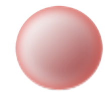
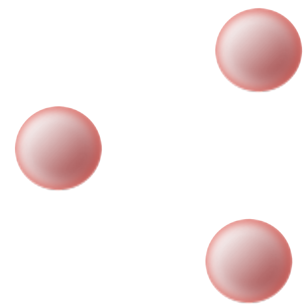
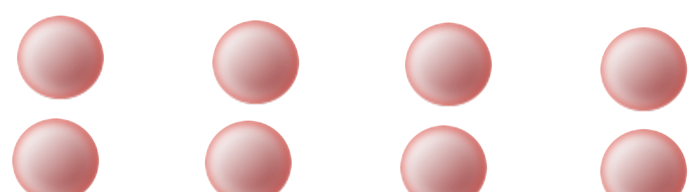
Impurity and few-to-many-body physics

Artem Volosniev

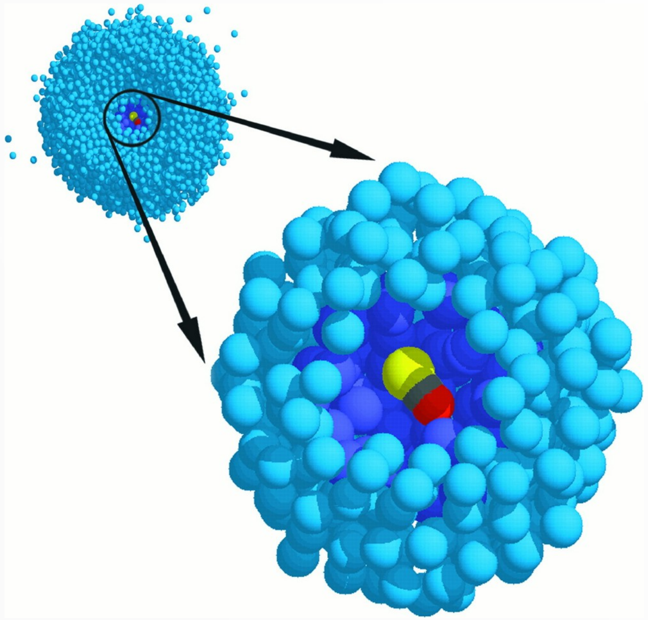
ISTA Institute of Science and Technology Austria

European conference on few-body problems in physics (EFB25) in Mainz

August 3, 2023

Physics		What is studied
One-Body		Laws of Motion
Few-Body		Interactions between particles
	Few-to-many-body physics	
Many-Body		Properties of matter

Motivational experiments

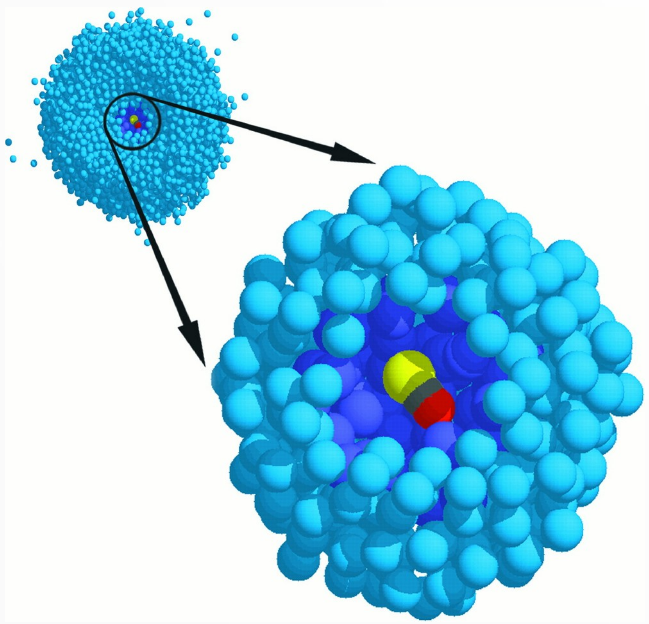


Superfluidity Within a Small Helium-4 Cluster: The Microscopic Andronikashvili Experiment

SLAVA GREBENEV, J. PETER TOENNIES, AND , ANDREI F. VILESOV Authors Info & Affiliations

SCIENCE • 27 Mar 1998 • Vol 279, Issue 5359 • pp. 2083-2086 • DOI: 10.1126/science.279.5359.2083

Motivational experiments



Transition N ~ 50

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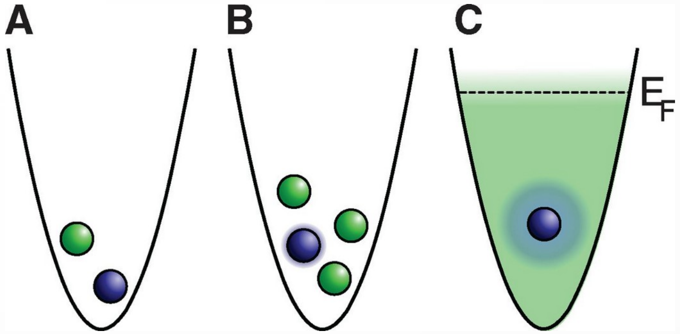


From Few to Many: Observing the Formation of a Fermi Sea One Atom at a Time

A. N. WENZ, G. ZÜRN, S. MURMANN, I. BROUZOS, T. LOMPE, AND, S. JOCHIM Authors Info & Affiliations

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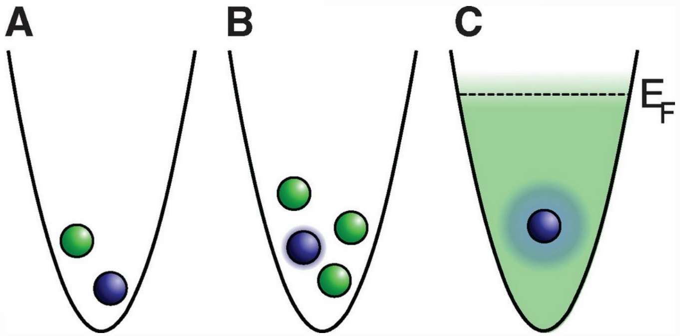
Motivational experiments



Transition N ~ 5

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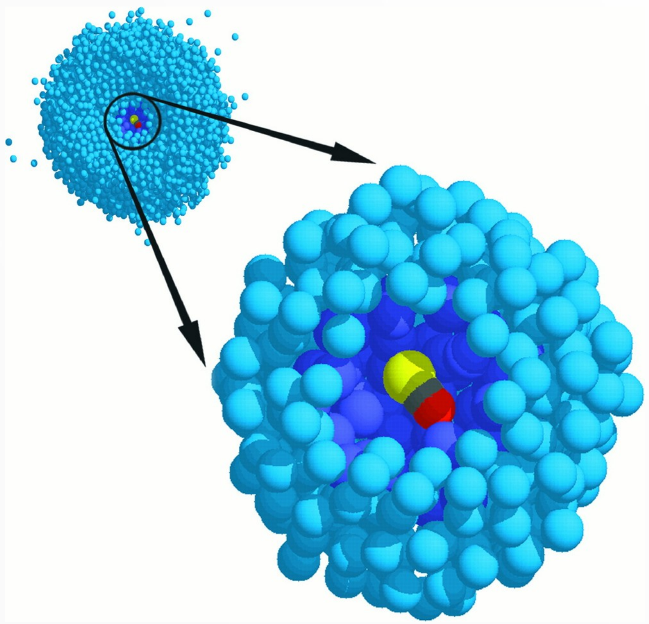

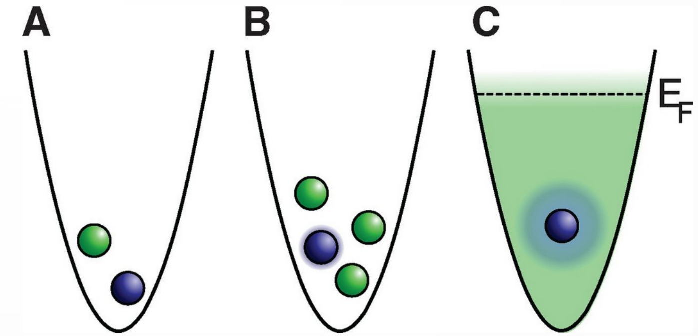

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Note: Impurity acts as a Probe (dressing)

