Understanding collective behavior in beryllium isotopes

Anna E. McCoy

Facility for Rare Isotope Beams, Michigan State University
Washington University in St. Louis

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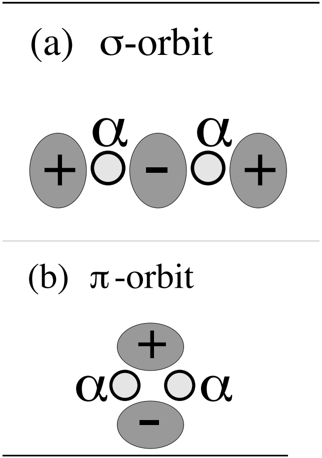
Neutron-rich beryllium isotopes

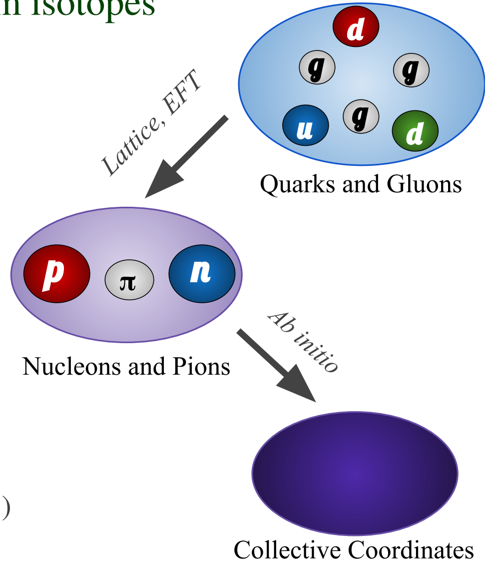
- Depicted as α dumbbell with valence neutrons
  Antisymmetrized molecular dynamics (AMD): valence neutrons in σ and π orbitals.
- Tension between shell effects and clustering
- Island of inversion near $N = 8$ shell closure.
  Parity inversion in ^{11}Be *and intruder ground state in* ^{12}Be.
- Appearance of shape coexistence
  Rotational bands with very different moment of inertia



Y. Kanada-En yo and H. Horiuchi. Phys. Rev. C **68 (2003) 014319**

Ab initio look at beryllium isotopes

- Effective field theory and lattice QCD provide a link between the quark scale and the nucleon scale.
- Solve nuclear many-body problem where protons and neutrons as degrees of freedom
  No-core shell model (NCSM)
- Signatures of collective behavior emerge in NCSM calculated spectrum
- Probe wave functions to gain insight into, e.g., underlying symmetries and correlations *Elliott's* SU(3)



No-core shell model

Solve many-body Schrodinger equation

$$\sum_{i}^{A} -\frac{\hbar^2}{2m_i}\nabla_i^2 \Psi + \frac{1}{2}\sum_{i,j=1}^{A} V(|r_i - r_j|)\Psi = E\Psi$$

Expanding wavefunctions in a basis

$$\Psi = \sum_{k=1}^{\infty} \alpha_k \phi_k$$

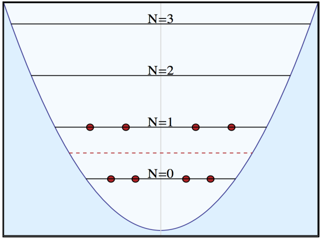
Reduces to Hamiltonian matrix eigenproblem

$$\begin{pmatrix} H_{11} & H_{12} & \dots \\ H_{21} & H_{22} & \dots \\ \vdots & \vdots & \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \end{pmatrix} = E \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \end{pmatrix}$$

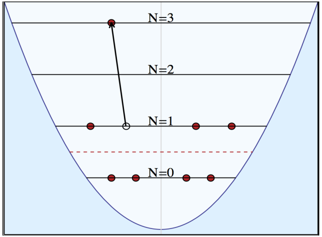
Harmonic oscillator basis

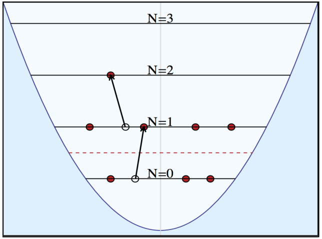
- Basis states are configurations, i.e., distributions of particles over harmonic oscillator shells (*nlj substates*)
- States are organized by total number of oscillator quanta above the lowest Pauli allowed number N_{ex}
- States with higher N_{ex} contribute less to the wavefunction
- Basis must be truncated:
  Restrict $N_{\mathrm{ex}} \leq N_{\max}$

$N = 2n + l$



$N_{\mathrm{ex}} = 0$



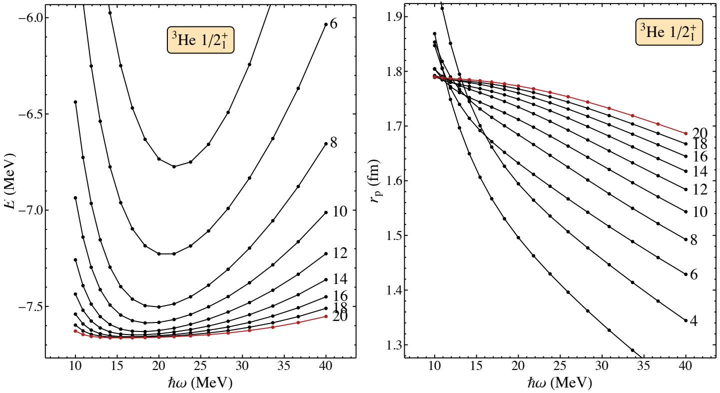


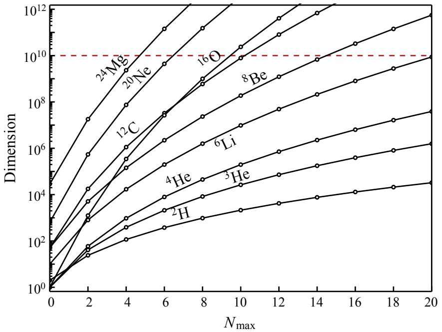
$N_{\mathrm{ex}} = 2$

Convergence Challenge

Results for calculations in a finite space depend upon:

