Light Λ hypernuclei within χ EFT and theoretical uncertainties

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Why is hypernuclear physics interesting?





• contribute to NS at $\rho = 2 - 3\rho_0$





- can 3BFs resolve the puzzle?
- scarce hyperon-nucleon (YN) scattering data
 difficult to constrain free-space YN interactions
- hypernuclei Iaboratory to explore YN (BB) interactions / test SU(3) symmetry of QCD
- ab initio (NCSM) treatment of hypernuclei —> connect underlying interactions & hypernuclear observables



$\sum_{i=1}^{N_{\text{p}}} \sum_{i=1}^{N_{\text{p}}} \sum_{i=1}^{N_{p}} \sum_{i=1}^{N_{\text{p}}} \sum_{i=1}^{N_{\text{$



LO: H. Polinder, J. Haidenbauer, U.-G. Meinner NPA 779(2006) ^{(p''} (MLO18: J. Haidenbauer) et al NPA 915(20/3); NLO19: J. Haidenbauer, U.-G. Meißner, A. Nogga EPJA 56(2020) SMS NLO, N²LO: J. Haidenbauer, U.-G. Meißner, A. Nogga, HL EPJA 59(2023)

 $\int_{a}^{b} \frac{\partial F}{\partial r} = \frac{\partial F}{\partial r} \int_{a}^{b} \frac{\partial F}{\partial r} \int_{a}^$

 $^{S_{\alpha}}\Lambda^{m}\Lambda^{m}$ SMS ^{m}N O, N²LO: employ a novel regularization scheme as in NN $\Sigma^{-}p \rightarrow \Lambda p$



	χ^2	$B_{\Lambda}(^3_{\Lambda}{ m H})$
250 - LO(600) - Jülich '04 - • Engelmann et al.	28.3	0.135
	16.8	0.09
	16.3	0.091
5 SMS NDO(550)	15.7	0.123
• SMS N ² LO	15.6	0.139

LO describes YN data poorly. Significant improvement at NLO (LO results are useful for truncation error estimation)**SMS** interactions lead to a slightly more attractive ΛN potential

 ${}^{1}_{1} \Lambda \mathcal{A}_{1}$ phase shift with (left) and without (right) ΣN coupling. Same description of curves as in Fig. 1



Extrapolation in ω & \mathcal{N} spaces

S. Liebig, U.-G. Meißner, A. Nogga, EPJA 52(2016); HL, J. Haidenbauer, U.-G. Meißner, A. Nogga EPJA 56 (2020)



Numerical uncertainties

- NCSM calculations for hypernuclei with bare SMS
 NN (3N) and YN interactions converge poorly
- NCSM uncertainties for **SRG-evolved** potentials:
 - ~ several keV for $A \leq 5$
 - \blacktriangleright ~ hundred(s) keV for $A=7\,(8)$
- FY uncertainties: ~ 1 (20) keV for ${}^{3}_{\Lambda}H({}^{4}_{\Lambda}He)$



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Similarity Renormalization Group (SRG)



are

Idea: continuously apply unitary transformation to H to suppress off-diagonal matrix elements

→ speed up the convergence of NCSM calculations (observables e.g. energies are conserved) F.J. Wegner NPB 90 (2000). S.K. Bogner, R.J. Furnstahl, R.J. Perry PRC 75 (2007)

$$\begin{split} \frac{dV(s)}{ds} &= \left[\left[T_{rel}, V(s) \right], H(s) \right], & H(s) = T_{rel} + V(s) + \Delta M \\ s &= 0 \to \infty & V(s) = V_{12}(s) + V_{13}(s) + V_{23}(s), \quad V_{123,s=0} \equiv V_{NNN}^{bare}; \quad (V_{YNN}^{bare} = 0) \end{split}$$

separate SRG flow equations for 2-body and 3-body interactions:

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S.K. Bogner et al PRC75 (2007), K. Hebeler PRC85 (2012)

$$\frac{dV^{NN}(s)}{ds} = [[T^{NN}, V^{NN}], T^{NN} + V^{NN}]
\frac{dV^{YN}(s)}{ds} = [[T^{YN}, V^{YN}], T^{YN} + V^{YN} + \Delta M]
\frac{dV_{123}}{ds} = [[T_{12}, V_{12}], V_{31} + V_{23} + V_{123}]
+ [[T_{31}, V_{31}], V_{12} + V_{23} + V_{123}]
+ [[T_{23}, V_{23}], V_{12} + V_{31} + V_{123}] + [[T_{rel}, V_{123}], H_s]$$
SRG-induced YNNs are generated even if $V_{YNN}^{bare} = 0$

