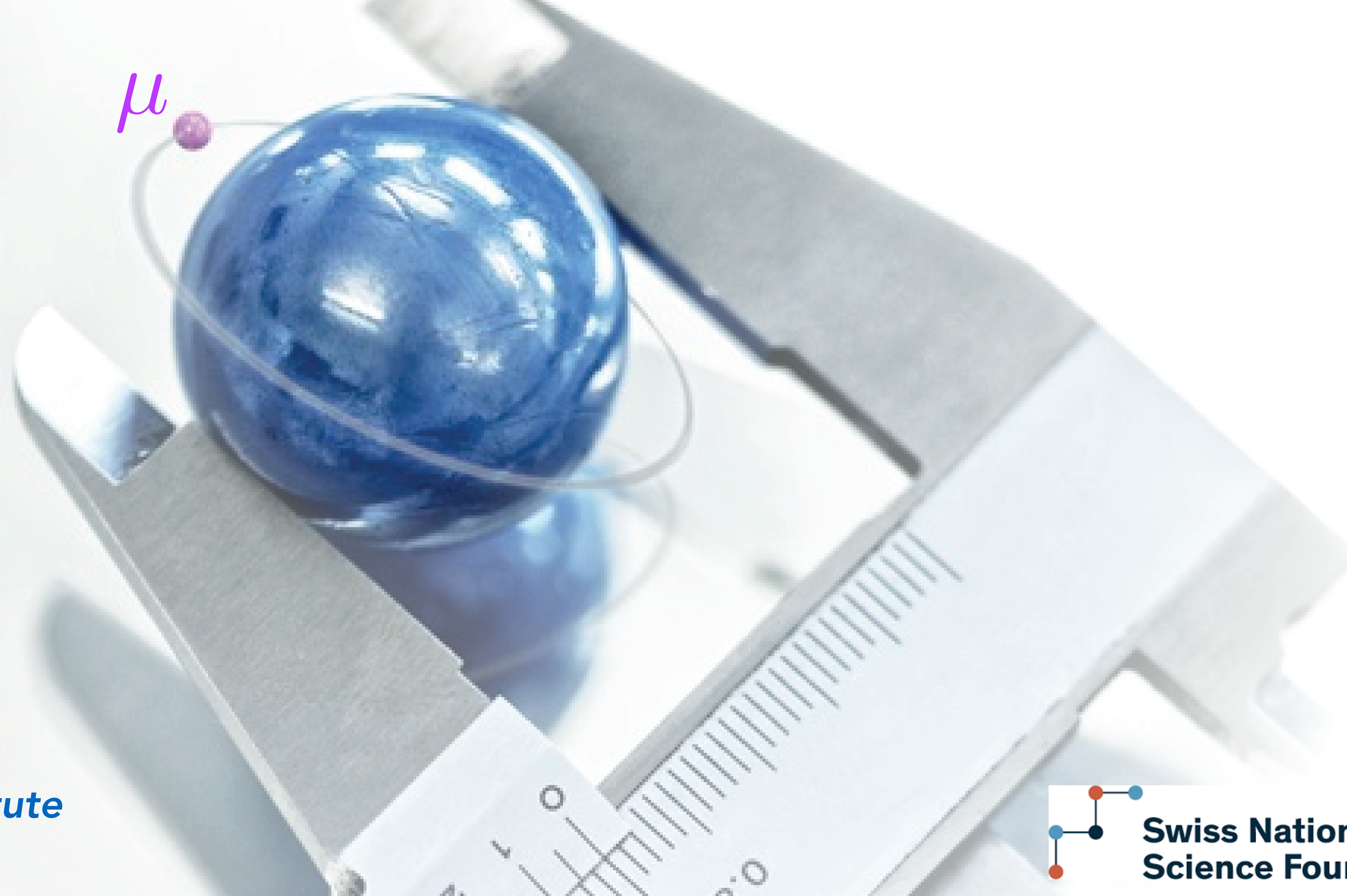
Benchmarks for nuclear and atomic physics from laser spectroscopy of muonic atoms

CREMA collaboration

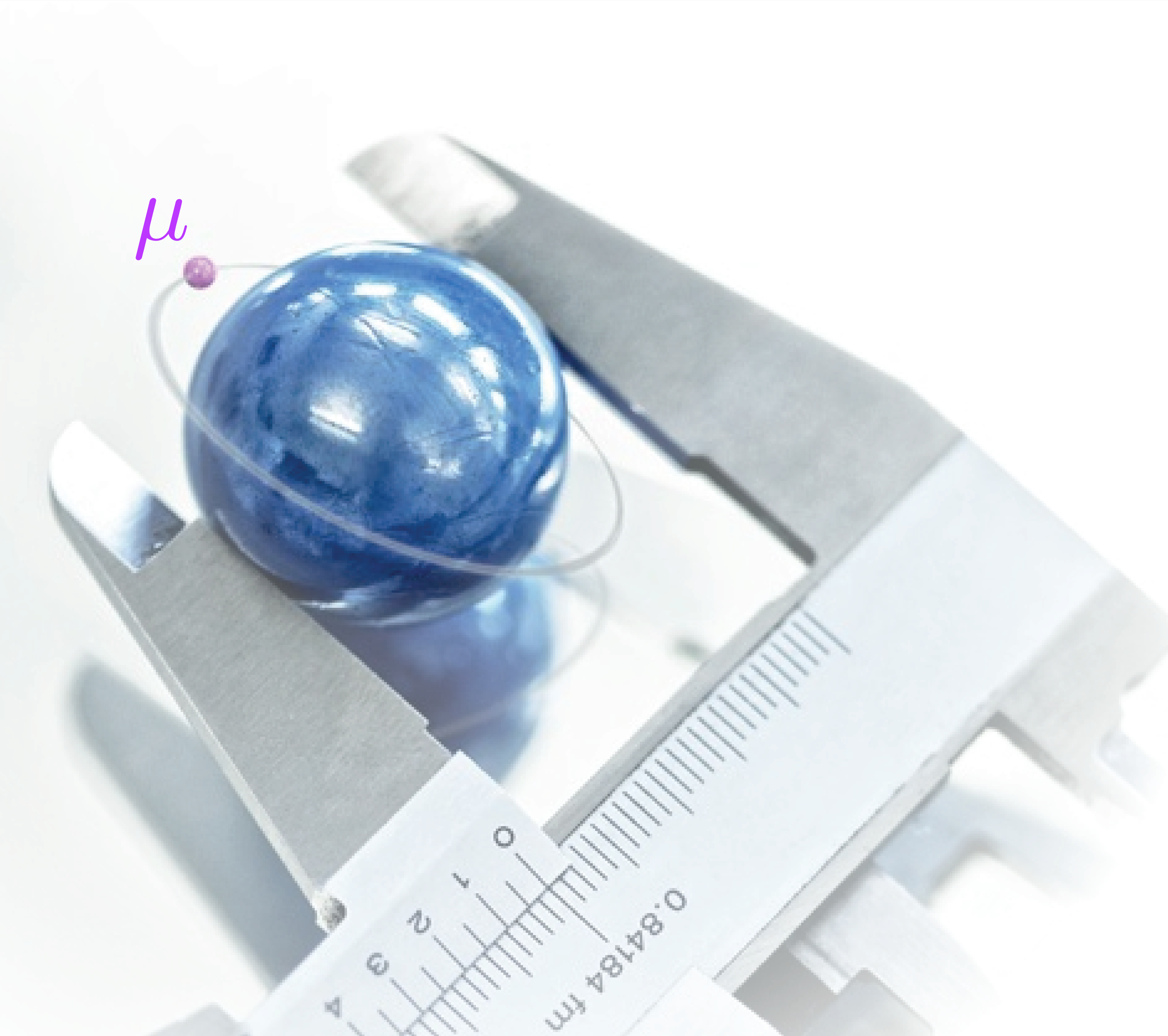


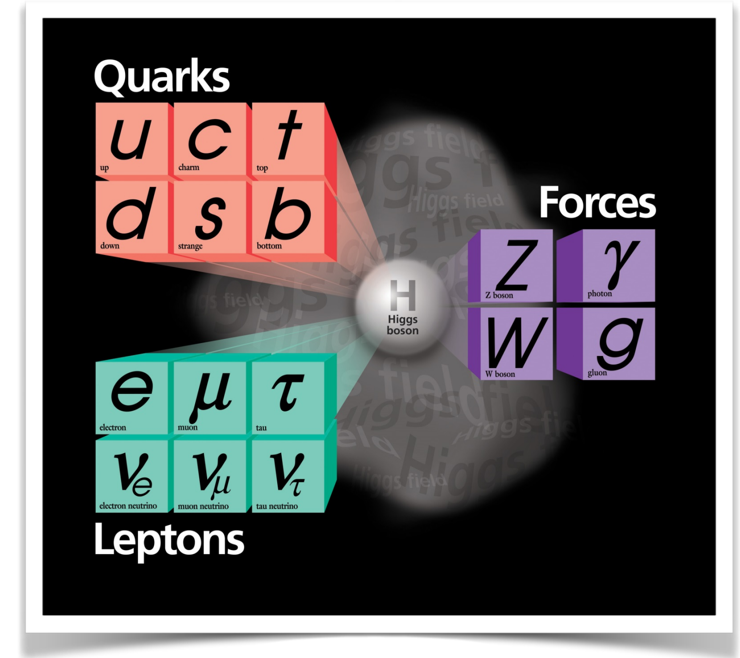
Aldo Antognini

Paul Scherrer Institute
ETH, Zurich

Swiss National Science Foundation

Laser spectroscopy of muonic atoms



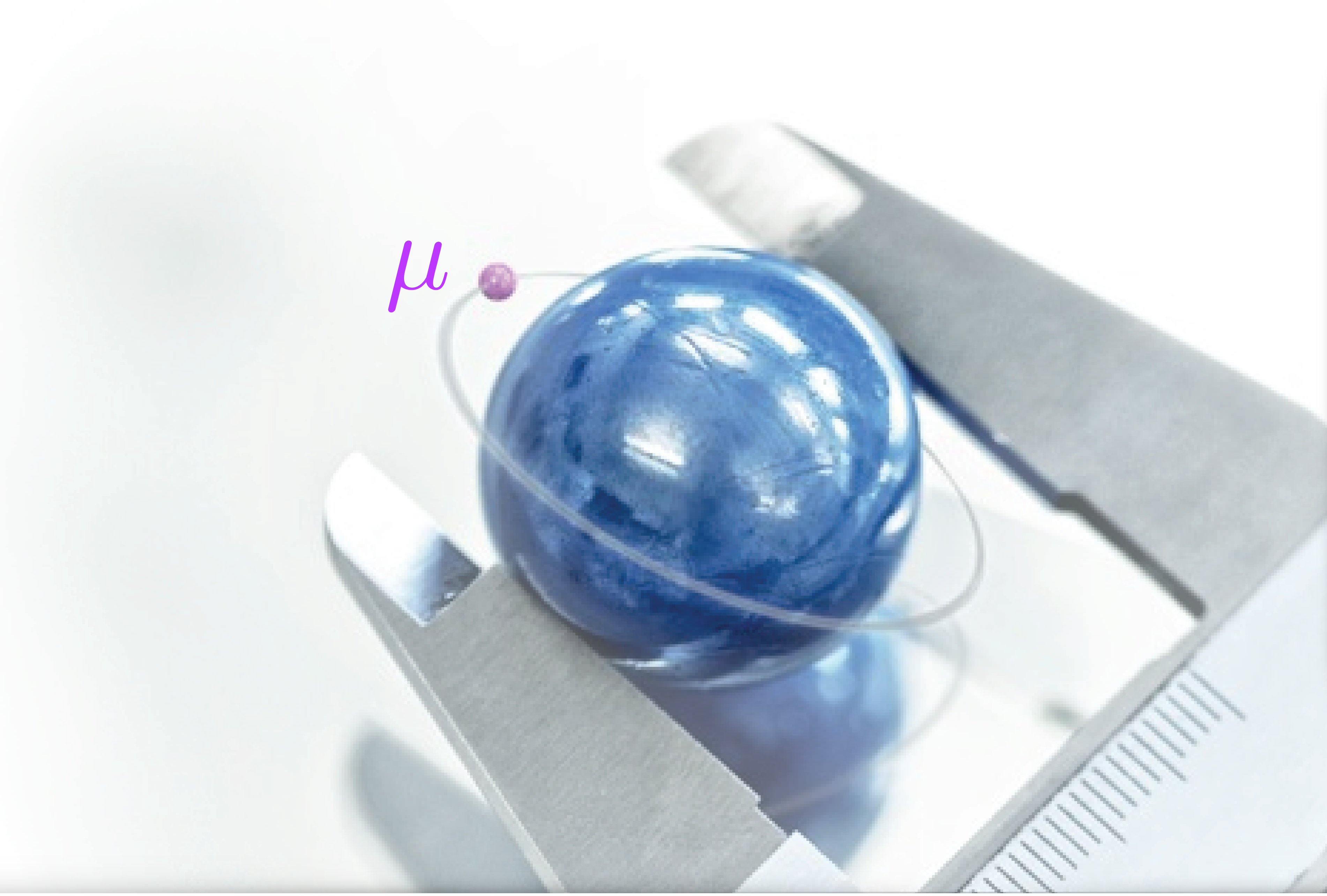


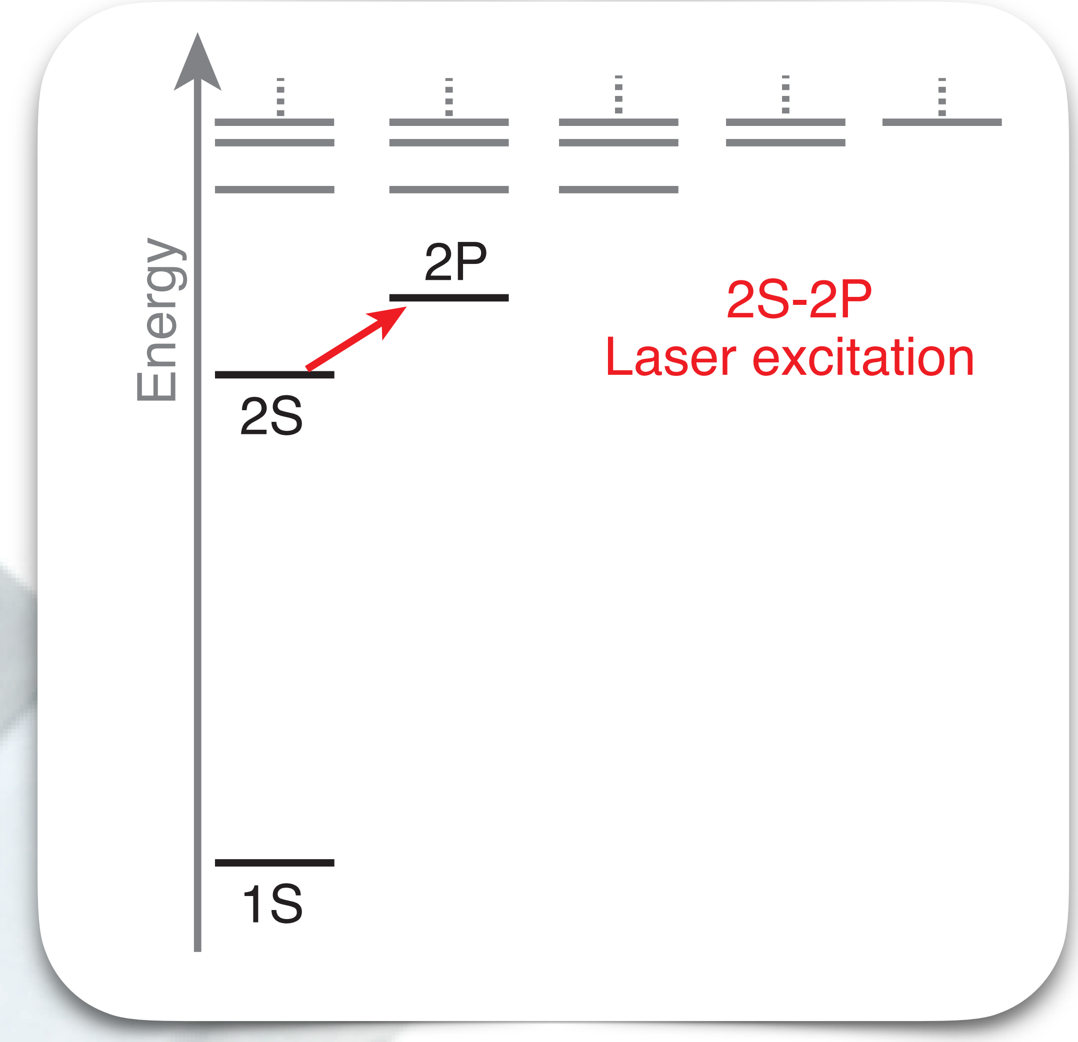
$$m_\mu \approx 200 m_e$$

Finite size effects

$$\Delta E_{\mathrm{FNS}} = \frac{2}{3n^3} Z^4 \alpha^4 m_r^3 r^2$$

Laser spectroscopy of light muonic atoms





We measured 10 2S-2P transitions in μp, μd, μ^3He^+, μ^4He^+

+

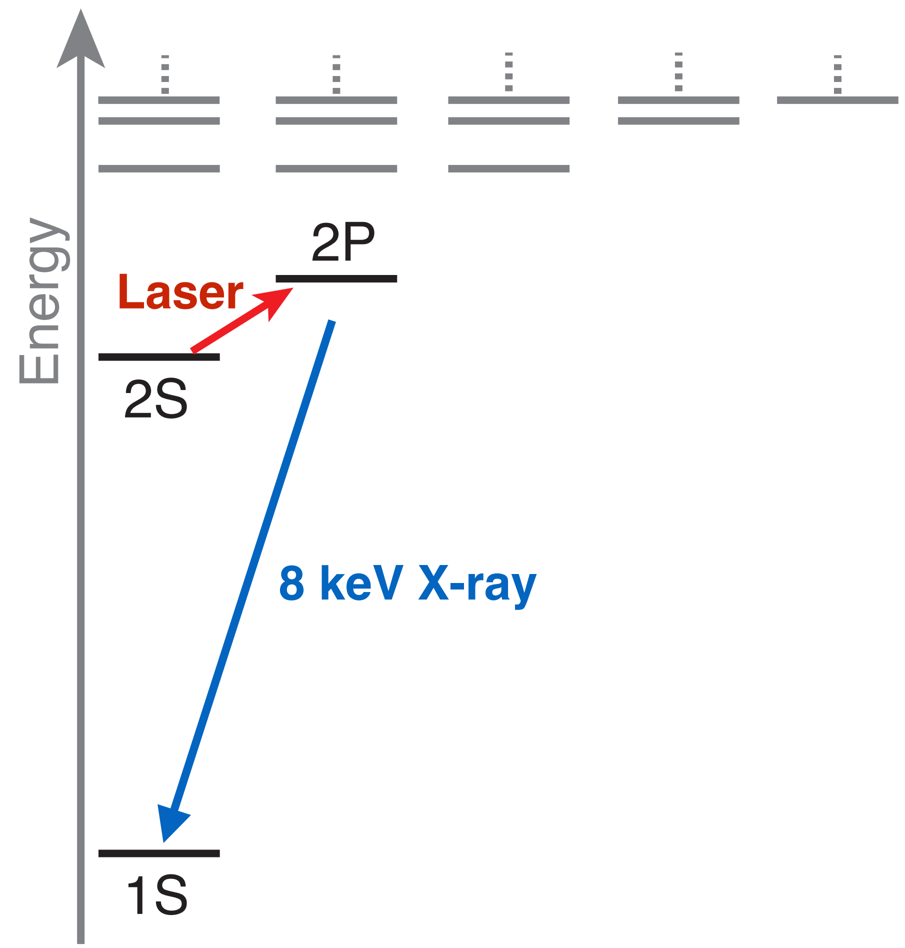
Theoretical predictions: QED + Nuclear structure

