Pseudoscalar mesons and Emergent Hadronic Mass in the Standard Model

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and many more…

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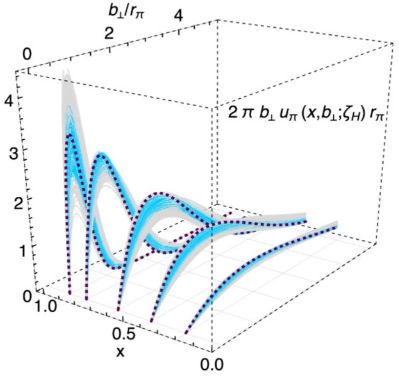
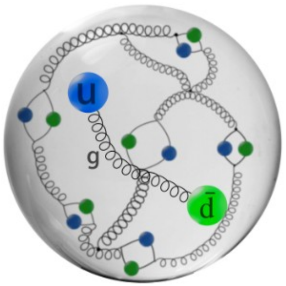




EFB25 - Mainz, Germany

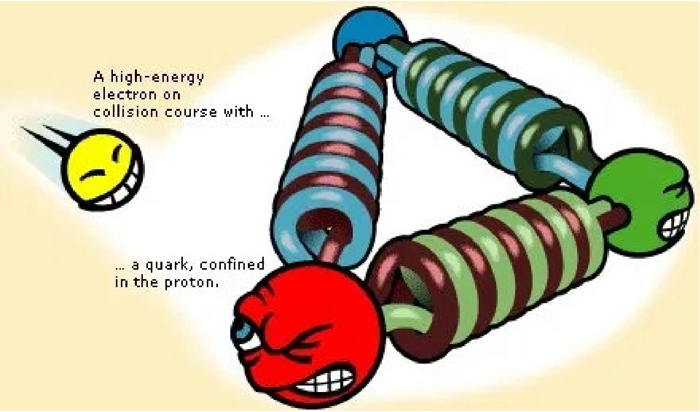
July 30 – August 4, 2023

QCD: Emergent Phenomena

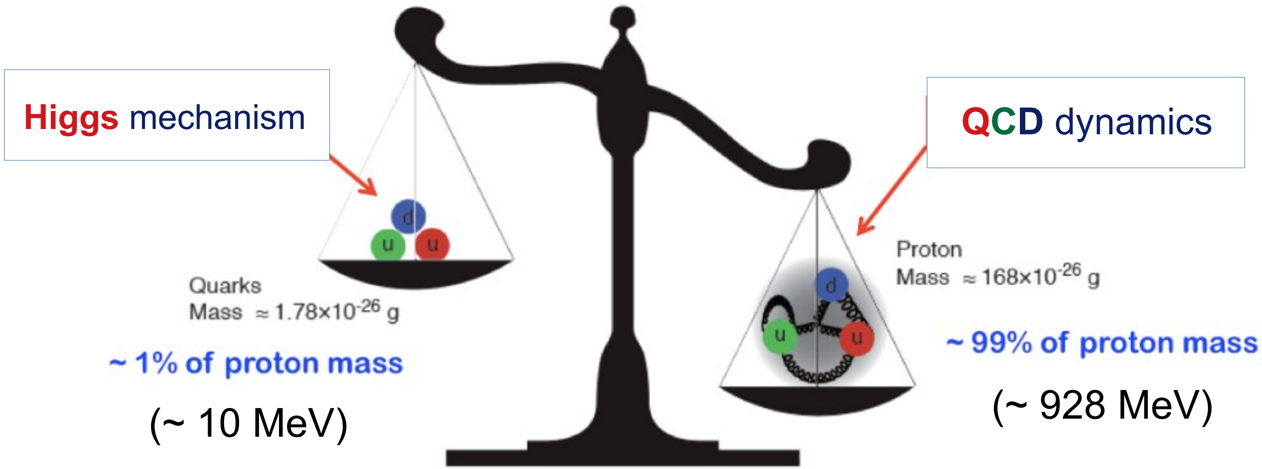
- **QCD** is characterized by two **emergent** phenomena: confinement and dynamical generation of mass (DGM).

$$\mathcal{L}_{\rm QCD} = \sum_{j=u,d,s,\ldots} \bar{q}_j[\gamma_\mu D_\mu + m_j]q_j + \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu},$$
$$D_\mu = \partial_\mu + ig\frac{1}{2}\lambda^a A^a_\mu\,,$$
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu + \partial_\nu A^a_\mu - \underline{gf^{abc}A^b_\mu A^c_\nu},$$

- Quarks and gluons not *isolated* in nature.
  - Formation of colorless bound states: "**Hadrons**"
  - **1-fm scale** size of hadrons?



- Emergence of hadron masses (EHM) from QCD **dynamics**



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Can we trace them down to fundamental d.o.f?

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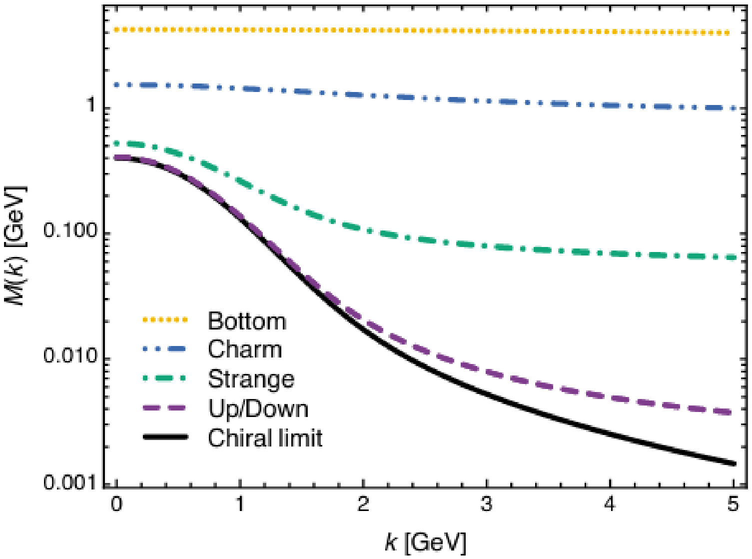
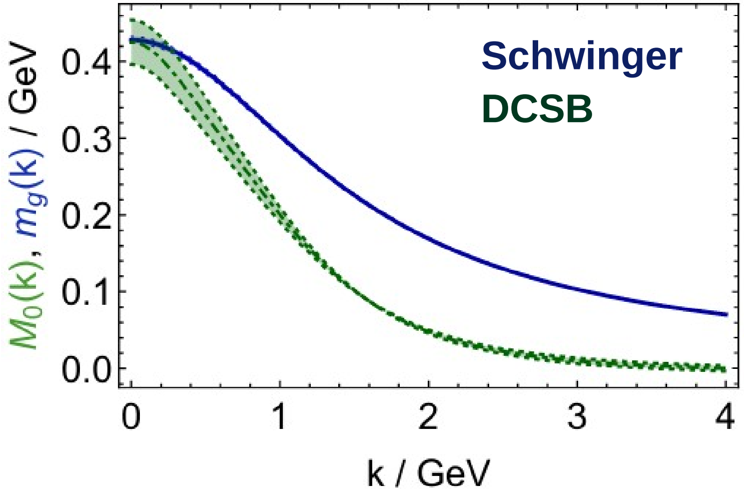
Dynamical masses



"Higgs" masses

$$S_f^{-1}(p) = Z_f^{-1}(p^2)(i\gamma\cdot p + \mathrm{M_f(p^2)})$$



Gluon and quark *running masses*

QCD: Emergent Phenomena

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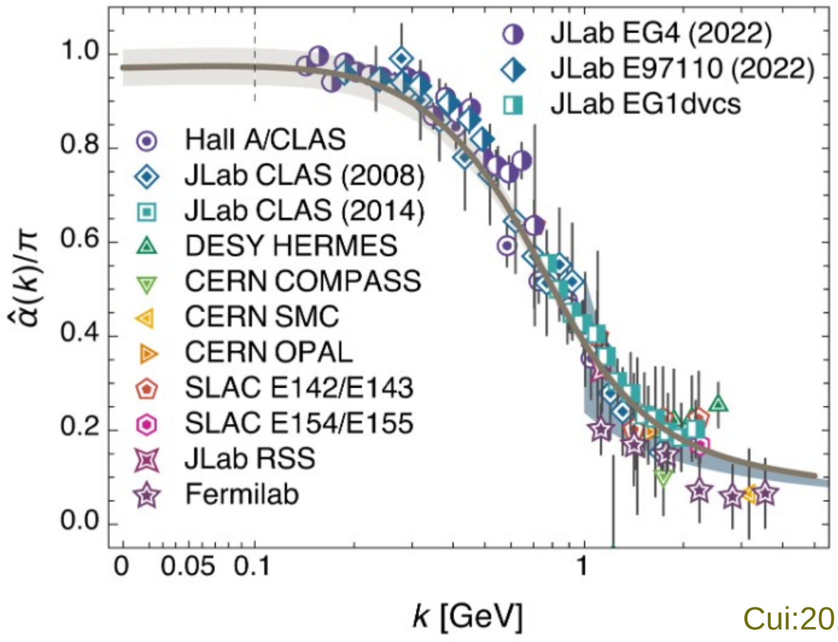
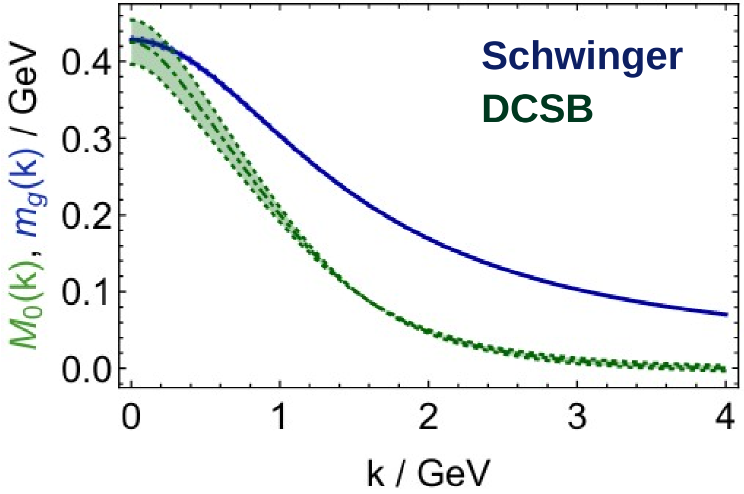


Cui:2019dwv



Gluon and quark *running masses*

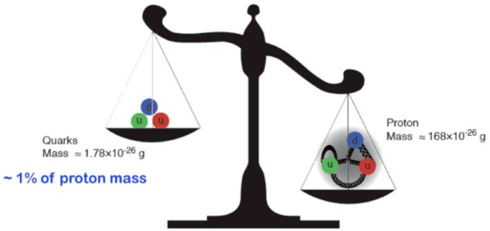
Mass **Budgets**

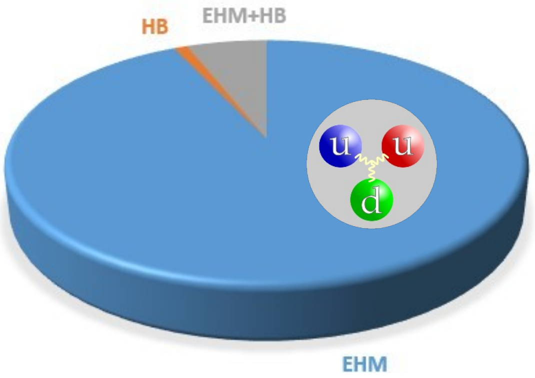
$$M_{u/d} \approx 0.3\,\mathrm{GeV}$$

- What is the origin of **EHM**?

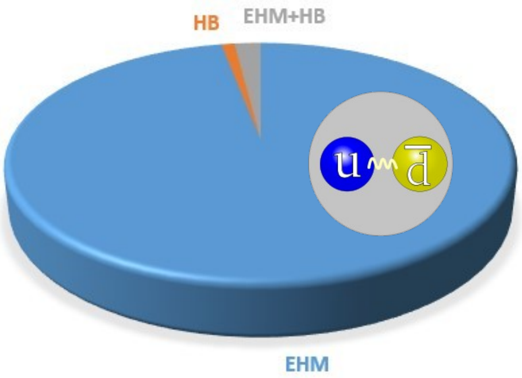
  … its connection with *e.g.* **confinement** and **DCSB**?

> - **Most** of the **mass** in the visible universe is contained within **nucleons**
>   - Which remain **pretty massive** whether there is Higgs mechanism or not...





Proton mass budget



Rho meson mass budget

$$m_p = 0.938\,\mathrm{GeV} \approx 2M_u + M_d$$

$$m_\rho = 0.775\,\mathrm{GeV} \approx M_u + M_d$$

Proton and **rho** meson mass budgets are practically **identical**

Mass **Budgets**

$m_s/m_u \sim 20$

$M_{u/d} \approx 0.3\,\mathrm{GeV}$ $\qquad$ $M_s/M_u \sim 1.2$

- What is the origin of **EHM**?

  … its connection with e.g. **confinement** and **DCSB**?

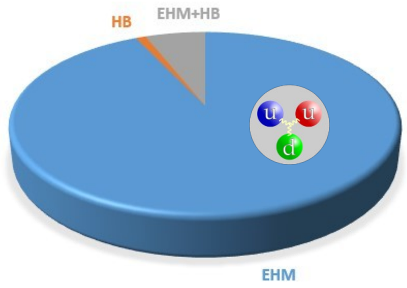
  And **Nambu-Goldstone** bosons?

- Unlike *e.g.* proton and ρ meson, **pion and Kaon** would be **massless** in the absence of **Higgs** mass generation.

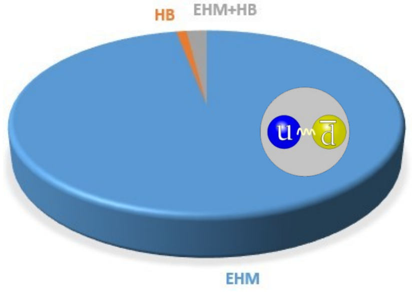
  - And **structurally alike**.

$$m_\pi = 0.14\,\mathrm{GeV} \neq M_u + M_d$$

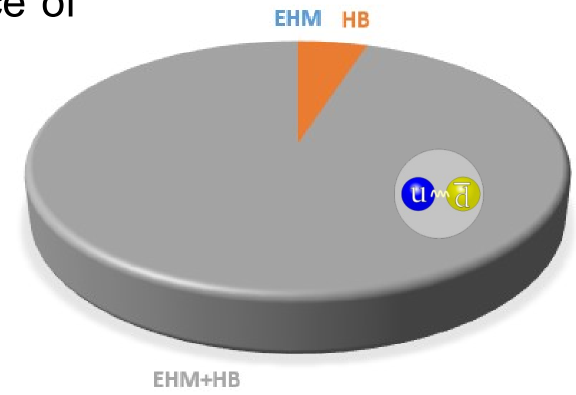
$$m_K = 0.49\,\mathrm{GeV} \neq M_u + M_s$$



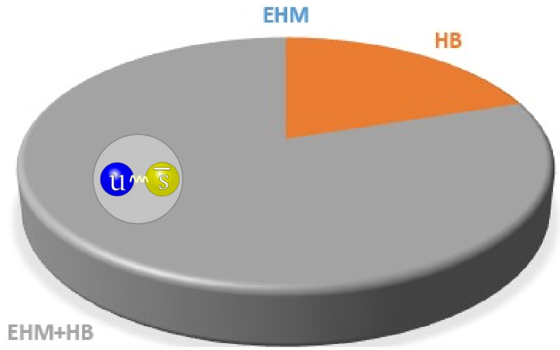
Proton mass budget



Rho meson mass budget



Pion mass budget



Kaon mass budget

Mass **Budgets**

$m_s/m_u \sim 20$

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$$m_\pi = 0.14\,\text{GeV} \neq M_u + M_d$$

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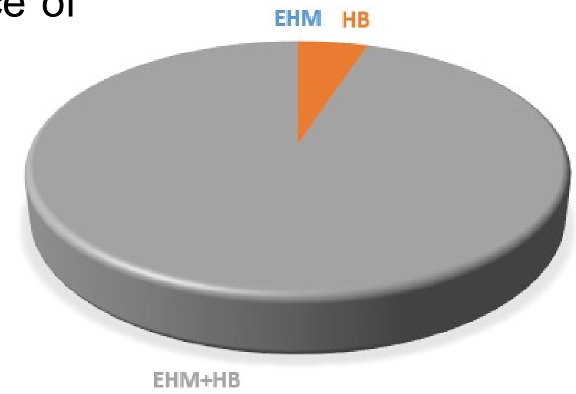
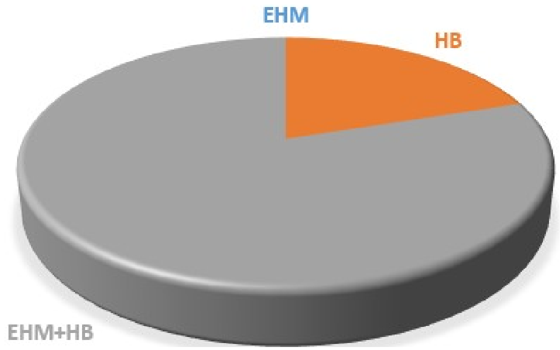
Pion and Kaon

- Both quark-antiquark **bound-states** and **NG bosons**
  - Their mere **existence** is connected with **mass** generation in the **SM**



Pion mass budget



Kaon mass budget

Mass **Budgets**

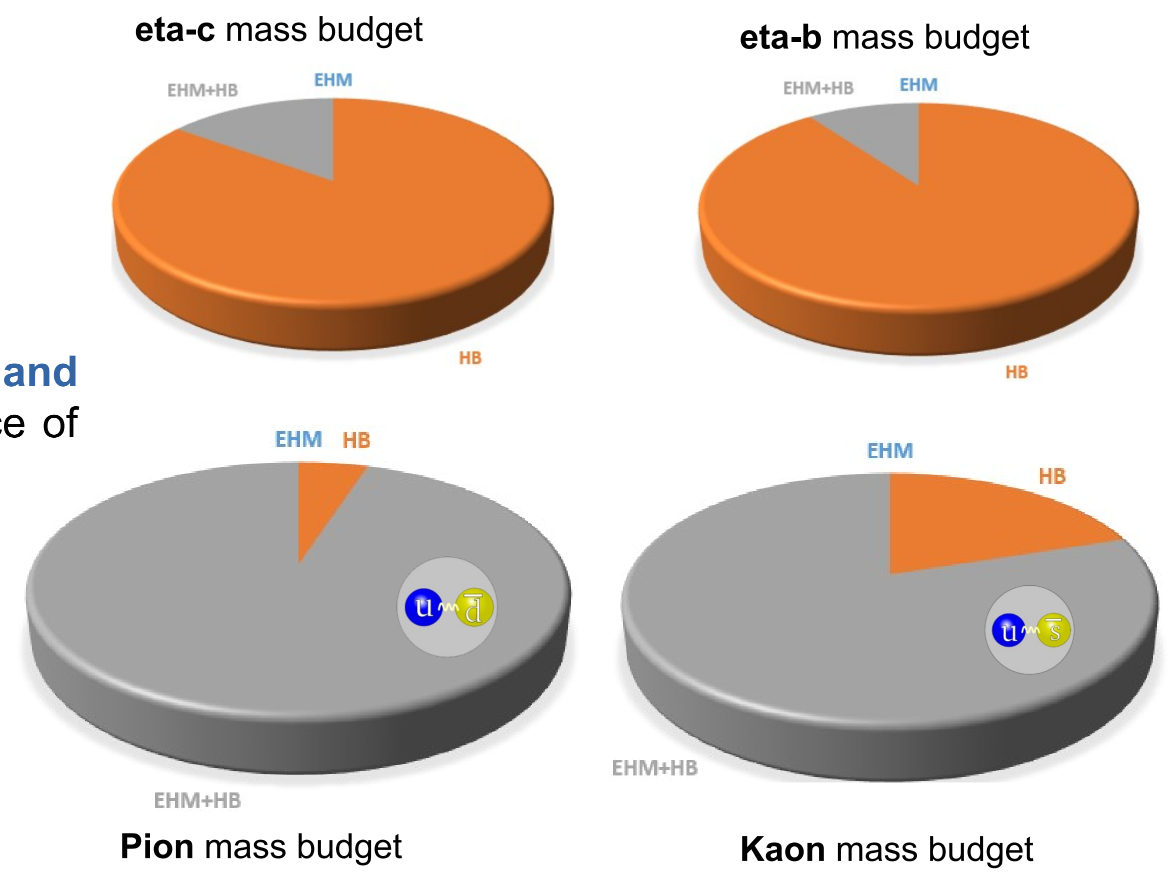
$M_{u/d} \approx 0.3\,\mathrm{GeV}$ $m_s/m_u \sim 20$ $M_s/M_u \sim 1.2$

- What is the origin of **EHM**?

