The ^{4}He spectrum

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Outline

1. Introduction
2. First excited state: $0+$
3. 2nd excited state: 0^-
4. Applications
5. Conclusions

Collaborators

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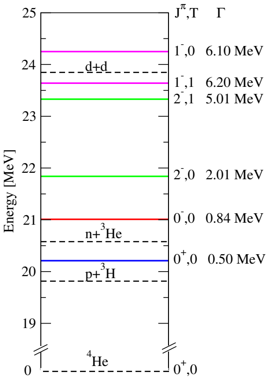
The ^{4}He spectrum



[Tilley, Weller, & Hale, 1992]

No true excited states – only resonances
1st and 2nd excited states "narrow" resonances

Interactions

- NN potentials
  - N3LO500, N3LO600 [Entem & Machleidt, 2003, 2011]
  - N4LO450, N4LO500, N4LO550 [Nosyk, Entem & Machleidt, 2017]
  - NVIa & NVIb [Piarulli *et al.*, 2018]
- + accompanying 3N potential at N2LO [Epelbaum *et al.*, 2002]

Numerical techniques for $A = 4$ for scattering

- Faddeev-Yakubovsky methods [Lazauskas & Carbonell, 2004], [Deltuva & Fonseca, 2007]
- Expansion on a basis: NCSM [Quaglioni, Navratil & Roth, 2010], Gaussians [Aoyama *et al.*, 2011], R-matrix [Descouvemont & Baye, 2010], HH [Kievsky, Marcucci, MV, *et al.*, 2008], . . .

HH method for continuum states

$$AB \to AB + CD + \cdots \text{ process}$$

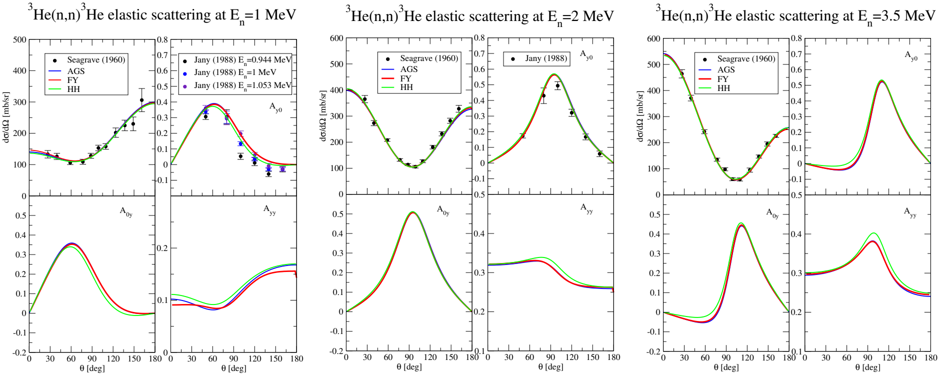
$$\Omega^{\pm}_{AB,LS}(q_{AB}) = \sqrt{\frac{1}{N}} \sum_{perm.=1}^{N} \Big[Y_L(\hat{\mathbf{y}}_p) \otimes [\phi_A \otimes \phi_B]_S \Big]_{JJ_z} \left(f_L(y_p) \frac{G_L(\eta, q_{AB} y_p)}{q_{AB} y_p} \pm \mathrm{i} \frac{F_L(\eta, q_{AB} y_p)}{q_{AB} y_p} \right)$$

$$|\Psi^{JJ_z}_{AB,LS}\rangle = \sum_{n,[K]} a_{AB,LS,[K]} |n,[K]\rangle + |\Omega^{-}_{AB,LS}(q_{AB})\rangle - \sum_{L'S'} \mathcal{S}^{(AB,AB),J}_{LS,L'S'} |\Omega^{+}_{AB,L'S'}(q_{AB})\rangle - \sum_{L'S'} \mathcal{S}^{(AB,CD),J}_{LS,L'S'} |\Omega^{+}_{CD,L'S'}(q_{CD})\rangle - \cdots$$

- $|n,[K]\rangle$ HH states – essentially, homegeneous polynomials of degree K
- Asymptotically $\Omega^{\pm}_{AB,LS}(q) \sim e^{\pm iqy}$
- $\mathcal{S}^{(AB,CD),J}_{LS,L'S'}$ = S-matrix ($T = (\mathcal{S} - I)2\pi$)
- $a_{AB,LS,[K]}$, $\mathcal{S}^{(AB,AB),J}_{LS,L'S'}$, $\mathcal{S}^{(AB,CD),J}_{LS,L'S'}$, ... computed using the Kohn variational principle
- For more details, see [MV *et al.* PRC **35**, 063101 (2020)]