- Many-body truncation $N_{\max}$
- Single-particle basis scale $\hbar\omega$





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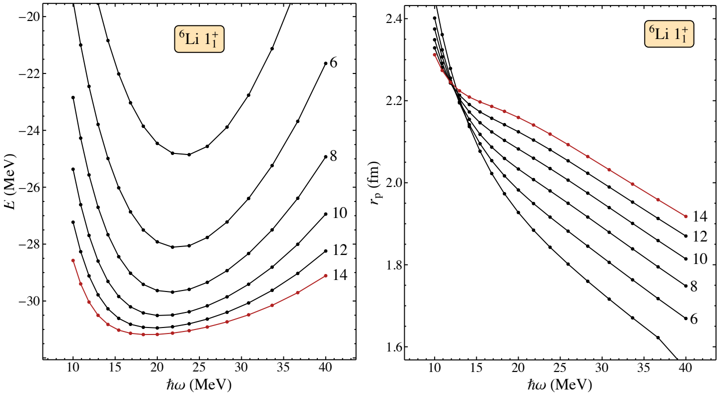
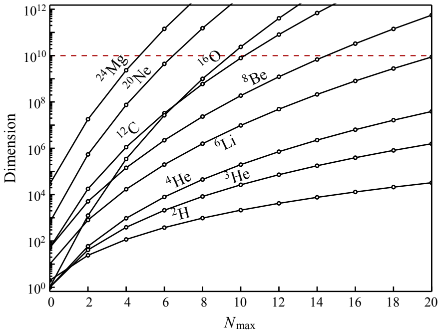




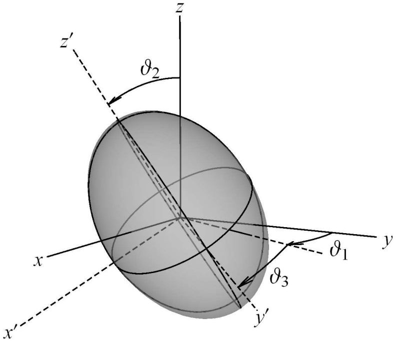
Even not fully converged, ab initio results can still tell us something about the underlying correlations

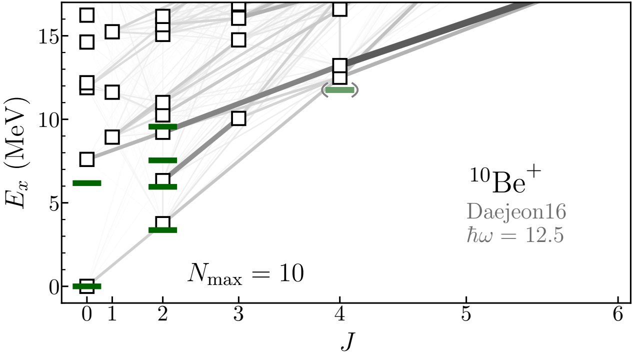
Nuclear rotations

Characterized by rotation of intrinsic state $|\phi_K\rangle$ by Euler angles ϑ $\quad (J = K, K+1, \ldots)$

$$|\psi_{JKM}\rangle \propto \int d\vartheta \left[\mathscr{D}^J_{MK}(\vartheta)|\phi_K;\vartheta\rangle + (-)^{J+K}\mathscr{D}^J_{M-K}(\vartheta)|\phi_{\bar{K}};\vartheta\rangle \right]$$

