## Light $\Lambda$ hypernuclei within $\chi$ EFT and theoretical uncertainties

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- contribute to NS at $\rho=2-3 \rho_{0}$

$\sim 40 \Lambda$-hypernuclei
${ }_{\Lambda}^{\mathbf{3}} \mathbf{H},{ }_{\Lambda}^{4} \mathbf{H e}, \cdots,{ }_{\Lambda}^{208} \mathbf{P b}$
D. Lonardoni et al. PRL 114(2015)

- can 3BFs resolve the puzzle?
- scarce hyperon-nucleon (YN) scattering data $\rightarrow$ difficult to constrain free-space YN interactions
- hypernuclei $\rightarrow$ laboratory to explore YN (BB) interactions / test SU(3) symmetry of QCD
- ab initio (NCSM) treatment of hypernuclei $\longrightarrow$ connect underlying interactions \& hypernuclear observables
$\Lambda \Lambda(\Xi)$-hypernuclei
${ }_{\Lambda \Lambda}^{6} \mathrm{He},{ }_{\Lambda \Lambda}^{10} \mathrm{Be},{ }_{\Lambda \Lambda}^{11} \mathrm{Be} ;{ }_{\Xi}^{15} \mathrm{C}$


## BB interactions in $\chi$ EFT

S. Weinberg PLB 251(1990), NPB 363(1991); E. Epelbaum, H.-W. Hammer, U.-G. Meißner RMP 81(2009)


2NN / 5YN LECs (short range parameters)
+7NN / 18YN LECs
no additional LECs for 2BFs
+2NNN / 5
+15NN LECs
+5NN LECs
$N^{4} \mathrm{LO}\left(Q^{5}\right)$

- $\sim 5000 \mathrm{NN}+\mathrm{Nd}$ scattering data $+{ }^{2} \mathrm{H},{ }^{3} \mathrm{H} /{ }^{3} \mathrm{He}$
$\rightarrow$ SMS NN forces up to $\mathbf{N}^{4} \mathbf{L} \mathbf{O}^{+}\left(\chi^{2} \sim 1\right), 3 N F$ up to $\mathbf{N}^{2} \mathbf{L O}$
P. Reinert et al. EPJA (2018), $\operatorname{LENPIC}(2021,2022)$, talk by Epelbaum (03.08)
- semi-local momentum space regularization (SMS):
- local regulator for pion exchange interactions


LO: H. Polinder, J. Haidenbauer, U.-G. Meißner NPA 779(2006)
NLO13: J. Haidenbauer et al NPA 915(2013); NLO19: J. Haidenbauer, U.-G. Meißner, A. Nogga EPJA 56(2020)
SMS NLO, N²LO: J. Haidenbauer, U.-G. Meißner, A. Nogga, HL EPJA 59(2023)

- most of s-wave LECs are fitted to 36 data points ( $\Lambda p \rightarrow \Lambda p, \Sigma N \rightarrow \Sigma N, \Sigma N \rightarrow \Lambda N$ ) $+B_{\Lambda}\left({ }_{\Lambda}^{3} \mathrm{H}\right)$
- at NLO: two realisations NLO13 \& NLO19 (phase equivalent; NLO13 leads to a larger $V_{\Lambda N \leftrightarrow \Sigma N}$ ) $\rightarrow$ tool to assess effect of YNN forces in many-body systems
- SMS NLO, $\mathrm{N}^{2}$ LO: employ a novel regularization scheme as in NN



|  | $\chi^{2}$ | $B_{\Lambda}\binom{3}{\Lambda}$ |
| :--- | :--- | :--- |
| $\mathrm{LO}(600)$ | 28.3 | 0.135 |
| NLO13(600) | 16.8 | 0.09 |
| $\mathrm{NLO} 19(600)$ | 16.3 | 0.091 |
| SMS NLO(550) | 15.7 | 0.123 |
| SMS N |  |  |

$\rightarrow$ • LO describes YN data poorly. Significant improvement at NLO
(LO results are useful for truncation error estimation)