Possible contribution from theorists

Conceptual:

→

←

Possible contribution from theorists

Conceptual: (for example)Proposals for what can be learned

→

←

Possible contribution from theorists

Conceptual: Proposals for what can be learned

→ Impurity to probe (bulk) properties of the system

←

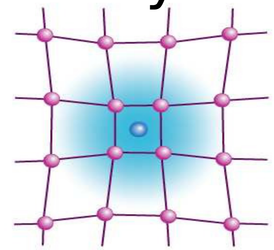
Possible contribution from theorists

Conceptual: Proposals for what can be learned

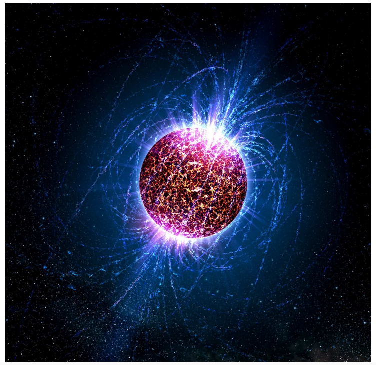
→ Impurity to probe (bulk) properties of the system

← Concept of a `polaron' quasiparticle (self-energy, effective mass)

Electron in a crystal



Helium3 in Helium4



Figures:wikimedia

Proton in a neutron star

Possible contribution from theorists

Conceptual: Proposals for what can be learned

→ Impurity to probe (bulk) properties of the system

← Concept of quasiparticles

Illustrative: Analysis of (toy or realistic) models

Possible contribution from theorists

Conceptual: Proposals for what can be learned

→ Impurity to probe (bulk) properties of the system

← Concept of quasiparticles

Illustrative: Analysis of (toy or realistic) models

Timely – novel numerical techniques for systems with N ~ 10-100

(Many-body techniques developed from few-body ones)

Possible contribution from theorists

Conceptual: Proposals for what can be learned

→ Impurity to probe (bulk) properties of the system

← Concept of quasiparticles

Illustrative: Analysis of (toy or realistic) models

Timely – novel numerical techniques for systems with N ~ 10-100

Helpful for conceptual development

Possible contribution from theorists

Conceptual: Proposals for what can be learned

→ Impurity to probe (bulk) properties of the system

← Concept of quasiparticles

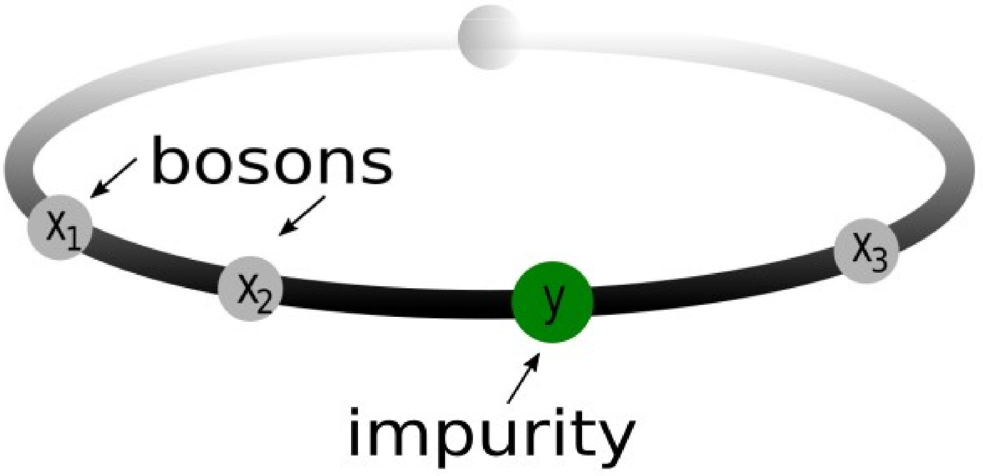
Illustrative: Analysis of (toy or realistic) models

Timely – novel numerical techniques for systems with N ~ 10-100

Helpful for conceptual development

This talk will illustrate these statements

One-dimensional Bose gas with an impurity



$$H = -\frac{\hbar^2}{2m_B}\sum_i \frac{\partial^2}{\partial x_i^2} - \frac{\hbar^2}{2m_I}\frac{\partial^2}{\partial y^2} + g_{BB}\sum_{i>j}\delta(x_i - x_j) + c\sum_i \delta(x_i - y)$$

Mean-Field approximation

$$-\frac{\hbar^2(m_I+m_B)}{2m_Bm_I}\frac{\partial^2\phi_{P_I}(z)}{\partial z^2}+i\frac{P_I}{2m_I}\frac{\partial\phi_{P_I}(z)}{\partial z}+g_{BB}N|\phi_{P_I}(z)|^2\phi_{P_I}(z)+c\delta(z)\phi(z)=\mu\phi_{P_I}(z)$$

‘Co-moving’ with the impurity frame of reference

Mean-Field approximation

$$-\frac{\hbar^2(m_I+m_B)}{2m_Bm_I}\frac{\partial^2\phi_{P_I}(z)}{\partial z^2}+i\frac{P_I}{2m_I}\frac{\partial\phi_{P_I}(z)}{\partial z}+g_{BB}N|\phi_{P_I}(z)|^2\phi_{P_I}(z)+c\delta(z)\phi(z)=\mu\phi_{P_I}(z)$$

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E. Gross, Annals of Physics 19, 234 (1962)

AGV and H.-W. Hammer, Phys. Rev. A 96, 031601 (2017)
S. I. Mistakidis, **AGV**, N. T. Zinner, P. Schmelcher, Phys. Rev. A 100, 013619 (2019)
G. Panochko and V. Pastukhov, Annals of Physics 409, 167933 (2019)
J. Jager, R. Barnett, M. Will, and M. Fleischhauer, Phys. Rev. Research 2, 033142 (2020)

[In 3D
M. Drescher, M. Salmhofer, and T. Enss, Phys. Rev. Research 2, 032011 (2020).
N.-E. Guenther, R. Schmidt, G. M. Bruun, V. Gurarie, and P. Massignan, Phys. Rev. A 103, 013317 (2021)]

Mean-Field approximation

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Impurity probes boundary energy of the Bose gas

Mean-Field approximation

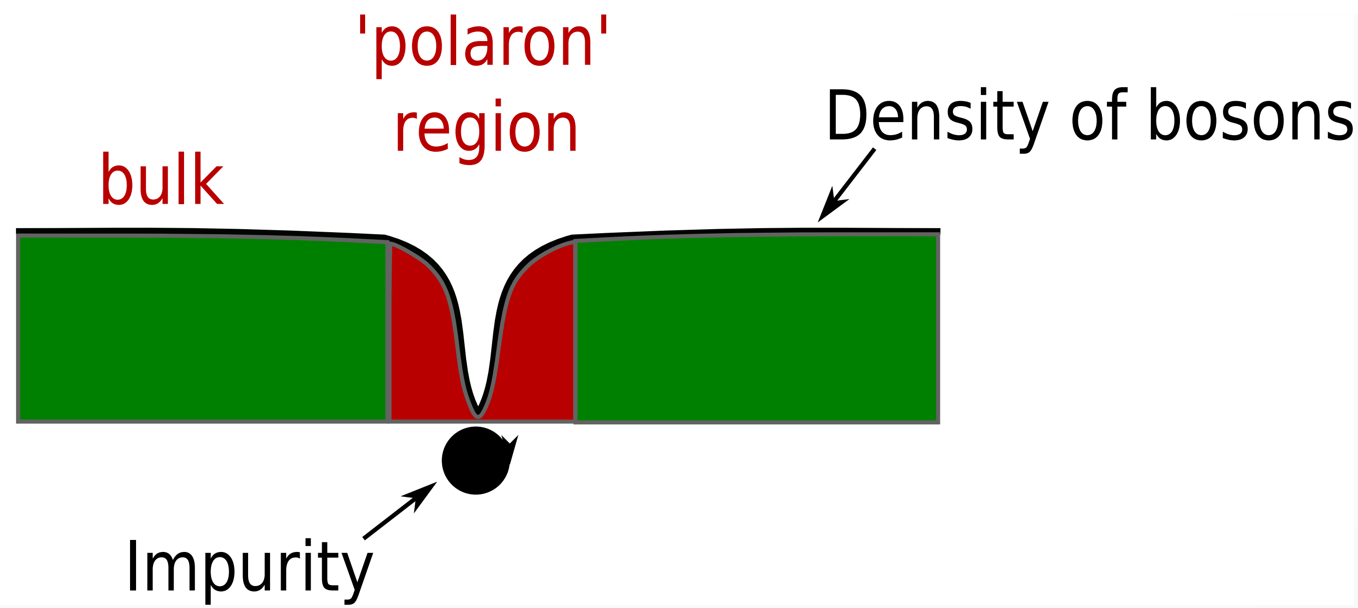
$$-\frac{\hbar^2(m_I + m_B)}{2m_B m_I}\frac{\partial^2 \phi_{P_I}(z)}{\partial z^2} + i\frac{P_I}{2m_I}\frac{\partial \phi_{P_I}(z)}{\partial z} + g_{BB}N|\phi_{P_I}(z)|^2\phi_{P_I}(z) + c\delta(z)\phi(z) = \mu\phi_{P_I}(z)$$