Rotational energy: $E(J) = E_0 + A[J(J+1)]$





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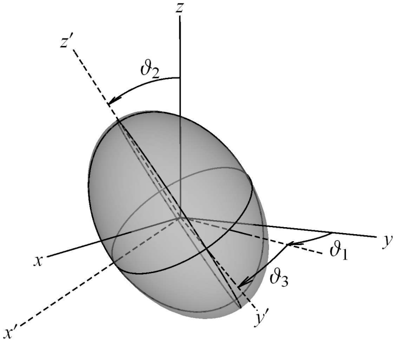
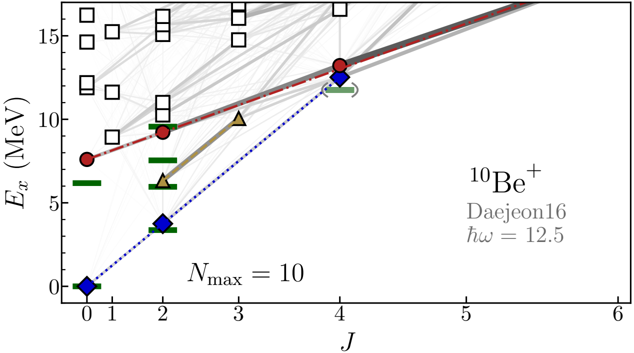
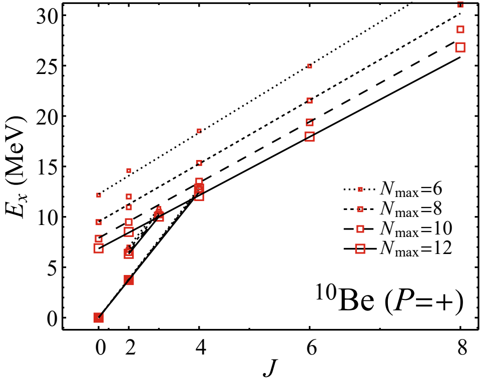
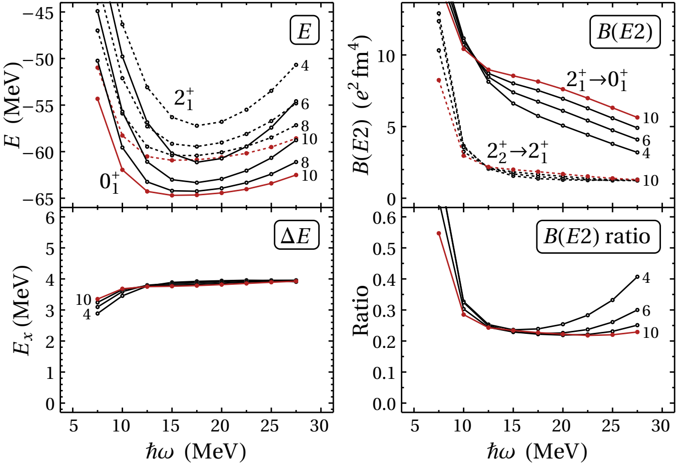




Robustness of band properties



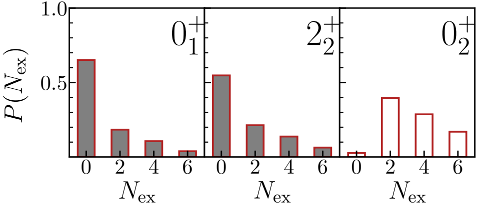


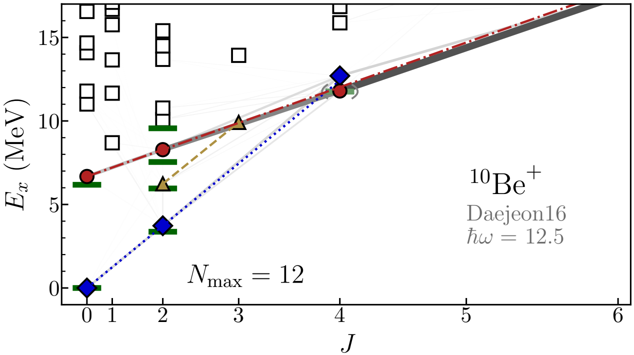
M. A. Caprio, P. J. Fasano, A. E. McCoy, P. Maris, J. P. Vary, Bulg. J. Phys. **46**, 455 (2019).

Probing underlying symmetries

– *Ab initio* calculations provides access to underlying wave functions of the collective states

– Using the "Lanczos trick" we can decompose the wave functions according to different symmetries
C. W. Johnson. Phys. Rev. C **91** *(2015) 034313.*





Elliott SU(3)

Elliott's SU(3): Irrep labels (λ, μ) associated with deformation parameters

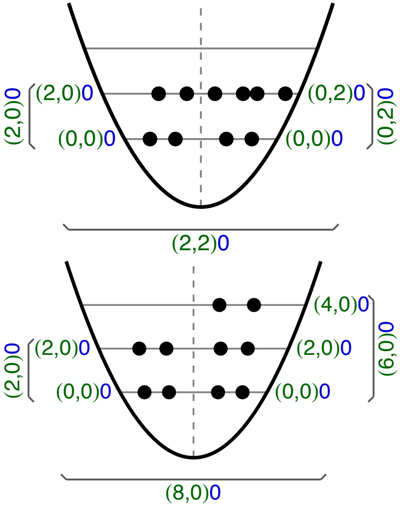
O. Castanos, J. P. Draayer, Y. Leschber, Z. Phys. A **329** *(1988) 3.*

$$\beta^2 \propto r^{-4}(\lambda^2 + \lambda\mu + \mu^2 + 3\lambda + 3\mu + 3)$$

$$\gamma \propto \tan^{-1}[\sqrt{3}(\mu+1)/(2\lambda+\mu)]$$

SU(3) symmetry of a configuration

- Each particle has SU(3) symmetry $(N, 0)$, $N = 2n + \ell$
- SU(3) couple particles to get total SU(3)
- Allowed spins dictated by antisymmetry constraints
- Final quantum numbers are $N_{\text{ex}}(\lambda\mu)S$.