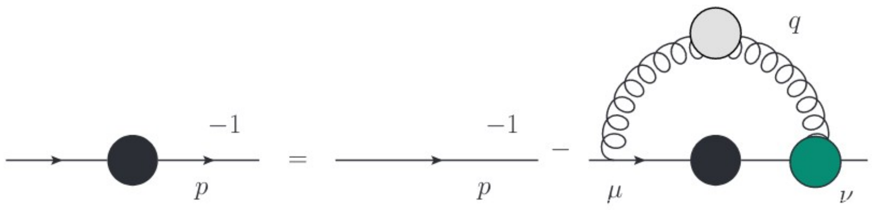
  … its connection with e.g. **confinement** and **DCSB**?

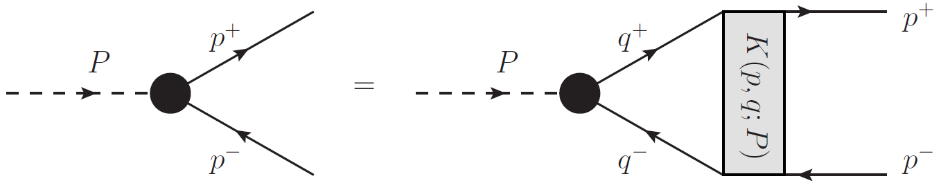
  And **Nambu-Goldstone** bosons?

- Unlike *e.g.* proton and ρ meson, **pion and kaon** would be **massless** in the absence of **Higgs** mass generation.
  - And **structurally alike**.
- The scrutiny of their heavier counterparts reveals the role of **weak mass generation** on the hadron **structural properties**.



Continuum Schwinger Methods (CSM)

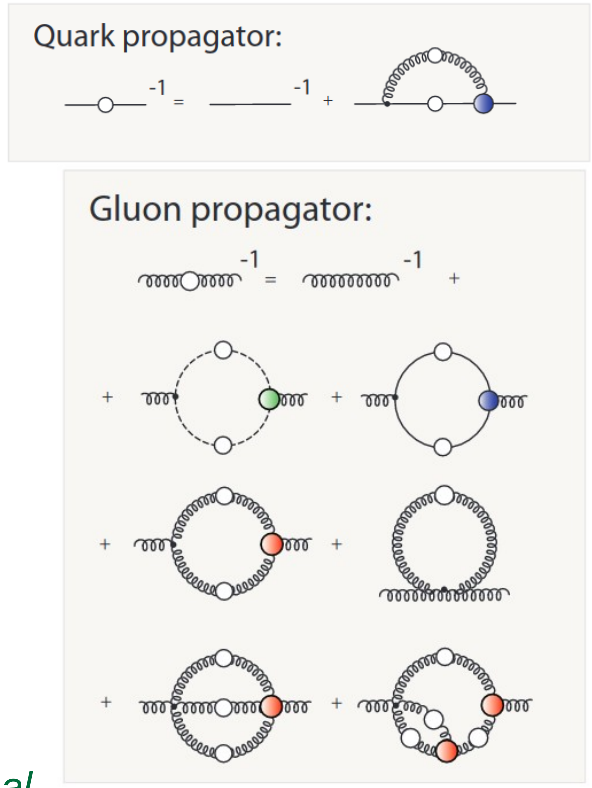




Dyson-Schwinger Equations

- Equations of motion of a **quantum field theory**
- Relate Green functions with higher-order Green functions
  - **Infinite** tower of coupled equations.
  - ✗ Systematic **truncation** required
- ✓ **No assumptions** on the **coupling** for their derivation.
  - ✓ Capture both **perturbative** and **non-perturbative** facets of **QCD**
- ✓ **Not limited** to a certain domain of current **quark masses**
- ✓ Maintain a **traceable connection** to QCD.

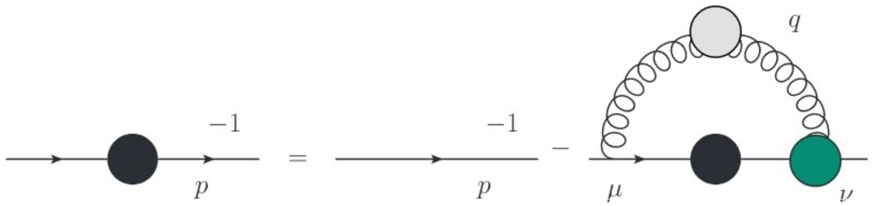
Example DSEs



C.D. Robert and A.G. Williams,
Prog.Part.Nucl.Phys. 33 (1994) 477-575

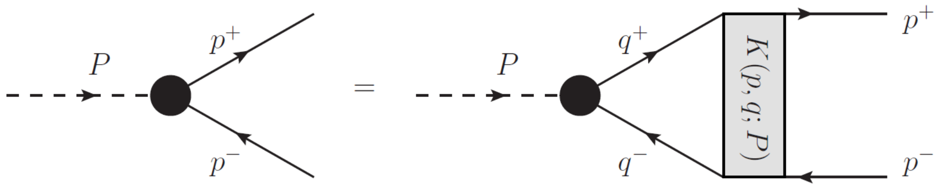
G. Eichmann, H. Sanchis-Alipus *et al.*
Prog.Part.Nucl.Phys. 91 (2016) 1-100

DSE-BSE approach



Quark DSE

- Relates the quark propagator with **QGV** and **gluon propagator**.



Meson BSE

- Contains **all interactions** between the valence quark and antiquark

- Any <u>sensible truncation</u> must preserve the **Goldstone's Theorem**, whose most fundamental expression is captured in:

"**Pions** exists, if and only if, **DCSB** occurs."

$$f_\pi E_\pi(k; P = 0) = B(k^2)$$

Leading BSA — "***Mass*** Function"

L. Chang *et al.*,
Phys.Rev.Lett. 110 (2013) 13, 132001

Valence-quark distribution amplitudes (PDAs)

$$f_M \phi_M^q(x) = \mathrm{tr} \int_{dk} \delta_n^x(k_M) \gamma_5 \gamma \cdot n \chi_M(k_-, P)$$

Light-front momentum fraction

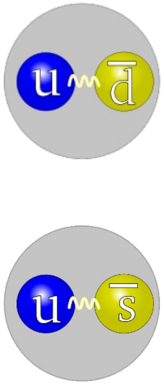
Written in terms of **BSWF**

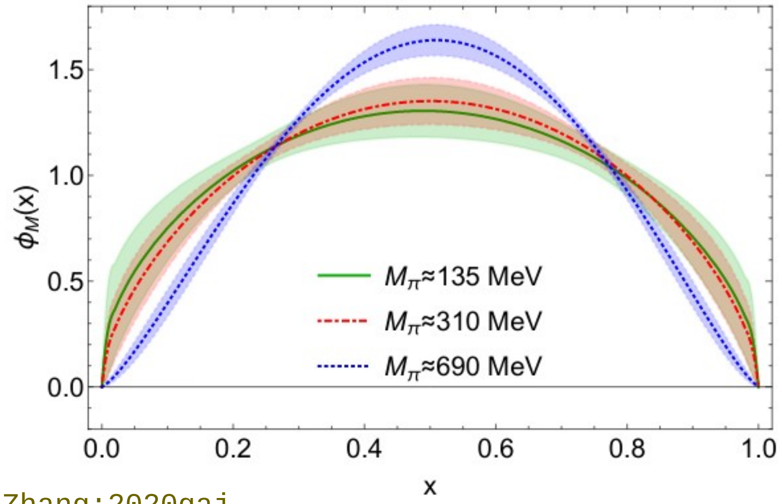
- Analogous with **quantum mechanic's** wave function (sort of).
- Clear **probe of EHM**, related with hard **exclusive processes**, etc.

π-K **PDAs**

$m_s/m_u \sim 20$
$M_s/M_u \sim 1.2$

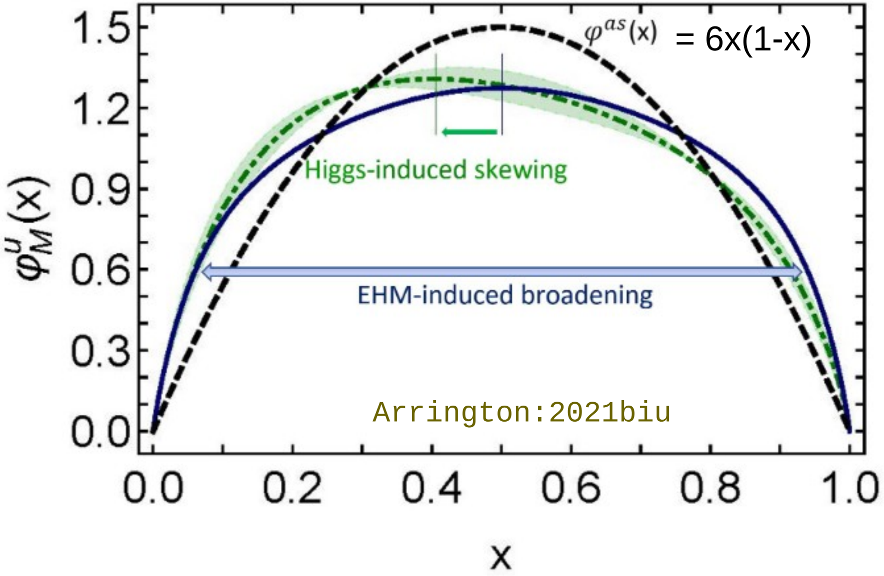
- **Broad** and concave **PDAs**.   @ real life (order of GeVs)
- Dominated by **QCD dynamics**.
- **Mild** skewing in **Kaon**: **strong** & **weak** interplay.





Zhang:2020gaj

[Blue]: **Pion**   [Green]: **Kaon**



- Lattice QCD supports those findings:

Zhang:2020gaj   Bali:2019dqc   Segovia:2013eca

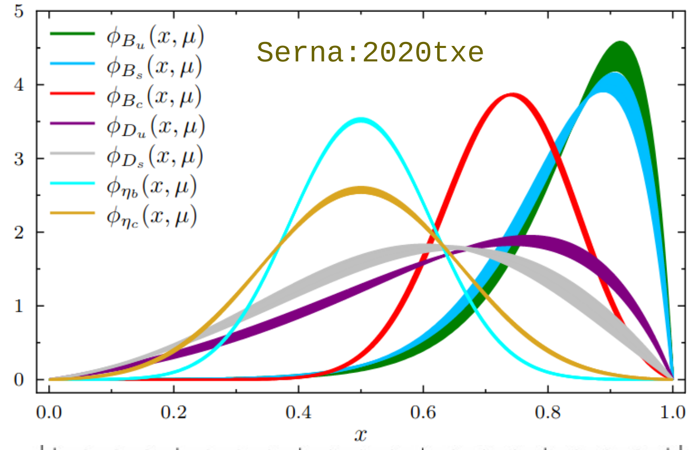
Pointwise form of the **PDA**!.

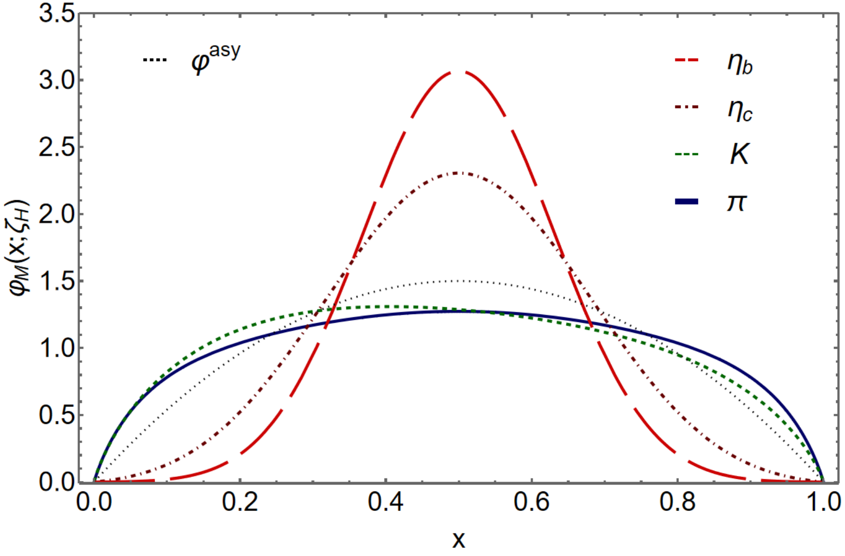
Heavy mesons PDAs

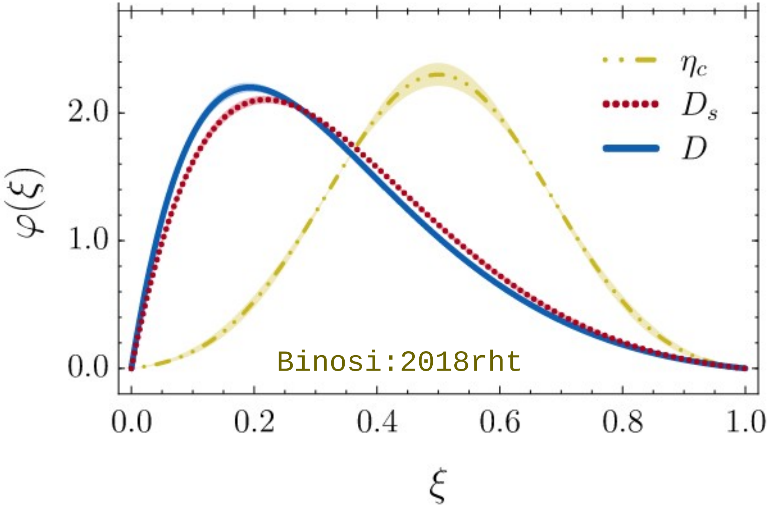
- Largely influenced by **Higgs** mass generation.
  - ➔ **Narrow** PDAs.

@ real life
(order of GeVs)

- Narrowness also observed in **heavy-light** mesons. →



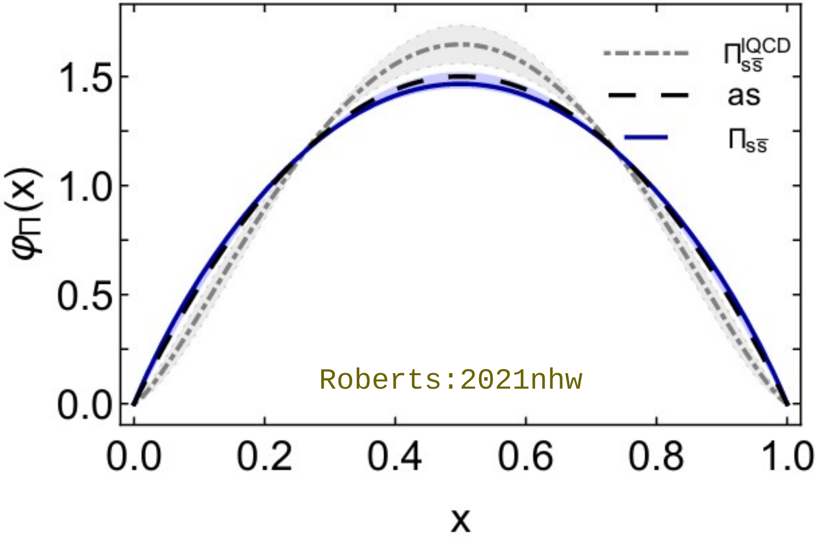




‘Strange’ PDAs

- **s-quark** mass: **interplay** between **strong** and **Higgs** mass-generation.
- **PDAs** lie **near** the **asymptotic distribution**.

@ any scale

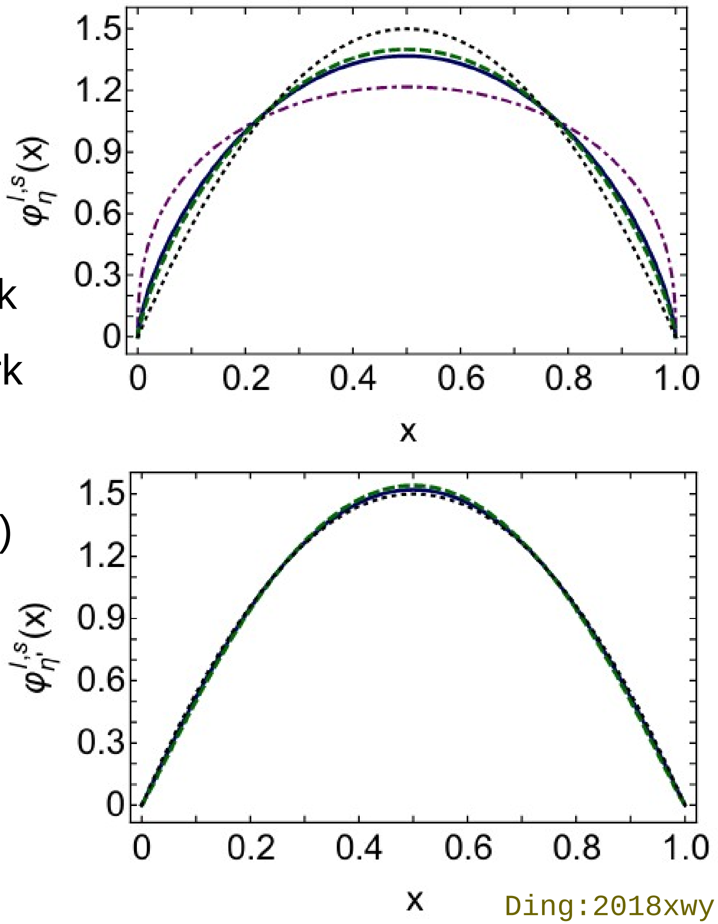


[Blue]: u-quark

[Green]: s-quark

[Purple]: Pion

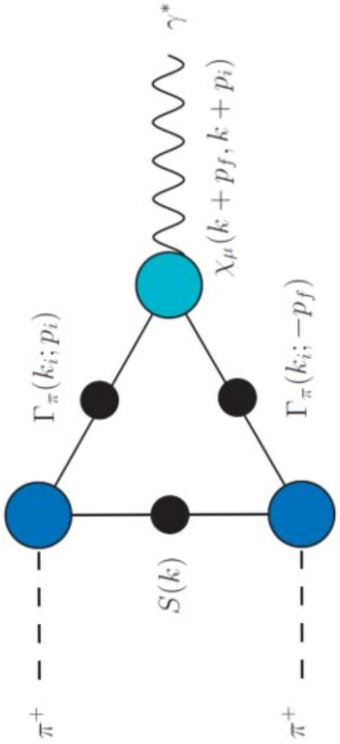
[Black]: 6x(1-x)



Ding:2018xwy

L. Chang *et al.*,
Phys.Rev.Lett. 111 (2013) 14, 141802

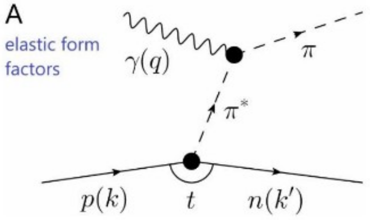
Electromagnetic Elastic Form Factors (EFFs)



$$K_\mu F_M(Q^2) = N_c \text{tr} \int_{dk} \chi_\mu(k+p_f, k+p_i)\Gamma_M(k_i;p_i)S(k)\gamma_M(k_f;-p_f)$$

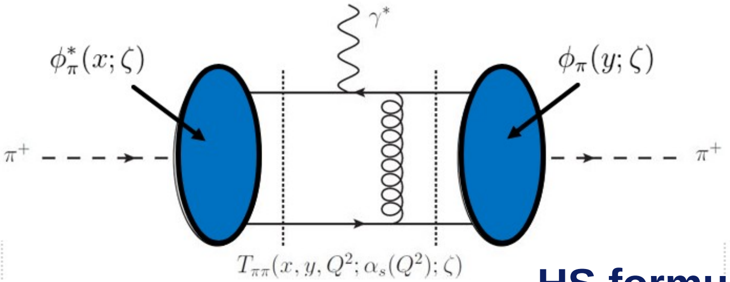
All can be written in terms of **propagators** and **vertices**

- Gives information on **momentum**/**charge** distribution.
- **Pion EFF** highly relevant for contemporary physics.



