Hypernuclear physics with pionless EFT

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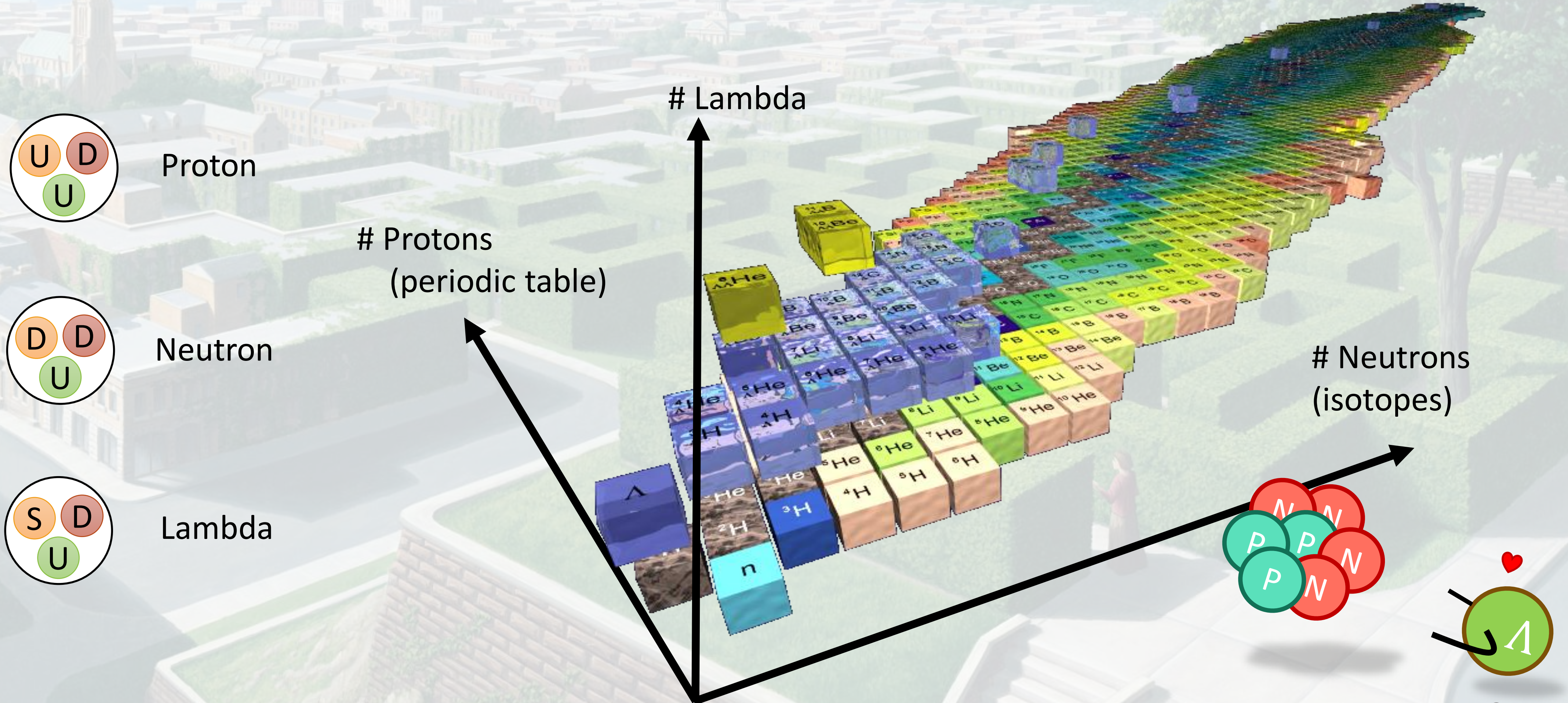
Λ - Hypernuclei



Experimental ferment

STAR collaboration

BES III

PANDA

J-parc

J-Lab

LHC

HALQCD

Abundant open queries

- **Description** of few-body hypernuclei
- **Double** Λ hypernuclei description
- **Life time** of $^{3}_{\Lambda}\mathrm{H}$ and $^{3}_{\Lambda}\mathrm{n}$
- **Charge symmetry breaking** ($^{4}_{\Lambda}\mathrm{H}$ - $^{4}_{\Lambda}$He)
- Λ^{*}(1405) **matter**
- **Neutron star** equation of state
- …

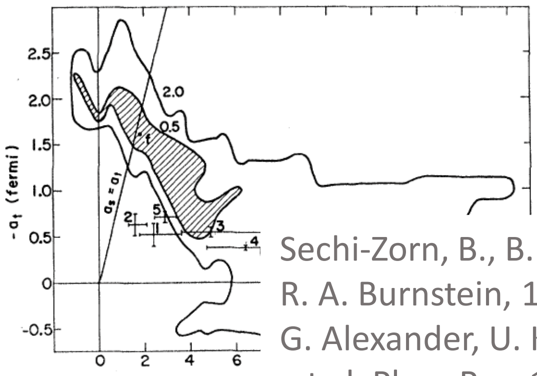
Theoretical and experimental challenge

Experiments are more complex to be performed than for nuclear physics

Less data to constrain theory. (Less precise predictions)

Theory likes few-body

Two-body scattering length



Sechi-Zorn, B., B. Kehoe, J. Twitty, and R. A. Burnstein, 1968, Phys. Rev. 175, 1735.
G. Alexander, U. Karshon, A. Shapira, et al. Phys. Rev. 173, 1452 (1968)

Few-body energy

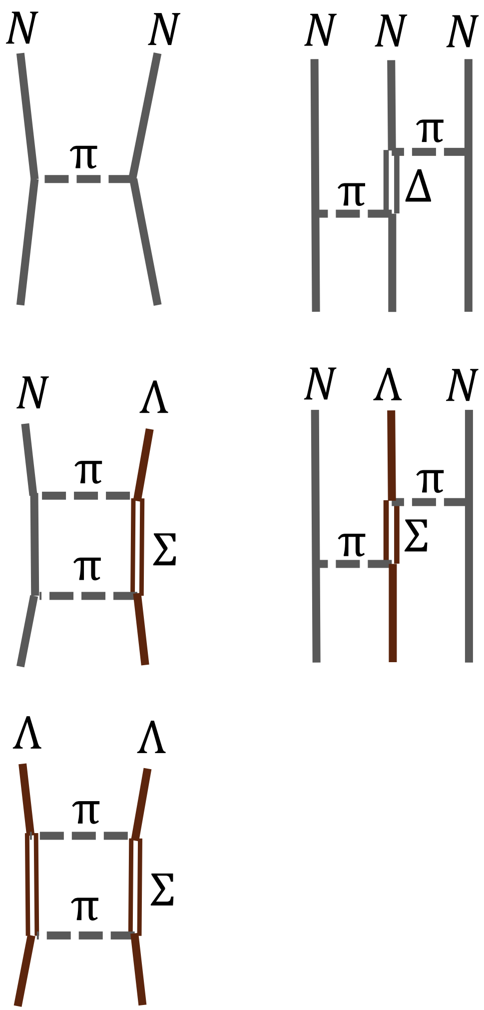
Emulsion: $E({}^{3}_{\Lambda}\mathrm{H}) = 0.13(5)$ MeV

M. Juric et al., Nucl. Phys. B 52, 1 (1973).

STAR: $E({}^{3}_{\Lambda}\mathrm{H}) = 0.41(12 + 11)$ MeV

J. Adam et al. (STAR Collaboration), Nature Physics 16, 409 (2020).

π-EFT (A)

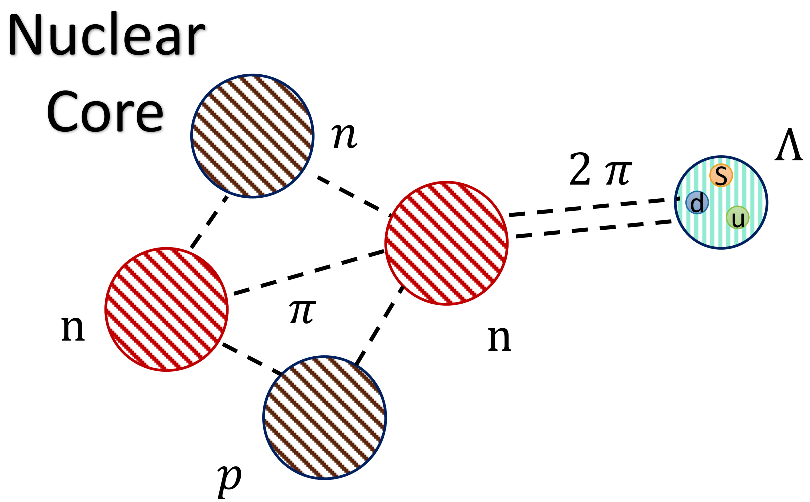


The interaction is mediated by pions

$m_\pi \sim 140$ MeV

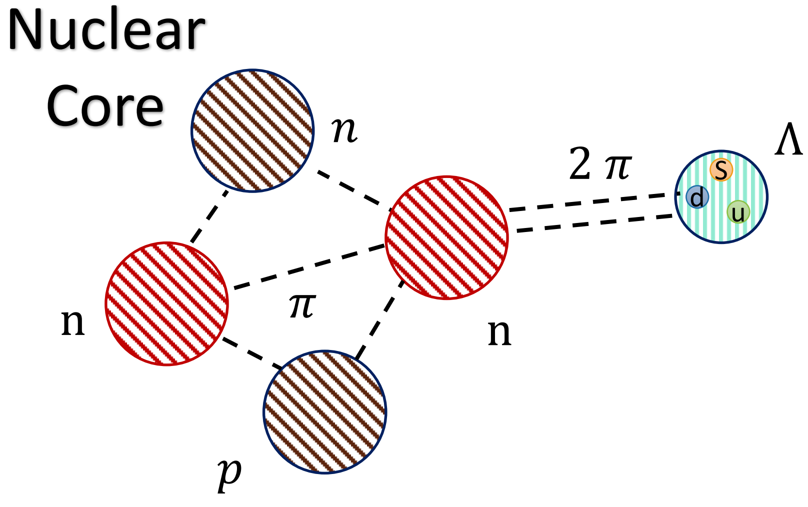
$\Lambda - \Sigma$ mixing

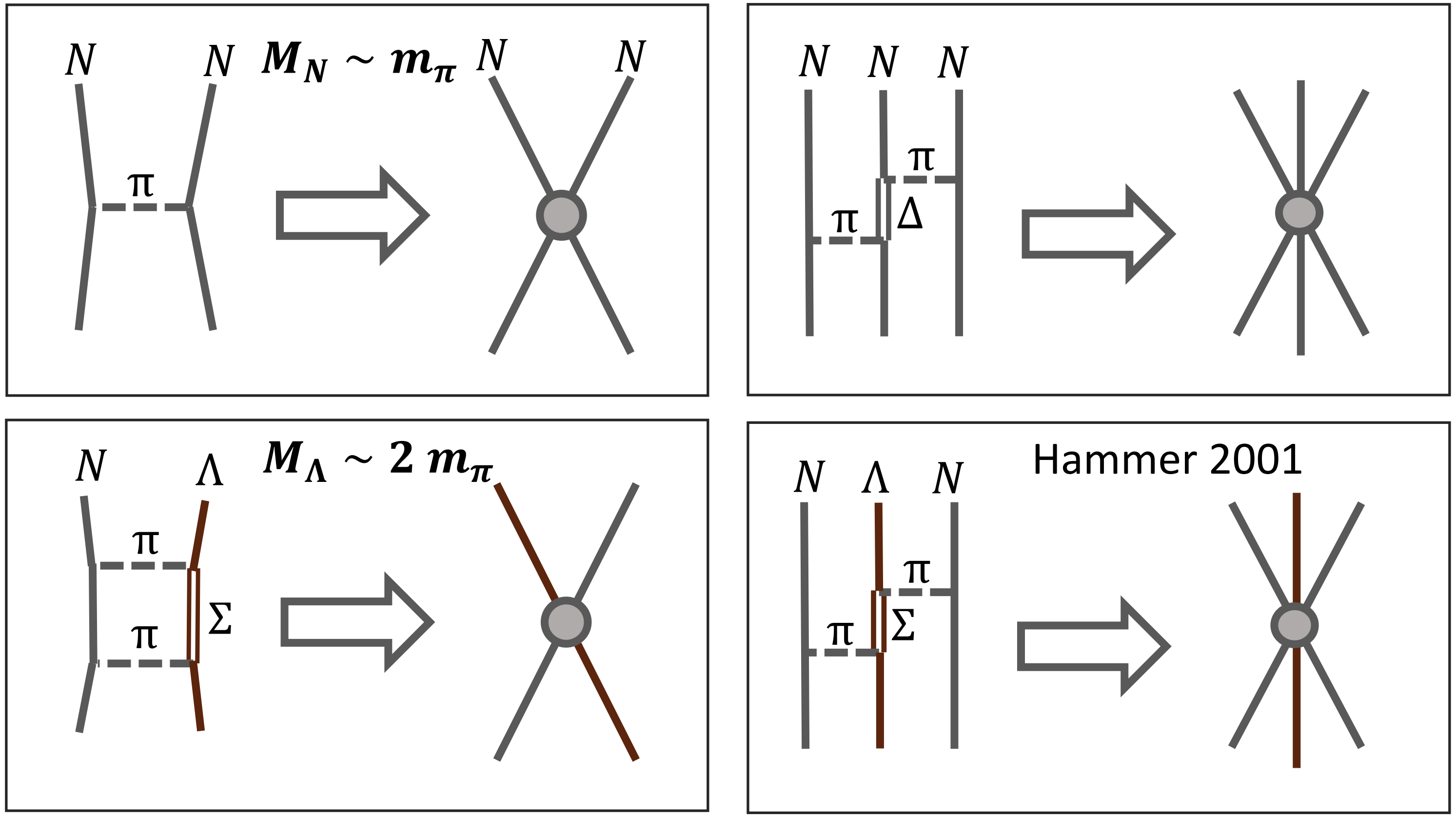
$m_\Sigma - m_\Lambda \sim 80$ MeV



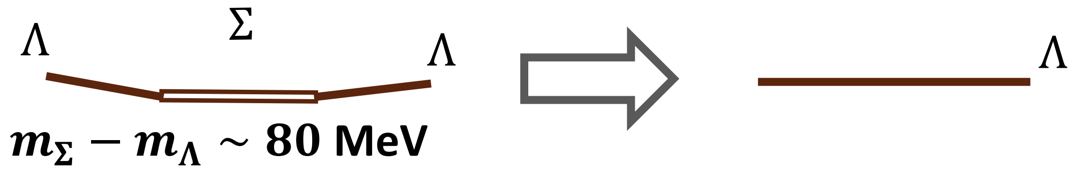
$\not{\pi}$-EFT

$\boldsymbol{M}$ = Theory break-scale
$\boldsymbol{Q}$ = Typical exchanged momentum





Thypical binding momentum of Λ is low
(few MeV per particle)

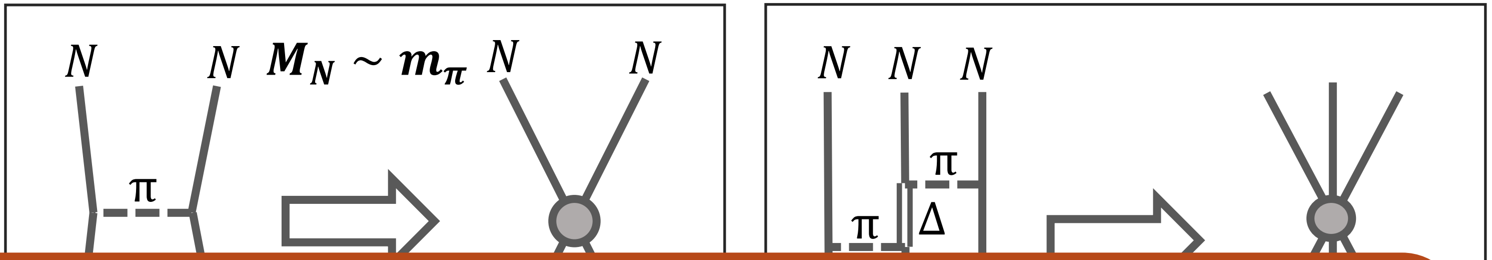


Larger wave function with respect the range of the interaction

2001 H.-W. Hammer
2018 LC, N. Barnea, A. Gal
2019 F. Hildenbrand, H.-W. Hammer
2019 LC, M. Schäfer, N. Barnea, A. Gal, J. Mareš

$\not{\pi}$-EFT

$\boldsymbol{M}$ = Theory break-scale
$\boldsymbol{O}$ = Typical exchanged momentum