Benchmark test of 4N scattering calculations - $n - {}^3\mathrm{He}$ scattering

N3LO500 potential – ^{3}He$(n,n)^3$He elastic scattering



AGS= Deltuva & Fonseca – FY= Lazauskas & Carbonell – HH= present work

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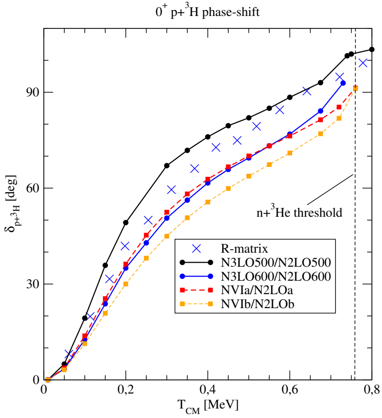
$p + {}^3\mathrm{H}$ scattering: 0^+ phase-shifts



Extraction of the resonance parameters
1) Time-delay [Thompson & Nunes, "Nuclear reactions for astrophysics" p. 301]
2) Poles of the S matrix [Rakityansky, Sofianos, & Elander, (2007)]

Interaction	E_R (MeV)	Γ (MeV)
N3LO500/N2LO500	0.09	0.26
N3LO600/N2LO600	0.10	0.39
NVIa/N2LOa	0.10	0.39
NVIb/N2LOb	0.08	0.47
Expt.	0.39	0.50

$$\frac{q^2}{2\mu} - \Delta B = \frac{(q')^2}{2\mu}$$

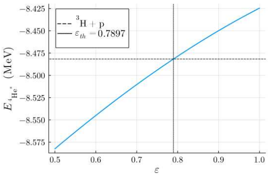
$$|\Psi^{JJ_z}_{pt,LS}\rangle = \sum_{n,[K]} a_{pt,LS,[K]} |n,[K]\rangle + |\Omega^-_{pt,LS}(q)\rangle - \sum_{L'S'} S^{(pt,pt),J}_{LS,L'S'} |\Omega^+_{pt,L'S'}(q)\rangle - \underbrace{\sum_{L'S'} S^{(pt,nh),J}_{LS,L'S'} |\Omega^+_{nh,L'S'}(iq')\rangle}_{\text{Vanishing as } \exp(-q'y)}$$

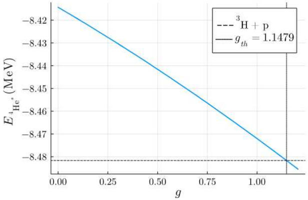
Extrapolation analysis

Extraction of the resonance parameters [Gattobigio & Kievsky, 2023]
based on the "Analytic continuation in the coupling constant" (ACCC) method
[Kukulin, Krasnopol'sky, & J. Hor'acek, Theory of Resonances (1989)]

Method 1: $H = H_{st} + \epsilon V_{Coul}$
For $\epsilon = 0$, the 1st excited state is more bound than $^3\mathrm{H}$
increase ϵ from 0 to ϵ_{thres}

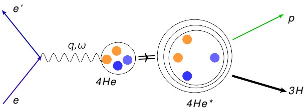
Method 2: $H' = H - gV_{4N}$
Add an actractive 4N interaction $g \geq 0$
decrease g from ∞ to g_{thres}





Using the ACCC formalism, one can extrapolate to the cases $\epsilon \to 1$ and $g \to 0$
Results quoted in [Gattobigio & Kievsky, 2023]: $E_R = 0.07(1)$ MeV

Electron scattering



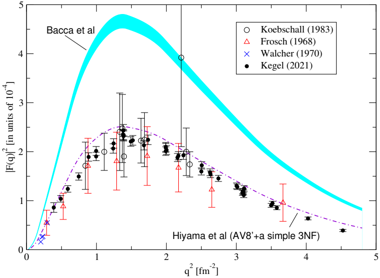
Transition form factor for $0^+_{gs} \to 0^+_{1st}$
See, for example, [Epelbaum, https://physics.aps.org/articles/v16/58]

Experimentally...

- [Walcher, 1970]
- [Frosh *et al.*, 1968]
- [Kobschall *et al.*, 1983]
- [Kegel *et al.*, 2021]

. . . and theoretically

- [Hiyama *et al.*, (2004)] treating the first excited state as a bound state
- [Bacca *et al.*, (2013)–(2015)] using the LIT and a χEFT interaction
- [Kamimura, 2023] seen yesterday...