- SMS interactions lead to a slightly more attractive $\Lambda N$ potential


## Jacobi-NCSM approach

S. Liebig, U.-G. Meißner, A. Nogga, EPJA 52(2016); HL, J. Haidenbauer, U.-G. Meißner, A. Nogga EPJA 56 (2020)

- Idea: represent the A-body translationally invariant hypernuclear Hamiltonian:

$$
\mathrm{H}=\mathrm{T}_{\text {rel }}+\mathrm{V}^{\mathrm{NN}}+\mathrm{V}^{\mathrm{YN}}+\mathrm{V}^{\mathrm{NNN}}+\mathrm{V}^{\mathrm{YNN}}+\Delta M+\cdots
$$

in a basis constructed from HO functions
$\Lambda \mathrm{N} \leftrightarrow \Sigma \mathrm{N}$

- Jacobi basis: depends on relative Jacobi coordinates of all particles


## (A-1)N

$$
|\underset{\Lambda(\Sigma)}{\longrightarrow}\rangle=|\mathcal{N} J T, \underbrace{\mathcal{N}_{A-1} J_{A-1} T_{A-1}}_{\text {antisym. }(A-1) N}, \underbrace{n_{Y} l_{Y} I_{Y} t_{Y}}_{\Lambda(\Sigma) \text { state }} ;\left(J_{A-1}\left(l_{Y} S_{Y}\right) I_{Y}\right) J,\left(T_{A-1} t_{Y}\right) T\rangle \quad \text { (independent of } \omega \text { ) }
$$



- intermediate bases for evaluating Hamiltonian:


- basis truncation: $\mathscr{N}=\mathcal{N}_{A-1}+2 n_{\lambda}+\lambda \leq \mathcal{N}_{\max } \Rightarrow E_{b}=E_{b}\left(\omega, \mathcal{N}_{\max }\right) \quad \xrightarrow{\text { extrapolation }} \mathbf{E}_{\mathbf{b}, \infty}$


## Extrapolation in $\omega \& \mathcal{N}$ spaces

S. Liebig, U.-G. Meißner, A. Nogga, EPJA 52(2016); HL, J. Haidenbauer, U.-G. Meißner, A. Nogga EPJA 56 (2020)

- $\mathbf{E}_{\mathbf{b}}(\omega, \mathcal{N})=\mathbf{E}_{\mathcal{N}}+\kappa\left(\log (\omega)-\log \left(\omega_{\text {opt }}\right)\right)^{\mathbf{2}}$

- $\mathbf{E}_{\mathcal{N}}=\mathbf{E}_{\infty}+\mathbf{A} \mathbf{e}^{-\mathbf{b} \mathcal{N}}$



## Numerical uncertainties

- NCSM calculations for hypernuclei with bare SMS NN (3N) and YN interactions converge poorly
- NCSM uncertainties for SRG-evolved potentials:
- ~ several keV for $\mathbf{A} \leq 5$
- ~hundred(s) keV for $\mathbf{A}=7$ (8)
- FY uncertainties: ~ $\mathbf{1} \mathbf{( 2 0 )}$ keV for ${ }_{\Lambda}^{3} \mathrm{H}\left({ }_{\Lambda}^{4} \mathrm{He}\right)$


Idea: continuously apply unitary transformation to H to suppress off-diagonal matrix elements
$\rightarrow$ speed up the convergence of NCSM calculations (observables e.g. energies are conserved)
F.J. Wegner NPB 90 (2000). S.K. Bogner, R.J. Furnstahl, R.J. Perry PRC 75 (2007)

$$
\frac{\mathbf{d V}(\mathbf{s})}{\mathbf{d s}}=\left[\left[\mathbf{T}_{\mathrm{rel}}, \mathbf{V}(\mathbf{s})\right], \mathbf{H}(\mathbf{s})\right]
$$

$$
\mathbf{H}(\mathbf{s})=\mathbf{T}_{\text {rel }}+\mathbf{V}(\mathbf{s})+\Delta \mathbf{M}
$$

$$
\mathbf{s}=\mathbf{0} \rightarrow \infty
$$

$$
\mathbf{V}(\mathbf{s})=\mathbf{V}_{12}(\mathbf{s})+\mathbf{V}_{13}(\mathbf{s})+\mathbf{V}_{23}(\mathbf{s})+\mathbf{V}_{123}(\mathbf{s}), \quad \mathbf{V}_{123, s=0} \equiv \mathbf{V}_{\mathrm{NNN}}^{\text {bare }} ; \quad\left(\mathbf{V}_{\mathrm{YNN}}^{\text {bare }}=0\right)
$$