Elliott's Rotational Model

- Bands arise from projecting out states with good L and K_L from intrinsic state with definite $(\lambda\mu)$

$$|(\lambda\mu)K_L L M_L\rangle$$

- Couple to spin to get good J states

$$L \times S \rightarrow J, \qquad K = K_L + K_S$$

- Lowest energies correspond to most deformed state

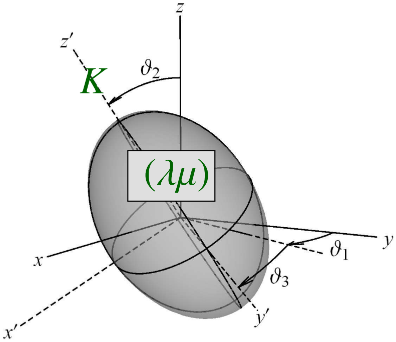
$$\beta^2 \propto \langle Q \cdot Q \rangle / \langle r^2 \rangle^2$$

$$H = \underbrace{H_0}_{\text{shell}} - \underbrace{\kappa Q \cdot Q}_{\text{correlations}}$$

D. J. Rowe, G. Thiamova, and J. L. Wood. Phys. Rev. Lett. **97** (2006) 202501.

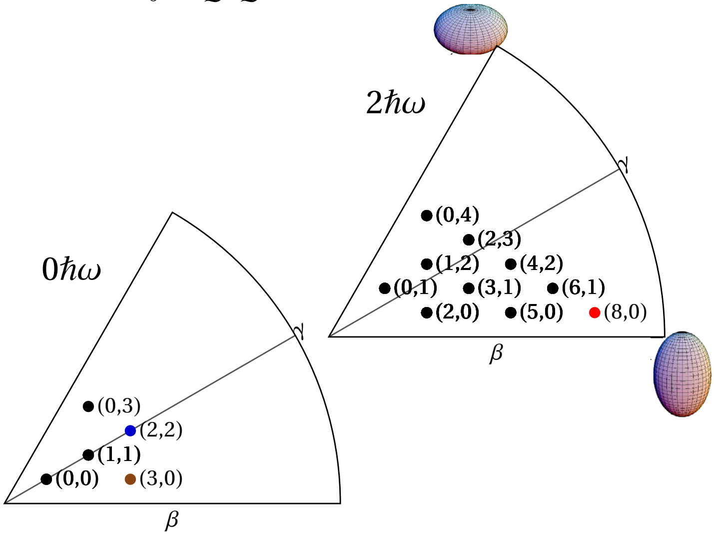
SU(3) generators

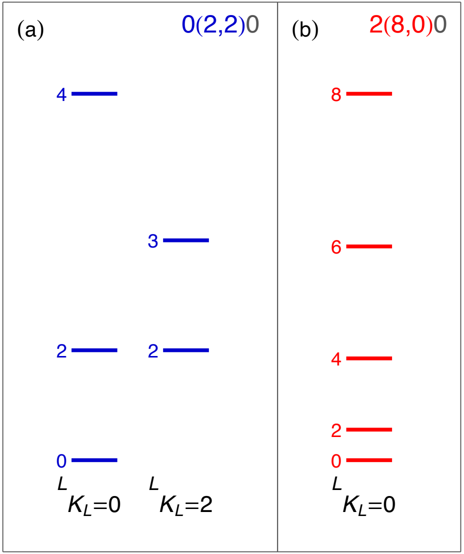
Q_{2M}	*Algebraic quadrupole*
L_{1M}	*Orbital angular momentum*



Elliott rotational bands: ^{10}Be

$$H = H_0 - \kappa Q \cdot Q$$





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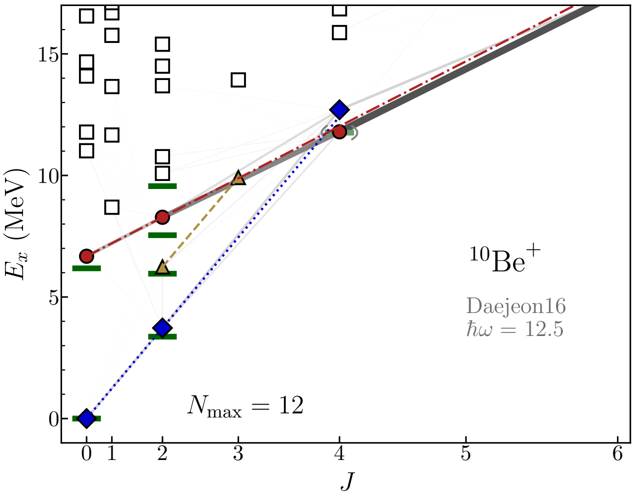

