine Geschichte der Irrungen und Wirrungen

Physiker sind bestrebt, die Natur unvoreingenommen zu beschreiben. Das Hin und Her um den Radius des Protons zeigt jedoch, dass sie diesem Anspruch nicht immer gerecht werden.

VON CHRISTIAN SPEICHER



ATOMIC AND MOLECULAR | FEATURE

## Solving the proton puzzle

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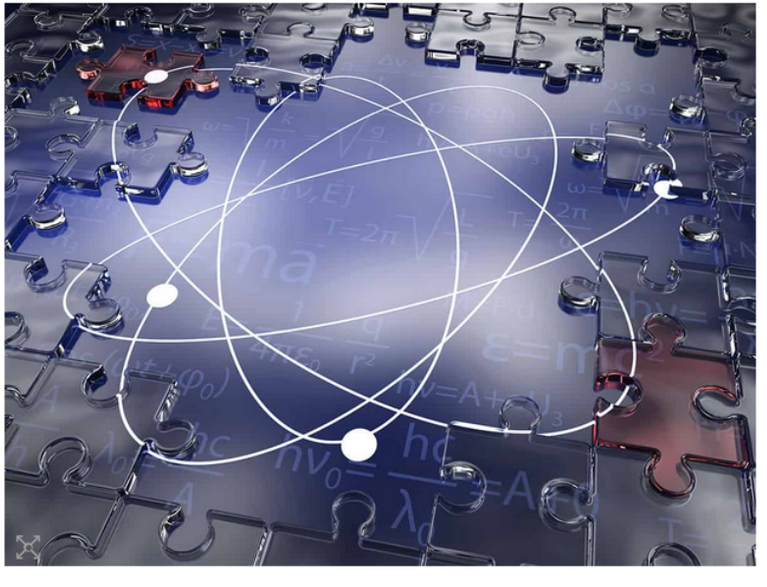
Taken from the June 2021 issue of *Physics World*. Members of the Institute of Physics can enjoy the full issue via the *Physics World* app.

**Why were so many physicists so wrong about the size of the proton for so long? As Edwin Cartlidge explains, the solution to this "proton radius puzzle" has as much to do with bureaucracy and politics as it does with physics**



– Ulf-G. Meißner, The Proton Radius and its Relatives A.D. 2023 – plenary talk, EFB25, Mainz, July 31stunzip figs-j , 2023 –