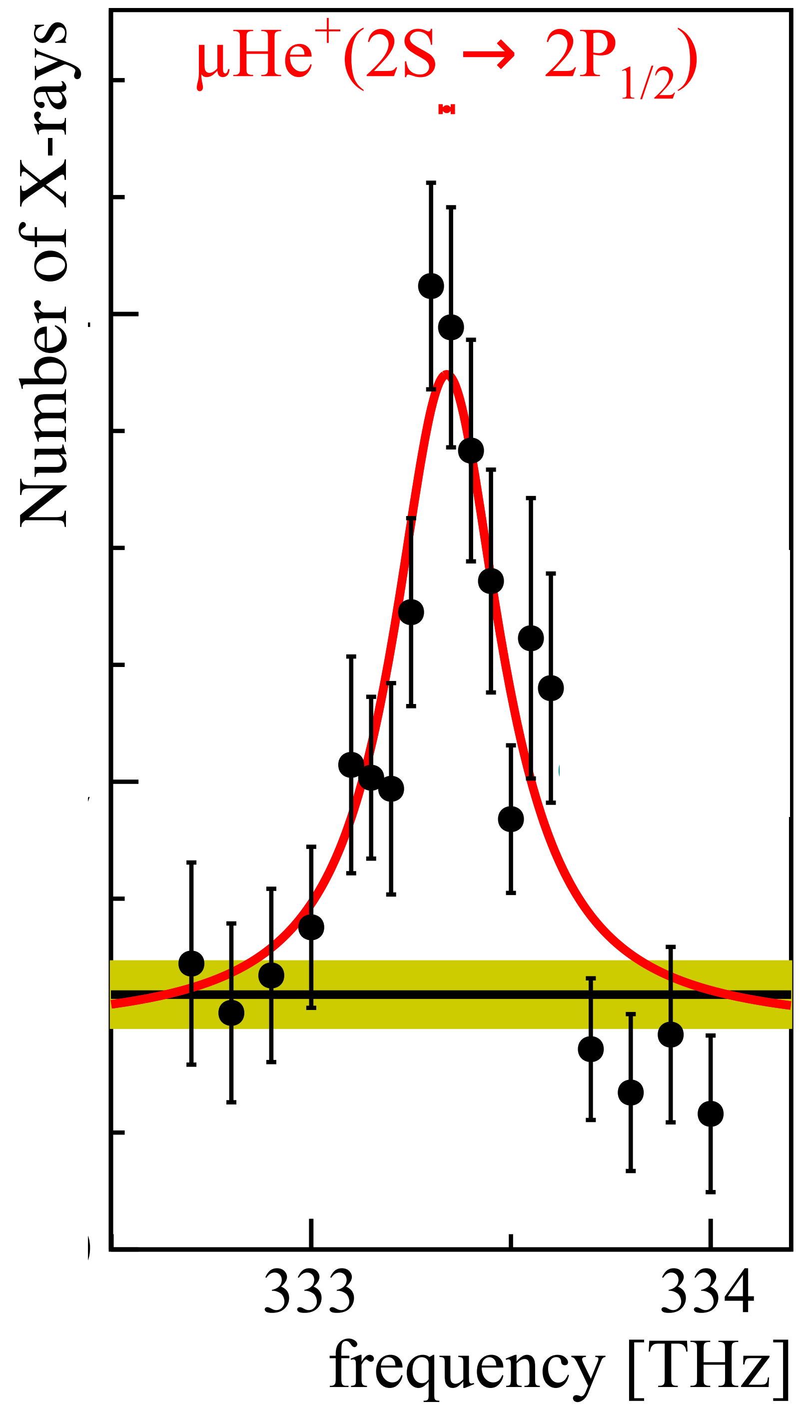
⇨

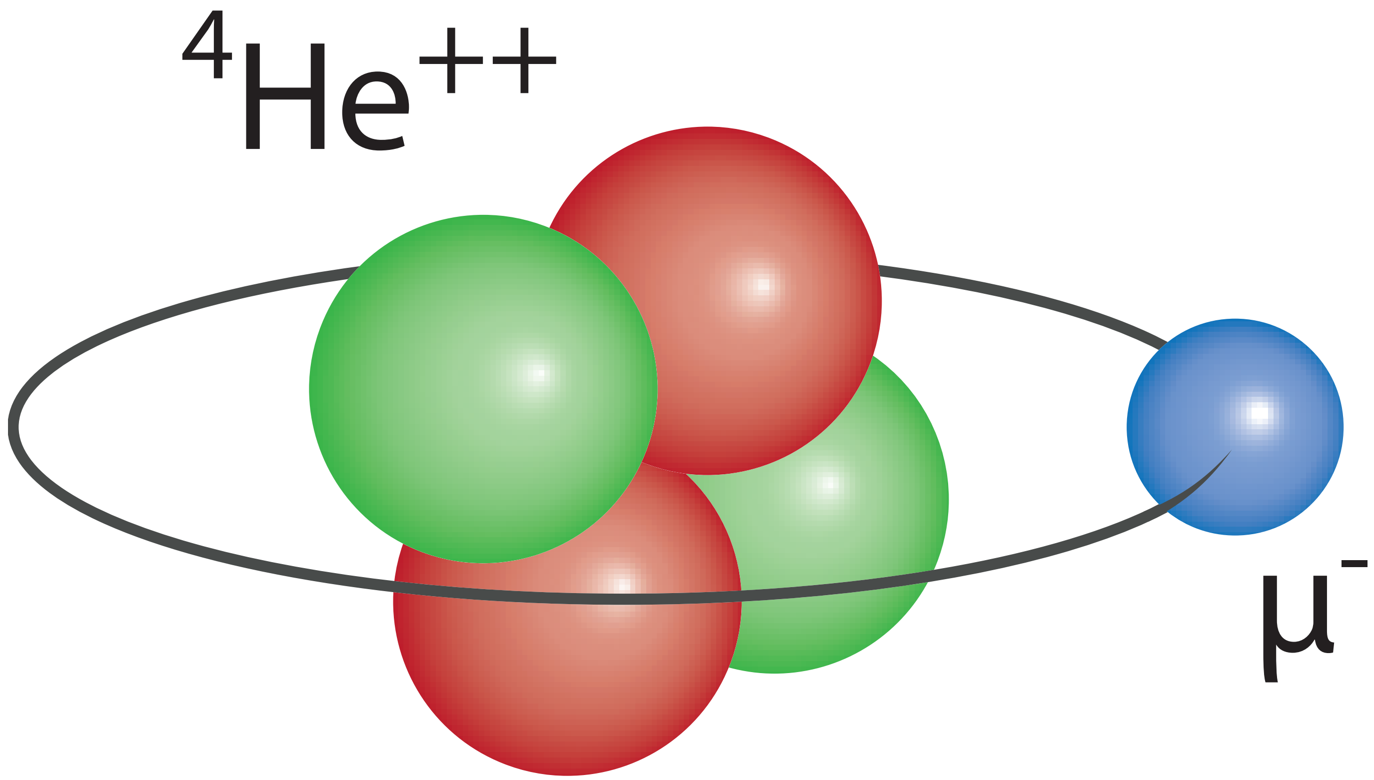
p, d, ^3He, ^4He charge radii

The μHe⁺ 2S-2P experiment

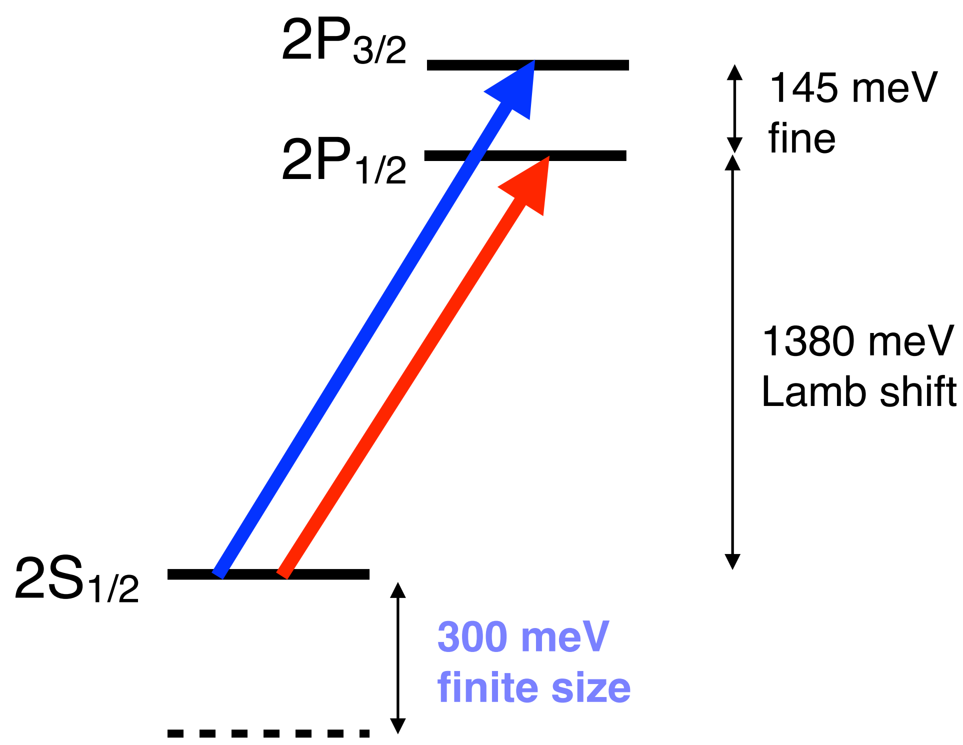
- Stop low-energy muons in 2 mbar Helium gas
- μHe^+ are formed (1% in the 2S-state)
- Excite 2S-2P transition with laser
- Detect X-ray from 2P-1S de-excitation
- Plot number of X-rays vs. laser frequency



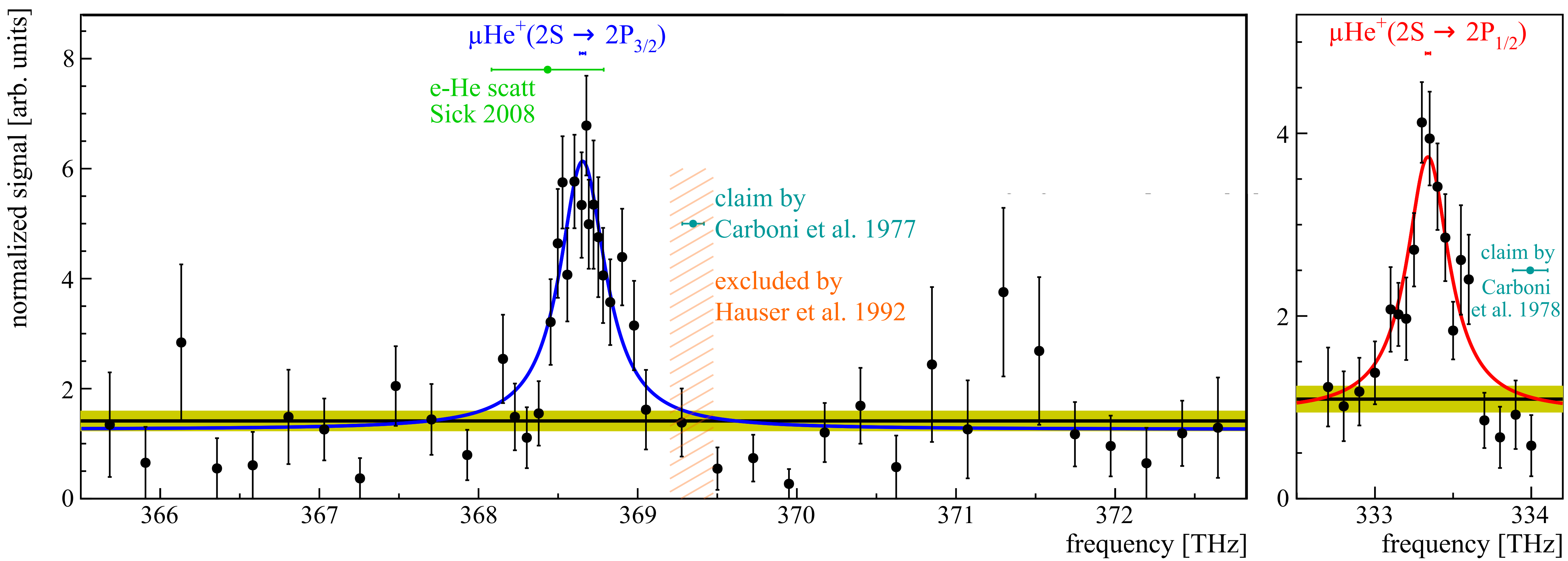



$^4He^{++}$
μ^-

The measured transitions in μ^4He^+



$$\nu^{exp}(2S \rightarrow 2P_{3/2}) = 368{,}653 \pm 18 \text{ GHz}$$
$$\nu^{exp}(2S \rightarrow 2P_{1/2}) = 333{,}339 \pm 15 \text{ GHz}.$$

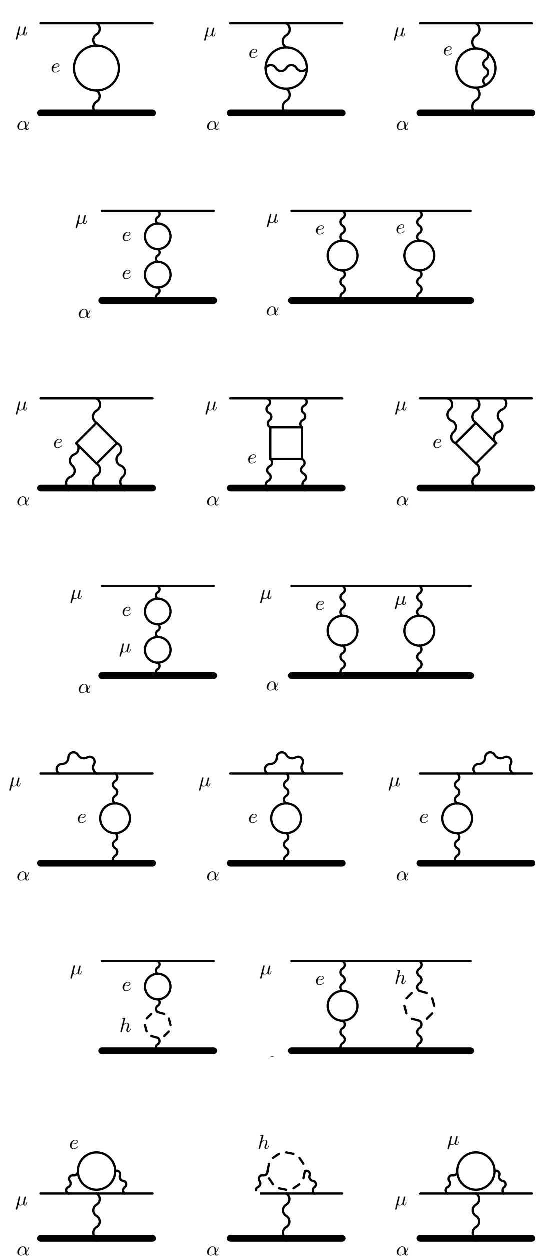


Numerous laser-off events used to determine the background

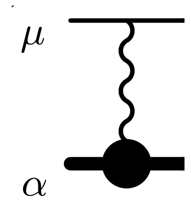
Theoretical prediction of the Lamb shift

$$E_{LS}^{th} = 1668.491(7) - 106.209\, r_\alpha^2 + 9.276(433)\ \text{meV}$$

QED



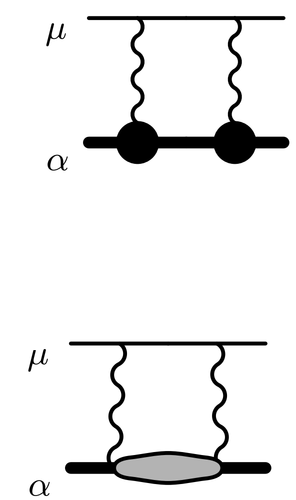
Finite size



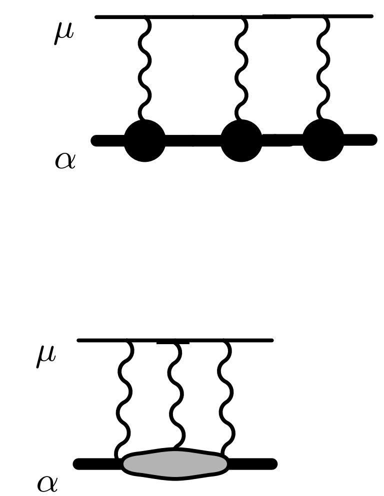
+ radiative-finite-size corrections

Can be calculated using form factor

2PE



3PE



- Elastic: can be calculated using form factor
- Inelastic: requires few-body theory

Chen Ji et al., J.Phys.G 45 (2018) 9, 093002
Nevo Dinur et al., PRC 99, 034004 (2019)
Li Muli et al., J.Phys.G 49 (2022) 10, 105101
Pachucki et al., PRA 97, 062511 (2018)
Pachucki et al., arXiv:2212.13782

The ^{4}He charge radius

QED | Finite size | nucl. str.

Theory

$$E_{\mathrm{LS}}^{\mathrm{th}} = 1668.491(7) - 106.209\, r_\alpha^2 + 9.276(433)\ \mathrm{meV}$$

Pachucki et al., arXiv:2212.13782

Experiment

$$E_{\mathrm{LS}}^{\mathrm{exp}} = 1378.521(48)\ \mathrm{meV}$$

Nature 589 (2021) 7843, 527-531

$$r_\alpha = 1.67824\,(13)_{\mathrm{exp}}(82)_{\mathrm{th}}\ \mathrm{fm}$$

Using 2021 theory

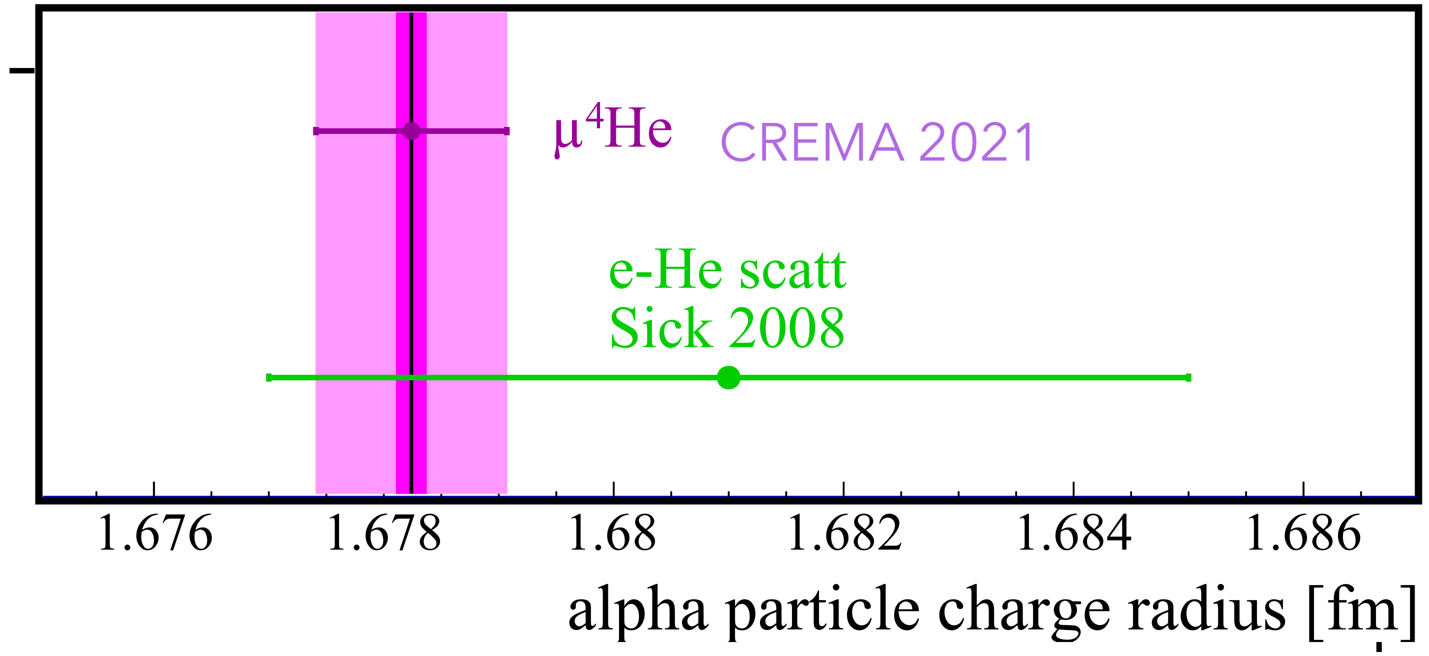
$$\boxed{r_\alpha = 1.6786\,(12)\ \mathrm{fm}}$$

Using 2023 theory

Uncertainty 2PE increased by 45%. No elastic/ inelastic separation. More consistent treatment.

