

# Near-term quantum simulation of nuclear dynamics

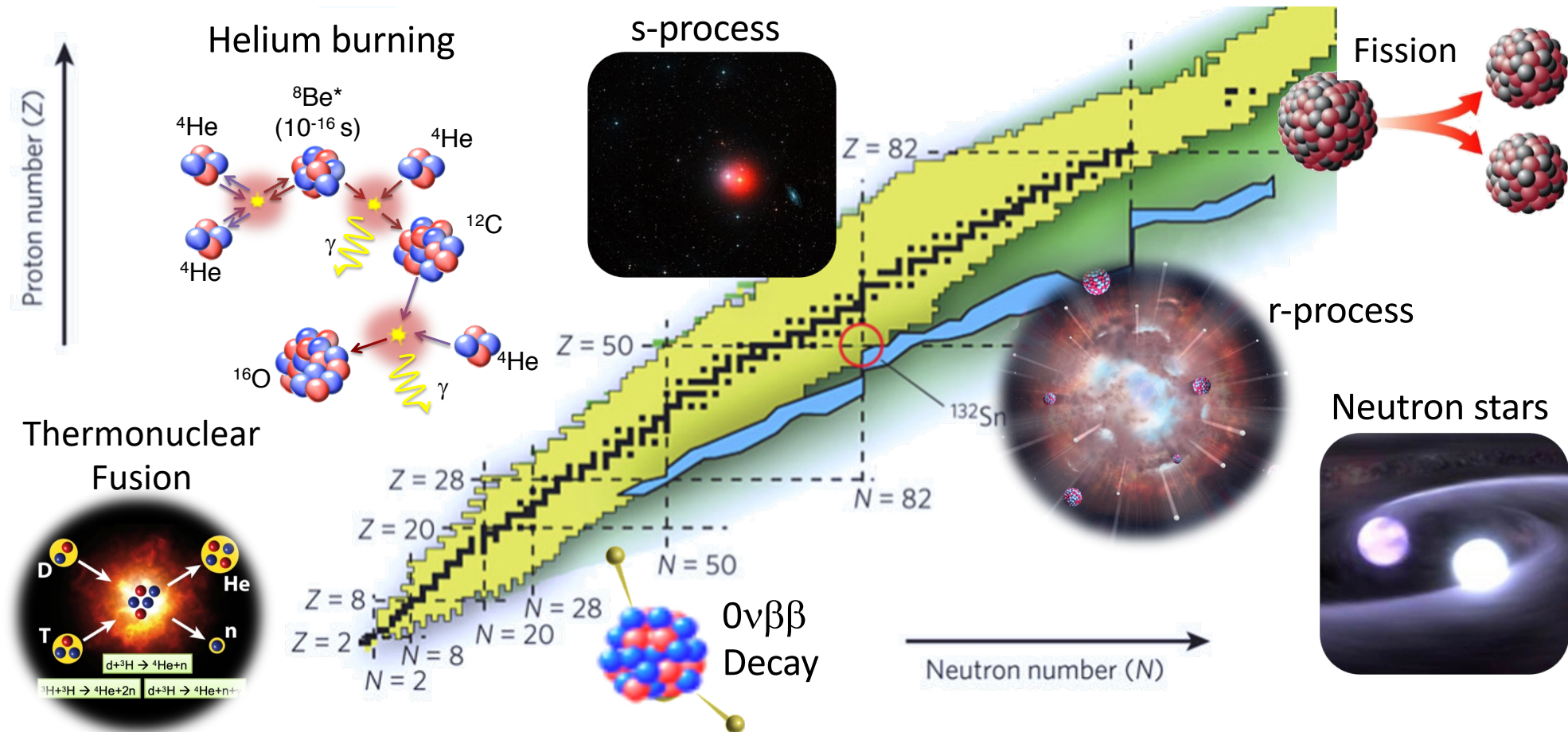
25<sup>th</sup> European Conference on Few-Body Problems in Physics

Mainz, August 2, 2023

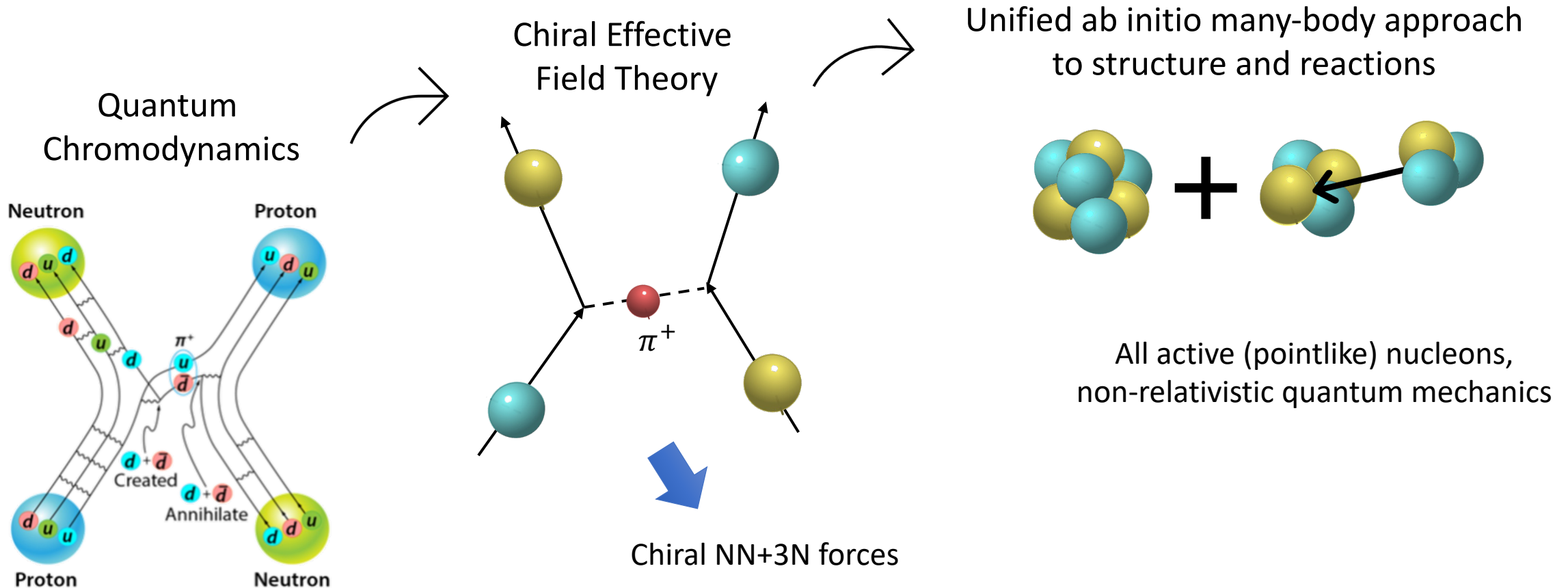
Sofia Quaglioni



# Goal: Predictive understanding of nuclei and their interactions

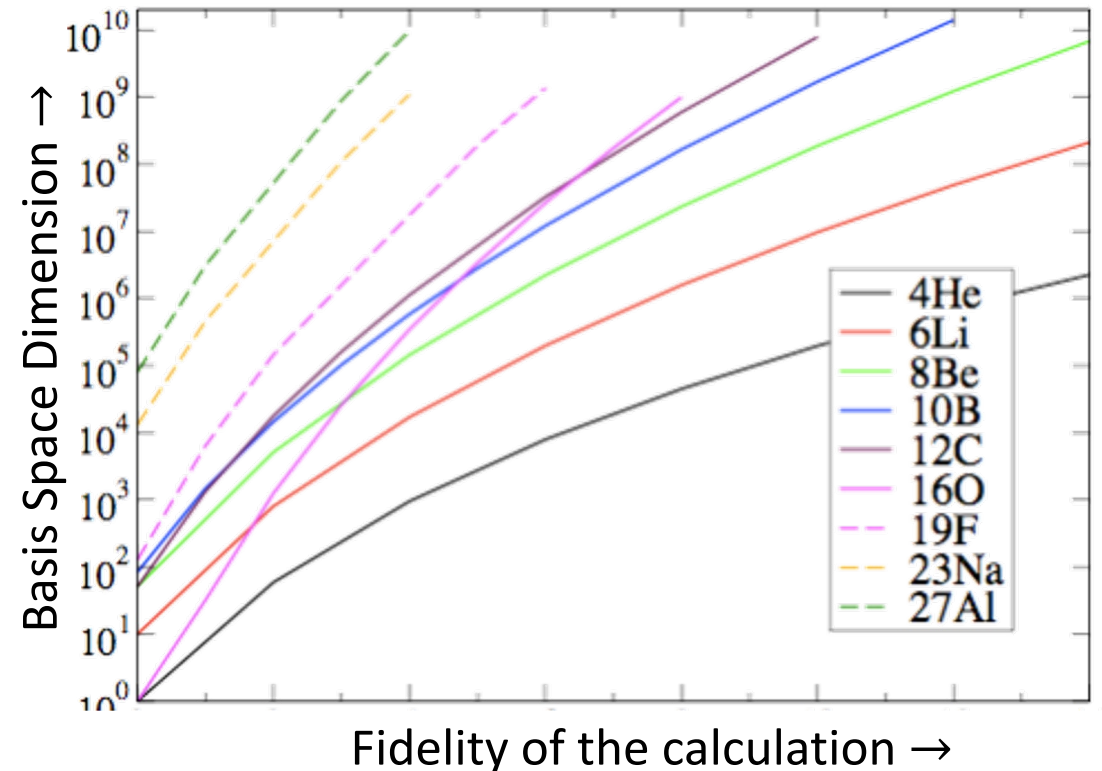
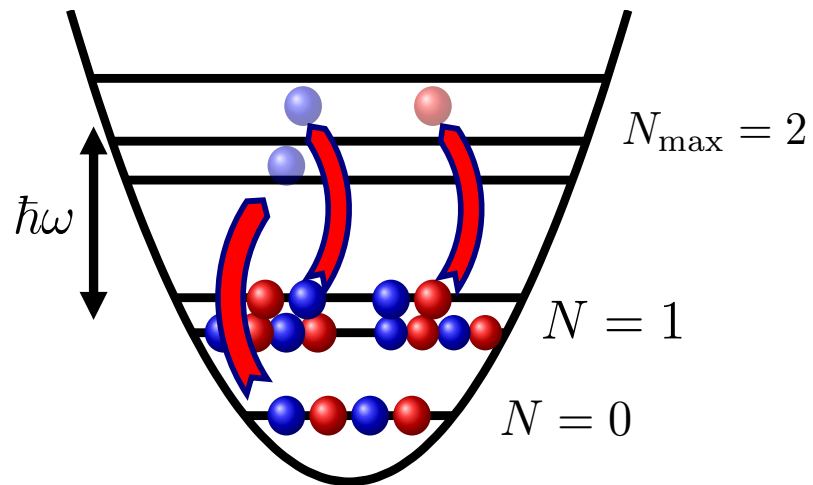


Currently best path to fundamental understanding combines effective field theory and ab initio methods



# Ab initio nuclear theory is among the most computationally intensive fields of science ...

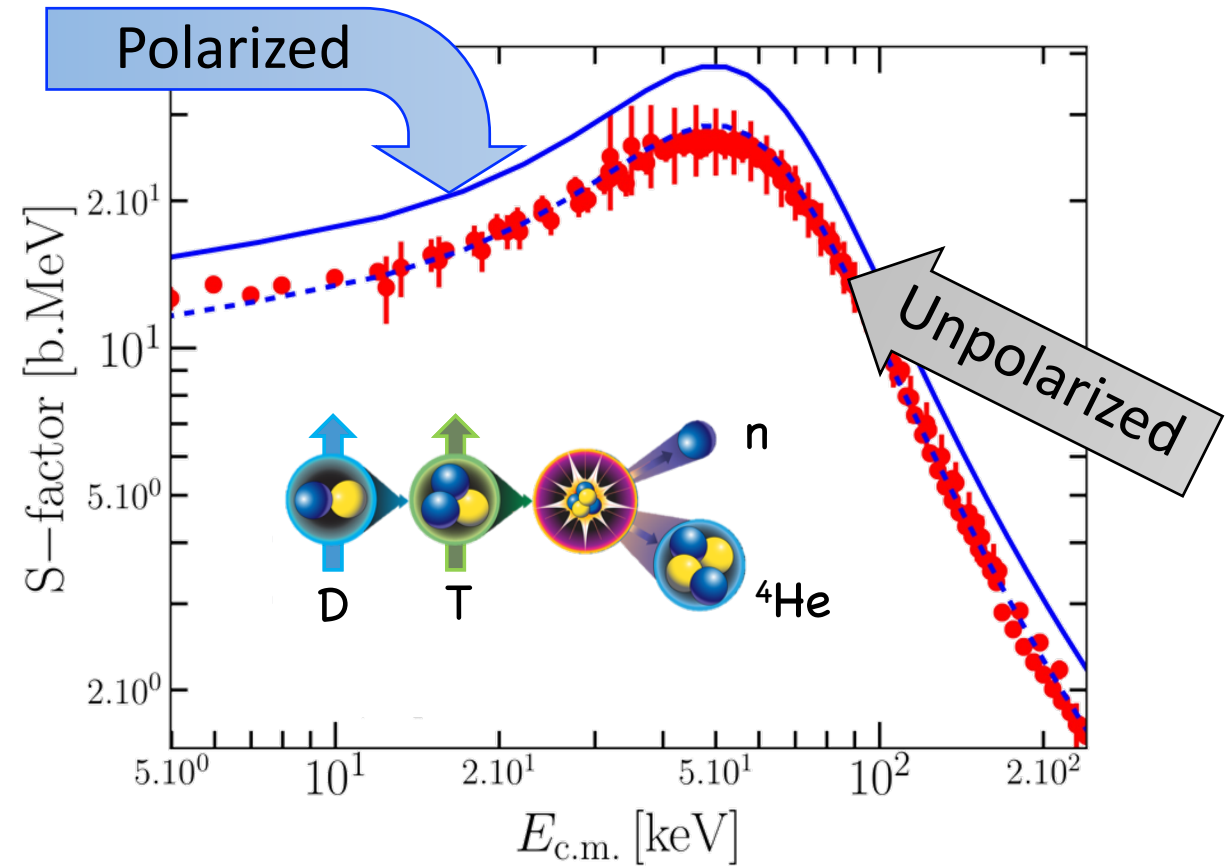
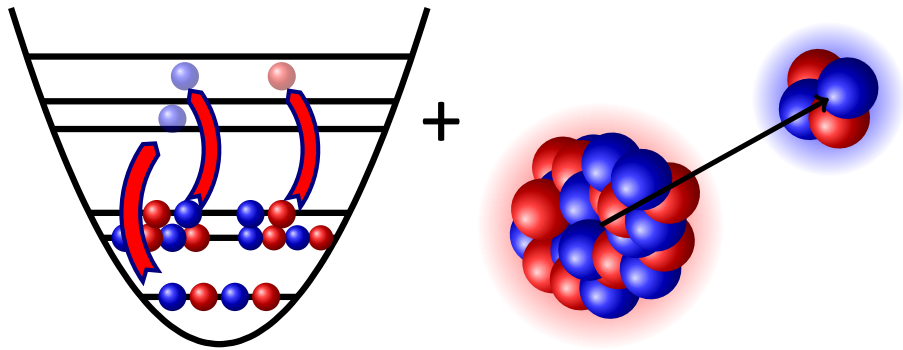
No-Core Shell Model:  
bound states, static properties