Two-body impurity-boson physics

Many-boson physics

Mean-Field approximation

$$-\frac{\hbar^2(m_I+m_B)}{2m_B m_I}\frac{\partial^2\phi_{P_I}(z)}{\partial z^2}+i\frac{P_I}{2m_I}\frac{\partial\phi_{P_I}(z)}{\partial z}+g_{BB}N|\phi_{P_I}(z)|^2\phi_{P_I}(z)+c\delta(z)\phi(z)=\mu\phi_{P_I}(z)$$



The size of the 'polaron' region is determined by two-body physics

Flow equations for bosons

$$\frac{\mathrm{d}\mathcal{H}(s)}{\mathrm{d}s} = \eta\mathcal{H} - \mathcal{H}\eta \equiv [\eta, \mathcal{H}(s)]$$

Similar to IM-SRG in nuclear physics

S. Kehrein, The Flow Equation Approach to Many-Particle Systems (2006)
K. Tsukiyama, S. K. Bogner, and A. Schwenk, PRL 106, 222502 (2011)

AGV and H.-W. Hammer, New J. Phys. 19, 113051 (2017)

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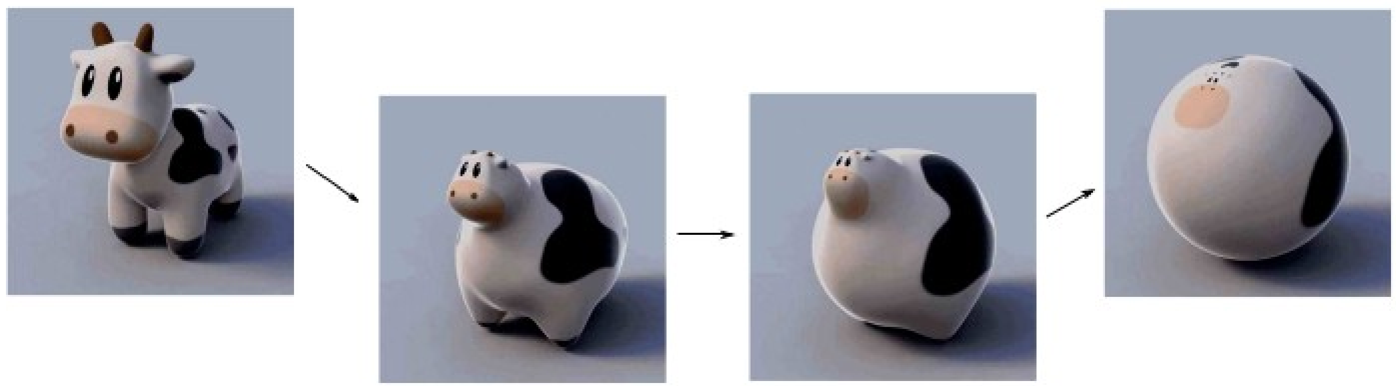
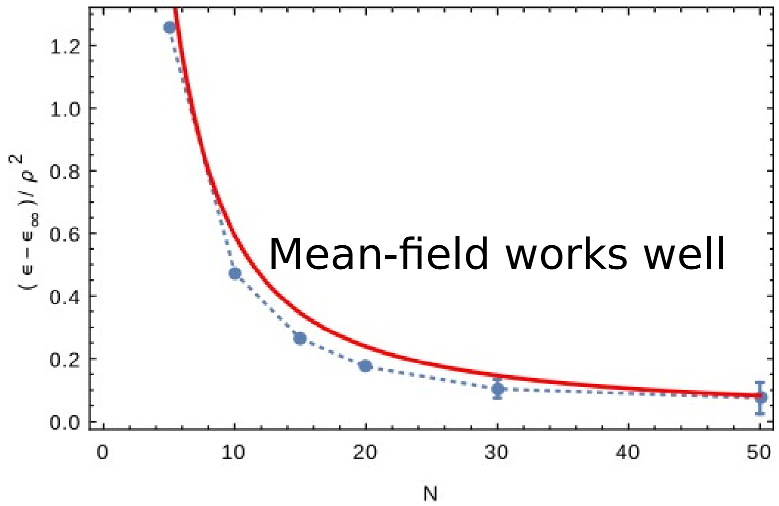


Ground-state energy

$N \to \infty$, $L \to \infty$ with $\frac{N}{L} = \rho$



$\frac{g_{BB}}{\rho} = 0.1$, $\frac{1}{c} = 0$ (impenetrable impurity).

Dots – the flow equations, curve – the Gross-Pitaevski equation.

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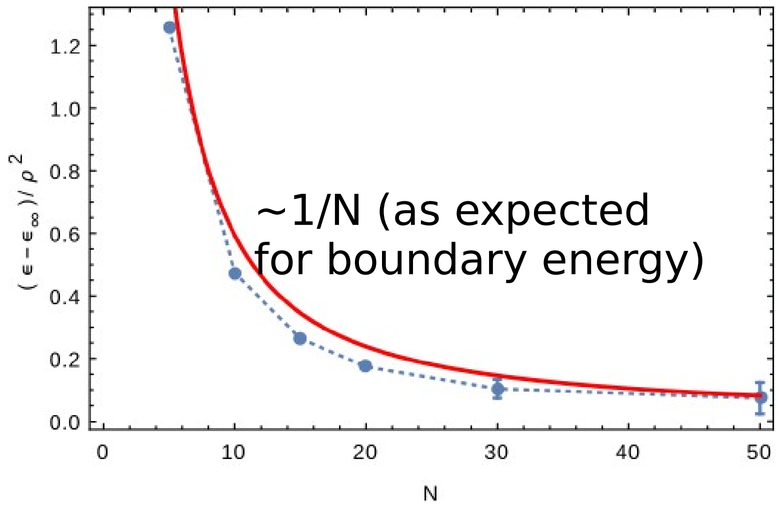

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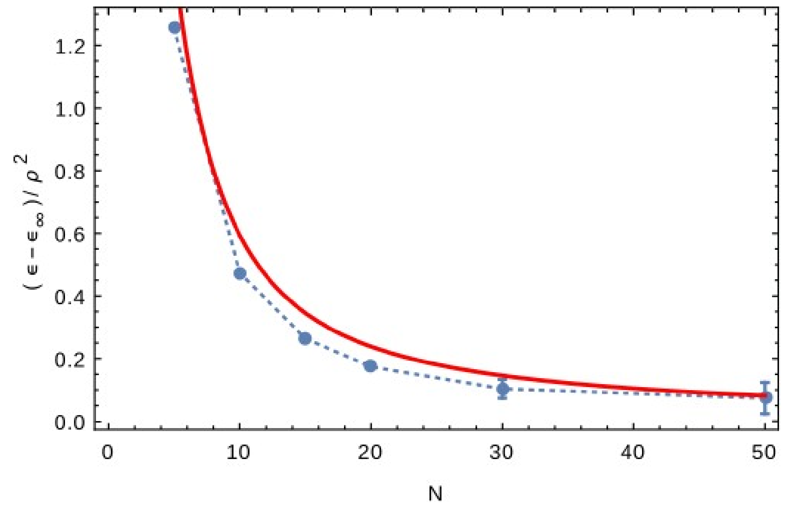