Looking for the **simplest theory** for Lambda hyperons

- Easier to be interpreted
- Can be fixed by small amount of experimental data

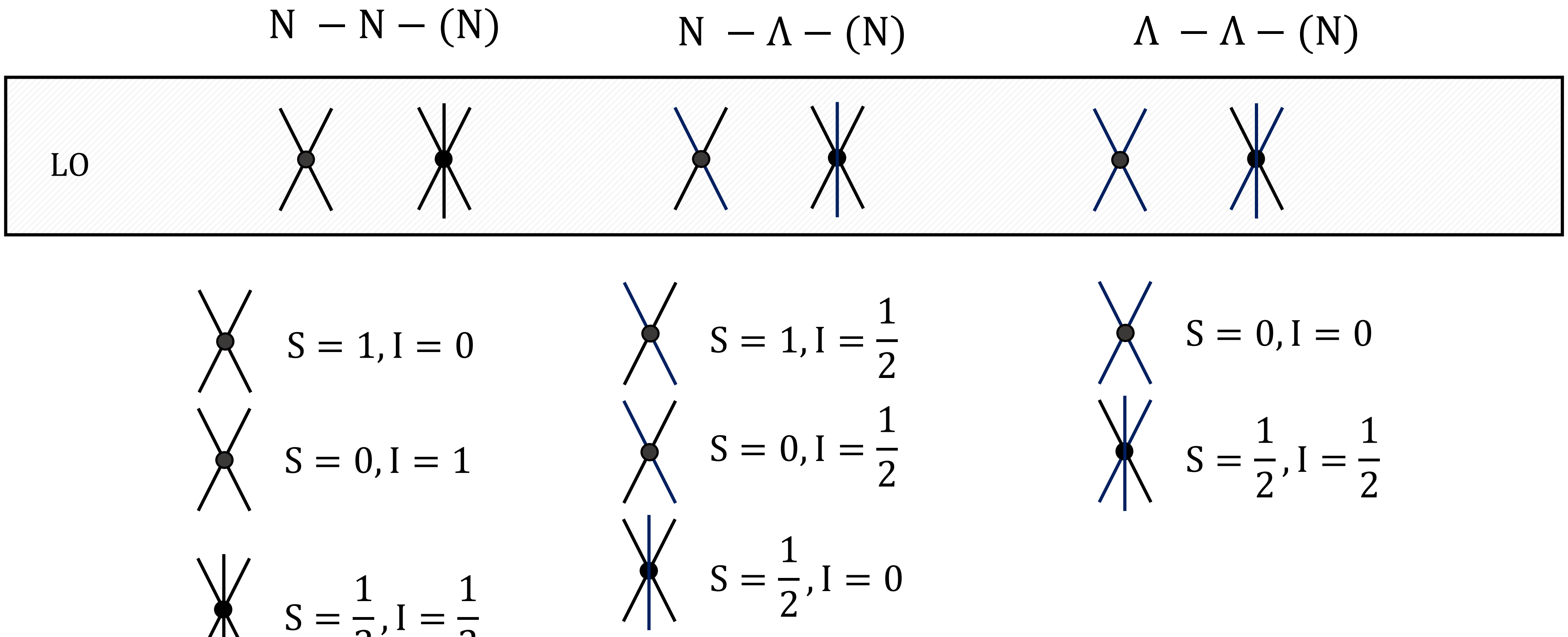
- Model independence? (Only if I can do an error estimation)

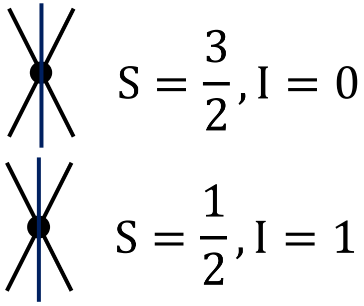
See. Hoai Le and Daniel's talks for a different approach

Larger wave function with respect the range of the interaction

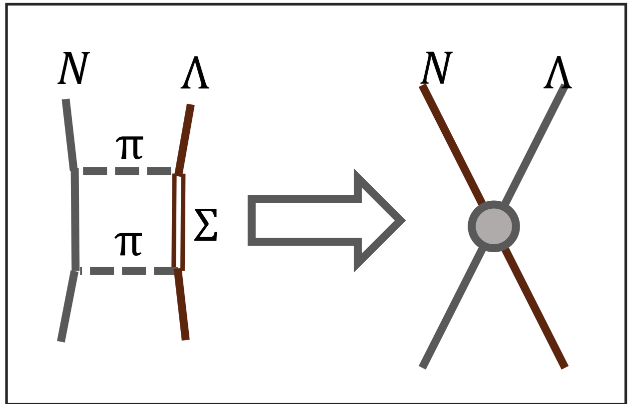
2019 L. C., M. Schäfer,
N. Barnea, A. Gal, J. Mareš

Pionless powercounting





The hypernuclear interaction if fitted on
$a_{N-\Lambda}$ and $a_{\Lambda-\Lambda}$
$\boldsymbol{B_\Lambda({}^{3}_{\Lambda}H)}$
$\boldsymbol{B_\Lambda({}^{4}_{\Lambda}H_{g.s.}),\ B_\Lambda({}^{4}_{\Lambda}H_{exc.})}$

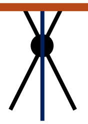


Pionless powercounting

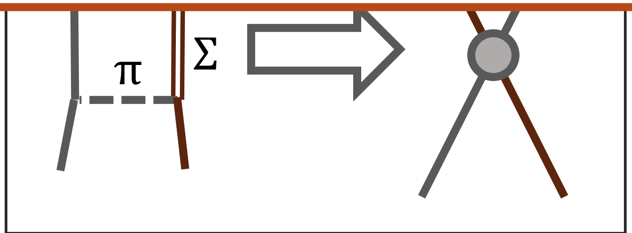
$N - N$ $N - \Lambda$ $\Lambda - \Lambda$

- Contact operators **can not be used directly**!
- Delta potentials need to be **smeared** and a **cutoff** introduced
- Results should be **“independent”** from the cutoff

$a_{N-\Lambda}$ and $a_{\Lambda-\Lambda}$
$B_\Lambda({}^{3}_{\Lambda}H)$
$B_\Lambda({}^{4}_{\Lambda}H_{g.s.}), B_\Lambda({}^{4}_{\Lambda}H_{exc.})$



$S = \frac{1}{2}, I = 1$

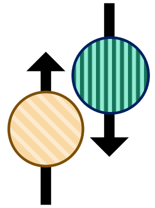


LEC	State	Fitting
C_{02}	$S = 1, I = 0$	$^{2}\mathrm{H}$
C_{20}	$S = 0, I = 1$	$\mathrm{N-N}$
C_{01}	$S = 1, I = \frac{1}{2}$	$\Lambda - \mathrm{N}$
C_{21}	$S = 0, I = \frac{1}{2}$	$\Lambda - \mathrm{N}$
C_{00}	$S = 0, I = 0$	$\Lambda - \Lambda$

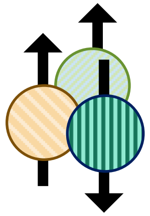
Boundstate (C_{02})

Scattering (C_{20}, C_{01}, C_{21})

Two body



Three body



Boundstates

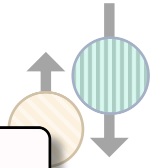
LEC	State		Fitting	
D_{11}	$S = \frac{1}{2}$,	$I = \frac{1}{2}$	$^{3}\mathrm{H}$	
D_{01}	$S = \frac{1}{2}$,	$I = 0$	$^{3}_{\Lambda}\mathrm{H}$	
D_{03}	$S = \frac{1}{2}$,	$I = 1$	$^{4}_{\Lambda}\mathrm{H}_{S=0, I=\frac{1}{2}}$	Λnn
D_{21}	$S = \frac{3}{2}$,	$I = 0$	$^{4}_{\Lambda}\mathrm{H}_{S=1, I=\frac{1}{2}}$	Λpn
D_{00}	$S = \frac{1}{2}$,	$I = 0$	$^{6}_{\Lambda\Lambda}\mathrm{He}$	$\Lambda\Lambda N$

LEC	State	Fitting
C_{02}	$S=1, I=0$	^2H
C_{20}	$S=0, I=1$	$N-N$
C_{01}	$S=1, I=\frac{1}{2}$	$\Lambda - N$
C_{21}	S =	
C_{00}	S =	

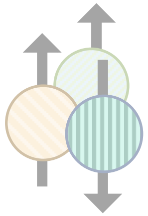
Boundstate

Scattering

Two body



Three bo



ΛN model	$a_s(NN)$	$a_s(\Lambda N)$	$a_t(\Lambda N)$
Alexander A	-23.72	-1.8	-1.6
Alexander B	-18.63	-1.8	-1.6
NSC97f	-18.63	-2.6	-1.7
χEFT(LO)	-18.63	-1.91	-1.23
χEFT(NLO)	-18.63	-2.91	-1.54

			Fitting	
			^3H	
			$^3_\Lambda H$	
			$S=0, I=\frac{1}{2}$	Λnn
D_{21}	$S=\frac{}{2},$	$I=0$	$S=1, I=\frac{1}{2}$	Λpn
D_{00}	$S=\frac{1}{2},$	$I=0$	$^6_{\Lambda\Lambda}He$	$\Lambda\Lambda N$

IN

^2H

$N-N$

$\Lambda - N$

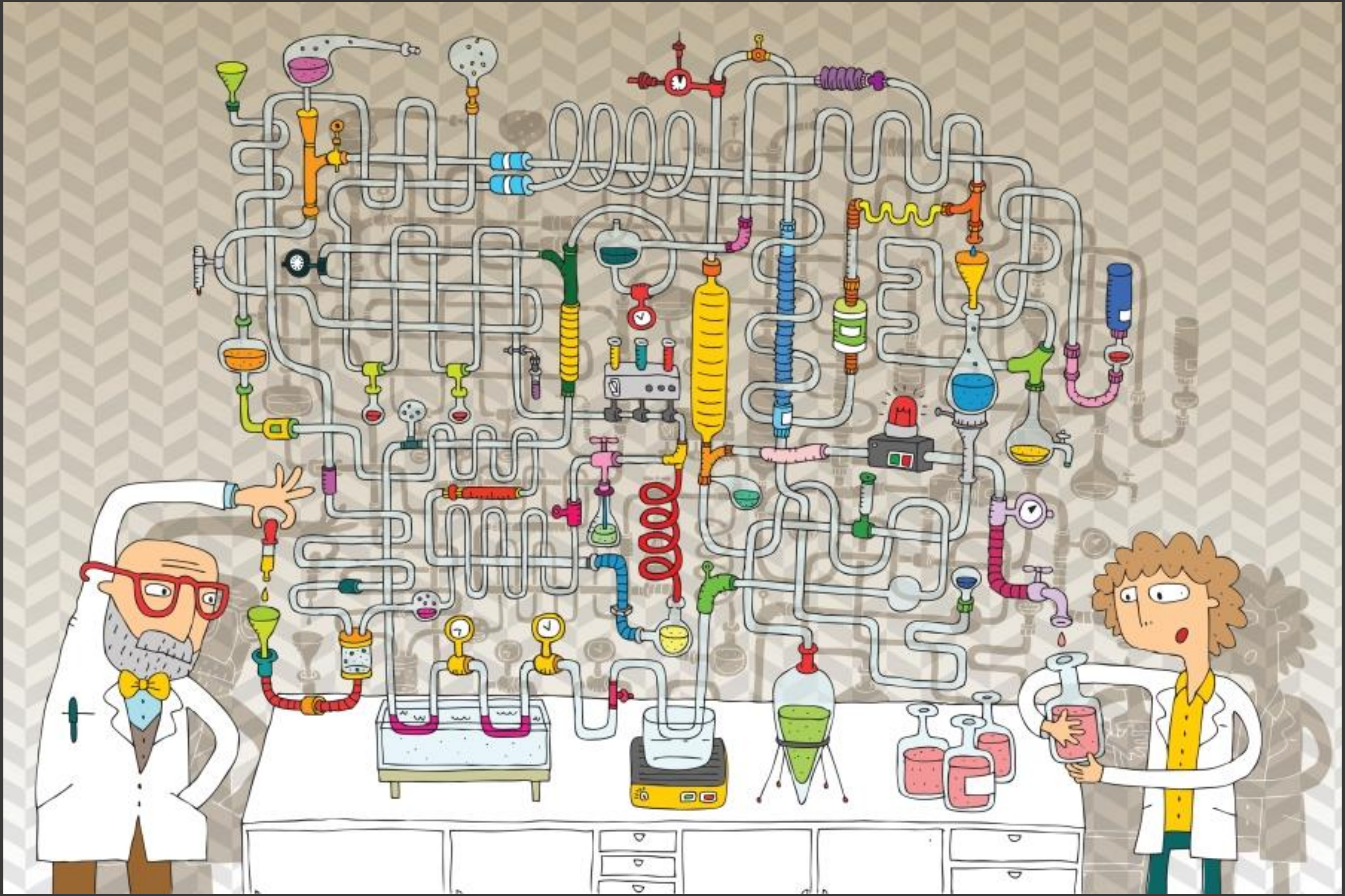
$\Lambda - \Lambda$

^3H

$^4_\Lambda H_{S=0,I=\frac{1}{2}}$

$^4_\Lambda H_{S=1,I=\frac{1}{2}}$

$^6_{\Lambda\Lambda}He$



OUT

$^5_\Lambda He$

$np\Lambda$

$nn\Lambda$

$^4_{\Lambda\Lambda}H$

$^5_{\Lambda\Lambda}H$

RESULTS: ${}^{5}_{\Lambda}\mathbf{He}$ Overbinding problem

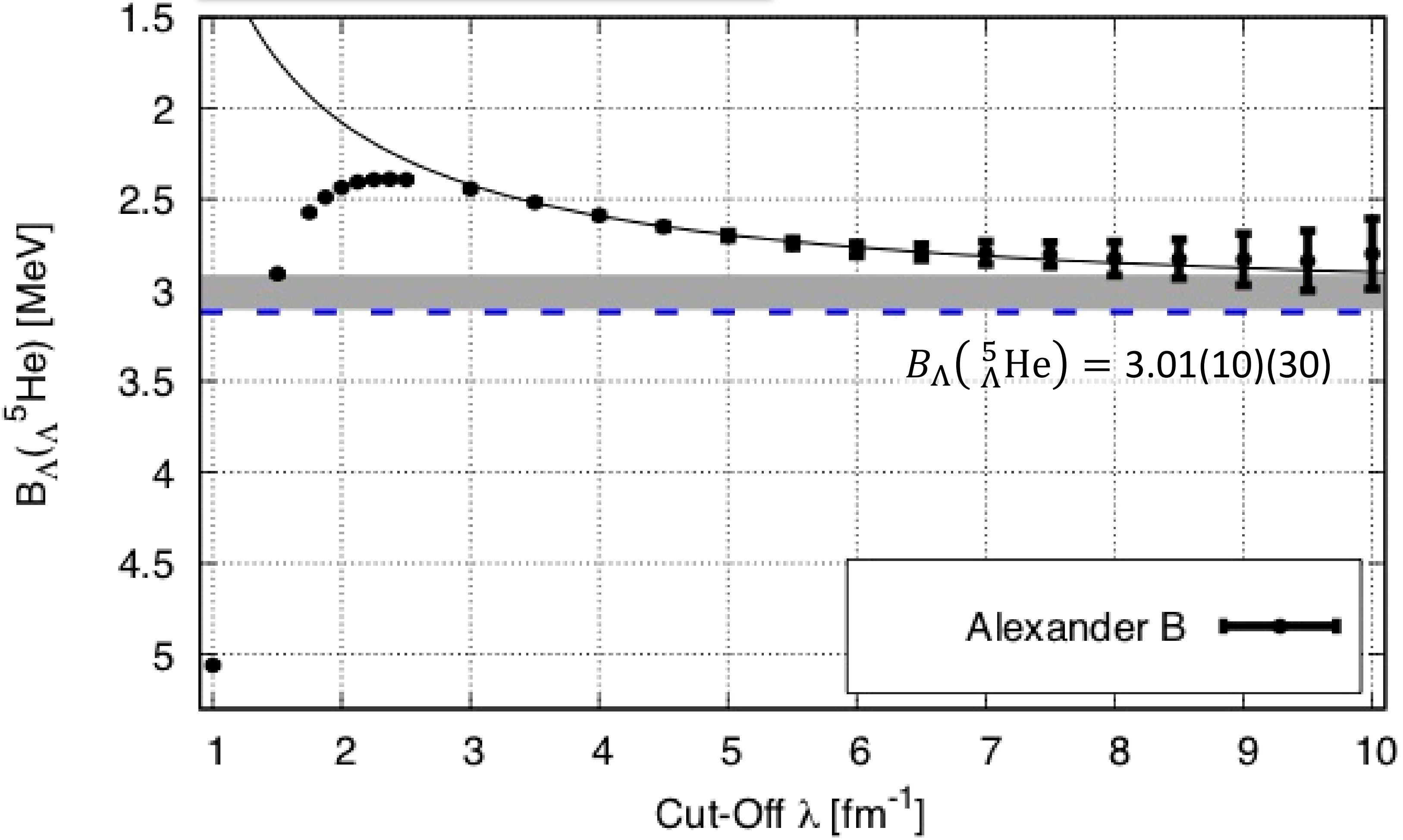
	$B_\Lambda({}^{3}_{\Lambda}H)$	$B_\Lambda({}^{4}_{\Lambda}H_{g.s.})$	$B_\Lambda({}^{4}_{\Lambda}H_{exc.})$	$B_\Lambda({}^{5}_{\Lambda}He)$
Exp.	0.13(5) [4]	2.16(8) [5]	1.09(2) [6]	3.12(2) [4]
DHT [7]	0.10	2.24	0.36	≥ 5.16
AFDMCa	-	1.97(11) [8]	-	5.1(1) [9]
AFDMCb’	0.23(9) [13]	1.95(9) [13]	-	2.60(6) [13]
χEFTa	0.11 [10]	2.31 (3) [11]	0.95(15) [11]	5.82(2) [12]
χEFTa	-	2.13 (3) [11]	1.39(15) [11]	4.43(2) [12]
χEFT NLO(2019)	0.091 [16]	1.462 [16]	1.055 [16]	2.16-5.63 [17]

All the energies are in MeV.

