

Chiral Symmetry and Nuclear Interactions



On the occasion of the
2021 Faddeev Medal Award



GORDON RESEARCH CONFERENCE
Proctor Academy
DYNAMICS OF SIMPLE SYSTEMS IN
CHEMISTRY & PHYSICS
R. Stephen Berry, chairperson
Colston Chandler, vice-chairperson
Aug. 11-16, 1996
Achber Studio, Laconia, NH

1990s: Chiral dynamics meets nuclear physics

EFFECTIVE CHIRAL LAGRANGIANS FOR NUCLEON-PION INTERACTIONS AND NUCLEAR FORCES

Steven WEINBERG*

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Received 2 April 1991

The general chiral invariant effective lagrangians is used to study the leading terms in powers of momenta in the S -matrix for a process involving arbitrary numbers of low-momentum pions and nucleons. This work extends and in part corrects an earlier report [S. Weinberg, Phys. Lett. B251 (1990) 288].

1. Introduction

Chiral invariant effective lagrangians were originally introduced as a labor-saving device, allowing a quick and easy derivation of the soft pion theorems that had earlier been deduced by the methods of current algebra. The lagrangians were highly non-linear and non-renormalizable, but this was not a problem because in those early days the lagrangians were only supposed to be used in the tree approximation. Later it was realized that effective chiral lagrangians could be used to calculate soft pion processes to any desired order in the pion energies, including loop as well as tree graphs; all ultraviolet divergences could be absorbed into a redefinition of the coupling constants of the lagrangian, provided one included in the lagrangian all terms consistent with chirality and other symmetry principles.

This widened view of the use of chiral effective lagrangians was first described in the context of purely pionic processes. It would clearly be to our advantage to be able to apply these methods to processes involving low-momentum nucleons as well as pions, where much more experimental data is available, but it takes a little

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S. Weinberg / Nucleon-pion interactions

preliminary treatment of such problems has already been presented*. Here we will give a fuller account, including a revised discussion of the case of three or more nucleons.

The picture of nuclear forces that emerges in the leading order in small momenta is quite crude, though not entirely unrealistic. The purpose of this work is not to improve our detailed picture of nuclear forces, which it is hardly likely could be accomplished with these methods, but rather to take a first step toward identifying those aspects of nuclear forces and pion-nucleon interactions that can be derived from the symmetry properties of quantum chromodynamics. An assessment of the success of these methods will have to wait for the evaluation of the

EFFECTIVE CHIRAL THEORY OF NUCLEAR FORCES

Lectures at the 7th Summer School & Symposium on Nuclear Physics
Seoul, June 1994

U. van KOLCK
Department of Physics, FM-15
University of Washington
Seattle, WA 98195

Abstract

An introduction to the concept of effective Lagrangians is presented, as it applies to QCD at low energies. The general chiral Lagrangian is used in a systematic expansion to derive nuclear forces in terms of a number of parameters that carry information about QCD dynamics. It is shown that the main features of nuclear physics regarding two- and few-body forces and isospin violation arise in a natural way.

1 Introduction

Nowadays there is little doubt that QCD is the correct theory of the strong forces that bind quarks into baryons and mesons and yet, despite many efforts, it has not been possible to derive from it the interaction among these bound states. At the root of the problem it is of course our inability to solve the dynamics of QCD at hadronic scales. Quantitative descriptions of hadronic properties and interactions have been achieved only in phenomenological models whose connections with QCD are unclear. One can pursue a more general alternative program, however, which is firmly based on symmetries and parametrizes ignorance of details of the dynamics in a number of undetermined constants.

In other words, I presented evidence to substantiate the hope that the chiral Lagrangian is the Lagrangian for nuclear physics. Of course, we are just at the beginning of the exploration of this paradigm and much is still to be done before we can make this claim. A list of possible future work includes: i) a more complete analysis of πN scattering, which would fix some of the parameters that were free in the NN fit of Sect.4.2; ii) a fit of parameters of the $3N$ potential to $3N$ data, which would completely determine the potential, and allow predictions for other light nuclei;

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Dear Prof. G. Bickel:
It was a pleasure talking
you in Staz.
I hope you find these
enjoyable.
Best regards,
Bickel

1990s: Chiral dynamics meets nuclear physics

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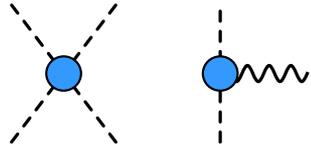
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So, what have we learned about the role of the χ symmetry?

Chiral Effective Field Theory

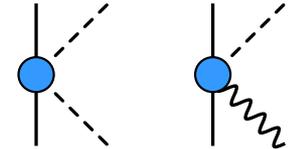
GB dynamics

Weinberg, Gasser, Leutwyler, ...



πN dynamics

Bernard-Kaiser-Meißner et al.



Chiral Perturbation Theory

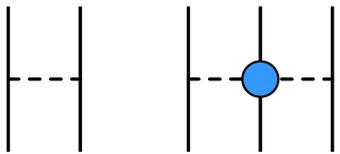
$$Q = \frac{\text{momenta of particles or } M_\pi}{\text{breakdown scale } \Lambda_b} \sim \frac{1}{4} \dots \frac{1}{3}$$

Effective Lagrangian:

$$\begin{aligned} \mathcal{L}_\pi &= \frac{F^2}{4} \text{Tr}(\nabla^\mu U \nabla_\mu U^\dagger + \chi_+) + \dots, \\ \mathcal{L}_{\pi N} &= \bar{N}(i v \cdot D + g_A u \cdot S) N + \dots, \\ \mathcal{L}_{NN} &= -\frac{1}{2} C_S (\bar{N} N)^2 + 2 C_T (\bar{N} S N)^2 + \dots \end{aligned}$$

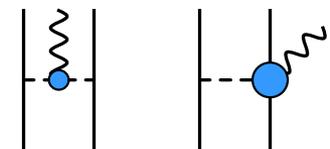
Nuclear forces

Weinberg, van Kolck, Kaiser, EGM, ...