Present work: 1) Compute the transition $\langle\Psi^{m_3,m_1}_{3+1}|\rho(\boldsymbol{q})|\Psi_4\rangle$ using accurate bound and continuum wave functions [Kievsky *et al.*, (2008)], [MV *et al.*, (2020)]
2) include the effects of MEC in $\rho(\boldsymbol{q})$ [Pastore *et al*, (2009)–(2011)]

Theoretical analysis (1)

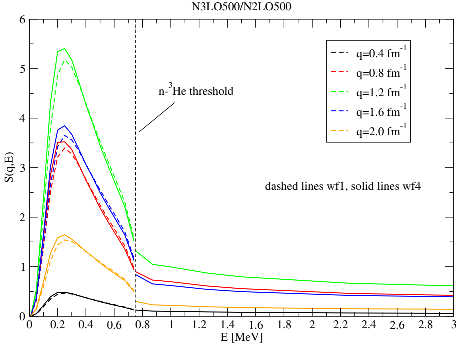
$$\Psi_{3+1}^{m_3,m_1} = \sum_{LSJJ_z} \cdots \Psi_{AB,LS}^{JJ_z} \sim \phi_3^{m_3}\phi_1^{m_1} e^{-i\boldsymbol{q}\cdot\boldsymbol{y}}$$

- We take into account only the component with $L = S = J = 0$
- $\Rightarrow$ RME $C_{AB}(q, E)$ calculated at various kinetic energies E
- q_{AB} = relative AB momentum; $E = \frac{q_{AB}^2}{2\mu_{AB}}$; μ = AB reduced mass
- (IA): $\rho(\boldsymbol{q}) = \sum_{j=1}^{A} \frac{1+\tau_z(j)}{2} e^{i\boldsymbol{q}\cdot\boldsymbol{r}_j} G_D(q)$, $G_D = \frac{1}{(1+0.056q^2)^2}$ Dipole FF

$$|F(q)|^2 = \frac{1}{16\pi}\sum_{m_1,m_3}\sum_{\boldsymbol{p}} |\langle\Psi_{3+1}^{m_3,m_1}|\rho(\boldsymbol{q})|\Psi_4\rangle|^2 = \frac{1}{16\pi}\int dE \underbrace{\sum_{AB=pt,nh,\ldots} 8\mu_{AB}q_{AB}|C_{AB}(q_{AB},E)|^2}_{S(q,E)}$$

Theoretical analysis (2)

wf1,. . .,wf5 = calculations performed using HH bases of increasing number of components

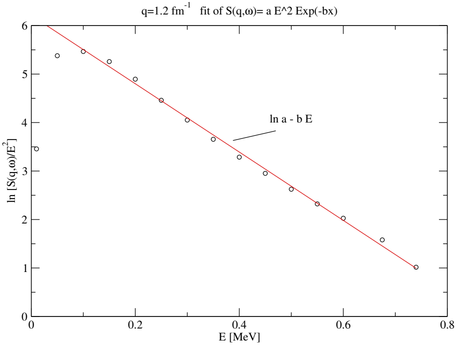


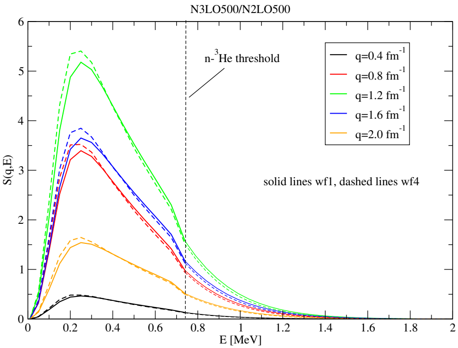
(PRELIMINARY)

- Slow convergence: at the peak, just below threshold
- Above threshold $|\mathcal{S}^{pt,pt}| \to 0$ while $|\mathcal{S}^{pt,nh}| \to 1$
- The charge exhange mechanism starts to be dominant – no formation of the resonance
- Plan: use the $S(q,E)$ below threshold and “complete” the peak

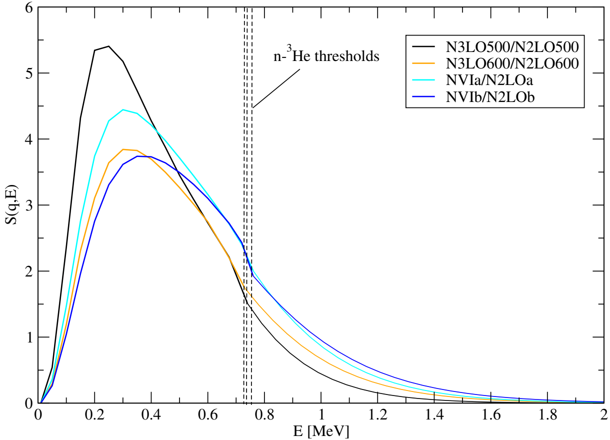
Theoretical analysis (3)