Elastic **Form Factors**

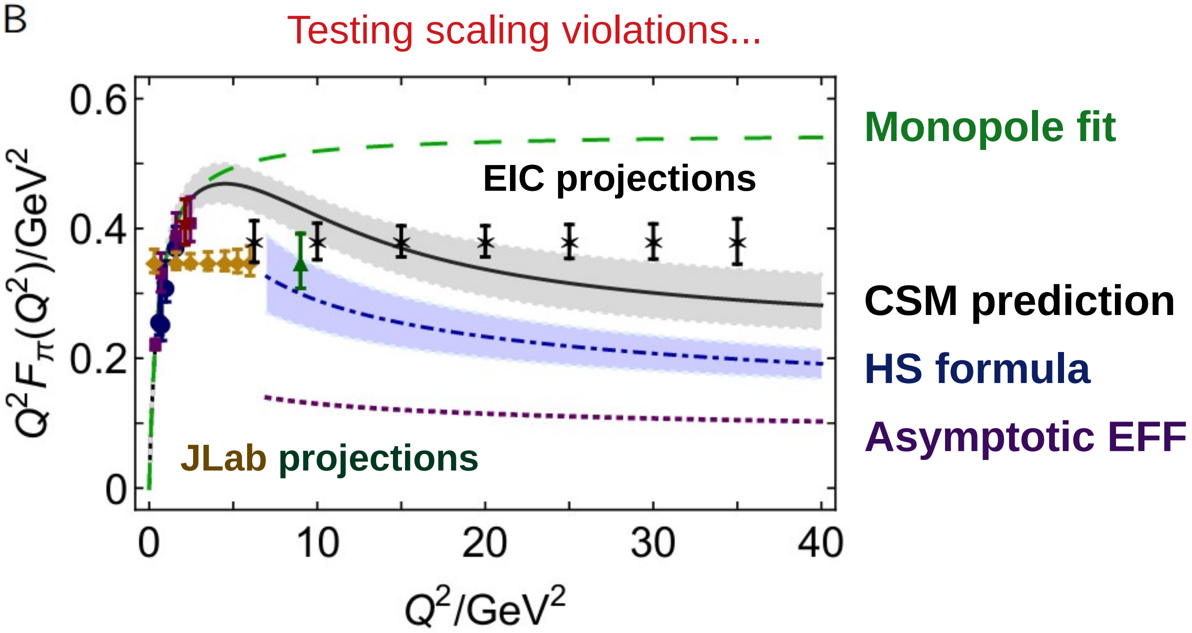
- Clear probe of the hadron's structure.
  - ➔ Structure manifests in $F(Q^2)$!= constant
- **Connected** with the **PDA**:



HS formula

(at sufficiently large Q^2)

- ➔ **Factorization** is a proof of the validity of **QCD itself.**



$$\exists Q_0 > \Lambda_{\mathrm{QCD}} \mid Q^2 F_\pi(Q^2) \overset{Q^2>Q_0^2}{\approx} 16\pi\alpha_s(Q^2) f_\pi^2 w_\varphi^2,$$

$$w_\varphi = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\varphi_\pi(x)$$ ← **PDA**

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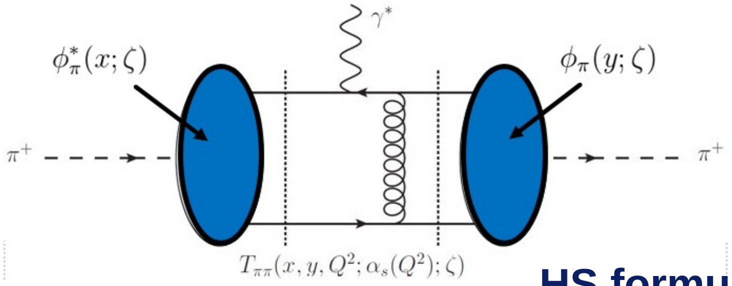

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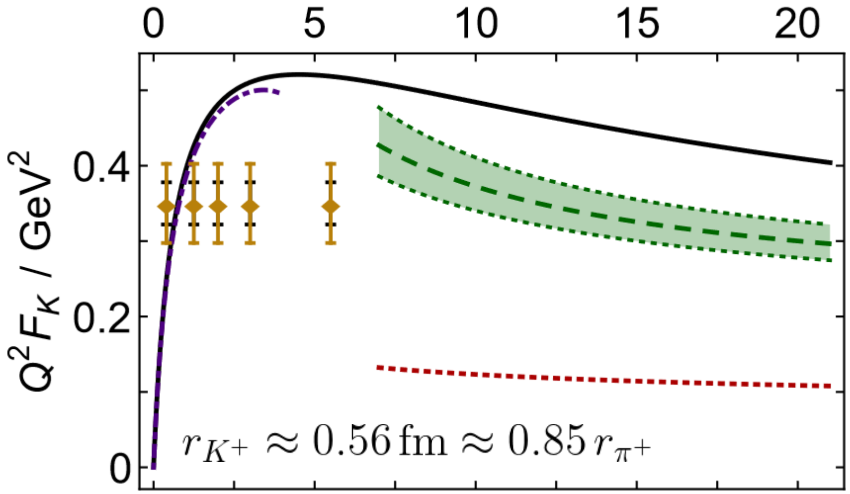
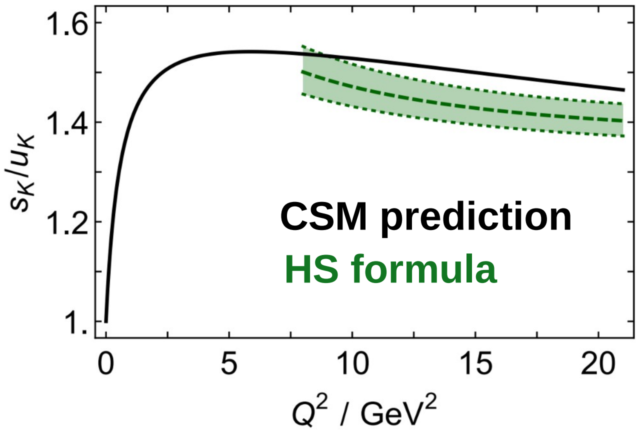


CSM prediction

HS formula

Asymptotic EFF

Gao:2017mmp

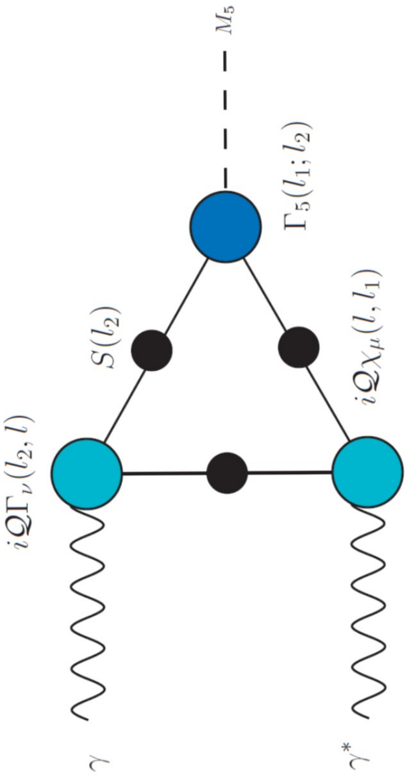


Flavor separation:

Height of ratio controlled by the interference of **weak** and **strong** mass generation.

KR, L. Chang *et al.*,
Phys.Rev.D 93 (2016) 7, 074017

Two-photon Transition Form Factors (TFFs)

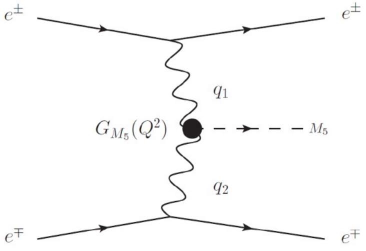


$$T_{\mu\nu}(k_1,k_2)=\epsilon_{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}G_{M_5}(k_1^2,k_1\cdot k_2,k_2^2)\ ,$$

$$T_{\mu\nu}(k_1,k_2)=\mathrm{tr}\int\frac{d^4l}{(2\pi)^4}iQ\chi_\mu(l,l_1)\Gamma_{M_5}(l_1,l_2)S(l_2)iQ\Gamma_\nu(l_2,l)$$

All can be expressed in terms of **propagators** and **vertices**

- Gives information on **momentum/charge** distribution.
- **Pion TFF** highly relevant for contemporary physics.



Two-photon **TFFs**

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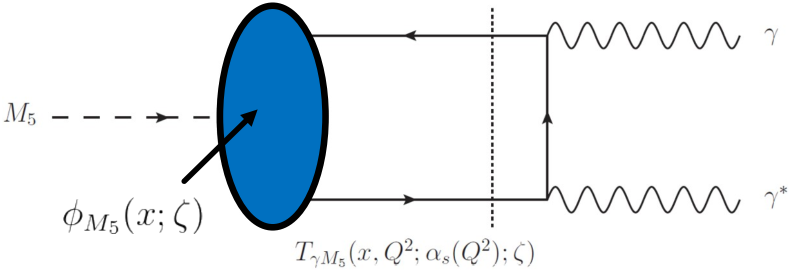

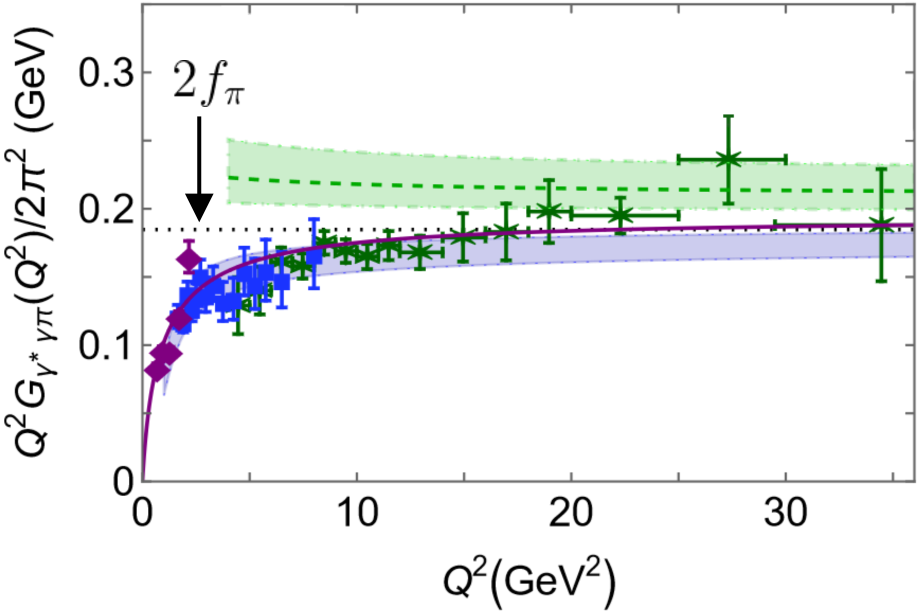

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(at sufficiently large Q^2)

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$$w_\varphi = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\varphi_\pi(x)$$ ← **PDA**

- ➔ **Factorization** is a proof of the validity of **QCD itself.**

Two-photon TFFs

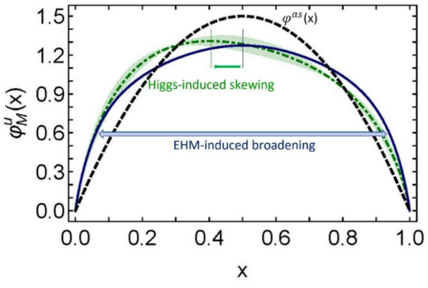
KR, L. Chang, A. Bashir *et al.*,
Phys.Rev.D 93 (2016) 7, 074017

- The **CSM prediction** satisfies the **Abelian anomaly**, while being faithfully recovering the **asymptotic limit**.

- A **dilated+concave PDA**, at the hadronic scale, connects both pion **EFF** and **TFF**.

- Precise **agreement** with ***all*** experimental data; except for Babar at large **Q²**.

  → Cannot conciliate with Babar...



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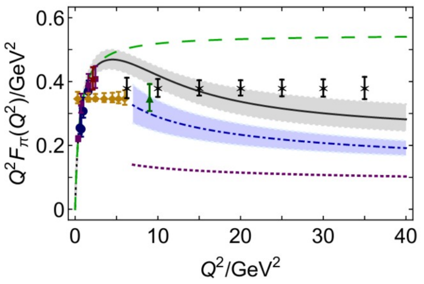
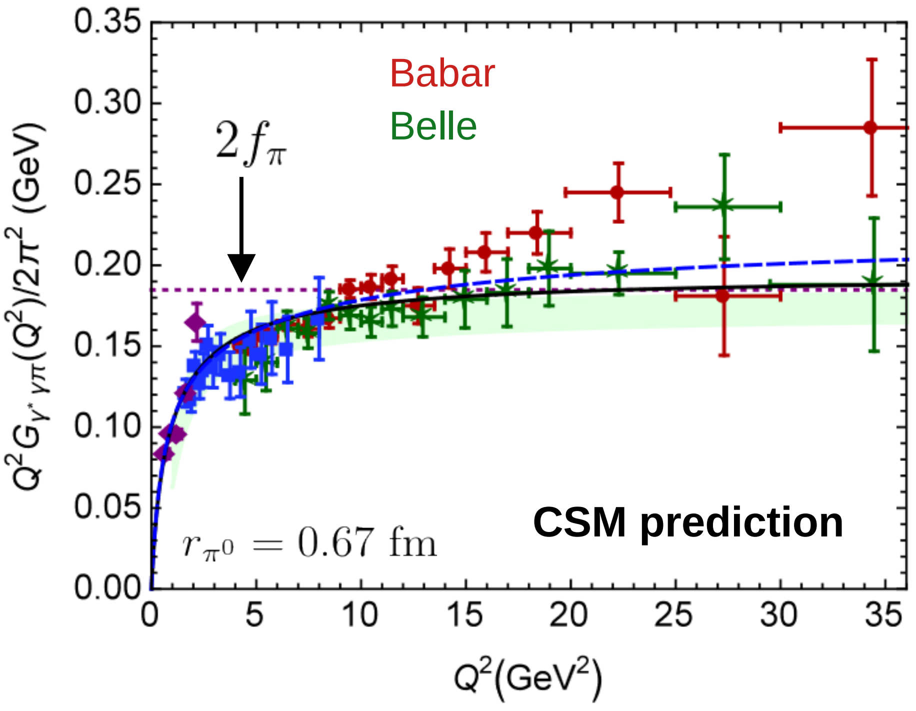




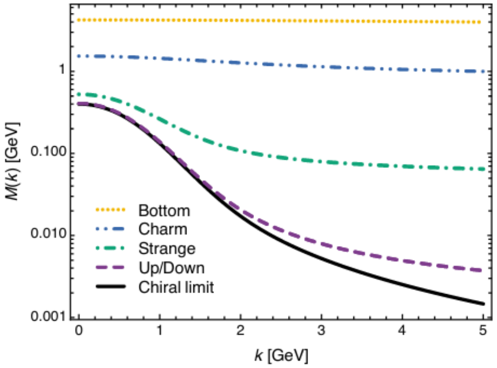
$$\exists \tilde{Q}_0 > \Lambda_{QCD} \,|\, Q^2 G_5(Q^2) \overset{Q^2>\tilde{Q}_0^2}{\sim} f_5\, w_\phi$$

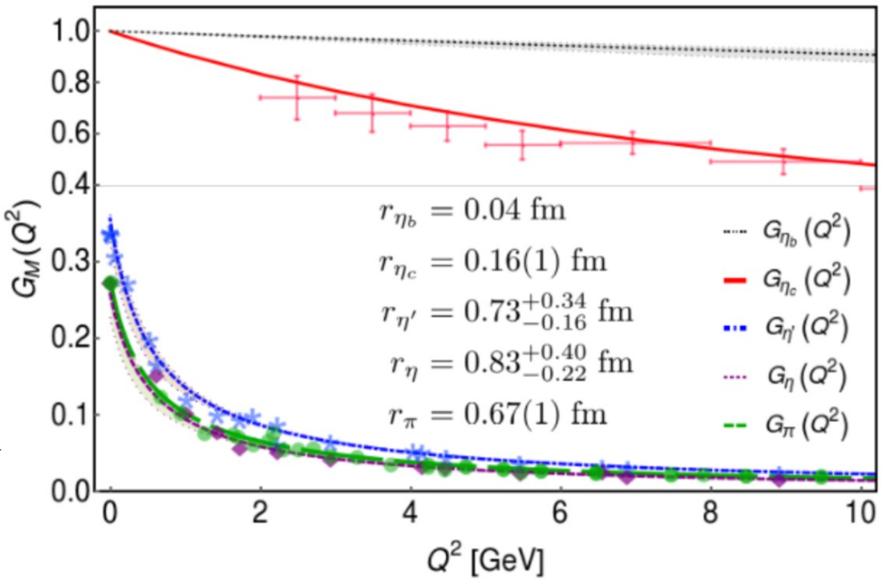
Two-photon **TFFs**

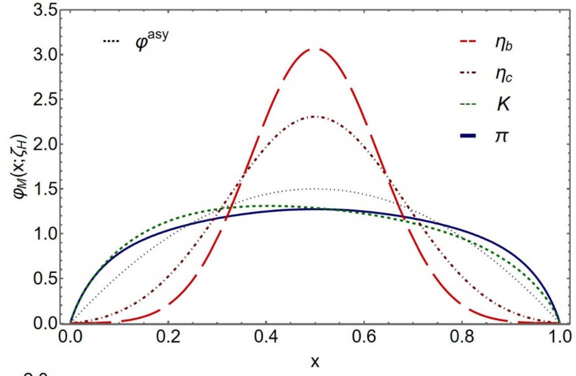
KR, A. Bashir, P. Roig
Phys.Rev.D 101 (2020) 7, 074021

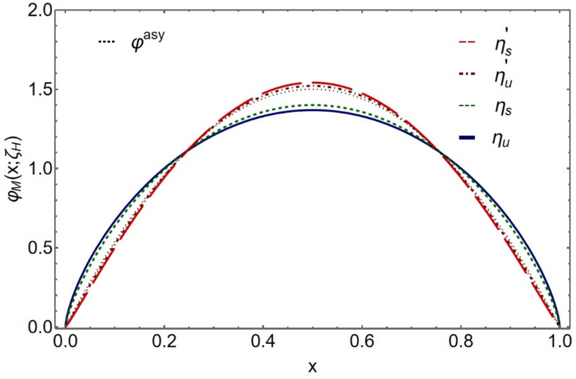
- ***All*** two-photon **TFFs** involving ground-state neutral pseudoscalars are within reach:
  - Invariably, **agreement** with the **experimental** data is found, with the exception of the large-Q^2 Babar data for the pion.

- Clearly, the shape of **M(k)** echoes in **TFFs** and **PDAs.**



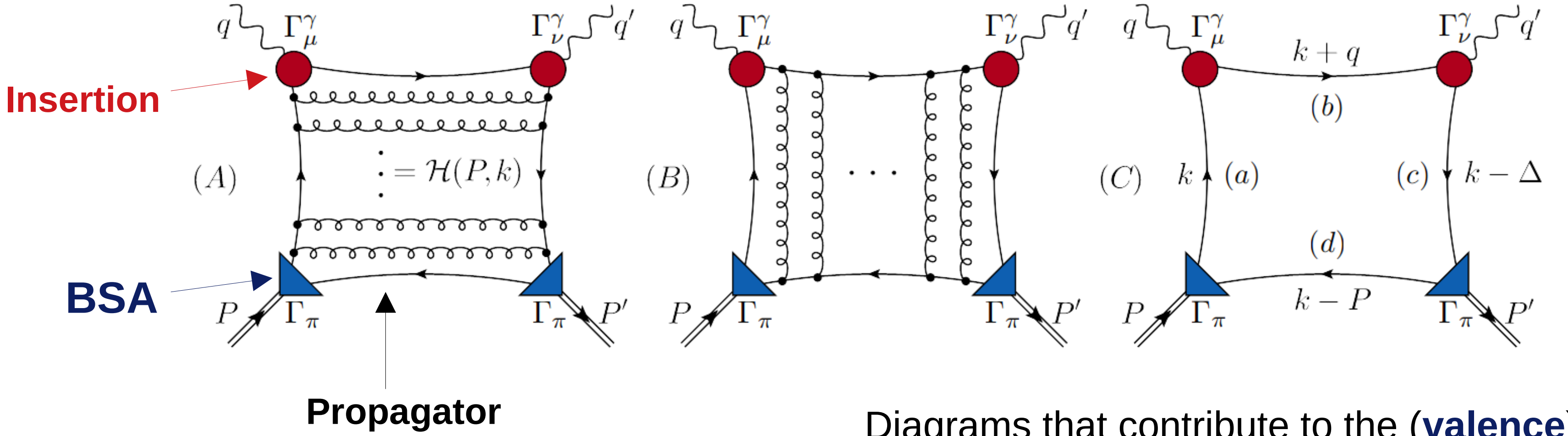




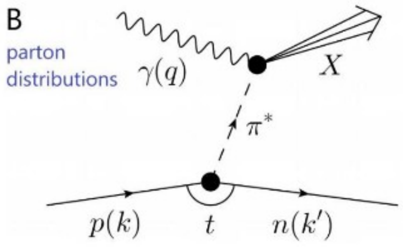


M. Ding *et al.*, PRD 101 5, 054014 (2020)
Z-F Cui *et al.*, EPJC 80 11, 1064 (2021)

Distribution functions (PDFs)



Diagrams that contribute to the (**valence**) **PDF**



- Yields information on **momentum** distribution.
- Evolution disentangles **valence**, **sea** and **gluon** contributions.