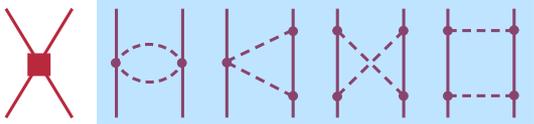
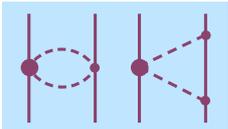
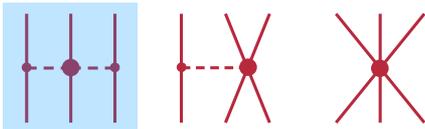
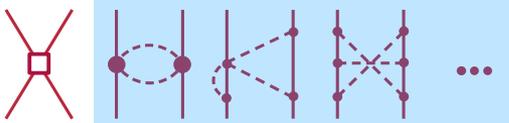
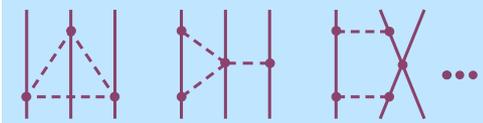
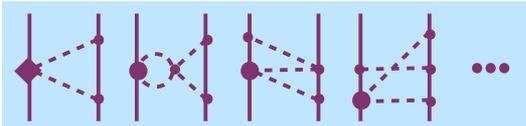
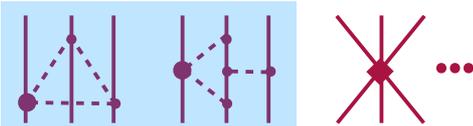
Nuclear currents

Park et al, Bochum-Bonn, JLab-Pisa



Enhanced ladder graphs are re-summed
by solving the many-body Schrödinger equation

Chiral expansion of the nuclear forces [NDA, DimReg]

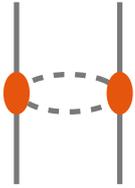
	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Q^0)			
	<small>Weinberg '90</small>		
NLO (Q^2)			
	<small>Ordonez, van Kolck '92</small>		
N ² LO (Q^3)			
	<small>Ordonez, van Kolck '92</small>	<small>van Kolck '94; EE et al. '02</small>	
N ³ LO (Q^4)			
	<small>Kaiser '00 - '02</small>	<small>Bernard, EE, Krebs, Meißner, '08, '11</small>	
N ⁴ LO (Q^5)			
	<small>Entem, Kaiser, Machleidt, Nosyk '15 EE, Krebs, Meißner '15</small>	<small>Girlanda, Kievsky, Viviani '11 Krebs, Gasparyan, EE '12, '13 (short-range loop contrib. still missing)</small>	

— explains the observed hierarchy of nuclear forces Weinberg, van Kolck, Friar

— chiral dynamics: long-range interactions are predicted in terms of on-shell amplitudes



Two-pion exchange and the πN amplitude



← exchanged pions can become on-shell for $q^2 \leq -(2M_\pi)^2$

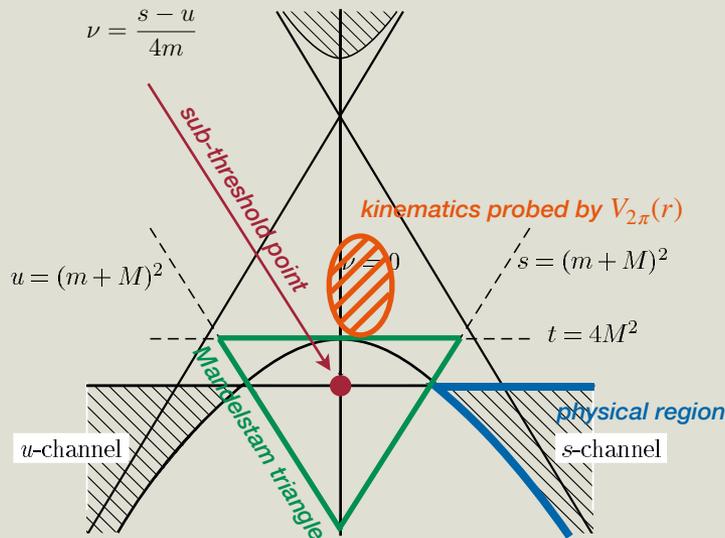
⇒ $V_{2\pi}(q^2)$ are analytic functions except for the branch cut $q^2 \in (-\infty, -4M_\pi^2]$

$$\Rightarrow V_{2\pi}^c(q^2) = \underbrace{\frac{2}{\pi} \int_{2M_\pi}^{\infty} d\mu \mu \frac{\rho(\mu)}{q^2 + \mu^2}}_{\text{in practice, subtractions are needed to make the integral convergent}} \quad \text{where } \rho(\mu) = \text{Im}[V_{2\pi}^c(q = 0^+ - i\mu)]$$

in practice, subtractions are needed to make the integral convergent

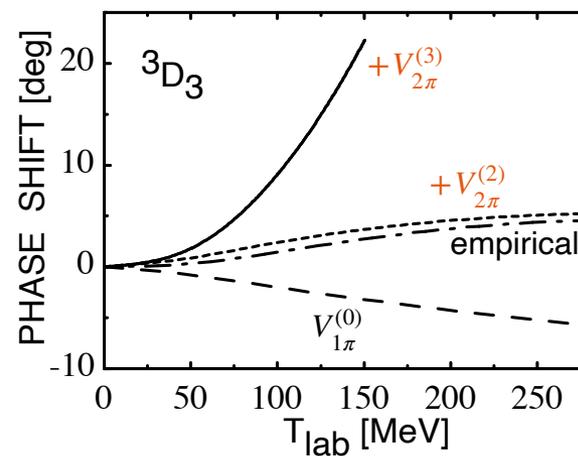
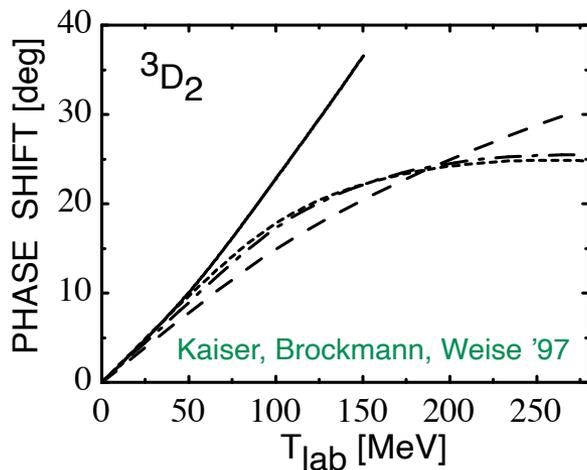
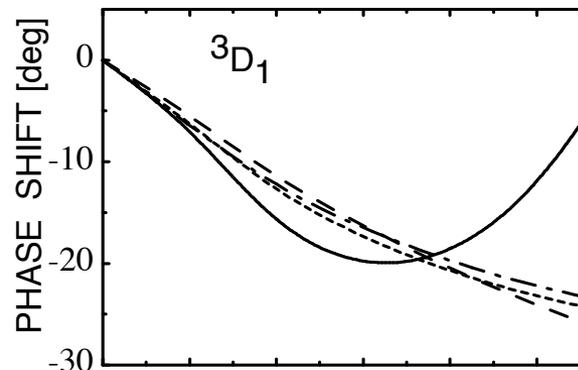
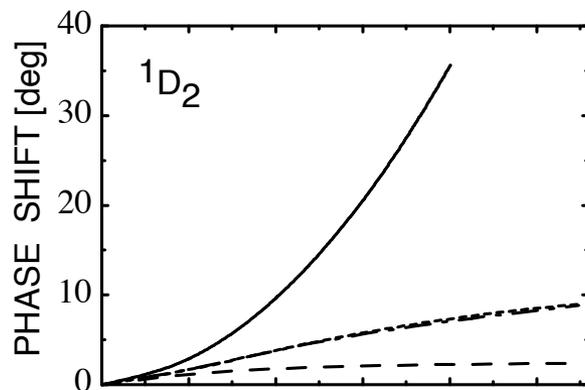
The spectral functions $\rho(\mu)$ determine the long-range behavior of $V_{2\pi}(r)$ and can be calculated from the **on-shell πN amplitude** (ChPT) using Cutkosky cutting rules (Norbert Kaiser, 1999)