- separate SRG flow equations for 2-body and 3-body interactions:

$$
\begin{aligned}
& \frac{\mathbf{d V}^{\mathbf{N N}}(s)}{\mathbf{d s}}=\left[\left[\mathbf{T}^{\mathrm{NN}}, \mathbf{V}^{\mathrm{NN}}\right], \mathbf{T}^{\mathrm{NN}}+\mathbf{V}^{\mathrm{NN}}\right] \\
& \frac{\mathbf{d V ^ { \mathbf { Y N } } ( \mathbf { s } )}}{\mathbf{d s}}=\left[\left[\mathbf{T}^{\mathbf{Y N}}, \mathbf{V}^{\mathbf{Y N}}\right], \mathbf{T}^{\mathbf{Y N}}+\mathbf{V}^{\mathbf{Y N}}+\Delta \mathbf{M}\right] \\
& \frac{\mathbf{d} \mathbf{V}_{\mathbf{1 2 3}}}{\mathbf{d s}=} {\left[\left[\mathbf{T}_{12}, \mathbf{V}_{12}\right], \mathbf{V}_{31}+\mathbf{V}_{23}+\mathbf{V}_{123}\right] } \\
&+\left[\left[\mathbf{T}_{31}, \mathbf{V}_{31}\right], \mathbf{V}_{12}+\mathbf{V}_{23}+\mathbf{V}_{123}\right] \\
&+\left[\left[\mathbf{T}_{23}, \mathbf{V}_{23}\right], \mathbf{V}_{12}+\mathbf{V}_{31}+\mathbf{V}_{123}\right]+\left[\left[\mathbf{T}_{\text {rel }}, \mathbf{V}_{123}\right], \mathbf{H}_{s}\right]
\end{aligned}
$$

S.K. Bogner et al PRC75 (2007), K. Hebeler PRC85 (2012)
$\rightarrow$ SRG-induced YNNs are generated even if $V_{Y N N}^{\text {bare }}=0$

- SRG-induced beyond 3BFs are not included. Estimate size of omitted 4 BFs by varying $\lambda=\left(4 \mu^{2} / s\right)^{1 / 4}\left[\mathrm{fm}^{-1}\right]$

HL, J. Haidenbauer, U.-G. Meißner, A. Nogga (in preparation)

- Estimate size of omitted 4BFs by varying $\lambda$

NN: $\mathrm{N}^{4} \mathrm{LO}^{+}, \mathbf{3 N}: \mathrm{N}^{2} \mathrm{LO}(450) ; \mathbf{Y N}: \mathrm{N}^{2} \mathrm{LO}(550)$

| $\lambda\left[\mathrm{fm}^{-1}\right]$ | $B_{\Lambda}\left({ }_{\Lambda}^{4} \mathrm{He}, 0^{+}\right)$ | $B_{\Lambda}\left({ }_{\Lambda}^{5} \mathrm{He}\right)$ |
| :--- | :--- | :--- |
| 1.88 | $1.992 \pm 0.002$ | $3.712 \pm 0.001$ |
| 2.00 | $1.991 \pm 0.005$ | $3.705 \pm 0.005$ |
| 2.236 | $1.990 \pm 0.007$ | $3.708 \pm 0.006$ |
| 2.60 | $1.989 \pm 0.014$ | $3.744 \pm 0.008$ |
| 3.00 | $1.985 \pm 0.024$ | $3.806 \pm 0.030$ |
| $\infty$ | $2.01 \pm 0.02$ |  |


$\lambda=\infty:$ FY calculation using bare $\mathbf{N N}+\mathbf{3 N}+\mathrm{YN}$ interactions