SRG-induced beyond 3BFs are not included. Estimate size of omitted 4BFs by varying $\lambda = (4\mu^2/s)^{1/4}$ [fm⁻¹]

Effect of SRG-induced 4BFs in A=4,5

HL, J. Haidenbauer, U.-G. Meißner, A. Nogga (in preparation)

• Estimate size of omitted 4BFs by varying λ

$oldsymbol{\lambda} \; [ext{fm}^{-1}]$	$B_{\Lambda}(^4_{\Lambda}{ m He},0^+)$	$B_{\Lambda}(^5_{\Lambda}{ m He})$
1.88	1.992 ± 0.002	3.712 ± 0.001
2.00	1.991 ± 0.005	$3.705{\pm}~0.005$
2.236	$1.990 \ \pm 0.007$	3.708 ± 0.006
2.60	$1.989 \ \pm \ 0.014$	$3.744\ \pm 0.008$
3.00	$1.985 \ \pm 0.024$	3.806 ± 0.030
∞	2.01 ± 0.02	

NN: N⁴LO⁺, **3N**: N²LO(450); **YN**: N²LO(550)



 $\lambda = \infty$: FY calculation using **bare NN + 3N + YN interactions**

• Variation of B_{Λ} for $1.88 \le \lambda \le 3.0 \text{ fm}^{-1}$: $\Delta B_{\Lambda}(^{4}_{\Lambda}\text{He}) = 10 \pm 25 \text{ KeV}$

 $\Delta B_{\Lambda}(^{5}_{\Lambda}\text{He}) = 90 \pm 30 \text{ KeV}$

→ contributions of induced 4BFs to $B_{\Lambda}({}^{4}_{\Lambda}\text{He}, {}^{5}_{\Lambda}\text{He})$ are negligible





Results for A=3-8 hypernuclei

NN: SMS $N^4LO^+(450)$ 3N: $N^2LO(450)$ YN: NLO13, NLO19(500); SMS NLO, $N^2LO(550)$

Predictions for $B_{\Lambda}(A \le 8)$ with NLO13 & NLO19



HL, J. Haidenbauer, U.-G. Meißner, A. Nogga PRC 107(2023)

- NLO13 & NLO19 build on non-local regulator; phase equivalent in 2-body sector
- NLO13 characterised by a stronger $\Lambda N \Sigma N$ transition potential $({}^{3}S_{1} {}^{3}D_{1})$



• ${}^{4}_{\Lambda}$ H(1⁺), ${}^{5}_{\Lambda}$ He, ${}^{7}_{\Lambda}$ Li, ${}^{8}_{\Lambda}$ Li are fairly well described by NLO19; NLO13 underestimates these B_{Λ} • signal for missing chiral YNN forces: ~ 50 KeV to $B_{\Lambda}({}^{3}_{\Lambda}$ H), ~ 200 – 300 KeV to $B_{\Lambda}({}^{4}_{\Lambda}$ He) ~ 0.7 – 1 MeV to $B_{\Lambda}({}^{5}_{\Lambda}$ He) (J. Haidenbauer et al EPJA 56(2020))

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Predictions for $B_{\Lambda}(A \le 5)$ with SMS NLO, N²LO



HL, J. Haidenbauer, U.-G. Meißner, A. Nogga (in preparation)

- NLO13 & NLO19 build on non-local regulator;
- SMS NLO, N²LO use local regulator for long-range interactions (milden cutoff artifacts)
 - -> SMS potentials lead to a more attractive ΛN (J. Haidenbauer, U.-G. Meißner, A. Nogga, HL EPJA 59(2023))

	$^{3}_{\Lambda}{ m H}$	$^4_{\Lambda}{ m He}(0^+)$	${}^4_{\Lambda}{ m He}(1^+)$	$^{5}_{\Lambda}{ m He}$
NLO13	0.090	1.48 ± 0.02	0.58 ± 0.02	2.22 ± 0.06
NLO19	0.091	1.46 ± 0.02	1.06 ± 0.02	3.32 ± 0.03
NLO	0.124	2.061 ± 0.001	1.087 ± 0.009	3.334 ± 0.008
N ² LO	0.139	1.992 ± 0.002	1.23 ± 0.009	3.710 ± 0.007
D *	0.148 ± 0.040	$2.169 \pm 0.042 \ ({}^4_{\Lambda}{ m H})$	$1.081 \pm 0.046 \ ({}^4_{\Lambda}{ m H})$	3.102 ± 0.030
Бхр		$2.347 \pm 0.036 \ ({}^4_{\Lambda}{ m He})$	$0.942 \pm 0.036 \ ({}^4_{\Lambda}{ m He})$	