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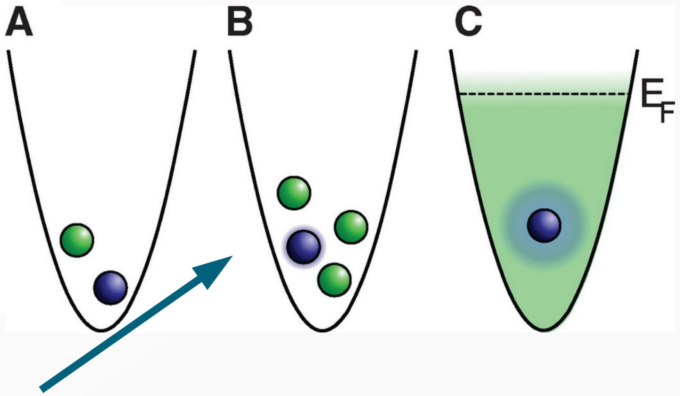


Many bosons to screen the impurity (cf. fermionic gas)

Effective mass

$$E = E_0 + \frac{P^2}{2m_{\text{eff}}}$$

F. Brauneis, A. Ghazaryan, H.-W. Hammer, **AGV**, arXiv:2301.10488 to appear in Commun. Physics (Nature)

Effective mass

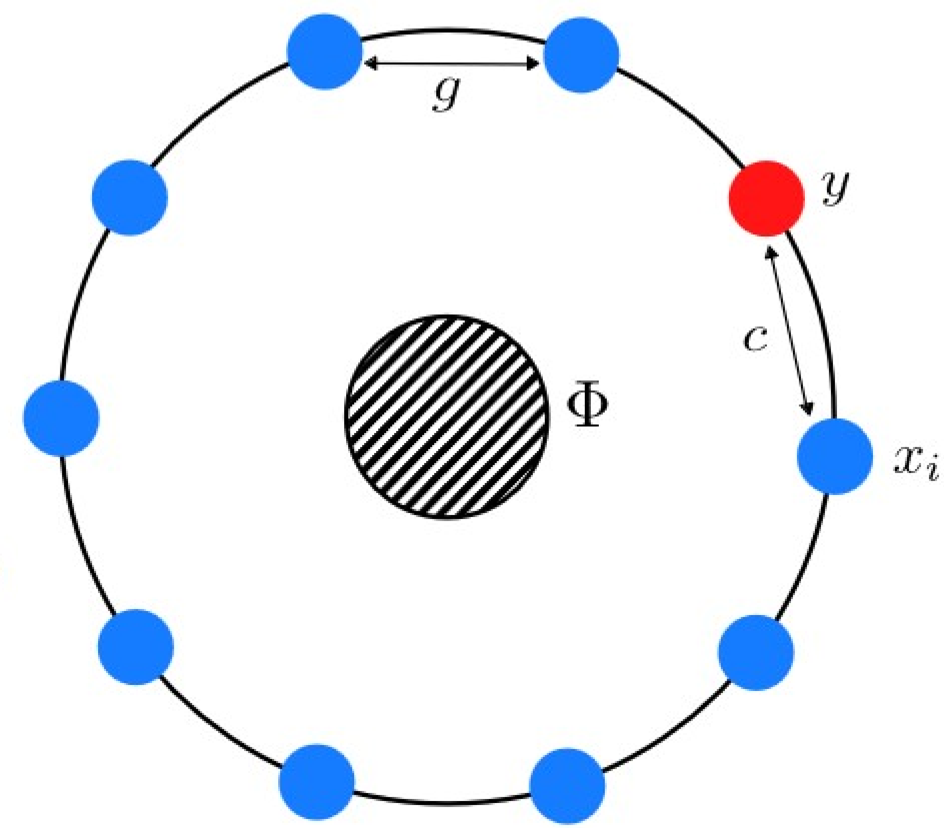
$$E = E_0 + \frac{P^2}{2m_{\text{eff}}}$$

Problem: Discrete variable for finite systems

Effective mass

$$E = E_0 + \frac{P^2}{2m_{\text{eff}}}$$

$$-i\frac{\partial}{\partial y} \to -i\frac{\partial}{\partial y} + \Phi$$



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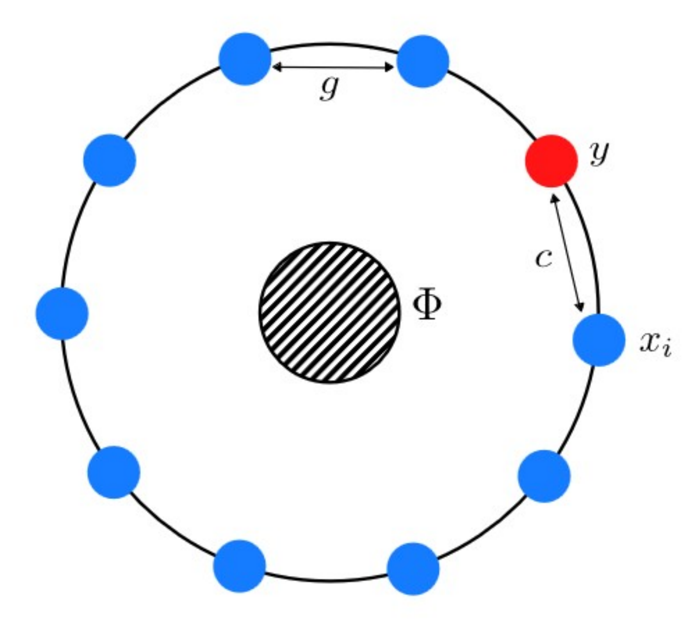
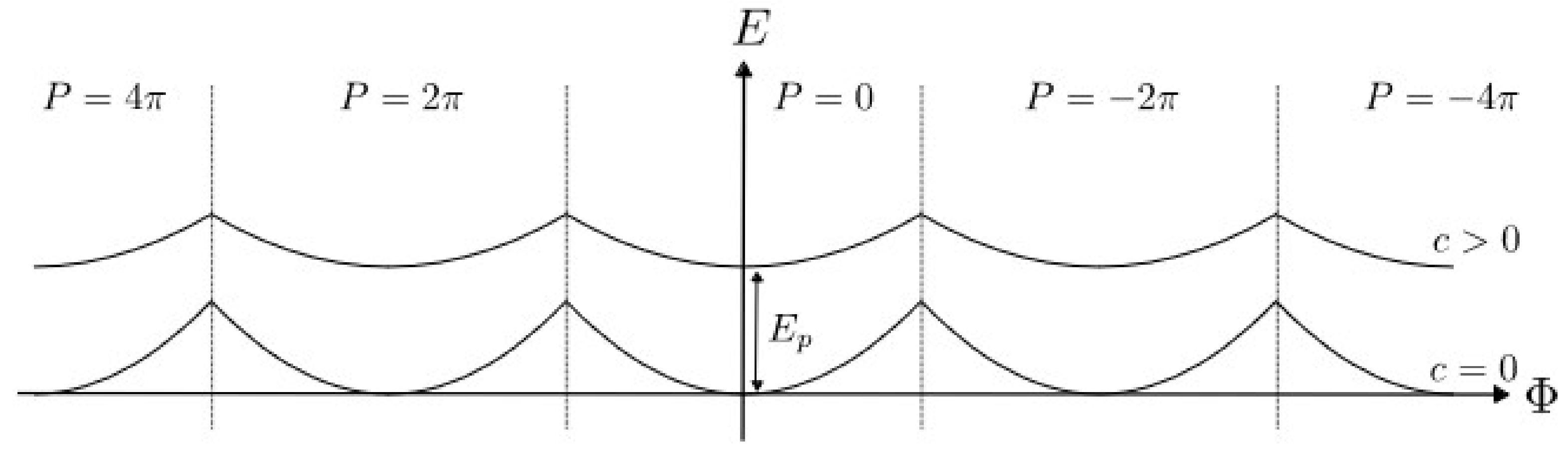
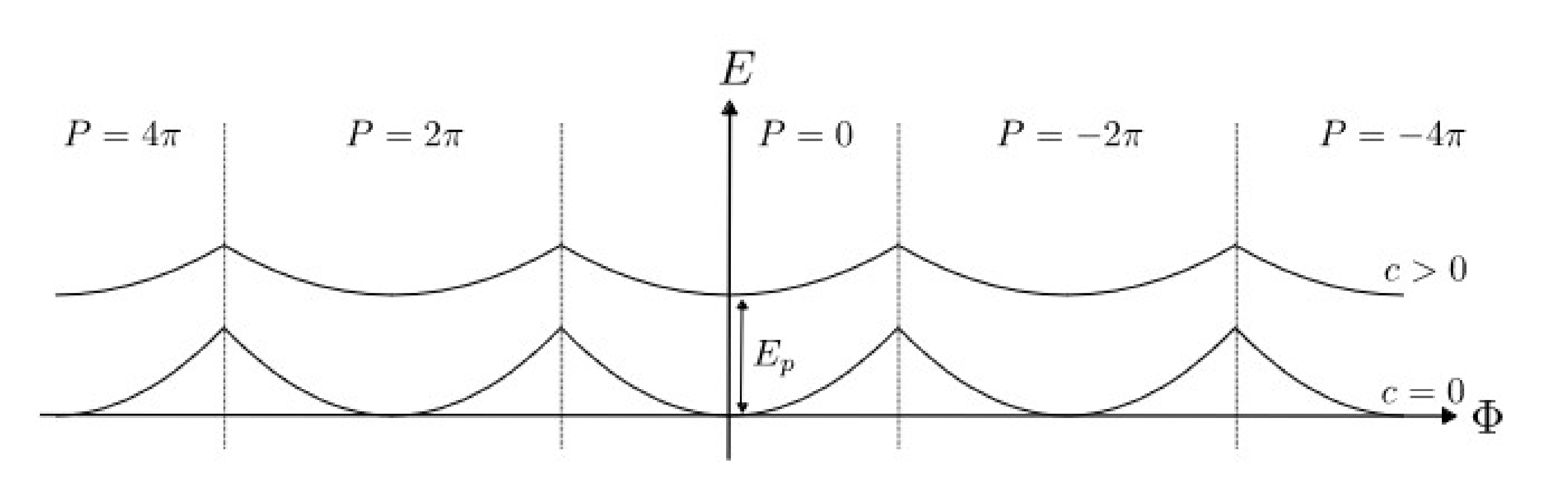