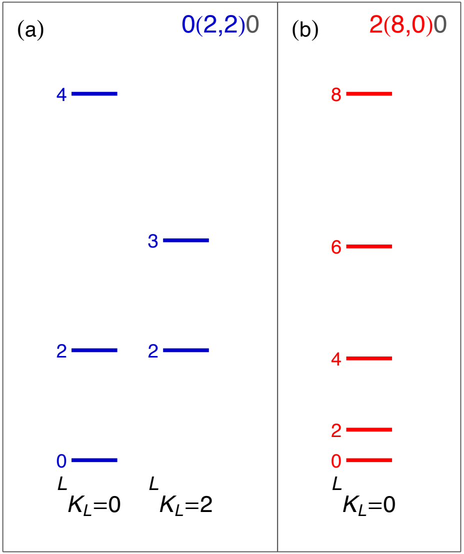

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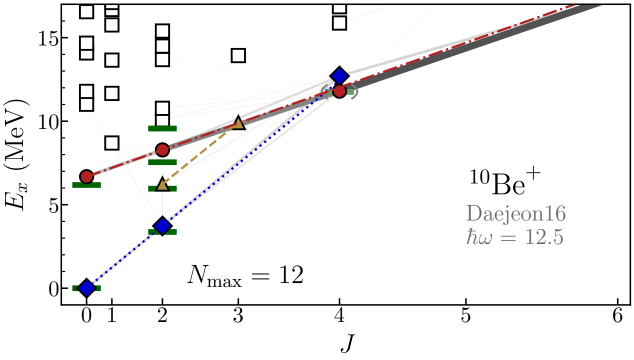
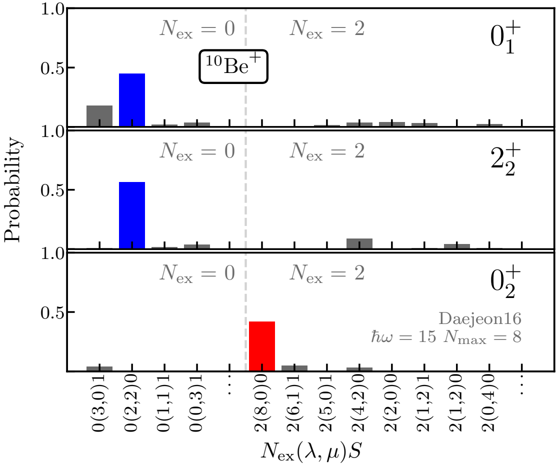
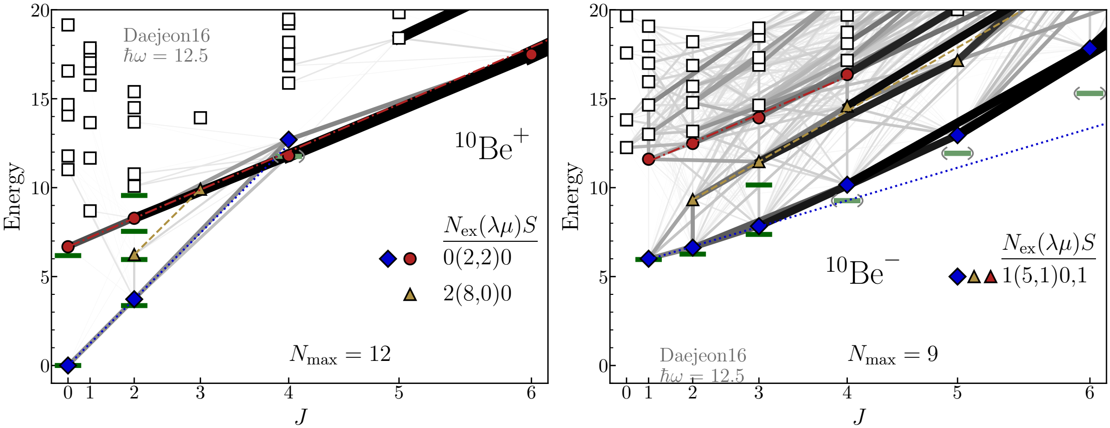


Approximate dynamical symmetry



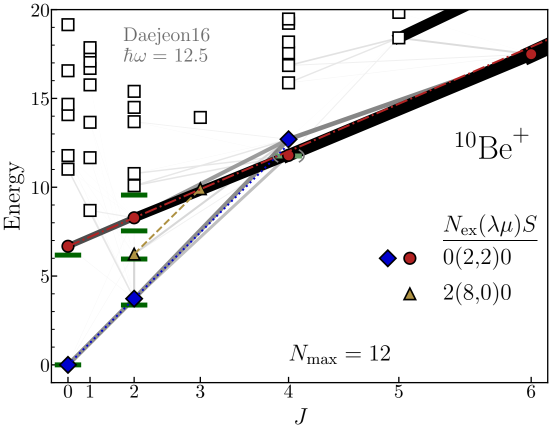
- Ground state band: $N_{\text{ex}}(\lambda\mu)S = 0(2,2)0$
- Intruder band: $N_{\text{ex}}(\lambda\mu)S = 2(8,0)0$

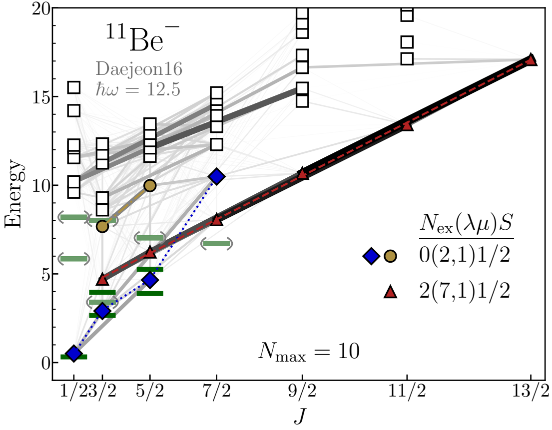
^{10}Be



Daejeon16
$\hbar\omega = 12.5$
^{10}Be^{+}
$N_{ex}(\lambda\mu)S$
0(2,2)0
2(8,0)0
$N_{max} = 12$
Energy
J
^{10}Be^{-}
1(5,1)0,1
$N_{max} = 9$