- Variation of $B_{\Lambda}$ for $1.88 \leq \lambda \leq 3.0 \mathrm{fm}^{-1}: \quad \Delta B_{\Lambda}\left({ }_{\Lambda}^{4} \mathrm{He}\right)=10 \pm 25 \mathrm{KeV}$

$$
\Delta B_{\Lambda}\left({ }_{\Lambda}^{5} \mathrm{He}\right)=90 \pm 30 \mathrm{KeV}
$$

$\longrightarrow$ contributions of induced 4 BFs to $B_{\Lambda}\left({ }_{\Lambda}^{4} \mathrm{He},{ }_{\Lambda}^{5} \mathrm{He}\right)$ are negligible

# Results for A=3-8 hypernuclei 

$\mathrm{NN}: \mathbf{S M S} \mathrm{N}^{4} \mathrm{LO}^{+}(\mathbf{4 5 0}) \quad 3 \mathrm{~N}: \mathbf{N}^{2} \mathbf{L O}(450)$
YN: NLO13, NLO19(500); SMS NLO, ${ }^{2}$ ²O(550)

HL, J. Haidenbauer, U.-G. Meißner, A. Nogga PRC 107(2023)

- NLO13 \& NLO19 build on non-local regulator; phase equivalent in 2-body sector
- NLO13 characterised by a stronger $\boldsymbol{\Lambda} \boldsymbol{N} \boldsymbol{-} \boldsymbol{\Sigma} \boldsymbol{N}$ transition potential $\left({ }^{3} S_{1}-{ }^{3} D_{1}\right)$
$\rightarrow$ manifest in higher-body observables (J. Haidenbauer U.-G. Meißner, A. Nogga EPJA 56(2020))



## NN:SMS $\mathrm{N}^{\mathbf{4}} \mathrm{LO}^{+}(450)$

 $+3 \mathrm{~N}: \mathrm{N}^{2} \mathrm{LO}(450)$+YN: NLO13,19(CSB)
+SRG-induced YNN
Experiment:
M. Agnello et al. PLB 681(2009)
M. Juric NPB 52(1973)

- ${ }_{\Lambda}^{4} \mathrm{H}\left(1^{+}\right),{ }_{\Lambda}^{5} \mathrm{He},{ }_{\Lambda}^{7} \mathrm{Li},{ }_{\Lambda}^{8} \mathrm{Li}$ are fairly well described by NLO19; NLO13 underestimates these $B_{\Lambda}$ $\rightarrow \bullet$ signal for missing chiral YNN forces: $\sim \mathbf{5 0} \mathrm{KeV}$ to $\mathbf{B}_{\boldsymbol{\Lambda}}\left({ }_{\boldsymbol{\Lambda}}^{\mathbf{3}} \mathrm{H}\right), \sim 200-\mathbf{3 0 0} \mathrm{KeV}$ to $\mathbf{B}_{\boldsymbol{\Lambda}}\left({ }_{\Lambda}^{\mathbf{4}} \mathrm{He}\right)$

HL, J. Haidenbauer, U.-G. Meißner, A. Nogga (in preparation)

- NLO13 \& NLO19 build on non-local regulator;
- SMS NLO, $\mathbf{N}^{2} \mathrm{LO}$ use local regulator for long-range interactions (milden cutoff artifacts)
$\rightarrow$ SMS potentials lead to a more attractive $\Lambda N$ (J. Haidenbauer, U.-G. Meißner, A. Nogga, HL EPJA 59(2023))