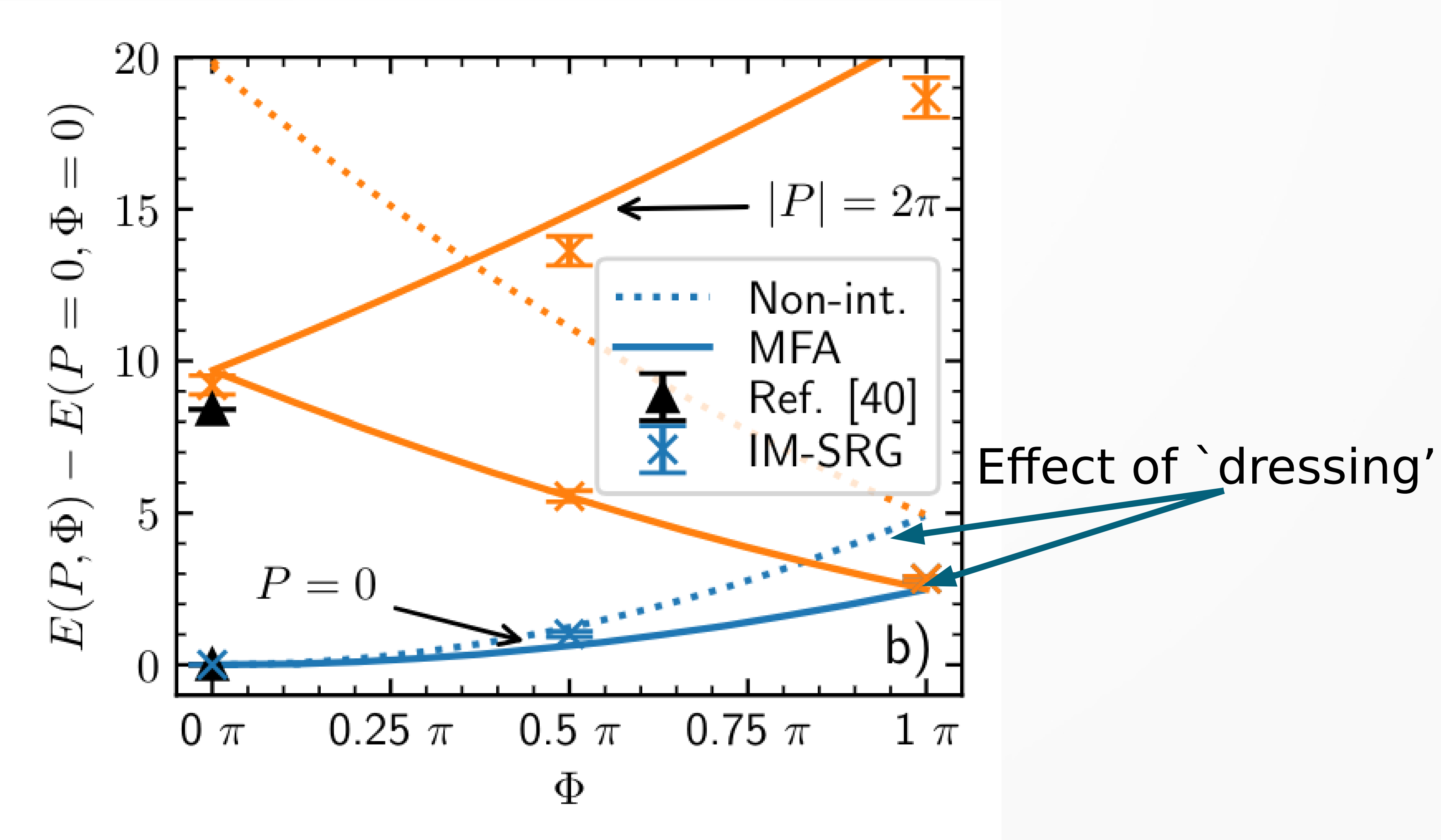
Effective mass



$$\lim_{\Phi \to 0} [E(P = 0, \Phi) - E(P = 0, \Phi = 0)] = \frac{\Phi^2}{2m_{\text{eff}}}$$