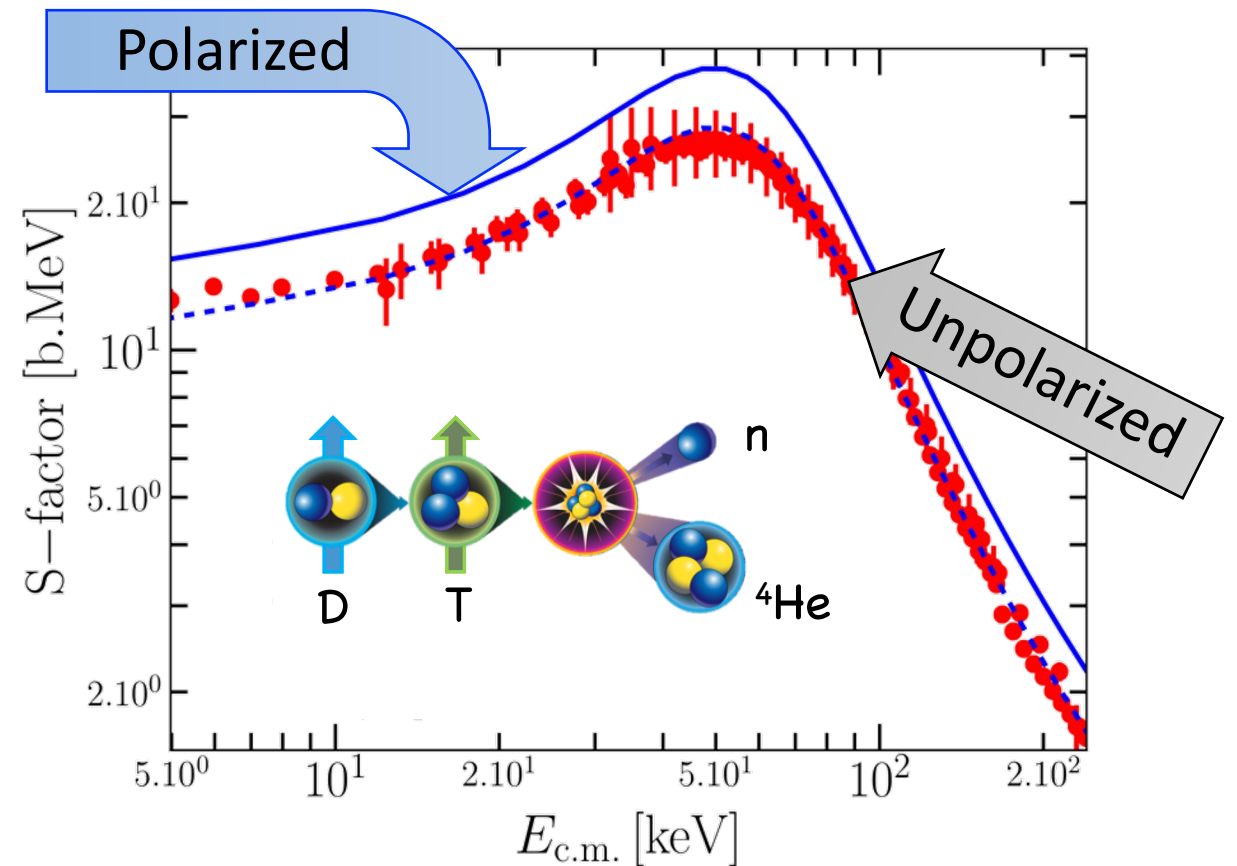


... and nuclear dynamical properties are among the most expensive to compute

No-Core Shell Model with Continuum: resonances, scattering, reactions



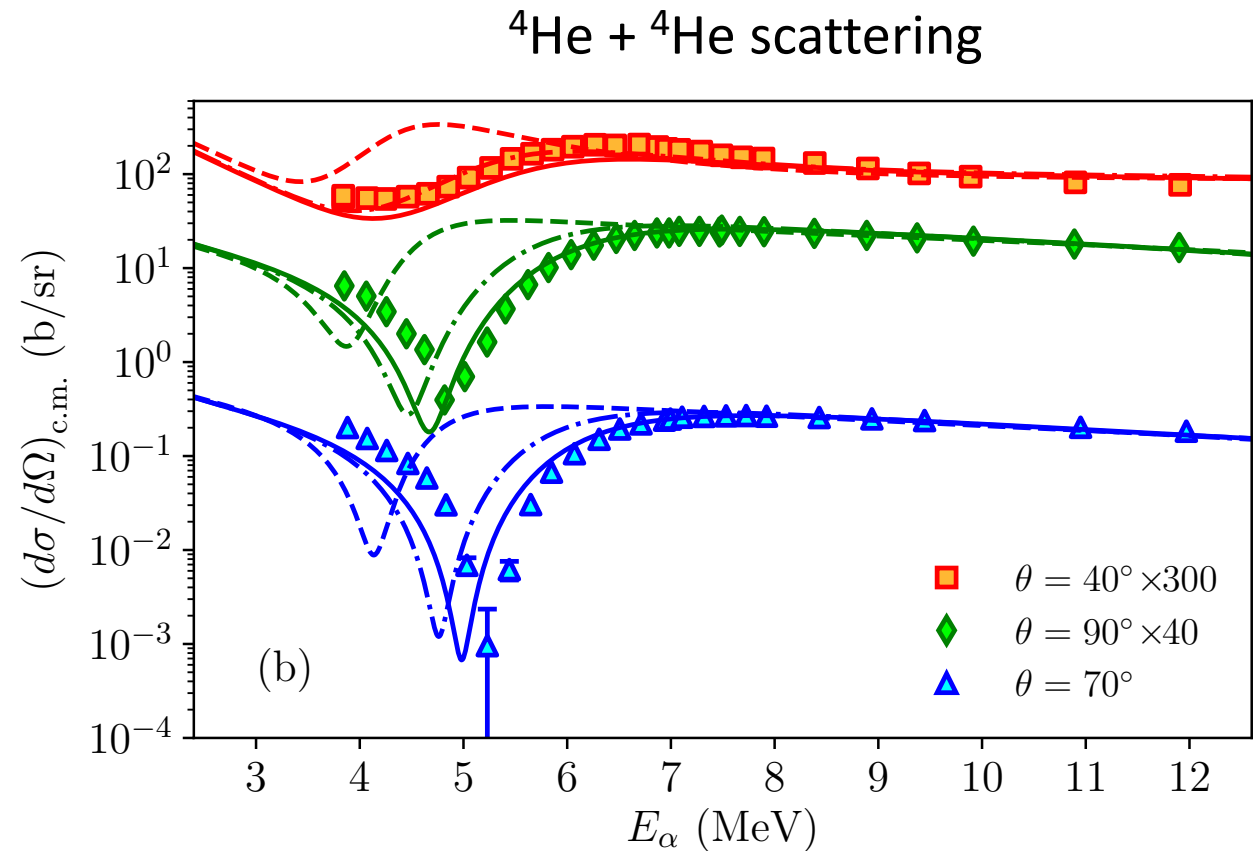
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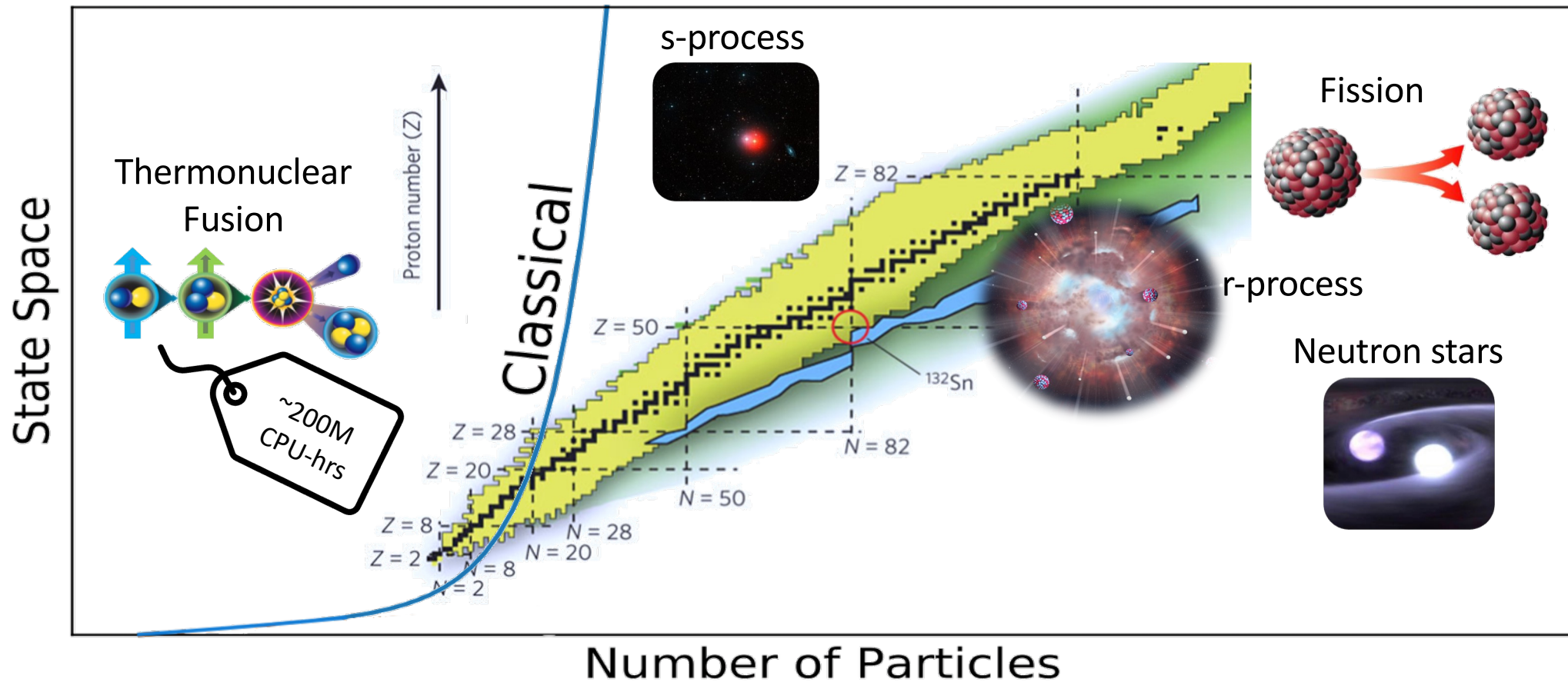
Advanced (CPU+GPU) architectures are enabling previously impossible ab initio reaction calculations



Memory of GPU cards present limiting factor

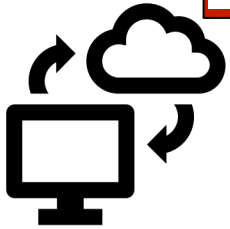
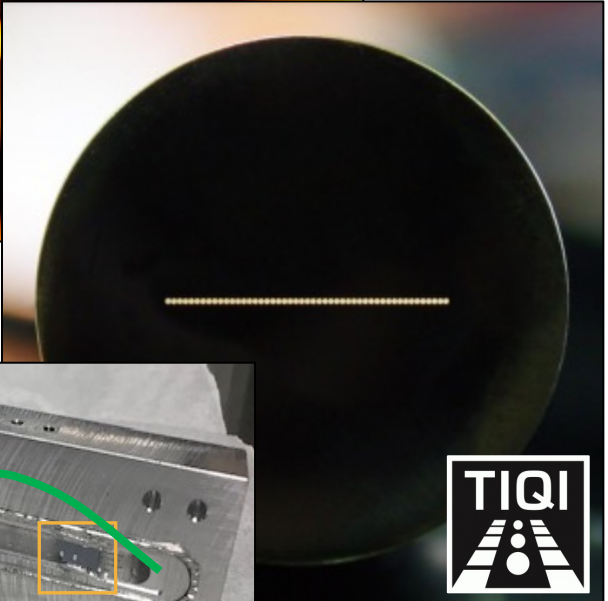
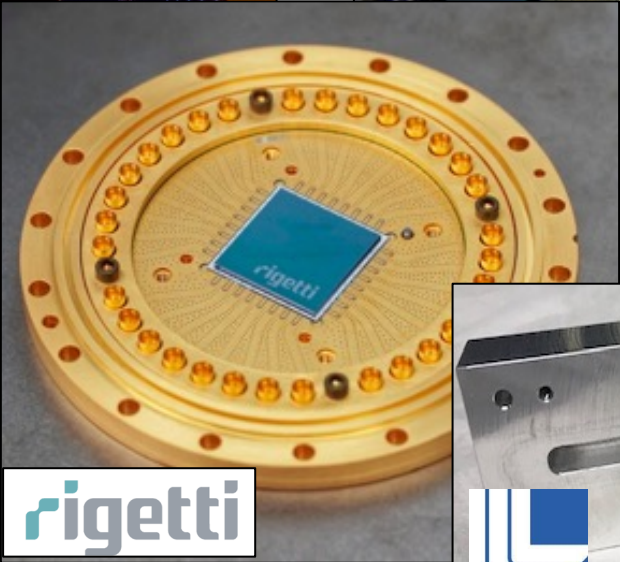
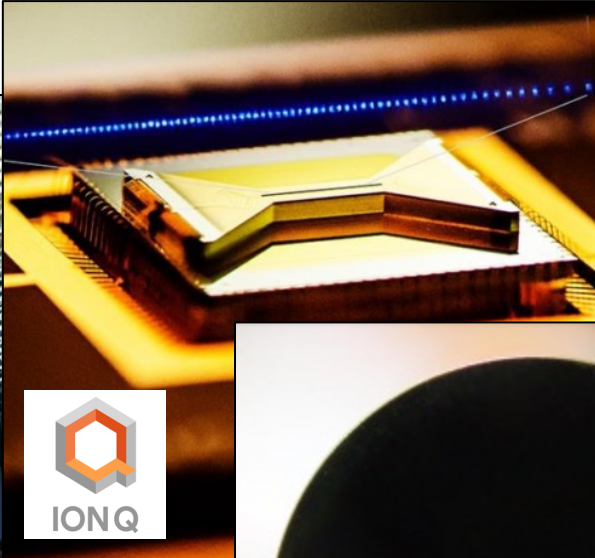
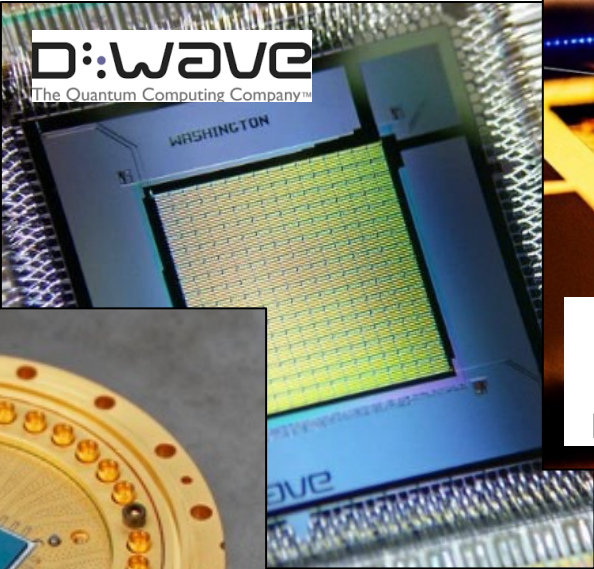
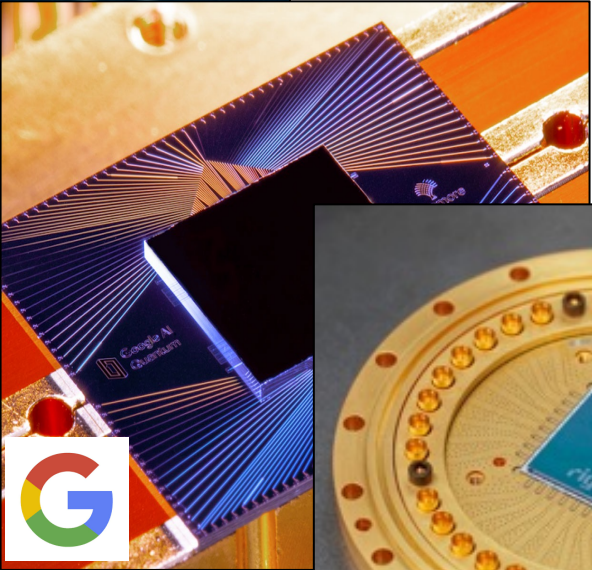
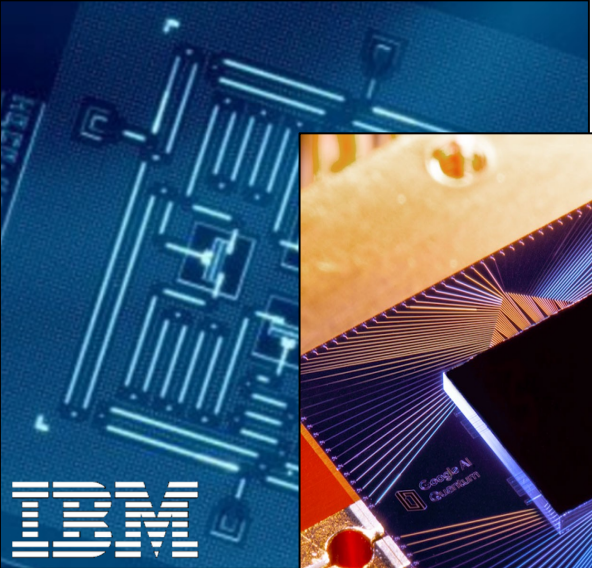


Fundamental description of most nuclear dynamics remains a major challenge even with next-gen HPC



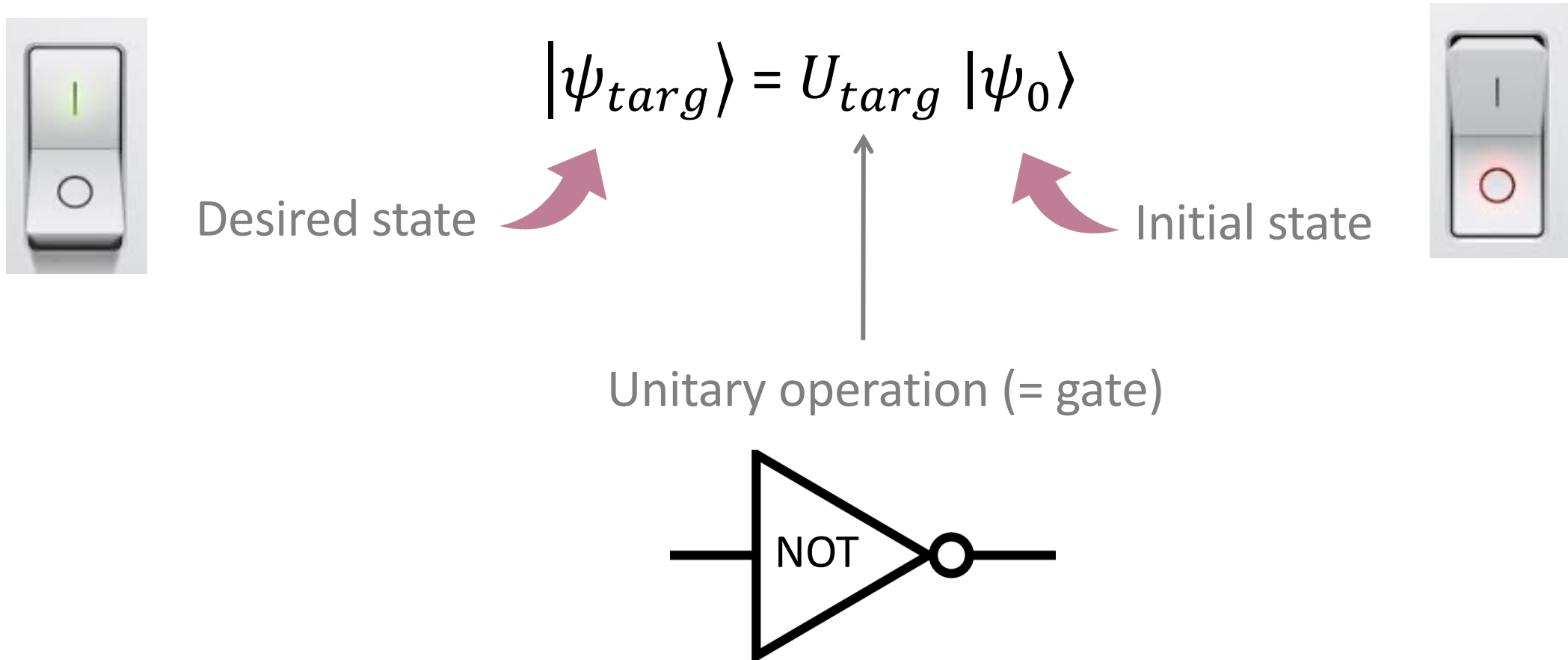


Several prototypes of quantum processors have emerged both in academia and industry