Bacca, Li Muli, Acharya, Ji, Hernandez, Barnea, ...

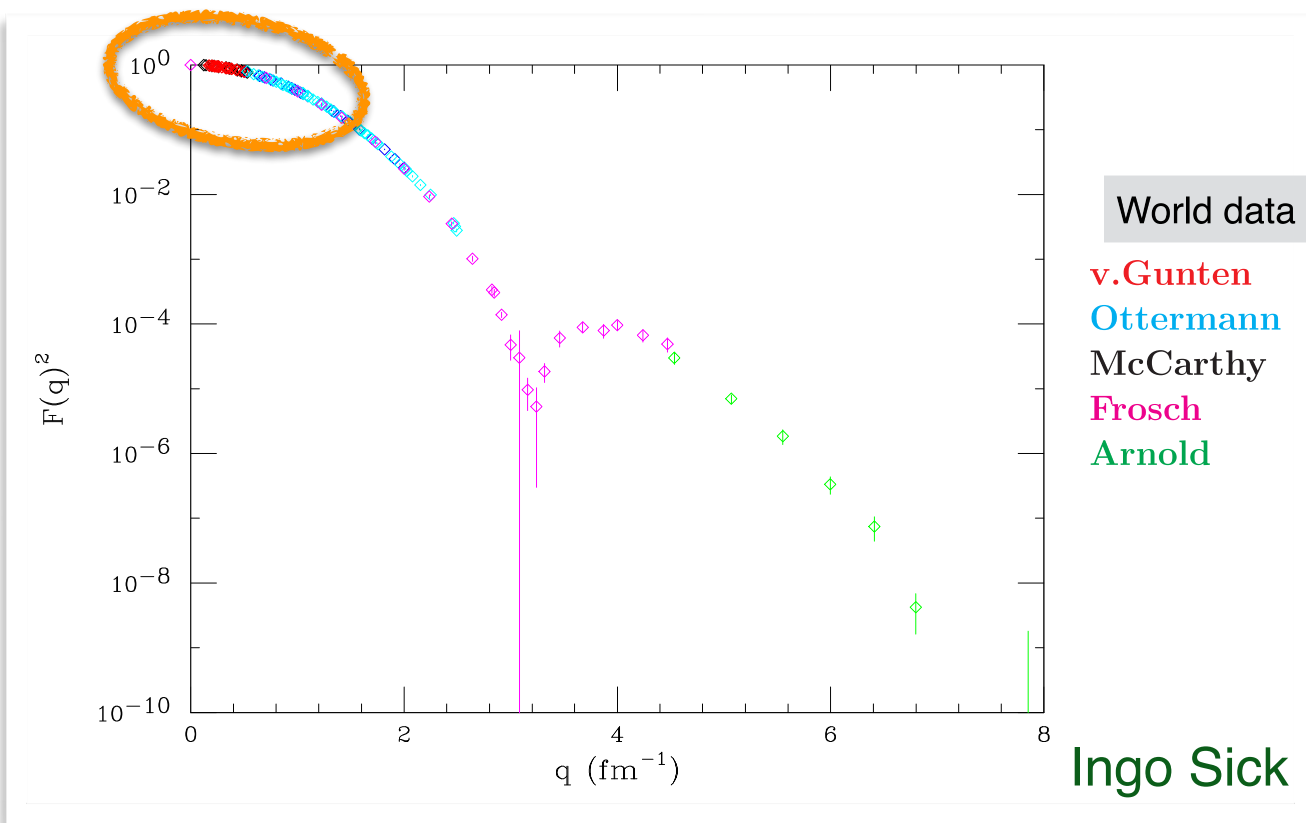
Perfect agreement with the electron scattering result



$$r_\alpha = 1.681(4)\text{fm}$$

PRC 77, 041302R (2008)

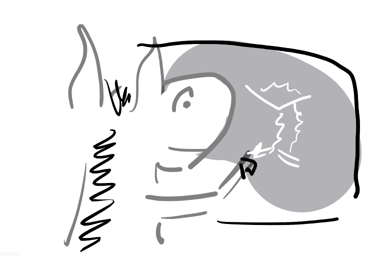
$$r_\alpha^2 = -6 \frac{dF(q^2)}{dq^2}\Big|_{q^2=0}$$



- ☑ Good data
- ☑ Only one form factor
- ☑ He-nucleous has smaller tail compared to proton
- ☑ Large-r behaviour of the charge distribution constrained from theory

- ☐ Proton information is used in this analysis
- ☐ Radiative corrections?

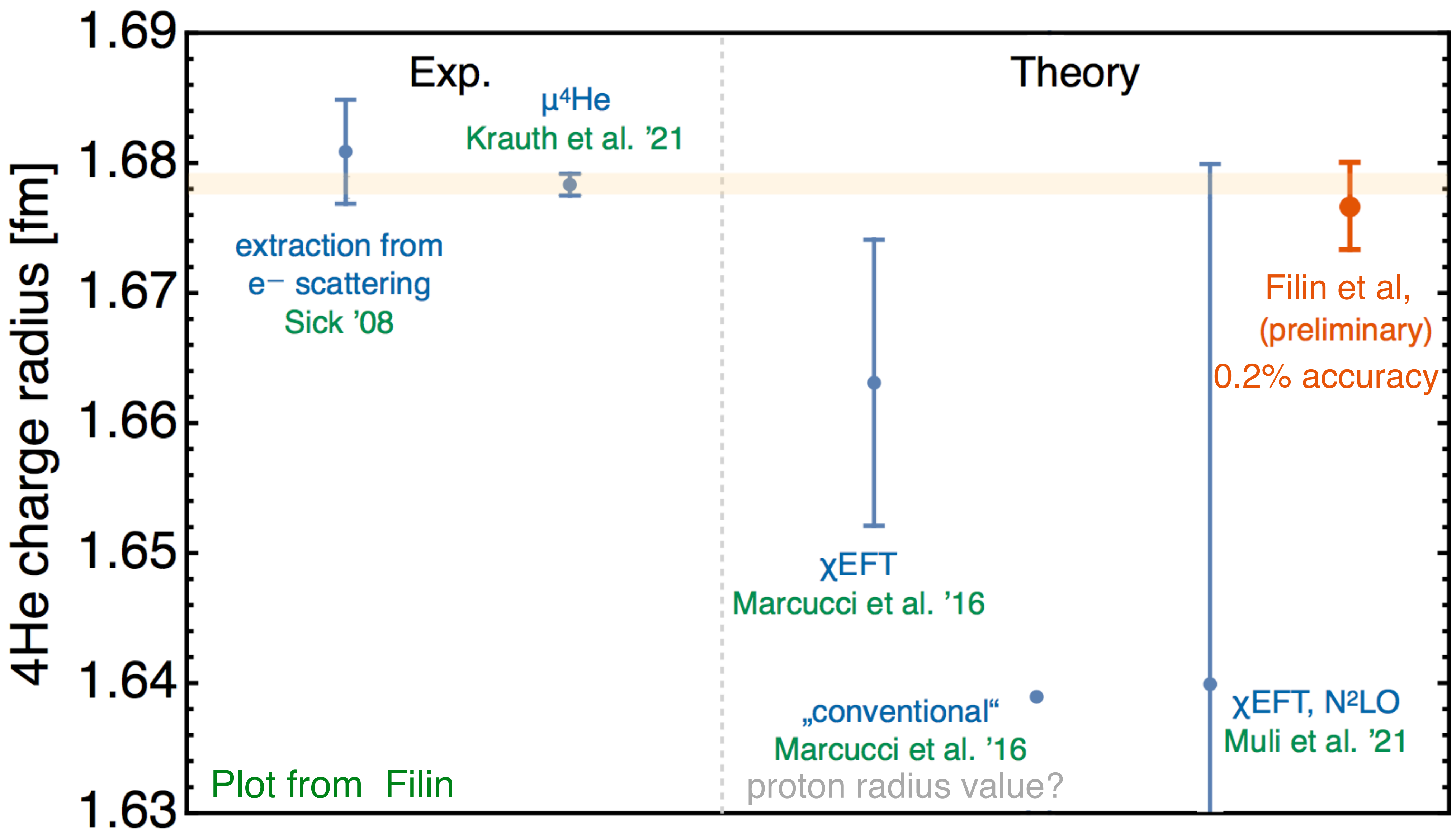
Radiative correction @ NNLO with McMule available
A. Signer et al., https://mule-tools.gitlab.io



Radius as a benchmark for *ab initio* few-nucleon theories

Measured — QED — Finite size — 2PE + 3PE

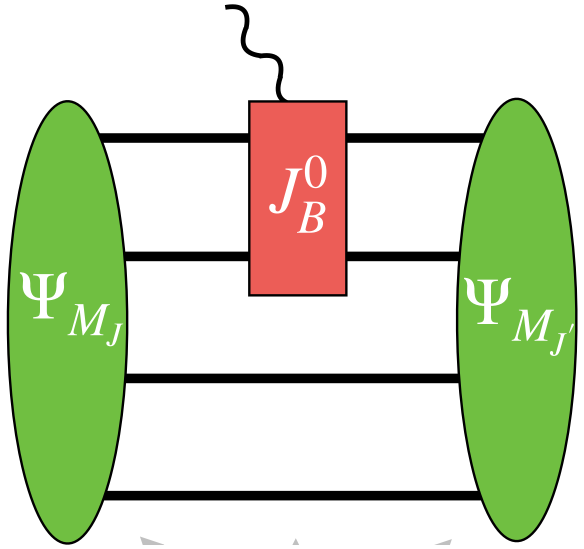
$$1378.521(48) = 1668.491(7) - 106.209\, r_\alpha^2 + 9.276(433)\ \text{meV}$$



Towards a consistent treatment of the nuclear effects: radii and 2PE+3PE

Radius as benchmark for *ab initio* few-nucleon predictions

Filin

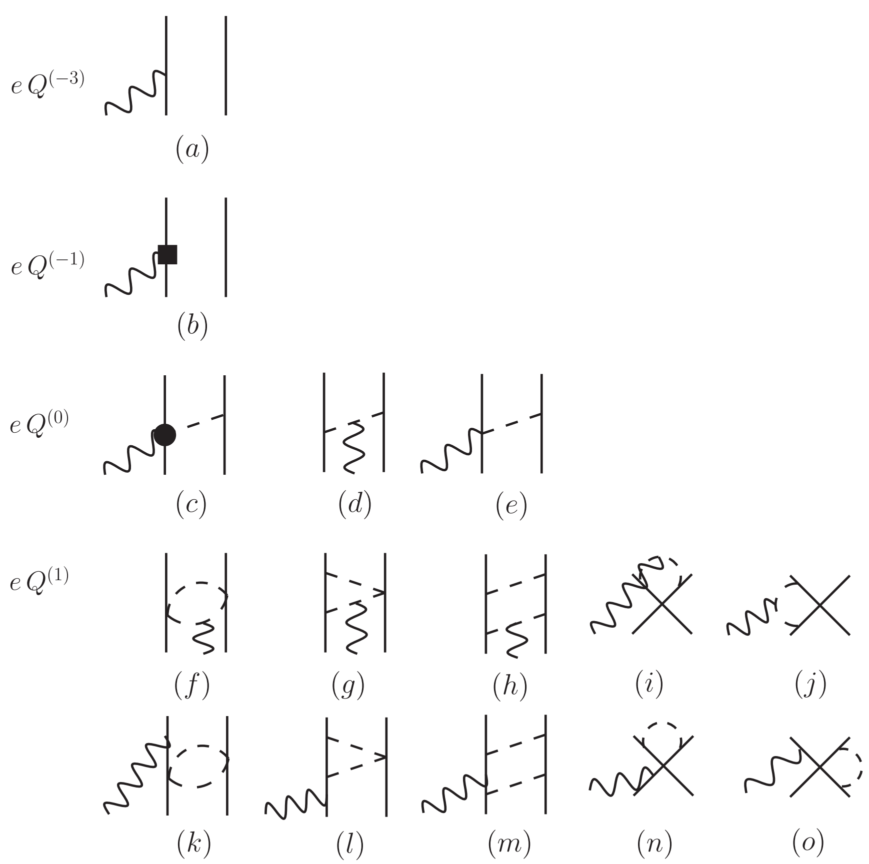


$$F_C(Q^2) = \frac{1}{2J+1} \sum_{M_J} < P', M_J | J_B^0 | P, M_J >$$

(in Breit frame)

$$r_C^2 = r_{str}^2 + \left(r_p^2 + \frac{3}{4m_p^2} \right) + \frac{A-Z}{Z} r_n^2$$

Current operator



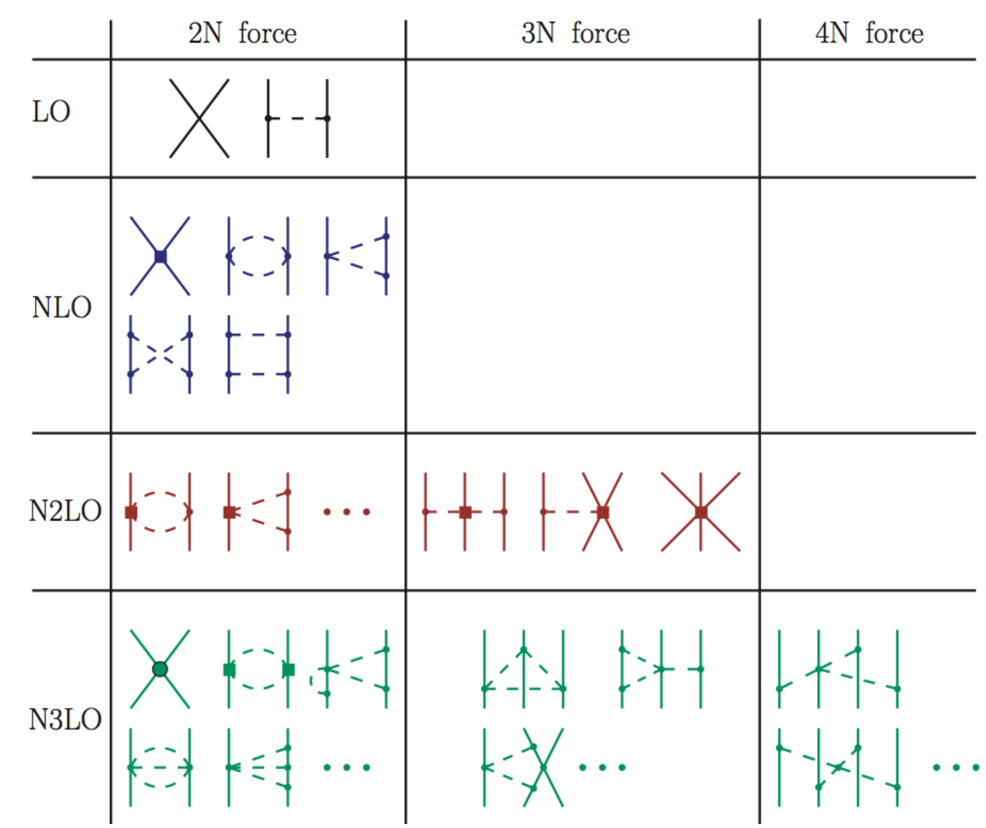
Li Muli

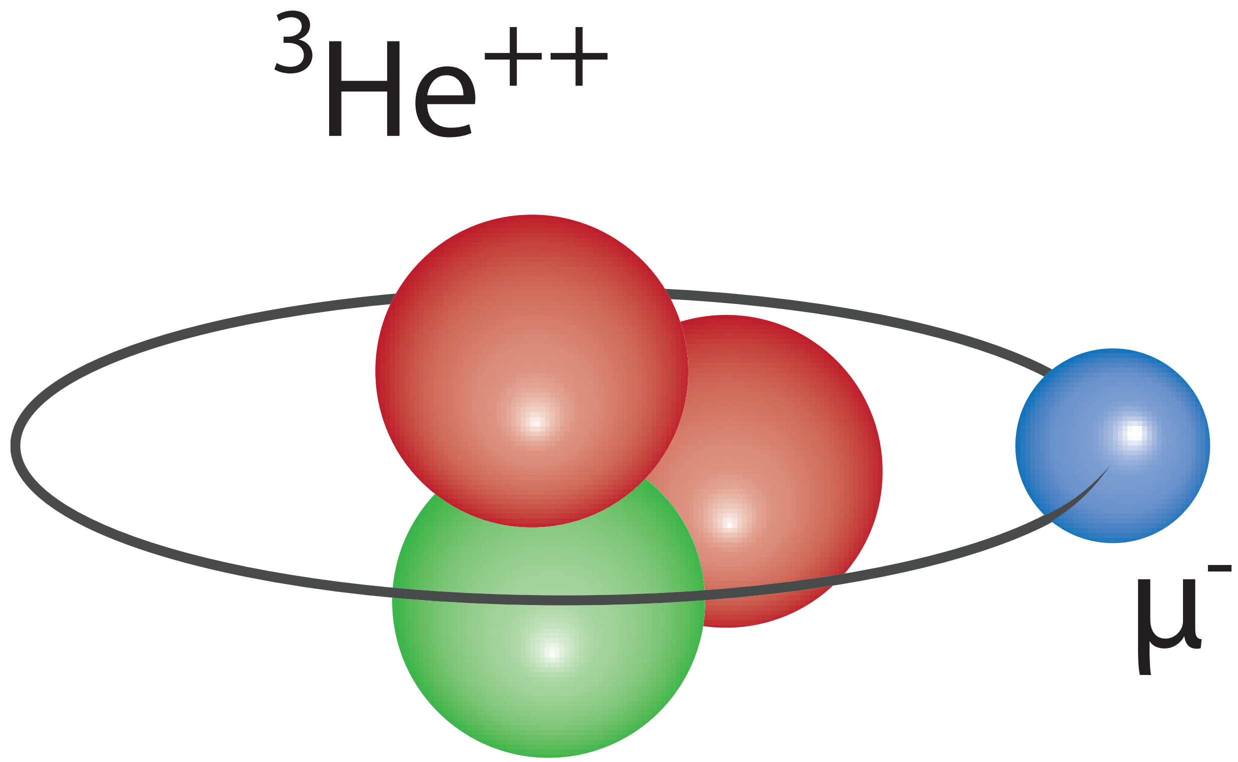
Consistent derivation and regularization of many body forces and nuclear current operators + Rel. Dynamics+ isospin breaking+…

Talk Epelbaum

Epelbaum, Krebs, Meissner, Reinert, Hammer, Maris, Lynn, Ekström, Bacca, Barnea, Li Muli,