IM-SRG vs mean-field

$N = 19$, $\gamma = 0.2$, and $c/g = 1$



IM-SRG vs mean-field

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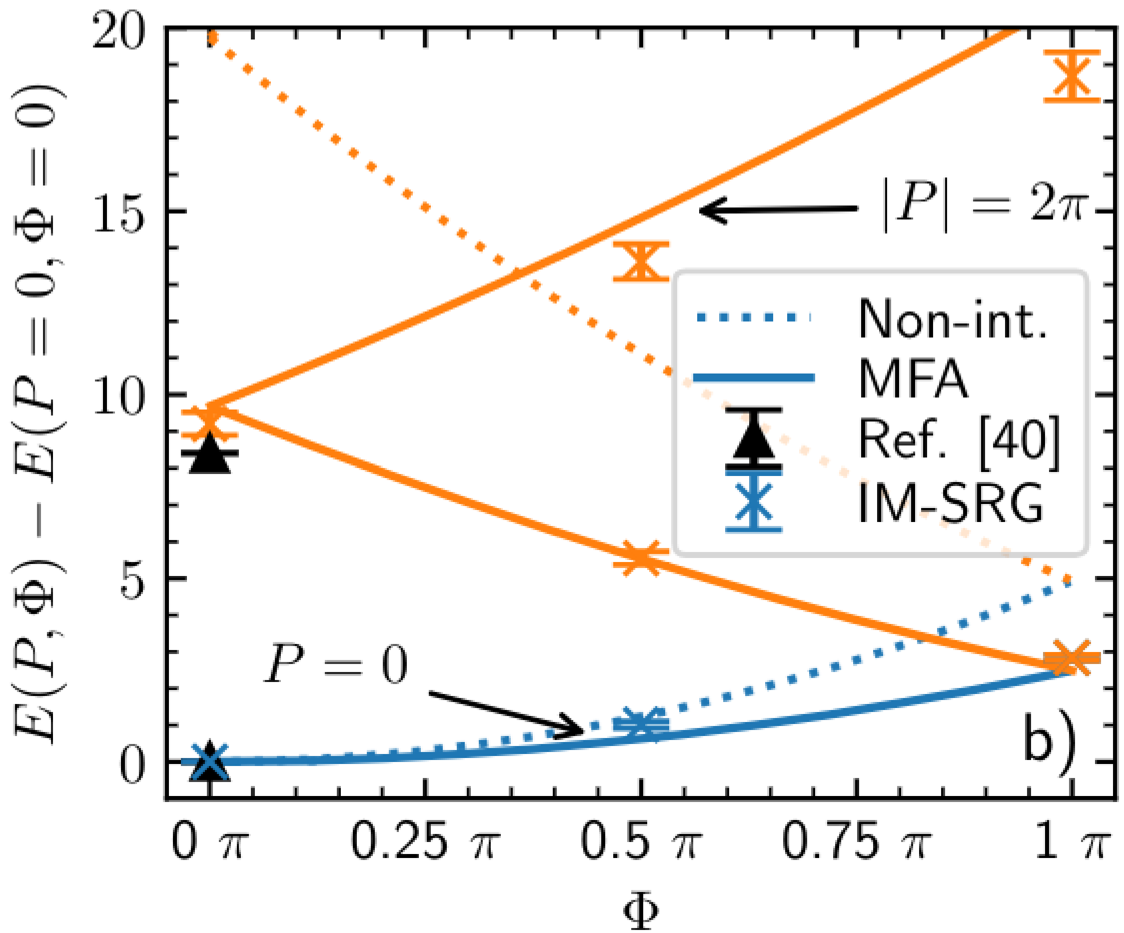
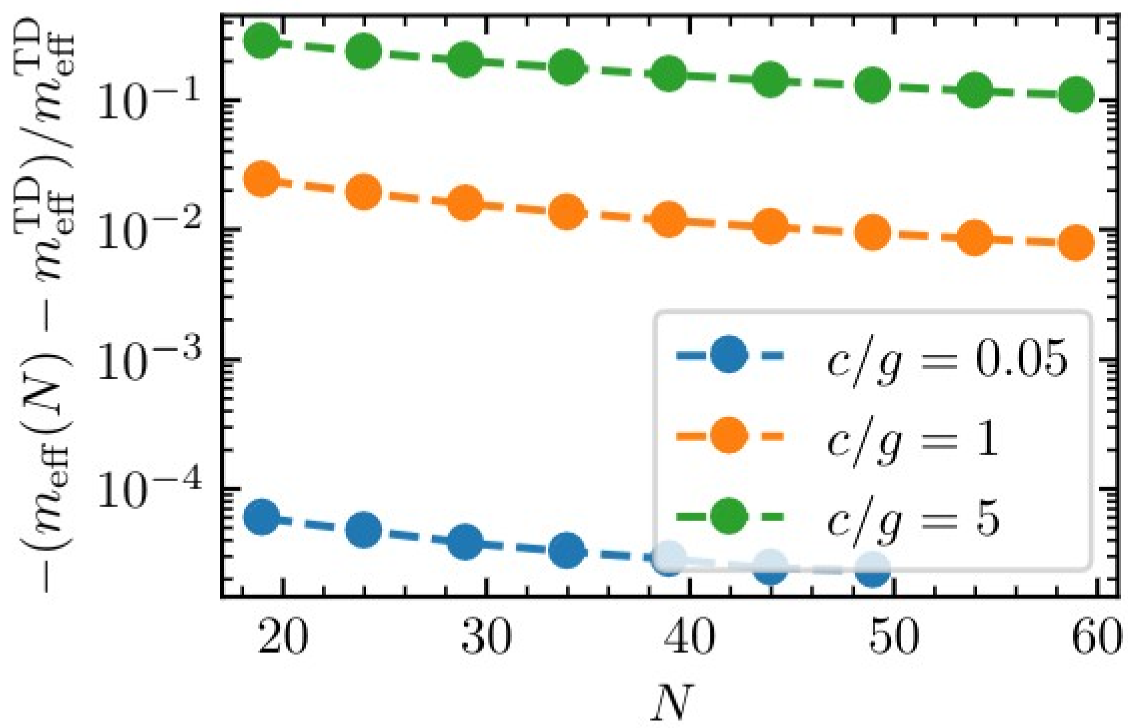


Mean-field works well

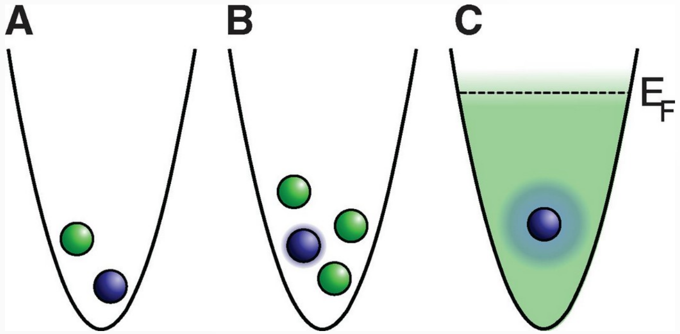
Effective mass (few→many)

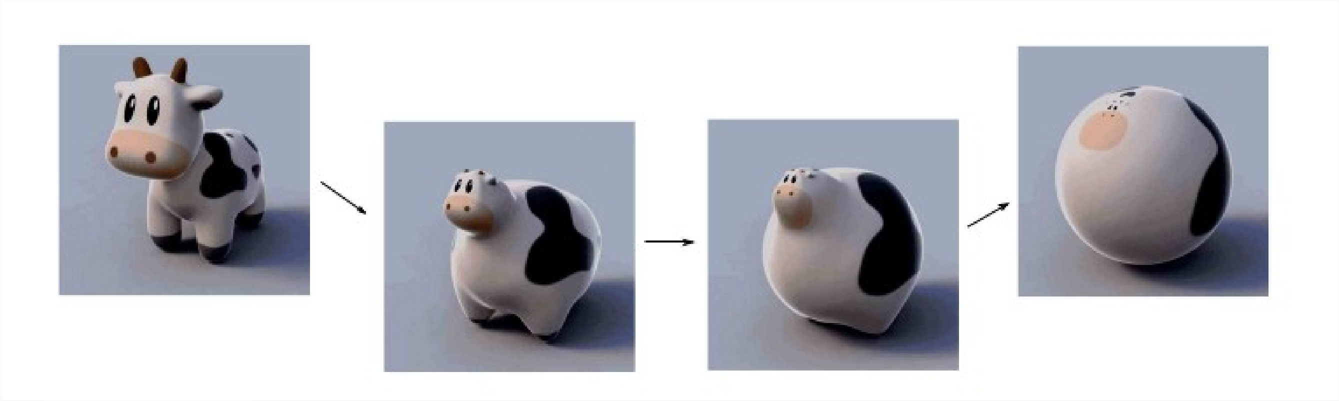


Slow convergence in comparison to that of the energy

Summary

Novel experiments and numerical methods motivate theoretical studies of systems with impurities where N ~ 10-100