$^{10}\mathrm{Be}+n$





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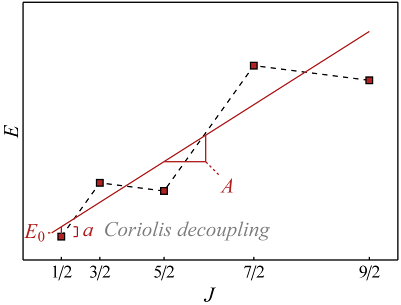
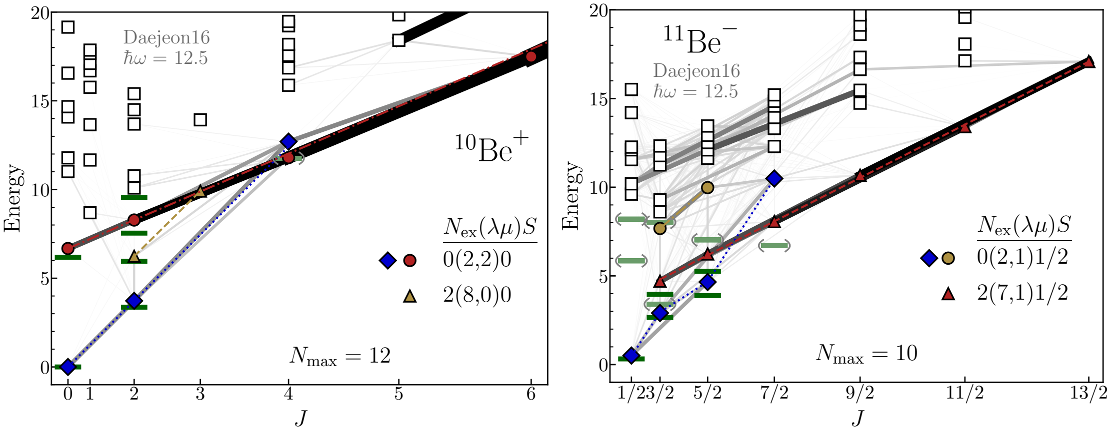
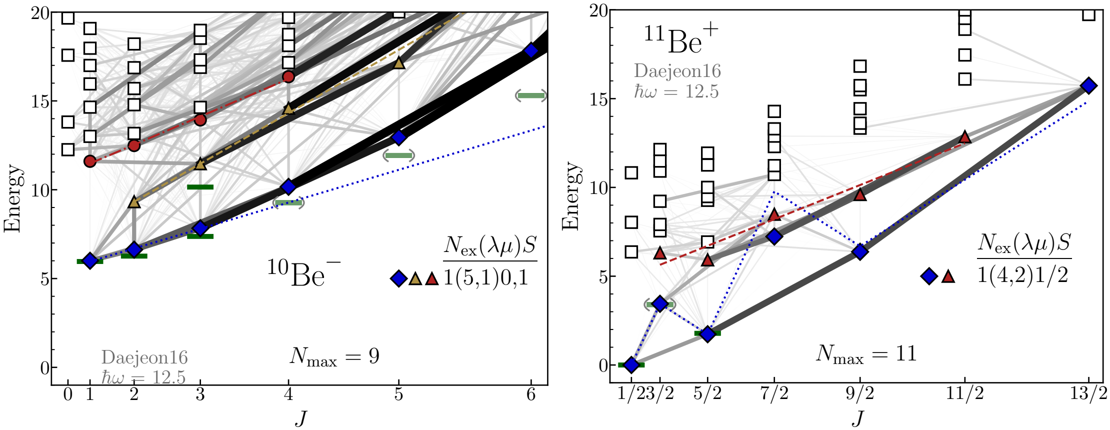