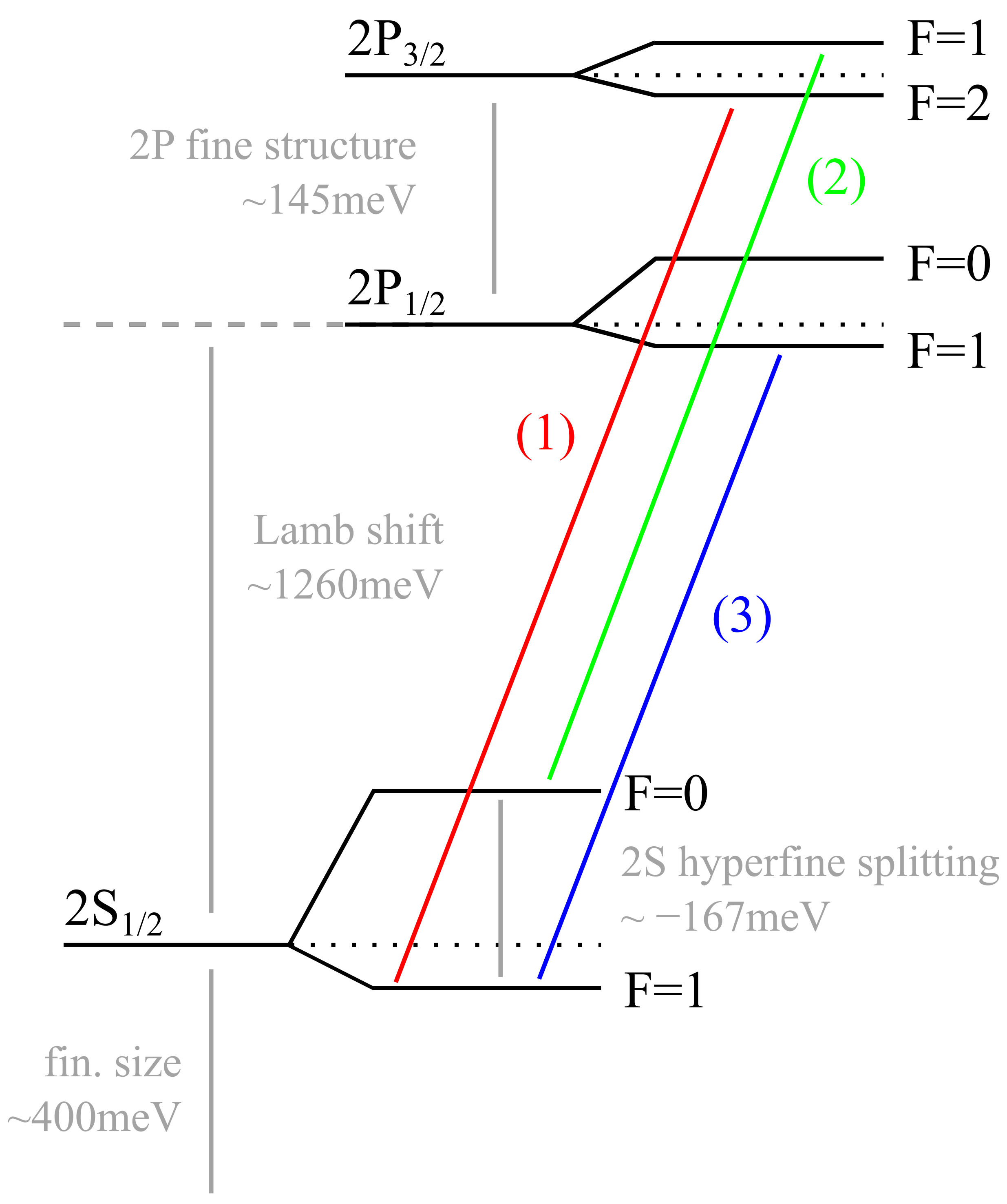
Nuclear forces

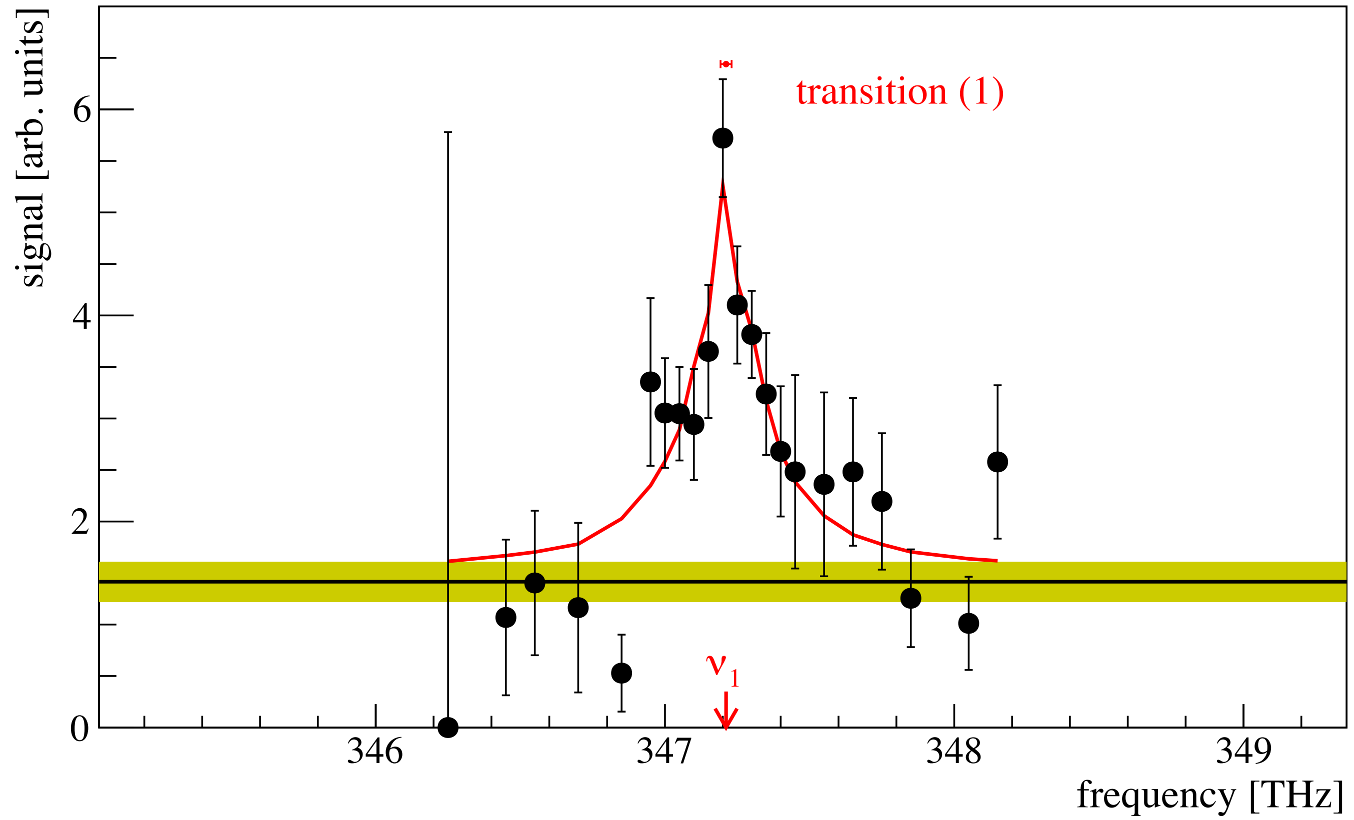


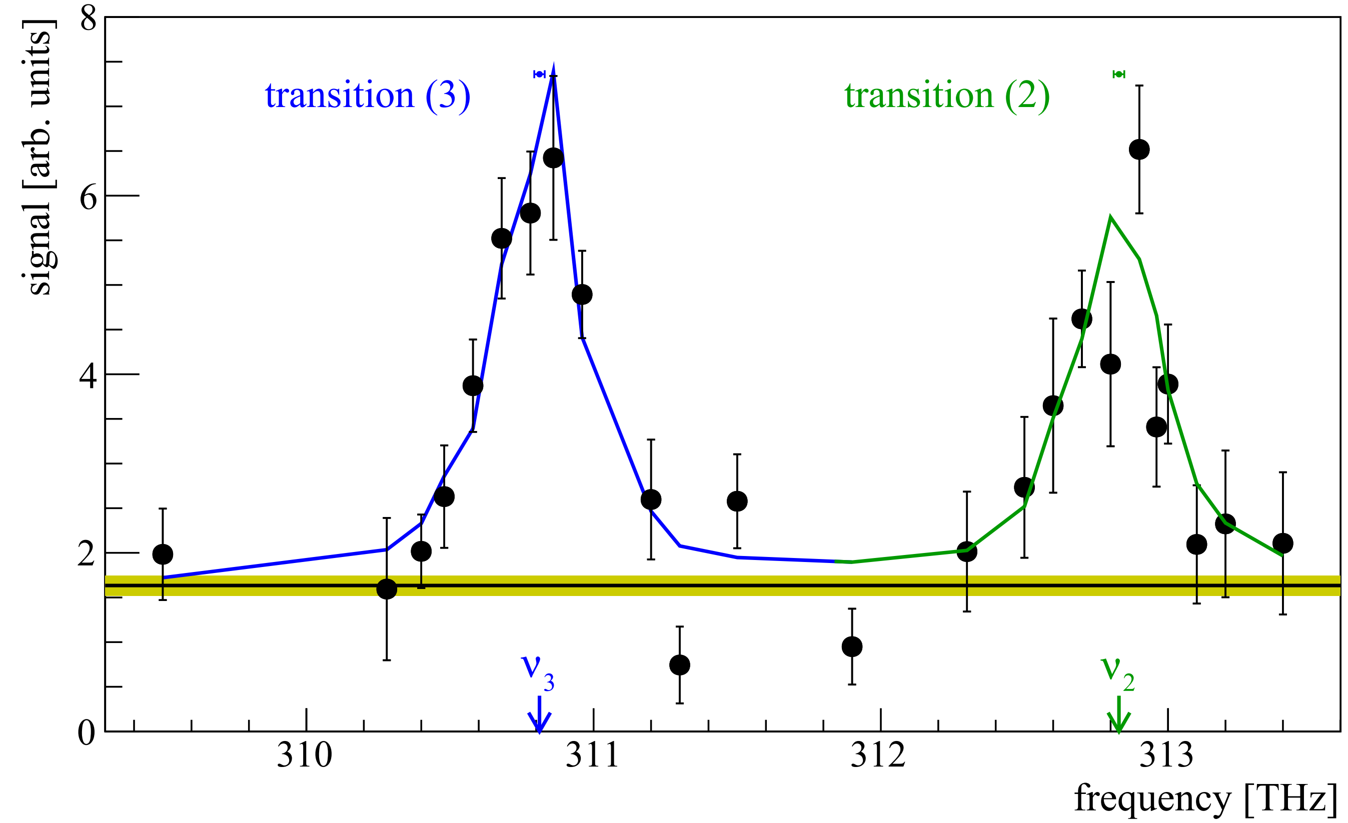


Schuhmann et al., arXiv 2305.11679

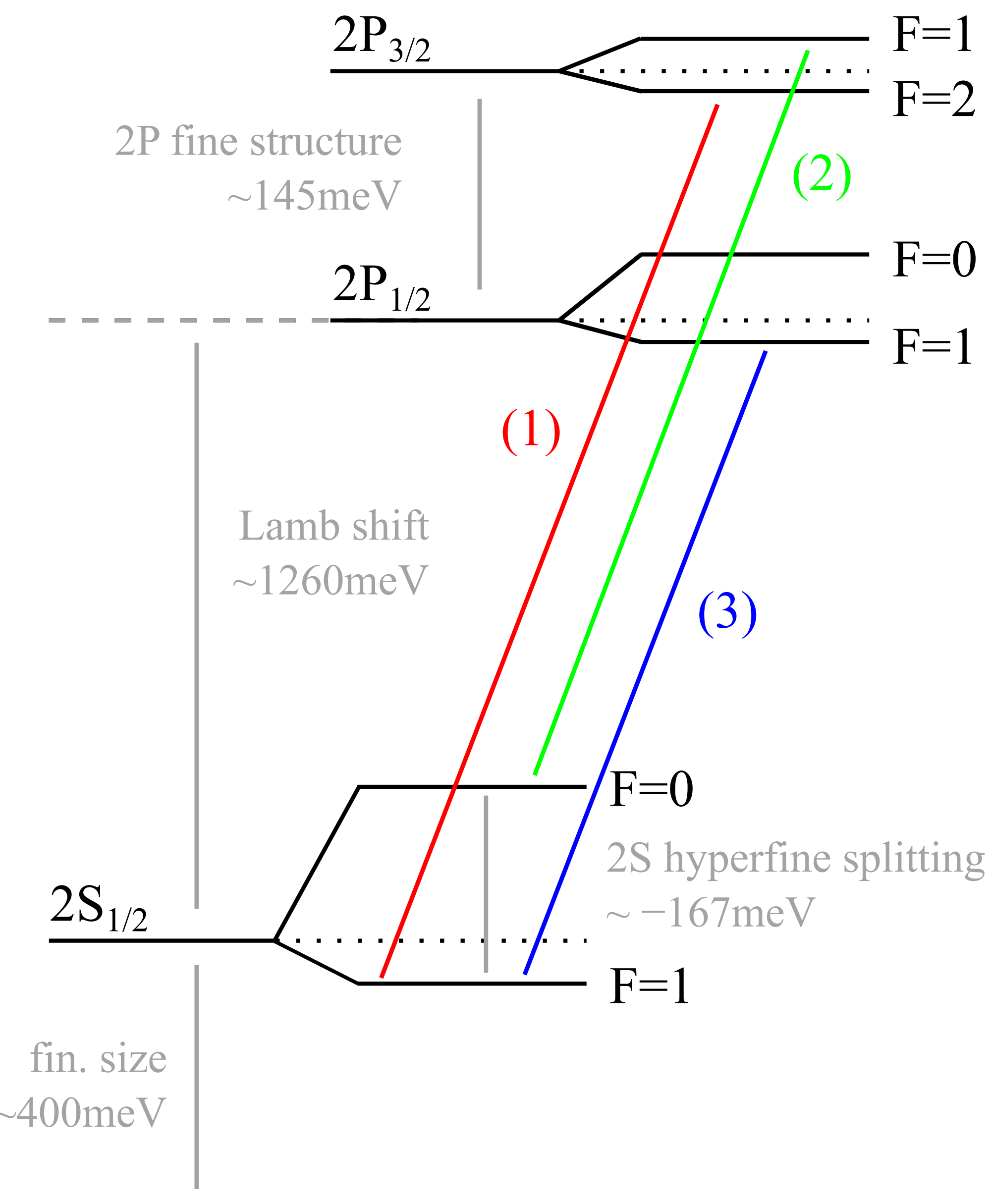
The measured transitions







Lamb shift and 2S-hyperfine splitting



Measured transition energies

$$\Delta E^{(1)}_{\text{exp}} = E_{\text{LS}} - \frac{1}{4}E_{\text{HFS}} + E_{\text{FS}} - 9.23945(26)\ \text{meV}$$

$$\Delta E^{(2)}_{\text{exp}} = E_{\text{LS}} + \frac{3}{4}E_{\text{HFS}} + E_{\text{FS}} + 15.05305(44)\ \text{meV}$$

$$\Delta E^{(3)}_{\text{exp}} = E_{\text{LS}} - \frac{1}{4}E_{\text{HFS}} - 14.80851(18)\ \text{meV}$$

$$E^{\text{exp}}_{\text{LS}} = 1258.598(48)^{\text{exp}}(3)^{\text{theo}}\ \text{meV}$$

$$E^{\text{exp}}_{\text{HFS}} = -166.496(104)^{\text{exp}}(3)^{\text{theo}}\ \text{meV}$$

$$E^{\text{exp}}_{\text{FS}} = 144.958(114)\ \text{meV}$$

The helion charge radius

QED | Finite size | nucl. str.

Theory

$$E_{LS}^{th} = 1644.348(8) - 103.383\, r_h^2 + 15.499(378) \text{ meV}$$

Pachucki et al., arXiv:2212.13782

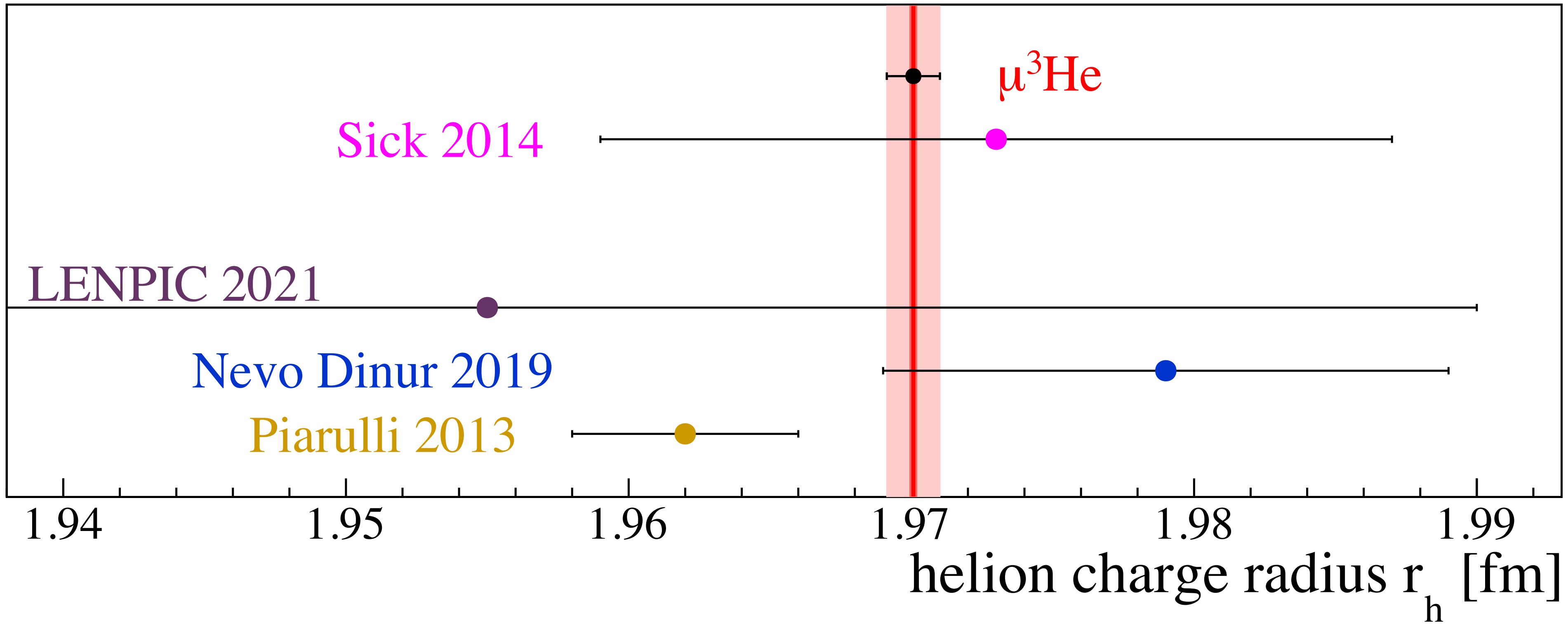
Experiment

$$E_{LS}^{exp} = 1258.598\,(48)^{exp}(3)^{theo} \text{ meV}$$

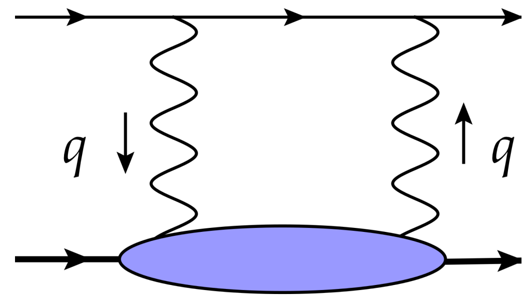
Contains **nuclear** and **nucleon** uncertainties

Pascalutsa, Birse, Hagelstein, Alarcon, Tomalak,….