M. Ding *et al.*, PRD 101 5, 054014 (2020)
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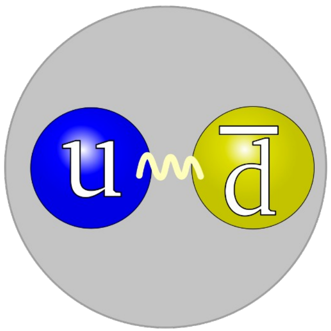
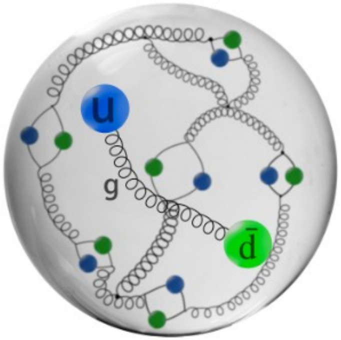


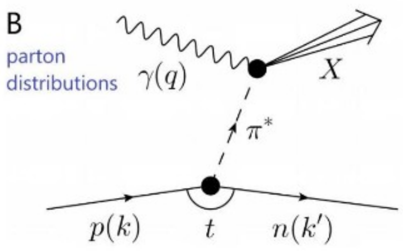
ζ_H

$\zeta > \zeta_H$

Resolution Scale

Evolution equations





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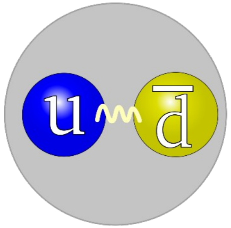
Pion PDF: hadronic scale



$(M_u = M_d)$

- **Fully-dressed valence quarks** (quasiparticles)

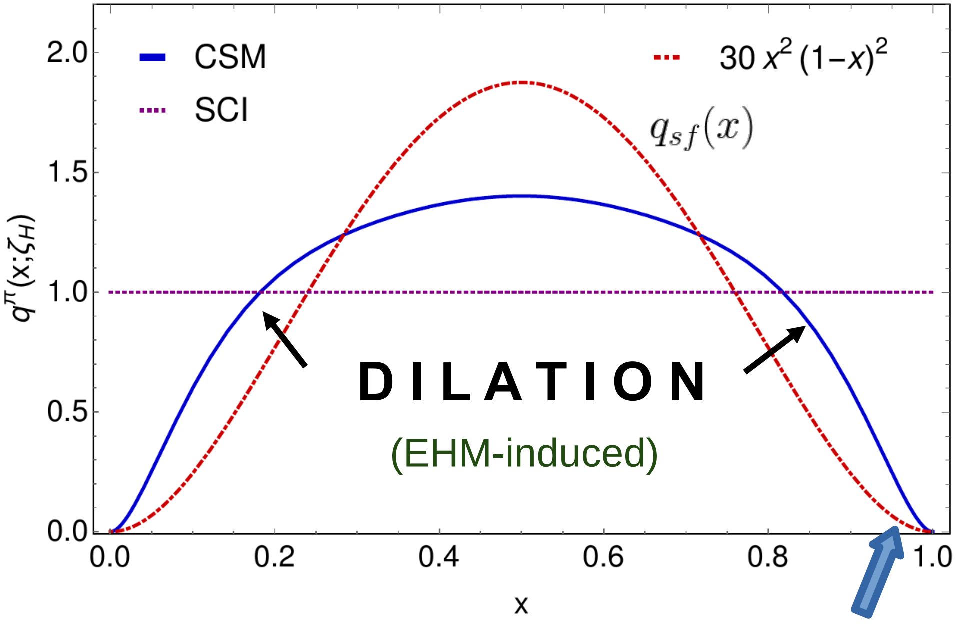
ζ_H : hadronic scale

- At this scale, **all properties** of the hadron are contained within their valence quarks.
  - Equally massive quarks means a **50-50** share of the total momentum:

$$<x(\zeta_H)>_q = 0.5$$

  - This implies symmetric distributions:

$$q(x;\zeta_H) = q(1-x;\zeta_H)$$

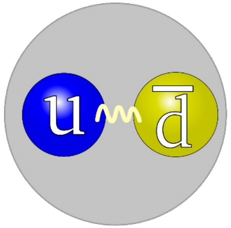


Endpoint **smoothness** is a reflection of the underlying interaction

$$1/(k^2)^\beta \rightarrow (1-x)^{2\beta}$$

Farrar:1975yb Berger:1979du Holt:2010vj

Pion PDF: hadronic scale



$(M_u = M_d)$

- **Fully-dressed valence quarks** (quasiparticles)

ζ_H : hadronic scale

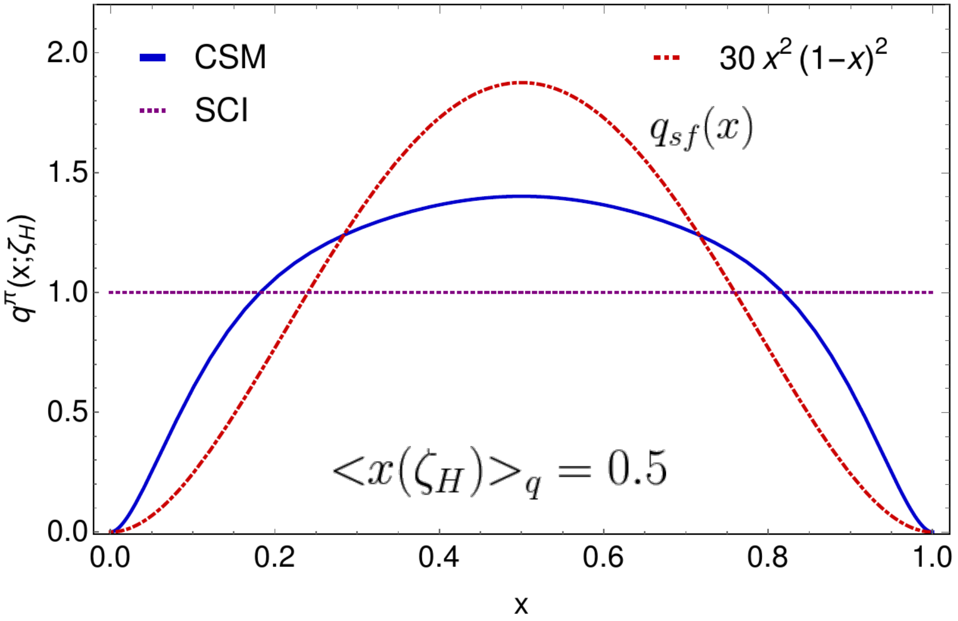
➢ At this scale, **all properties** of the hadron are contained within their valence quarks.

"**Physical**" boundaries:

$$\frac{1}{2^n} \overset{(i)}{\leq} \langle x^n \rangle_{u_\pi}^{\zeta_H} \overset{(ii)}{\leq} \frac{1}{1+n}$$

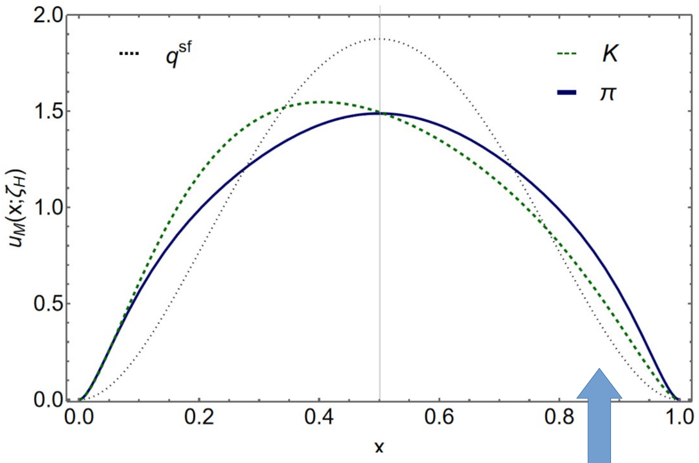
Produced by $q(x;\zeta_H) = \delta(x - 1/2)$ (infinitely heavy valence quarks)

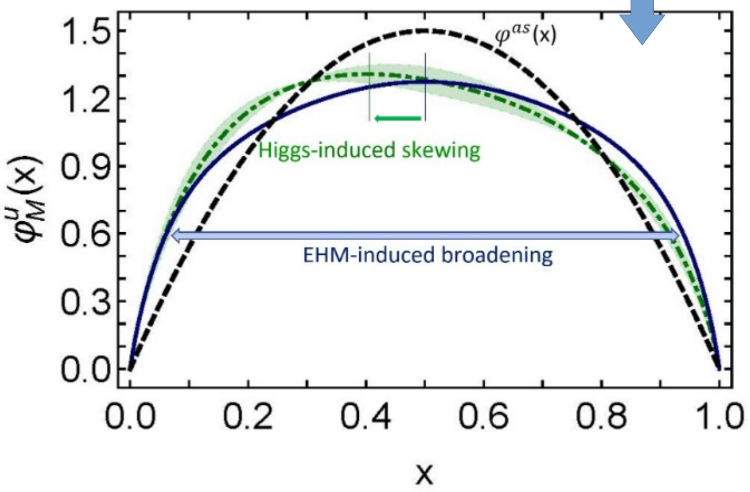
Produced by $q(x;\zeta_H) = 1$ (massless SCI case)



- Equally massive quarks means a **50-50** share of the total momentum.
- This implies symmetric distributions.

Kaon PDF: hadronic scale





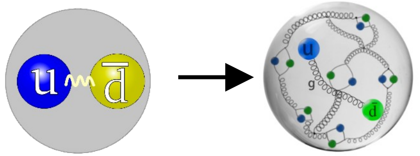
- As the **PDAs**, the **π-K PDFs** are **dilated**.
- The kaon distributions are only-shifted by a **few-percentage**.
  - ➔ QCD's EHM is still dominant. $m_s/m_u \sim 20$ $M_s/M_u \sim 1.2$
  - ➔ The momentum fractions at ζ_H:
    $< x >_u^\pi = 0.5 \quad < x >_u^K = 0.47\,, \; < x >_s^K = 0.53$
- The **bridge** between **PDA** and **PDF** is the **light-front wavefunction**:

$$f_P \varphi_P^u(x,\zeta_{\mathcal{H}}) = \int \frac{dk_\perp^2}{16\pi^3} \psi_P^u\left(x, k_\perp^2; \zeta_{\mathcal{H}}\right)$$

$$u^P(x;\zeta_{\mathcal{H}}) = \int \frac{d^2k_\perp}{16\pi^3} \left|\psi_P^u\left(x, k_\perp^2; \zeta_{\mathcal{H}}\right)\right|^2$$

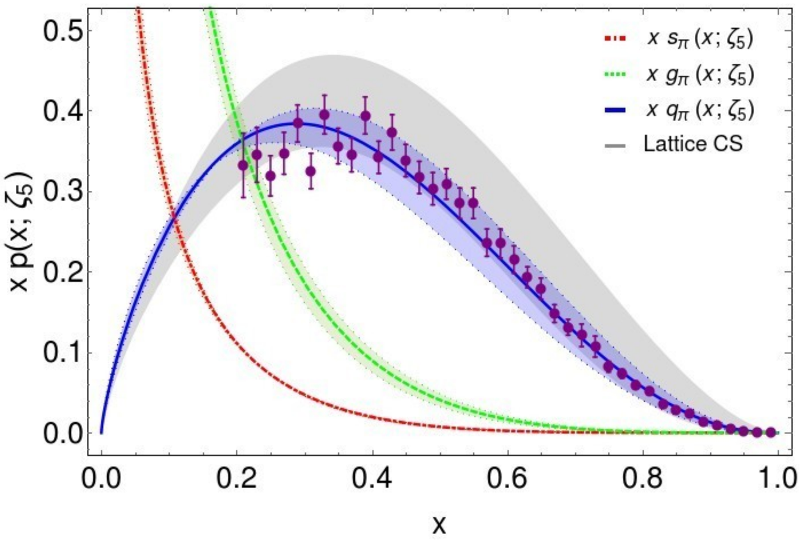
EHM manifests in **PDAs, PDFs, LFWFs...**

Pion PDFs: Lattice & Experiment



- At **5.2 GeV**, the experimental scale, our predictions matches that from Aicher *et al.* Aicher:2010cb

$$\langle x_{\text{gluon}} \rangle = 0.45(1)\ ,\ \langle x_{\text{sea}} \rangle = 0.14(2)$$

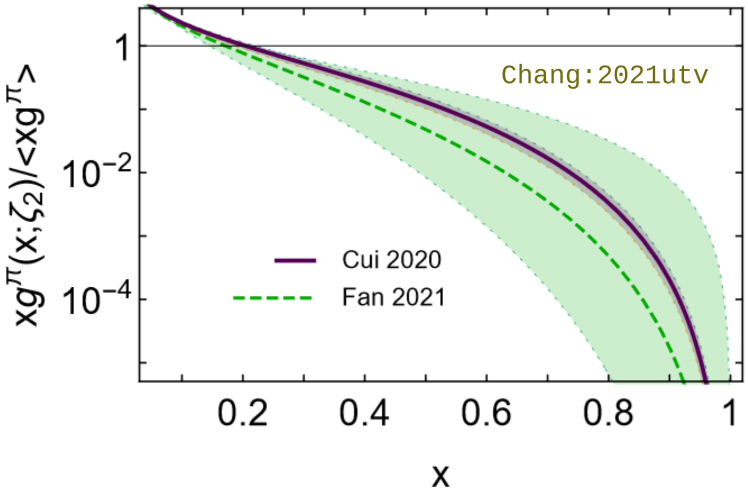


- An agreement with novel **lattice** "Cross Section" results is also obtained. Sufian:2019bol

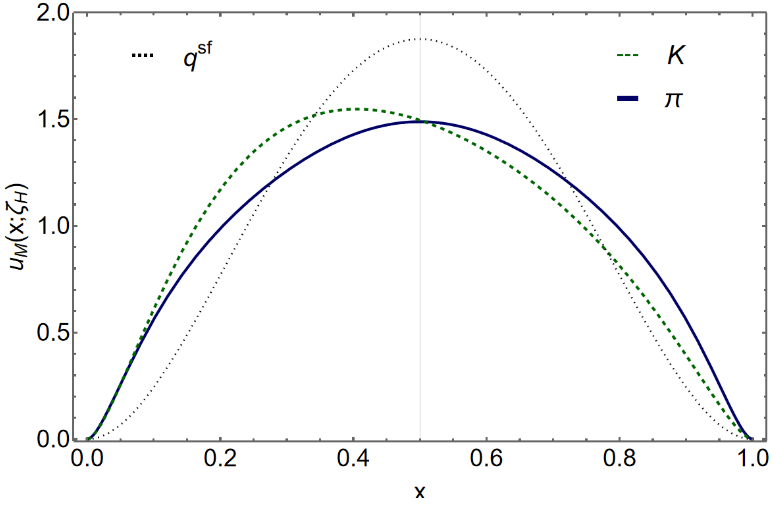
- At **2 GeV**, the valence DF shows agreement with lattice moments:

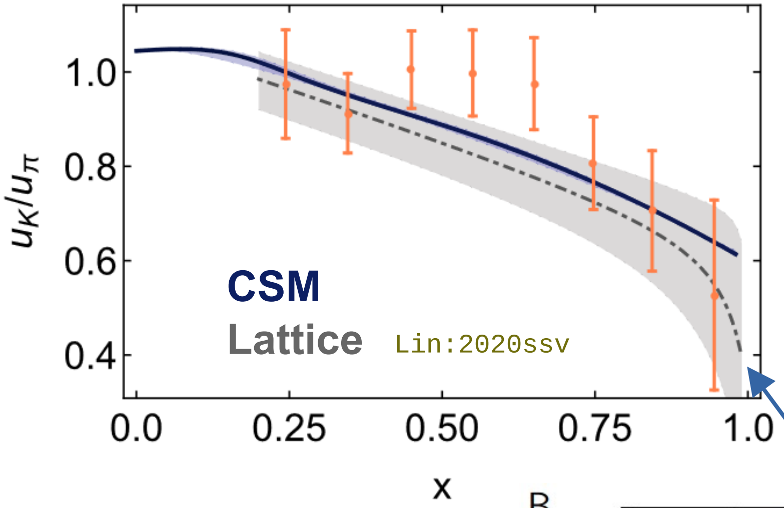
ζ_2	$\langle x\rangle_u^\pi$	$\langle x^2\rangle_u^\pi$	$\langle x^3\rangle_u^\pi$
Ref. [34]	0.24(2)	0.09(3)	0.053(15)
Ref. [35]	0.27(1)	0.13(1)	0.074(10)
Ref. [36]	0.21(1)	0.16(3)	
Herein	0.24(2)	0.098(10)	0.049(07)

- The **Gluon DF** profiles matches **lattice** expectations:



Kaon PDFs: Lattice & Experiment





- Ratio is good but **too forgiving!**
- Besides, there are only **few data points**

➢ At the hadronic scale ζ_H, one has:

$$< x >^{\pi}_{u} = 0.5 \qquad < x >^{K}_{u} = 0.47\,, \; < x >^{K}_{s} = 0.53$$

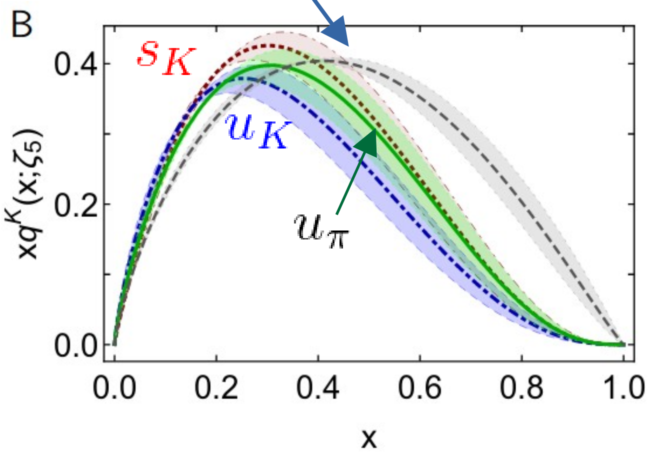
➢ **Above ζ > ζ_H**, there is slightly more valence content in K.

Massiveness of the s-quark is considered in the evolution equations.

$$< \mathbf{x} >^{\text{val}}_{\pi} = 0.41(4)$$

$$< \mathbf{x} >^{\text{val}}_{K} = 0.43(4)$$

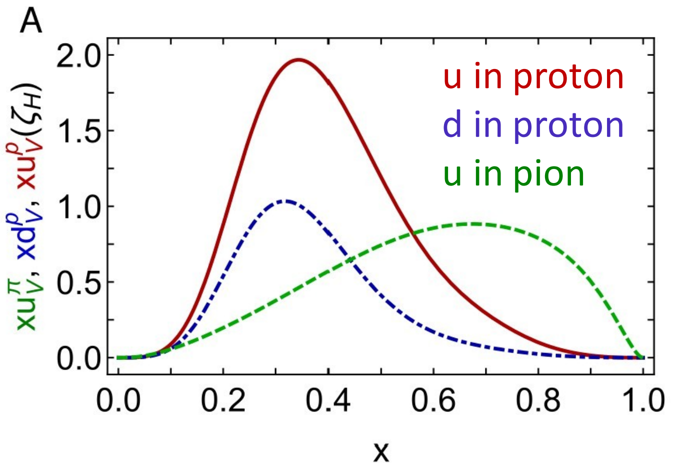
$$\zeta = 5.2\,\text{GeV}$$



Pion vs Proton

Y. Lu *et al.*
Phys.Lett.B 830 (2022) 137130

- The (nearly) massless **pion DFs** differs vastly from the massive **proton**. For instance:



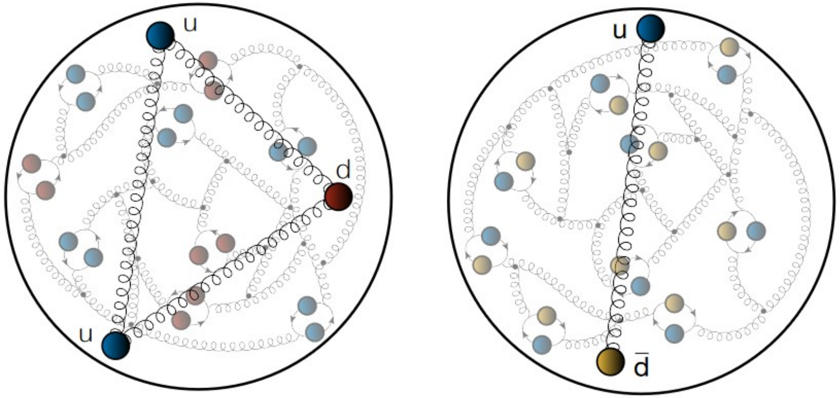
  - The momentum fractions at ζ_H: $(M_u = M_d)$

$$\langle x \rangle_{u_p}^{\zeta_{\mathcal{H}}} = 0.687\,,\ \langle x \rangle_{d_p}^{\zeta_{\mathcal{H}}} = 0.313\,,\ \langle x \rangle_{u_\pi}^{\zeta_{\mathcal{H}}} = 0.5$$

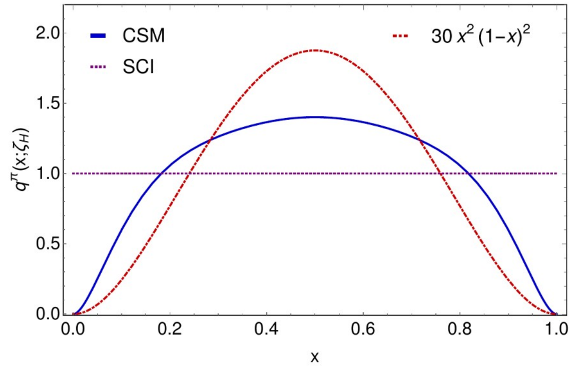
$$\Rightarrow u_V(x) \neq 2 d_V(x)$$

EHM induced diquark correlations inside the proton:

→ No equitable distribution of momentum!



  - Marked dilation of the **pion PDF** at ζ_H.