[7] R.H. Dalitz, R.C. Herndon, and Y.C. Tang, Nucl. Phys. B 47, 109 (1972).
[8] D. Lonardoni, F. Pederiva, and S. Gandolfi, Phys. Rev. C 89, 014314 (2014).
[9] D. Lonardoni, S. Gandolfi, and F. Pederiva, Phys. Rev. C 87, 041303(R) (2013).
[10] R. Wirth et al., Phys. Rev. Lett. 113, 192502 (2014).
[11] D. Gazda and A. Gal, Phys. Rev. Lett. 116, 122501 (2016); D. Gazda and A. Gal, Nucl. Phys. A 954, 161 (2016).
[12] R. Wirth and R. Roth, Phys. Lett. B 779, 336 (2018). We thank Roland Wirth for providing us with these values.
[13] D. Lonardoni arXiv:1711.07521v2 & Private comunication.
[15] H. Nemura, Y. Akaishi, and Y. Suzuki, Phys. Rev. Lett. 89, 142504 (2002); see also Y. Akaishi, T. Harada.
[16] J. Haidenbauer 1 , U.-G. Meißner 2 , 1 , 3 , 4 , A. Nogga 1 , 4 (only cut-ff 600 MeV)
[17] Johann Haidenbauer and Isaac Vidana Eur.Phys.J.A 56 (2020) 2, 55 (cut-offs 500 to 700 MeV)

${}^{5}_{\Lambda}\mathrm{He}$: Λ separation energy

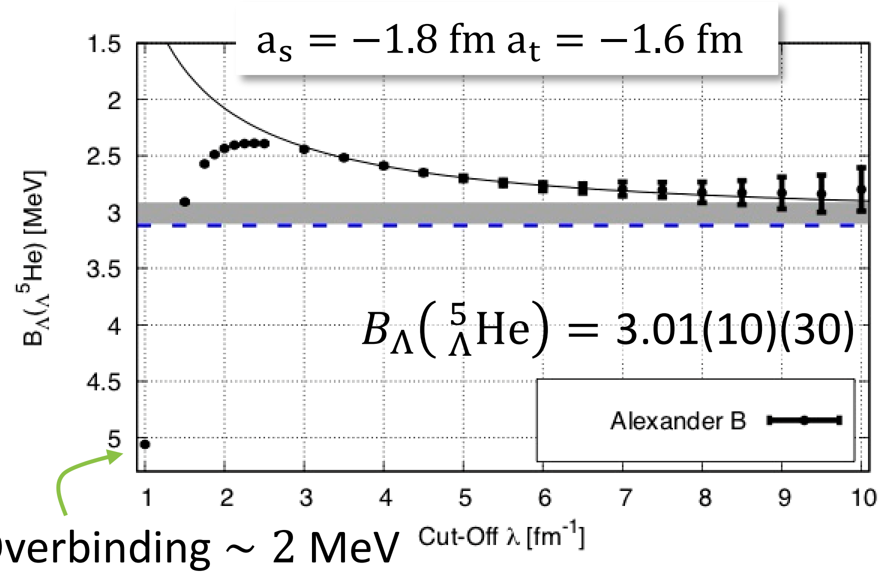
$a_s = -1.8$ fm $a_t = -1.6$ fm



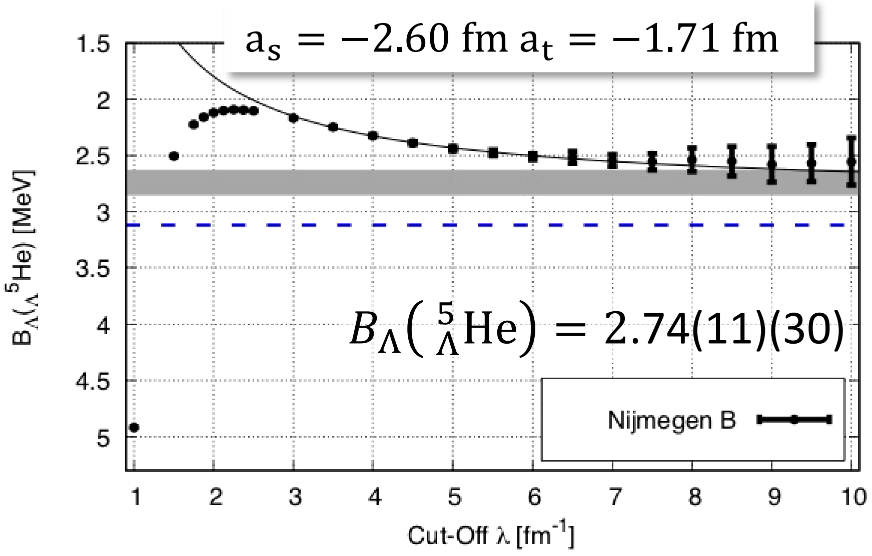
$B_\Lambda({}^{5}_{\Lambda}\mathrm{He}) = 3.01(10)(30)$

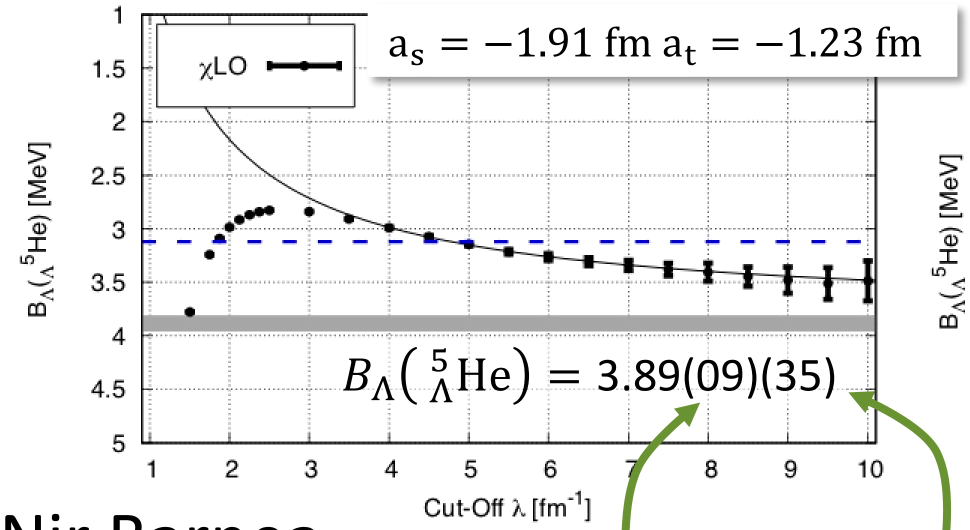
2018 L. C., Nir Barnea, Avraham Gal

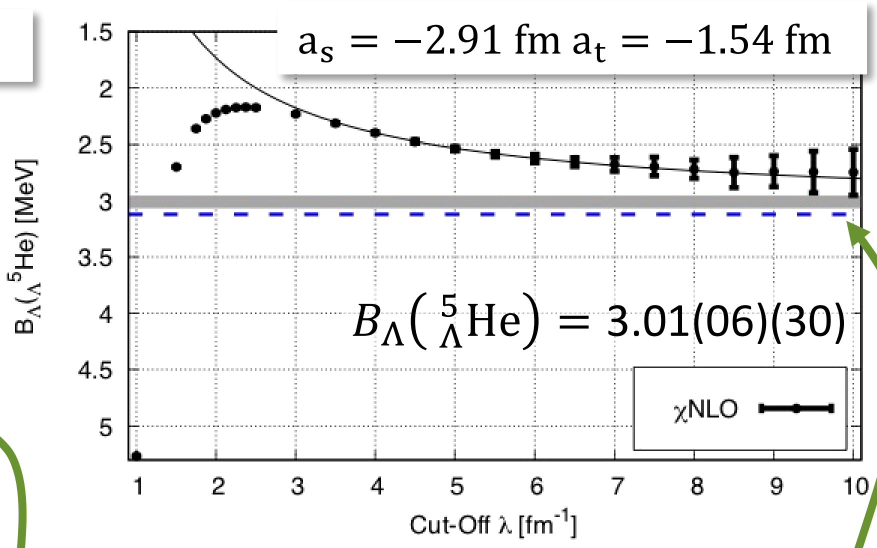
${}^{5}_{\Lambda}\mathrm{He}$: Λ separation energy



Overbinding ~ 2 MeV







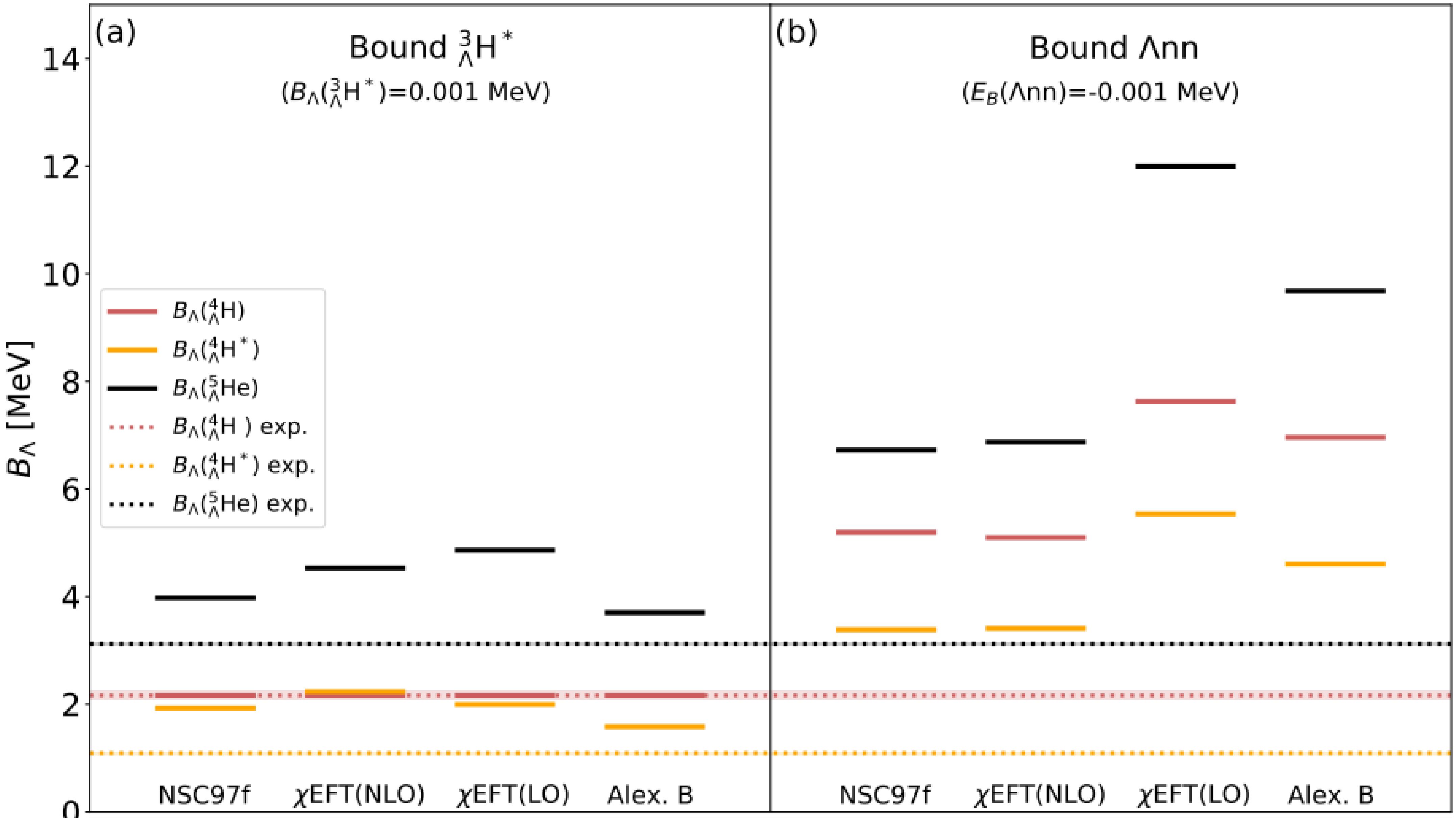
2018 L. C., Nir Barnea, Avraham Gal

Numerical uncertainty

Theory uncertainty

Experimental: 3.12(2) MeV

What is the nature of ${}^{3}_{\Lambda}\mathrm{H}^{*}$ and Λnn?



Schäfer, Bazak, Barnea, Mares (2021)

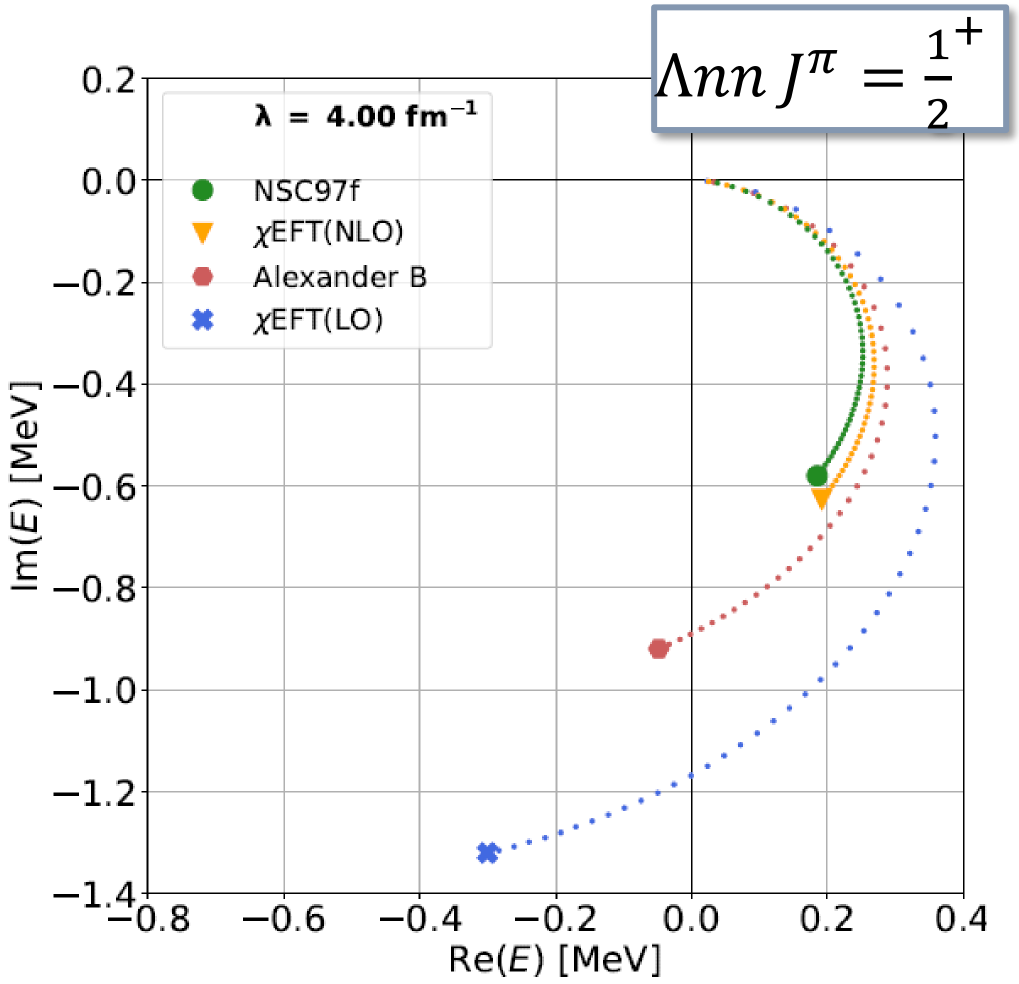


FIG. 2. Trajectories of the Λnn resonance pole in the complex energy plane determined by a decreasing attractive strength $d_{\lambda}^{I=1,S=1/2}$ for selected sets of ΛN scattering length, calculated at $\lambda = 4.00\ \mathrm{fm}^{-1}$. Larger symbols stand for the physical position of the Λnn pole ($d_{\lambda}^{I=1,S=1/2} = 0$).