(PRELIMINARY) Fit with $S(q,E) = aE^2 e^{-bE}$; note that $\ln[S(q,E)/E^2] = \ln a - bE$



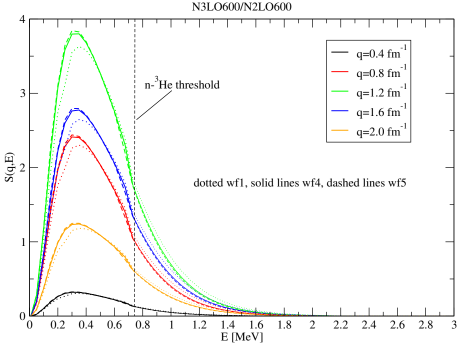


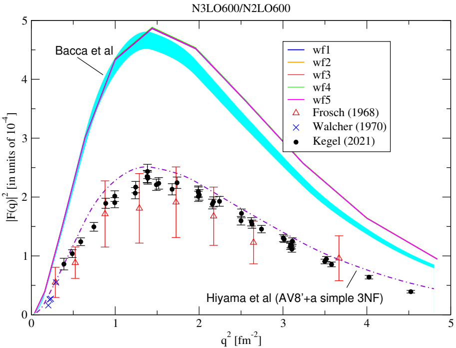
Comparison of the $S(q, E)$ at $q = 1.2\ \text{fm}^{-1}$



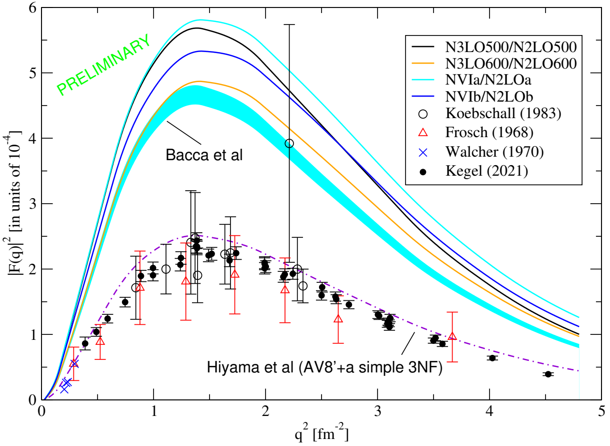
NVIa: short-range cutoff in r-space $R_S = 1.2$ fm
NVIb: short-range cutoff in r-space $R_S = 1.0$ fm

Convergence



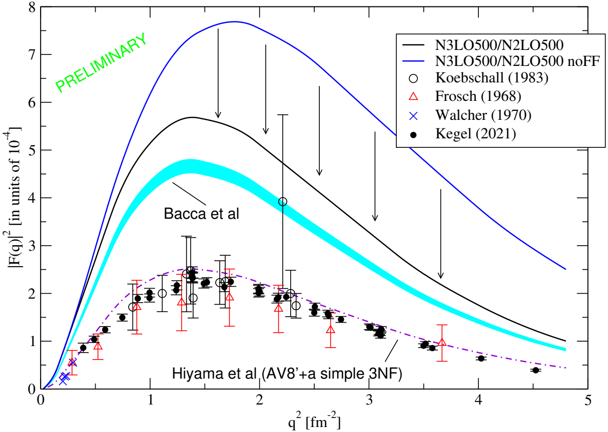


Results

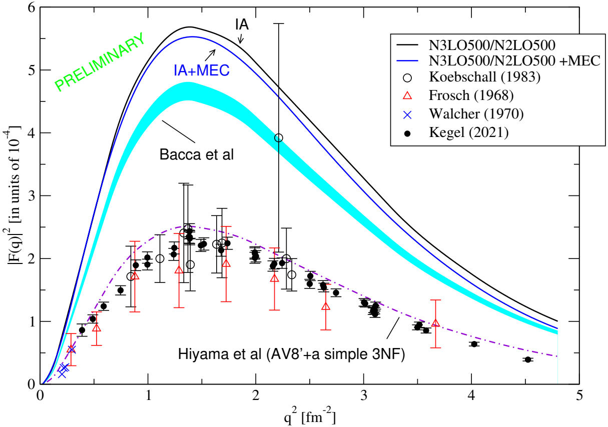


(PRELIMINARY)