|  | ${ }_{\Lambda}^{3} \mathrm{H}$ | ${ }_{\Lambda}^{4} \mathrm{He}\left(0^{+}\right)$ | ${ }_{\Lambda}^{4} \mathrm{He}\left(1^{+}\right)$ | ${ }_{\Lambda}^{5} \mathrm{He}$ |
| :--- | :--- | :--- | :--- | :--- |
| NLO13 | 0.090 | $1.48 \pm 0.02$ | $0.58 \pm 0.02$ | $2.22 \pm 0.06$ |
| NLO 19 | 0.091 | $1.46 \pm 0.02$ | $1.06 \pm 0.02$ | $3.32 \pm 0.03$ |
| NLO | 0.124 | $2.061 \pm 0.001$ | $1.087 \pm 0.009$ | $3.334 \pm 0.008$ |
| $\mathrm{~N}^{2} \mathrm{LO}$ | 0.139 | $1.992 \pm 0.002$ | $1.23 \pm 0.009$ | $3.710 \pm 0.007$ |
| Exp $^{*}$ | $0.148 \pm \mathbf{0 . 0 4 0}$ | $2.169 \pm 0.042\left({ }_{\Lambda}^{4} \mathrm{H}\right)$ | $1.081 \pm 0.046\left({ }_{\Lambda}^{4} \mathrm{H}\right)$ | $3.102 \pm 0.030$ |
|  |  | $2.347 \pm 0.036\left({ }_{\Lambda}^{4} \mathrm{He}\right)$ | $0.942 \pm 0.036\left({ }_{\Lambda}^{4} \mathrm{He}\right)$ |  |

## NN:SMS $\mathrm{N}^{4} \mathrm{LO}^{+}(450)$

$+3 \mathrm{~N}: \mathrm{N}^{2} \mathrm{LO}(450)$
+SRG-induced YNN

* https://hypernuclei.kph.uni-mainz.de/
- all s-shell states are fairly well described with SMS NLO \& $\mathbf{N}^{2}$ LO
- chiral YNN forces contribute at $\mathrm{N}^{2} \mathrm{LO}(5$ LECs for $\Lambda N N) \rightarrow$ not possible to fix with decuplet saturation at NLO: 2 LECs $(1 \Lambda N N+1 \Sigma N N) \rightarrow$ fit to $\boldsymbol{B}_{\boldsymbol{\Lambda}}\left({ }_{\Lambda}^{4} \mathbf{H e}\left(0^{+}, 1^{+}\right)\right)$or $\boldsymbol{B}_{\boldsymbol{\Lambda}}\left(\mathbf{H e}\left(0^{+}\right),{ }_{\boldsymbol{\Lambda}}^{5} \mathbf{H e}\left(1 / 2^{+}\right)\right)$


## Variation due to NN \& YN interactions







## Variation due to NN interactions

$$
\mathbf{N N}: \mathbf{N}^{2} \mathrm{LO}(450-500), \mathbf{N}^{3} \mathrm{LO}(450), \mathrm{N}^{4} \mathrm{LO}^{+}(400-550) ; 3 \mathrm{~N}: \mathrm{N}^{2} \mathrm{LO}
$$

| considered NN + YN potentials | $\mathbf{B}_{\Lambda}\left({ }_{\Lambda}^{3} \mathrm{H}\right)$ | $\mathbf{B}_{\boldsymbol{\Lambda}}\left({ }_{\Lambda}^{4} \mathrm{He}, \mathbf{0}^{+}\right)$ | $\mathbf{B}_{\Lambda}\left({ }_{\Lambda}^{4} \mathrm{He}, \mathbf{1}^{+}\right)$ | $\mathbf{B}_{\Lambda}\left({ }_{\Lambda}^{5} \mathrm{He}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{N}^{4} \mathrm{LO}^{+}(400-550)+\mathrm{N}^{2} \mathrm{LO}(550)$ | 3 | 43 | 44 | 44 |
| $\mathrm{N}^{4} \mathrm{LO}^{+}(400-550)+\mathrm{NLO}(550)$ | 14 | 110 | 25 | 90 |
| $\left(\mathbf{N}^{2} \mathbf{L O}, \mathbf{N}^{3} \mathrm{LO}, \mathbf{N}^{4} \mathrm{LO}^{+}\right)+\mathrm{N}^{2} \mathrm{LO}(550)$ | 11 | 114 | 114 | 295 |
| $\left(\mathbf{N}^{2} \mathbf{L O}, \mathbf{N}^{3} \mathbf{L O}, \mathbf{N}^{4} \mathrm{LO}^{+}\right)+\mathbf{N L O}(550)$ | 14 | 147 | 88 | 273 |
| $\mathrm{N}^{4} \mathrm{LO}^{+}(400-550)+\mathrm{LO}(600)$ | 25 | 194 | 223 | 970 |
| Gazda* variation with cutoff | 50 | 270 | 240 | 1150 |
| Gazda* variance $\sigma_{\text {model }}$ | 20 | 100 | 100 | 400 |