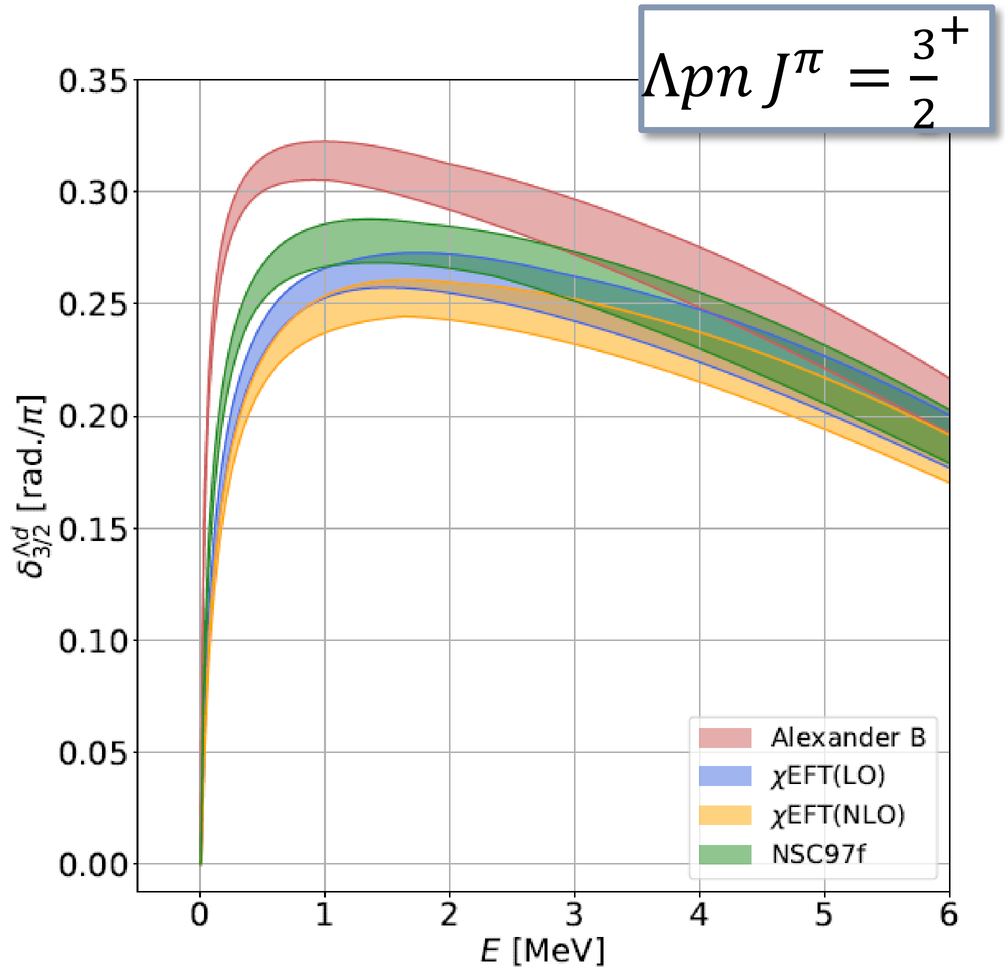
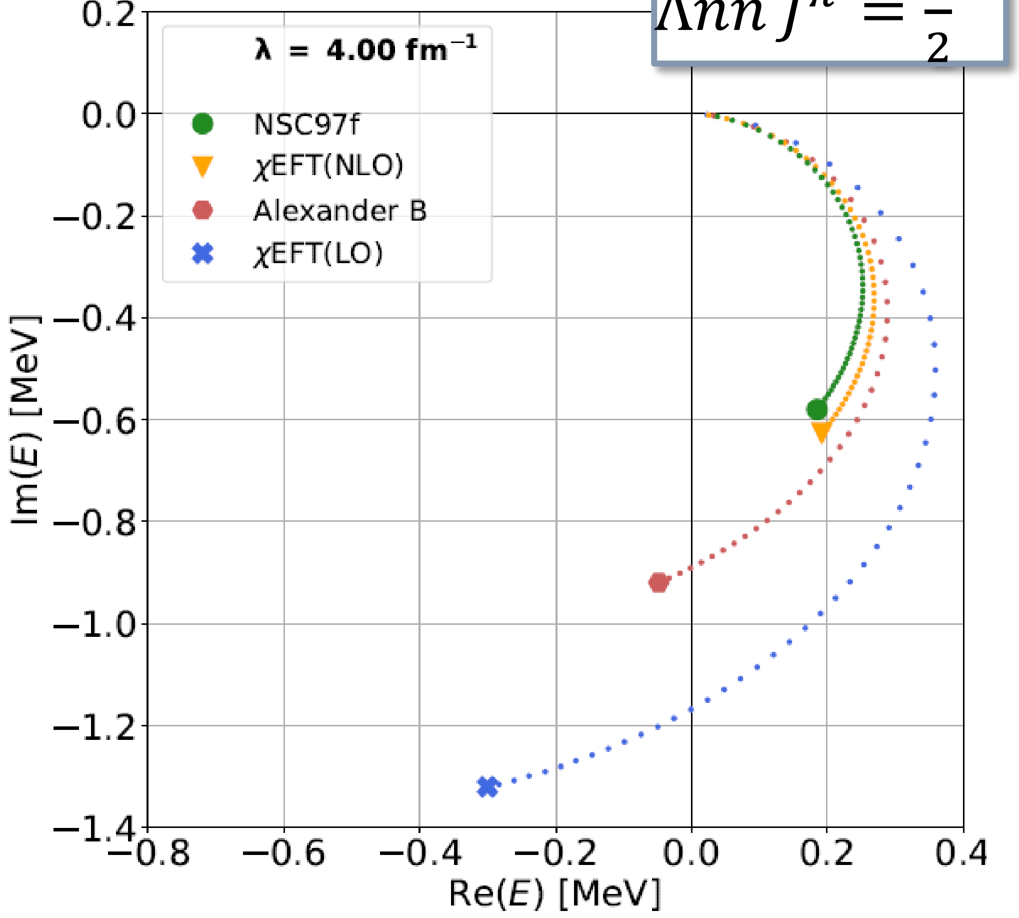


FIG. 4. S-wave Λd phase-shifts in the $J^{\pi} = 3/2^{+}$ channel $\delta_{3/2}^{\Lambda d}$ as a function of energy E above the $\Lambda + d$ threshold, extracted from the continuum level density of the rotated CSM spectra. The phase-shifts are calculated for cut-off $\lambda = 6\ \mathrm{fm}^{-1}$ and several ΛN interaction strengths. Shaded areas mark uncertainty introduced by the rotation angle θ within interval $15^{\circ} < \theta < 20^{\circ}$.

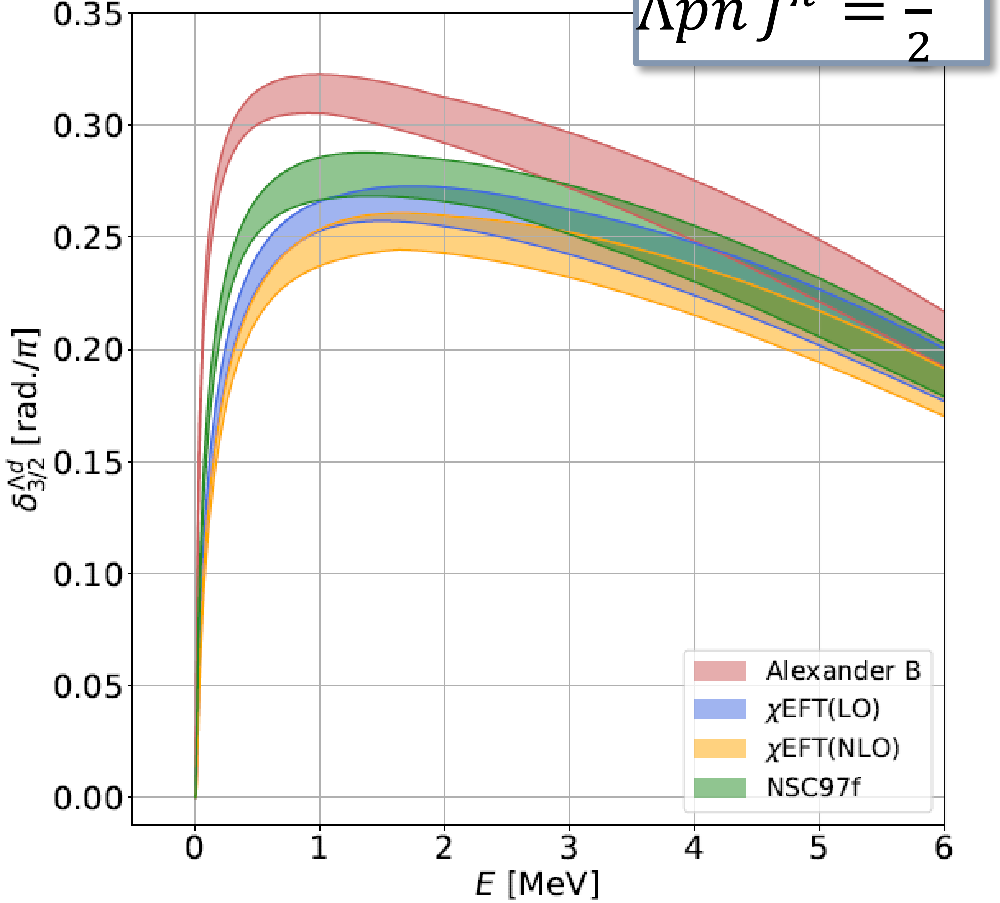
2020 - M. Shäfer, B. Bazack, N. Barnea, J. Mareš