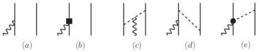
Effect of the nucleon form factor



Effect of MEC



[Pastore *et al.*, 2011]



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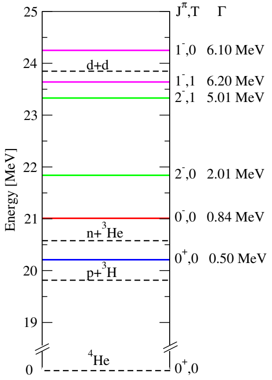
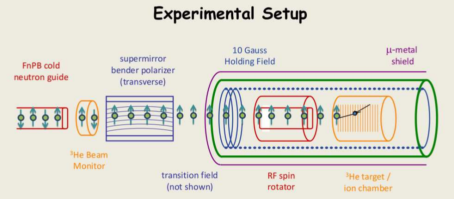
Study of the 2nd excited state

ORNL "n3he" experiment: $n + {}^3\mathrm{He} \to p + {}^3\mathrm{H}$+765 keV with SNS cold neutrons

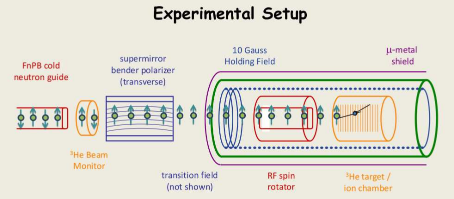




$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_u \left(1 + A_{\rm PV}\hat{s}_n \cdot \hat{k}_p + A_{\rm PA}(\hat{s}_n \times \hat{k}_n)\cdot \hat{k}_p\right)$$

- A_{PV} = parity-violating analyzing power (A_x)
- $\boldsymbol{k}_p$ in the vertical plane: $A_{\rm PV}$
  - Sensitive to V_{PV} (still unknown)
  - $A_{\rm PV} = (1.55 \pm 0.97 \text{ (stat)} \pm 0.24 \text{ (sys)}) \times 10^{-8}$
  - [Gericke *et al.*, 2020]

ORNL "n3he" experiment: $n + {}^3\mathrm{He} \to p + {}^3\mathrm{H}$+765 keV with SNS cold neutrons



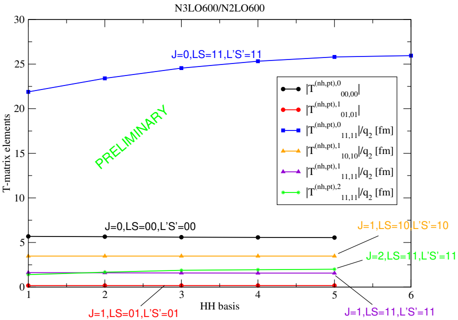
- A_{PA} = parity-allowed analyzing power (A_y)
- $\boldsymbol{k}_{\mathrm{p}}$ in the horizontal plane : A_{PA}
  - Sensitive to $V_{\mathrm{strong}} + V_{\mathrm{EM}}$ (they should be known ...)
  - Interference between S- and P-waves, $A_{\mathrm{PA}} \sim q_{nh}$
  - $E_n \sim 5meV$, $q_{nh} \sim 10^{-5}$ fm^{-1}
  - $A_{\mathrm{PA}} = (-43.9 \pm 6.0\,(\mathrm{stat}) \pm 0.2\,(\mathrm{sys}))$
  - (PRELIMINARY) [n3he Coll. in preparation]

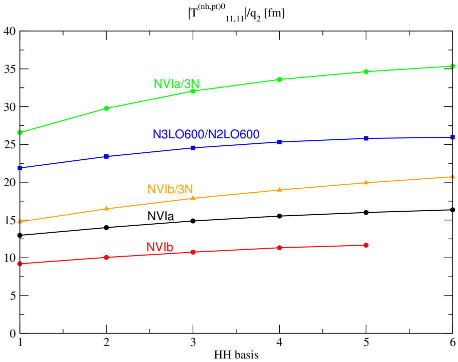
Theoretical treatment
in terms of the matrix elements T

$$\frac{A_{PA}}{q_{nh}} \sim \Im\left[\frac{({}^0T^{2,1}_{11,11})^*}{q_{nh}}\;{}^1T^{2,1}_{01,01}\right] + \cdots$$