*D. Gazda et al PRC 106(2022): based on LO YN + 42 non-local regularized $\mathrm{NN}(3 \mathrm{~N}) \mathrm{N}^{2} \mathrm{LO}_{\text {sim }}$

$$
\sigma_{\text {model }} \equiv\left[\sigma^{2}\left(\mathrm{~N}^{2} \mathrm{LO}_{\mathrm{sim}}\right)\right]^{1 / 2} \quad \text { (talk by Gazda) }
$$

$\rightarrow$ • for YN NLO, $\mathbf{N}^{2}$ LO: NN variations are "small", especially when SMS $\mathrm{N}^{4} \mathrm{LO}^{+}$are employed

- NN variations are expected to be counterbalanced by chiral YNN forces


## Truncation error estimation

- NN \&YN interactions are truncated at certain orders $\rightarrow$ higher-order contributions?


## - Epelbaum, Krebs and Meißner:

- cutoff variations $\rightarrow$ no reliable estimate for truncation errors
- estimate truncation error at each order via expected + actual size of higher-order corrections:

$$
\begin{aligned}
& X^{(k)}=X^{(0)}+\sum_{i=2}^{k} \Delta X^{(i)} ; \\
& \delta X^{(0)}=Q^{2}\left|X^{\mathrm{LO}}\right| ; \\
& \delta X^{(\mathrm{i})}=\max _{2 \leq j \leq i}\left(Q^{i+1}\left|X^{(0)}\right|, Q^{i+1-j}\left|X^{(\mathrm{j})}-X^{(\mathrm{j}-1)}\right|\right) ; \quad Q=M_{\pi}^{e f f} / \Lambda_{b} \quad \text { E. Epelbaum, H. Krebs, U.-G. Meißner EPJA 51(2015) } \\
&
\end{aligned}
$$

- Furnstahl, Klco, Phillips, Melendez, Weslowski : R. J. Furnstahl et al PRC 92(2015)
J. A. Melendez et al PRC 96 (2017), 100(2019)

$$
\begin{aligned}
X^{(k)} & =X^{(0)}+\sum_{i=2}^{k} \Delta X^{(i)}=: X_{r e f}\left(c_{0}+c_{2} Q^{2}+c_{3} Q^{3}+\cdots\right) \\
\delta X^{(k)} & =X_{r e f}\left(\sum_{n=k+1}^{\infty} c_{n} Q^{n}\right) ; \quad c_{n} \sim \mathcal{O}(1) ; \quad c_{n} \mid \bar{c}^{2} \stackrel{\mathrm{iid}}{\sim} \mathcal{N}\left(0, \bar{c}^{2}\right) ; \quad \bar{c}^{2} \sim \chi^{-2}\left(\nu_{0}, \tau_{0}^{2}\right) \quad \text { (pointwise model) }
\end{aligned}
$$

Truncation error estimation







- NN convergence is "faster" than YN
- uncertainty due to YN truncation is dominant
- YNN contribution (half of $68 \%$ DoB interval at NLO):
$\sim 0.15,0.24,900 \mathrm{KeV}$ for ${ }_{\Lambda}^{3} \mathbf{H},{ }_{\Lambda}^{3} \mathrm{He},{ }_{\Lambda}^{5} \mathrm{He}$
$\rightarrow$ consistent with estimates using NLO13 \& NLO19