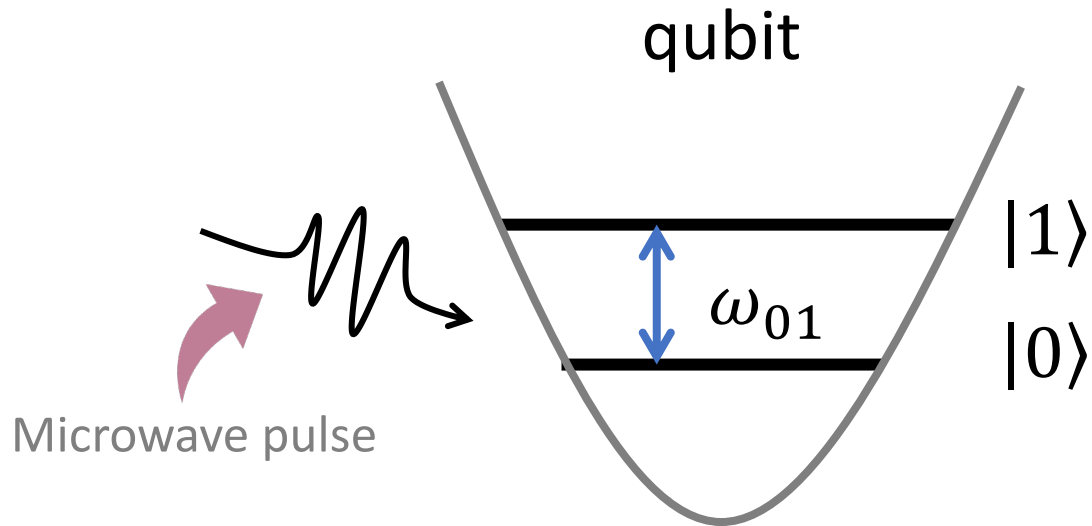




# Quantum computers perform calculations by manipulating quantum states



Most quantum computers perform calculations by manipulating 2-level quantum systems or 'qubits'



$$|\psi_{qubit}\rangle = \alpha|0\rangle + \beta|1\rangle$$

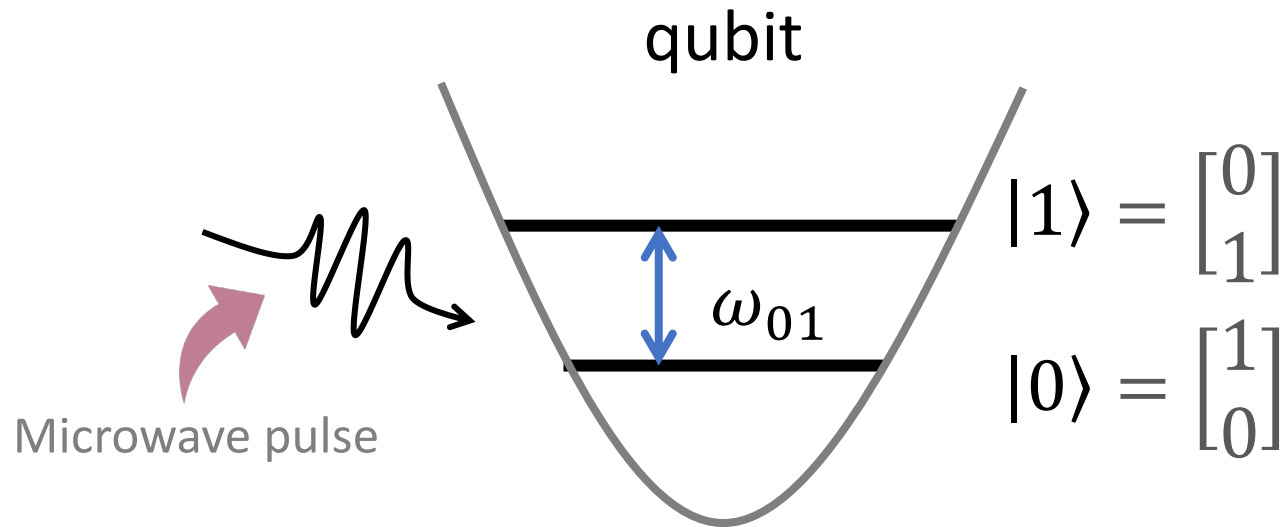
$$(|\alpha|^2 + |\beta|^2 = 1)$$

bit



Either 0 or 1

Most quantum computers perform calculations by manipulating 2-level quantum systems or 'qubits'



$$|\psi_{qubit}\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$(|\alpha|^2 + |\beta|^2 = 1)$$

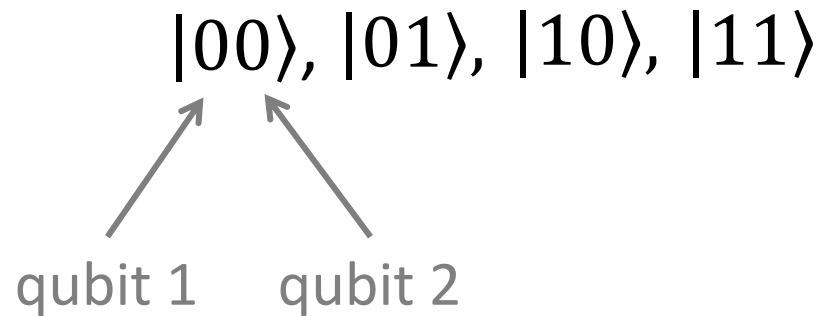
bit



Either 0 or 1

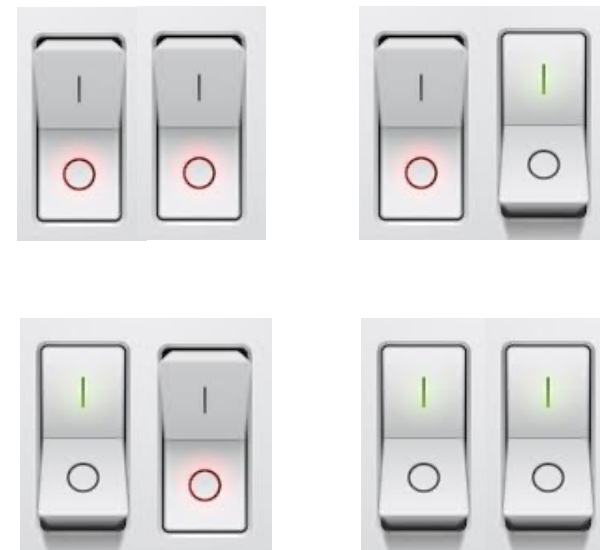
A register of  $n$  qubits  
has  $2^n$  possible basis states (here  $n = 2$ ) ...

2-qubit register:



$2^n$  basis states

2-bit register:



$2^n$  possible configurations

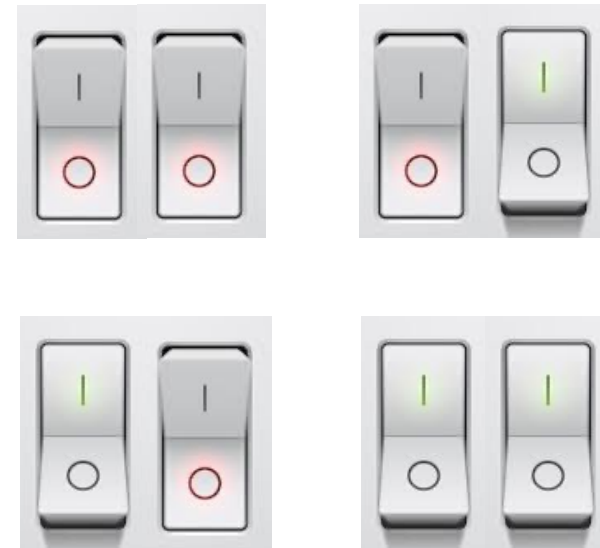
... can store, process  $2^n$  basis states simultaneously.  
Well suited for many-particle entangled quantum states!

2-qubit register:

$$|\psi_{2-qubit}\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle \\ + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

Can store superpositions of  
 $2^n$  basis states simultaneously

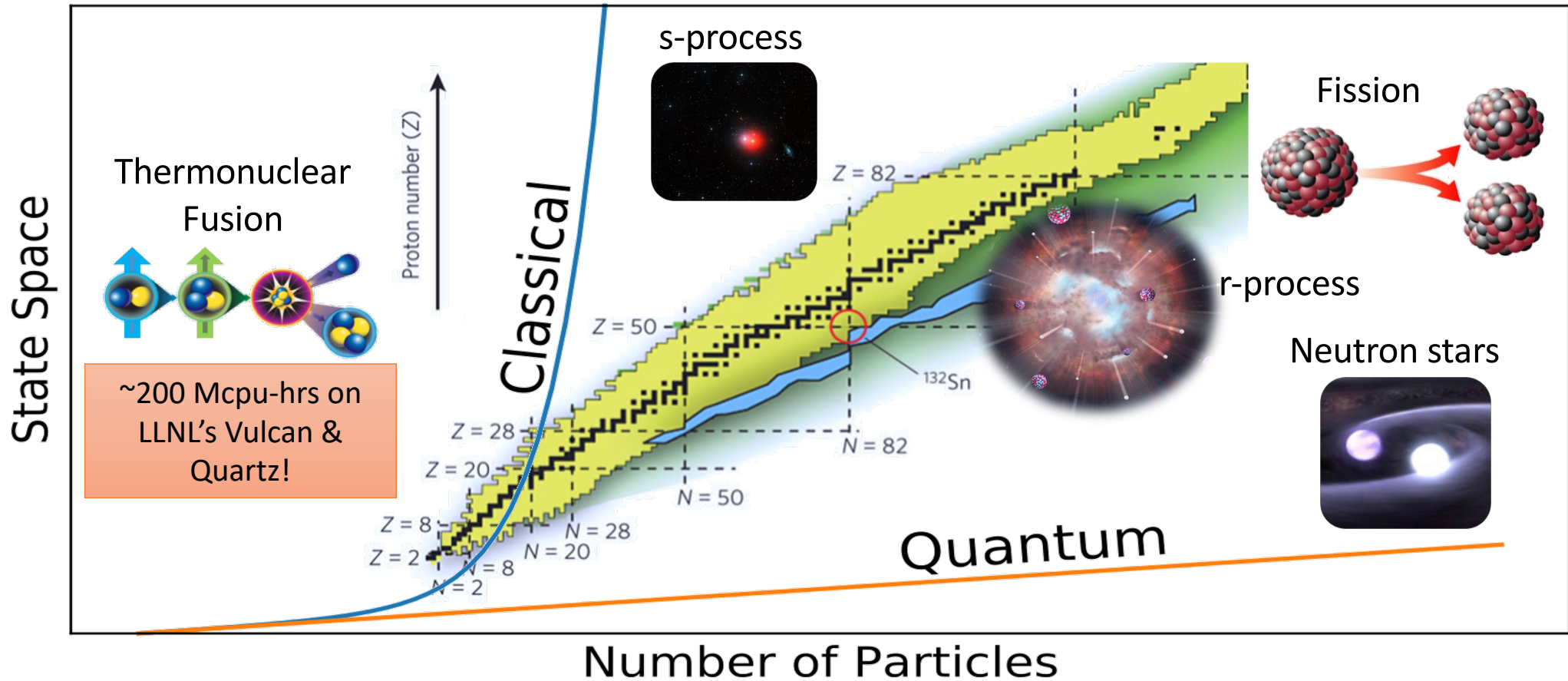
2-bit register:



can only store one configuration

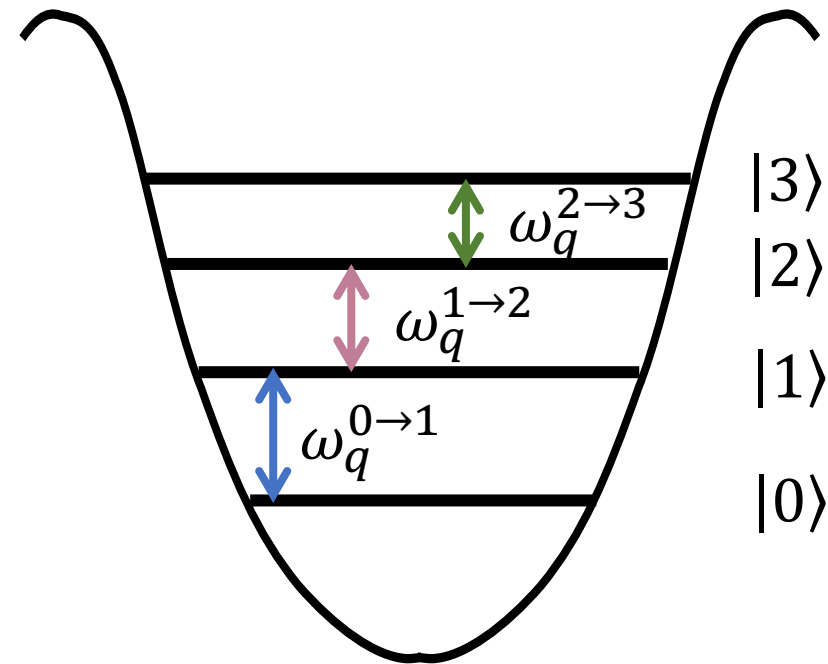
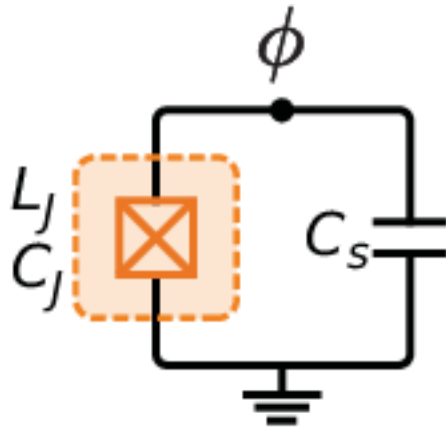


# Quantum computing holds the promise of exact simulations of nuclear matter and dynamics



# A physical realization of a qubit is a transmon

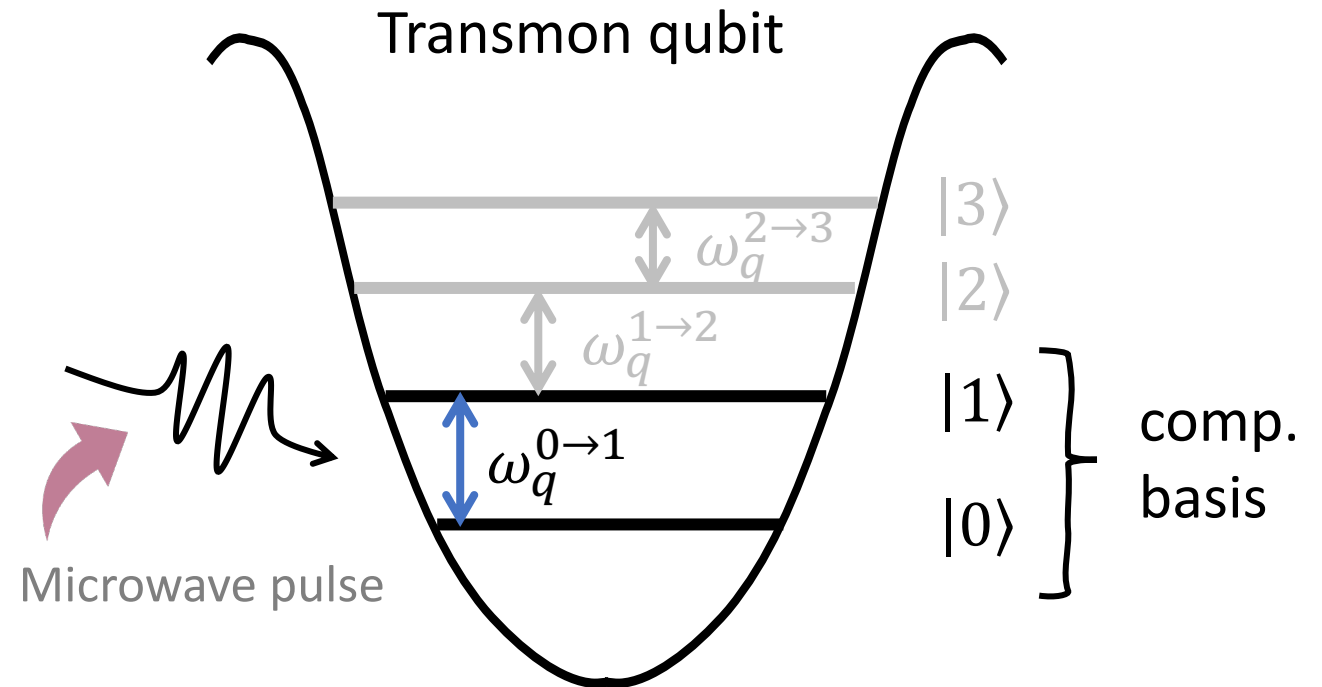
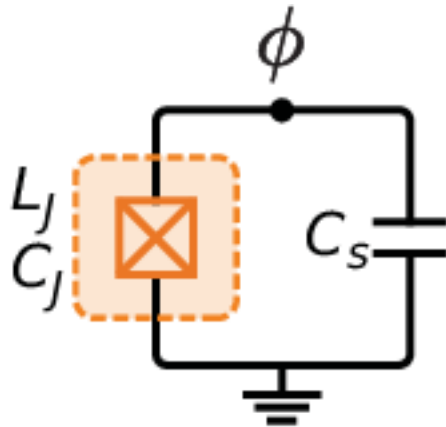
Quantized transmon:



$$H = \hbar\omega_q a^\dagger a + \frac{\alpha}{2} a^\dagger a^\dagger a a$$

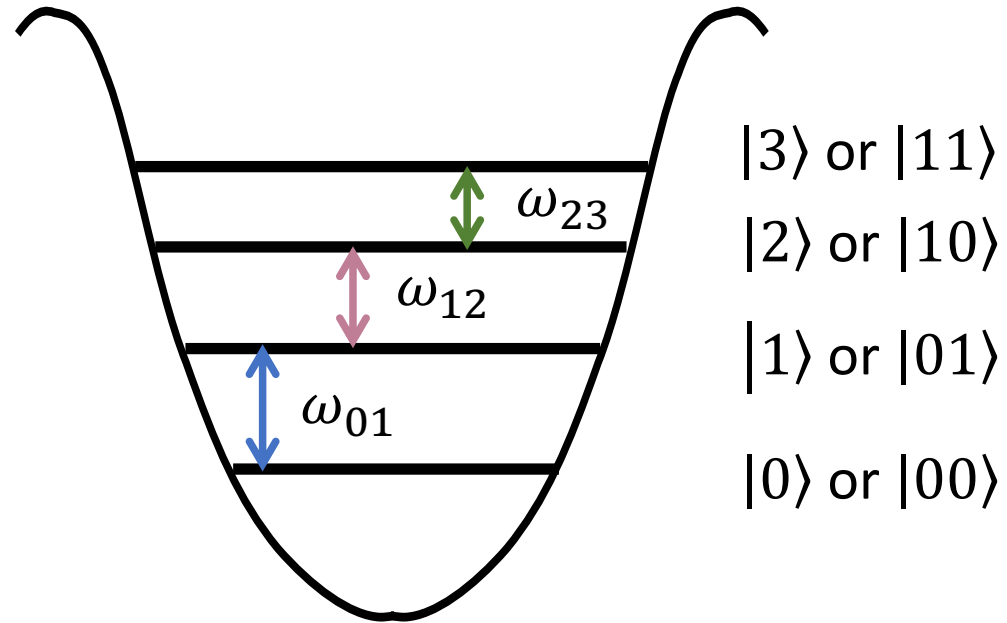
# A physical realization of a qubit is a transmon