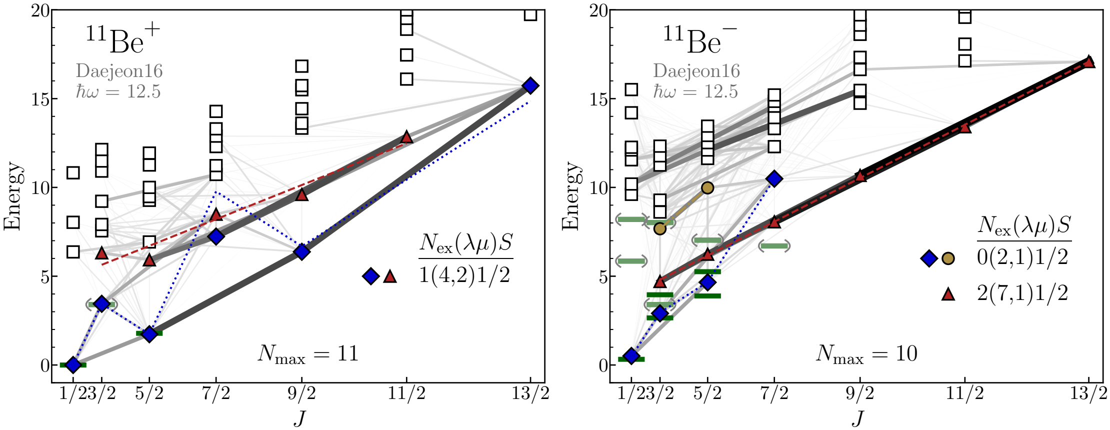
$^{10}\mathrm{Be} + n$



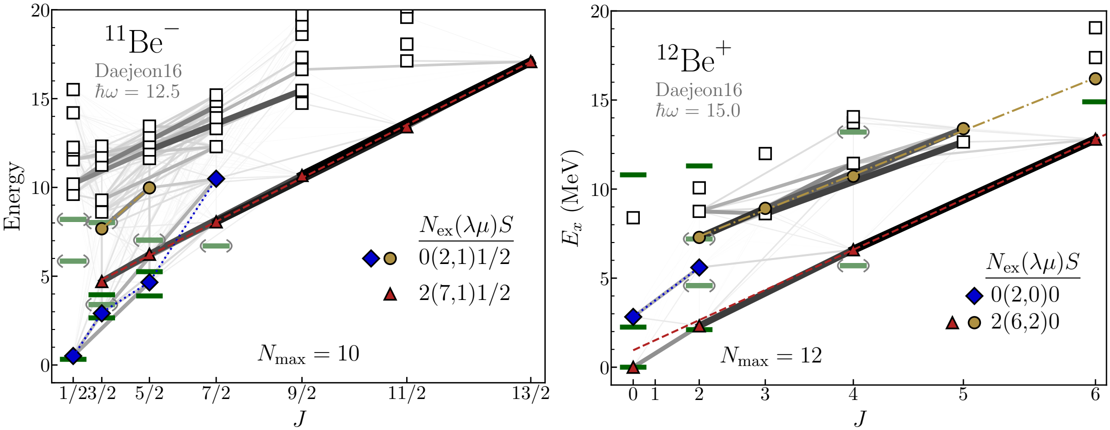
$^{10}\mathrm{Be}+n$



Parity inversion in ^{11}Be



$^{11}\mathrm{Be} + n$



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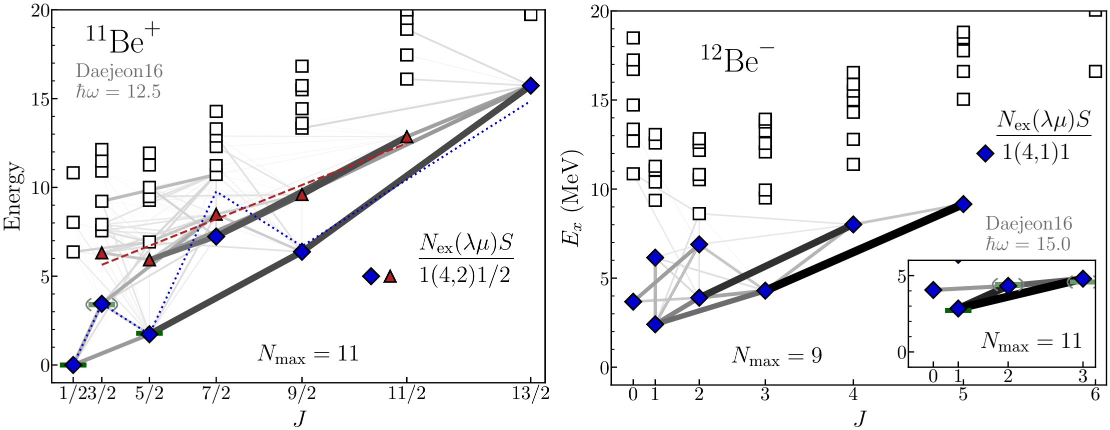
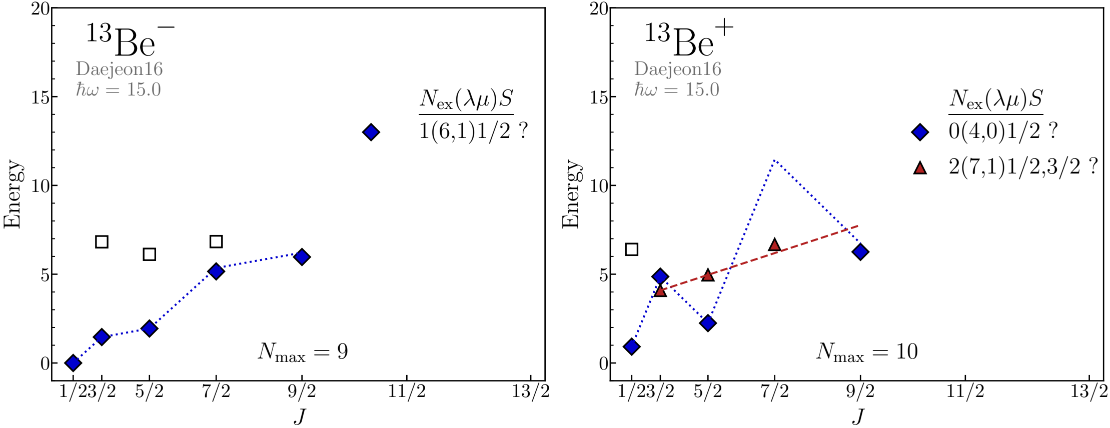
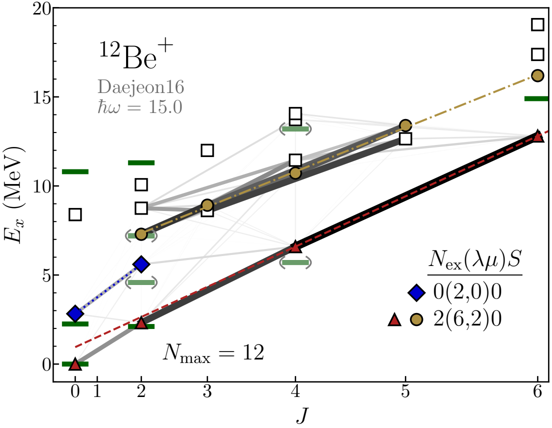