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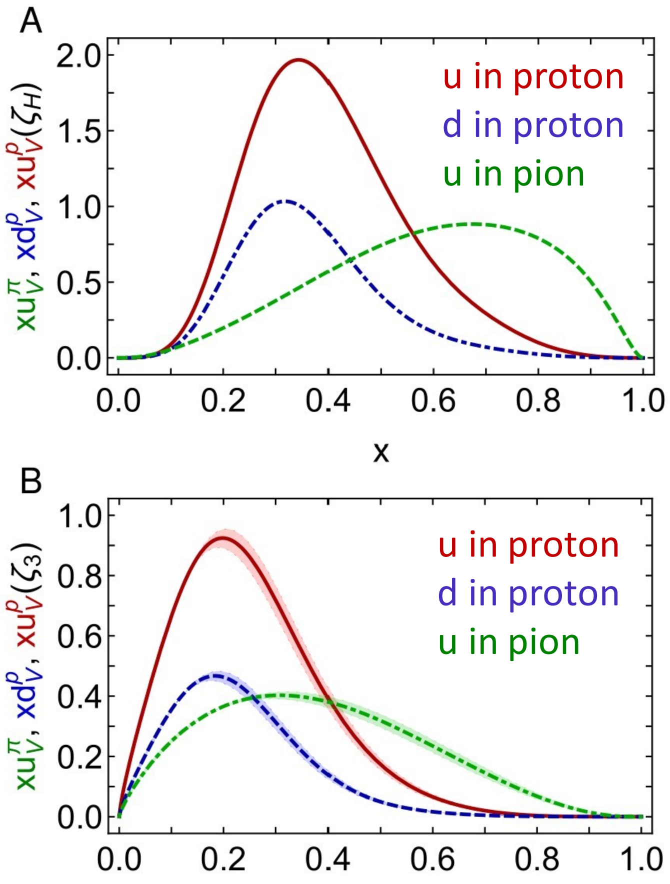
EHM induced diquark correlations inside the proton:

→ No equitable distribution of momentum!

  - Counting rules entail large-x behaviors **(1-x)²** and **(1-x)³** for the **pion** and **proton**, respectively.
  - Marked dilation of the **pion PDF** at ζ_H.
  - Differences are **preserved** after evolution.



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EHM induced diquark correlations inside the proton:

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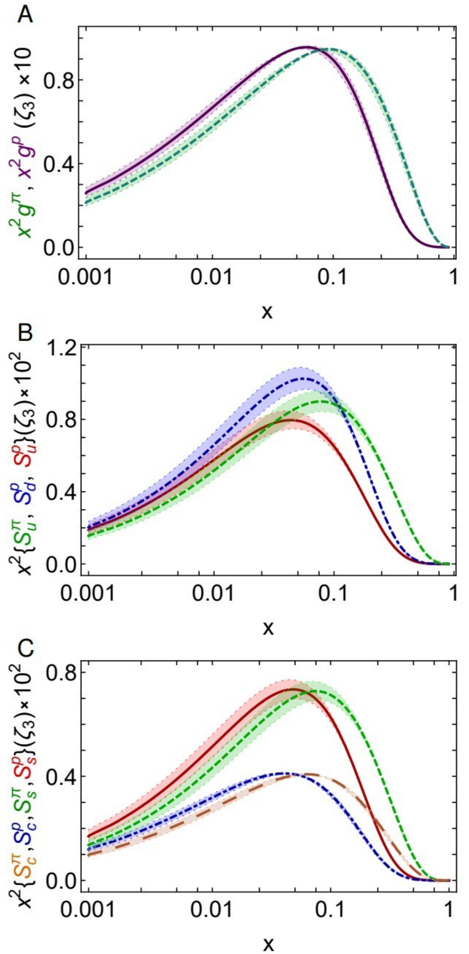
  - Counting rules entail large-x behaviors **$(1-x)^2$** and **$(1-x)^3$** for the **pion** and **proton**, respectively.
  - Marked dilation of the **pion PDF** at ζ_H.
  - Differences are **preserved** after evolution.
    - which results in different profiles of glue and sea PDFs.



KR, Z-F Cui *et al.*,
Chin.Phys.C 46 (2022) 1, 013105

Light-front wave functions (LFWF)

"One ring to rule them all"

$$\psi_{\mathrm{M}}^{q}\left(x, k_{\perp}^{2}\right)=\operatorname{tr} \int_{d k_{\|}} \delta_{n}^{x}\left(k_{\mathrm{M}}\right) \gamma_{5} \gamma \cdot n\, \chi_{\mathrm{M}}\left(k_{-}, P\right)$$

Bethe-Salpeter wave function

- **Intrinsic** of the hadron's nature.
- Yields a **variety** of **distributions**.

LFWFs: Connecting the dots

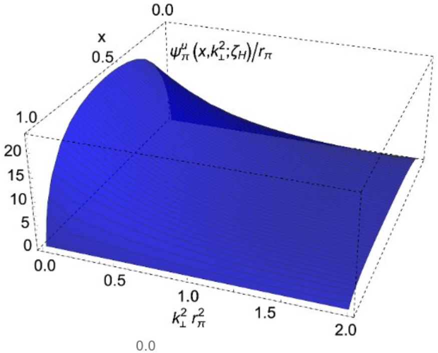
The idea: Connect ***everything*** through the **LFWF.**

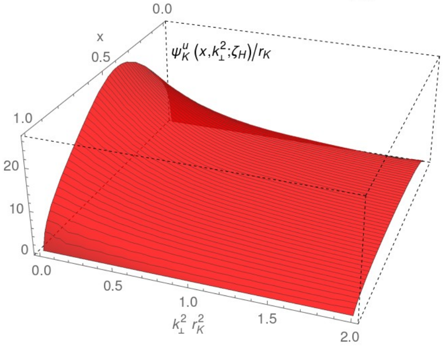
LFWFs → **PDAs**

$$f_{\mathsf{P}}\varphi_{\mathsf{P}}^{u}(x,\zeta_{\mathcal{H}}) = \int \frac{dk_\perp^2}{16\pi^3}\psi_{\mathsf{P}}^{u}\left(x,k_\perp^2;\zeta_{\mathcal{H}}\right)$$

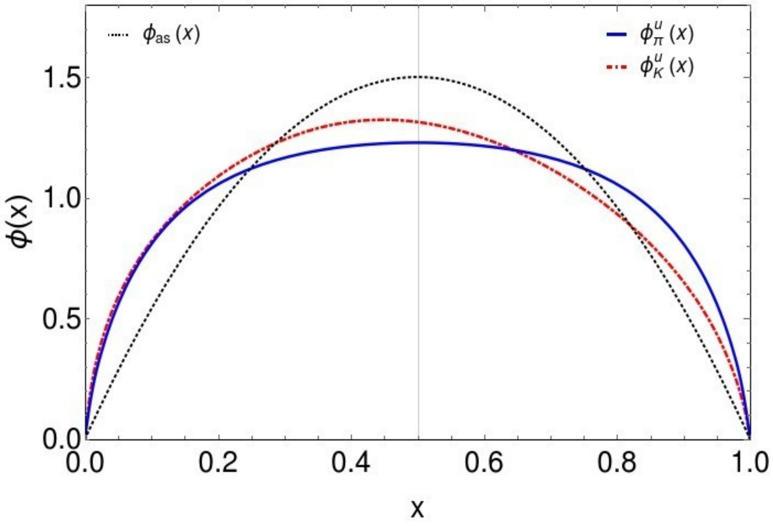
$\sim \psi^*\psi$

TMDs





- **1-D projection** of the hadron's **LFWF**.



$\int dx$ (red) $\int dk_\perp$ (green)

$t = 0, \xi = 0$ (purple)

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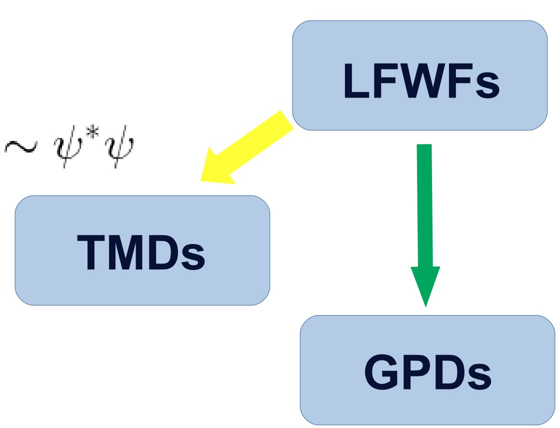


$$f_{\mathrm{P}} \varphi_{\mathrm{P}}^u(x, \zeta_{\mathcal{H}}) = \int \frac{dk_\perp^2}{16\pi^3} \psi_{\mathrm{P}}^u \left(x, k_\perp^2; \zeta_{\mathcal{H}}\right)$$

Overlap representation:

$$H_{\mathrm{M}}^q(x, \xi, t) = \int \frac{d^2 k_\perp}{16\pi^3} \psi_{\mathrm{M}}^{q*} \left(x^-, (\mathbf{k}_\perp^-)^2\right) \psi_{\mathrm{M}}^q \left(x^+, (\mathbf{k}_\perp^+)^2\right)$$

Sufficient to sketch charge, mass and spatial distributions...

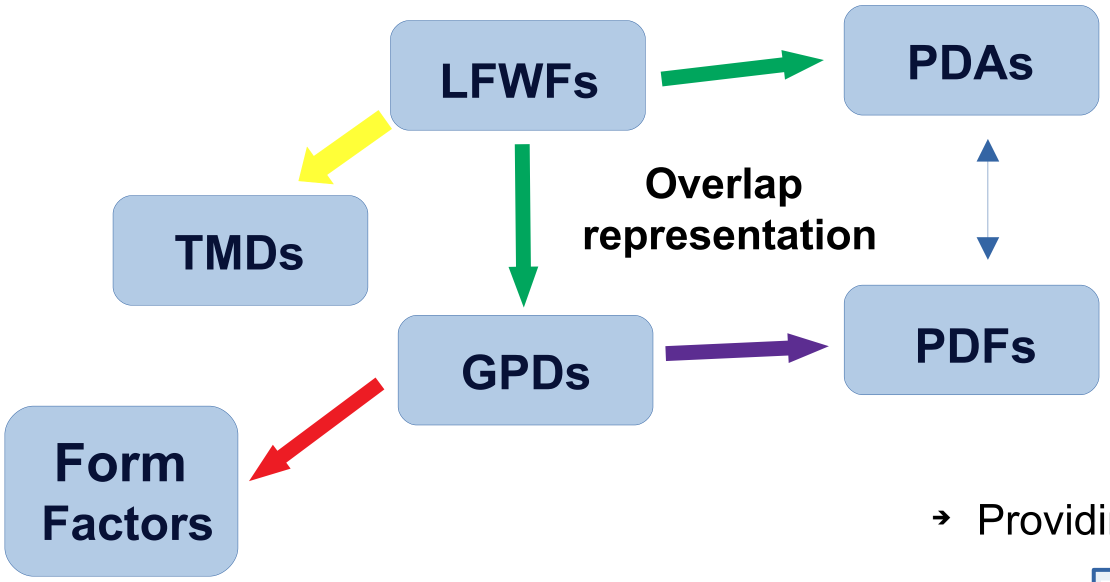
- ✓ ***Valid*** in the **DGLAP** region
- ✓ ***Positivity*** fulfilled
- ✓ Can be ***extended*** to the **ERBL** region Chavez:2021llq

Red: $\int dx$ Green: $\int dk_\perp$

Purple: $t = 0, \xi = 0$

LFWFs: Connecting the dots

The idea: Connect ***everything*** through the **LFWF.**



$$f_{\mathsf{P}}\varphi_{\mathsf{P}}^{u}(x,\zeta_{\mathcal{H}}) = \int \frac{dk_\perp^2}{16\pi^3}\psi_{\mathsf{P}}^{u}\left(x,k_\perp^2;\zeta_{\mathcal{H}}\right)$$

PDFs are understood as the **forward limit** of the **GPD**

$$q(x) = H(x,0,0)$$

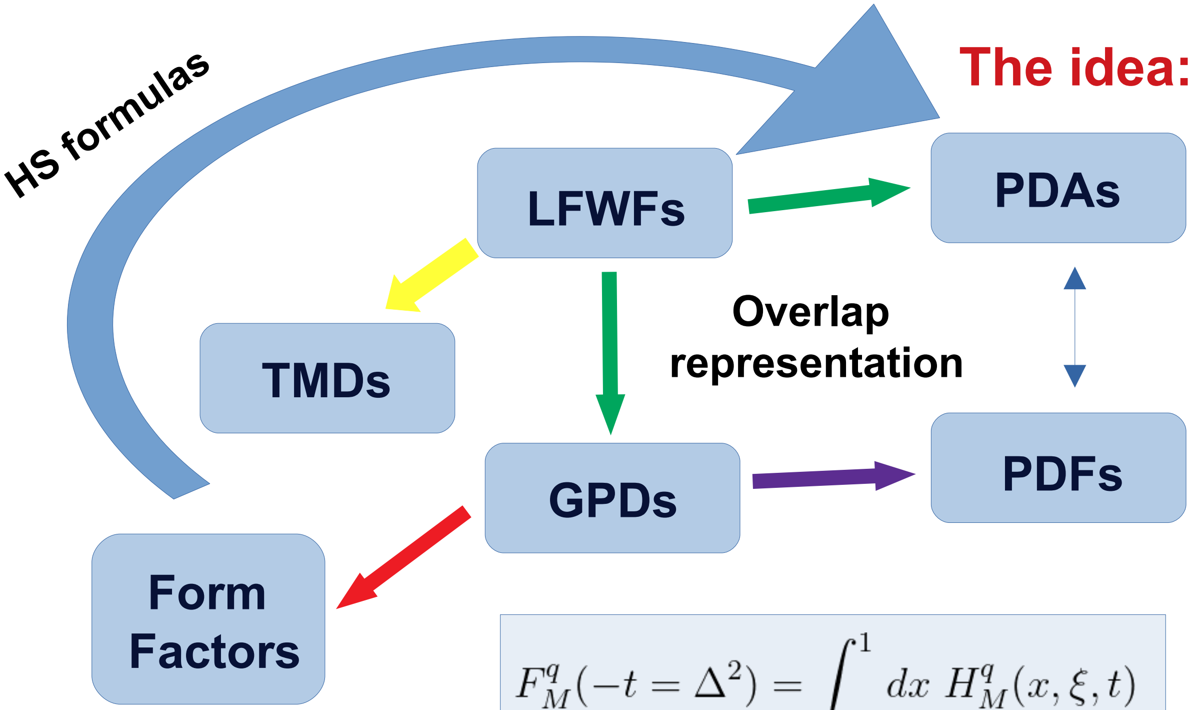
➔ Providing another **connection** with the **PDA**.

$$u^{\mathsf{P}}(x;\zeta_{\mathcal{H}}) = \int \frac{d^2k_\perp}{16\pi^3}\left|\psi_{\mathsf{P}}^{u}\left(x,k_\perp^2;\zeta_{\mathcal{H}}\right)\right|^2$$

Red: $\int dx$ Green: $\int dk_\perp$

Purple: $t=0, \xi=0$

LFWFs: Connecting the dots



The idea: Connect ***everything*** through the **LFWF.**

$$f_{\mathsf{P}}\varphi_{\mathsf{P}}^{u}(x,\zeta_{\mathcal{H}}) = \int \frac{dk_{\perp}^{2}}{16\pi^{3}}\psi_{\mathsf{P}}^{u}\left(x,k_{\perp}^{2};\zeta_{\mathcal{H}}\right)$$

PDFs are understood as the **forward limit** of the **GPD**

$$q(x) = H(x,0,0)$$

$$u_M(x;\zeta_H) \sim \int d^2k_{\perp}\psi^{*}\psi$$

Form factors are expressed as **sum rules** of the **GPD**

$$F_M^q(-t=\Delta^2) = \int_{-1}^{1} dx\ H_M^q(x,\xi,t)$$

Electromagnetic FF

$$J_M(t,\xi) = \int_{-1}^{1} dx\ xH_M(x,\xi,t) = \Theta_2^M(t) - \xi^2\Theta_1^M(t)$$

Gravitational FFs

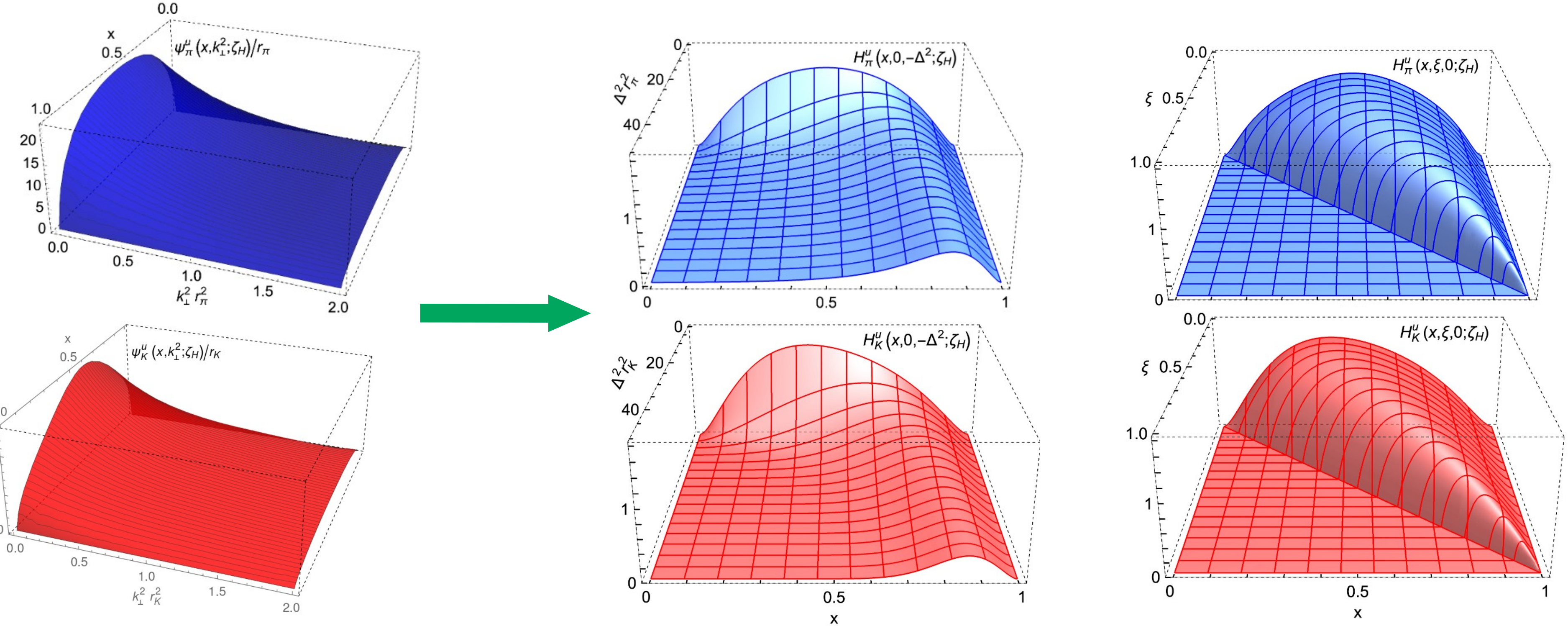
(red) $\int dx$ (green) $\int dk_{\perp}$

(purple) $t=0, \xi=0$

π-K: LFWFs & GPDs

LFWFs → GPDs

$$H_{\mathrm{M}}^{q}(x,\xi,t) = \int \frac{d^2 k_\perp}{16\pi^3} \psi_{\mathrm{M}}^{q*}\left(x^-, (\mathbf{k}_\perp^-)^2\right) \psi_{\mathrm{M}}^{q}\left(x^+, (\mathbf{k}_\perp^+)^2\right)$$



Pion Gravitational FFs

GPD → FFs

- **Gravitational** form factors are obtained from the **t-dependence** of the **1-st moment**:

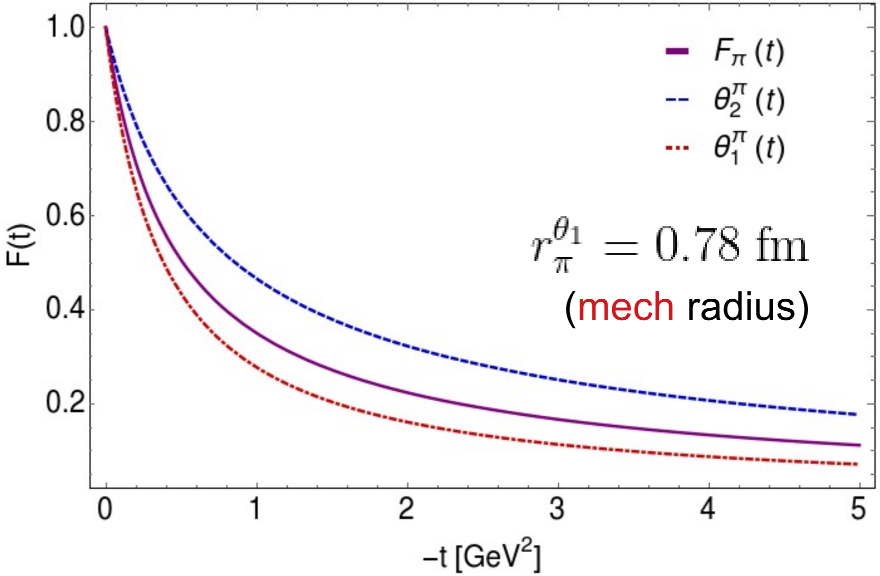
$$J_M(t,\xi) = \int_{-1}^{1} dx\, x H_M(x,\xi,t) = \Theta_2^M(t) - \xi^2 \Theta_1^M(t)$$