Quantized transmon:

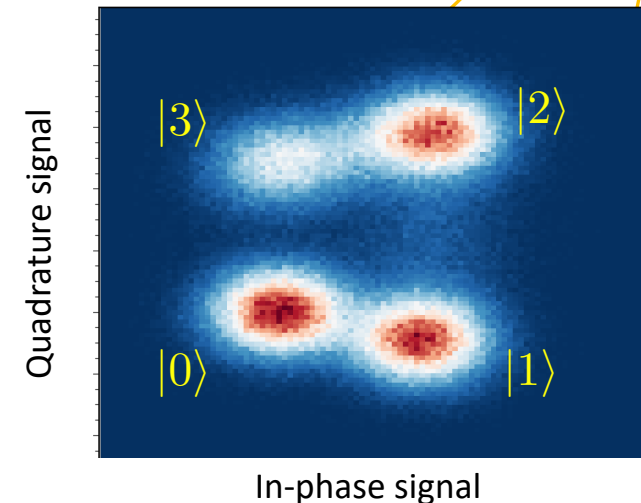
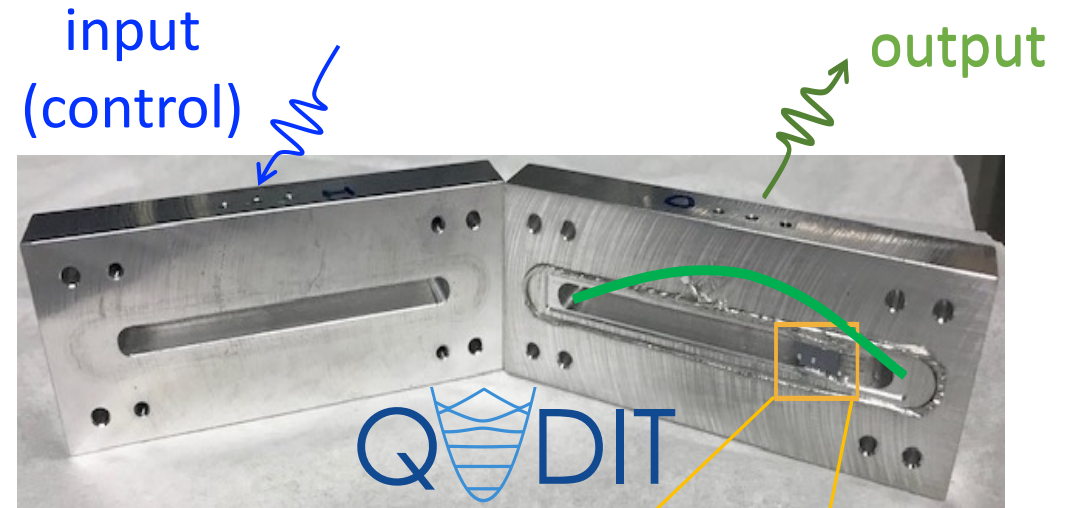


$$H = \hbar\omega_q a^\dagger a + \frac{\alpha}{2} a^\dagger a^\dagger a a$$

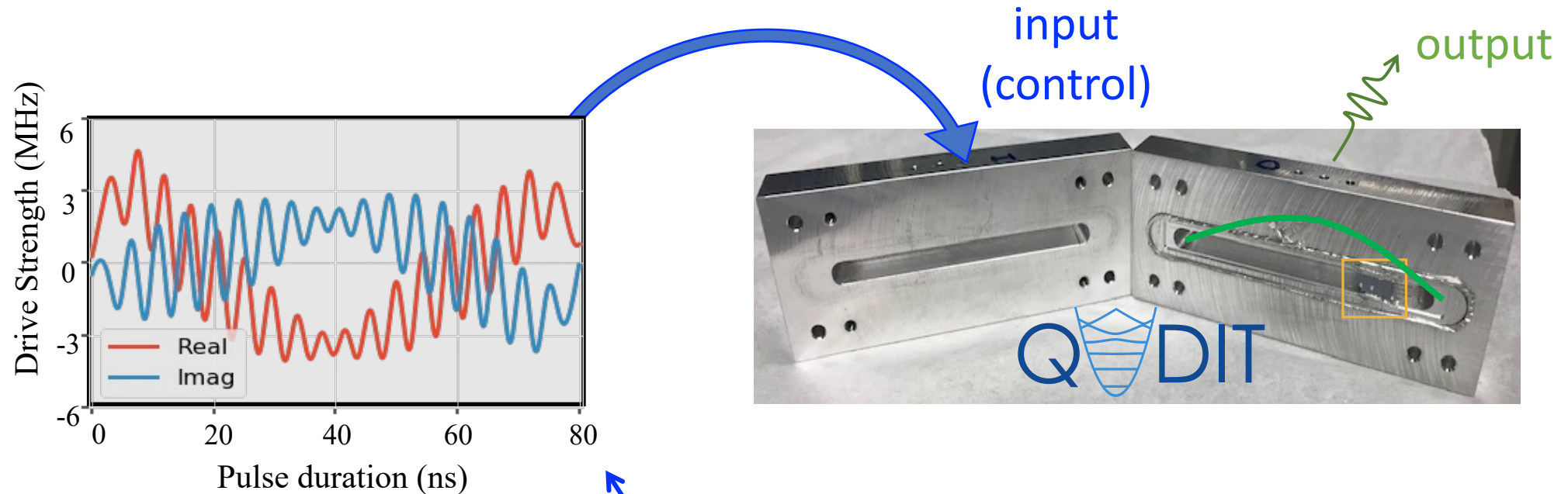
One can also work with registers of d-level quantum systems or ‘qudits’



$$|\psi_{qudit}\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle + \alpha_3|3\rangle + \dots + \alpha_{d-1}|d-1\rangle$$



Physically, quantum gates are realized by controlling the device with microwave pulses engineered with optimization techniques



$$U_{targ} \approx U_{opt} = \mathcal{T} \exp \left[ -\frac{i}{\hbar} \int_0^{T_g} \left( H_0 + \sum_{k=1}^{2n} u_k(t) H_k \right) dt \right]$$

Solve time evolution for quantum device

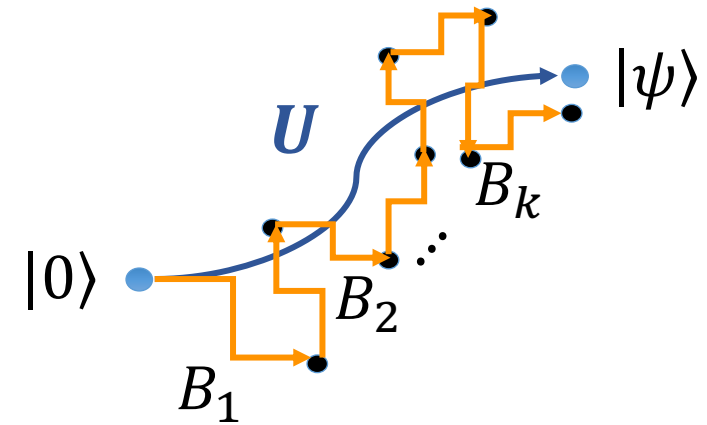


Quantum programs (sequence of unitaries) can be compiled through digital or analog gates, or a combination of both



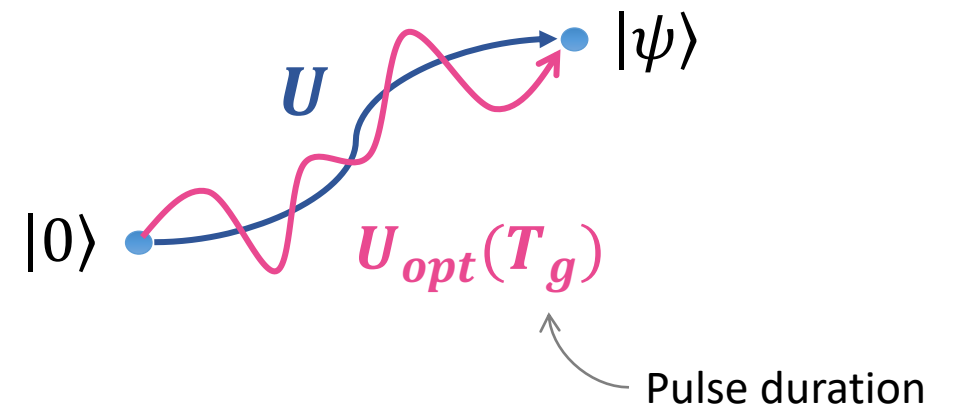
$$U \approx B_k \cdots B_2 B_1$$

Universal set of gates,  
Digital simulation

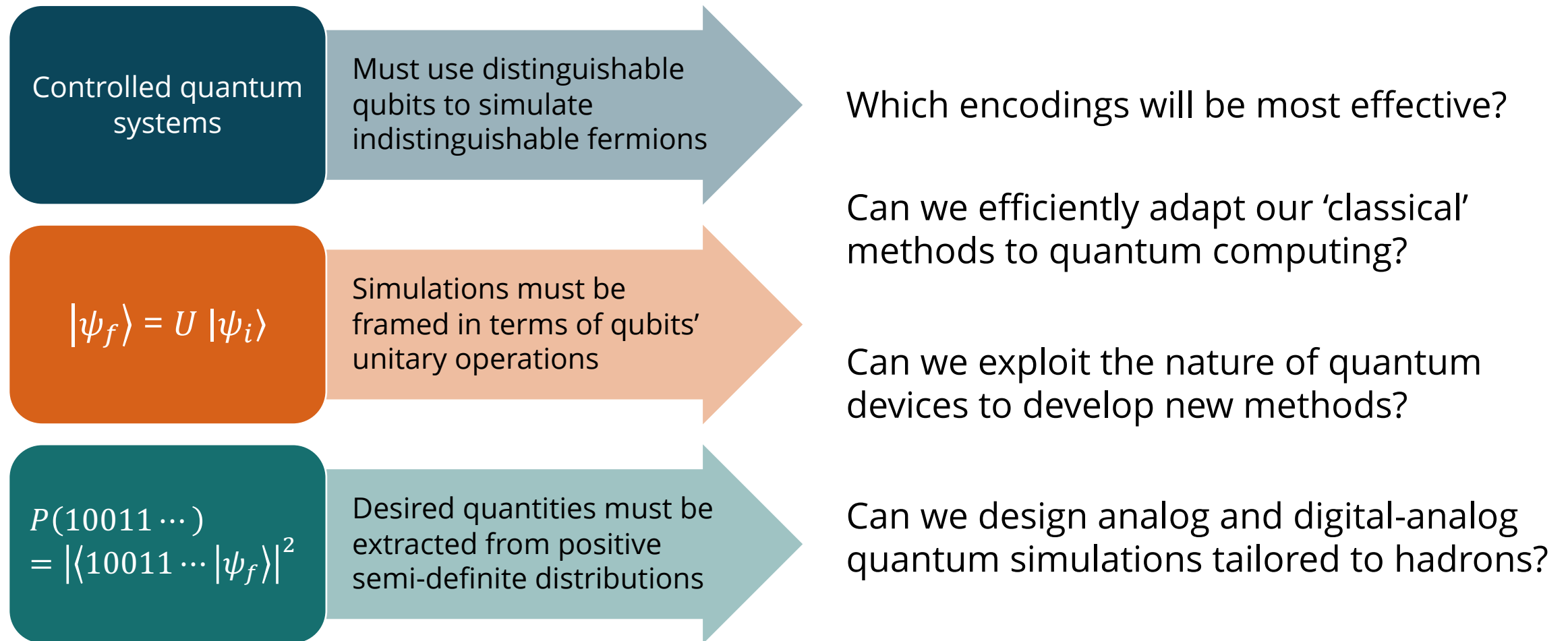


$$U \approx U_{opt}(T_g)$$

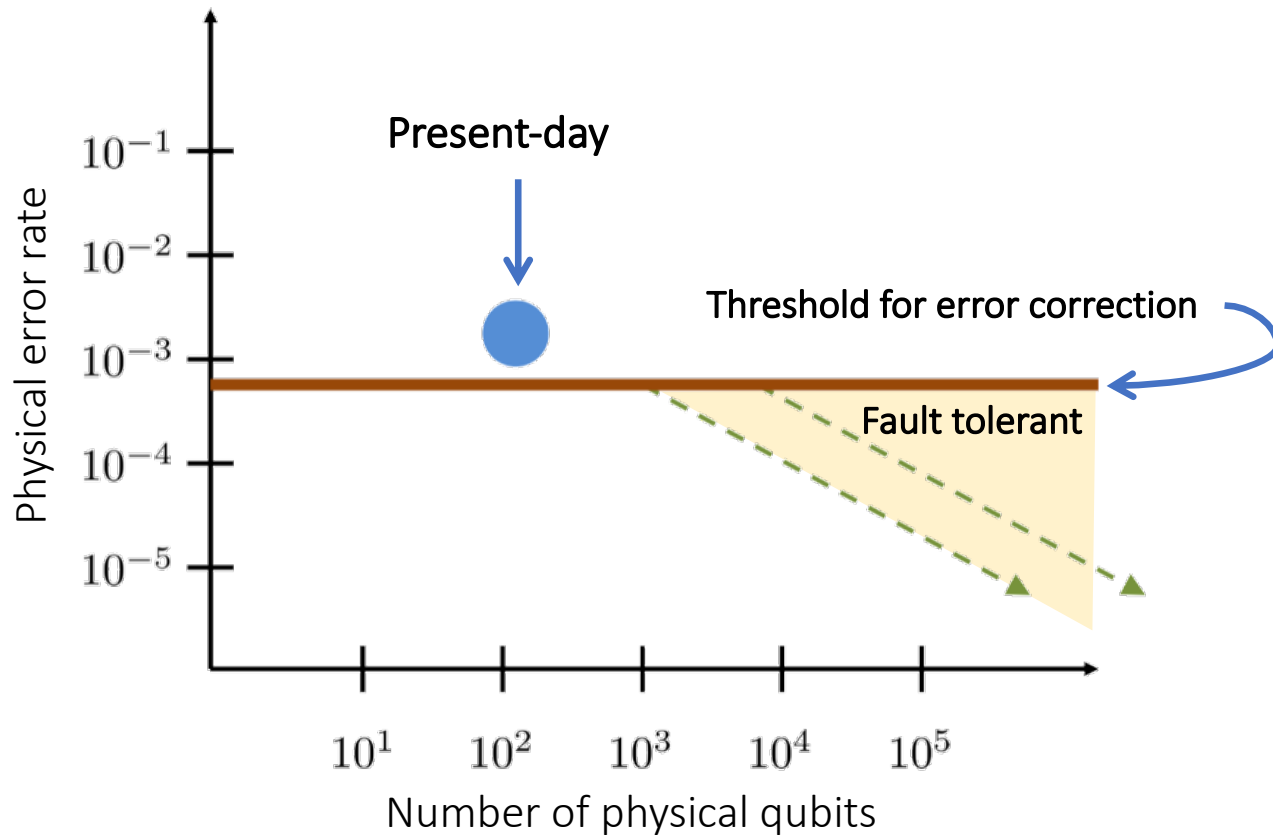
Custom gate,  
Analog simulation



# A challenge is to formulate quantum algorithms, measurement schemes to simulate nuclear physics



Another challenge is to realize useful quantum simulations in the noisy and intermediate scale (NISQ) quantum era

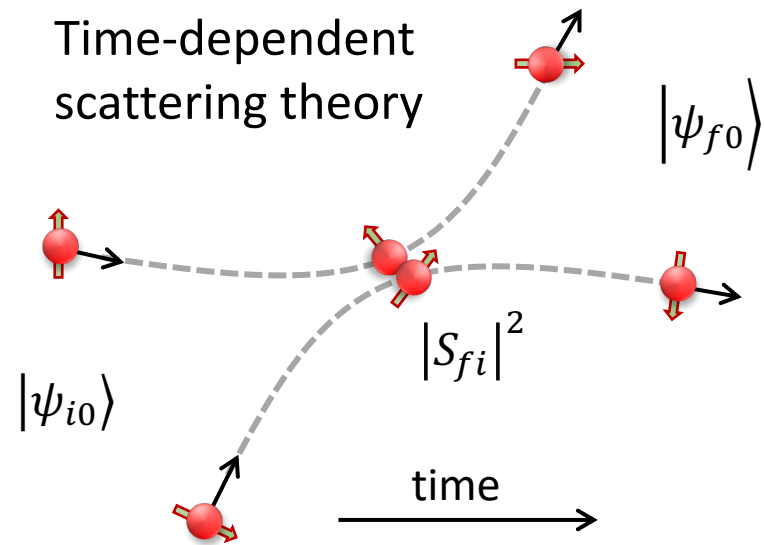


Decoherence



What are noise-resilient protocols that will enable useful quantum simulations of hadron dynamics in the near term?