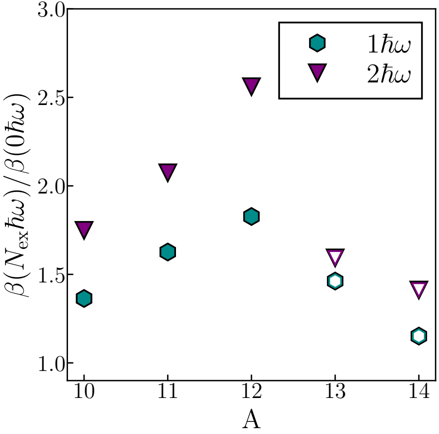
Parity inversion ^{13}Be



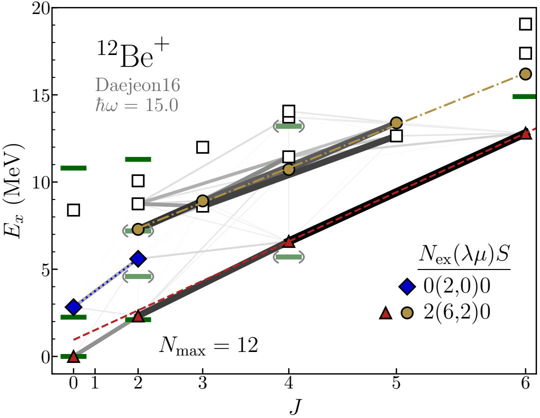
SU(3) shape coexistence

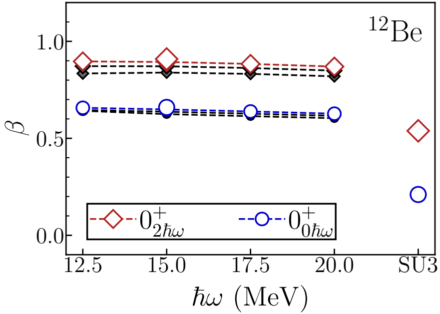


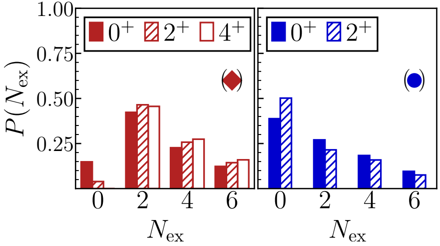
$$\beta^2 \propto r^{-4}(\lambda^2 + \lambda\mu + \mu^2 + 3\lambda + 3\mu)$$



SU(3) only approximate







Acknowledgements

In collaboration with...

Mark Caprio *Univ. Notre Dame*
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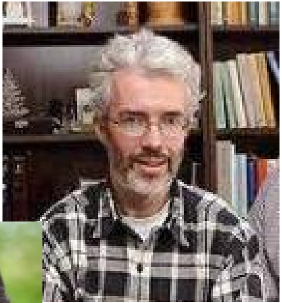
Pieter Maris *Iowa State Univ.*



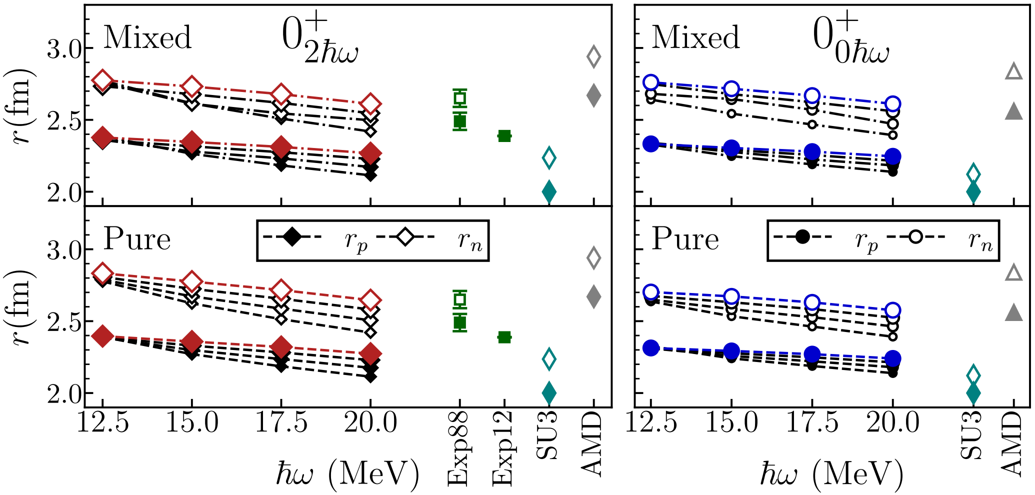
Summary

– Rotational bands emerge in *ab initio* predictions of In $^{10-13}$Be.

– Symmetry decompositions of calculated wave functions provide insight into the intrinsic structure of the states
Approximate SU(3) *symmetry*

– SU(3) provides a simple framework for competition between collectivity and shell effects
Shape coexistence, parity inversion and intruder ground state

^{12}Be radii


Mixed
$0^+_{2\hbar\omega}$
Mixed
$0^+_{0\hbar\omega}$
Pure
Pure
r_p r_n
r_p r_n
r (fm)
3.0
2.5
2.0
12.5
15.0
17.5
20.0
Exp88
Exp12
SU3
AMD
$\hbar\omega$ (MeV)

$$\beta^2 \sim \langle Q \cdot Q \rangle / \langle r^2 \rangle^2$$