  - ✓ Directly obtained if **ξ = 0**
  - ✓ Only **DGLAP** GPD is required
  - ✗ **ERBL** GPD needed

- Sophisticated techniques exist.
- But a sound expression can be constructed:

$$\theta_1^{\mathsf{P}_q}(\Delta^2) = c_1^{\mathsf{P}_q}\theta_2^{\mathsf{P}_q}(\Delta^2) + \int_{-1}^{1} dx\, x \left[H_{\mathsf{P}}^q(x,1,0)P_{M_q}(\Delta^2) - H_{\mathsf{P}}^q(x,1,-\Delta^2)\right]$$

"Soft pion theorem"



$r_\pi = 0.66\,\text{fm}$ (charge radius) $r_\pi^{\theta_2} = 0.56\,\text{fm}$ (mass radius)

Impact parameter space GPDs

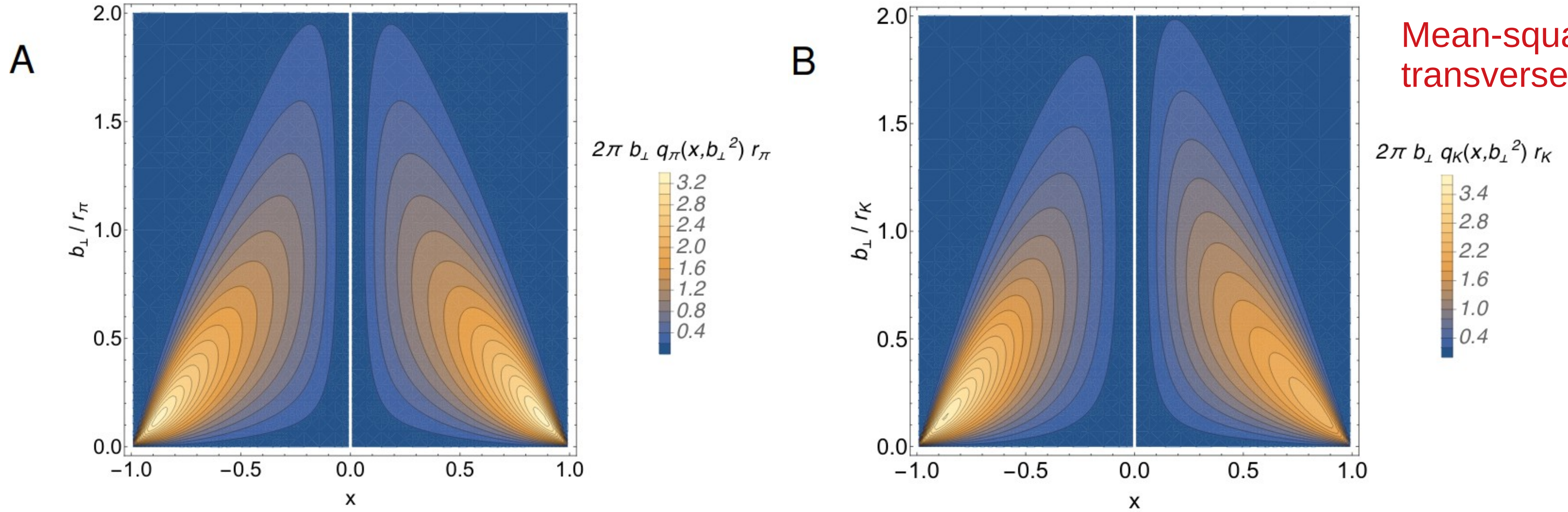
$$u^{\mathrm{P}}(x, b_\perp^2; \zeta_{\mathcal{H}}) = \int_0^\infty \frac{d\Delta}{2\pi} \Delta J_0(b_\perp \Delta)\, H_{\mathrm{P}}^u(x, 0, -\Delta^2; \zeta_{\mathcal{H}})$$

Algebraic derivation!

$$\langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_u^\pi = \boxed{\frac{2}{3}} r_\pi^2 = \langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_{\bar{d}}^\pi, \approx [r_\pi^{\theta_2}]^2$$

$$\langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_u^K = 0.71 r_K^2, \langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_{\bar{s}}^K = 0.58 r_K^2.$$

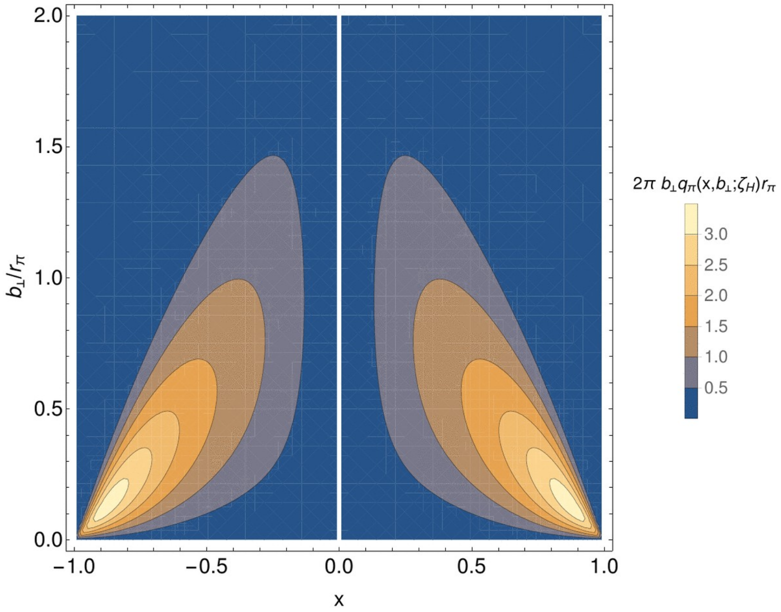
Mean-square transverse extent



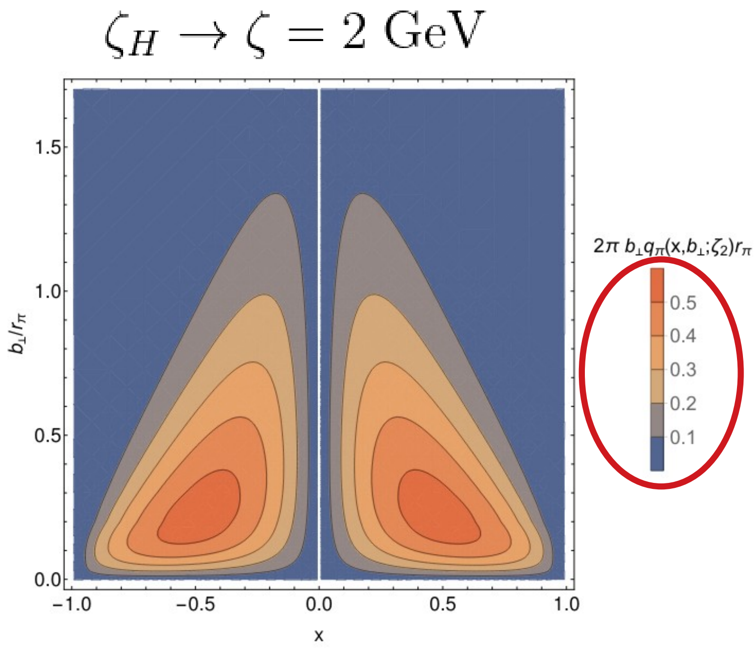
- Likelihood of finding a valence-**quark** with **momentum fraction x**, at **position b**.

Evolved IPS-GPD: Pion Case

$$u^{P}(x, b_{\perp}^{2}; \zeta_{\mathcal{H}}) = \int_{0}^{\infty} \frac{d\Delta}{2\pi} \Delta J_{0}(b_{\perp}\Delta)\, H_{P}^{u}(x, 0, -\Delta^{2}; \zeta_{\mathcal{H}})$$



- **Likelihood** of finding a parton with LF momentum **x** at transverse position **b**



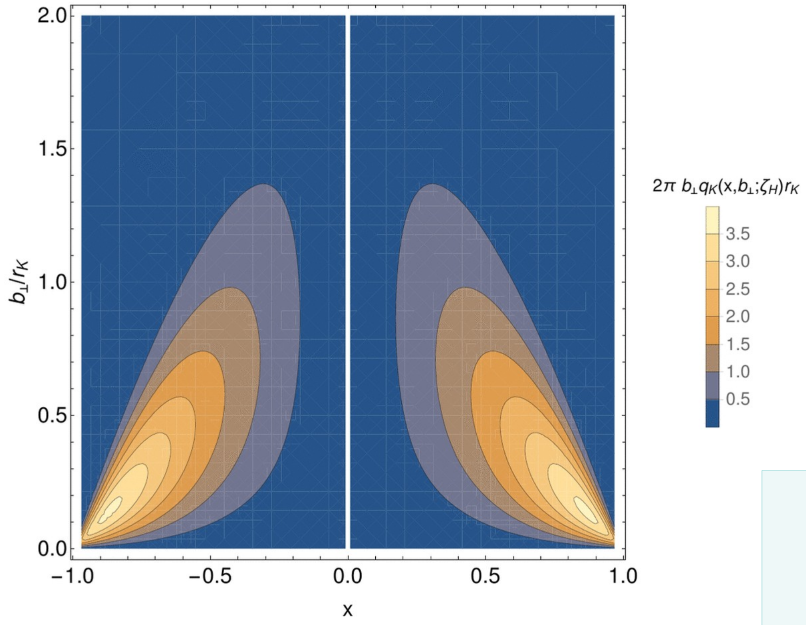
- Peaks **broaden** and **maximum drifts:**

$$\max: \; 3.29 \to 0.55$$

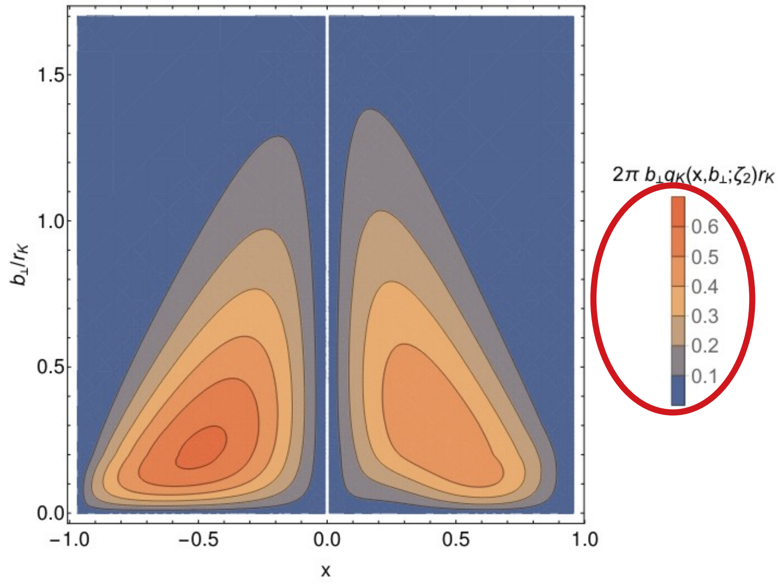
$$(|x|, b) = (0.88, 0.13) \to (0.47, 0.23)$$

Evolved IPS-GPD: Kaon Case

$$u^{\mathsf{P}}(x, b_\perp^2; \zeta_{\mathcal{H}}) = \int_0^\infty \frac{d\Delta}{2\pi} \Delta J_0(b_\perp \Delta)\, H_{\mathsf{P}}^u(x, 0, -\Delta^2; \zeta_{\mathcal{H}})$$



$\zeta_H \to \zeta = 2$ GeV



- **Likelihood** of finding a parton with LF momentum **x** at transverse position **b**

$$\max_{(s,u)} : (3.61, 2.38) \to (0.61, 0.49)$$
$$(x, b)_u = (0.84, 0.17) \to (0.41, 0.28)$$
$$(x, b)_s = (-0.87, 0.13) \to (-0.48, 0.22)$$

Distributions: Charge & Mass

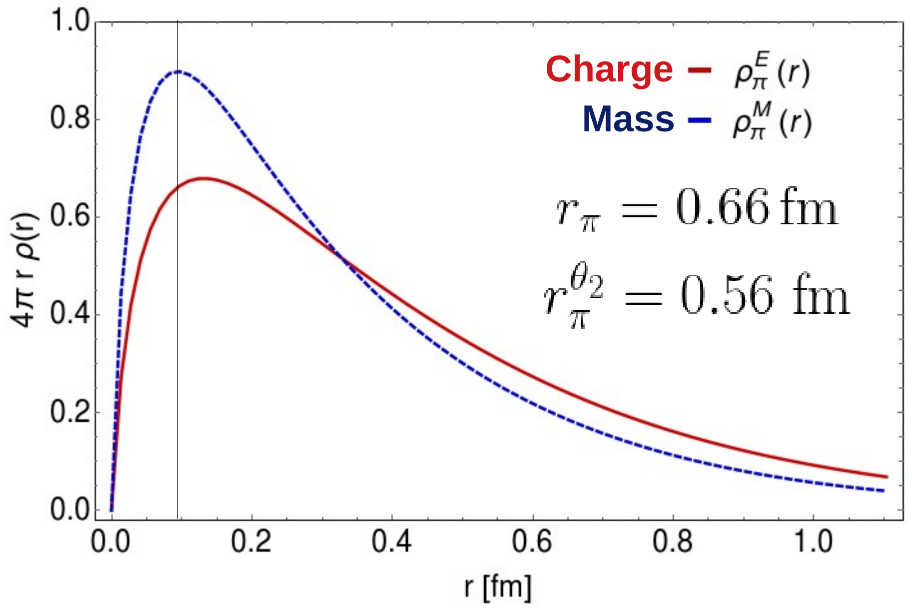
$$\rho_{\mathrm{P}}(b) = \frac{1}{2\pi}\int_0^\infty d\Delta\, \Delta J_0(\Delta\, b) F_{\mathrm{P}}(\Delta^2)$$

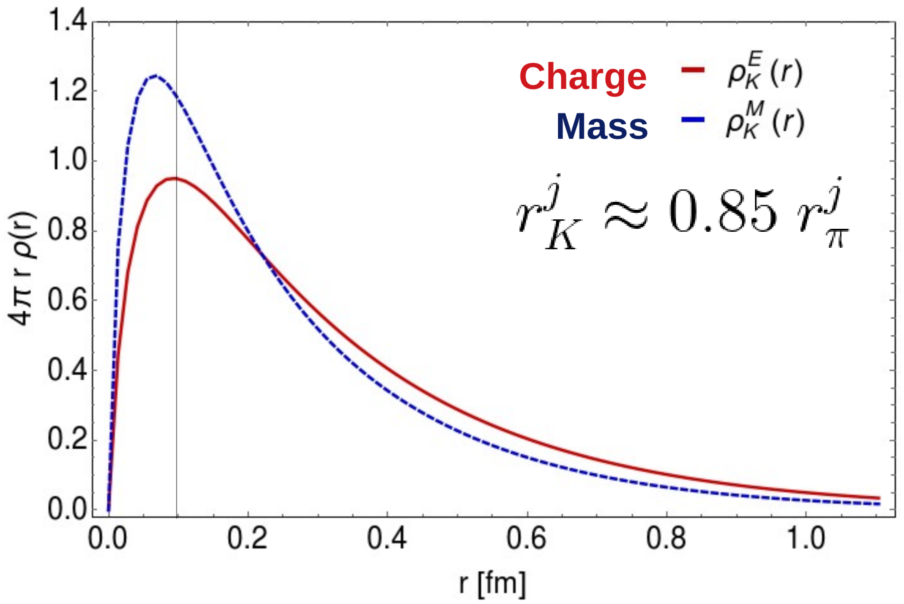
$$F_{\mathrm{P}}^{E}(\Delta^2) \to \rho_{\mathrm{P}}^{E}(b)$$

$$\theta_2^{\mathrm{P}}(\Delta^2) \to \rho_{\mathrm{P}}^{M}(b)$$

- **Intuitively**, we expect the meson to be localized at a **finite space**.
- **Charge** effect span over a **larger domain** than **mass** effects.

More **massive** hadron → More **compressed**

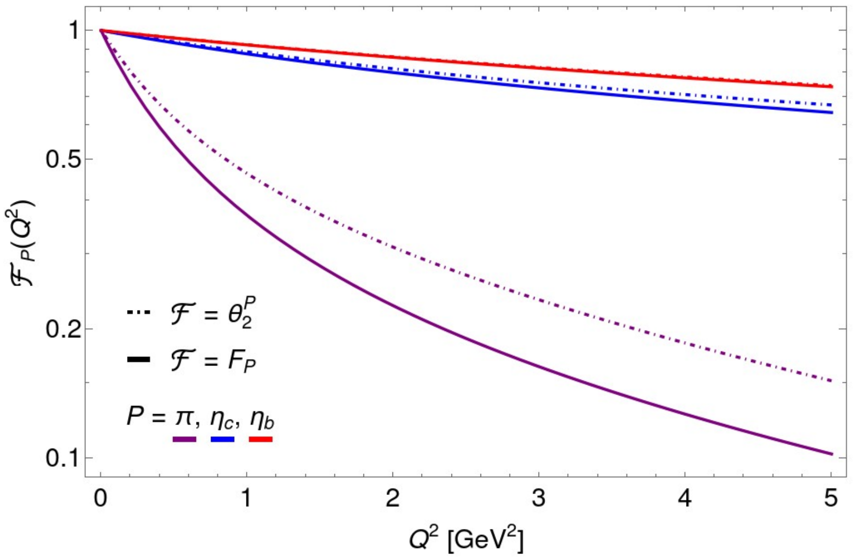


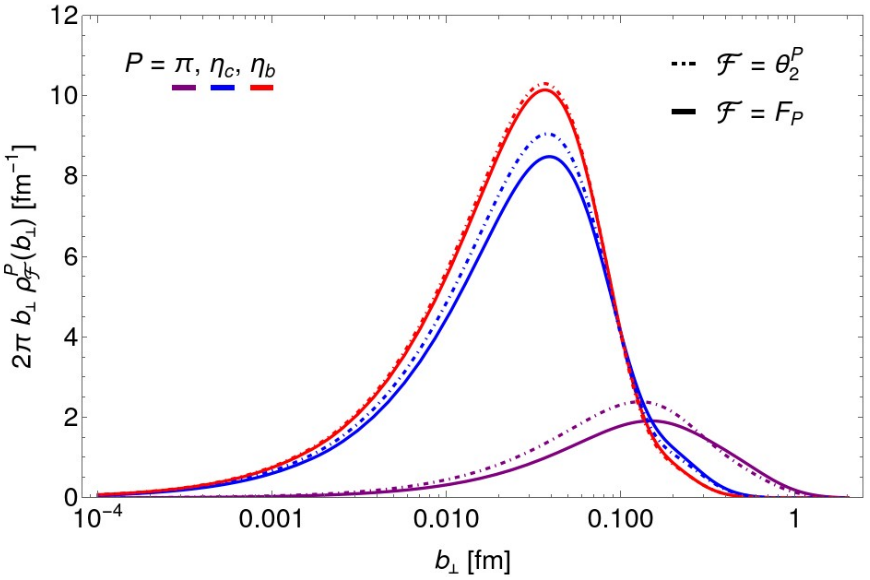


Distributions: Charge & Mass

$$\rho_{\rm P}(b) = \frac{1}{2\pi}\int_0^\infty d\Delta\, \Delta J_0(\Delta\, b) F_{\rm P}(\Delta^2)$$

- **Charge** effects span over a larger domain than **mass** effects.
- **Weak** mass generation dominance translates into **hardening** of the form factors and **compression** of the hadrons. $r_K^j \approx 0.85\, r_\pi^j$
- As the meson **mass increases**, the classical limit is recovered, thus **charge and mass** (and corresponding FFs) distributions exhibit the **same profiles.**