Quantum computing offers a natural framework for simulating nuclear dynamics, classically very hard!



$$P = |S_{fi}|^2 = |\langle \psi_f(0) | \psi_i(0) \rangle|^2$$

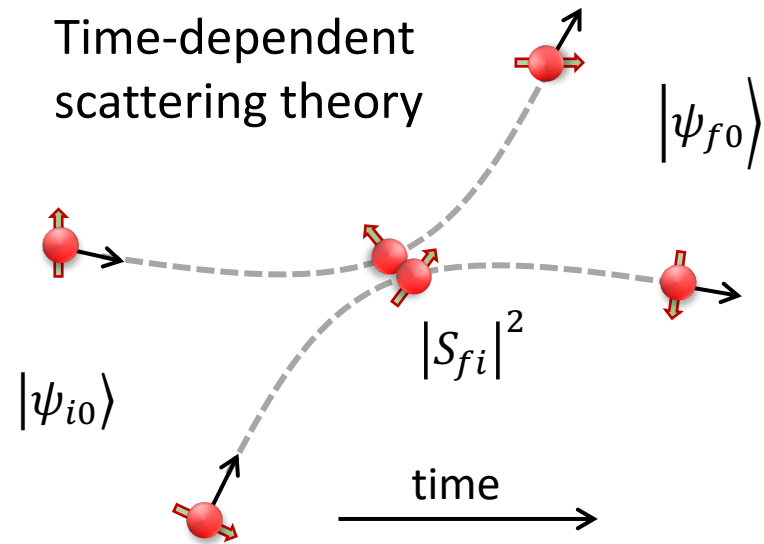
$$\approx |\langle \psi_{f0}(0) | \Omega_+^\dagger \Omega_- | \psi_{i0}(0) \rangle|^2$$

↑

$$e^{-iHt} e^{iH_0 t}$$

Application of unitary transformations (= gates)

Quantum computing offers a natural framework for simulating nuclear dynamics, classically very hard!



$$P = |S_{fi}|^2 = |\langle \psi_f(0) | \psi_i(0) \rangle|^2$$

$$\approx |\langle \psi_{f0}(t) | e^{-2iHt} | \psi_{i0}(-t) \rangle|^2$$

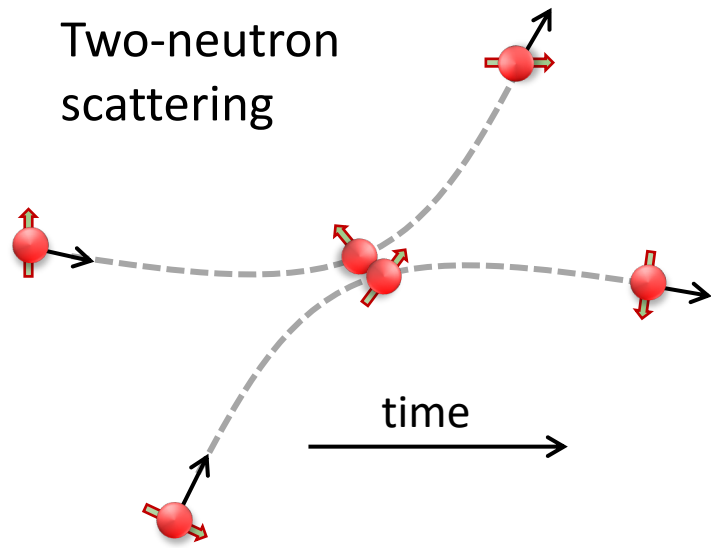
$U(2t)$

Real-time evolution

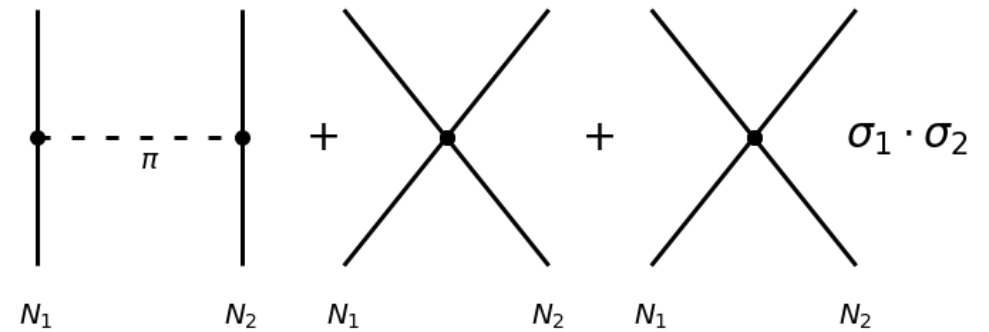
State preparation



# Target problem: prototypical two-neutron scattering

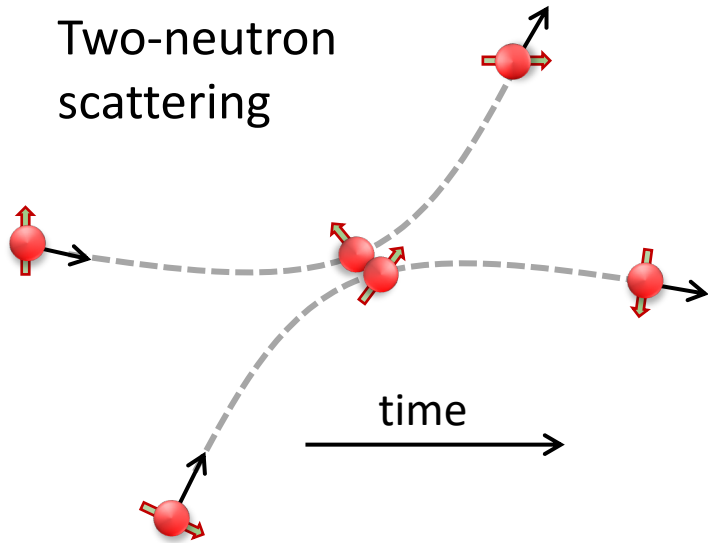


NN force @LO of chiral EFT

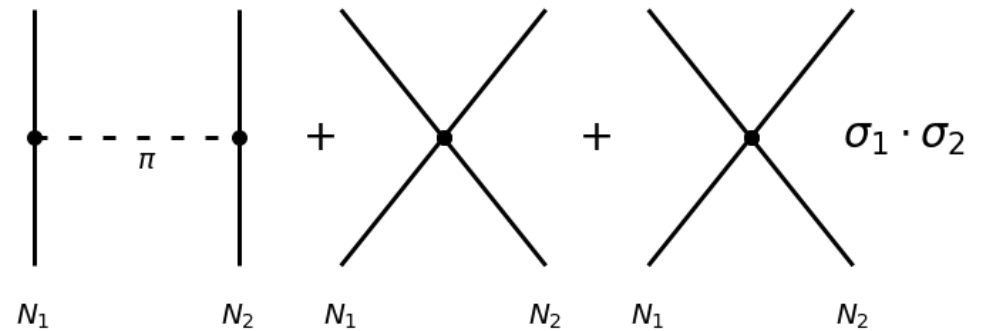


$$S_{12} = 3 (\vec{\sigma}^1 \cdot \hat{r}) (\vec{\sigma}^2 \cdot \hat{r}) - \vec{\sigma}^1 \cdot \vec{\sigma}^2$$

# Target problem: prototypical two-neutron scattering



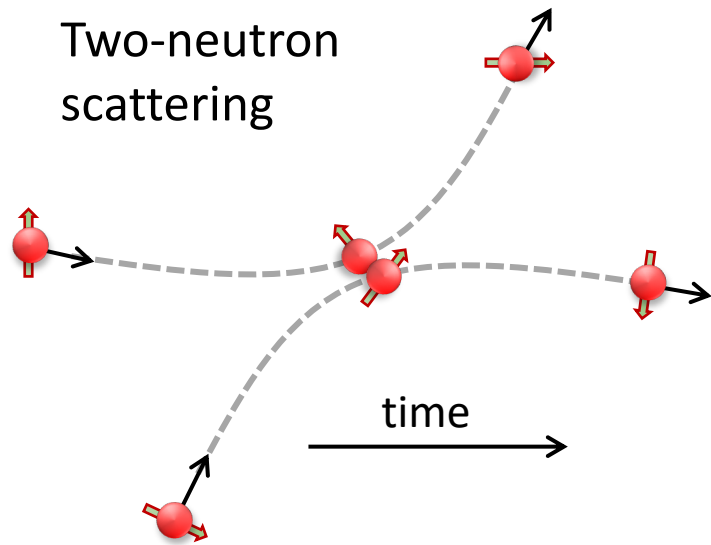
NN force @LO of chiral EFT



$$H_{LO}(\vec{r}, \vec{\sigma}_1, \vec{\sigma}_2) = T(\vec{r}) + V_{SI}(\vec{r}) + V_{SD}(\vec{r}, \vec{\sigma}_1, \vec{\sigma}_2)$$

$$U(\Delta t) \cong U_{spatial}(\Delta t) U_{spin}(\vec{r}, \Delta t)$$

# Target problem: prototypical two-neutron scattering

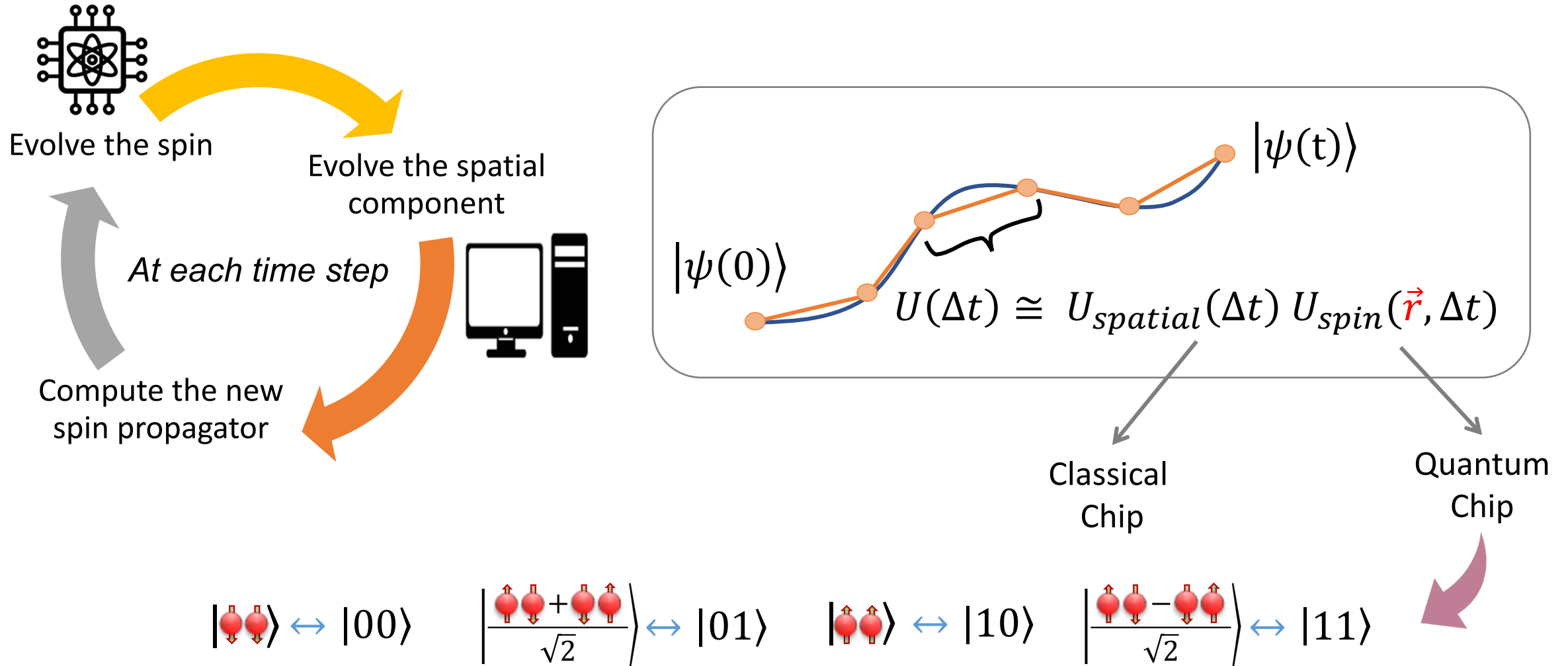


$|\psi(0)\rangle$   $|\psi(t)\rangle$

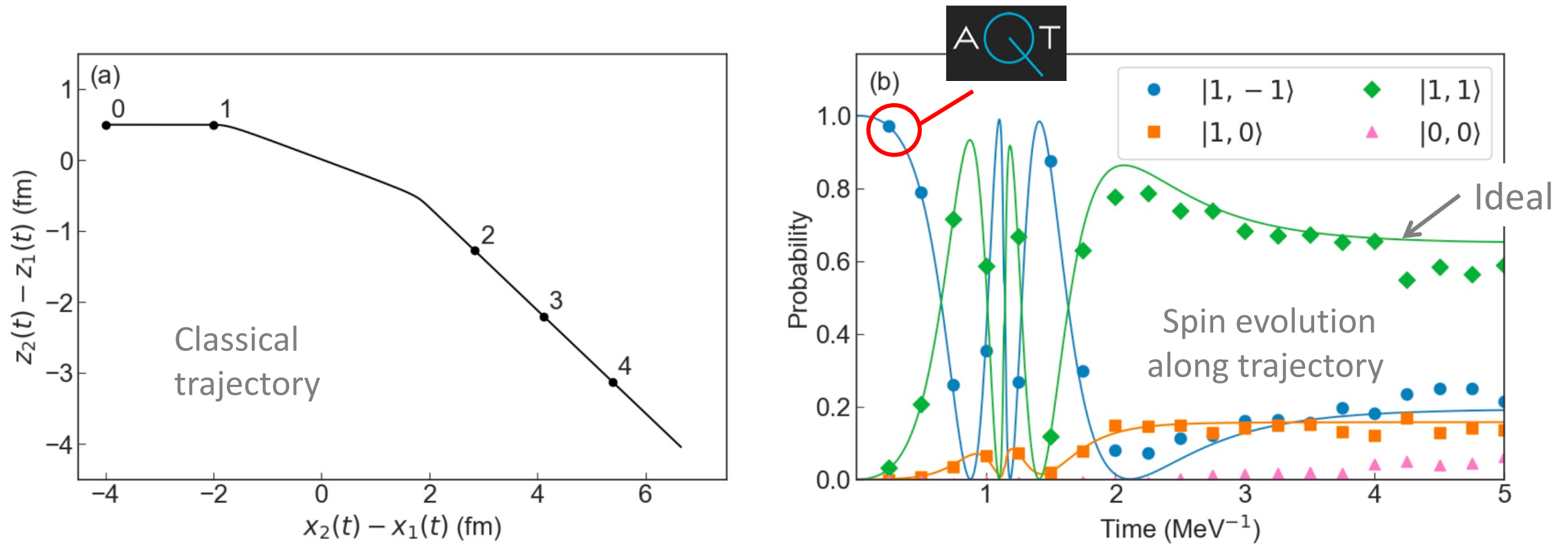
$$U(\Delta t) \cong U_{spatial}(\Delta t) U_{spin}(\vec{r}, \Delta t)$$

The diagram shows a sequence of five orange dots representing the state of a system at discrete time intervals. A blue line connects these dots, showing a path that starts at the left, rises, dips, and then rises again. A black bracket is drawn under the second and third dots. The text  $|\psi(0)\rangle$  is positioned to the left of the first dot, and  $|\psi(t)\rangle$  is to the right of the fifth dot. Below the dots, the equation  $U(\Delta t) \cong U_{spatial}(\Delta t) U_{spin}(\vec{r}, \Delta t)$  is displayed.

# Target problem: prototypical two-neutron scattering

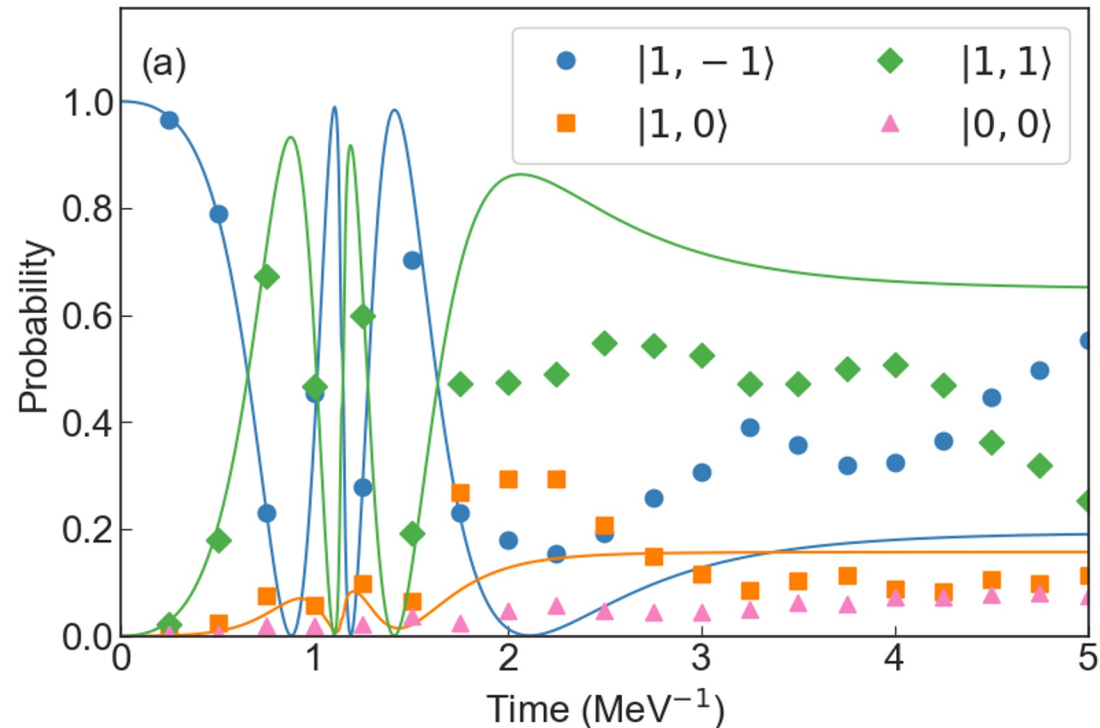


Quantum-classical coprocessing scheme works on present hardware, could enable path-integral simulations of scattering