$\Lambda nn\ J^{\pi} = \frac{1}{2}^{+}$



Resonance

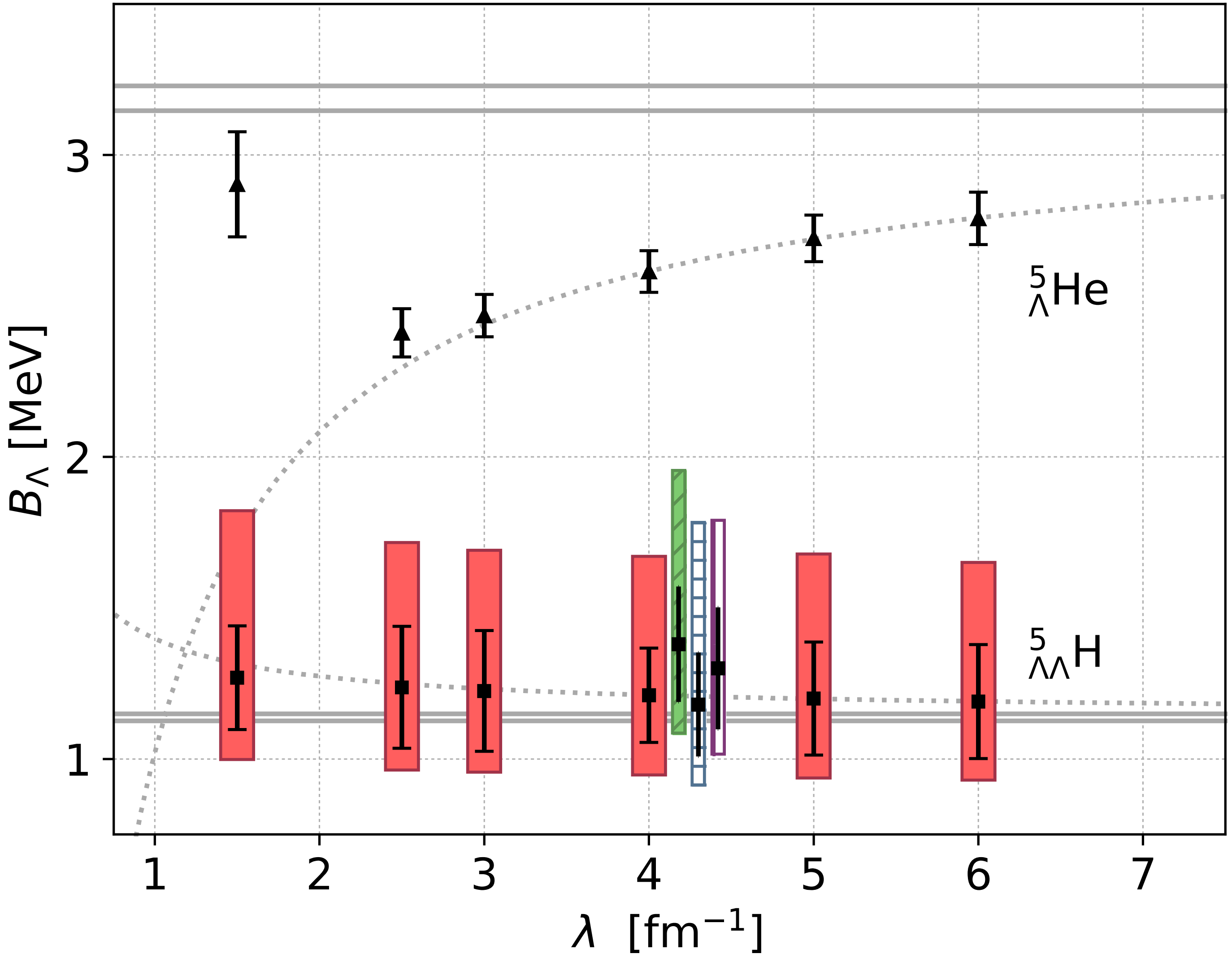
$\Lambda pn\ J^{\pi} = \frac{3}{2}^{+}$



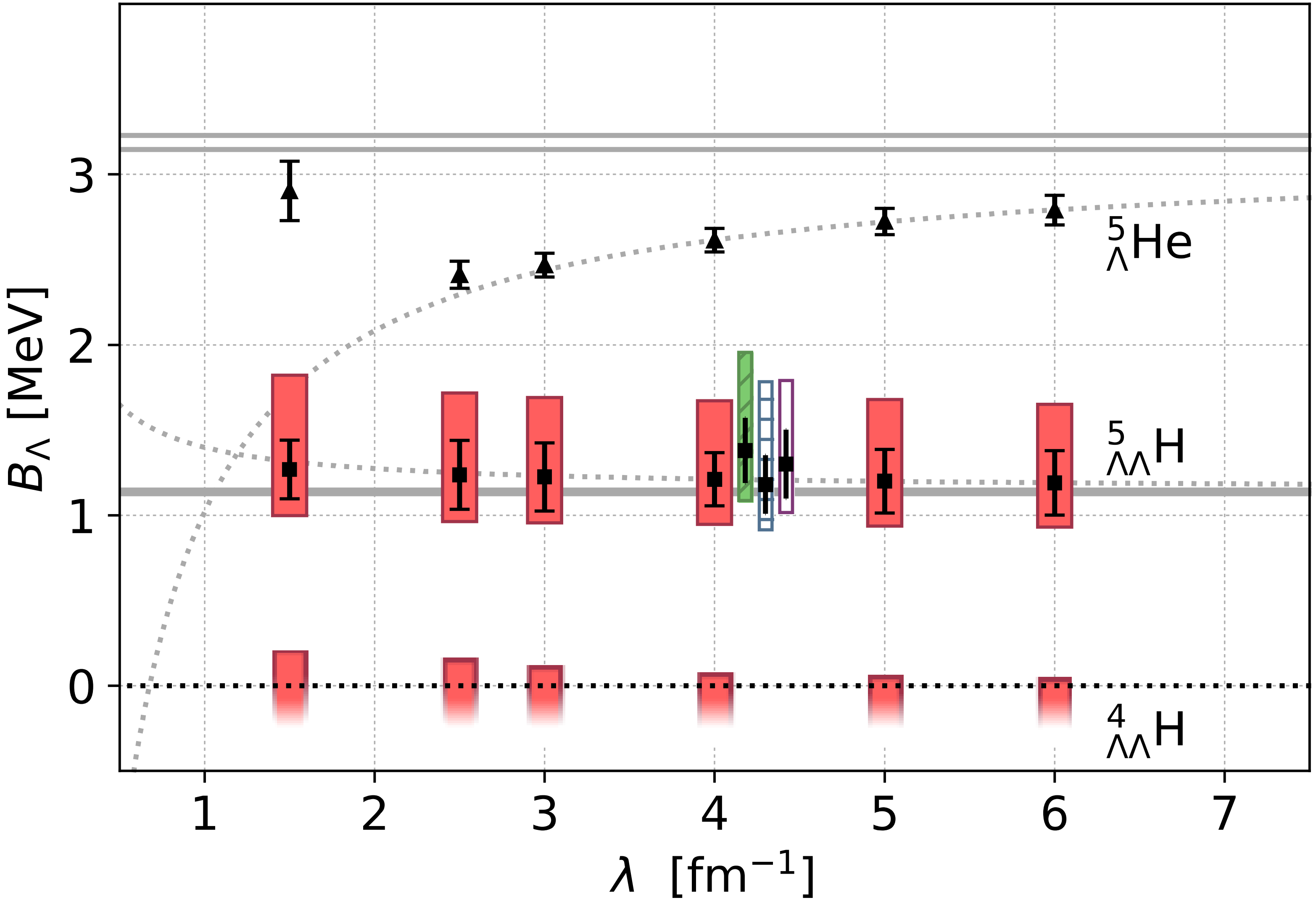
Virtual state

2020 - M. Shäfer, B. Bazack, N. Barnea, J. Mareš

$^{5}_{\Lambda\Lambda}H$



$^{4}_{\Lambda\Lambda}\mathrm{H}$



2022 - M. Schäfer, N. Barnea, and A. Gal

See. Talk of Emiko Hiyama of Monday

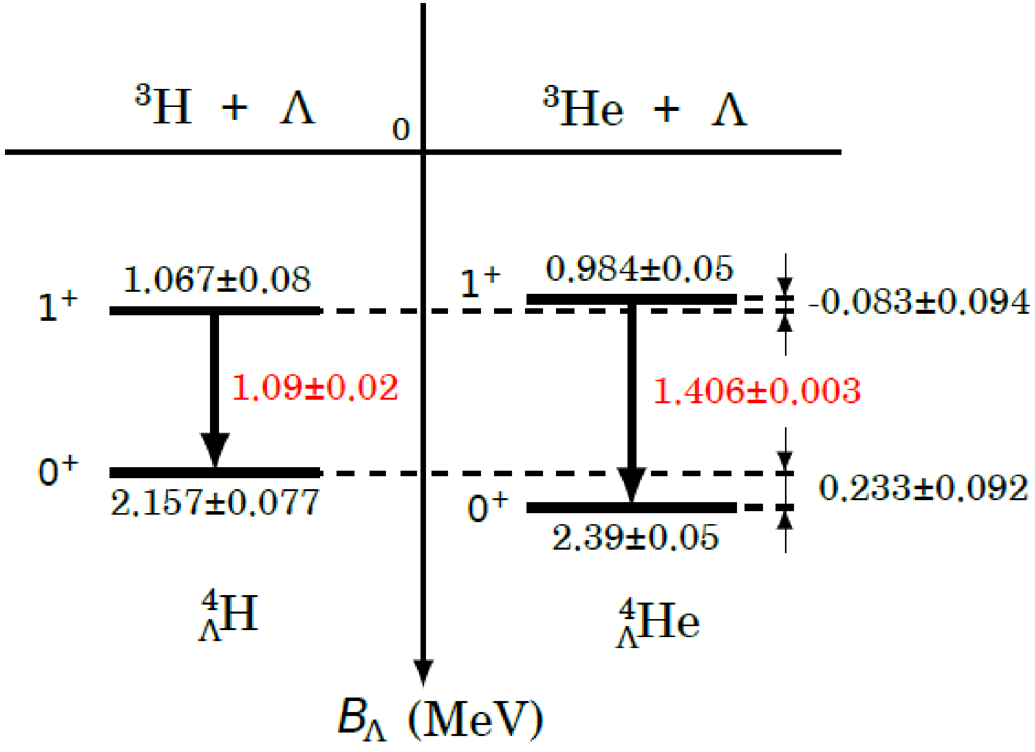
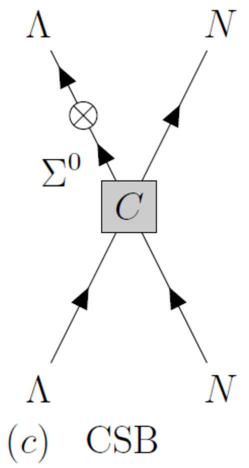
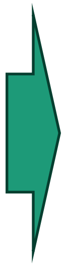
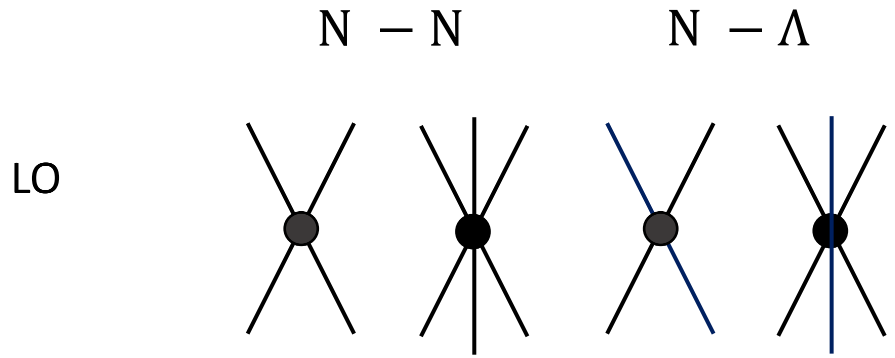


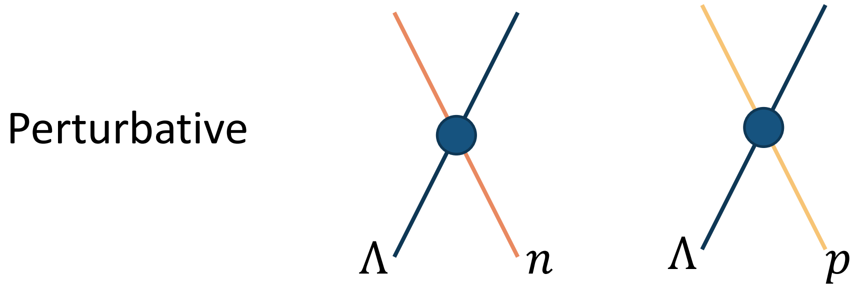
FIG. 1: $A = 4$ hypernuclear level scheme [11–13] with γ-ray energies [11] marked in red. CSB splittings are shown in MeV to the right of the $^4_\Lambda$He levels. Figure adapted from Ref. [13].



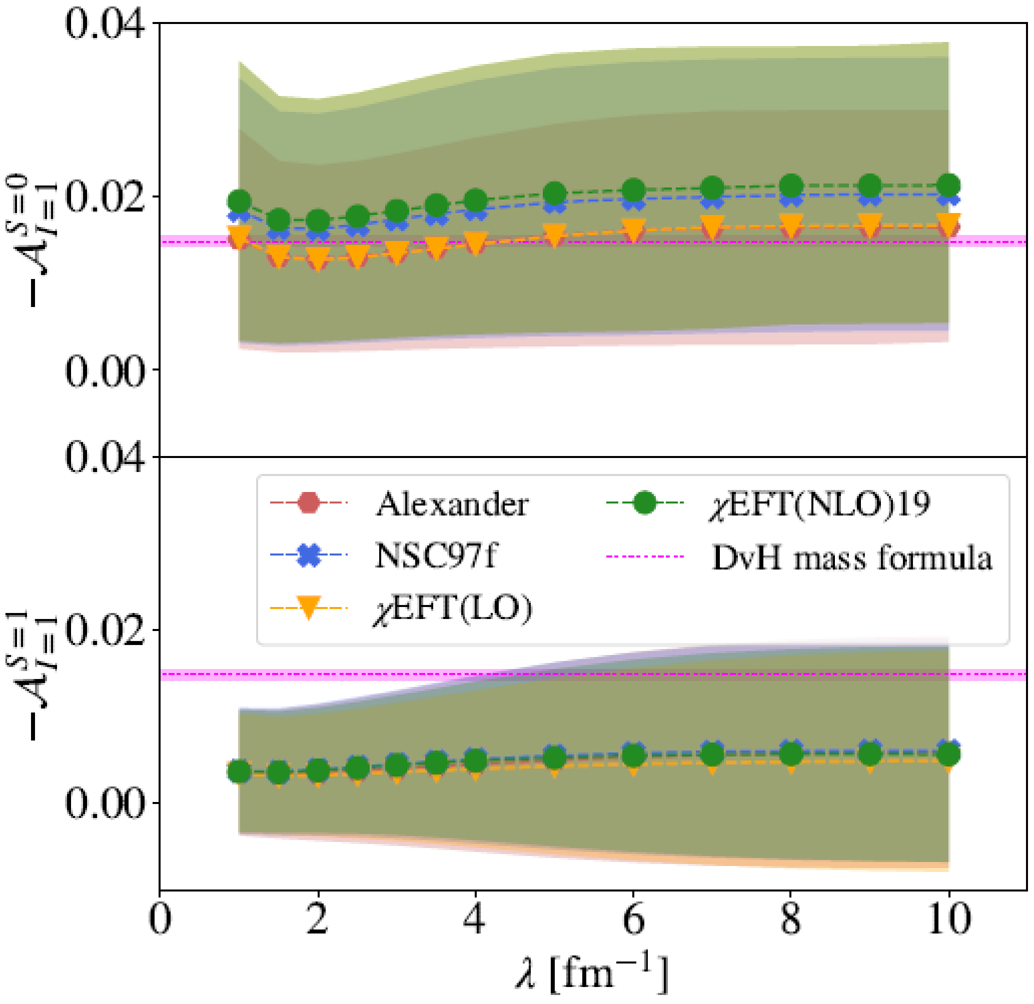


Difference between $a(\Lambda\, p)$ and $a(\Lambda\, n)$

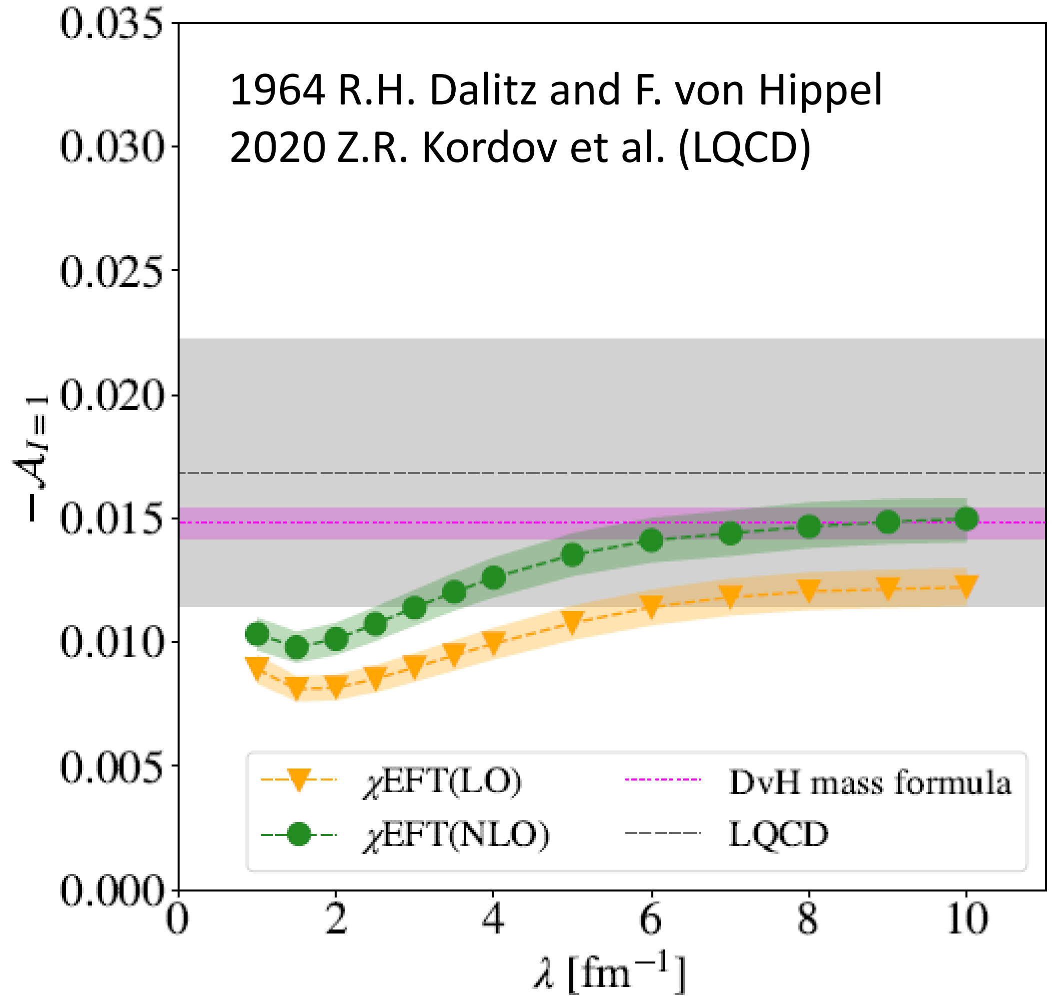




Calculated by energy difference between G. and EX. states



Calculated by energy difference between emitted γs



Charge symmetry breaking amplitude can be calculated from the pionless perturbative corrections:

- It is in agreement with microscopic calculations
- It is simple and do not require new degrees of freedom

Conclusions

General

- Λ **hypernuclei** can be described by contact interactions (in first approximation).
- **7** new input data that **can** be fix on **experimental** data!*

*Scattering lengths are not very well known

Predictions

- **Overcomes overbinding** problem (comprehensive description of $A \leq 5$ Λ-hyperons)
- **No** boundstate in $\boldsymbol{nn\Lambda}, \boldsymbol{np\Lambda}\left(\mathrm{S} = \frac{3}{2}\right), \boldsymbol{n\Lambda\Lambda}$ or $\boldsymbol{nn\Lambda\Lambda}$
  - $\boldsymbol{nn\Lambda}$ is a **virtual state** $\boldsymbol{np\Lambda}\left(\mathrm{S} = \frac{3}{2}\right)$ is a **resonance**
- $^{5}_{\Lambda\Lambda}\mathbf{He}$ **bound** ($B\left({}^{5}_{\Lambda\Lambda}\mathrm{He}\right) = \mathbf{1.14(1)}^{+(44)}_{-(26)}$ MeV)
- **Charge symmetry braking** are easy to calculate and in agreement with previous calculations

Prospective

- **Three-body forces** and their interplay are essential in the this framework
- How to access **next orders** (increase precision) for few-body?
- **Many-body** systems?
- Looking forward for new exciting **experimental results**.

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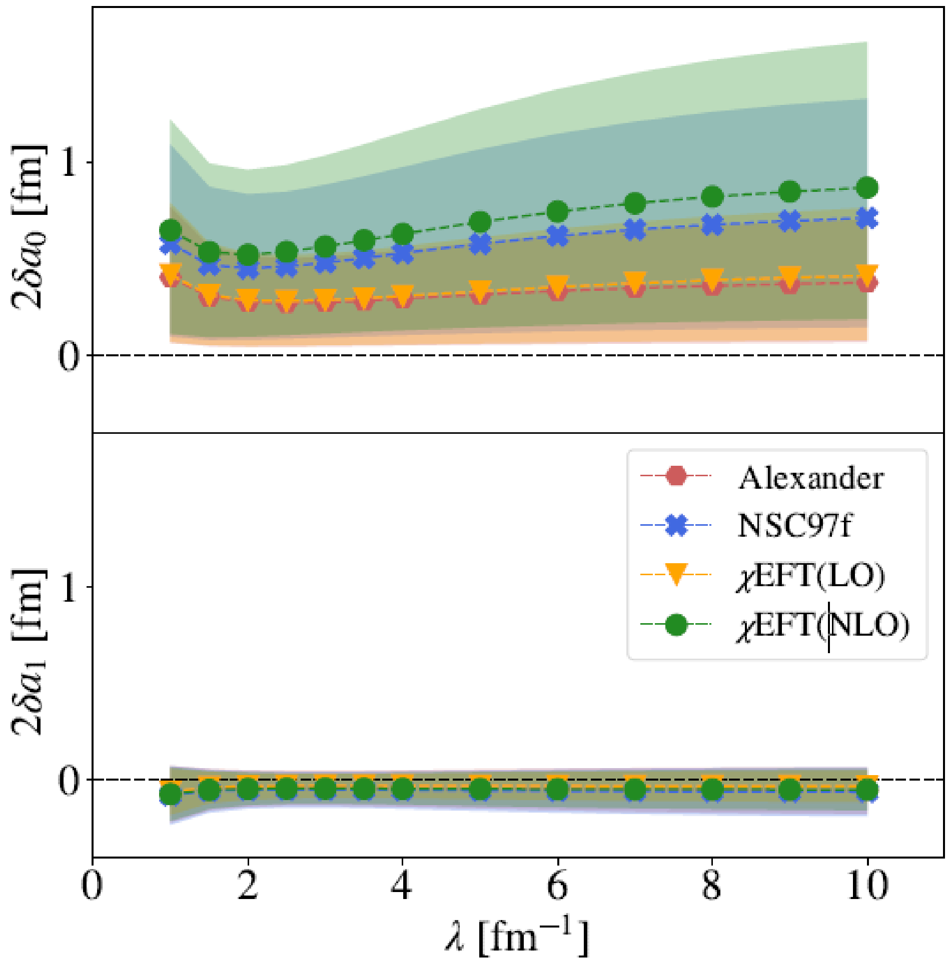
Thanks



Charge symmetry breaking amplitude can be calculated from the pionless perturbative corrections:

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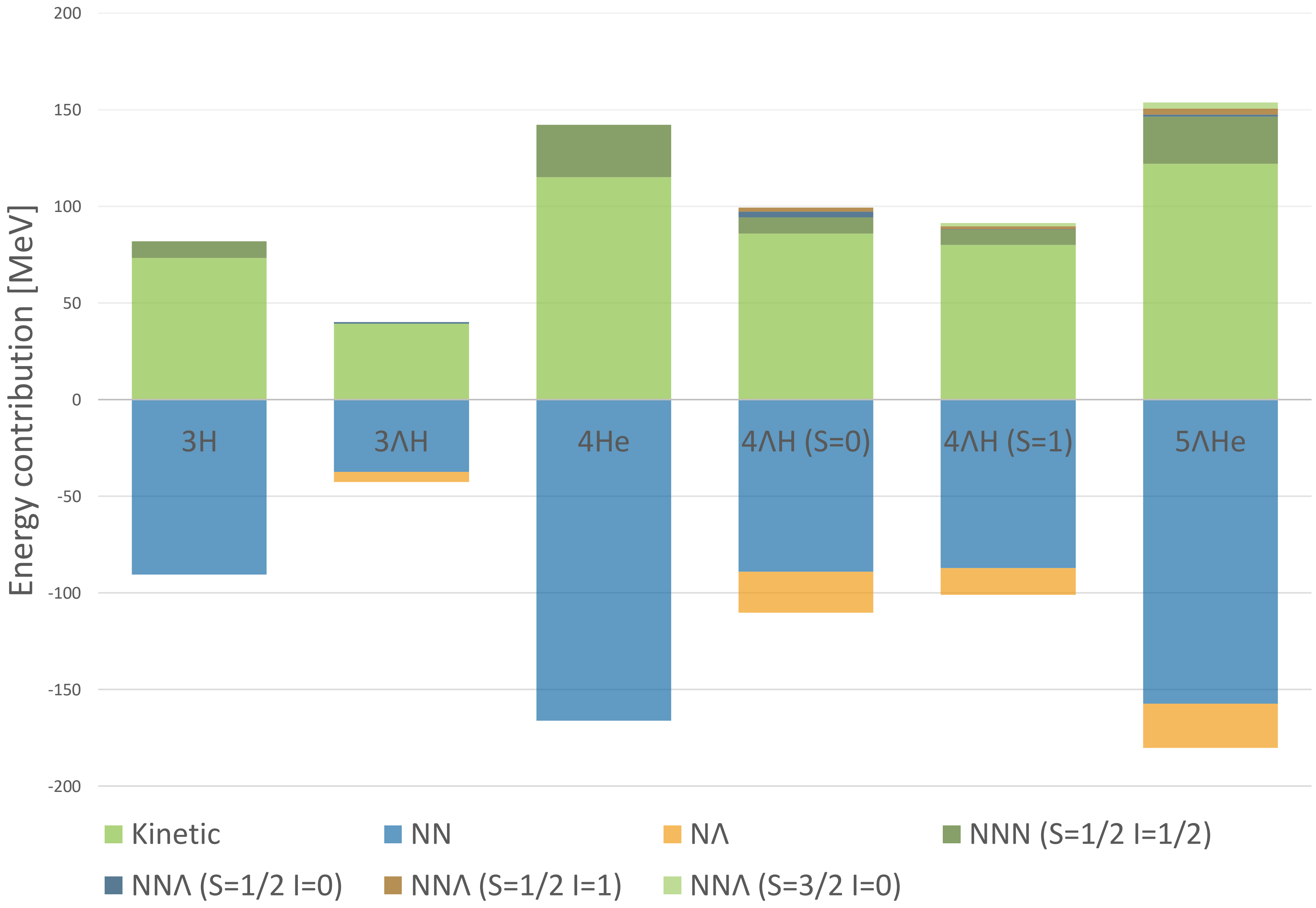
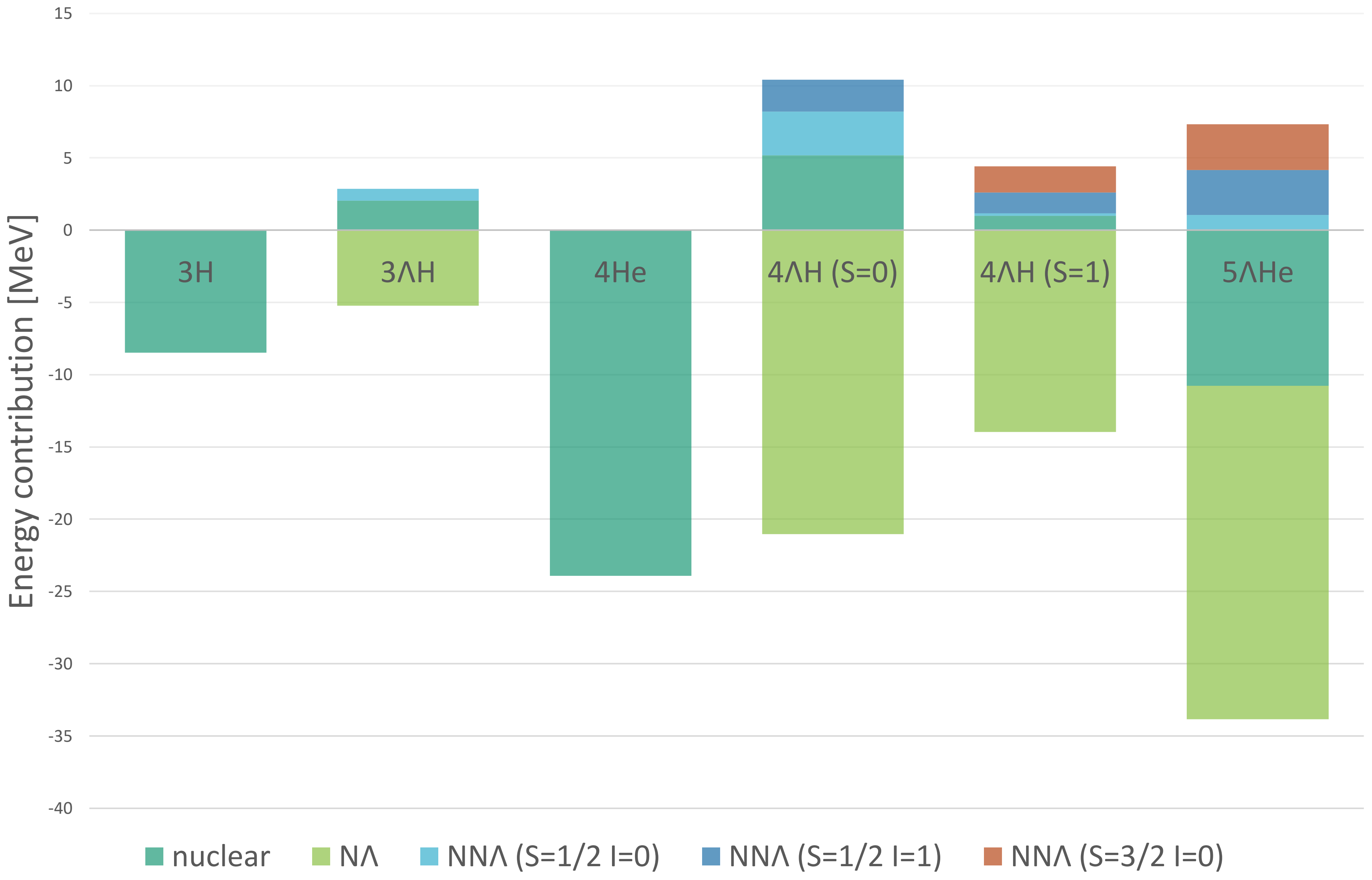
- It is simple and do not require new degrees of freedom

Energy contributions for $\lambda = 4\ \mathrm{fm}^{-1}$

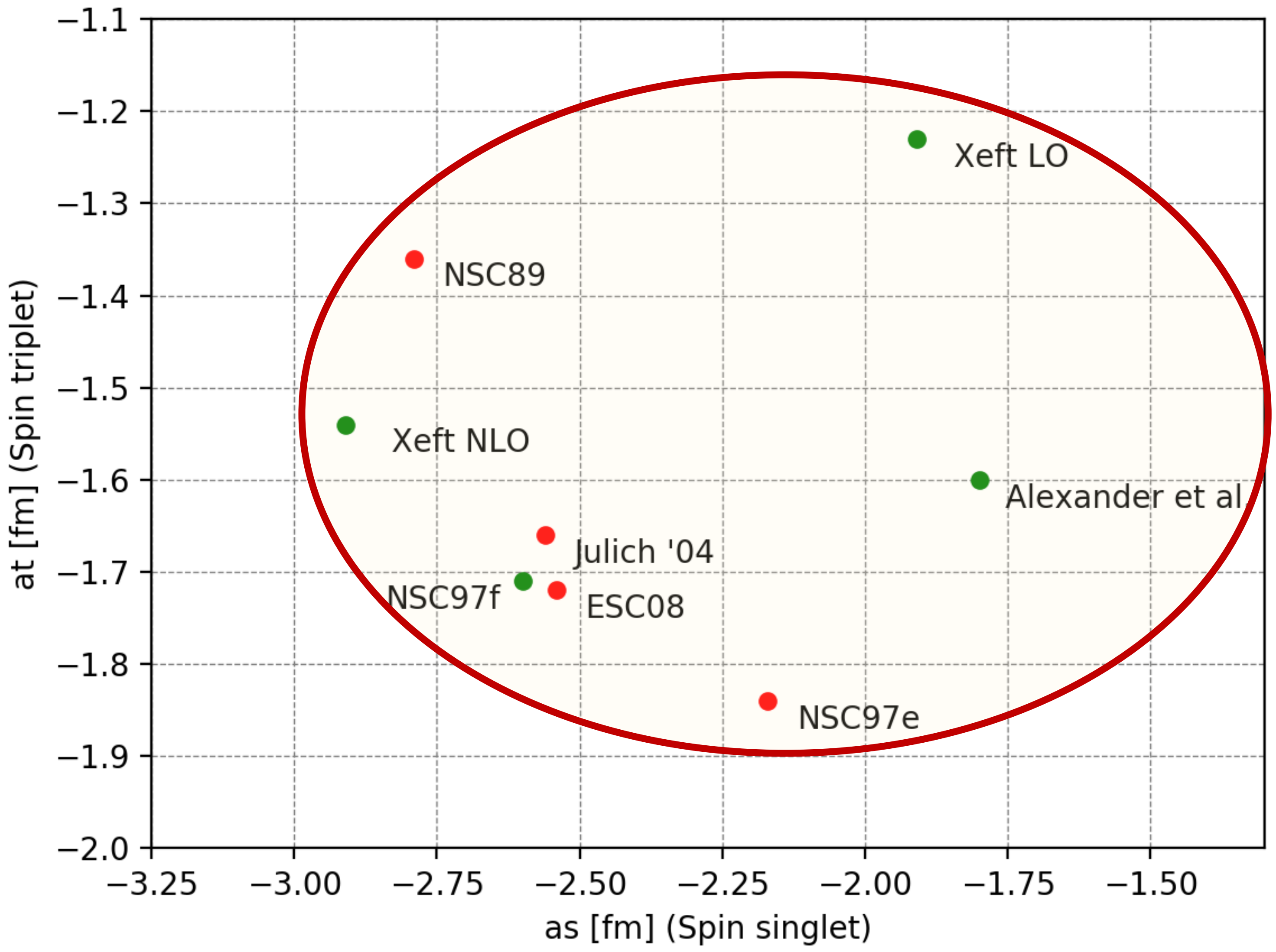


Energy contributions for $\lambda = 4\ \mathrm{fm}^{-1}$

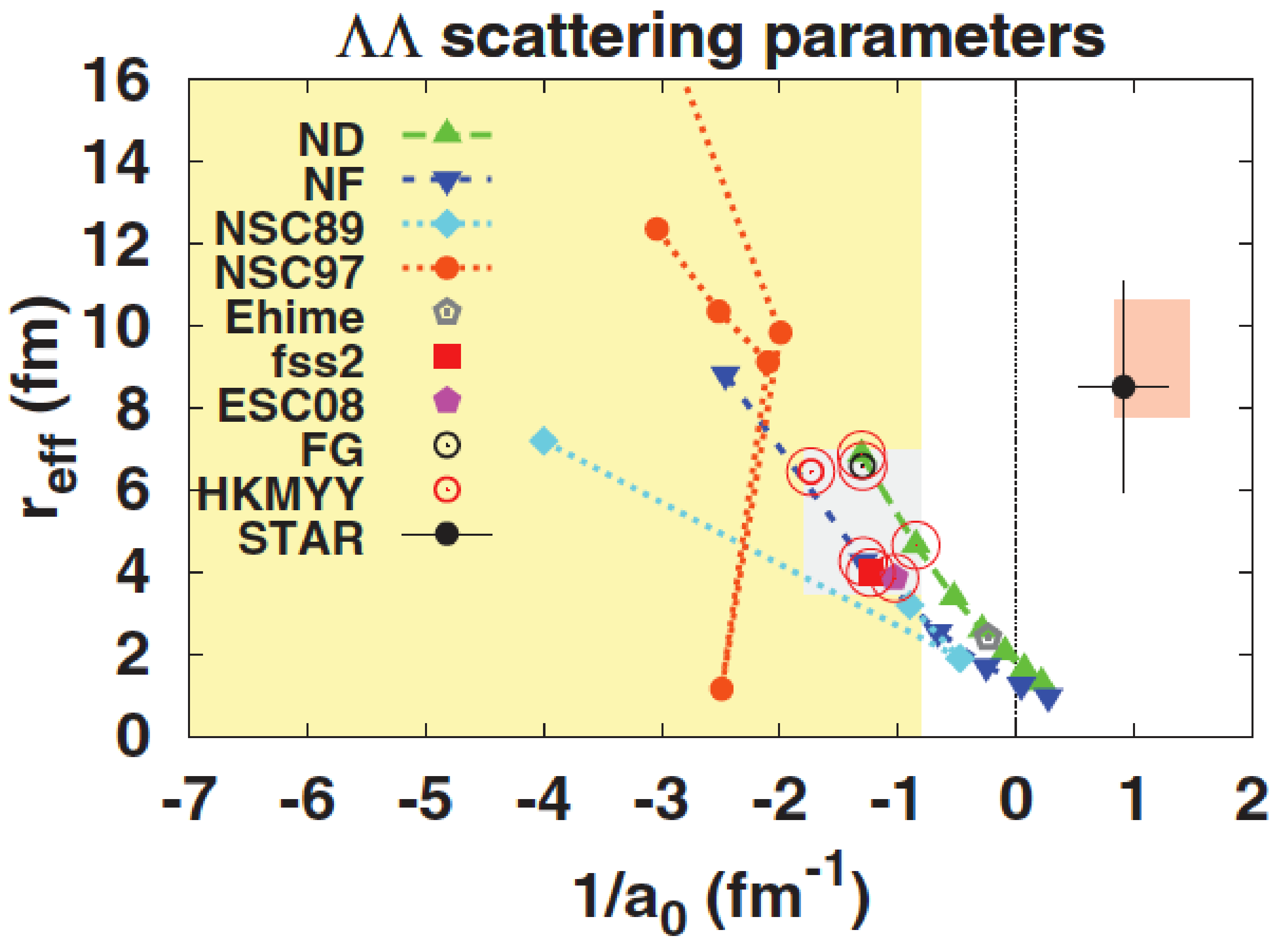


N-Λ scattering length

A. Gal et al. - Strangeness in nuclear physics - Rev.Mod.Phys. 88 (2016) no.3, 035004



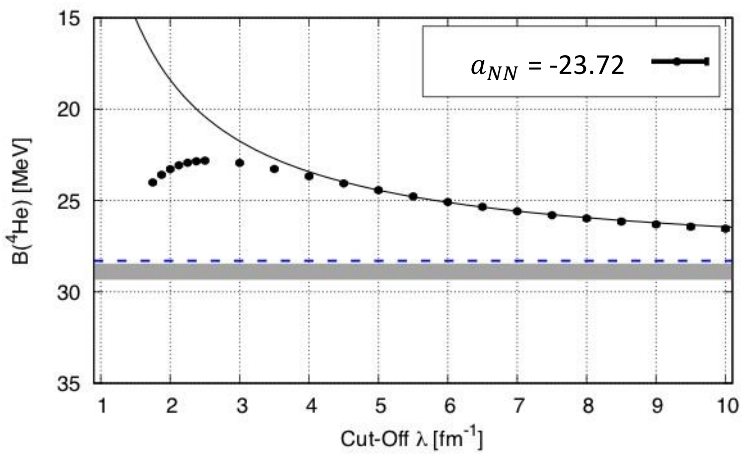
ΛΛ Scattering data

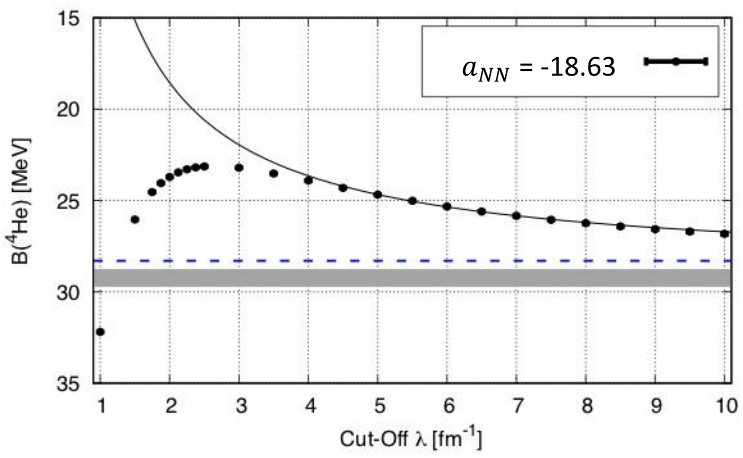


PHYSICAL REVIEW C 91, 024916 (2015)