$$r_h = 1.97007\,(12)^{exp}(93)^{theo} \text{ fm} = 1.97007(94) \text{ fm}$$



Two ways to the two-photon exchange

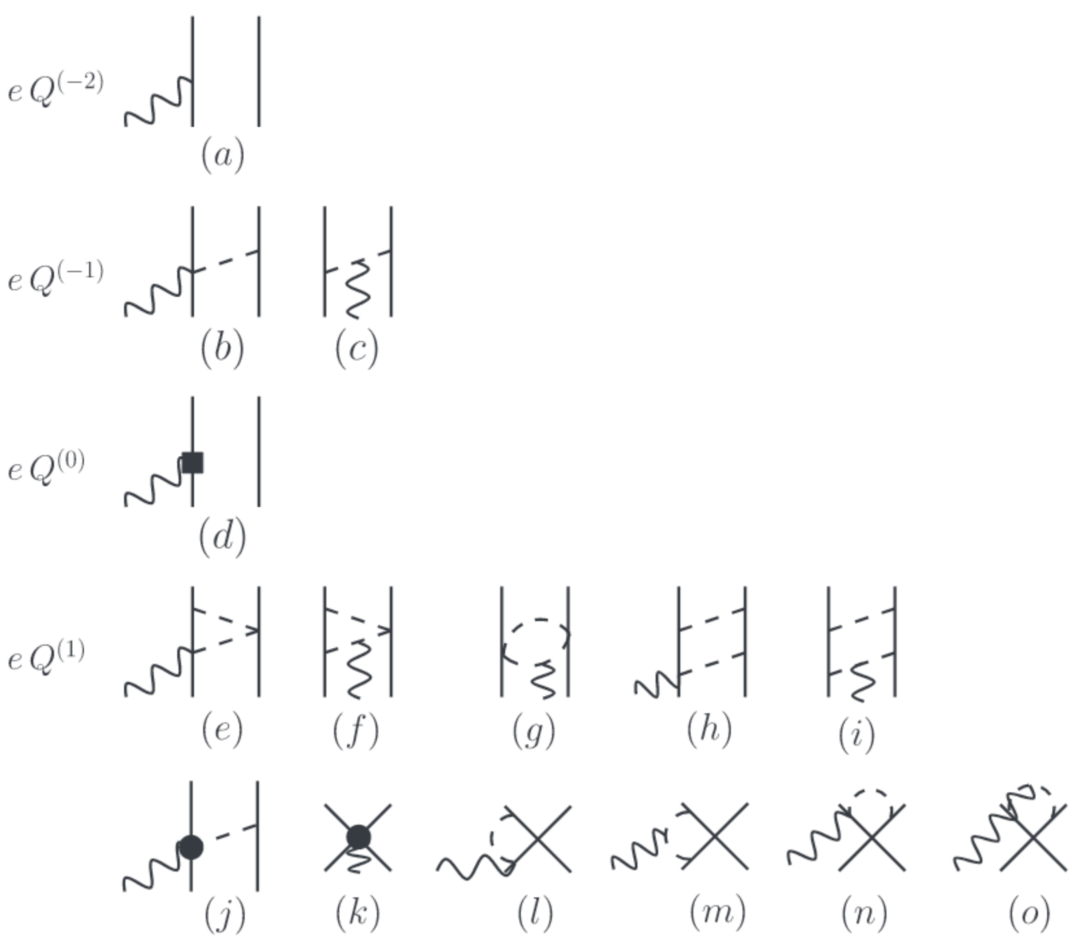


Barnea, Bacca, Nevo Dinur, Ji, Hernandez, Li Muli …

few-nucleon ab initio theory
with chiral-inspired potentials

Dispersion relation and data

Carlson, Gorchtein, Vanderhaeghen



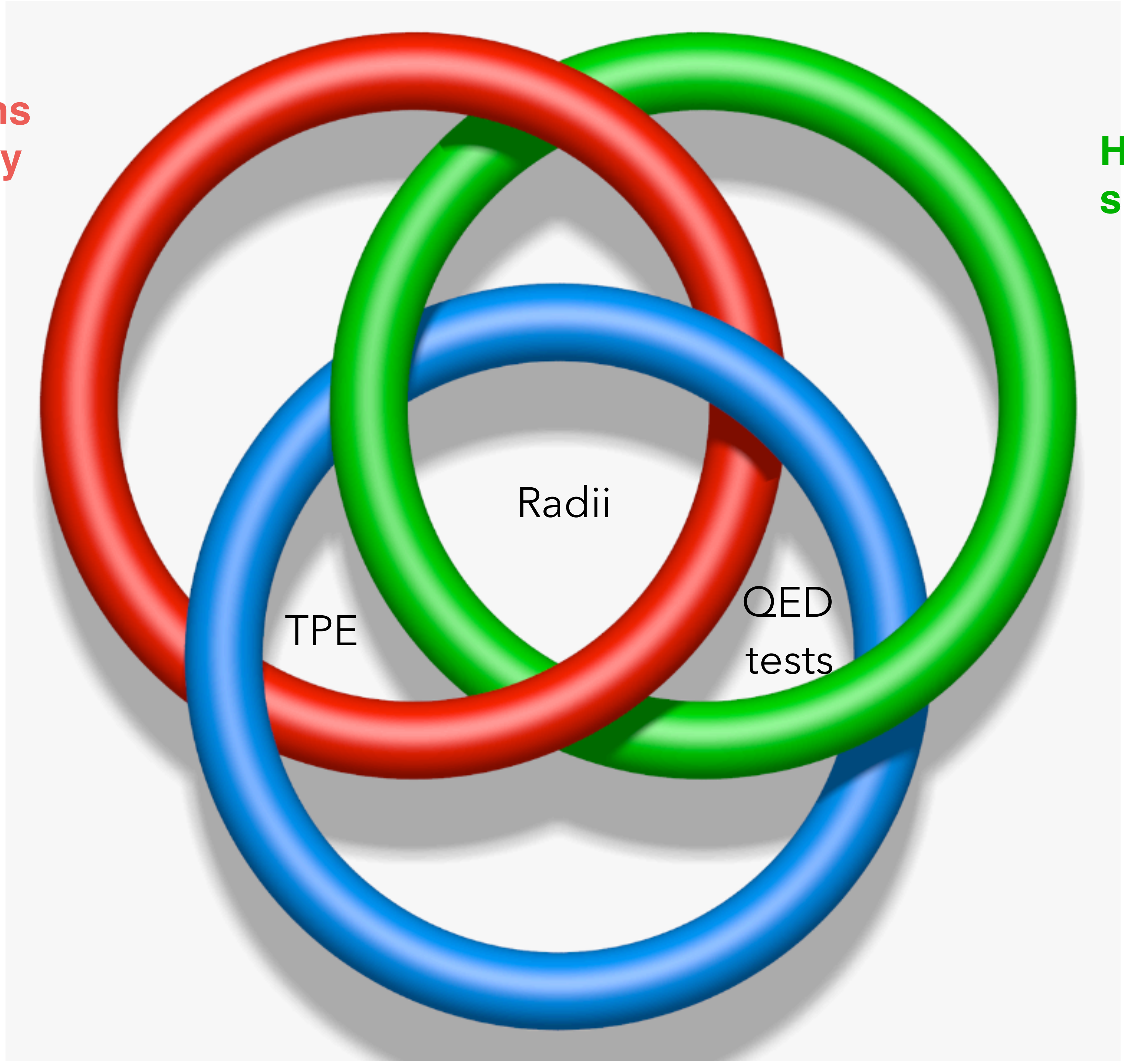
Agreement

For μ^3He

Dispersion:	15.14 (49) meV
Few-nucleon th.:	16.38 (31) meV

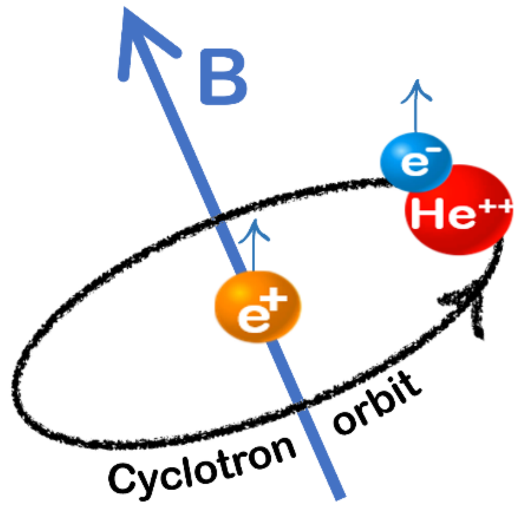
Muonic atoms spectroscopy

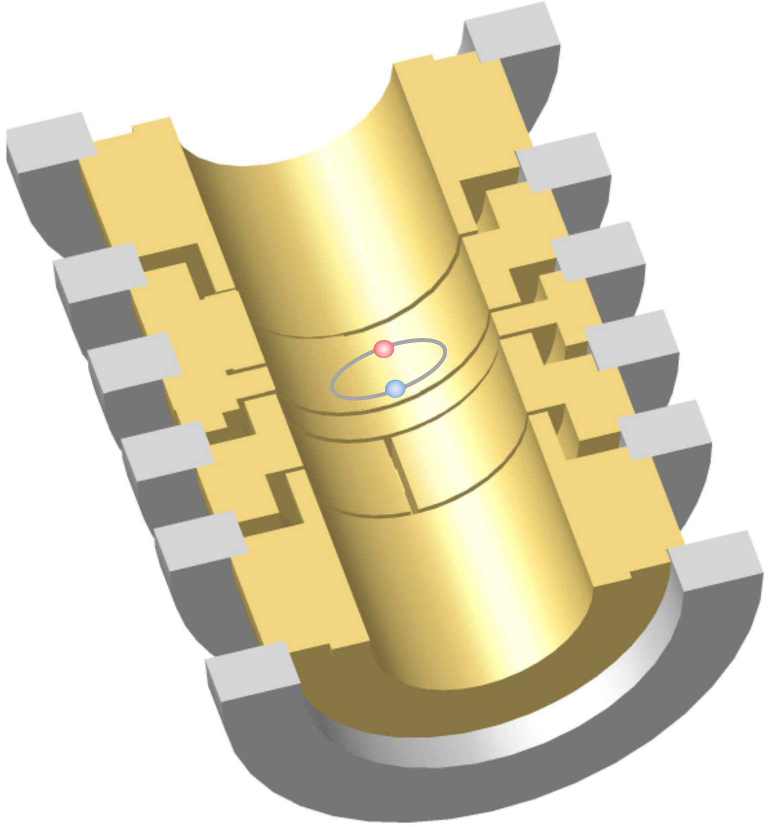
High-precision laser spectroscopy in He and He^+



Nuclear and hadron theories

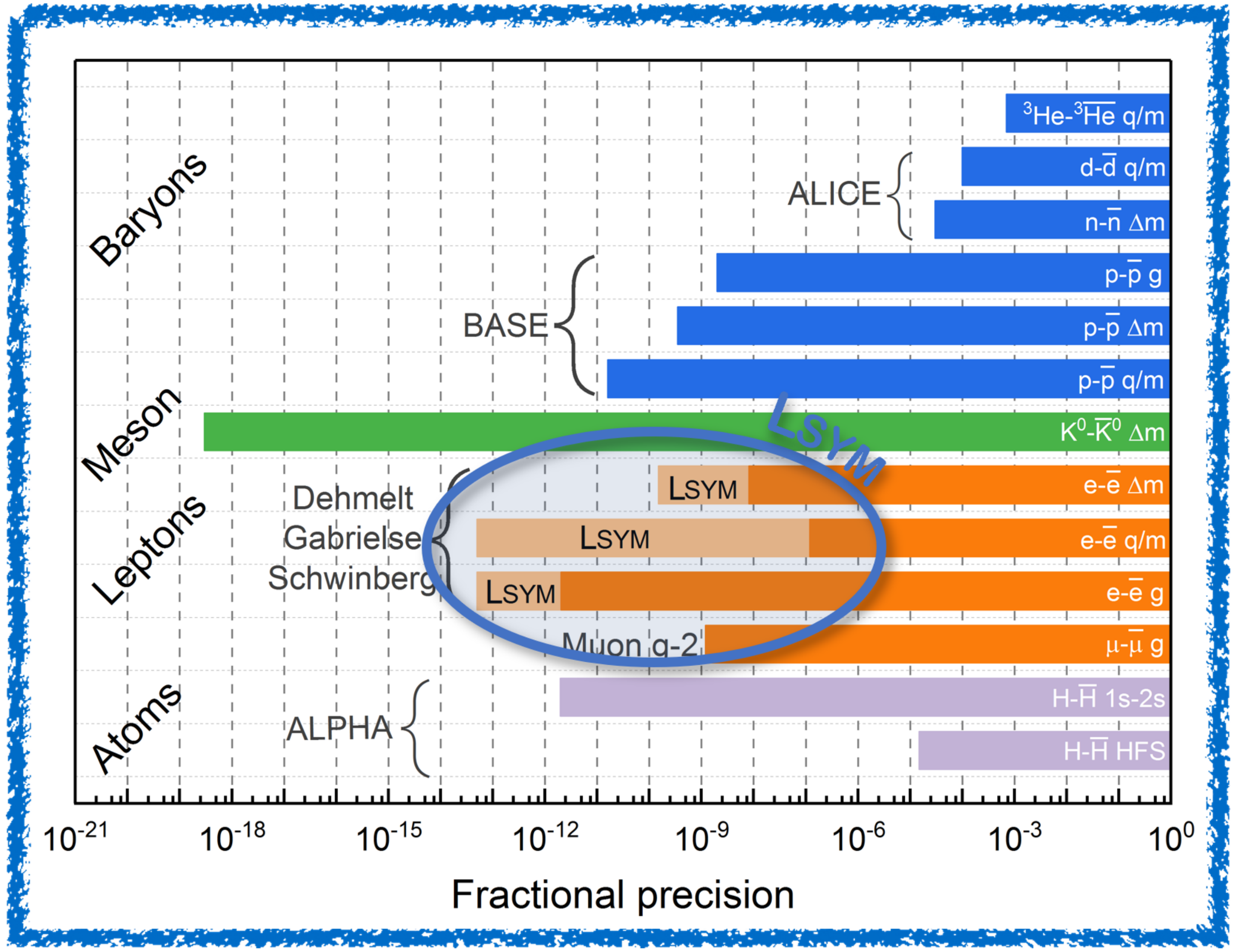
CPT tests (Lsym-project)





$$g_{e^-} - g_{e^+} \sim \omega_{L,e^-} - \omega_{L,e^+} \sim \frac{q}{m} g_{e^-} - \frac{q}{m} g_{e^+}$$

He charge radius needed to extract the electron g-factor
→ CPT test



S. Sturm

The hydrogen-like He^+

Karshenboim et al., PLB795:432(2019)
Yerokhin et al., *Ann.Phys.*531(5):1800324(2019)

Theory sensitive to higher order contributions

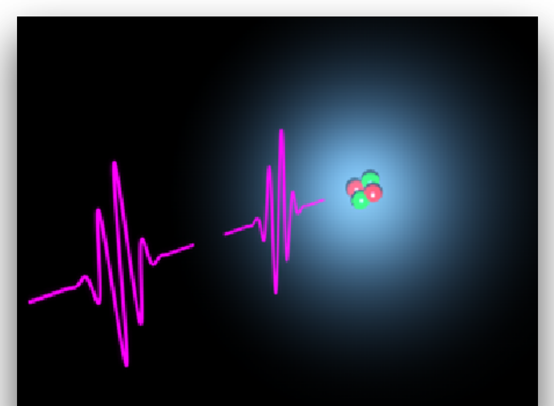
$$f_{2S-1S}(\mathrm{He}^+) \approx \frac{3Z^2 cR_\infty}{4}\frac{1}{1+\frac{m_e}{M_\alpha}} + \mathrm{QED}_{\mathrm{He}^+}\left(Z^{3.7}, Z^{5\ldots7}\right) - \frac{7(Z\alpha)c^4}{24\pi\, a_B^3\, \hbar^3} r_\alpha^2 + \mathrm{nucl.struct.}$$

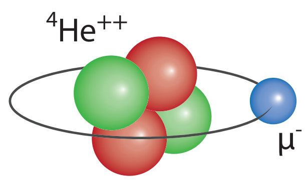
Uncertainty	$(1\,\mathrm{kHz})$	$(9\,\mathrm{kHz})$	$(40\,\mathrm{kHz})$	$(61\,\mathrm{kHz})$	$(3\,\mathrm{kHz})$

Prospected experimental accuracy $u_r = 1 \times 10^{-13}$

Using R_∞ from H+μH

Using r_α from $\mu^4\mathrm{He}^+$





Moreno, et al. EPJD 77, 67 (2023).
Krauth et al. PoS FFK2019, 49 (2019).

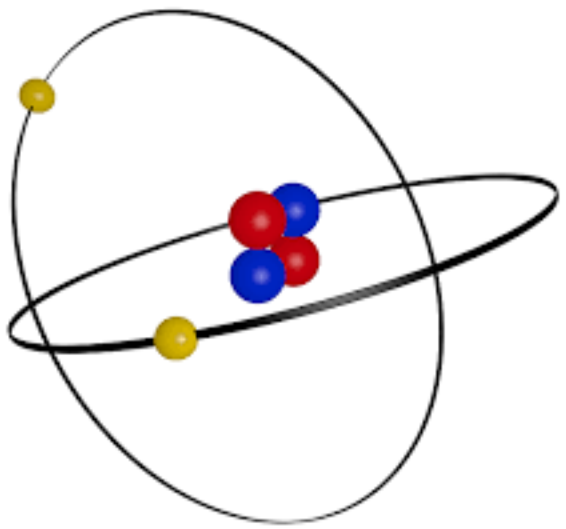
Pachucki et al., arXiv:2212.13782 90 kHz
Li Muli, Bacca (in preparation) >90 kHz

The He atom

Two electrons are much more than one electron

$m\alpha^7$ contributions completed

Patkos et al, PRA 103, 042809 (2021)
Yerokhin et al., PRA **107**, 012810 (2023)

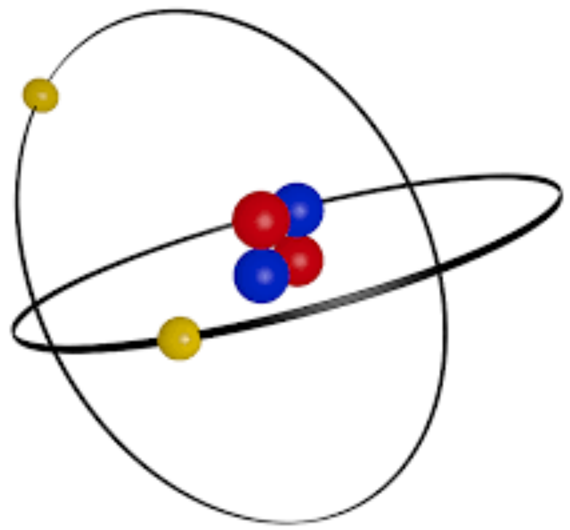


For some transitions there is perfect agreement, for others perfect disagreement

Clausen et al, PRL 127, 093001 (2021)
Zheng, et al, PRL 119, 263002 (2017)

From He spectroscopy a charge radius can not yet be extracted due to theory uncertainties

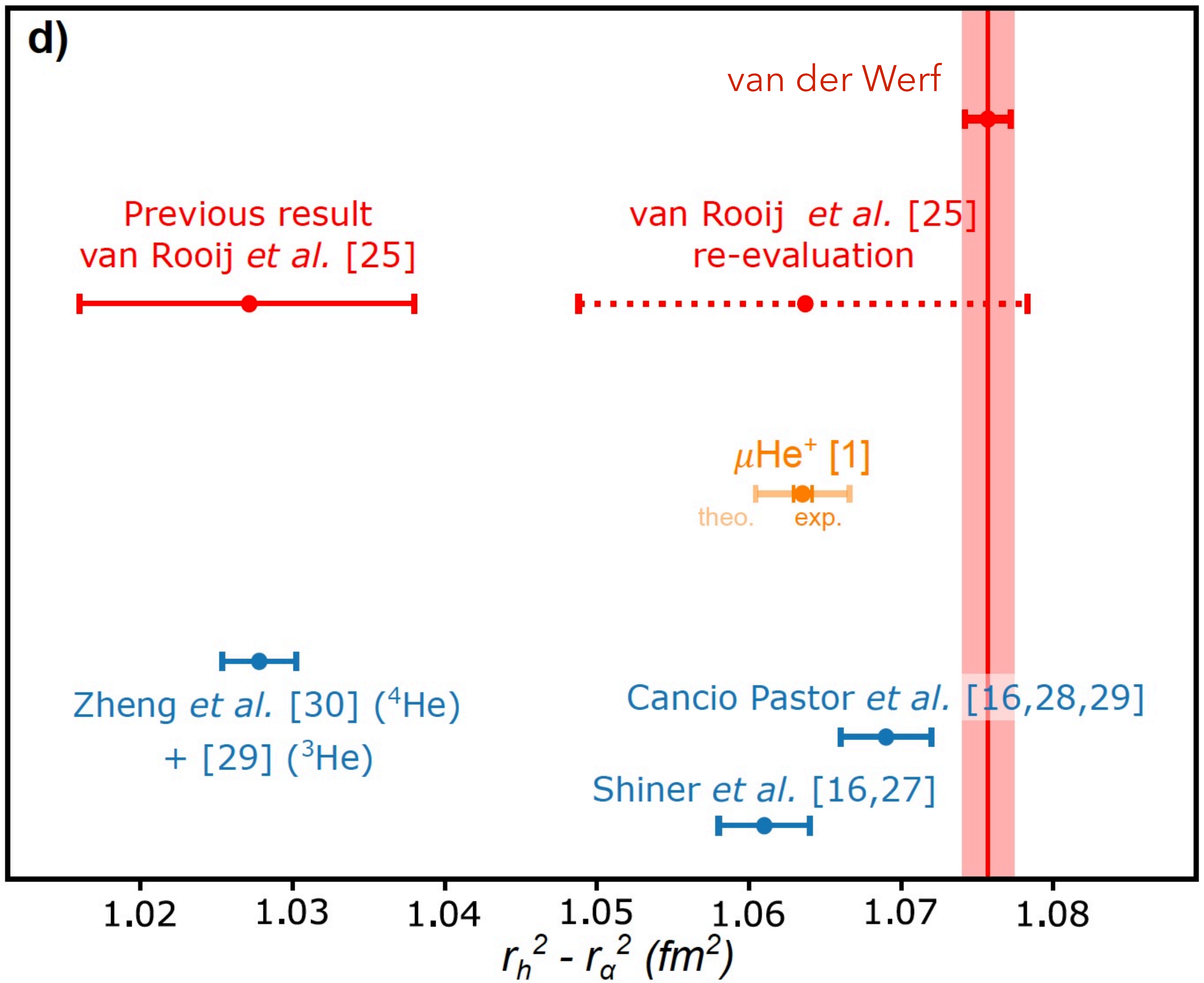
Isotopic shift in He



But **large** theory cancellations are taking place in the isotopic shift of regular He.