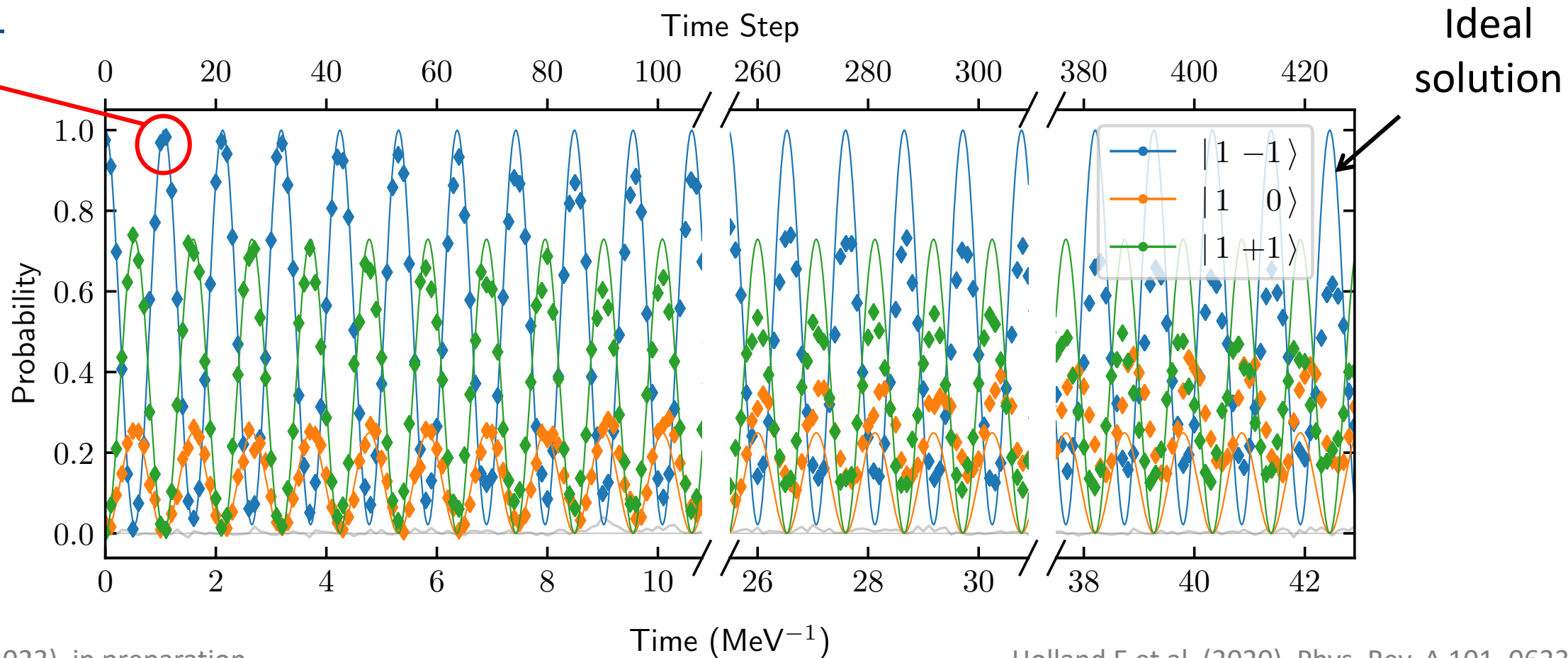
However, it first required (scalable) error mitigation:  
randomized compiling, state purification

Before error mitigation

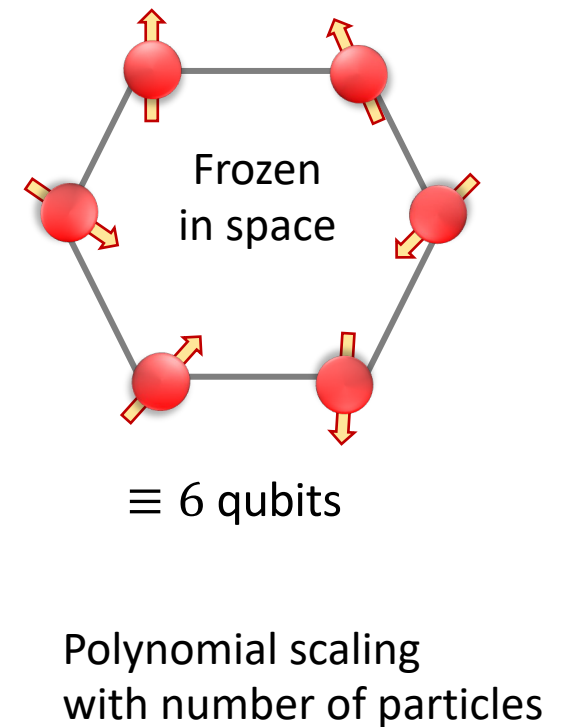
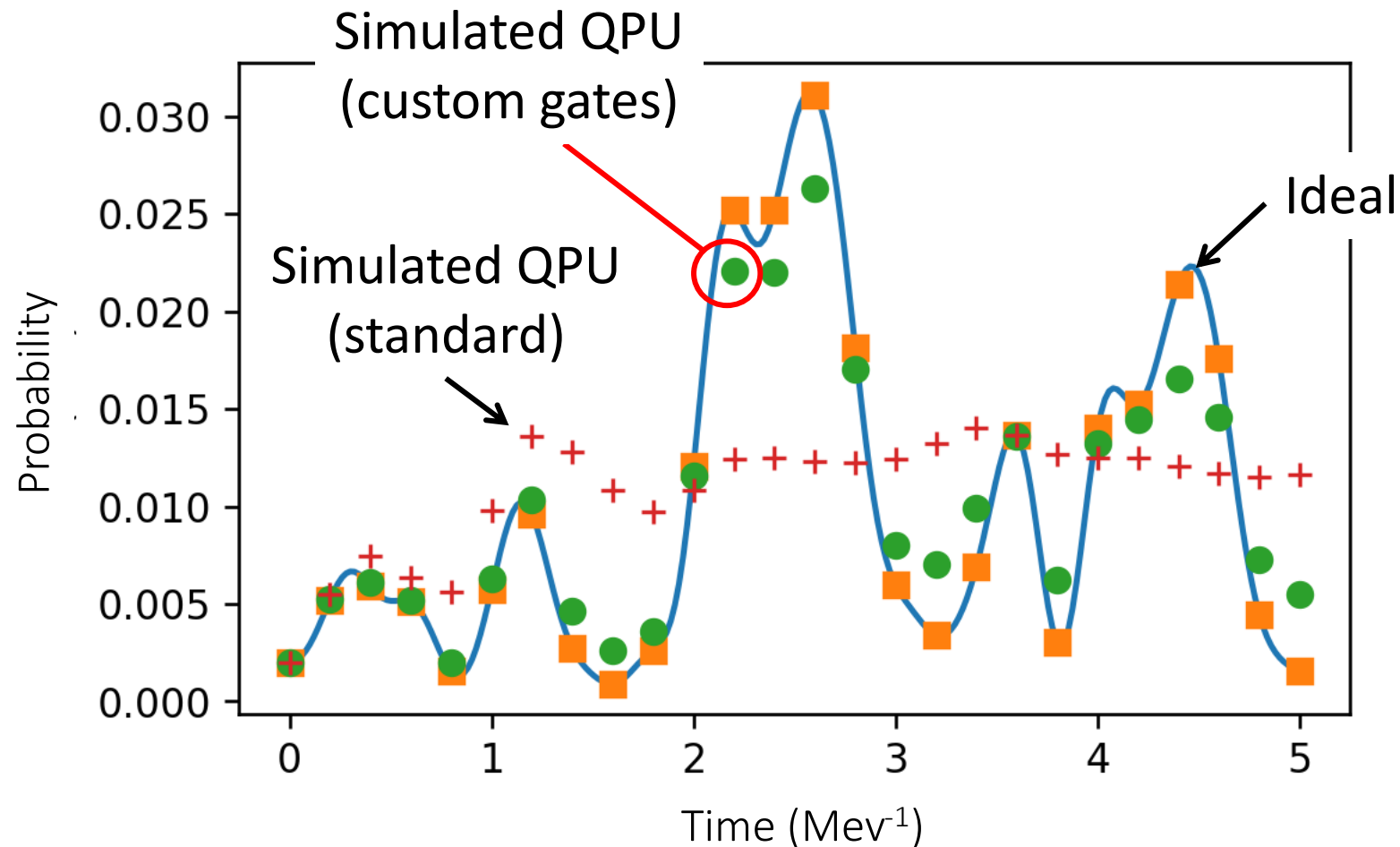


Time step	# 2-qubit gates	# Total gates
1	3	27
4	12	135
8	24	243
12	36	351
16	48	459
20	60	540

# Digital-analog simulation with custom two-spin short-time propagator gates provides enhanced resilience to noise

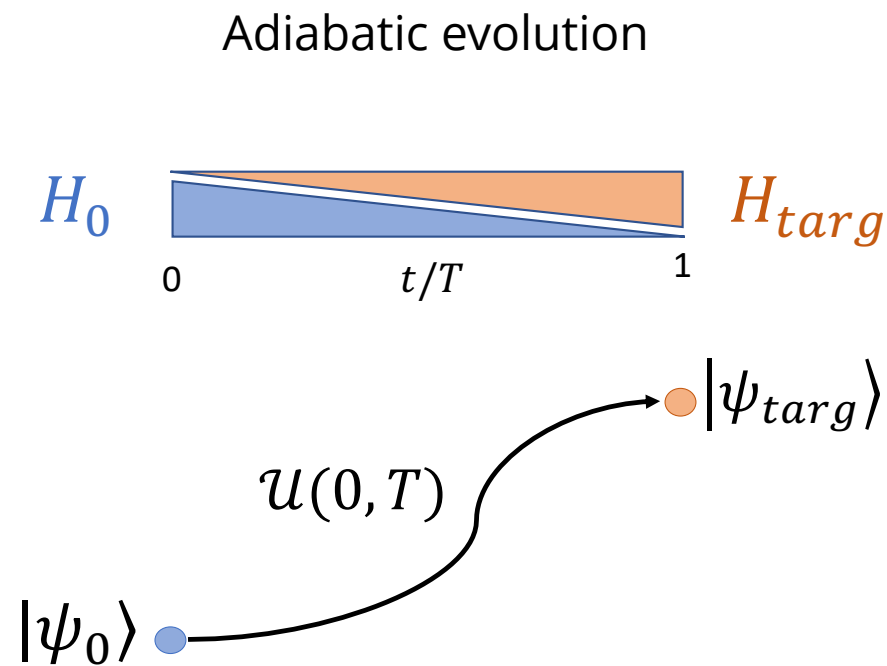


# With custom gates, designed noise-resilient algorithm for quantum simulation of multi-nucleon spin dynamics

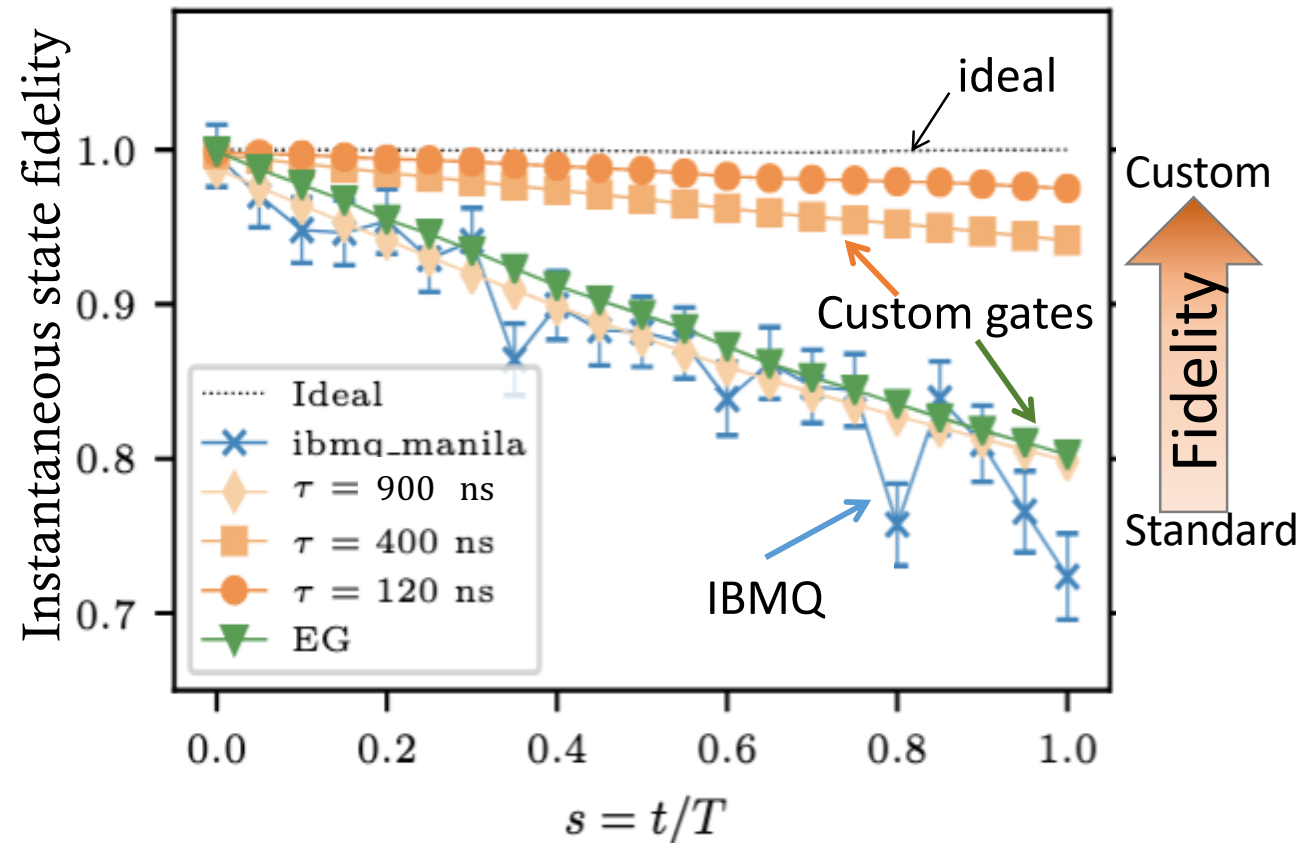




# Digital-analog simulations with custom gates enable major performance improvements for state preparation

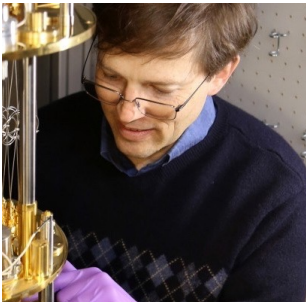


E.g.: 2-spin system



# The LLNL/Trento QC team and collaborators

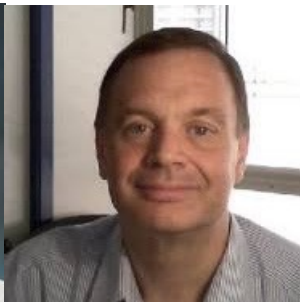
Jonathan DuBois



Yaniv Rosen



Francesco Pederiva



Francesco Turro



Piero Luchi



Valentina Amitrano



Kostas Kravvaris



Erich Ormand



Sofia Quaglioni



Kyle Wendt



Tono Coello-Perez



Alessandro Roggero

Joey Bonitati (MSU)  
Collaborator



Dean Lee (MSU)  
Collaborator



Trevor  
Chistolini  
& AQT Team



This is an exciting time for paving the way to exact quantum simulations of few- & many-body dynamics

In the near term, noise-resilient quantum simulations will require hybrid algorithms and customized gates

Understanding, exploiting the underlying characteristics of near-term quantum devices will be key

Lots of (few-body) work ahead to develop noise-resilient quantum simulations that scale with particle number

**PROGRAM** Upcoming @INT

OCTOBER 7 - NOVEMBER 8, 2024

**Quantum Few- and Many-  
Body Systems in Universal  
Regimes (INT)**

A. Bergschneider, S. Gandolfi,  
M. Gattobigio, S. Quaglioni

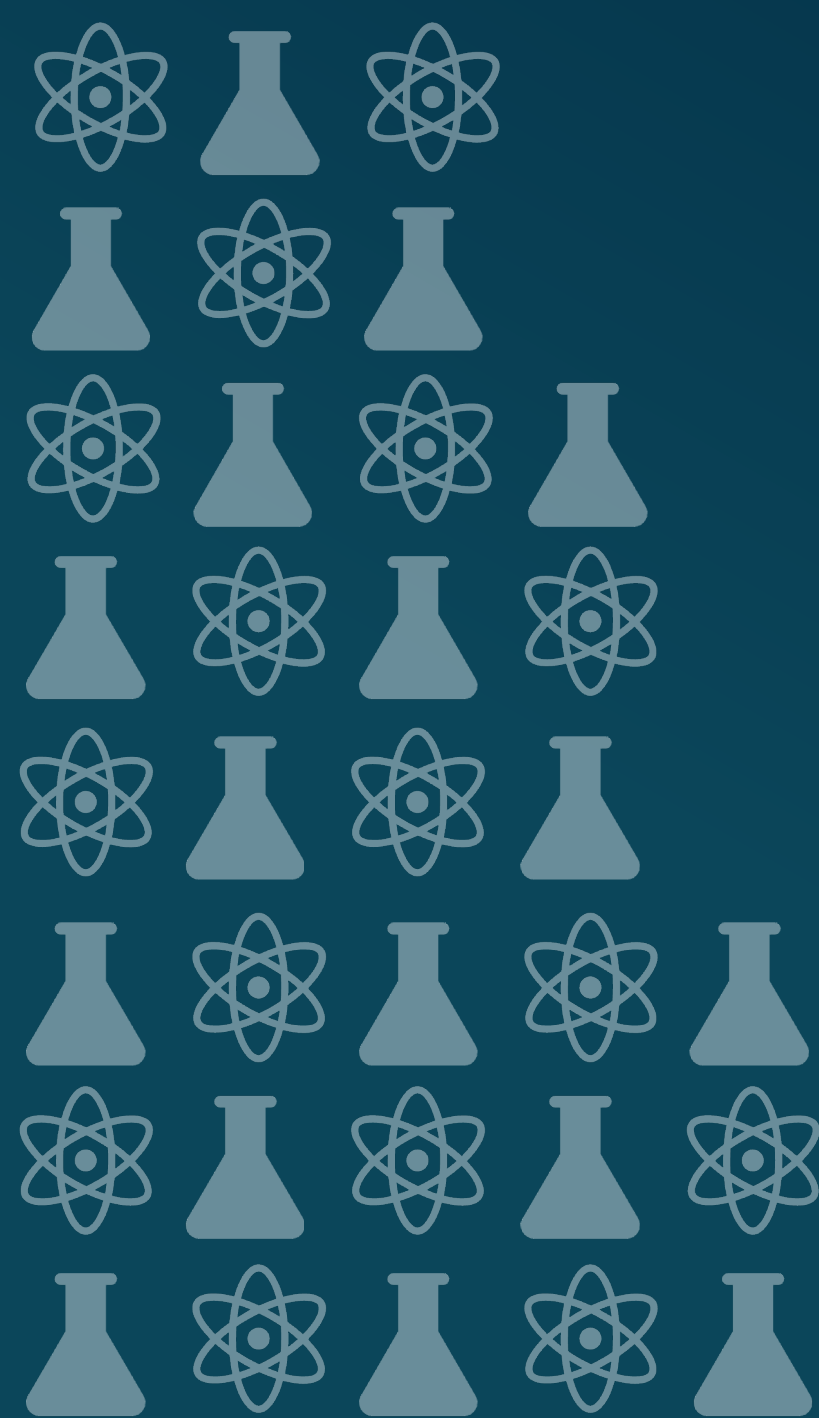
*Embedded Workshop: October 21 - 25*



# The APS Topical Group On Few-Body Systems and Multiparticle Dynamics

Graduate/Postdoctoral Travel Grants  
International, European, Asian-Pacific Conferences  
on Few-body Problems in Physics  
APS Fellowship, Faddeev Medal