Kinematic regions for πN scattering



- πN amplitude from the numerical solution of the dispersive Roy-Steiner equations
Ditsche et al., 2012
- Extract LECs from matching ChPT at the **sub-threshold point** Hoferichter et al., 2015
- Closer to the **kinematics probed by $V_{2\pi}(r)$** than the physical region
- Beyond HBChPT: $\Delta, 1/m$ Siemens et al., 2017
→ also talks by Tae-Sun Park and Xiu-Lei Ren

Probing the 2π -exchange in peripheral NN scattering



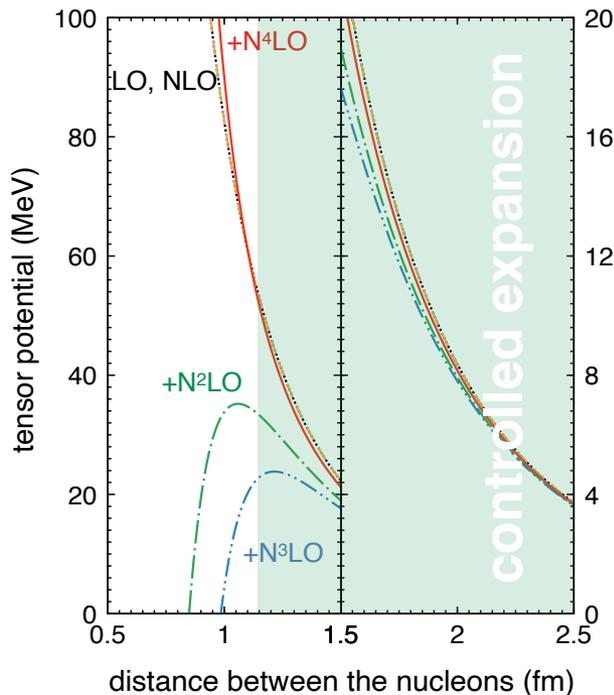
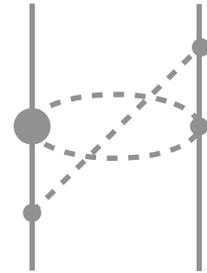
The sub-leading (i.e., N^2LO , Q^3) 2π -exchange is a way too attractive... Predictive power?

The long and short of the nuclear force

- m-pion exchange starts contributing at order Q^n , $n \geq 2m - 2$
- static contributions at order Q^n lead to local potentials of the form

$$V_{m\pi}^{(n)}(\mathbf{r}) = \frac{M_\pi^3}{F_\pi^2} \left[\frac{M_\pi}{\Lambda_b} \right]^n f(x) O_{\text{spin}}(\hat{\mathbf{r}}) O_{\text{isospin}}$$

$\xrightarrow{M_\pi r}$ $f(x)$ $\xrightarrow{\text{set by } 4\pi F_\pi, \sqrt{4\pi F_\pi} \text{ and/or scales entering } \pi\text{N LECs}}$



The functions $f(x)$ are predicted within ChPT, e.g.: $f_{1\pi, \text{tensor}}(x) = x^{-1} + 3x^{-2} + 3x^{-3}$

— controlled ChPT expansion for $r \gtrsim M_\pi^{-1}$:
 $f(x) \sim \mathcal{O}(1)$

— meaningless for $r \lesssim \Lambda_b^{-1}$ as it is driven by $\rho(\mu)$ with $\mu \sim \Lambda_b$:

$$f(x) \xrightarrow{x \ll 1} x^{-(3+n)}$$

(The short-distance part of the force is modeled by all possible contact terms.)

Unveiling the chiral symmetry

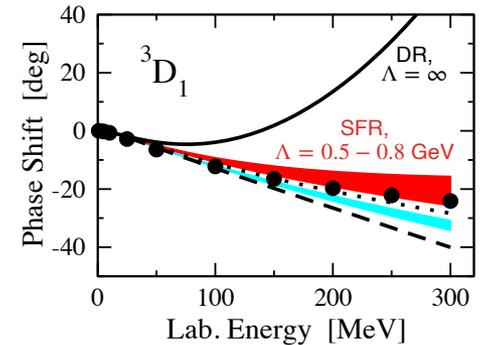
2003: The actual trouble-maker is the (uncontrolled) **short-range** part of the TPEP

⇒ spectral-function regularization EE, Glöckle, Meißner '03

$$V_{2\pi}(q) = \frac{2}{\pi} \int_{2M_\pi}^{\infty} d\mu \mu \frac{\rho(\mu)}{\mu^2 + q^2} \rightarrow \frac{2}{\pi} \int_{2M_\pi}^{\infty} d\mu \mu \frac{\rho(\mu)}{\mu^2 + q^2} f_\Lambda(\mu)$$

+ additional non-local regulator in p, p'

Used in the 2004 EE-Glöckle-Meißner and in 2017 Entem-Machleidt-Nosyk potentials.