Convergence

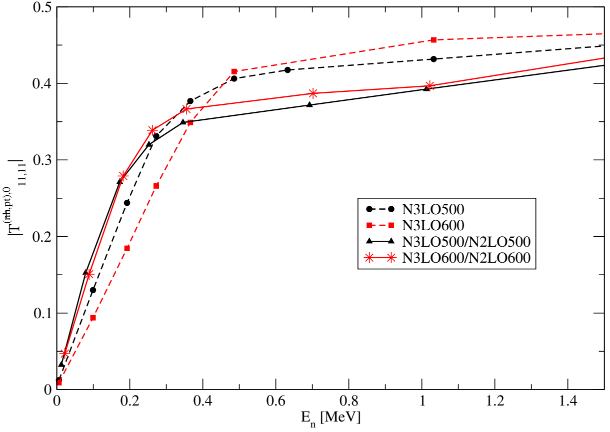
T-matrix elements $T^{(nh,pt),J}_{LS,L'S'}$ $[T = (\mathcal{S} - 1)/2i]$





Energy dependence of $T^{(nh,pt),0}_{11,11}$

$T^{(nh,pt),0}_{11,11}$ as function of E_n



From $\mathcal{S}^{(nh,pt),0}_{11,11}$ we can also extract the energy and width of the 0^- resonance
Poles of the S matrix [Rakityansky, Sofianos, & Elander, (2007)]

Results

A_{PA}/q_{nh} is independent on the energy for very small q_{nh}

Interaction	$E_{\rm R}$ [MeV]	Γ [MeV]	$\lvert T^{(nh,pt),0}_{11,11} \rvert / q_{nh}$ [fm]	$A_{PA}/q_{nh} \times 10$ [fm]
N3LO500	0.16	0.41	20.7	-1.03
N3LO600	0.24	0.51	16.9	-0.57
NVIa	0.31	0.53	17.3	-0.55
NVIb	0.30	0.54	13.0	-0.09
N3LO500/N2LO500	0.06	0.26	30.1	-2.13
N3LO600/N2LO600	0.09	0.30	25.9	-1.12
NVIa/N2LOa	0.04	0.36	35.4	-2.49
NVIb/N2LOb	0.12	0.40	23.9	-1.09
Experimental	0.44	0.84		-0.44 ± 0.06 (stat) ± 0.03 (sys)

Experimental value of A_{PA} still (PRELIMINARY)
Determination of corrections and systematic uncertainties still in progress [McCrea *et al.*, 2023]

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Evidences of a new boson? Search of the X17

- ^{7}Li(p, $e^{+}e^{-}$)^{8}Be [Krasznahorkay *et al.*, 2016]
- ^{3}H(p, $e^{+}e^{-}$)^{4}He [Krasznahorkay *et al.*, 2019-2021]
- ^{11}B (p, $e^{+}e^{-}$)^{12}C [Krasznahorkay *et al.*, 2022]

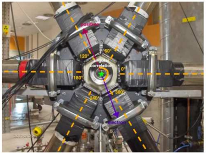
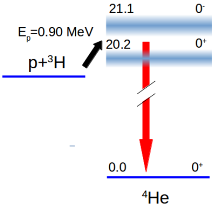
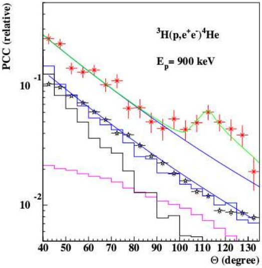


Figure 3. The Atomki nuclear spectrometer. This is an upgraded detector



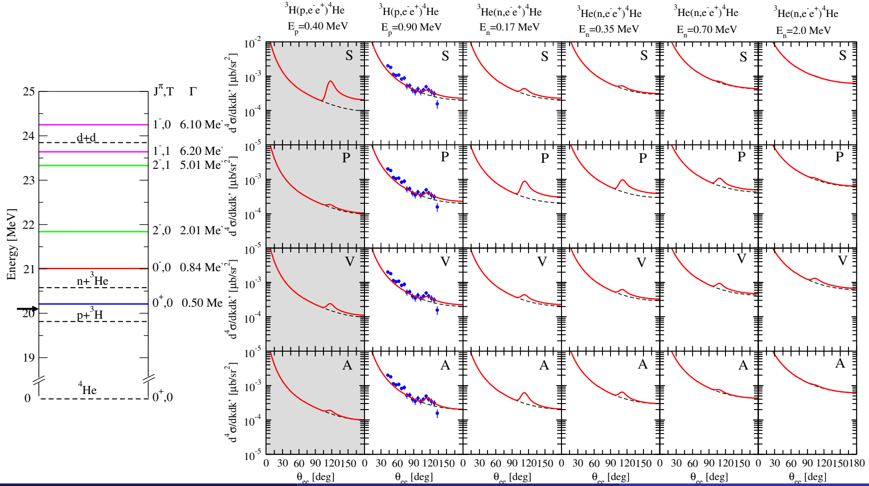
Angular distribution of the $e^{-}e^{+}$ pair



$$p + {}^{3}\mathrm{H} \to ({}^{4}\mathrm{He})^{*} \to {}^{4}\mathrm{He} + X \to {}^{4}\mathrm{He} + e^{+} + e^{-}$$