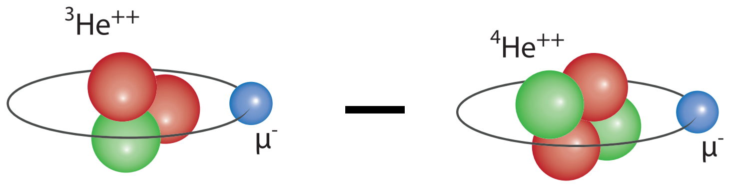
The isotopic shift is sensitive to $r_h^2 - r_\alpha^2$

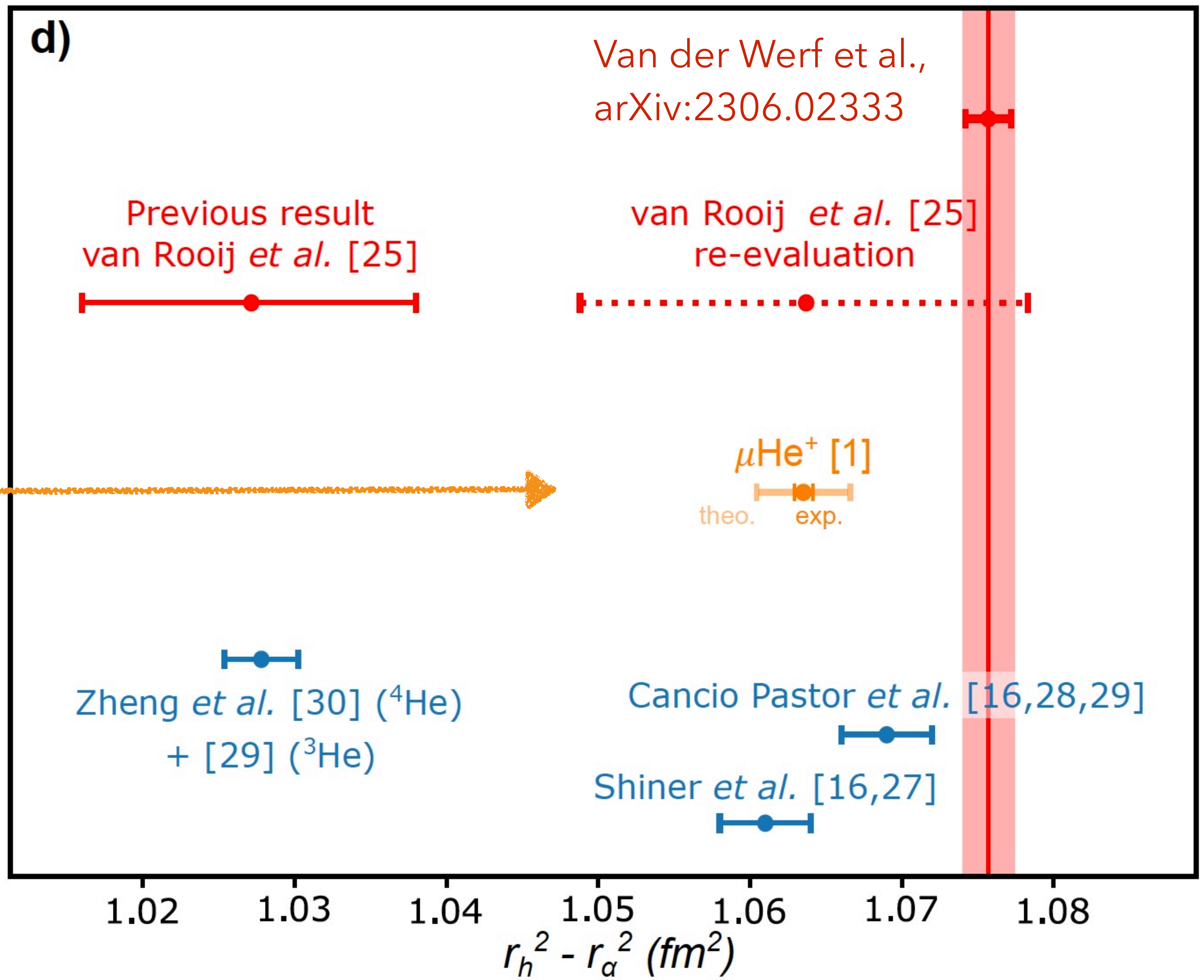
Van der Werf et al., arXiv:2306.02333



Isotopic shift in He

Between $\mu\mathrm{He}^+$ and van der Werf results there is a 3.6σ tension



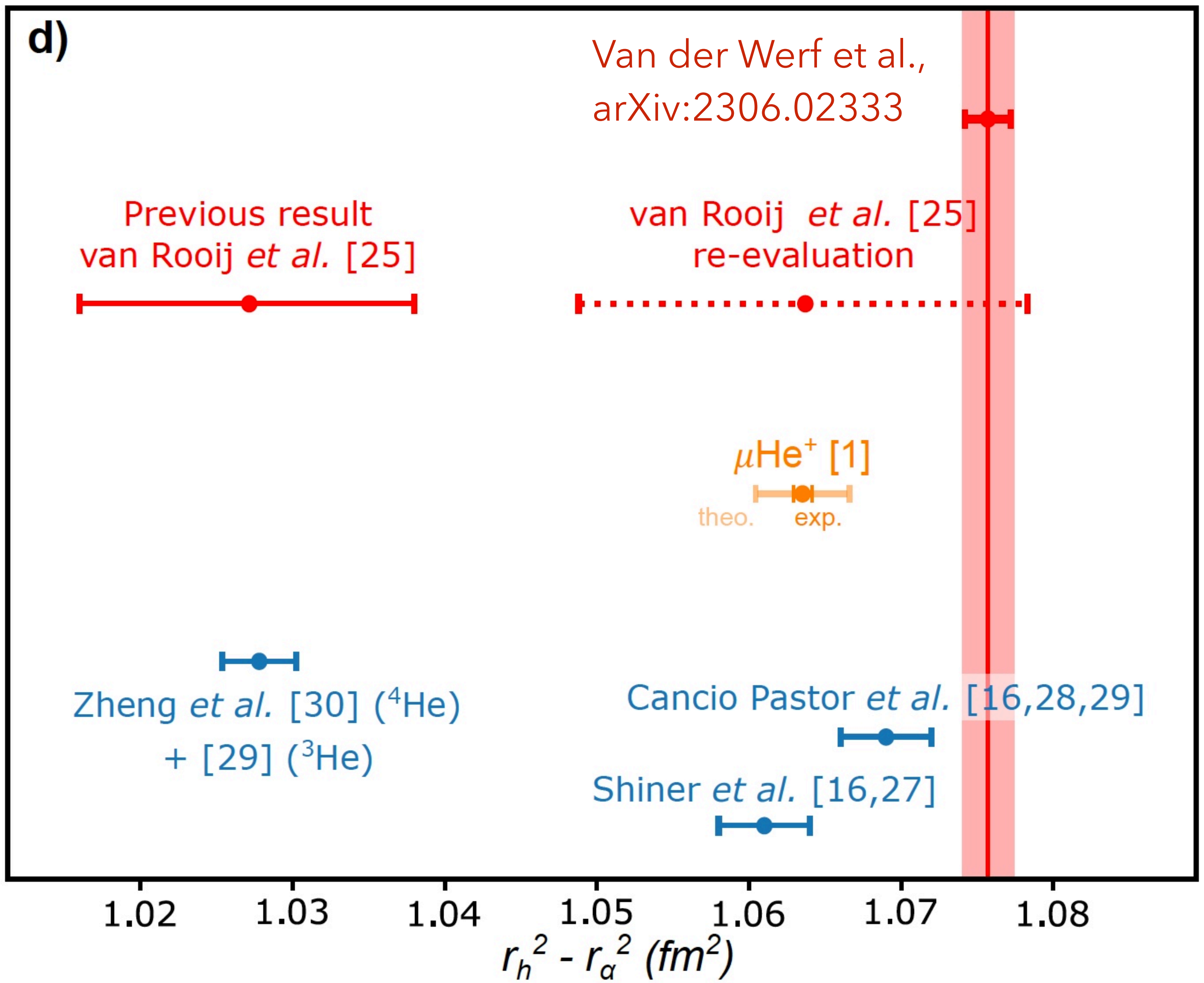


Isotopic shift in He

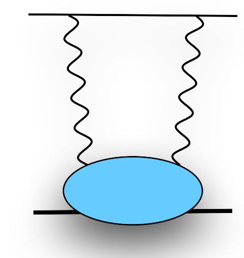
- Van Rooij results corrected to include a differential ac-stark shift effect. The van der Werf result makes use of the magic wavelength to eliminate this systematic

- The values of Shiner and Pastor may lack a systematic correction due to quantum interference effects
  Marsman et al., PRA 89, 043403 (2014)

- The value of Zheng may have to be corrected for a systematic Doppler shift
  Wen, et al. PRA 107, 042811 (2023).



Another benchmark for nuclear theories: 2S HFS



Fermi+QED | nucl. str.

Theory

$$E^{\mathrm{th}}_{\mathrm{HFS}} = -\,172.7457(89)\ \mathrm{meV} + E^{\mathrm{nucl.struct.}}_{\mathrm{HFS}}$$

Franke et al., EPJD 71, 341 (2017).

Experiment

$$E^{\mathrm{exp}}_{\mathrm{HFS}} = -\,166.496(104)^{\mathrm{exp}}(3)^{\mathrm{theo}}\ \mathrm{meV}$$

Schuhmann et al., arXiv 2305.11679

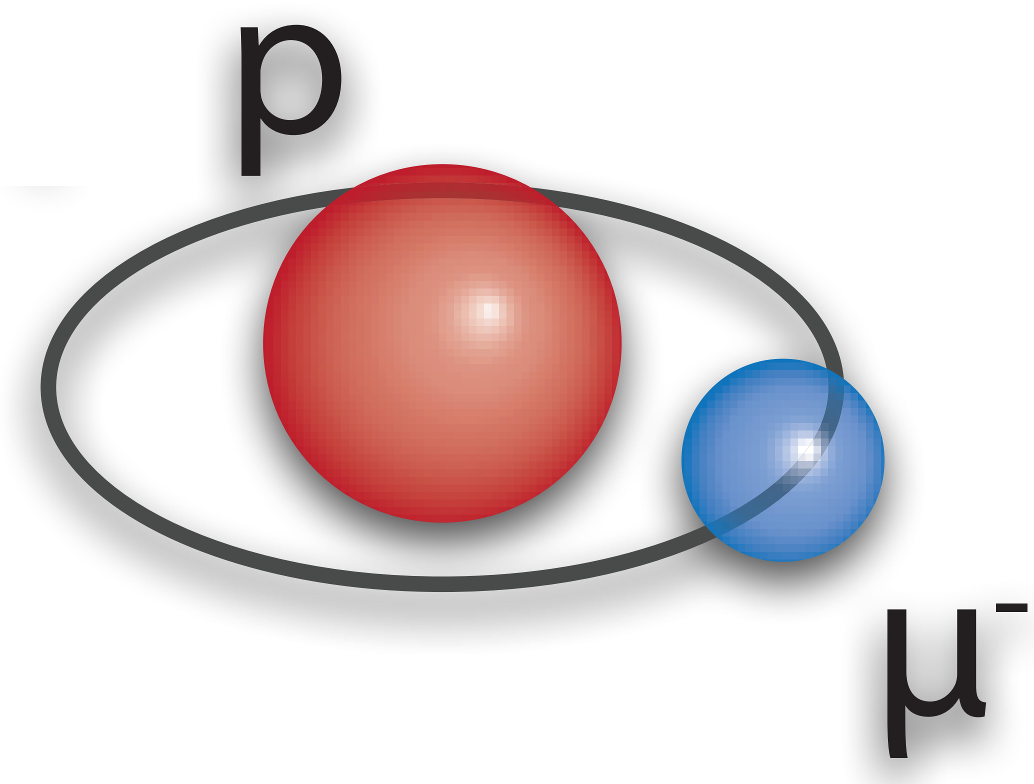
Nuclear structure contribution (2PE+…)

$$E^{\mathrm{nucl.struct.}}_{\mathrm{2S-HFS}} = 6.25(10)\ \mathrm{meV}$$

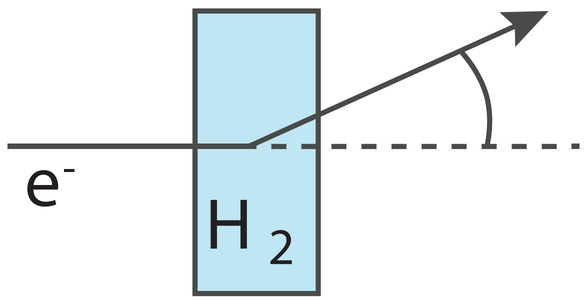
No theory predictions exist to date for the inelastic part

$$\Delta E^{\mathrm{Zemach}}_{\mathrm{2PE}} = 6.53(4)\ \mathrm{meV}$$

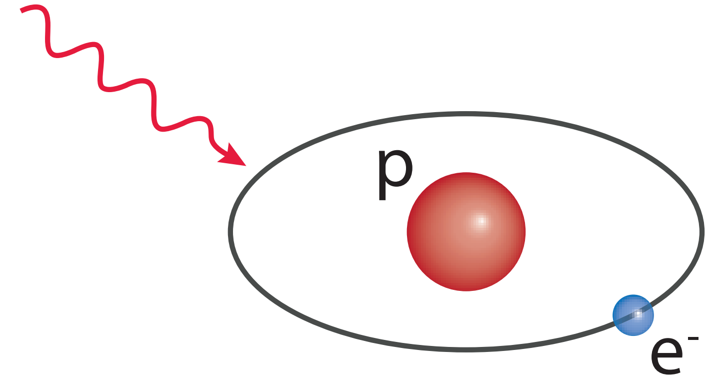
using $r_Z = 2.528(16)\ \mathrm{fm}$ Sick. PRC 90, 064002 (2014).