2014: The SFR removes the unphysical short-range components but one also distorts some good long-range physics... Can one do better?

⇒ local regularization in r-space EE, Krebs, Meißner, EPJA 51 (15) 53
PRL 115 (15) 122301



2018: The r-space regulator turned out to be inconvenient for 3N forces and currents

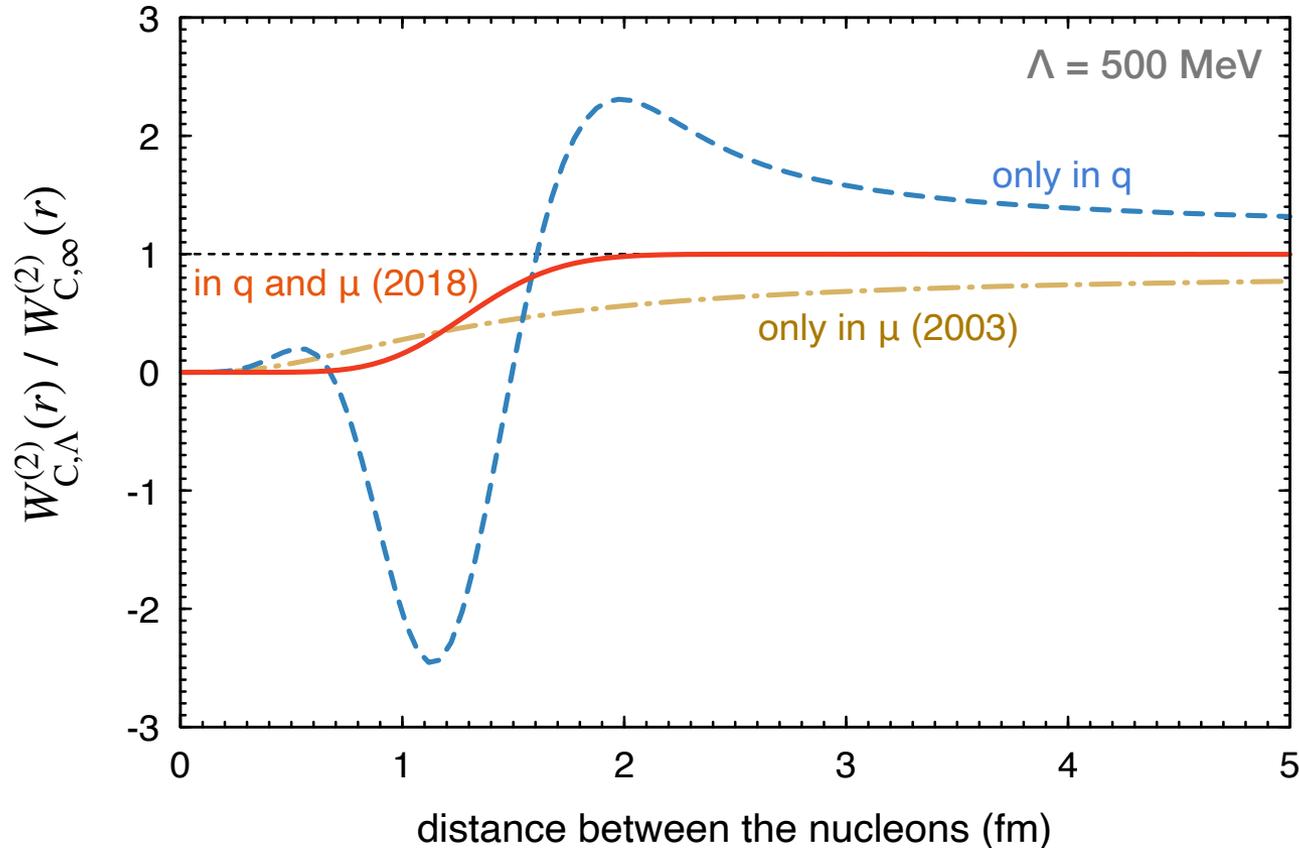
⇒ local regularization in momentum space Reinert, Krebs, EE, EPJA (18) 85

$$V_{1\pi}(q) = \frac{\alpha}{\vec{q}^2 + M_\pi^2} e^{-\frac{\vec{q}^2 + M_\pi^2}{\Lambda^2}} + \text{subtraction,} \quad V_{2\pi}(q) = \frac{2}{\pi} \int_{2M_\pi}^{\infty} d\mu \mu \frac{\rho(\mu)}{\vec{q}^2 + \mu^2} e^{-\frac{\vec{q}^2 + \mu^2}{2\Lambda^2}} + \text{subtractions}$$

Unveiling the chiral symmetry

Regularized 2π -exchange potential (central isospin-dependent part of $\left[\begin{array}{c} | \text{---} | \\ | \text{---} | \end{array} \right] + \left[\begin{array}{c} | \text{---} | \\ | \times \text{---} | \end{array} \right]$):