## Charge symmetry breaking (CSB) in $A=4$


Schulz et al (2016); Yamamoto et al (2015); Juric et al $(1973)$; Bedjidian et al $(1976,1979)$
$\Delta E\left(1^{+}\right)=B_{\Lambda}\left({ }_{\Lambda}^{4} \mathrm{He}, 1^{+}\right)-B_{\Lambda}\left({ }_{\Lambda}^{4} \mathrm{H}, 1^{+}\right)=-83 \pm 94 \mathrm{keV}$
$\Delta E\left(0^{+}\right)=B_{\Lambda}\left({ }_{\Lambda}^{4} \mathrm{He}, 0^{+}\right)-B_{\Lambda}\left({ }_{\Lambda}^{4} \mathrm{H}, 0^{+}\right)=233 \pm 92 \mathrm{keV}$
$\Delta E\left({ }^{3} \mathrm{H},{ }^{3} \mathrm{He}\right) \sim 683+81 \mathrm{keV}$ (R. Brandenburg et al NPA 294(1978))
Coulomb $\Delta M(\boldsymbol{p}, \boldsymbol{n})$

## CSB YN interactions at NLO

(J. Haidenbauer, U.-G. Meißner, A. Nogga FBS 62(2021))

- sub-leading contributions are dominant:


$$
C_{s}^{C S B}, C_{t}^{C S B} \text { adjusted to } \Delta E\left(0^{+}, 1^{+}\right)
$$

| $(\mathrm{fm} / / \mathrm{keV})$ | $a_{s}^{\Lambda p}$ | $a_{s}^{\Lambda n}$ | $\delta a_{s}$ | $a_{t}^{\Lambda p}$ | $a_{t}^{\Lambda n}$ | $\delta a_{t}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{NLO} 19(\mathbf{5 0 0})$ <br> no CSB | $\mathbf{- 2 . 9 1}$ | $\mathbf{- 2 . 9 1}$ | $\mathbf{0}$ | $\mathbf{- 1 . 4 2}$ | $\mathbf{- 1 . 4 1}$ | $\mathbf{- 0 . 0 1}$ |
| $\operatorname{CSB}(\mathbf{5 0 0})$ | $\mathbf{- 2 . 6 5}$ | $\mathbf{- 3 . 2 0}$ | $\mathbf{0 . 5 5}$ | $\mathbf{- 1 . 5 8}$ | $\mathbf{- 1 . 4 7}$ | $\mathbf{- 0 . 1 1}$ |
| $\operatorname{CSB}(550)$ | -2.64 | $\mathbf{- 3 . 2 1}$ | 0.57 | -1.52 | $\mathbf{- 1 . 4 1}$ | -0.11 |
| $\operatorname{CSB}(600)$ | -2.63 | -3.23 | 0.6 | $\mathbf{- 1 . 4 7}$ | $\mathbf{- 1 . 3 6}$ | -0.09 |
| $\operatorname{CSB}(650)$ | -2.62 | -3.23 | 0.61 | -1.46 | -1.37 | -0.09 |

## CSB results in $A=4-8$

HL, J. Haidenbauer, U.-G. Meißner, A. Nogga PRC 107(2023)


NN:SMS $\mathrm{N}^{4} \mathrm{LO}+(450)$
$+3 \mathrm{~N}: \mathrm{N}^{2} \mathrm{LO}(450)$
+YN: NLO13,19(CSB)
+SRG-induced YNN

- CSB predictions for $A=7$ are comparable to experiment.
- both potentials predict a somewhat larger CSB in A=8 doublet as compared to experiment
$\rightarrow$ experimental CSB splitting for A=8 larger than $40 \pm 60 \mathrm{keV}$ ?
- CSB estimated for $\mathrm{A}=4$ : too large? different spin-dependence?

Recent STAR measurement suggests somewhat different CSB in $A=4$ :

$$
\begin{aligned}
\Delta E\left(1^{+}\right) & =B_{\Lambda}\left({ }_{\Lambda}^{4} \mathrm{He}, 1^{+}\right)-B_{\Lambda}\left({ }_{\Lambda}^{4} \mathrm{H}, 1^{+}\right) \\
& =-83 \pm 94 \mathrm{keV} \Rightarrow(\mathrm{CSB}) \\
& =\mathbf{- 1 6 0} \pm \mathbf{1 4 0} \pm \mathbf{1 0 0} \mathrm{keV} \Rightarrow\left(\mathrm{CSB}^{*}\right)
\end{aligned}
$$

$$
\begin{aligned}
\Delta E\left(0^{+}\right) & \left.=B_{\Lambda}\left({ }_{\Lambda}^{4} \mathrm{He}, 0^{+}\right)-B_{\Lambda}{ }_{\Lambda}^{4} \mathrm{H}, 0^{+}\right) \\
& =233 \pm 92 \mathrm{keV} \Rightarrow(\mathrm{CSB}) \\
& =\mathbf{1 6 0} \pm \mathbf{1 4 0} \pm \mathbf{1 0 0} \mathrm{keV} \Rightarrow\left(\mathrm{CSB}^{*}\right)
\end{aligned}
$$