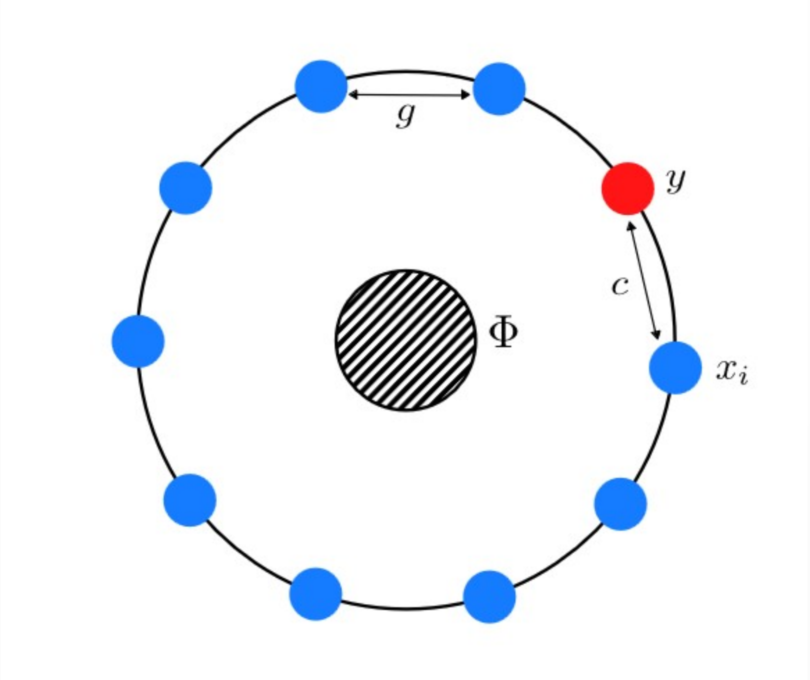
1D Reviews:

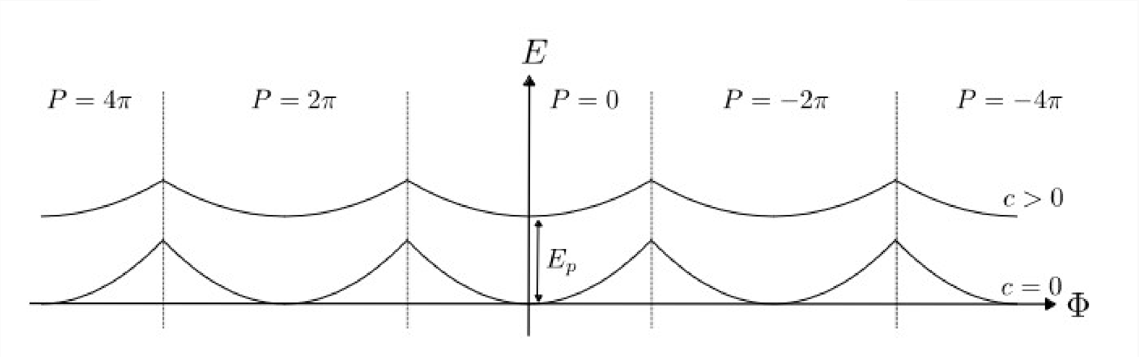
T. Sowiński and M. Á. García-March, Rep. Prog. Phys. 82 104401 (2019)

S. I. Mistakidis, **AGV**, R. E. Barfknecht, T. Fogarty, Th. Busch, A. Foerster, P. Schmelcher, and N. T. Zinner arXiv:2202.11071