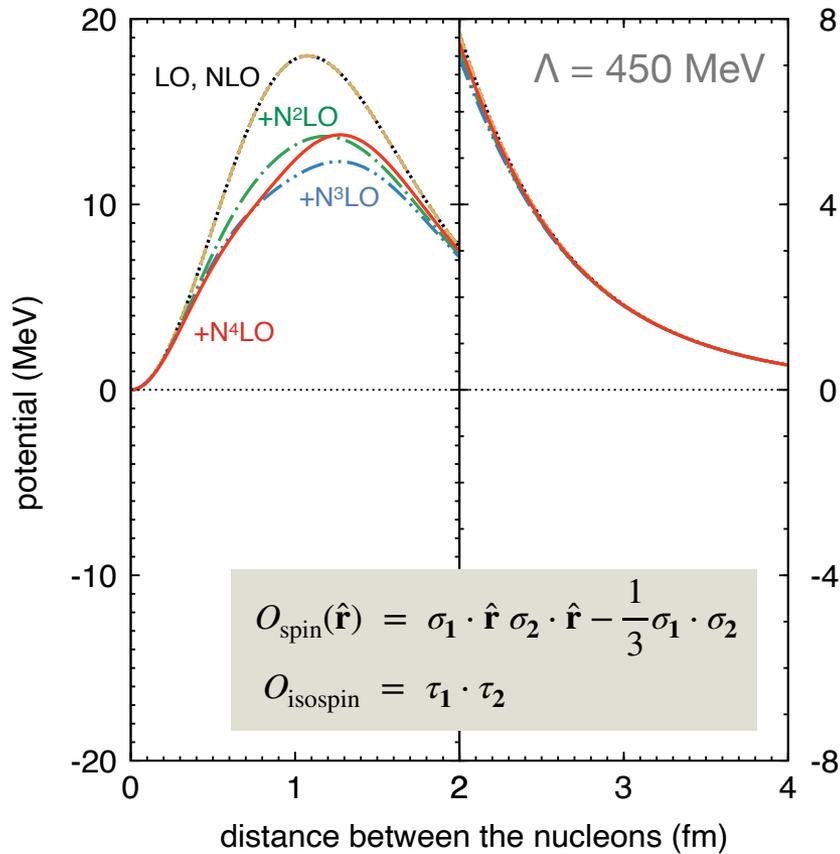
Various regularization approaches



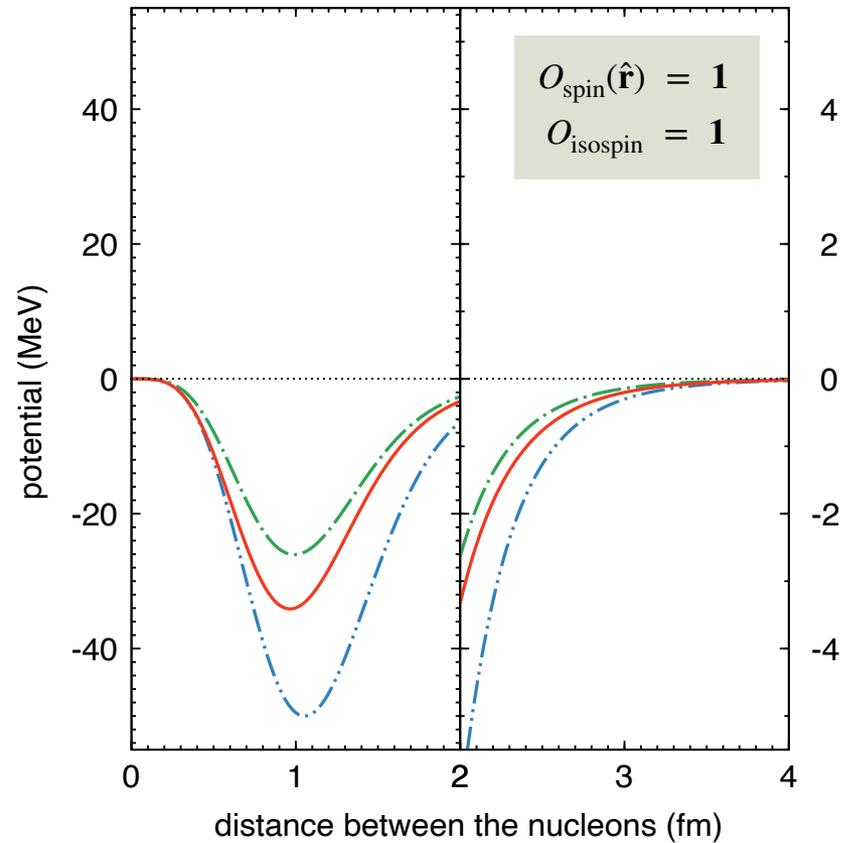
$$V_{2\pi}(q) = e^{-\frac{\vec{q}^2}{2\Lambda^2}} \frac{2}{\pi} \int_{2M_\pi}^{\infty} d\mu \mu \frac{\rho(\mu)}{\vec{q}^2 + \mu^2} e^{-\frac{\mu^2}{2\Lambda^2}} + \text{subtractions}$$