$$a_{\Lambda\Lambda} \sim \{-0.5 \ldots -1.9\} \text{ fm}$$

Results: $^4\mathrm{He}$

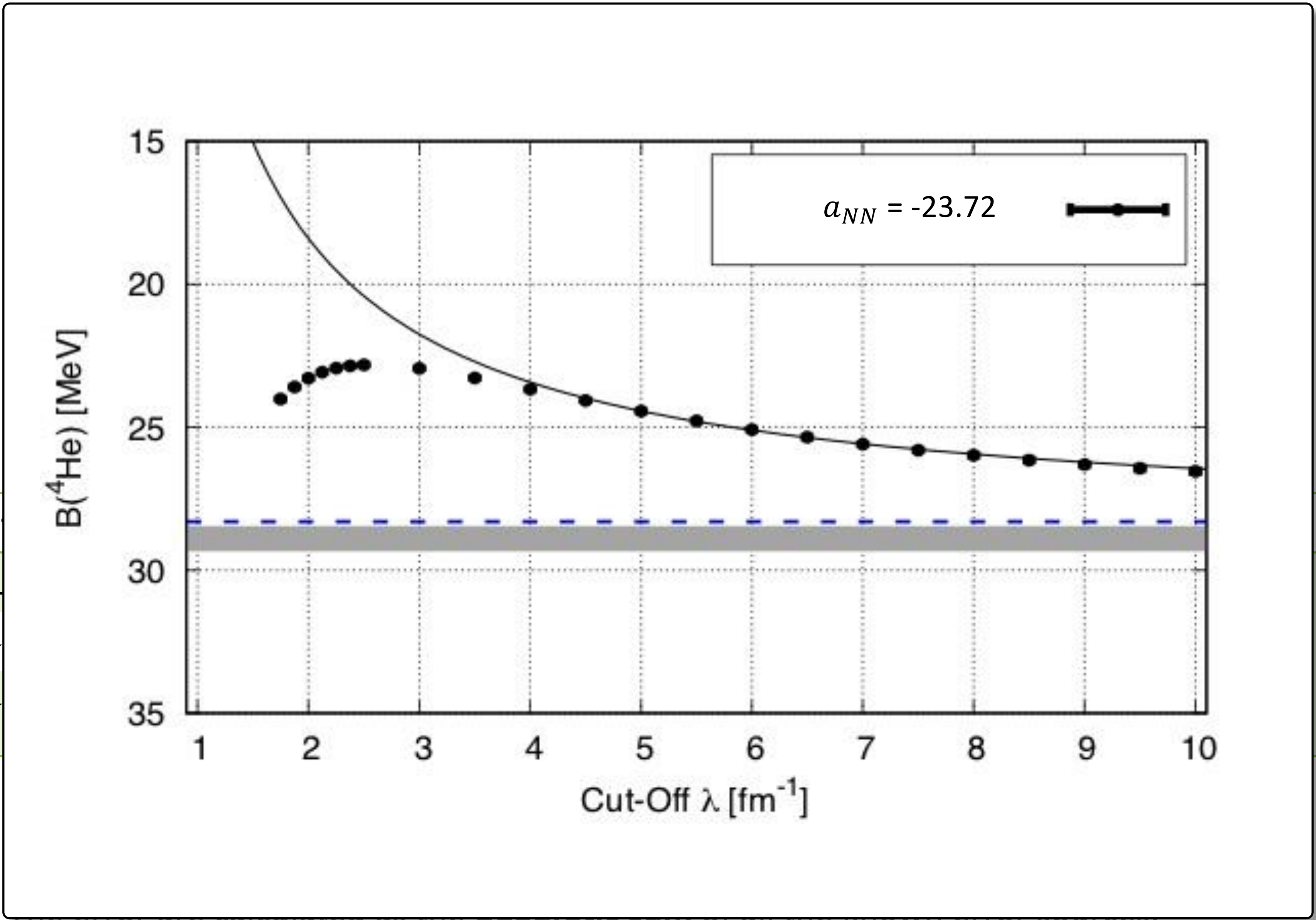




Cut-off λ	**2**	**4**	**6**	**8**	**∞**	**Exp.**
P. L. B 772, 839 (2017).	-23.17(2)	-24.63(3)	-24.06(2)	-26.04(5)	-30 (2)	
$a_{^1S_0}$ = -23.72 fm	-23.28	-23.67	-25.08(1)	-25.99(1)	-28.5 (2)	-28.296
$a_{^1S_0}$ = -18.63 fm	-23.71	-26.50	-25.33(1)	-26.25(5)	-28.8 (3)	

- Cut-off in $[\mathrm{fm}^{-1}]$.
- Energies in [MeV].
- The error are calculated as the **quadratic sum** of all the known error sources.
- In P. L. B 772, 839 (2017) the LECs are calculated using the $a_0^{NN}(\,^3S_1)$ instead of B(d).

Results: $^4\mathrm{He}$



Cu

P. L

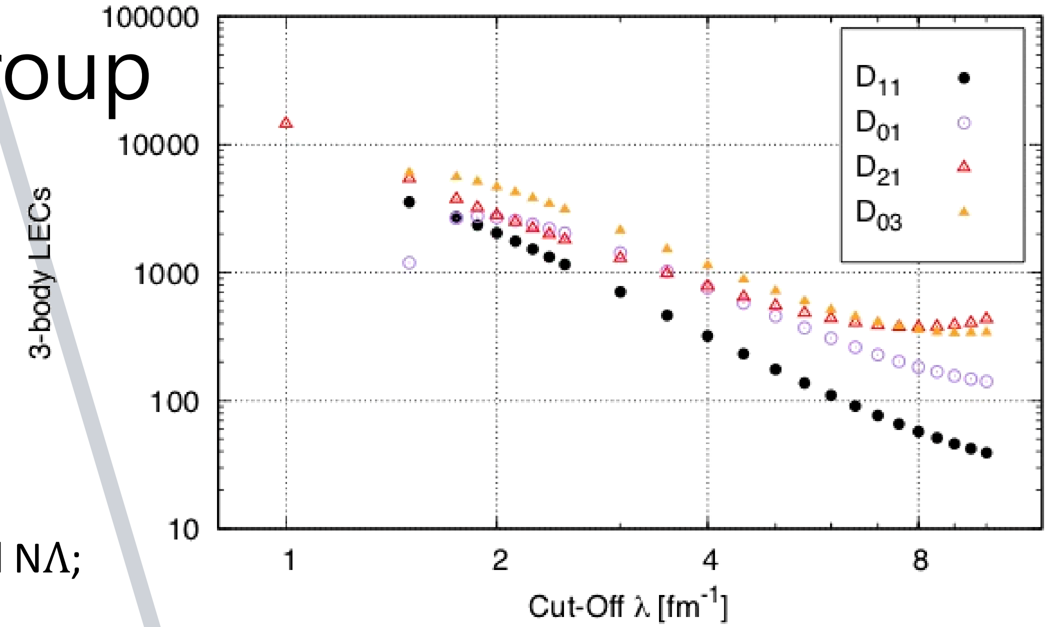
a_N

a_N

- In P. L. B 772, 839 (2017) the LECs are calculated using the $a_0^{NN}(\,^3S_1)$ instead of B(d).

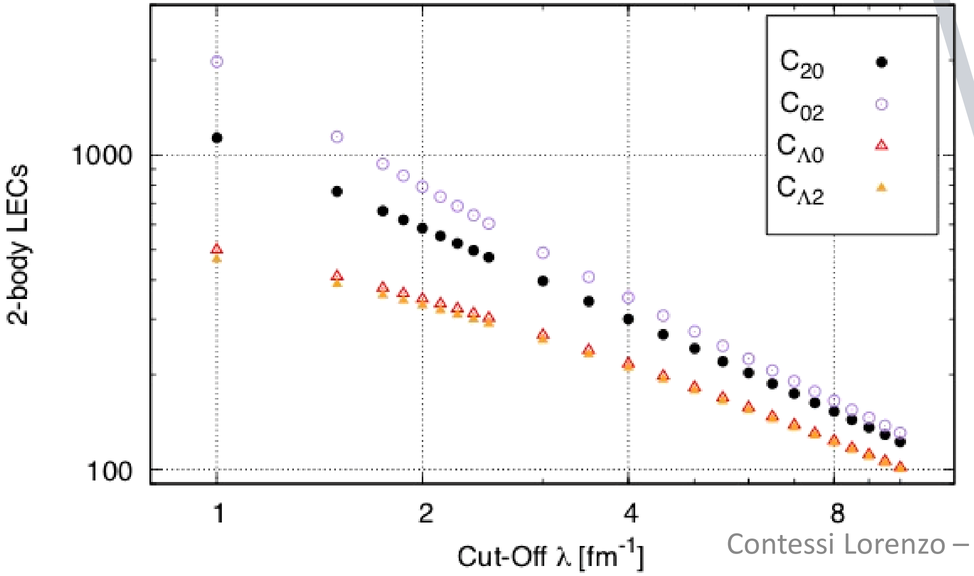
Low Energy Constants Renormalization Group

$$C(\lambda)\,\delta_\lambda = C(\lambda)\frac{\lambda^3}{8\pi^{3/2}}e^{\frac{-\lambda^2 r_{ij}^2}{4}}$$



2B LECs:

- **SU(4) symmetry** for NN and NΛ;
- $\boldsymbol{\Lambda N}$ **softer than** $\boldsymbol{NN}$;
- Smooth.



3B LECs:

- D_{21}-D_{03} behave differently than D_{11}-D_{01};
- For $\Lambda < 2$ fm $^{-1}$ few 3b-LECs have a flex;
- Attractive three body for $\lambda = 1$.

LO Interaction

$\alpha \rightarrow$ N – N states
$\beta \rightarrow$ N – Λ states
$\gamma \rightarrow$ N – Λ –N states

$\overrightarrow{r_{ij}} = \overrightarrow{r_i}$ - $\overrightarrow{r_j}$

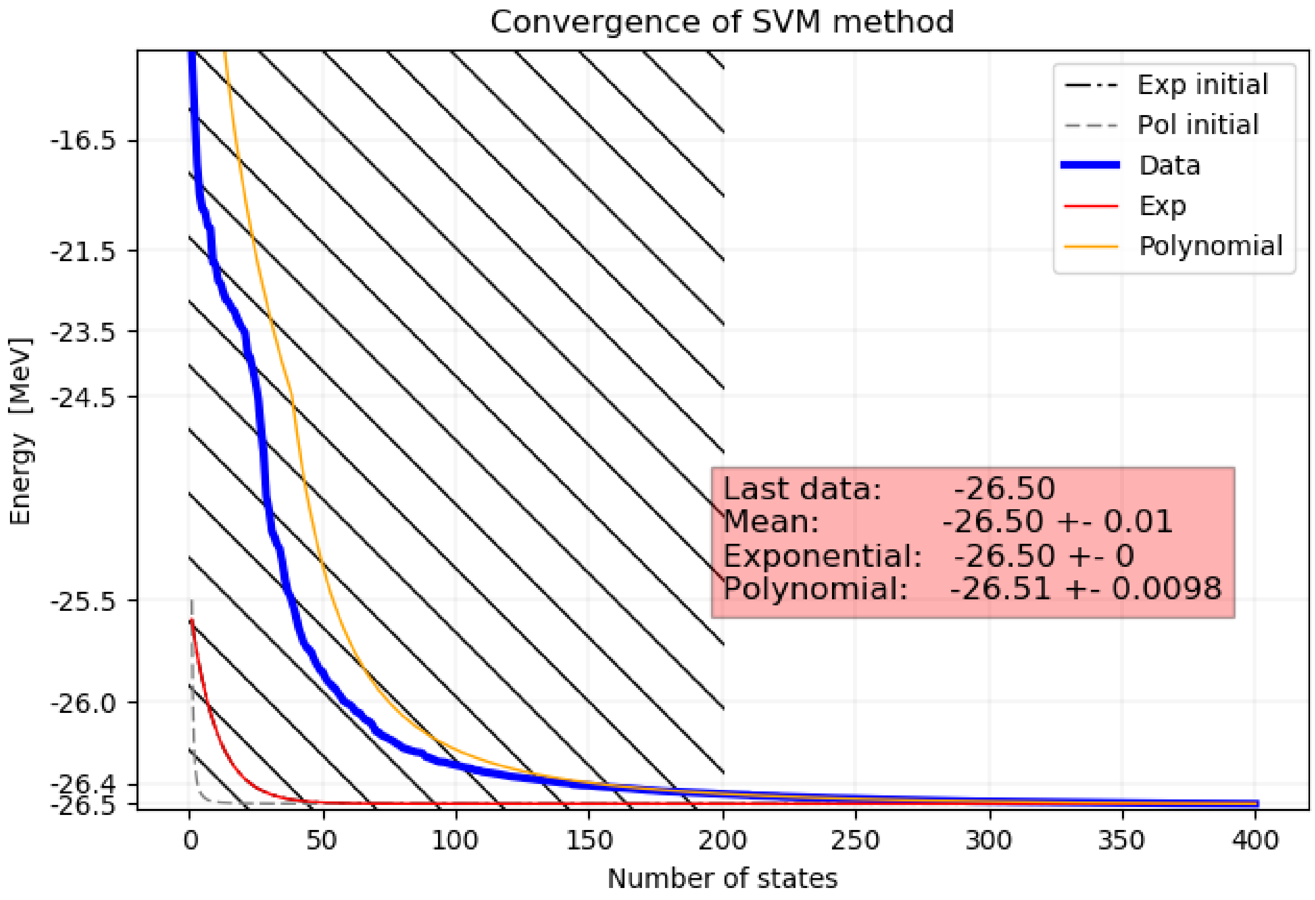
$$V^{LO} =$$

$$= \sum_{i<j} \sum_{\alpha} \left[C_\alpha(\lambda)\; P_\alpha \; e^{\frac{-\lambda^2 r_{ij}^2}{4}} \right] + \sum_{i} \sum_{\beta} \left[C_\beta(\lambda) P_\beta \; e^{\frac{-\lambda^2 r_{i\Lambda}^2}{4}} \right]$$

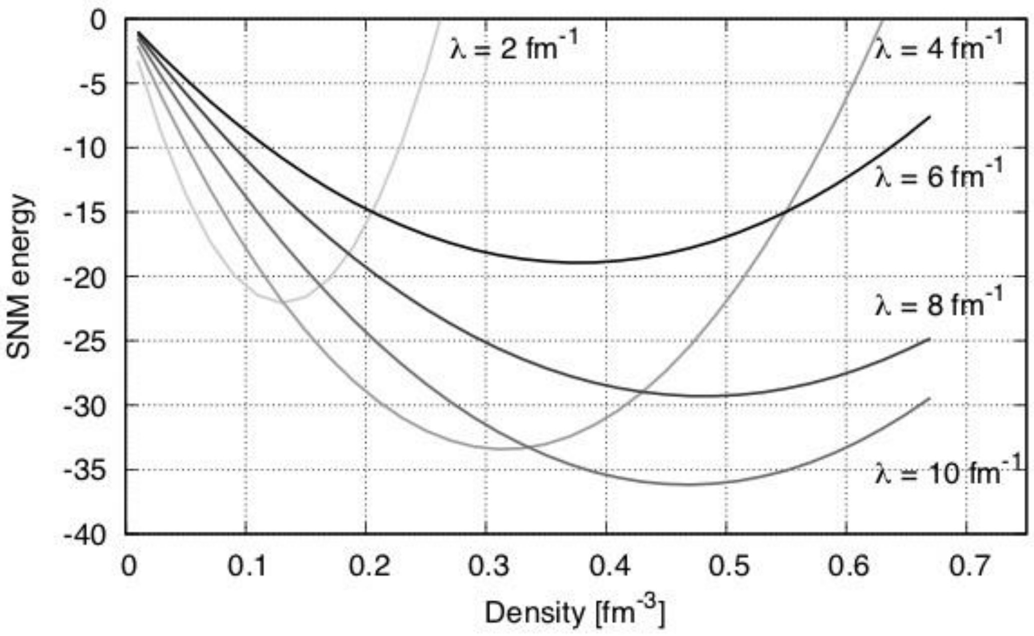
$$+ \sum_{(i<j)\neq k} \sum_{\beta} D_0(\lambda) \sum_{cyc} \left[e^{\frac{-\lambda^2 \left(r_{ij}^2 + r_{jk}^2 \right)}{4}} \right]$$

$$+ \sum_{i<j} \sum_{\gamma} D_\gamma(\lambda) P_\gamma \sum_{cyc} \left[e^{\frac{-\lambda^2 \left(r_{ij}^2 + r_{j\Lambda}^2 \right)}{4}} \right]$$