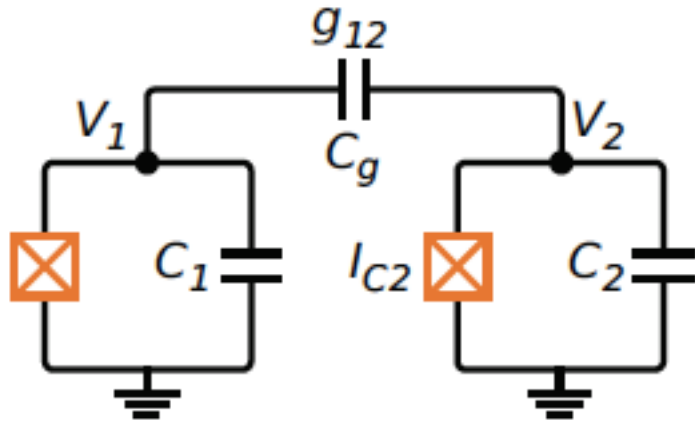
<https://engage.aps.org/gfb/home>



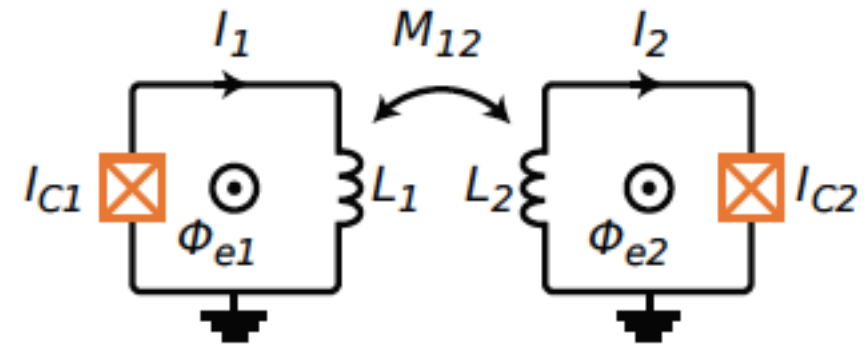
Thank You

To generate entanglement (build registers  $n$  of qubits), superconducting qubits must be coupled (here  $n = 2$ )

Ex: Direct capacitive coupling



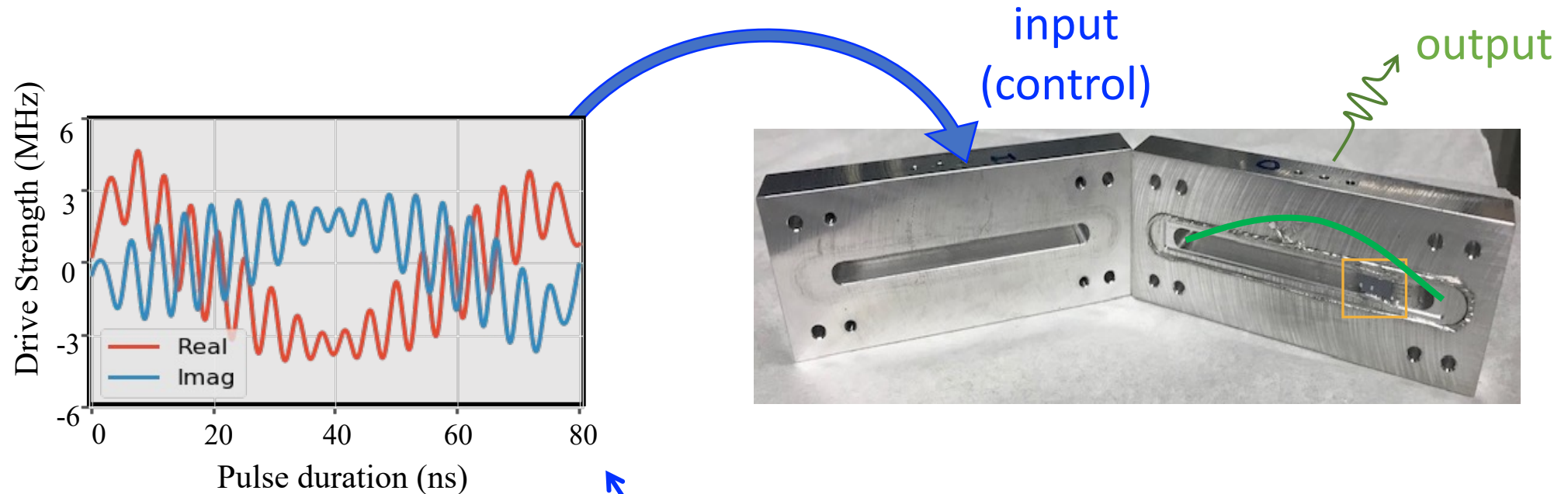
Ex: Direct inductive coupling



$$H = H_0 + H_{int} = \sum_{i \in \{1,2\}} \left[ \hbar \omega_i a_i^\dagger a_i + \frac{\alpha_i}{2} a_i^\dagger a_i^\dagger a_i a_i \right] - g(a_1 - a_1^\dagger)(a_2 - a_2^\dagger)$$



‘Basic’ quantum gates are realized with optimization techniques, hinging on a realistic model of the physical quantum device

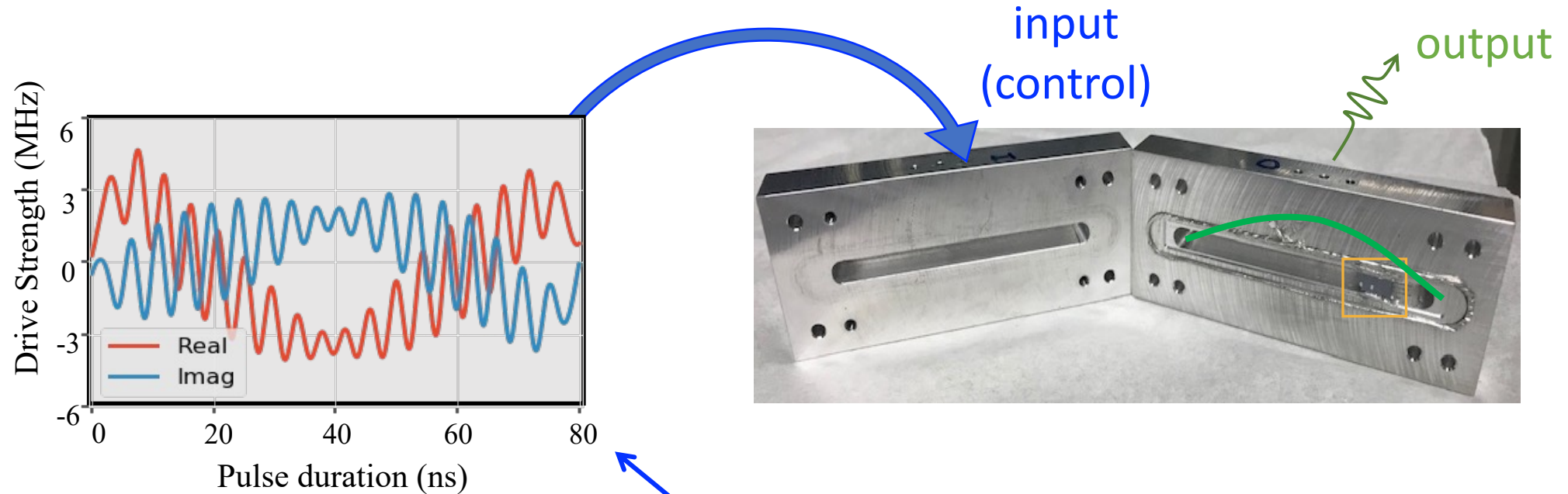


$$U_{targ} \approx U_{opt} = \mathcal{T} \exp \left[ -\frac{i}{\hbar} \int_0^{T_g} \left( H_0 + \sum_{k=1}^{2n} u_k(t) H_k \right) dt \right]$$

Solve time evolution for quantum device



‘Basic’ quantum gates are realized with optimization techniques, hinging on a realistic model of the physical quantum device



$$U_{targ} \approx U_{opt} = \mathcal{T} \exp \left[ -\frac{i}{\hbar} \int_0^{T_g} [H_0 + u_R(t)(a + a^\dagger) + iu_I(t)(a - a^\dagger)] dt \right]$$

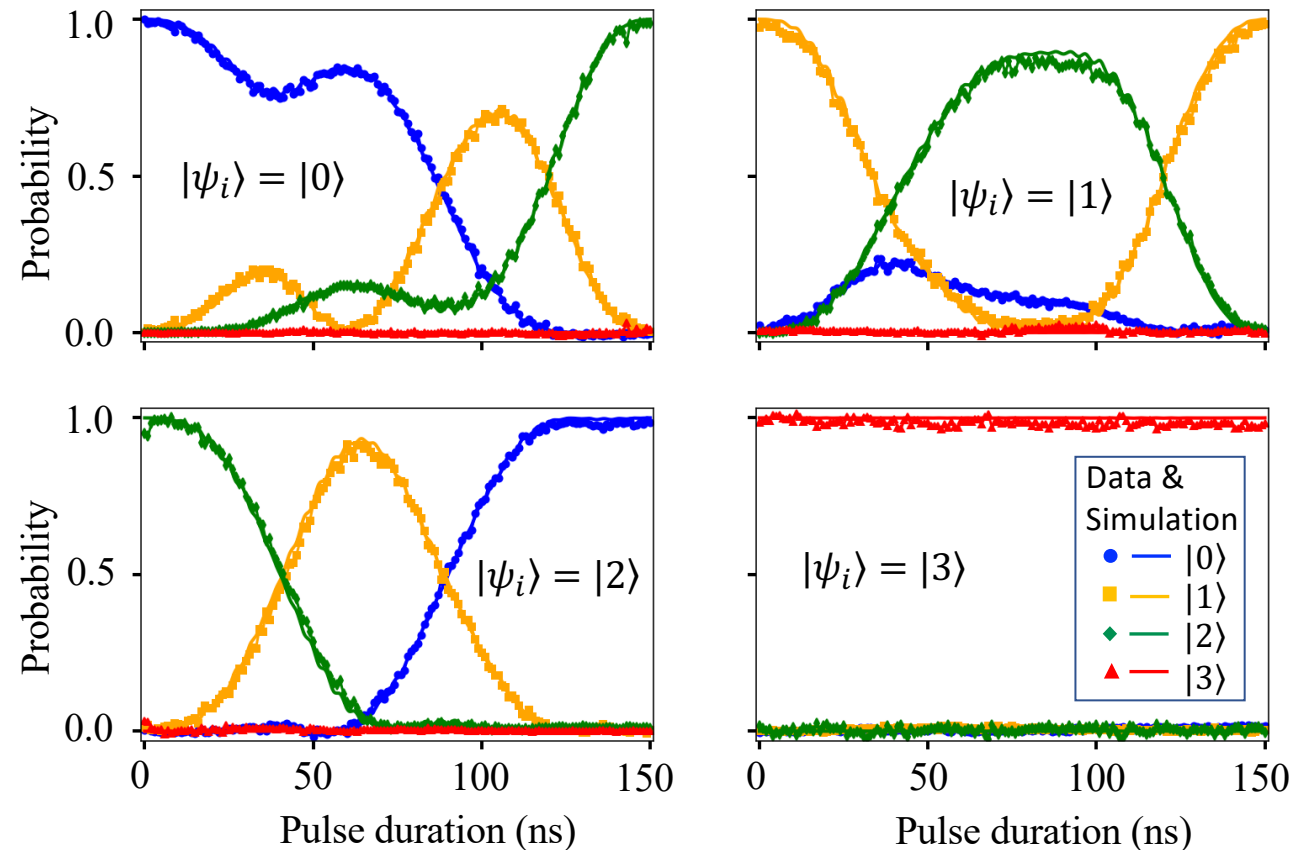
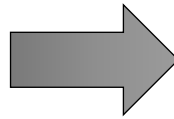
# The $0 \leftrightarrow 2$ swap gate was realized with high fidelity on the LLNL qudit

$$|0\rangle \rightarrow |2\rangle$$

$$|1\rangle \rightarrow |1\rangle$$

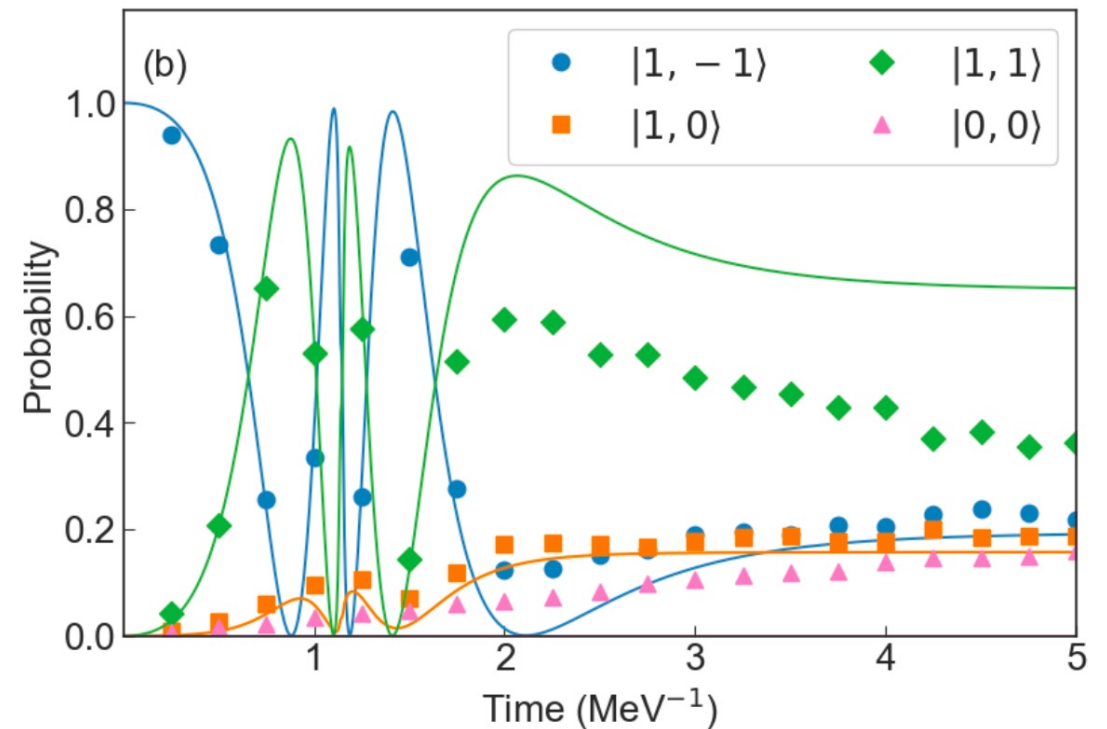
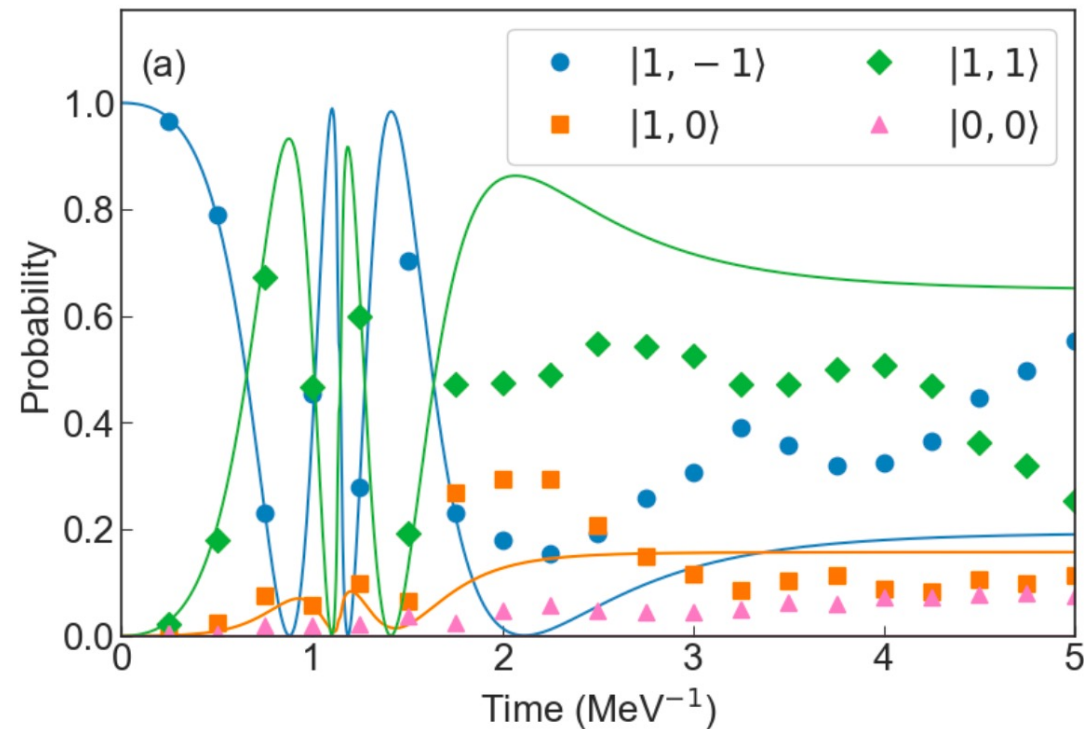
$$|2\rangle \rightarrow |0\rangle$$