Unveiling the chiral symmetry

Isovector tensor potential



Isoscalar central potential



The two-nucleon system

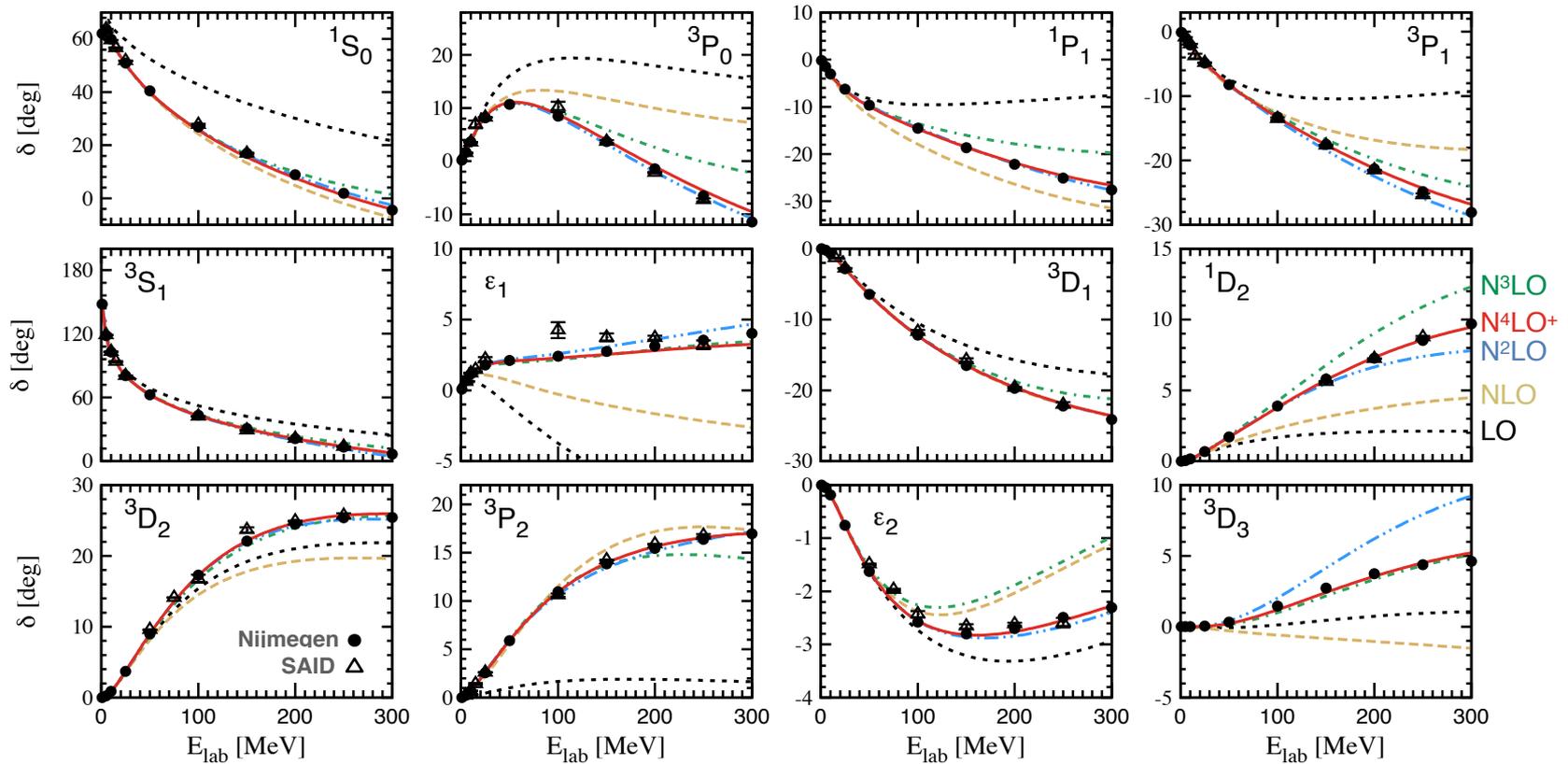
Results for $\Lambda = 450$ MeV

from: P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

	LO (Q^0)	NLO (Q^2)	N ² LO (Q^3)	N ³ LO (Q^4)	N ⁴ LO (Q^5)	N ⁴ LO ⁺
χ^2/datum (np, 0 – 300 MeV)	75	14 no new LECs	4.1	2.01	1.16	1.06
χ^2/datum (pp, 0 – 300 MeV)	1380	91 no new LECs	41	3.43	1.67	1.00
	2 LECs	+ 7 + 1 IB LECs		+ 12 LECs	+ 1 LEC (np)	+ 4 LECs

Beautiful evidence of the parameter-free chiral 2π -exchange!
(similar results found by Rentmeester et al. and by Birse et al.)

Chiral expansion of the neutron-proton phase shifts [$\Lambda = 450$ MeV]



Renormalization and Cutoff

- in HBChPT, the amplitude can be made regulator-independent at any order
- but for NN, we need to re-sum pion-exchange iterations...

Renormalization: Exact Λ -independence at every order, $\Lambda \gg \Lambda_b$
→ dictates power counting
van Kolck, Long, Yang, Valderrama ...

- consistent in the EFT sense?
EE, Gegelia, Meißner, Gasparyan
- achievable at all?
Gasparyan, EE, PRC 107 (23) 034001

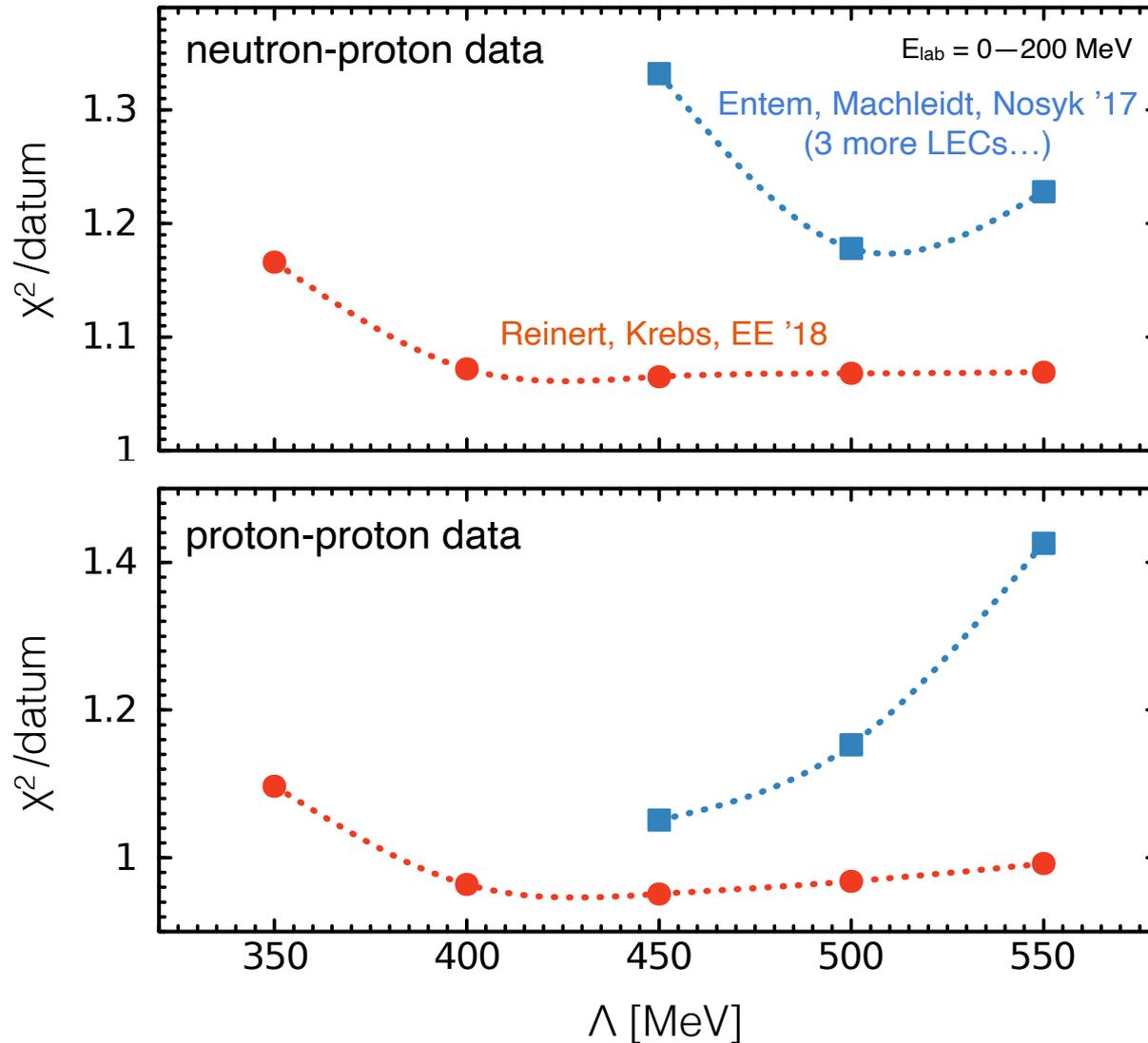
Renormalization: Exact Λ -independence only at ∞ order, $\Lambda \sim \Lambda_b$
Lepage, EE, Gegelia, Meißner, ...