p
μ-

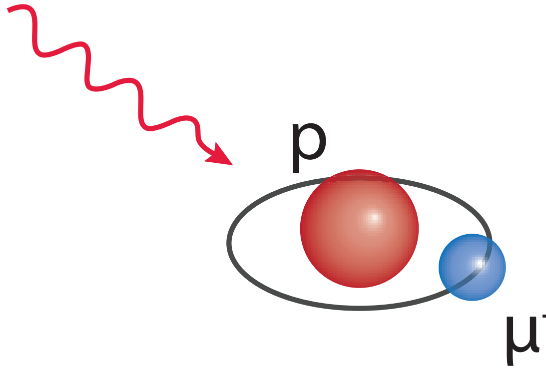
The proton radius puzzle



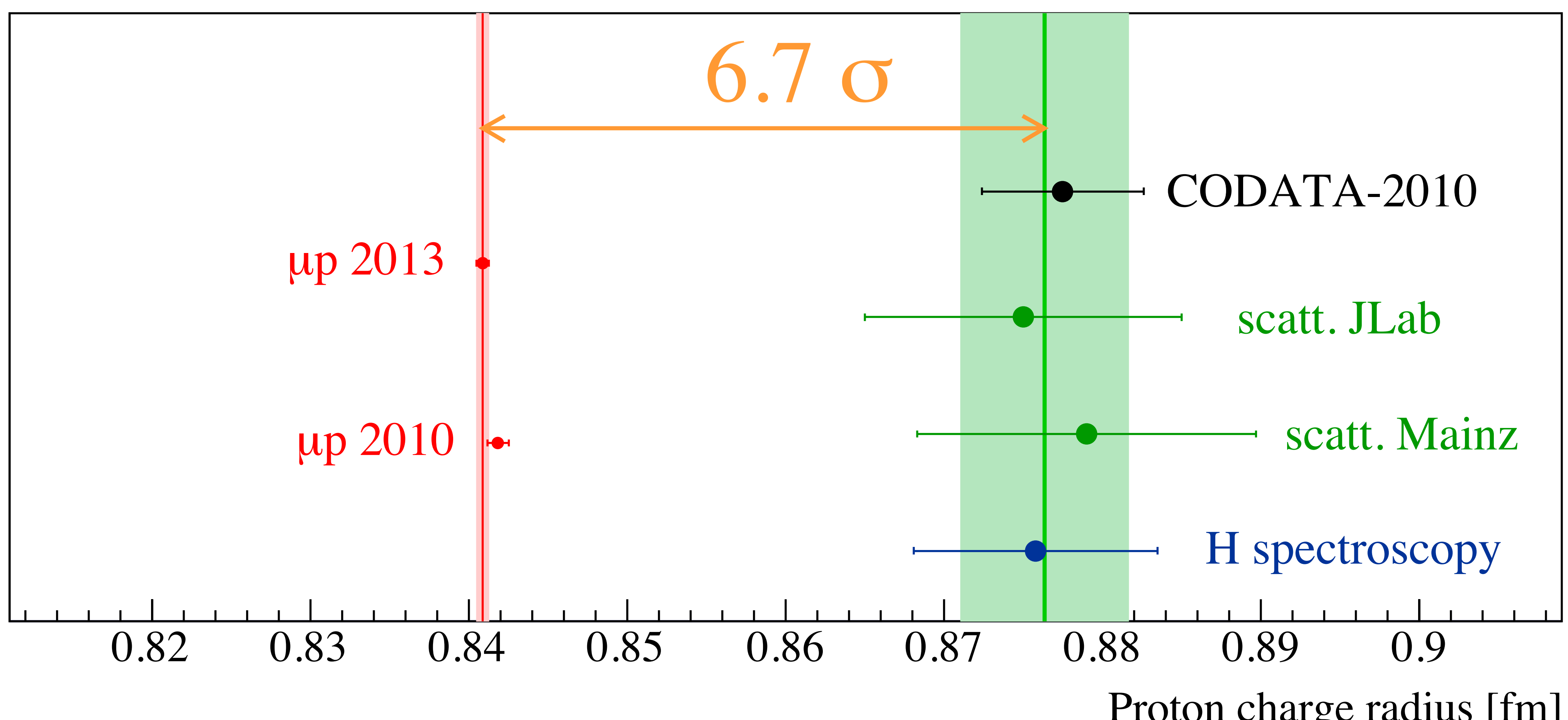
e^--p scattering



H spectroscopy

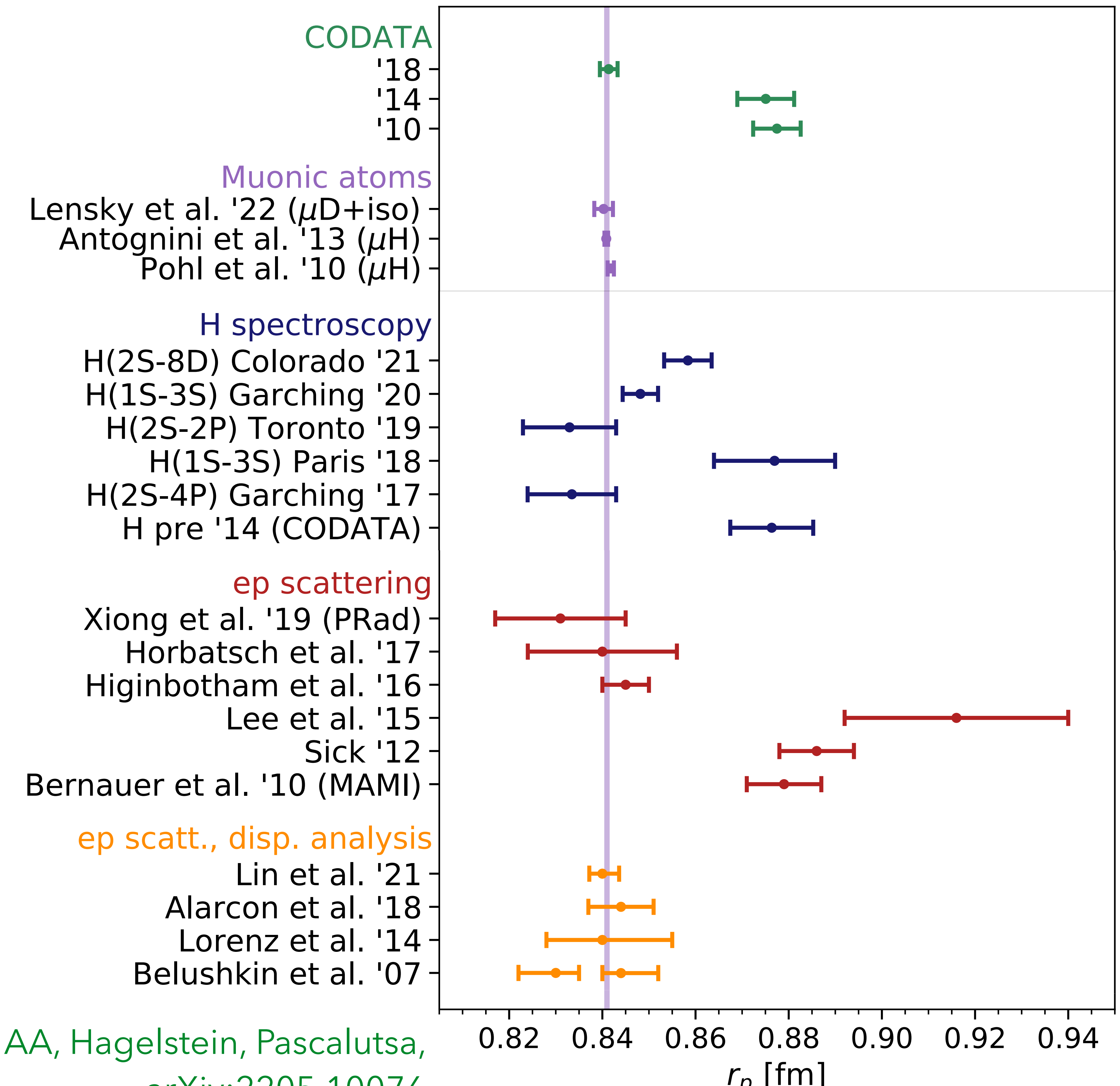


μp spectroscopy



Pohl et al., Nature 466, 213 (2010)
Antognini et al., Science 339, 417 (2013)

Status of the proton charge radius puzzle in 2 minutes



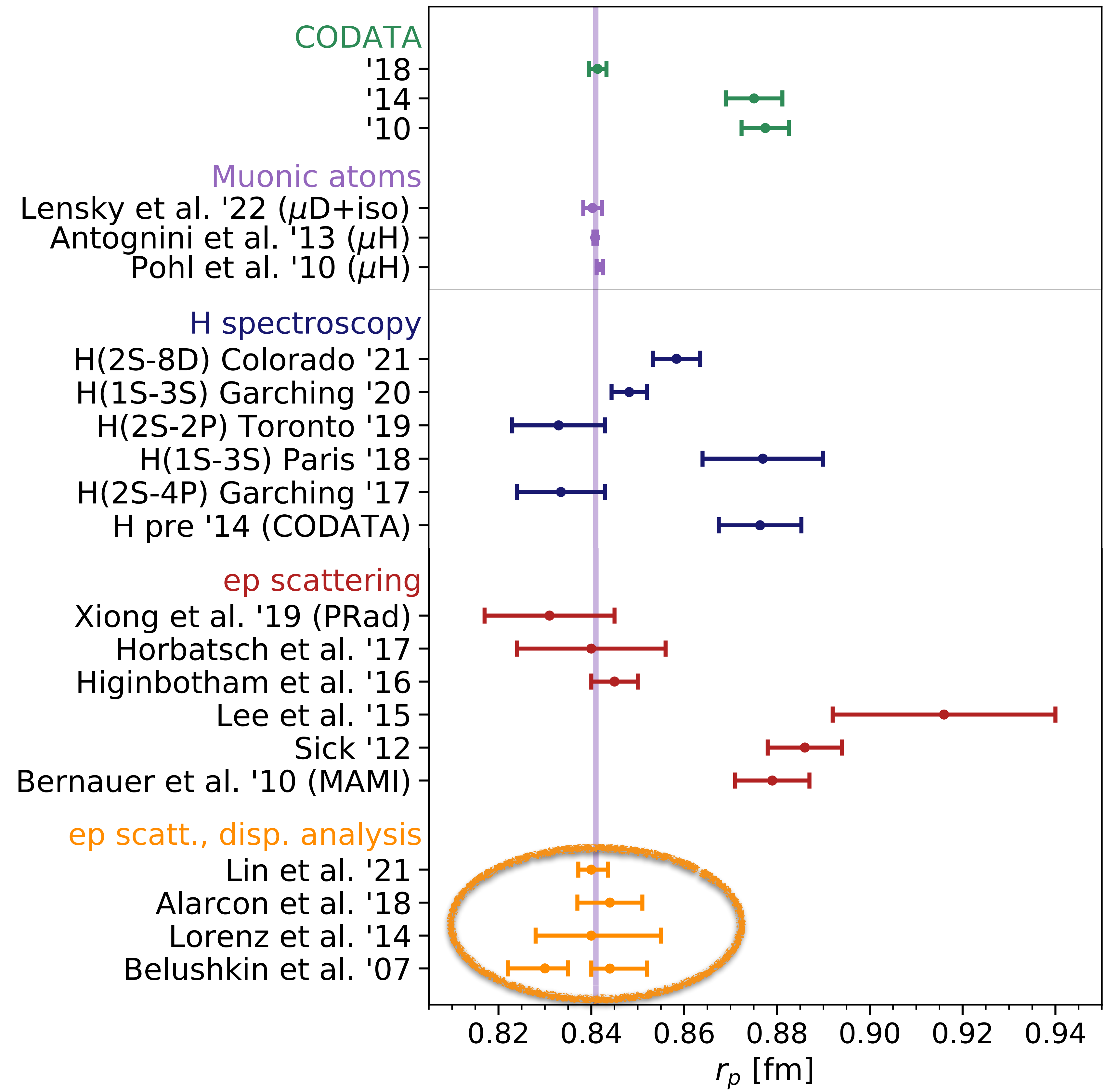
AA, Hagelstein, Pascalutsa, arXiv:2205.10076

Status of the proton charge radius puzzle in 2 minutes

Dispersion-based analysis

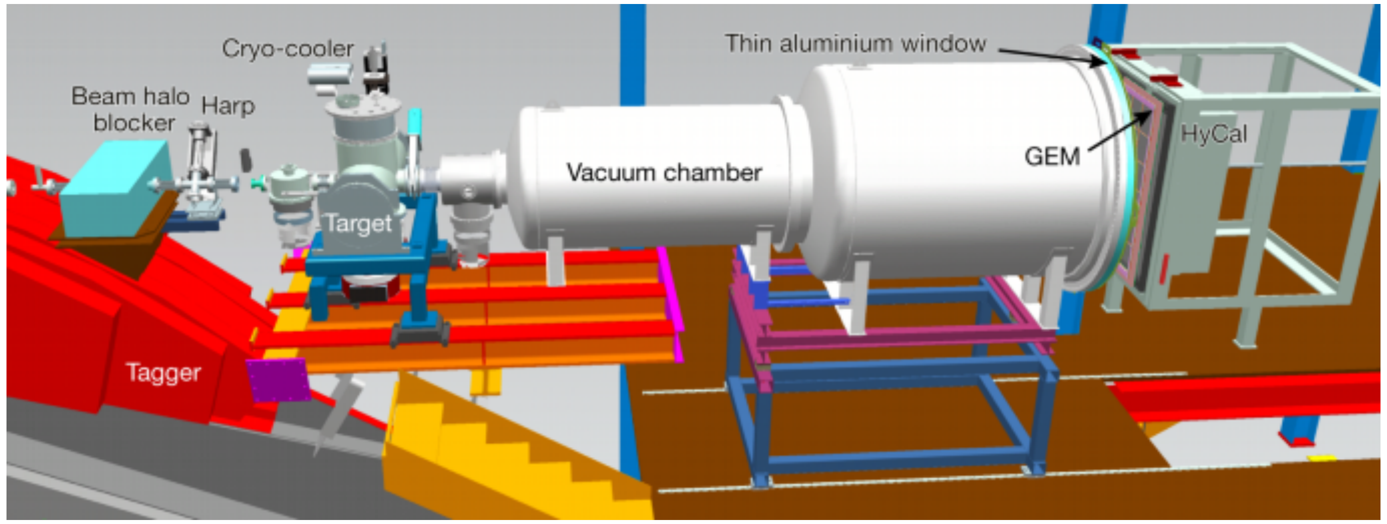
- Allow for a consistent description of all data (including neutron) in the space- and time-like regions based on fundamental principles.
- Always led to a small proton charge radius

Talk U. Meissner

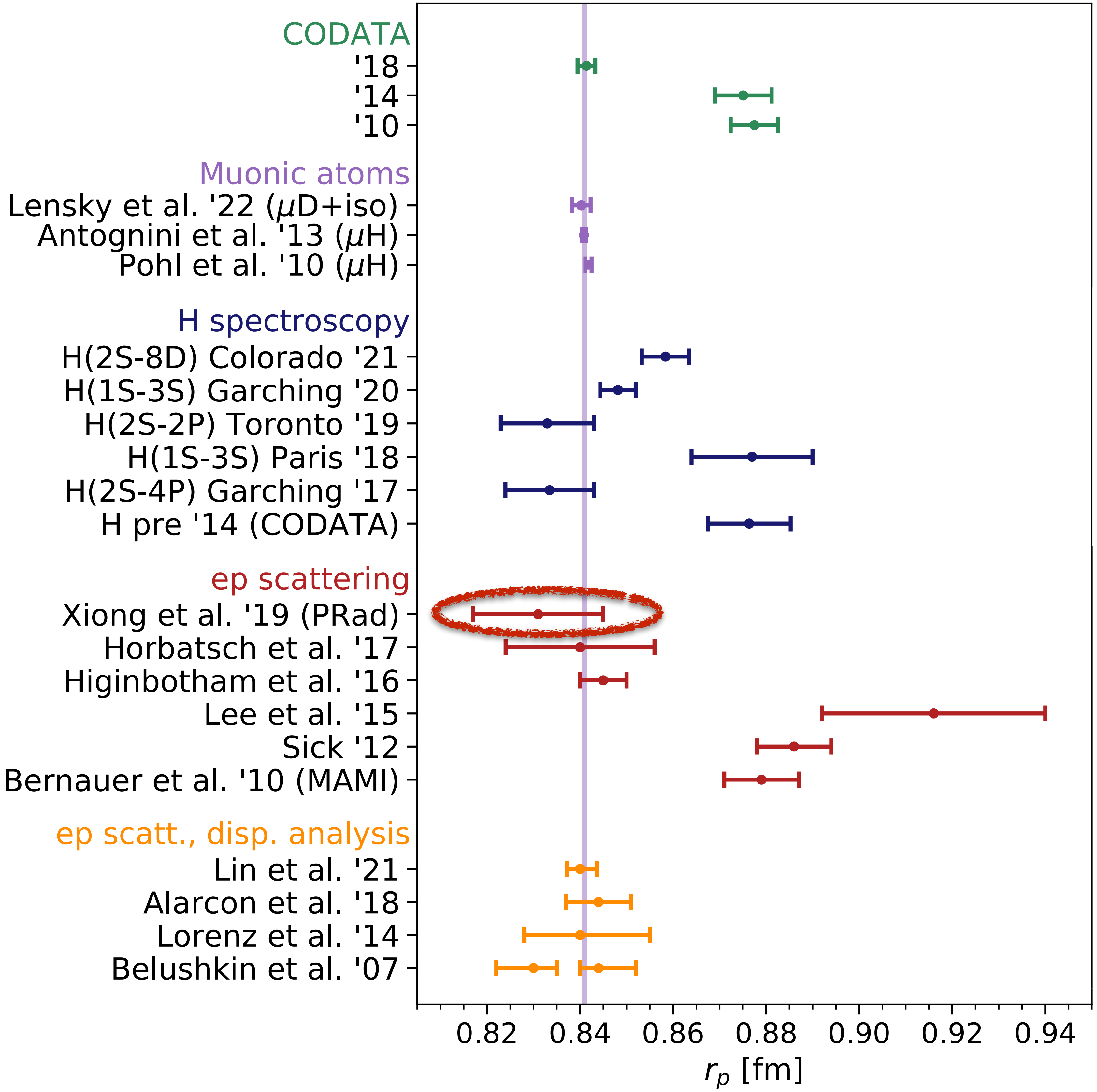


Status of the proton charge radius puzzle in 2 minutes

The PRAD experiment



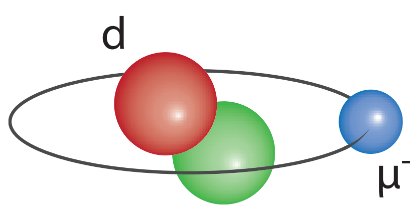
Talk H. Gao



Hessels

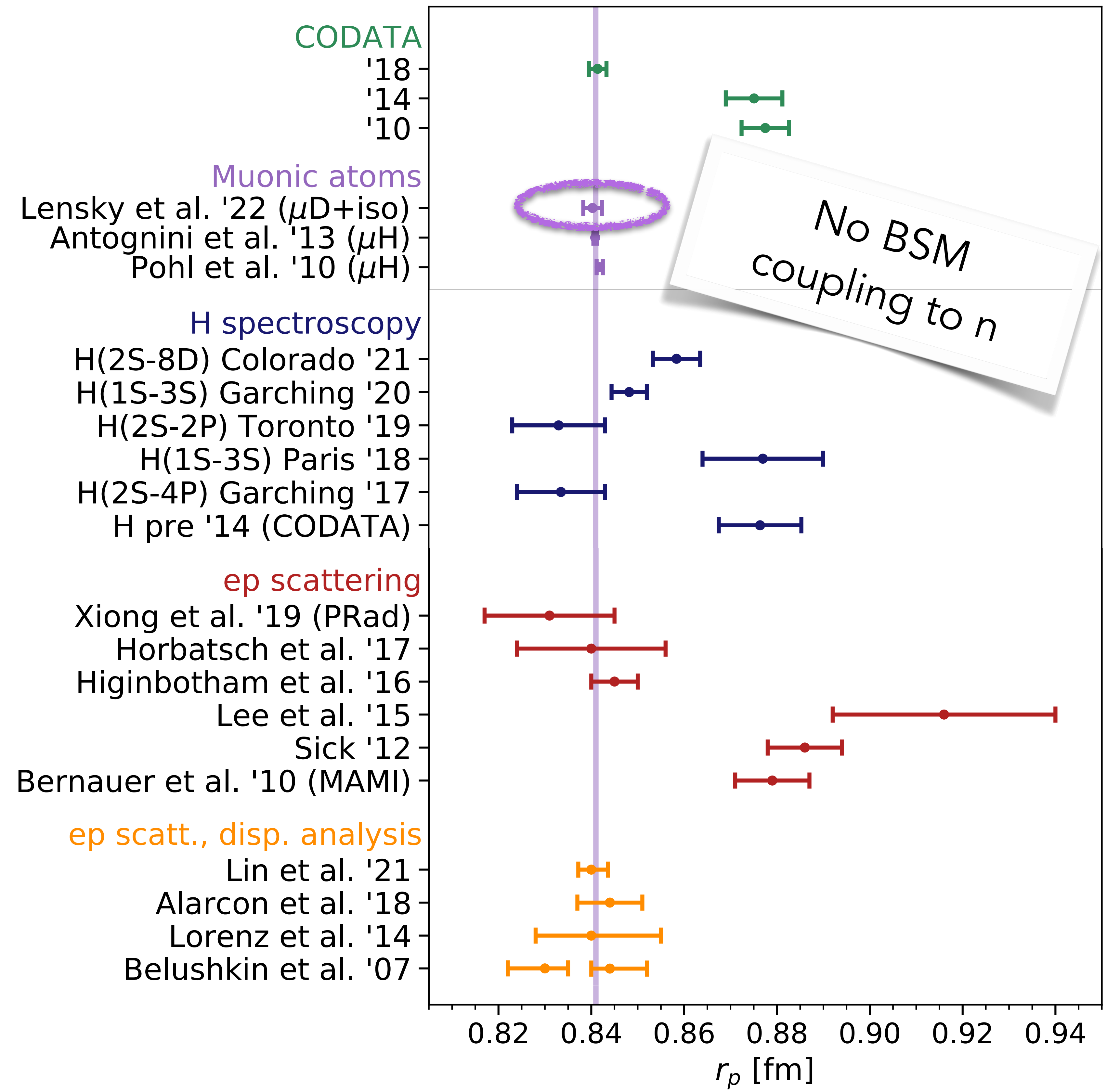
Status of the proton charge radius puzzle in 2 minutes

Talks
F. Hagelstein
V. Lensky
T. Richardson
B. Acharya



μD and H-D isotopic shift

$$\text{H/D shift:} \quad r_d^2 - r_p^2 = 3.820\,61(31)\ \mathrm{fm}^2$$
$$\mu d: \quad r_\mathrm{d} = 2.12763(78)\ \mathrm{fm}$$