NN:SMS N⁴LO⁺(450) +3N: N²LO(450) +SRG-induced YNN

* https://hypernuclei.kph.uni-mainz.de/

 \rightarrow • all s-shell states are fairly well described with SMS NLO & N²LO

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• chiral YNN forces contribute at N²LO (5 LECs for ΛNN) \rightarrow not possible to fix

with decuplet saturation at NLO: 2 LECs $(1 \Lambda NN + 1 \Sigma NN) \rightarrow \text{fit to } B_{\Lambda}(^{4}_{\Lambda}\text{He}(0^{+}, 1^{+})) \text{ or } B_{\Lambda}(\text{He}(0^{+}), ^{5}_{\Lambda}\text{He}(1/2^{+}))$ (work in progress)

Variation due to NN & YN interactions





variations due to NN potentials < variations due to YN & differences to experiment;

Variation due to NN interactions



NN: $N^{2}LO(450 - 500)$, $N^{3}LO(450)$, $N^{4}LO^{+}(400 - 550)$; 3N: $N^{2}LO$

considered $NN + YN$ potentials	$\mathbf{B}_{\Lambda}(^{3}_{\Lambda}\mathbf{H})$	$\mathbf{B}_{\Lambda}(^4_{\Lambda}\mathrm{He},0^+)$	$\mathbf{B}_{\Lambda}(^4_{\Lambda}\mathrm{He},1^+)$	$\mathbf{B}_{\Lambda}(^{5}_{\Lambda}\mathrm{He})$
$N^{4}LO^{+}(400 - 550) + N^{2}LO(550)$	3	43	44	44
$N^{4}LO^{+}(400 - 550) + NLO(550)$	14	110	25	90
$(\mathbf{N^{2}LO},\mathbf{N^{3}LO},\mathbf{N^{4}LO^{+}}) + \mathbf{N^{2}LO}(550)$	11	114	114	295
$(\mathbf{N^2LO}, \mathbf{N^3LO}, \mathbf{N^4LO^+}) + \mathbf{NLO}(550)$	14	147	88	273
$N^4LO^+(400-550) + LO(600)$	25	194	223	970
Gazda [*] variation with cutoff	50	270	240	1150
${ m Gazda}^*$ variance $\sigma_{ m model}$	20	100	100	400

*D. Gazda et al PRC 106(2022): based on LO YN + 42 non-local regularized NN(3N) N^2LO_{sim}

 $\sigma_{\text{model}} \equiv [\sigma^2 (N^2 LO_{\text{sim}})]^{1/2}$ (talk by Gazda)

• for YN NLO, N^2LO : NN variations are "small", especially when SMS N^4LO^+ are employed

• NN variations are expected to be counterbalanced by chiral YNN forces

Truncation error estimation



- NN &YN interactions are truncated at certain orders higher-order contributions?
- Epelbaum, Krebs and Meißner:
 - cutoff variations no reliable estimate for truncation errors
 - estimate truncation error at each order via expected + actual size of higher-order corrections:

$$\begin{split} X^{(k)} &= X^{(0)} + \sum_{i=2}^{k} \Delta X^{(i)}; \\ \delta X^{(0)} &= \mathcal{Q}^{2} |X^{\text{LO}}|; \\ \delta X^{(i)} &= \max_{2 \leq j \leq i} \left(\mathcal{Q}^{i+1} |X^{(0)}|, \mathcal{Q}^{i+1-j} |X^{(j)} - X^{(j-1)}| \right); \\ \mathcal{Q} &= M_{\pi}^{eff} / \Lambda_{b} \quad (\mathcal{Q}: \text{ expansion parameter}) \end{split}$$