- cutoff independence proven?
Towards a formal proof:
Gasparyan, EE, PRC 105 (22) 024001
PRC 107 (23) 044002
→ talks by A. Gasparyan and J. Gegelia

Still under debate, see Ingo Tews et al., „Nuclear Forces for Precision Nuclear Physics: A Collection of Perspectives“, FBS 63 (2022)

Regulator (in)dependence

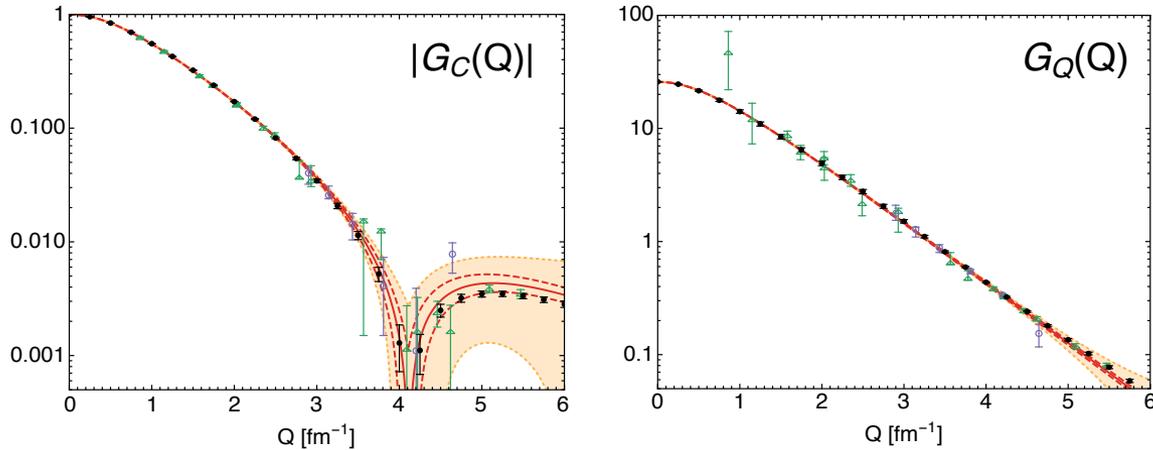
χ^2/datum for the description of the NN data in the range of 0 – 200 MeV at N⁴LO+



Precision physics with χ EFT: Deuteron FFs

Filin, Möller, Baru, EE, Krebs, Reinert, PRL 124 (2020) 082501; PRC 103 (2021) 024313

Charge and quadrupole form factors of the deuteron at N⁴LO



Extracted quadrupole moment:

$$Q_d = 0.2854^{+0.0038}_{-0.0017} \text{ fm}^2$$

EFT truncation, choice of fitting range,
NN, π N and γ NN LECs

to be compared with experiment

$$Q_d^{\text{exp}} = 0.285\,699(15)(18) \text{ fm}^2$$

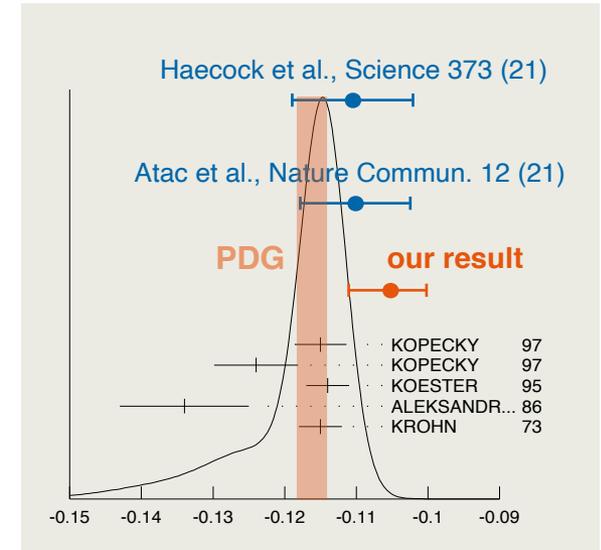
Puchalski et al., PRL 125 (2020)

The charge and **structure radius**:

$$r_d^2 = (-6) \left. \frac{\partial G_C(Q^2)}{\partial Q^2} \right|_{Q^2=0} = r_{\text{str}}^2 + r_p^2 + r_n^2 + \frac{3}{4m_p^2}$$

Combining our result $r_{\text{str}} = 1.9729^{+0.0015}_{-0.0012} \text{ fm}$ with very precise isotope-shift spectroscopy data for $r_d^2 - r_p^2$, we determine the neutron m.s. charge radius:

$$r_n^2 = -0.105^{+0.005}_{-0.006} \text{ fm}^2$$



The three-nucleon force challenge

Precision physics beyond $A = 2$?

The 3NF challenge: No Hamiltonian exists that can describe both the 2N and 3N data!

→ talk by Kimiko Sekiguchi

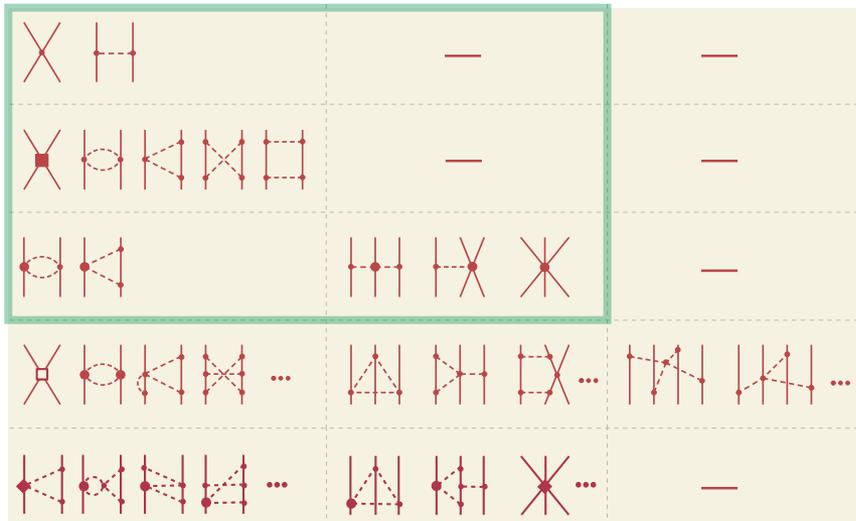
Energy	potential	A_y	iT_{11}	T_{20}	T_{21}	T_{22}
10 MeV	AV18	288	29	10	6.2	24
	AV18+UR	224	23	13	6.1	7.6