$$|3\rangle \rightarrow |3\rangle$$



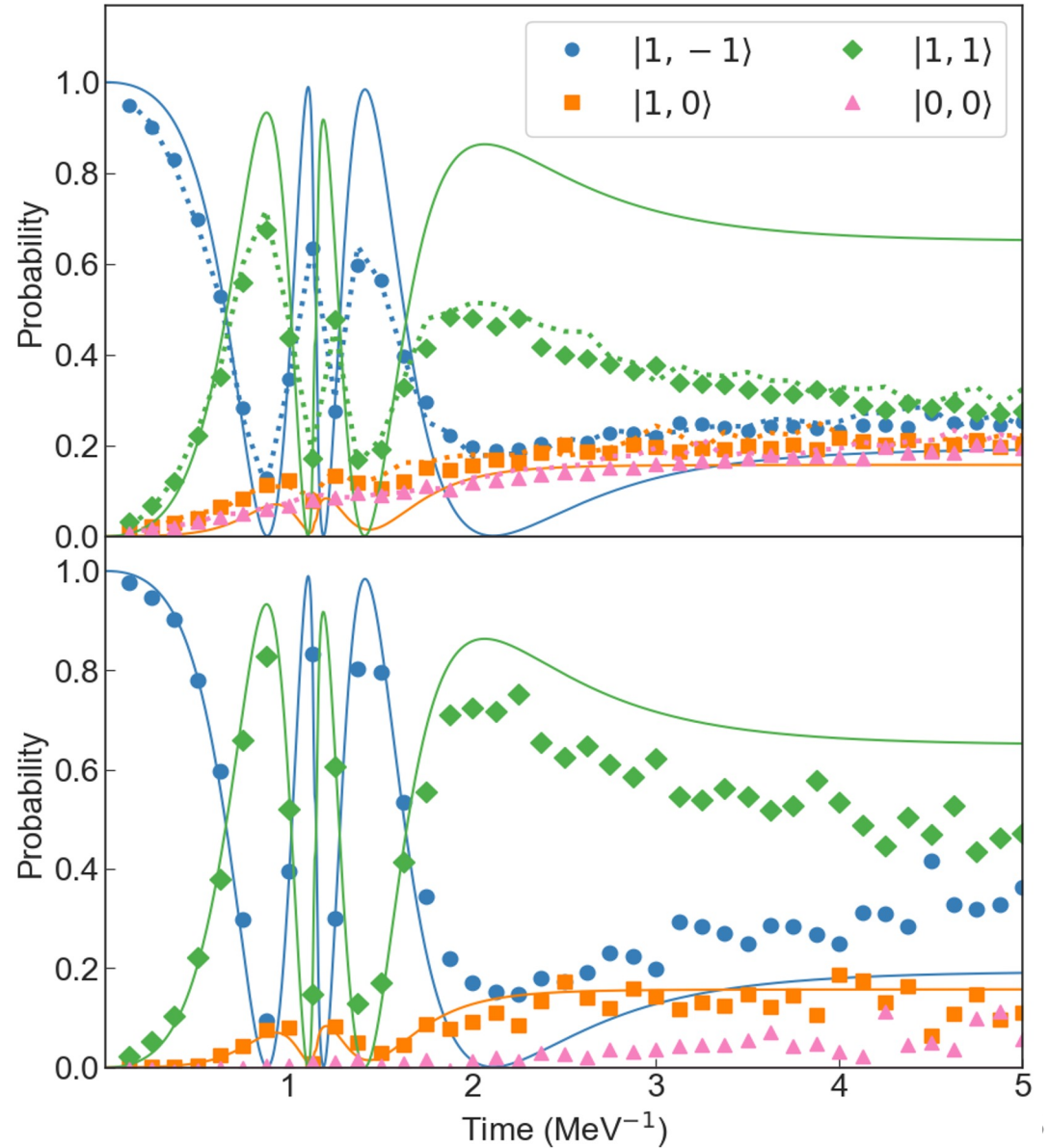
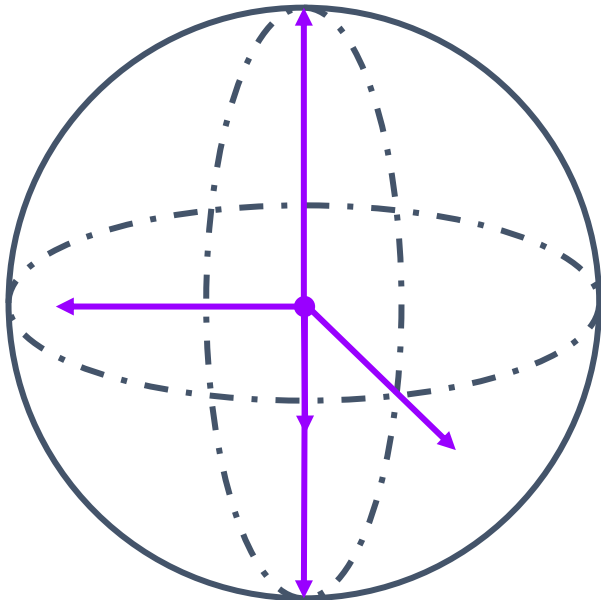
# Randomized compiling

- New circuit by inserting random virtual twirling gates, corresponding inverting gates
- Translate coherent error to stochastic error using multiple random, logically equivalent circuits



# State purification

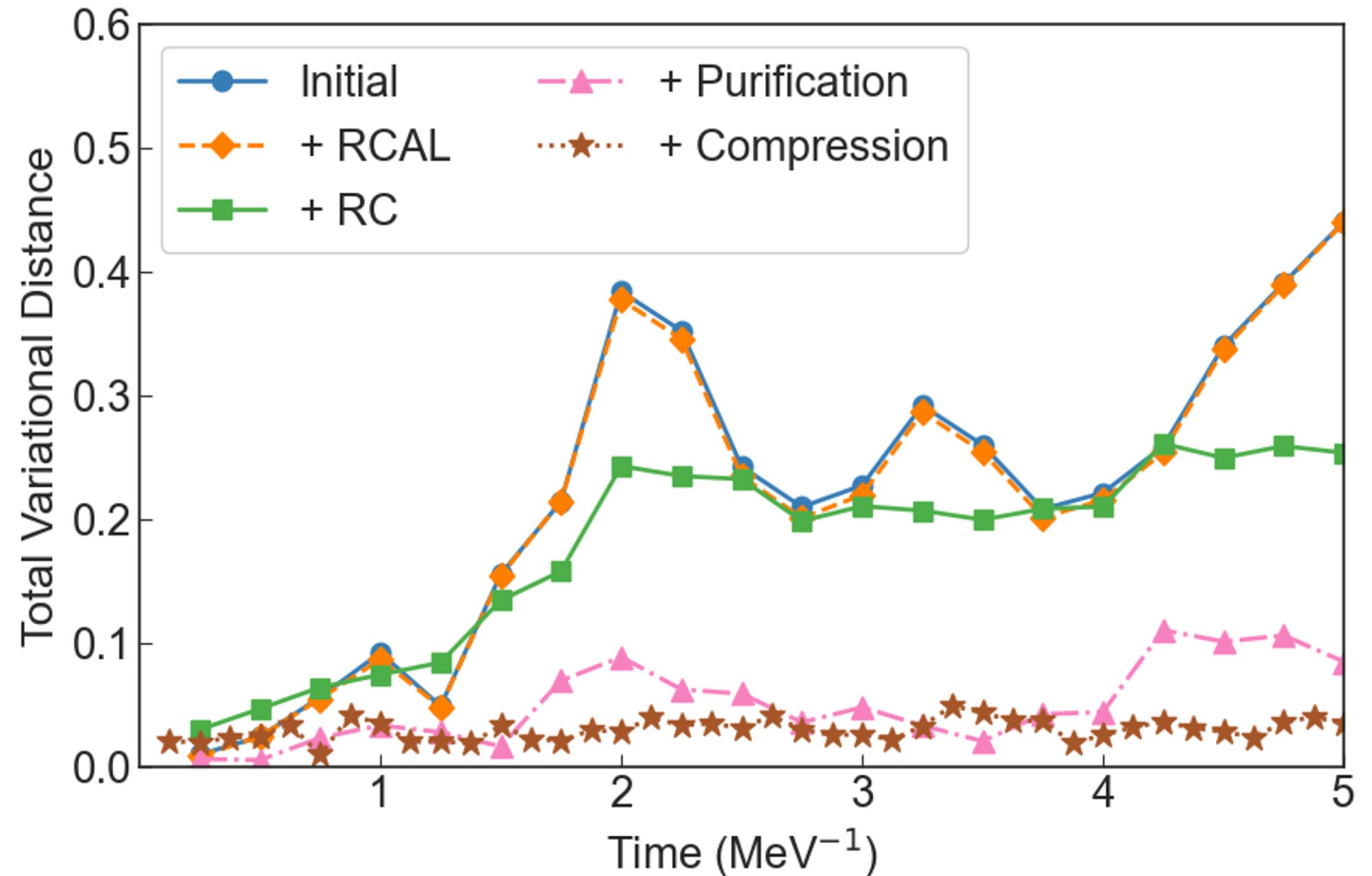
1. Estimate extent that Bloch vector shrinks upon each gate application
2. Renormalize length to unity in post-processing



# Summary of results: Total variational distance

## Total variational distance

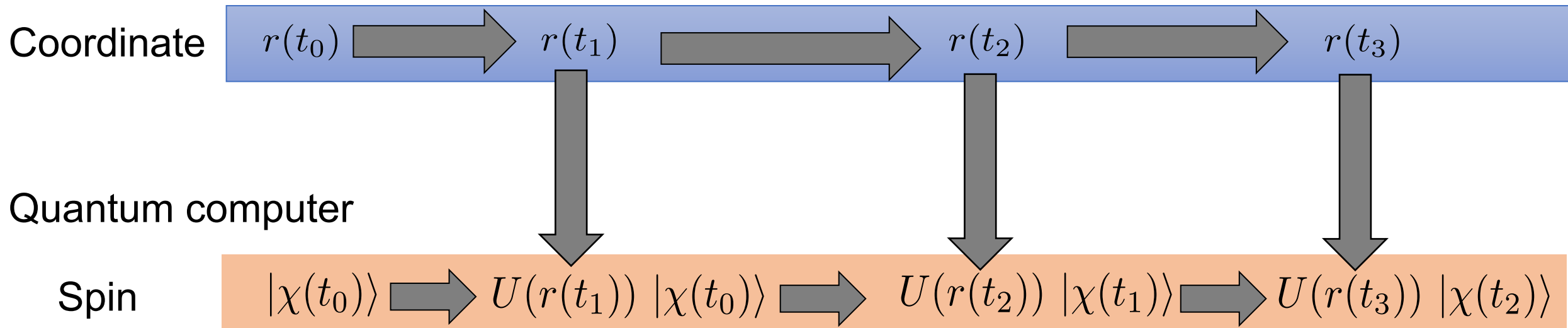
- Metric of absolute difference between two probability distributions
- Here: experimental versus ideal, to evaluate accuracy



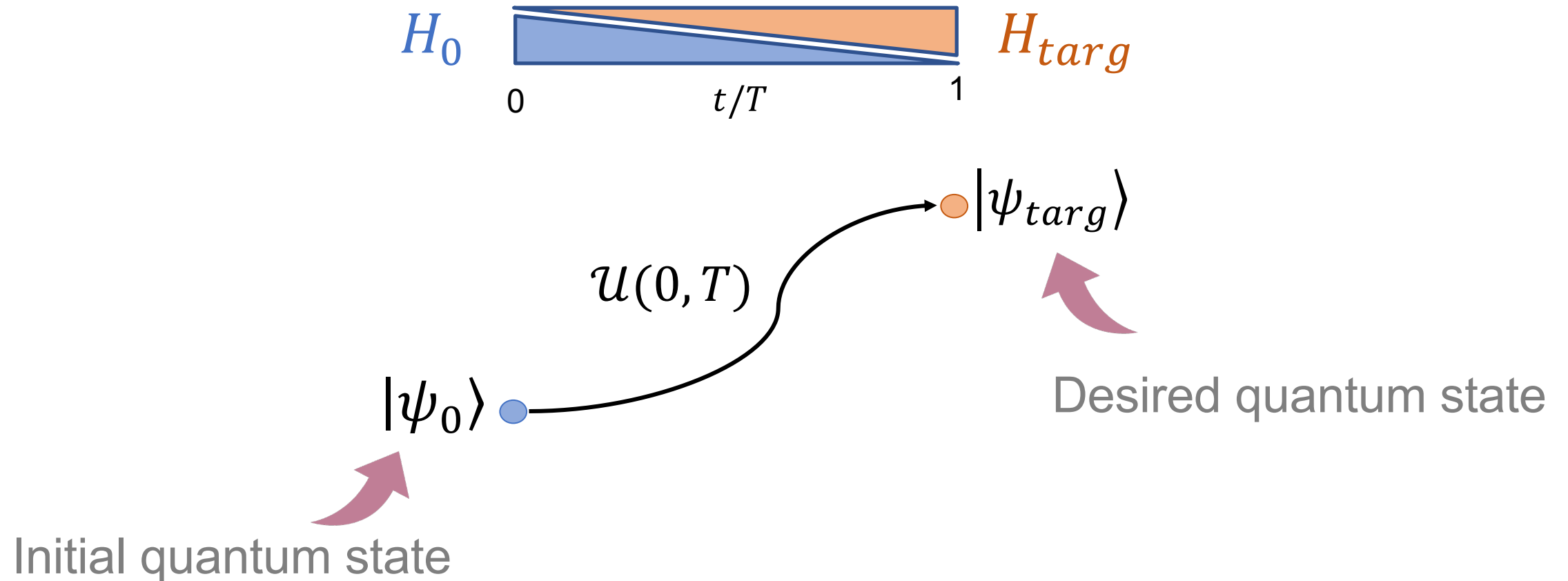
# Further approximation: Semiclassical approximation

**Approximation:** the spatial evolutions are computed on a classical device by solving the Newton equation (**saddle point approximation**)

Classical device: Solving the Newton equation

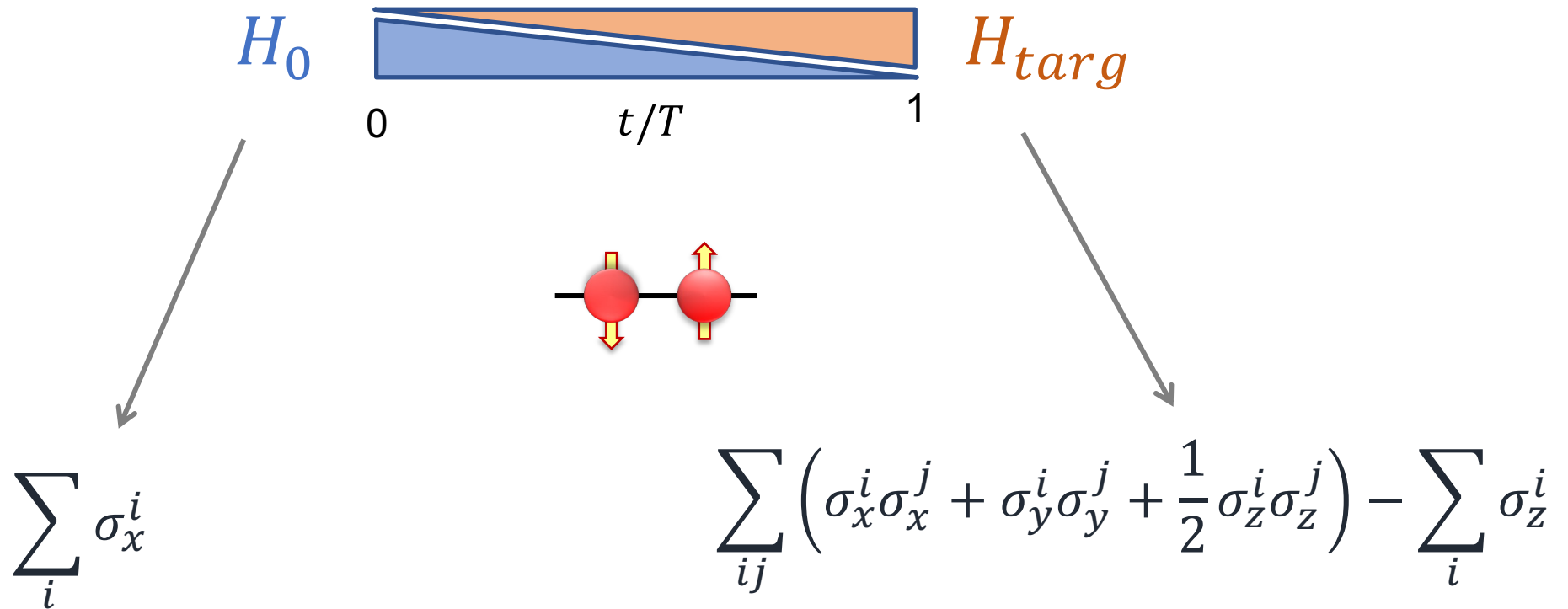


To simulate scattering also need state preparation approach, e.g., adiabatic evolution





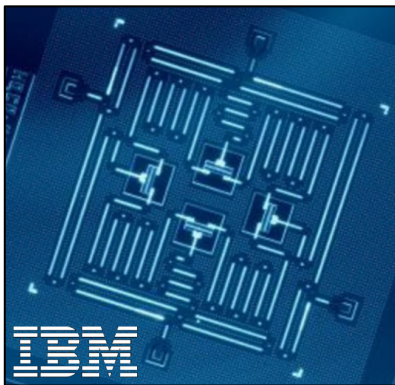
As an initial target problem, we consider the adiabatic evolution of a simple two-spin system



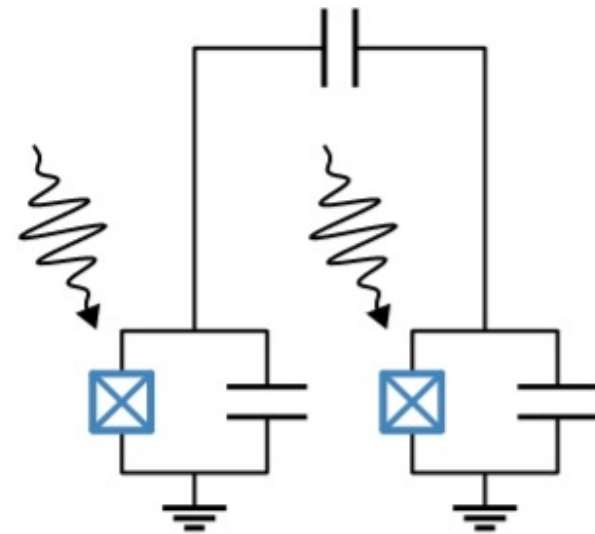
As before, we consider two strategies:  
1) gate-based approach; and 2) customized gates approach

$$\mathcal{U}(0, T) \approx \prod_{k=1}^n U(t_k) = \prod_{k=1}^n e^{-iH(t_k)\Delta t}$$

1) Experimental quantum simulations



2) Classical device-level simulations



Customized gates significantly improve state fidelity, allow for accurate extraction of state properties