- Furnstahl, Klco, Phillips, Melendez, Weslowski :
- R. J. Furnstahl et al PRC 92(2015) J. A. Melendez et al PRC 96 (2017), 100(2019)

$$X^{(k)} = X^{(0)} + \sum_{i=2}^{k} \Delta X^{(i)} =: X_{ref} (c_0 + c_2 Q^2 + c_3 Q^3 + \cdots)$$

$$\delta X^{(k)} = X_{ref} \left(\sum_{n=k+1}^{\infty} c_n Q^n \right); \qquad c_n \sim \mathcal{O}(1); \quad c_n | \overline{c}^2 \stackrel{\text{iid}}{\sim} \mathcal{N}(0, \overline{c}^2); \quad \overline{c}^2 \sim \chi^{-2}(\nu_0, \tau_0^2) \quad \text{(pointwise model)}$$

 Q, \overline{c}^2 : learn from order-by-order calculations together with prior expectations + consistency plots

Truncation error estimation



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Exp

 $^{4}_{\Lambda}$ He(0⁺)

N²LO

NLO

LO



CSB results in A=4-8

HL, J. Haidenbauer, U.-G. Meißner, A. Nogga PRC 107(2023)



- CSB predictions for A=7 are comparable to experiment.
- both potentials predict a somewhat larger CSB in A=8 doublet as compared to experiment
- → experimental CSB splitting for A=8 larger than 40 ± 60 keV?
 - CSB estimated for A=4: too large? different spin-dependence?

Fitting LECs to new Star measurement



Recent STAR measurement suggests somewhat different CSB in A=4:

$\Delta E(1^+) = B_{\Lambda}(^4_{\Lambda}\text{He}, 1^+) - B_{\Lambda}(^4_{\Lambda}\text{H}, 1^+)$	
$= -83 \pm 94 \text{ keV} \Rightarrow (\text{CSB})$	a_s^A
$= -160 \pm 140 \pm 100 \text{ keV} \Rightarrow (\text{CSB}^*)$	a_s^{Λ}
$\Delta E(0^+) = B_{\Lambda}(^4_{\Lambda}\text{He}, 0^+) - B_{\Lambda}(^4_{\Lambda}\text{H}, 0^+)$	δa a_t^A
$= 233 \pm 92 \text{ keV} \Rightarrow (\text{CSB})$	a_t^{Λ}
$= 160 \pm 140 \pm 100 \text{ keV} \Rightarrow (\text{CSB}^*)$	δα
* STAR Collaboration PLB 834 (2022)	→ δa

	NLO19(500)	CSB	CSB*
$a_s^{\Lambda p}$	-2.91	-2.65	-2.58
$a_s^{\Lambda n}$	-2.91	-3.20	-3.29
δa_s	0	0.55	0.71
$a_t^{\Lambda p}$	-1.42	-1.57	-1.52
$a_t^{\Lambda n}$	-1.41	-1.45	-1.49
δa_t	-0.01	-0.12	-0.03

 $\rightarrow \delta a({}^{1}S_{0})$ increases while $\delta a({}^{3}S_{1})$ decreases

→ How does the STAR measurement affect the predictions of CSB in A=7,8 multiplets ?

Impact of Star measurement on CSB in A=7,8





NN:SMS N⁴LO+(450)
+YN: NLO13,19(CSB)
$$\lambda_{NN} = 1.6 \text{ fm}^{-1}$$

 $\lambda_{NN}^{opt} = 0.823 \text{ fm}^{-1}$

$$B_{\Lambda}({}^{\mathbf{5}}_{\Lambda}\mathbf{He}, \lambda_{YN}^{opt}) = B_{\Lambda}({}^{\mathbf{5}}_{\Lambda}\mathbf{He}, 3BFs)$$

- CSB* fit predicts reasonable CSB in both A=7 and A=8 systems
- CSB in $A=4(0^+)$ and A=8, and in $A=4(1^+)$ and A=7 are correlated
 - accurate CSB in A=7 & 8 may allow for an independent check of A=4 CSB



Summary

- At our disposal we have 3 tools to tackle light (hyper)nuclear systems:
 - s-shell (hyper)nuclei: Faddeev-Yakubovsky, NLEFT (D. Frame et al EPJA 56(2020))
 - s-shell & light p-shell: Jacobi NCSM approach; numerical uncertainties (s-shell) ~ few keV
 - \rightarrow establish a direct link between chiral YN interactions and observables ($A \leq 9$)
- YN at NLO & N²LO yields reasonable B_{Λ} in A = 3 8 hypernuclei. NLO13 & NLO19 results show a clear signal of missing chiral YNN forces
- study convergence w.r. to NN & YN orders in A = 3 5 hypernuclei
- CSB NLO interactions reproduce experimental CSB for A = 4,7 multiplets,
 - A = 8 CSB prediction is larger than experiment

Thank you for the attention!