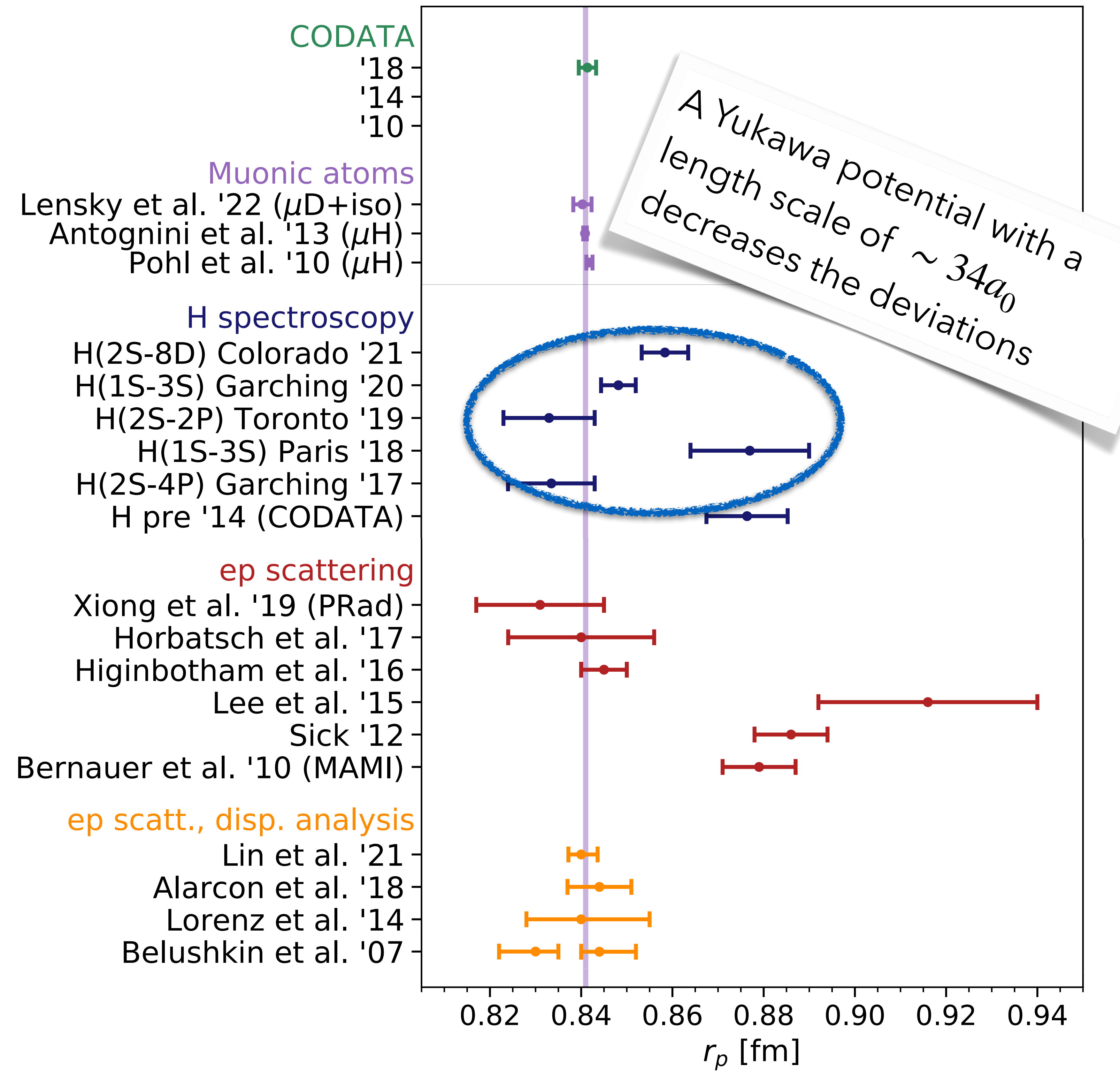


Pachucki et al., PRA 97, 062511 (2018)
Kalinowsiet al., PRA 99, 030501 (2019)
Lensky et al., PLB 835 (2022) 137500
Lensky et al., EPJA 58, 224 (2022)

Status of the proton charge radius puzzle in 2 minutes

New measurements in H

- Values have moved towards $r_p(\mu H)$, yet, some deviations still exist.
- Deviations tends to decrease as n increases.



Impact of precise nuclear radii?

- A simplified story (neglecting least square adjustment)
- Muonic-atom centric approach
- Just consider the proton

Muonic hydrogen

$$E_{2S-2P}(\mu H) \approx \mathrm{QED} + \kappa r_p^2 + \mathrm{NS}$$

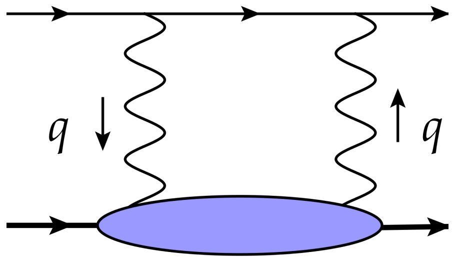
$(\delta = 1 \times 10^{-5})$

Combining μH and H(1S-2S) measurements

Muonic hydrogen

$$E_{2S-2P}(\mu H) \approx \text{QED} + \kappa r_p^2 + \text{NS}$$

Input from proton structure



$+3\text{PE}$

Extract proton radius

$\delta = 4 \times 10^{-4}$

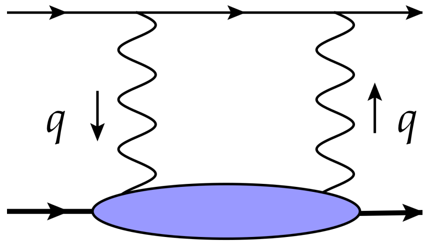
Benchmark for hadron theories

Combining μH and H(1S-2S) measurements

Muonic hydrogen

$$E_{2S-2P}(\mu H) \approx \mathrm{QED} + \kappa r_p^2 + \mathrm{NS}$$

Input from proton structure



$+3\mathrm{PE}$

Hydrogen

$$E_{1S-2S}(H) \approx \frac{3}{4} R_\infty + \mathrm{QED}' + k' r_p^2$$

$(\delta = 4 \times 10^{-15})$

Extract Rydberg constant

$$R_\infty = \frac{\alpha^2 m_e c}{2h}$$

$$\delta = 8 \times 10^{-13}$$

Fundamental constant needed for precision predictions in atoms, molecules, ions.

Adding for example the H(1S-3S).....

Muonic hydrogen

$$E_{2S-2P}(\mu H) \approx \text{QED} + \kappa r_p^2 + \text{NS}$$

$(\delta = 1 \times 10^{-5})$

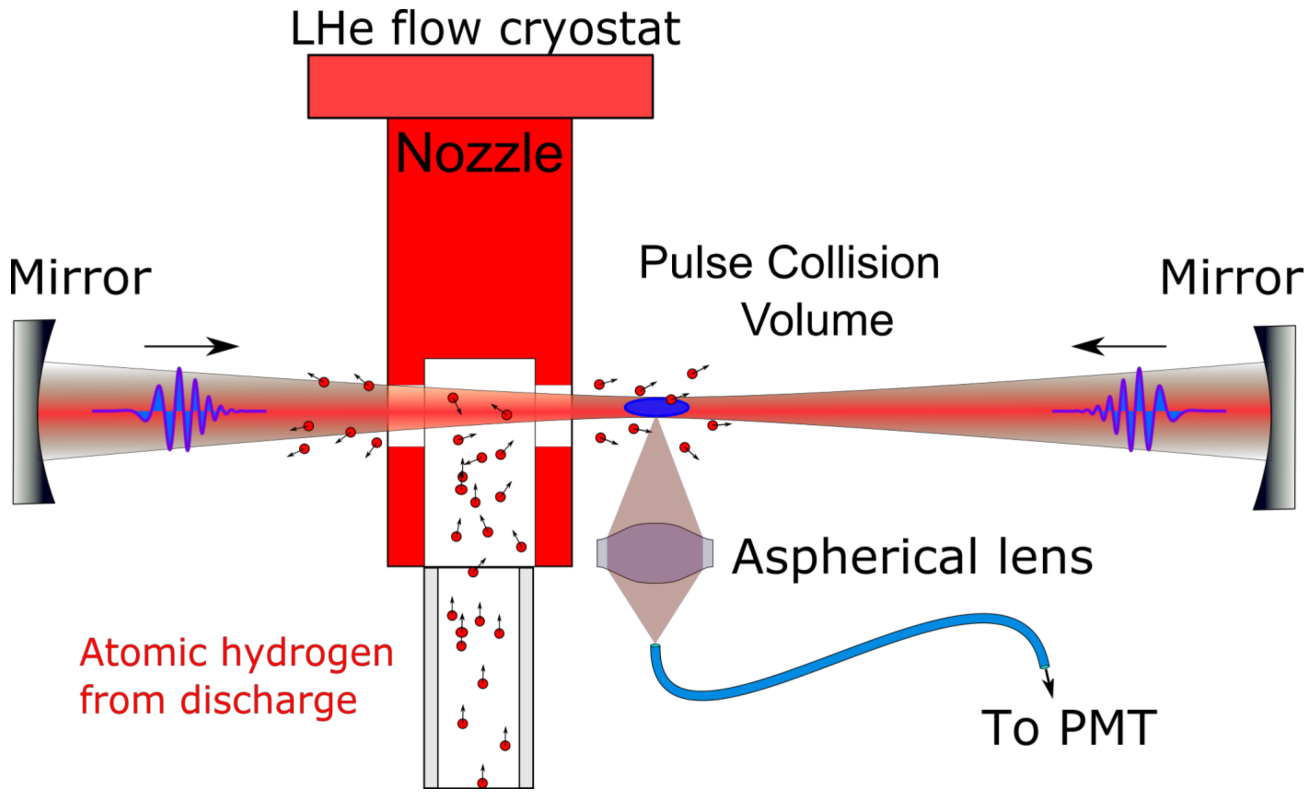
Hydrogen

$$E_{1S-2S}(H) \approx \frac{3}{4} R_\infty + \text{QED}' + k' r_p^2$$

$(\delta = 4 \times 10^{-15})$

$$E_{1S-3S}(H) \approx \frac{8}{9} R_\infty + \text{QED}'' + k'' r_p^2$$

$(\delta = 2.5 \times 10^{-13})$



Grinin et al. Science 370(6520):1061–1066 (2020)

Adding H(1S-3S).....

Muonic hydrogen

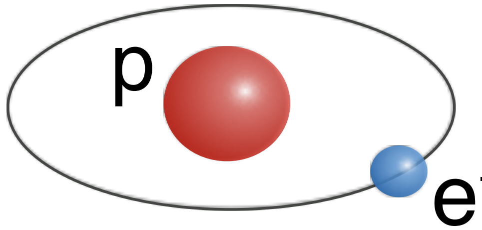
$$E_{2S-2P}(\mu H) \approx QED + \kappa r_p^2 + NS$$

Input from proton structure

Hydrogen

$$E_{1S-2S}(H) \approx \frac{3}{4} R_\infty + QED' + k' r_p^2$$

$$E_{1S-3S}(H) \approx \frac{8}{9} R_\infty + QED'' + k'' r_p^2$$



Test of H theory
Test of bound-states QED
$\delta \sim 1 \times 10^{-12}$

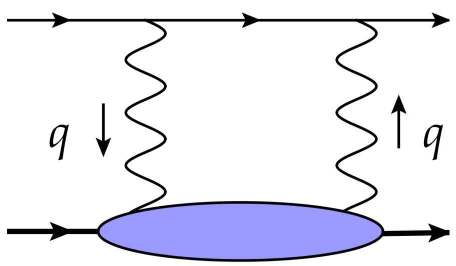
BSM limits

Adding H(1S-3S).....

Muonic hydrogen

$$E_{2S-2P}(\mu H) \approx QED + \kappa r_p^2 + NS$$

Test proton structure



$+3PE$

Theoretical tools

- dispersive
- sum rules
- chiral perturbation th.
- lattice QCD
- Nuclear structure contribution

Hydrogen

$$E_{1S-2S}(H) \approx \frac{3}{4}R_\infty + QED' + k' r_p^2$$

Combine

$$E_{1S-3S}(H) \approx \frac{8}{9}R_\infty + QED'' + k'' r_p^2$$

Adding HD$^+$ measurements

Muonic hydrogen

$$E_{2S-2P}(\mu H) \approx \mathrm{QED} + \kappa r_p^2 + \mathrm{NS}$$

$(\delta = 1 \times 10^{-5})$

Hydrogen

$$E_{1S-2S}(H) \approx \frac{3}{4} R_\infty + \mathrm{QED}' + k' r_p^2$$

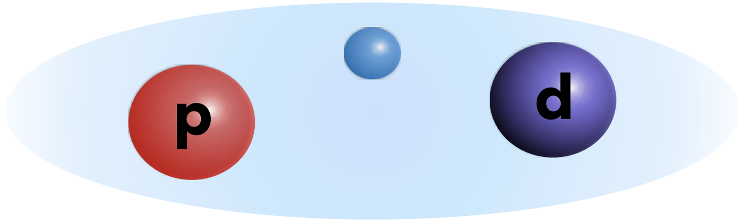
$(\delta = 4 \times 10^{-15})$

HD isotope shift

HD$^+$

$$E(\nu\, L \rightarrow \nu'\, L') = R_\infty \mathrm{QED}_{3-\mathrm{body}}\left(\alpha, \frac{m_e}{m_p}, \frac{m_p}{m_d}, r_p, r_d\right)$$

$\delta = \mathcal{O}(10^{-11} - 10^{-12})$



Karr et al., Springer Proc. Phys. 238:75–81 (2020)
Alighanbari et al., Nature 581(7807):152–158 (2020)
Patra et al., Science 369(6508):1238–1241 (2020)

Adding Penning traps measurements

Muonic hydrogen

$$E_{2S-2P}(\mu H) \approx \text{QED} + \kappa r_p^2 + \text{NS}$$

$(\delta = 1 \times 10^{-5})$

Hydrogen

$$E_{1S-2S}(H) \approx \frac{3}{4} R_\infty + \text{QED}' + k' r_p^2$$

$(\delta = 4 \times 10^{-15})$

HD isotope shift

HD+

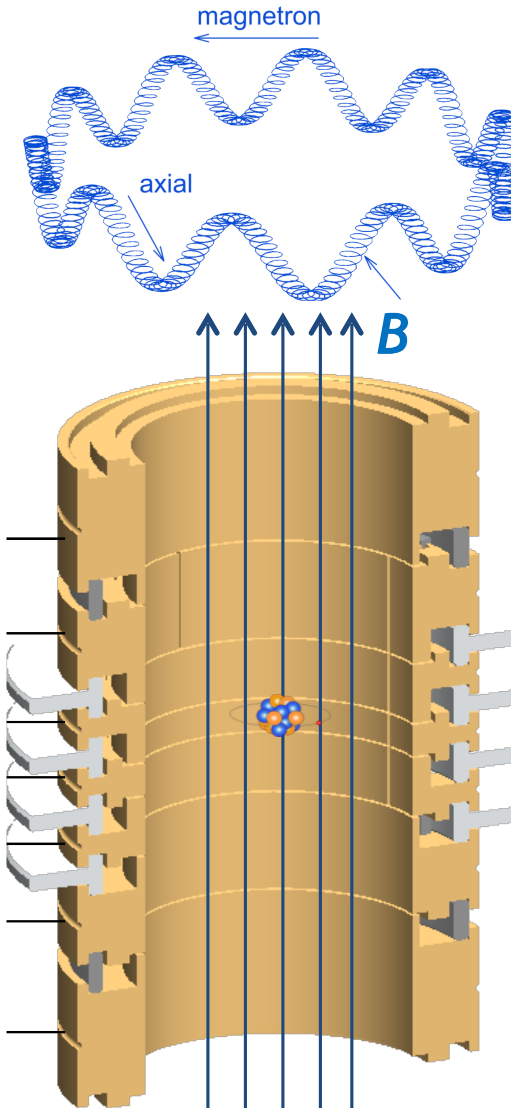
$$E(\nu\, L \to \nu'\, L') = R_\infty \text{QED}_{3-\text{body}}\left(\alpha, \frac{m_e}{m_p}, \frac{m_p}{m_d}, r_p, r_d\right)$$

$\delta = \mathcal{O}(10^{-11} - 10^{-12})$

Penning traps

g-factors, $\frac{\omega_p}{\omega_{^{12}C^{5+}}}$ ...

$\delta \sim \mathcal{O}(10^{-11})$



Heiße et al. Phys. Rev. A 100(2):022518 (2019)
Sturm et al. Nature 506(7489):467–470 (2014)

Combining measurements in µp, H, HD⁺ and Penning-traps

Muonic hydrogen

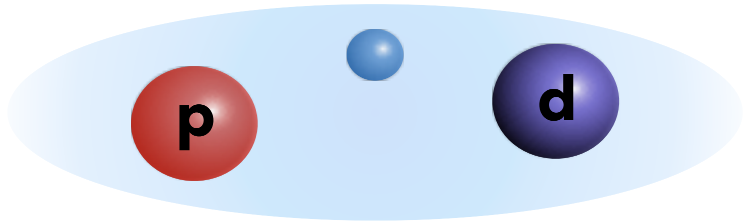
$$E_{2S-2P}(\mu H) \approx \mathrm{QED} + \kappa r_p^2 + \mathrm{NS}$$

Hydrogen

$$E_{1S-2S}(H) \approx \frac{3}{4} R_\infty + \mathrm{QED}' + k' r_p^2$$

HD+

$$E(\nu\, L \to \nu'\, L') = R_\infty \mathrm{QED}_{3-\mathrm{body}}\left(\alpha, \frac{m_e}{m_p}, \frac{m_p}{m_d}, r_p, r_d\right)$$



Test of HD+ theory
Test of 3-body QED
$\delta \sim 10^{-11}$

BSM limits

Penning traps

g-factors, $\frac{\omega_p}{\omega_{^{12}C^{5+}}}$ ...

Combining measurements in μp, H, HD+ and Penning-traps

Muonic hydrogen

$$E_{2S-2P}(\mu H) \approx \mathrm{QED} + \kappa r_p^2 + \mathrm{NS}$$

Hydrogen

$$E_{1S-2S}(H) \approx \frac{3}{4} R_\infty + \mathrm{QED}' + k' r_p^2$$

HD+

$$E(\nu\, L \to \nu'\, L') = R_\infty \mathrm{QED}_{3-\mathrm{body}}\left(\alpha, \frac{m_e}{m_p}, \frac{m_p}{m_d}, r_p, r_d\right)$$

Extract m_e/m_p ratio

$\delta = 2 \times 10^{-11}$

Best determination of the electron mass

Combining measurements in μp, H, HD^+ and Penning-traps

Muonic hydrogen

$$E_{2S-2P}(\mu H) \approx \text{QED} + \kappa r_p^2 + \text{NS}$$

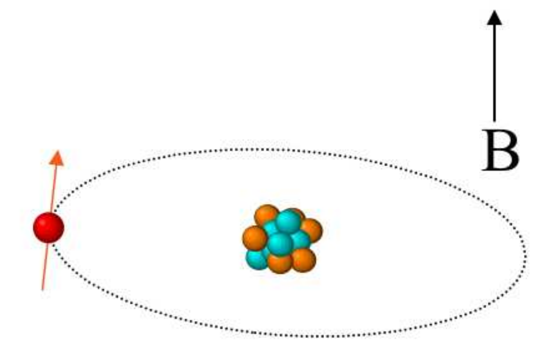
Hydrogen

$$E_{1S-2S}(H) \approx \frac{3}{4}R_\infty + \text{QED}' + k' r_p^2$$

HD+

$$E(\nu\, L \to \nu'\, L') = R_\infty \text{QED}_{3-\text{body}}\left(\alpha, \frac{m_e}{m_p}, \frac{m_p}{m_d}, r_p, r_d\right)$$

m_e/m_p



Penning traps measurements

Test of bound g-factors

$$\delta \sim 4 \times 10^{-11}$$

Two-body QED tests in H-like systems
QED test in simple molecules
BSM searches
Fundamental constants
Three-body QED tests in He-like systems
Hadron structure
Radiative corrections
Nuclear structure
High-precision laser spectroscopy
Penning traps program
Scattering experiments
Muonic atom spectroscopy

The CREMA collaboration

JG|U
JOHANNES GUTENBERG UNIVERSITÄT MAINZ

MAX-PLANCK-INSTITUT FÜR QUANTENOPTIK GARCHING
MPQ

Laboratoire Kastler Brossel
Physique quantique et applications

國立清華大學
NATIONAL TSING HUA UNIVERSITY

ETH

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