Theoretical study of ^{3}H(p, e^+e^-)^{4}He & ^{3}He(n, e^+e^-)^{4}He

pair emission in the perpendicular plane – peak fitted at 0.90 MeV



Theoretical study of $^3\mathrm{H}(p, e^+e^-)^4\mathrm{He}$ & $^3\mathrm{He}(n, e^+e^-)^4\mathrm{He}$

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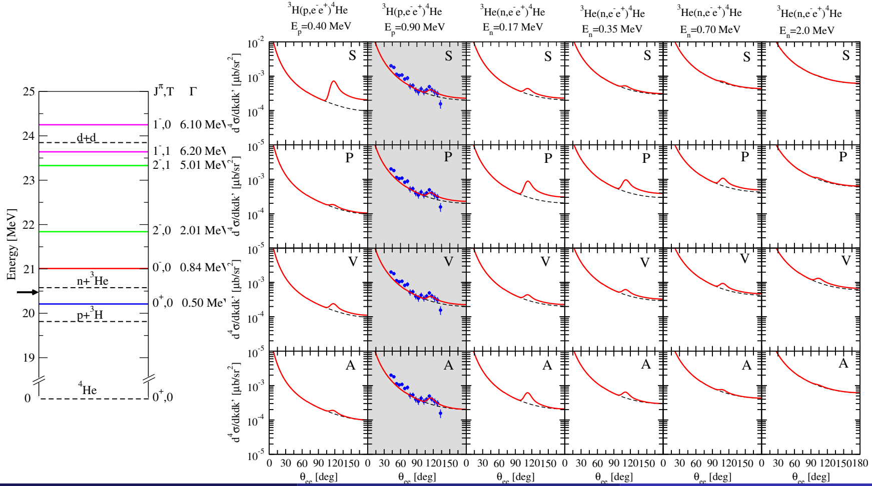

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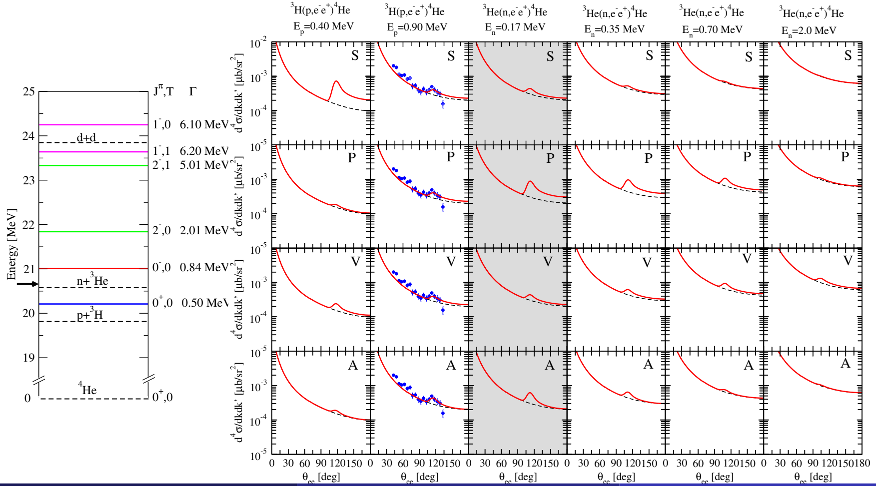