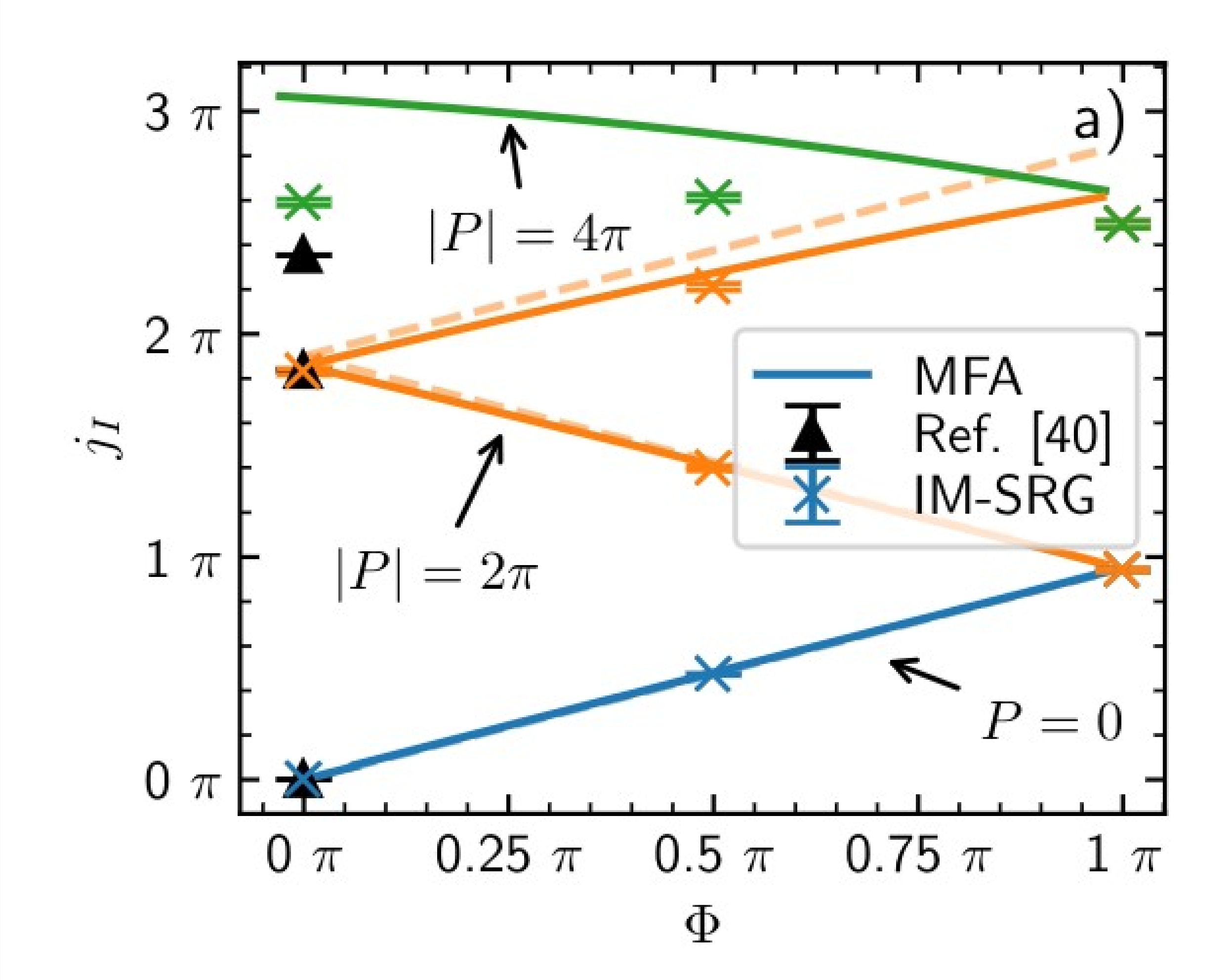
Summary

We analyzed one of the simplest systems

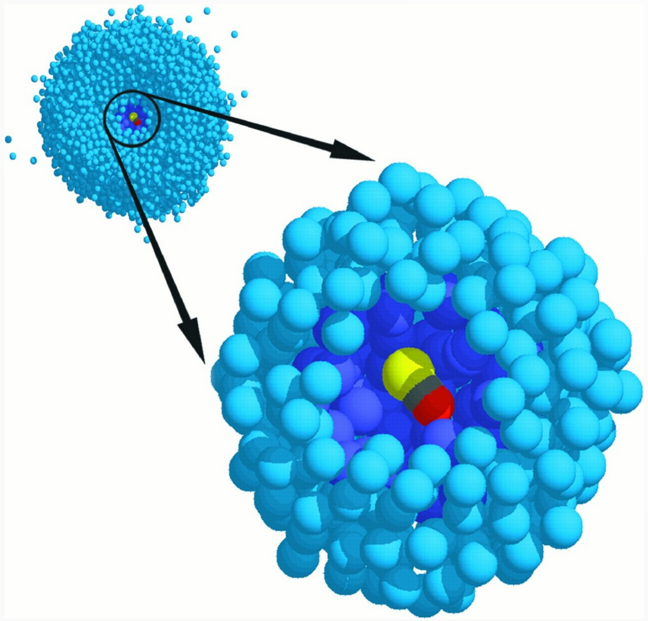


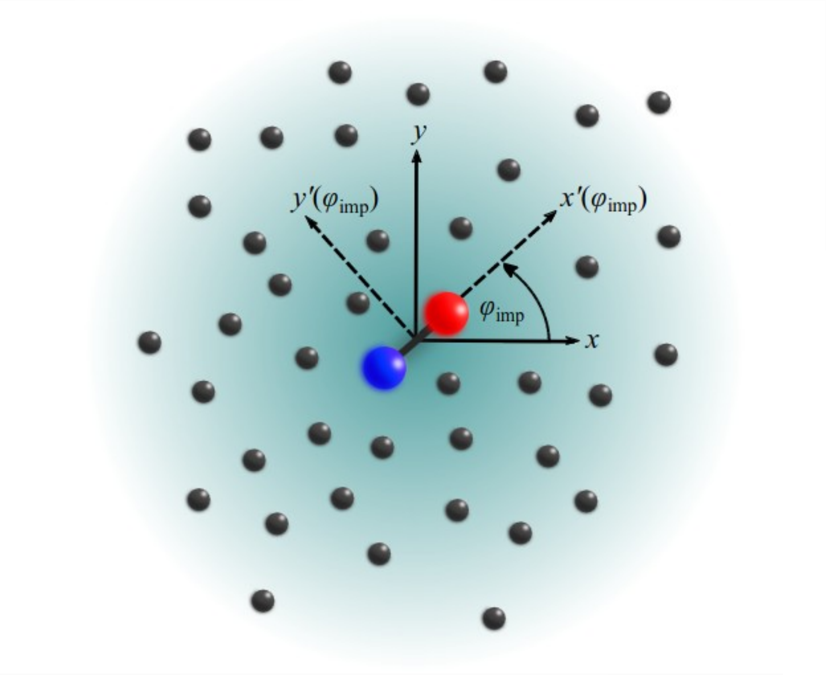


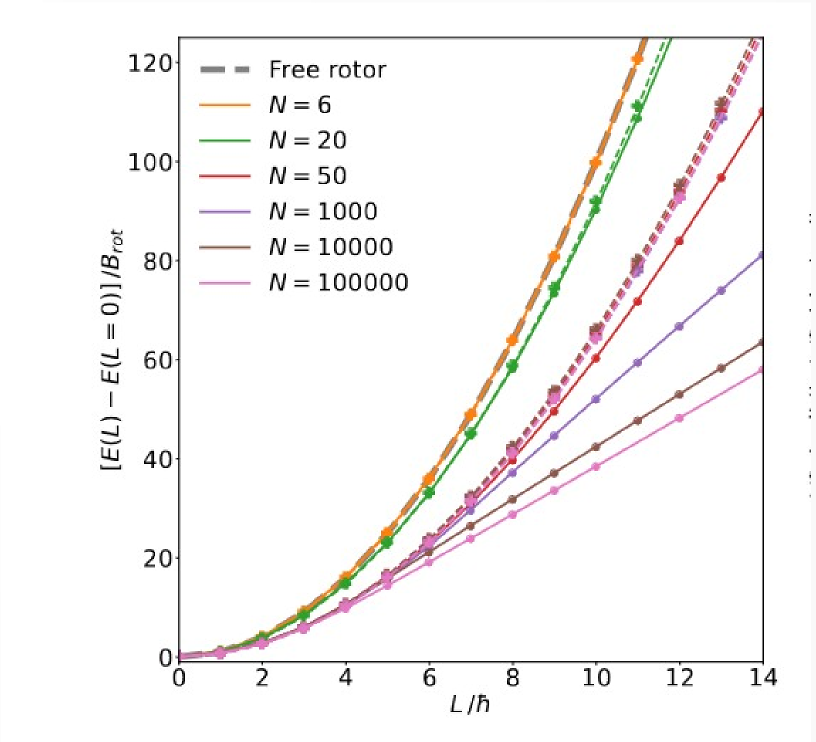
Outlook (critical velocity)



Outlook (angulon)







M. Suchorowski, A. Badamshina, M. Tomza, M. Lemeshko, **AGV** (in preparation)

Acknowledgements

Fabian Brauneis

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Areg Ghazaryan

TECHNISCHE UNIVERSITÄT DARMSTADT

DFG Deutsche Forschungsgemeinschaft

ISTA Institute of Science and Technology Austria

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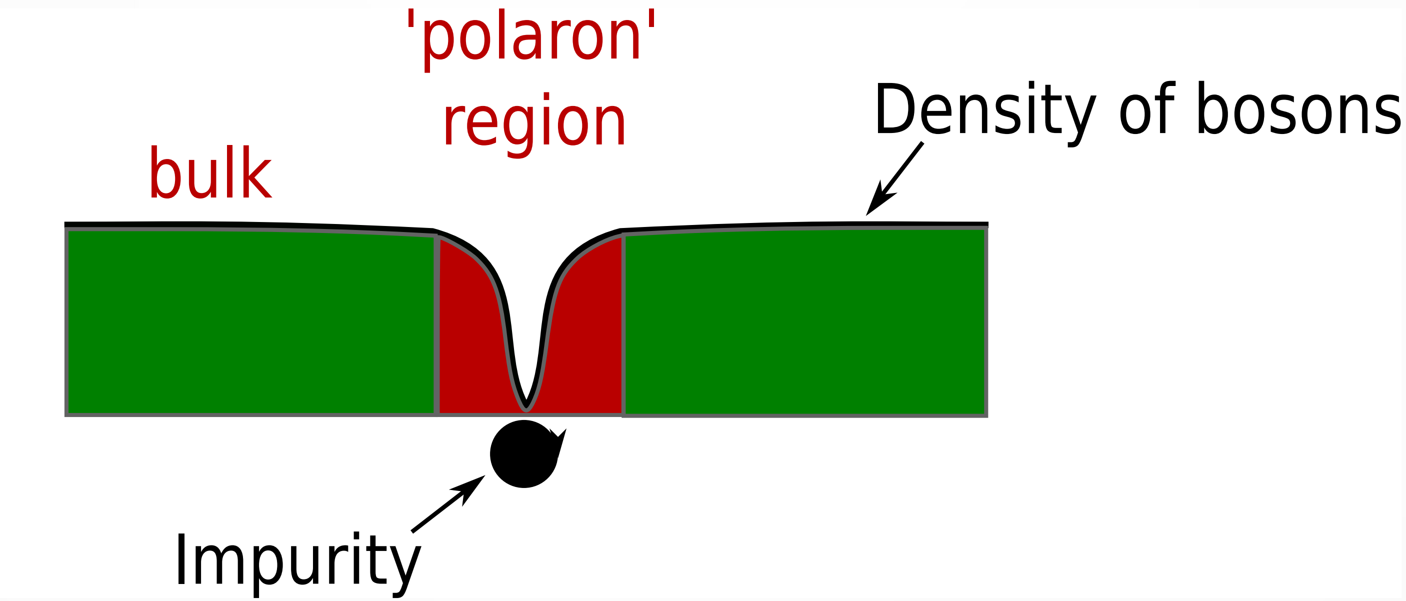
Areg Ghazaryan

Thank you!

Few-to-many-body



Few-to-many crossover (impurity) → physics in the ‘polaron’ region