π-K: Pressure profiles

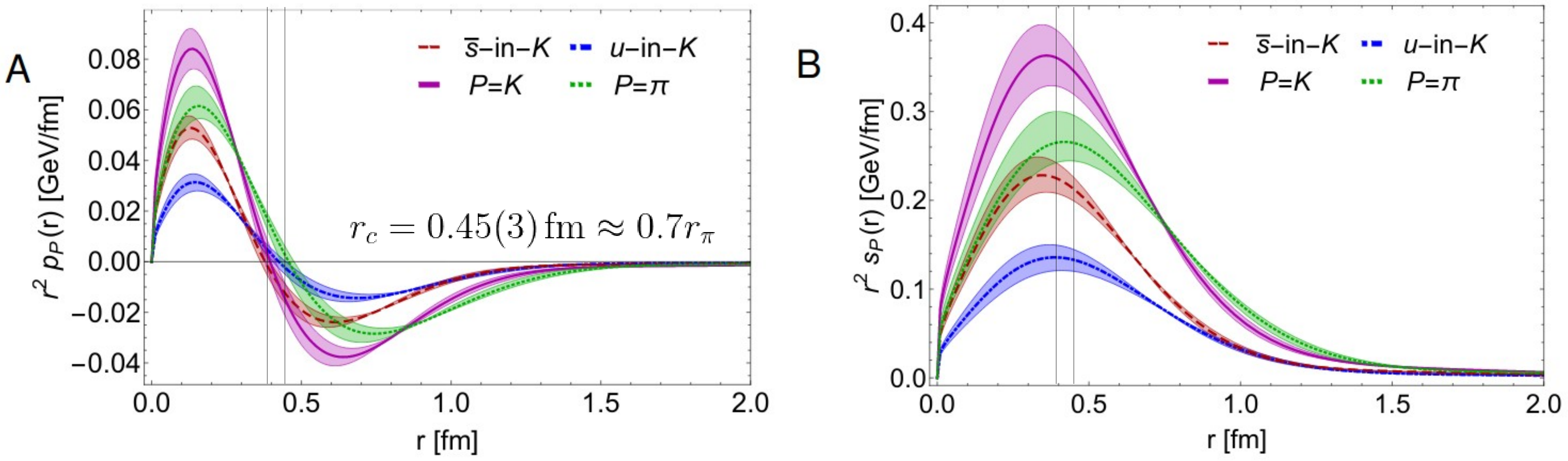
$$p_K^u(r) = \frac{1}{6\pi^2 r}\int_0^\infty d\Delta \, \frac{\Delta}{2E(\Delta)} \, \sin(\Delta r)[\Delta^2 \theta_1^{K_u}(\Delta^2)]\,,$$

$$s_K^u(r) = \frac{3}{8\pi^2}\int_0^\infty d\Delta \, \frac{\Delta^2}{2E(\Delta)} j_2(\Delta r)\,[\Delta^2 \theta_1^{K_u}(\Delta^2)]\,,$$

"Pressure" Quark attraction/repulsion

CONFINEMENT

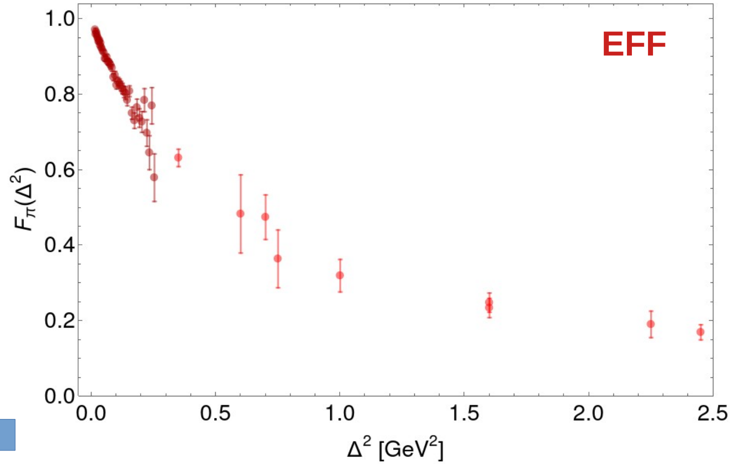
"Shear" *Deformation* QCD forces

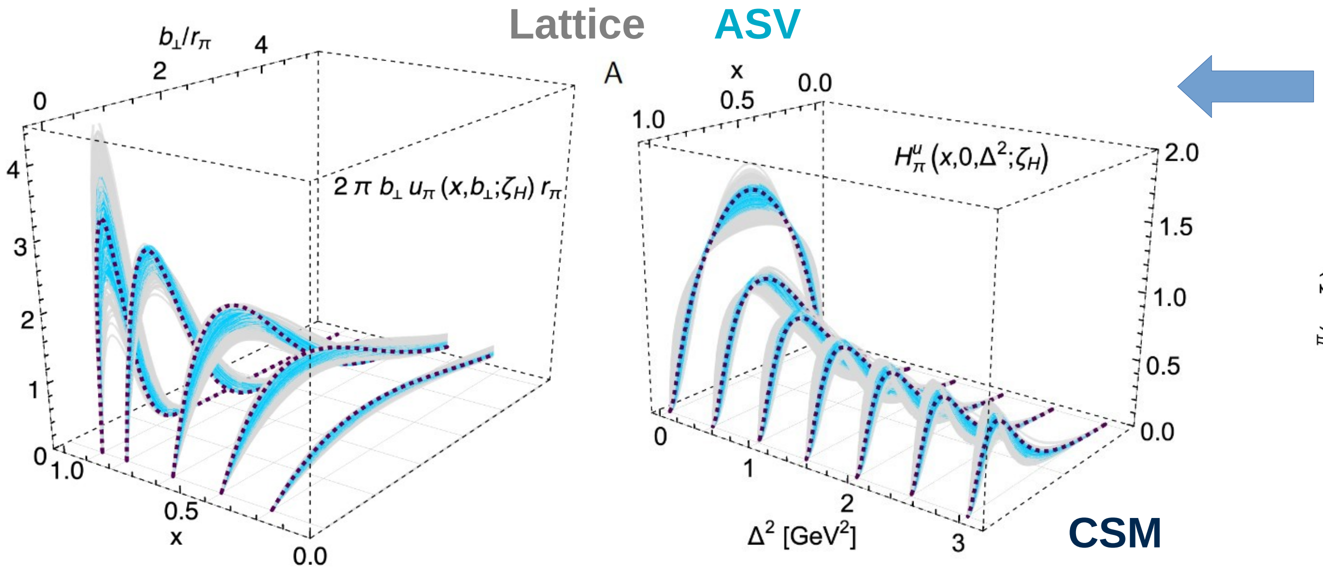


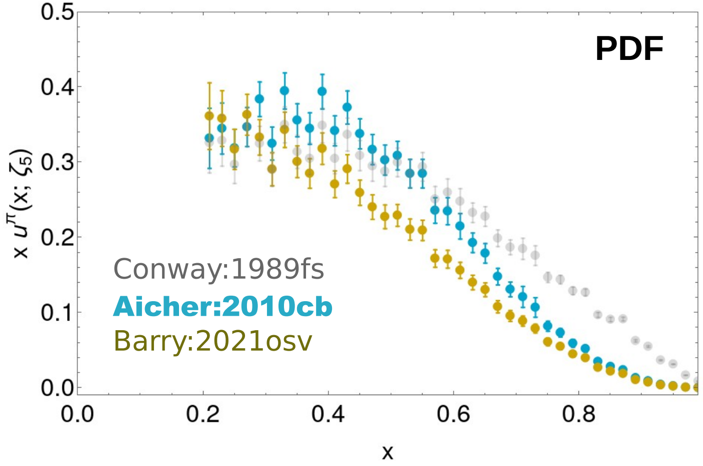
GPDs: Empirical determination

Pion GPD: Empirical determination

- The connection of **GPDs** with PDFs and EFFs enable us to use existing data on those quantities to **reconstruct** the **pion GPD**.
- Using a chi^2-based probabilistic selection procedure, an ensemble of **representations** for the **pion GPD** is generated.





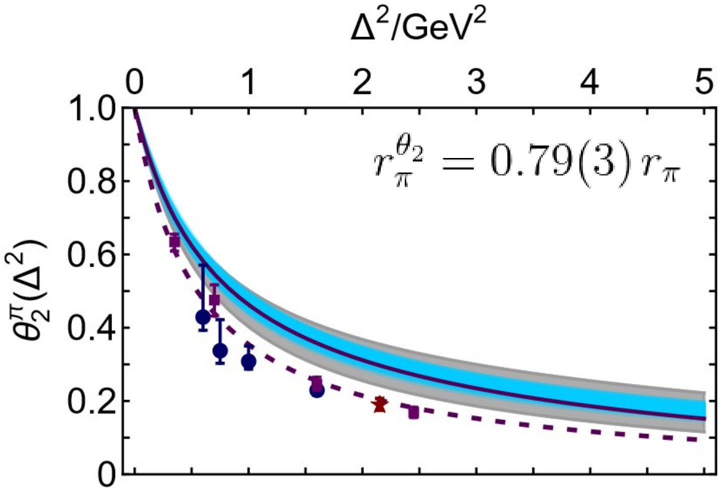


Pion GPD: Empirical determination

- The connection of **GPDs** with PDFs and EFFs enable us to use existing data on those quantities to **reconstruct** the **pion GPD**.

- Using a chi^2-based probabilistic selection procedure, an ensemble of **representations** for the **pion GPD** is generated.

- The produced ensemble turns out to be in **agreement** with previous **CSM predictions**.



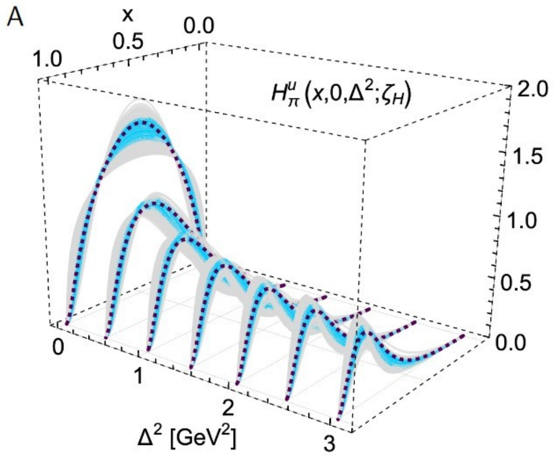
- Proving, once again, that θ_2 is harder than the EFF:

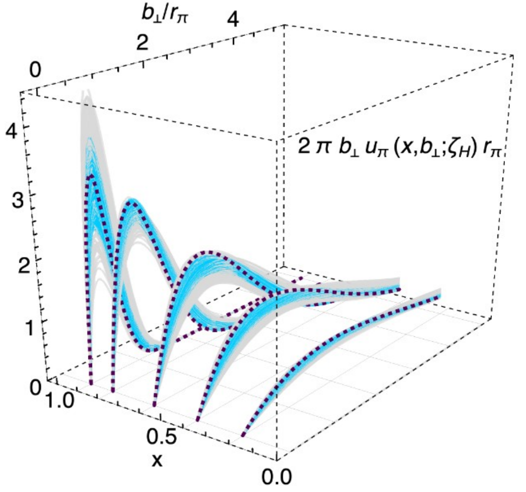
  i.e. the **mass distribution** is **more compact** than the **charge** one.

The physical boundaries:

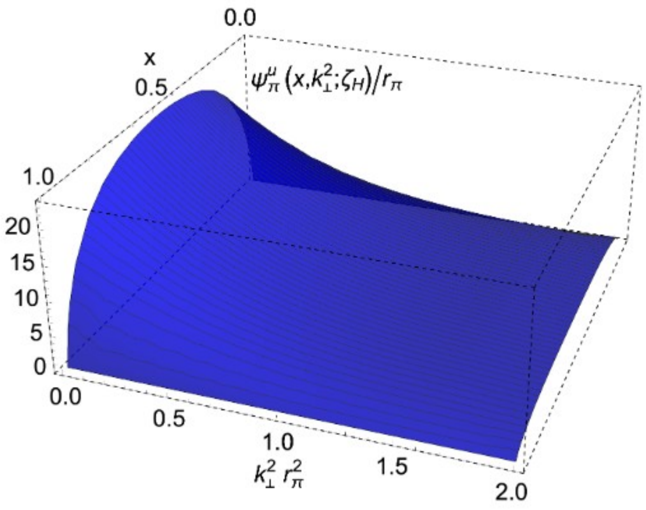
$$\frac{1}{\sqrt{2}} \leq r_\pi^{\theta_2} / r_\pi \leq 1$$

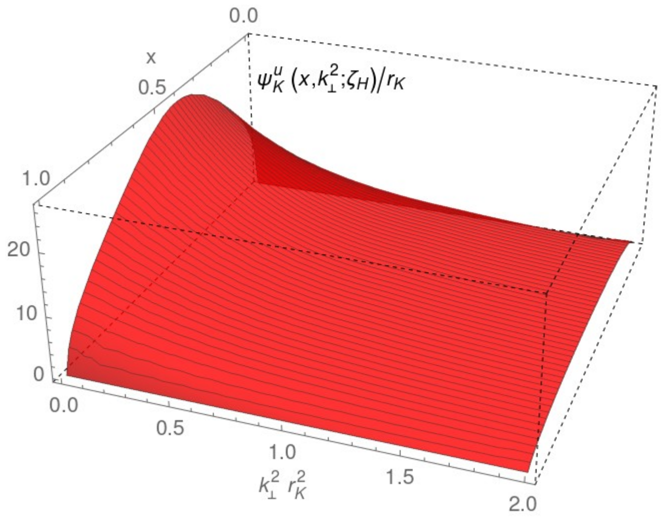
Xu, KR *et al.*
Chin.Phys.Lett. 40 (2023) 4, 041201





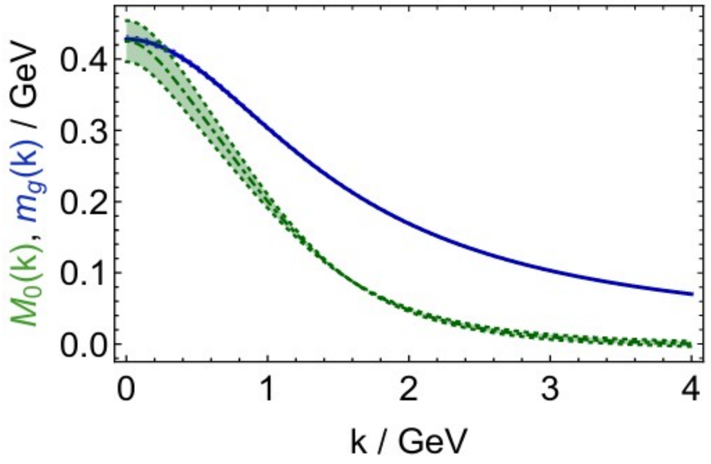
Final Highlights

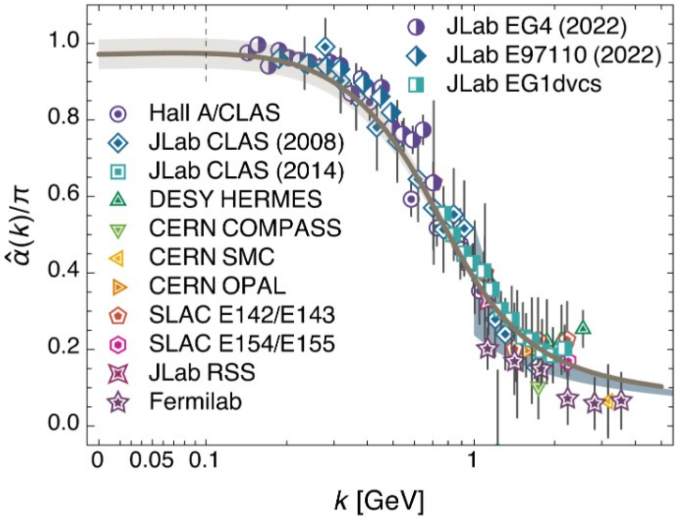




Final **Highlights**

- The **emergent** **phenomena** in **QCD** produces unique outcomes:
  - The degrees-of-freedom are not directly accessible, we get to observe hadrons (**confinement**).
  - Through their own mechanisms, **dynamical mass generation** is present in both **matter** and **gauge** sectors of QCD; the later yielding a running **coupling** that saturates at infrared momenta.





M. Ding *et al.*
Particles 6 (2023) 57-120

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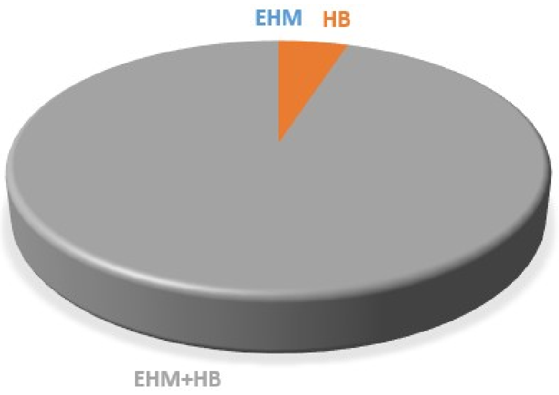
- **Pseudoscalar** mesons are an ideal platform to inquire on these facets of **QCD**:
  - Their mere **existence and properties** are connected with the **mass generation** in the Standard Model and, potentially, confinement.
  - Modern facilities are **capable** to address the properties of **NG bosons** and it's connection with QCD's emergent phenomena.

→ Jlab, EIC, EicC, Amber, etc.

J. Arrington *et al.*
J.Phys.G 48 (2021) 7, 075106



Pion mass budget

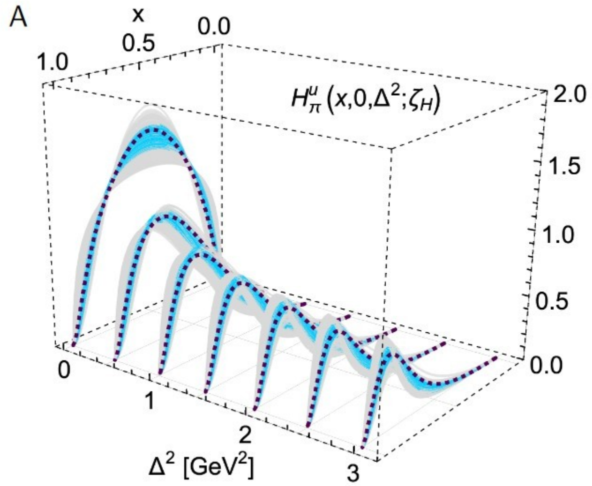
Final **Highlights**

- The **emergent** **phenomena** in **QCD** produces unique outcomes:
  - The degrees-of-freedom are not directly accessible, we get to observe hadrons (**confinement**).
  - Through their own mechanisms, **dynamical mass generation** is present in both **matter** and **gauge** sectors of QCD; the later yielding a running **coupling** that saturates at infrared momenta.

- **Pseudoscalar** mesons are an ideal platform to inquire on these facets of **QCD**:
  - Their mere **existence and properties** are connected with the **mass generation** in the Standard Model and, potentially, confinement.
  - Modern facilities are **capable** to address the properties of **NG bosons** and it's connection with QCD's emergent phenomena.

- Theory has evolved to the point where **all sorts of** parton **distributions** within pseudoscalar mesons are **within reach**.

➔ Many of them connected via **LFWF**

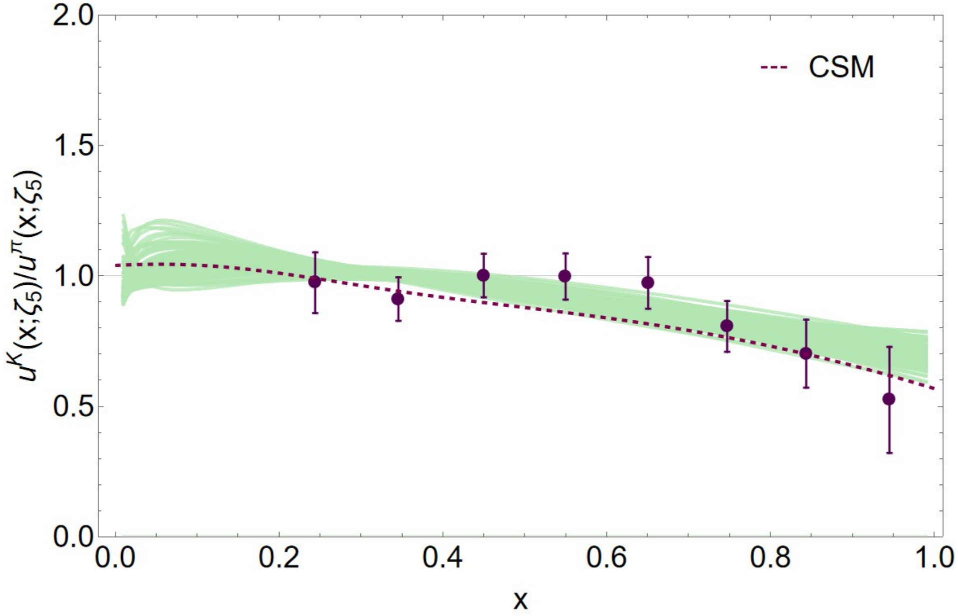


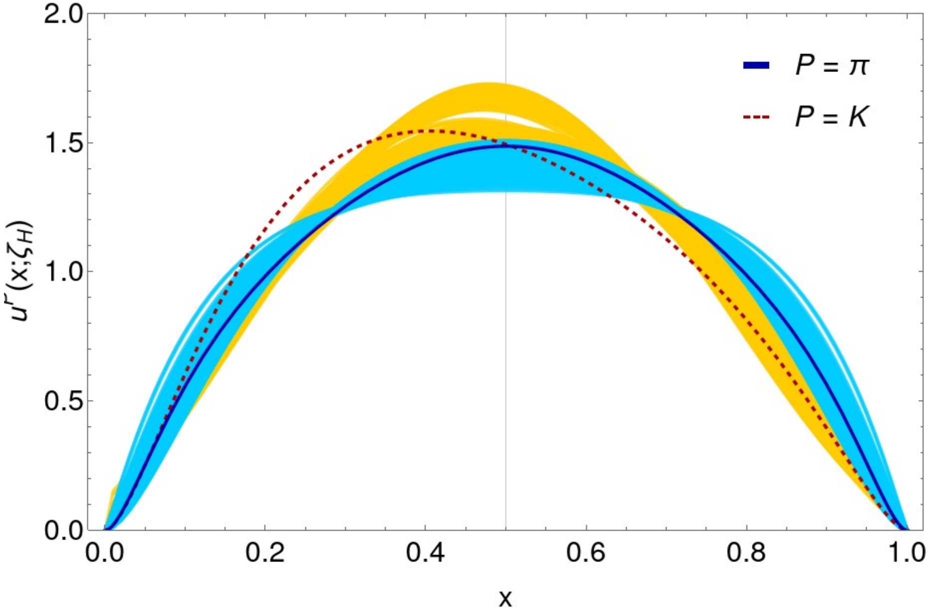
Backup slides

Kaon Data-Driven GPD

Kaon: Data-Driven GPD

- Even though analogous empirical information on the kaon is scarce, we can perform an **analogous exploration** of the **kaon**.