← χ^2/datum (Nd scattering)

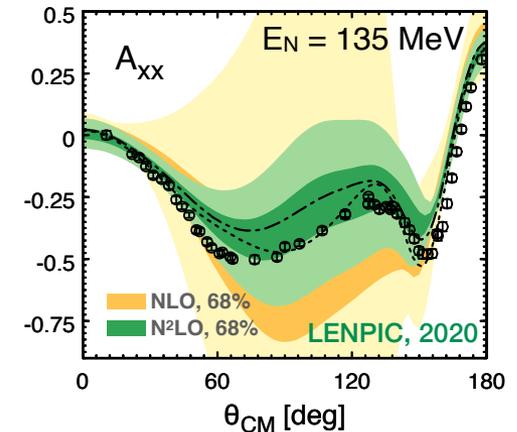
Alejandro Kievsky, in I. Tews et al., FBS 63 (22) 4, 67

Why is it so difficult to phenomenologically parametrize the 3NF?

Computational cost [→ emulators] + scarce data base [→ talk by Kimiko] + extremely rich structure [→ theory needed] ⇒ a great opportunity for χ EFT



N²LO →

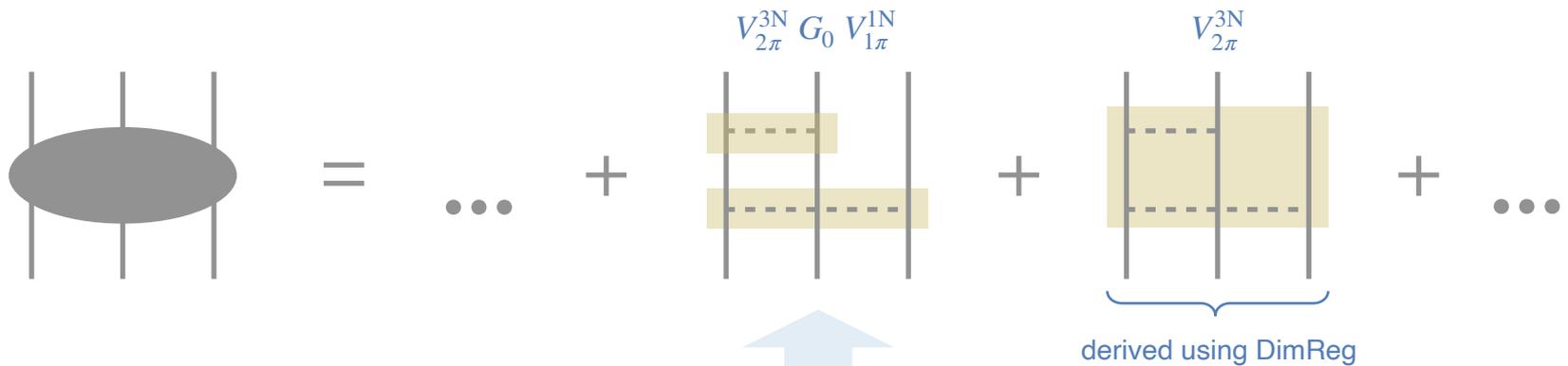


⇒ looks promising, but not yet high-precision...

3NF and the chiral symmetry

Chiral symmetry is even more important than for the 2NF (2π -exchange is the *leading* and *longest-range* 3NF mechanism). But do we have it under control?

Faddeev equation for 3N scattering:



$$-\Lambda \frac{g_A^4}{96\sqrt{2\pi^3} F_\pi^6} \left[\underbrace{\tau_1 \cdot \tau_3 (\vec{q}_3 \cdot \vec{\sigma}_1)}_{\text{absorbable into } c_D: \times} - \underbrace{\frac{4}{3} (\tau_2 \cdot \tau_3 - \tau_1 \cdot \tau_3) (\vec{q}_2 \cdot \vec{\sigma}_3)}_{\text{violates chiral symmetry}} \right] \frac{\vec{q}_3 \cdot \vec{\sigma}_3}{q_3^2 + M_\pi^2} + \dots$$

⇒ mixing DimReg and Cutoff regularization breaks chiral symmetry

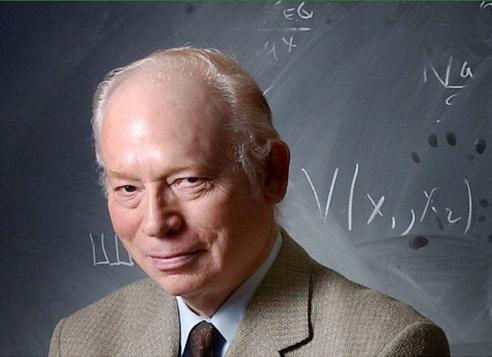
3NF and the chiral symmetry

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Q^0)			
NLO (Q^2)			
N ² LO (Q^3)			
N ³ LO (Q^4)			
N ⁴ LO (Q^5)			

While NN interactions are not affected (at a fixed M_π), 3NF, 4NF and exchange currents must be re-derived using a symmetry-preserving regulator [Hermann Krebs, EE, work in progress](#)

→ new path integral approach using gradient flow regularization [Lüscher, 2010](#)

Instead of a summary



„The purpose of this work is not to improve our detailed picture of nuclear forces, which is hardly likely could be accomplished with these methods, but rather to take a first step toward identifying those aspects of nuclear forces [...] that can be derived from the symmetry properties of quantum chromodynamics.“

Steven Weinberg, NPB 363 (1991)

- Good quantitative understanding achieved for the 2N interaction: Clear evidence of the 2π -exchange, **fixed in a parameter-free way by the χ symmetry** (and π N data).
- Works even better than anticipated by Weinberg: χ EFT has already been developed to **a precision tool in the 2N sector**.
- The **3NF problem** may become a „Holy Grail“ for χ EFT. **The challenge is to maintain the chiral symmetry beyond N²LO level** (gradient flow regularization).

We are still at the very beginning of a long journey towards developing nuclear χ EFT into a precision science

I am deeply indebted to:

- my colleagues & collaborators
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