* STAR Collaboration PLB 834 (2022)

|  | NLO19(500) | CSB | CSB $^{*}$ |
| :--- | :---: | :---: | :---: |
| $a_{s}^{\Lambda p}$ | -2.91 | -2.65 | -2.58 |
| $a_{\boldsymbol{s}}^{\Lambda n}$ | -2.91 | -3.20 | -3.29 |
| $\boldsymbol{\delta} \boldsymbol{a}_{\boldsymbol{s}}$ | $\mathbf{0}$ | $\mathbf{0 . 5 5}$ | $\mathbf{0 . 7 1}$ |
| $a_{t}^{\Lambda p}$ | -1.42 | -1.57 | -1.52 |
| $a_{t}^{\Lambda n}$ | -1.41 | -1.45 | -1.49 |
| $\boldsymbol{\delta} \boldsymbol{a}_{\boldsymbol{t}}$ | $\mathbf{- 0 . 0 1}$ | $\mathbf{- 0 . 1 2}$ | $\mathbf{- 0 . 0 3}$ |

$\rightarrow \delta a\left({ }^{1} S_{0}\right)$ increases while $\delta a\left({ }^{3} \mathbf{S}_{\mathbf{1}}\right)$ decreases
$\rightarrow$ How does the STAR measurement affect the predictions of CSB in A=7,8 multiplets ?


NN:SMS $\mathbf{N}^{\mathbf{4}} \mathbf{L O + ( 4 5 0 )}$
+YN: NLO13,19(CSB)
$\lambda_{N N}=1.6 \mathrm{fm}^{-1}$
$\lambda_{Y N}^{o p t}=0.823 \mathrm{fm}^{-1}$
$B_{\Lambda}\left({ }_{\boldsymbol{\Lambda}}^{\mathbf{5}} \mathbf{H e}, \lambda_{Y N}^{\text {opt }}\right)=B_{\Lambda}\left({ }_{\boldsymbol{\Lambda}}^{\mathbf{5}} \mathbf{H e}, 3 \mathrm{BFs}\right)$

- CSB* fit predicts reasonable CSB in both $A=7$ and $A=8$ systems
- $\operatorname{CSB}$ in $A=4\left(\mathbf{0}^{+}\right)$and $A=8$, and in $A=4\left(\mathbf{1}^{+}\right)$and $A=7$ are correlated
$\rightarrow$ accurate CSB in $A=7$ \& 8 may allow for an independent check of $A=4$ CSB


## Summary

- At our disposal we have 3 tools to tackle light (hyper)nuclear systems:
- s-shell (hyper)nuclei: Faddeev-Yakubovsky, NLEFT (D. Frame et al EPJA 56(2020))
- s-shell \& light p-shell: Jacobi NCSM approach; numerical uncertainties (s-shell) ~ few keV
$\rightarrow$ establish a direct link between chiral YN interactions and observables $(A \leq 9)$
- YN at NLO \& $\mathbf{N}^{2} \mathrm{LO}$ yields reasonable $\mathbf{B}_{\Lambda}$ in $\boldsymbol{A}=\mathbf{3 - 8}$ hypernuclei.

NLO13 \& NLO19 results show a clear signal of missing chiral YNN forces

- study convergence w.r. to NN \& YN orders in $A=3-5$ hypernuclei
- CSB NLO interactions reproduce experimental CSB for $A=4,7$ multiplets, $\boldsymbol{A}=\mathbf{8}$ CSB prediction is larger than experiment


## Thank you for the attention!