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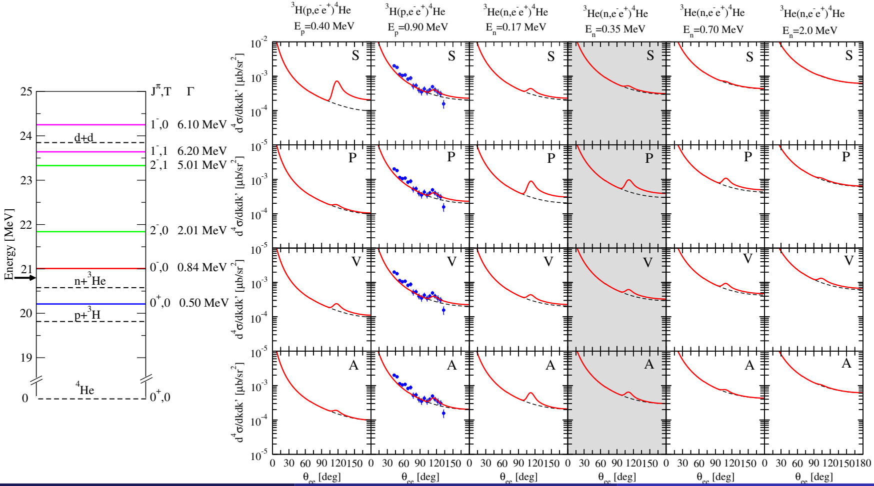

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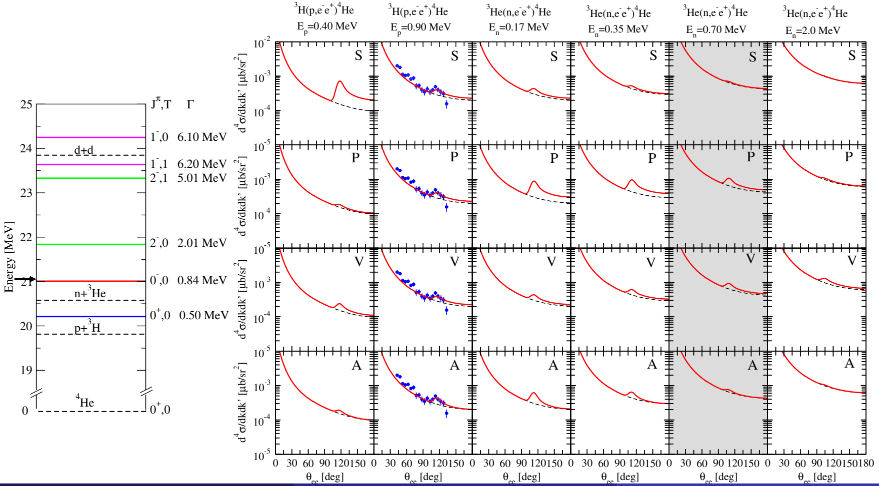

Theoretical study of ^{3}H$(p, e^+e^-)^4$He & ^{3}He$(n, e^+e^-)^4$He

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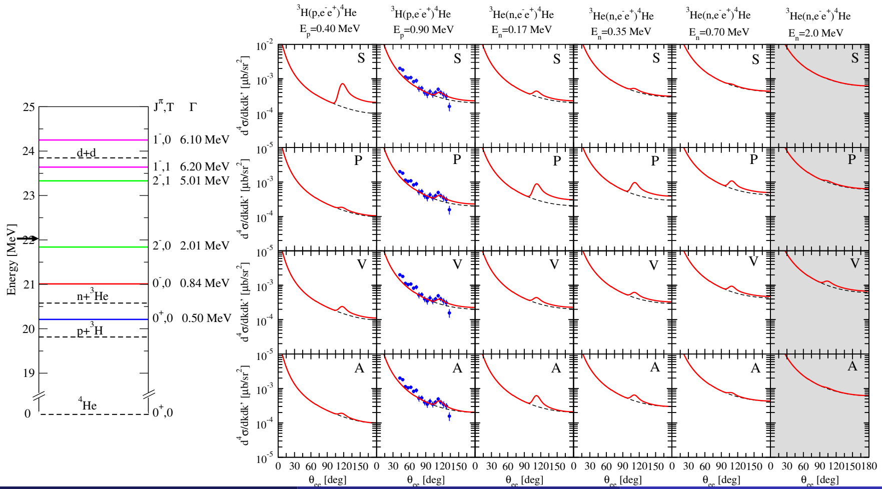

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Conclusions and perspectives

Conclusions:

- Still problems with the first two excited states
- Current interactions predicts different positions and widths
- Interesting “handles”
  - Transition form factor
  - A_{PA} at ORNL

Perspectives:

- Tuning of the 3N force?
- Cutoff dependence of the 3N force?
- “Unitary ambiguity of NN contact interactions and the 3N force” [Girlanda *et atl.*]
  - 5 unknown LECs at N3LO
  - some of them can be used to solve $n-d$ and $p-d$ A_y “puzzle”
- Still works to do . . .

Acknowlegements: We thanks for useful discussion S. Bacca, N. Barnea, G. Orlandini, & W. Leidemann . . .

. . . and thank you for your attention!