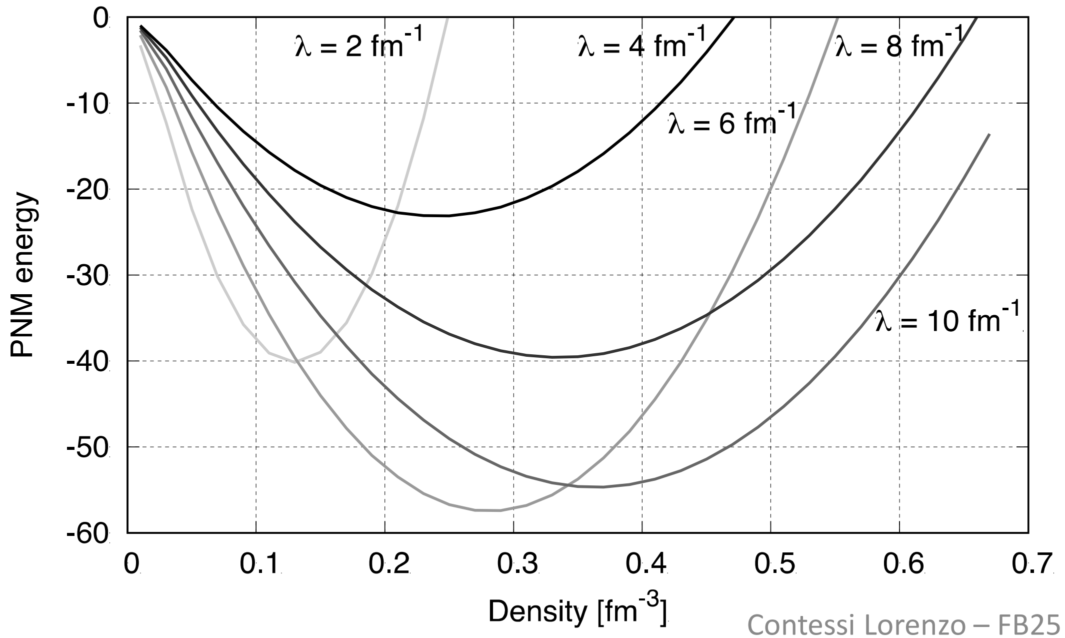
SVM convergence



SNM and PNM

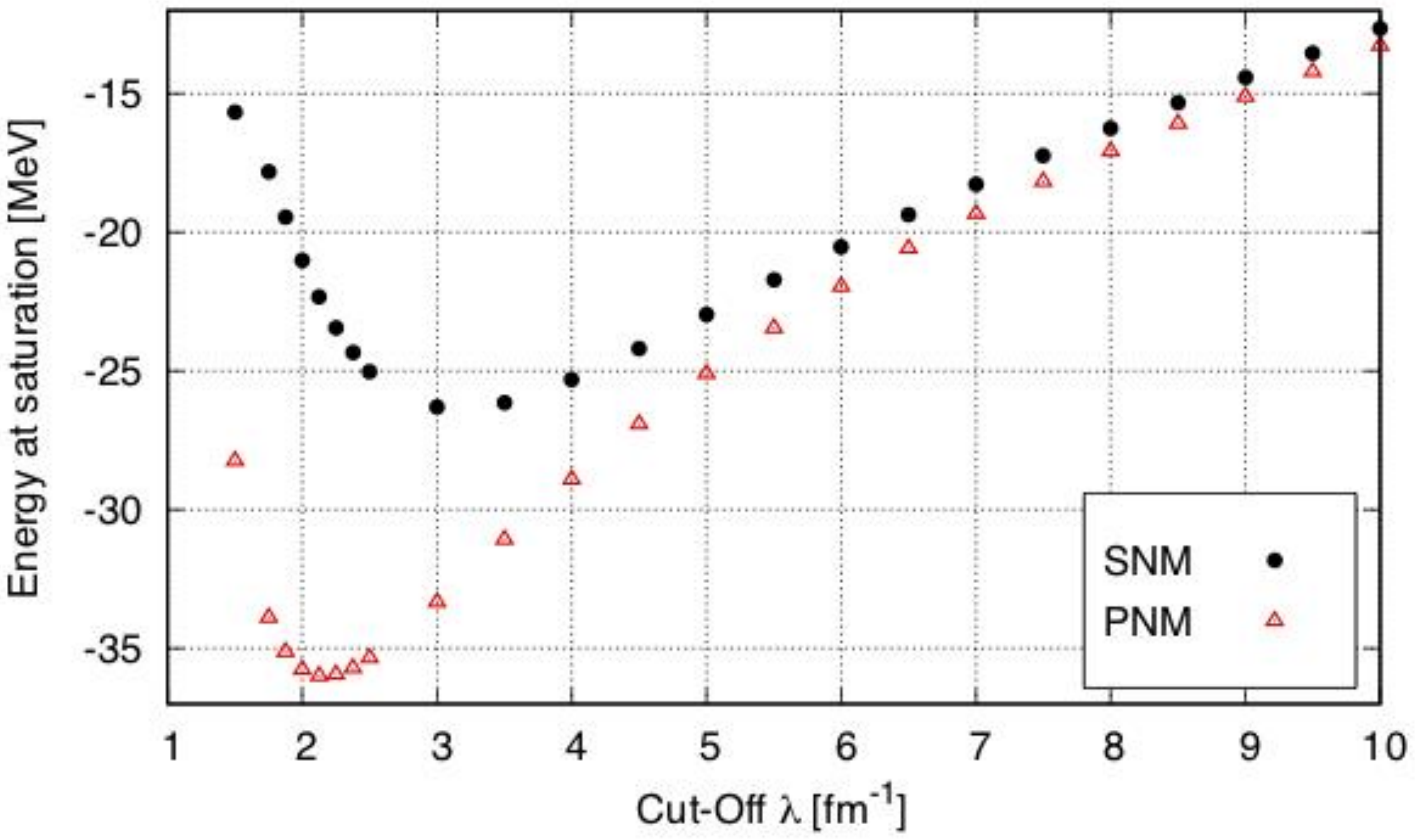


λ = 2 fm^-1
λ = 4 fm^-1
λ = 6 fm^-1
λ = 8 fm^-1
λ = 10 fm^-1
SNM energy
Density [fm^-3]
0
-5
-10
-15
-20
-25
-30
-35
-40
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7



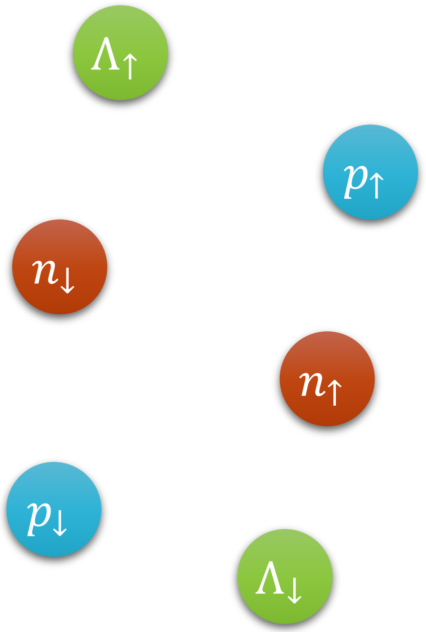
λ = 2 fm^-1
λ = 4 fm^-1
λ = 8 fm^-1
λ = 6 fm^-1
λ = 10 fm^-1
PNM energy
Density [fm^-3]
0
-10
-20
-30
-40
-50
-60
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

SNM and PNM energy at saturation

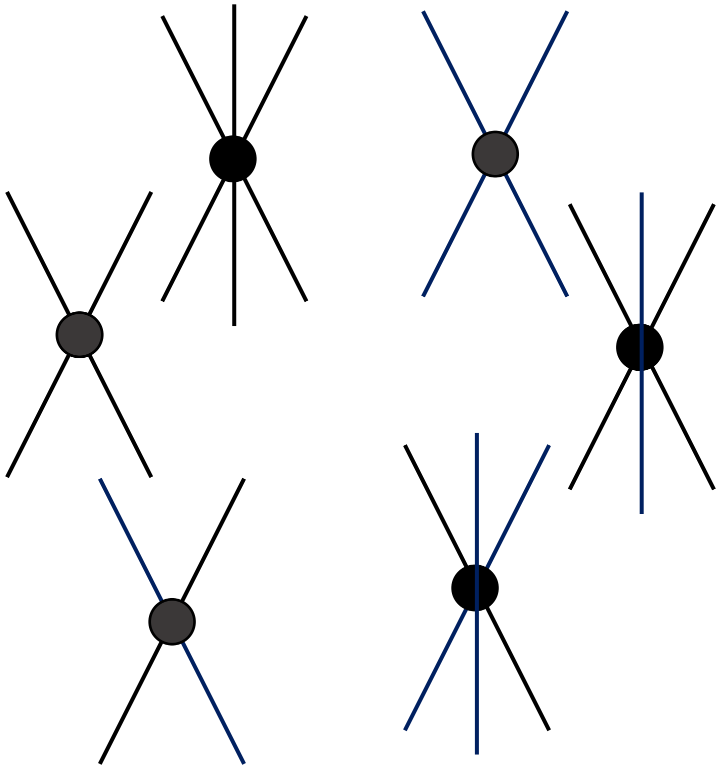


The simplest theory to be defined

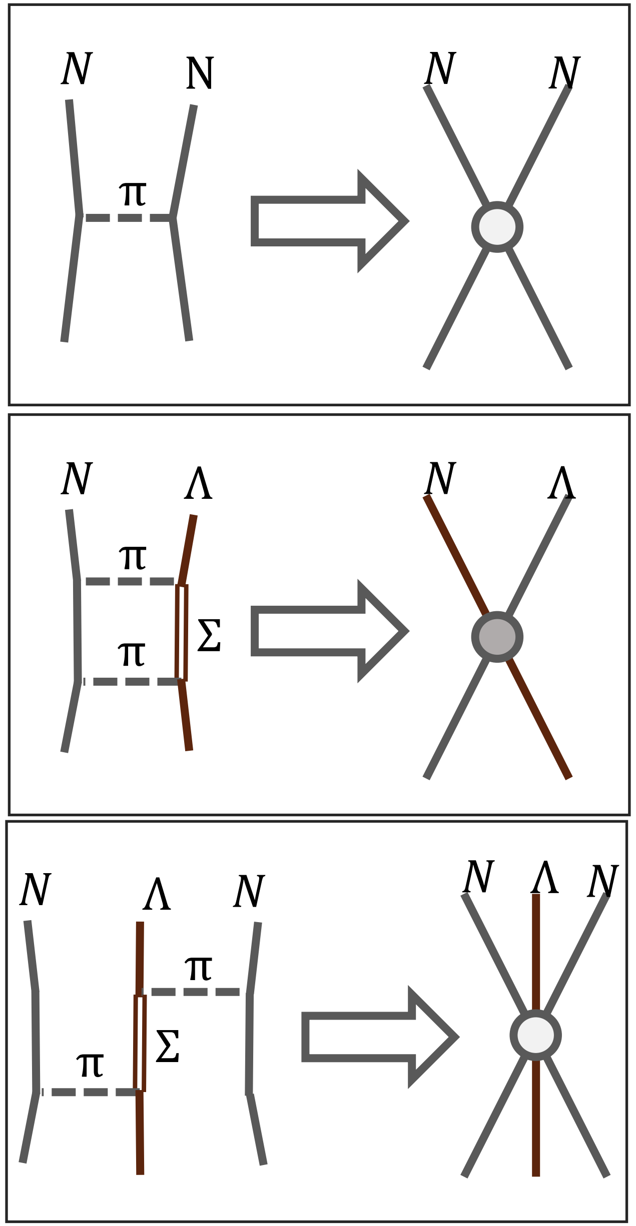
6–kinds
of fermions



How many operators?



3 two-body and
4 three-body



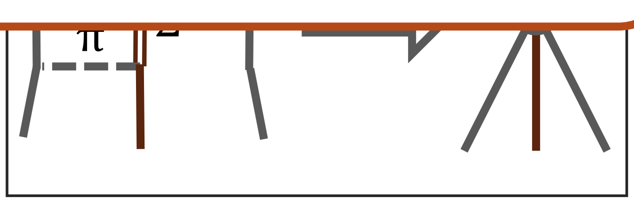
The simplest theory to be defined

How many operators?

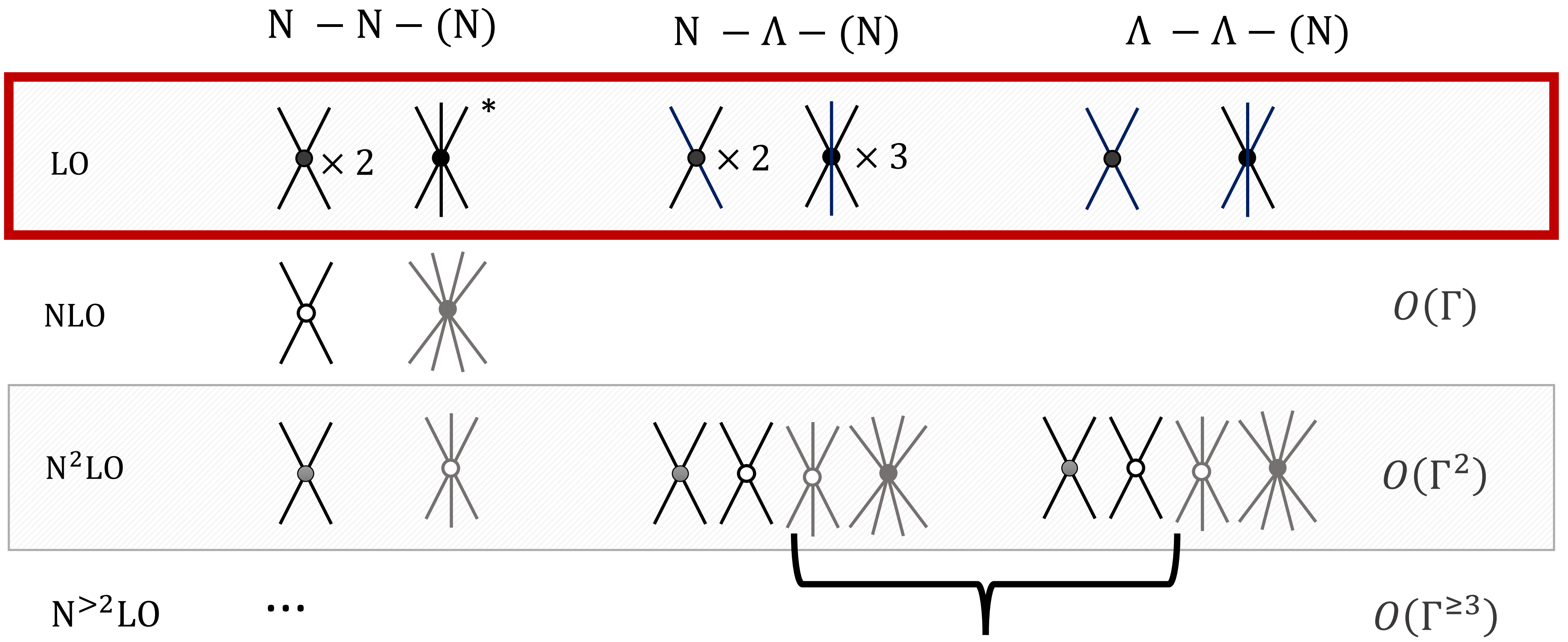
Looking for the **simplest theory** for Lambda hyperons

- Easier to be interpreted
- Can be fixed by small amount of experimental data
- Model independence? (Only if I can do an error estimation)

5 three-body



Pionless powercounting



$$\Gamma_{NN} = \frac{Q}{m_\pi} = 0.5 \sim 0.8$$

$$\Gamma_{N\Lambda} = \frac{Q}{2\ m_\pi^{**}} \sim 0.2/0.3$$

B. Bazak, Four-Body Scale in Universal Few-Boson Systems, PRL 122.143001 (2019)
G.P. Lepage, How to renormalize the Schrodinger equation (1997)
van Kolck, U. Nucl.Phys. A645 (1999) 273-302
Chen, Jiunn-Wei et al. Nucl.Phys. A653 (1999)
S. K˙onig, H. W. Grießhammer, H. W. Hammer, and U. van Kolck J. Phys. G43, 055106 (2016)

* Three body force is necessary to avoid Thomas collapse
** OPE not allowed