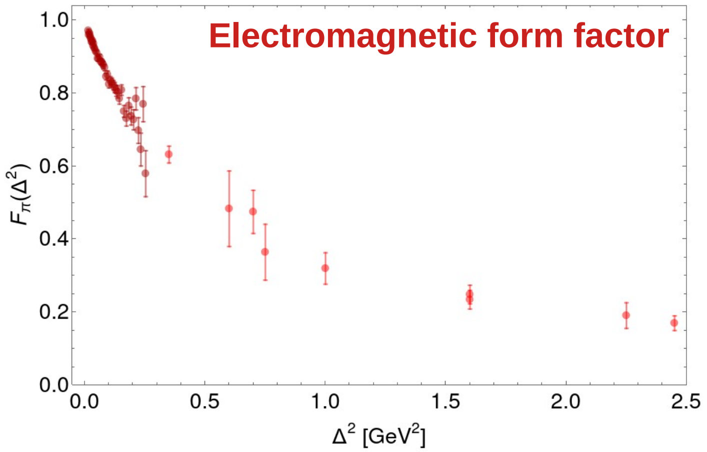


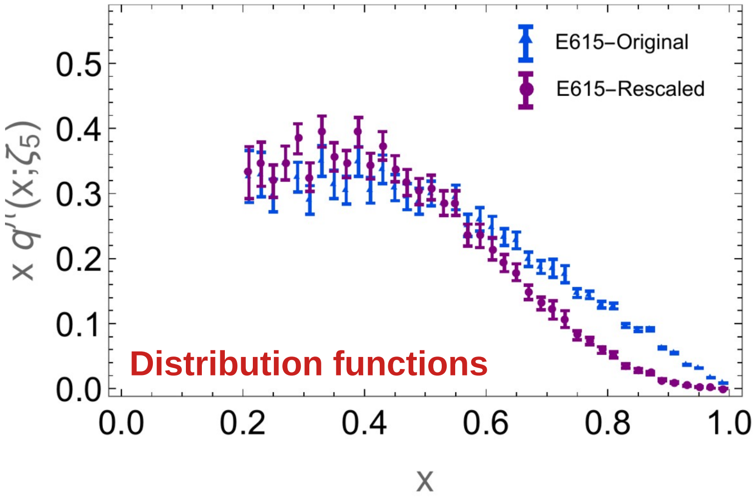
Pion Data-Driven GPD

Pion: Data-Driven GPD

- **Question**:
  From the empirical knowledge of 1-dimensional distributions (**EFF** and **PDF**), can we obtain the 3-dimensional **GPD**?

$$u^\pi(x;\zeta_{e/l})\,,\ F_\pi(\Delta^2) \longrightarrow H_\pi(x,\xi,-\Delta^2;\zeta)$$ **???**





Pion: Data-Driven GPD

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$$u^\pi(x;\zeta_{e/l})\,,\;F_\pi(\Delta^2)\;\longrightarrow\;H_\pi(x,\xi,-\Delta^2;\zeta)$$

???

- ~~Partial~~ **Answer:**

$u^\pi(x;\zeta_{e/l})$

All orders evolution

DGLAP GPD

$$H_\pi^u(x,\xi,-\Delta^2;\zeta_H) = \theta(x_-)\sqrt{u^\pi(x_-;\zeta_H)u^\pi(x_+;\zeta_H)}\,\Phi^\pi(z^2;\zeta_H)$$

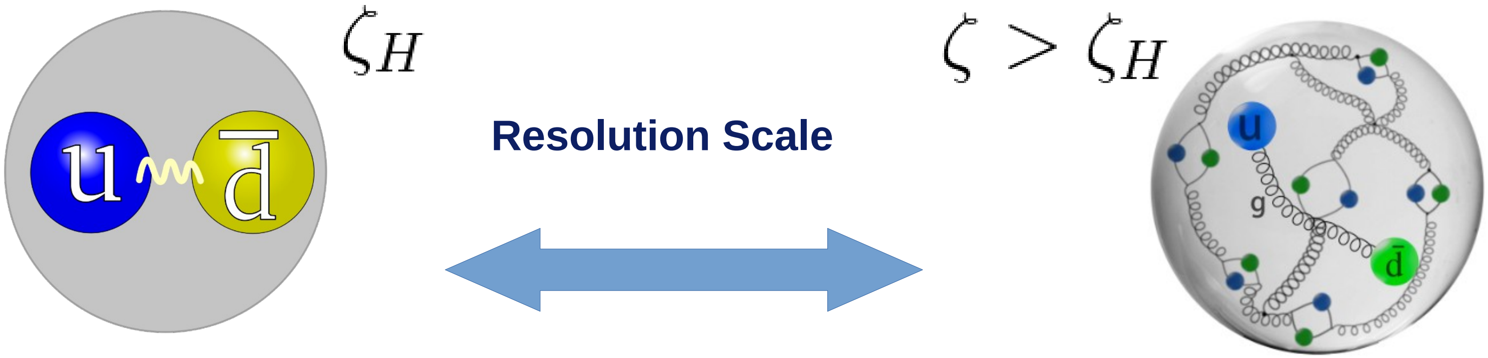
Factorized LFWF

Sum rule

$$H_\pi^u(x,\xi,-\Delta^2;\zeta_H) \sim \int_{k_\perp} \psi^*\psi$$

$$F_\pi(\Delta^2) = \int_0^1 dx\, H_\pi^u(x,0,-\Delta^2)$$

PARTON DISTRIBUTIONS



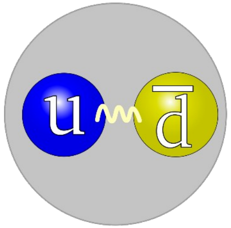
- **Fully-dressed valence quarks**

  (quasiparticles)

- **Unveiling of glue and sea d.o.f**

  (partons)

Pion PDF: hadronic scale



$(M_u = M_d)$

- **Fully-dressed valence quarks** (quasiparticles)

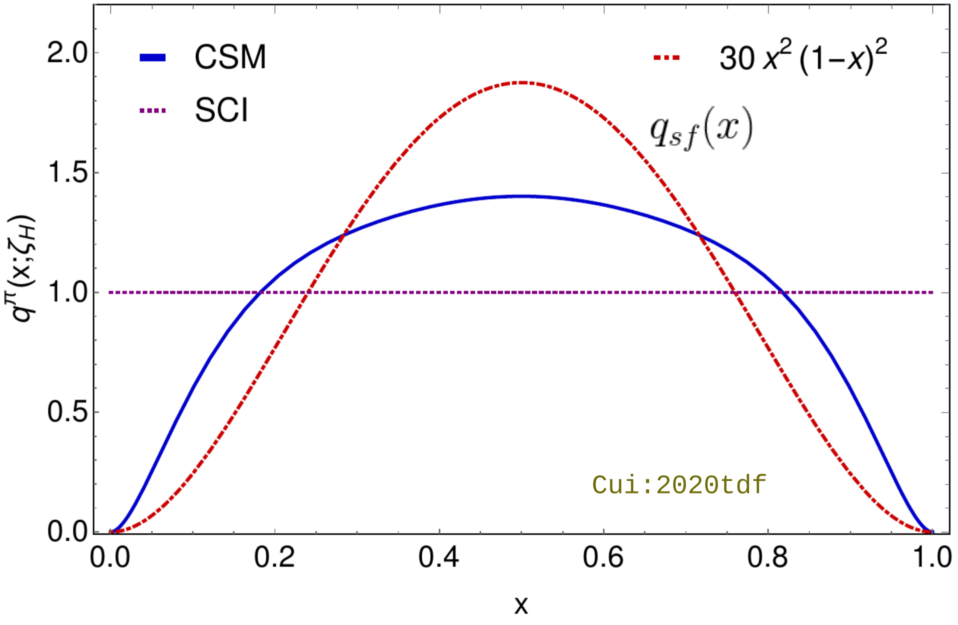
ζ_H : hadronic scale

- At this scale, **all properties** of the hadron are contained within their valence quarks.
  - Equally massive quarks means a **50-50** share of the total momentum:

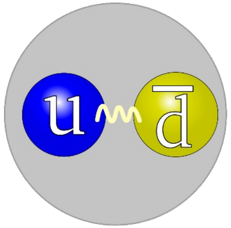
$$<x(\zeta_H)>_q = 0.5$$

  - This implies symmetric distributions:

$$q(x;\zeta_H) = q(1-x;\zeta_H)$$



Pion PDF: hadronic scale



$(M_u = M_d)$

- **Fully-dressed valence quarks** (quasiparticles)

ζ_H : hadronic scale

➢ At this scale, **all properties** of the hadron are contained within their valence quarks.

"**Physical**" boundaries:

$$\frac{1}{2^n} \overset{(i)}{\leq} \langle x^n \rangle_{u_\pi}^{\zeta_H} \overset{(ii)}{\leq} \frac{1}{1+n}$$

Produced by

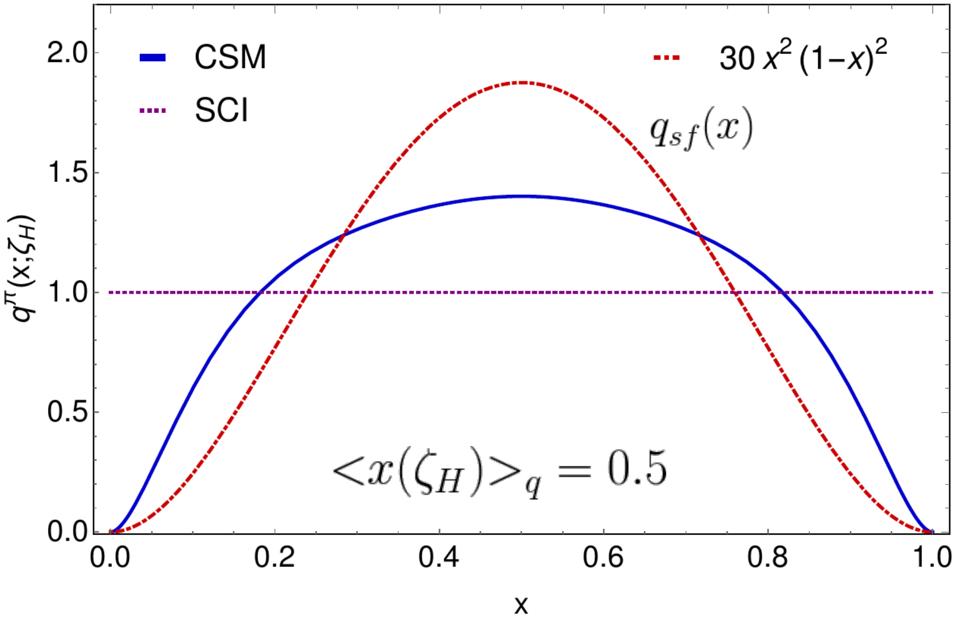
$$q(x;\zeta_H) = \delta(x - 1/2)$$

(infinitely heavy valence quarks)

Produced by

$$q(x;\zeta_H) = 1$$

(massless SCI case)

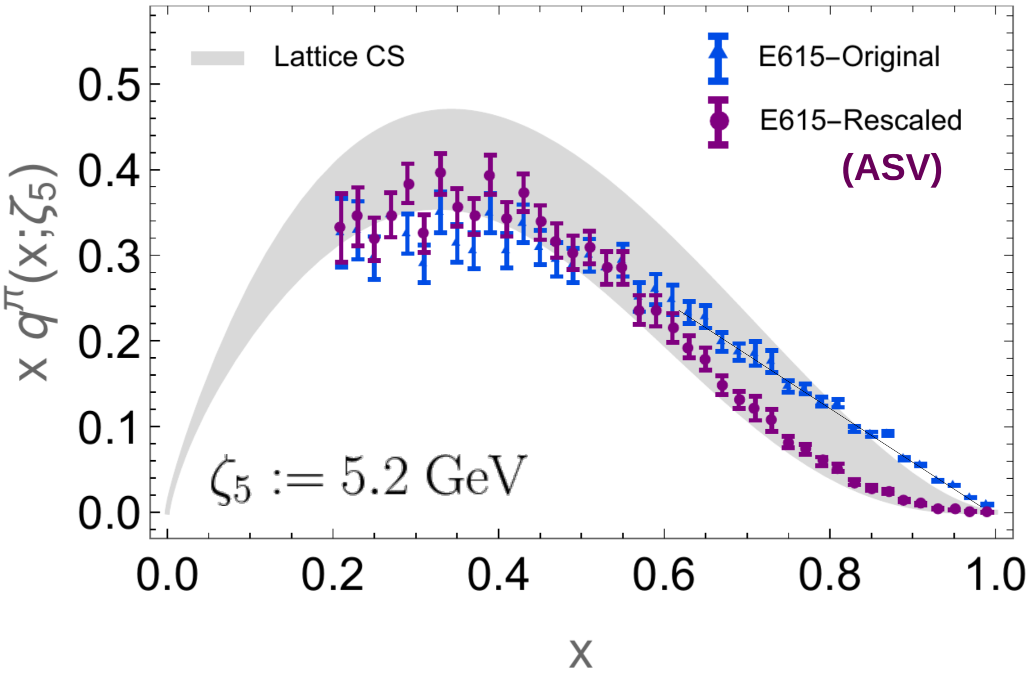


➔ Equally massive quarks means a **50-50** share of the total momentum.

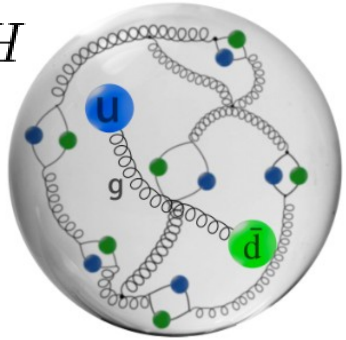
➔ This implies symmetric distributions.

Pion PDF: experimental scale

Conway:1989fs Aicher:2010cb Sufian:2020vzb



$\zeta > \zeta_H$

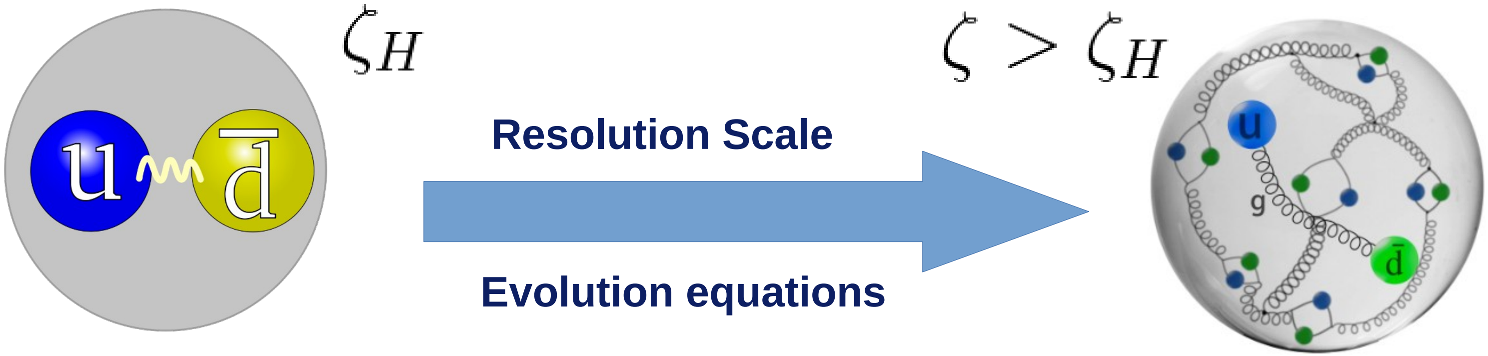


- **Unveiling of glue and sea d.o.f** (partons)

- **Experimental** data is given **here**.
- **Lattice QCD** results are also quoted beyond the **hadronic scale**.
  - The interpretation of parton distributions from cross sections demands **special care**.

Barry:2021osv

Pion PDF: energy scales



- **Fully-dressed valence quarks**

  (quasiparticles)

  - Theoretical calculations are perfomed at *some* low energy scale.

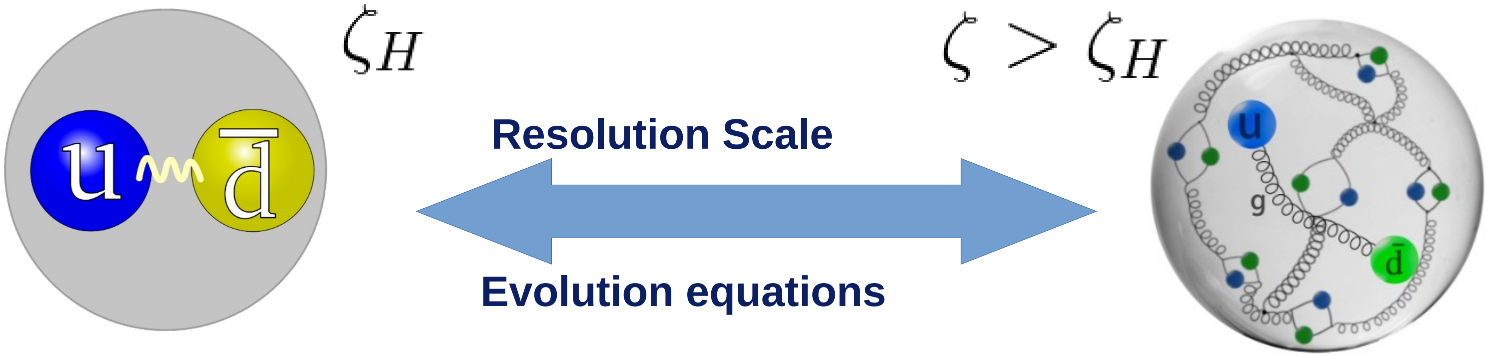
- **Unveiling of glue and sea d.o.f**

  (partons)

  - Then evolved via **DGLAP** equations to compare with experiment and lattice.

Pion PDF: energy scales

Rodriguez-Quintero:2019fyc



- **Fully-dressed valence quarks**

  (quasiparticles)

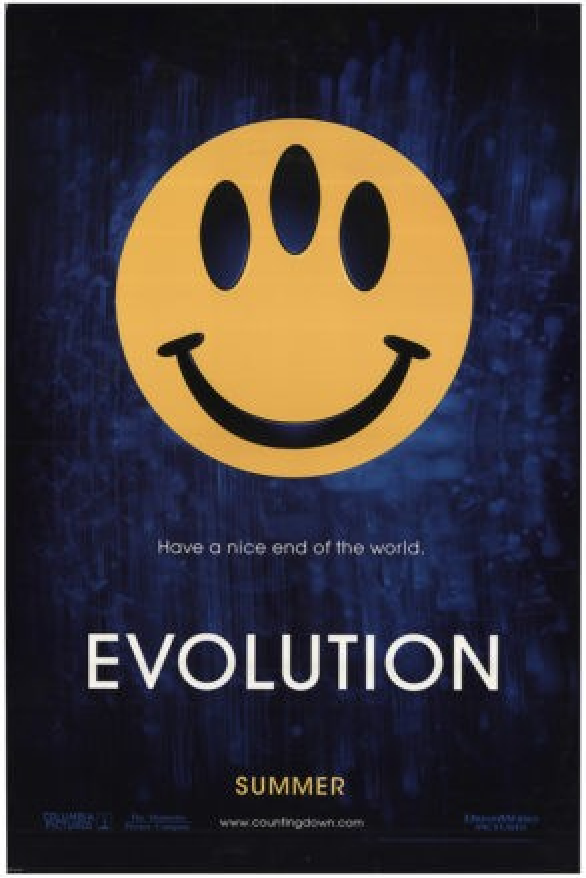
  - Theoretical calculations are perfomed at *some* low energy scale.

- **Unveiling of glue and sea d.o.f**

  (partons)

  - Then evolved via **DGLAP** equations to compare with experiment and lattice.

> - Following our **all orders** evolution, we can go **either way**.
> - Besides, the **hadronic scale** becomes unambigously **determined**.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EVOLUTION
SUMMER
www.countingdown.com

DGLAP: All orders evolution

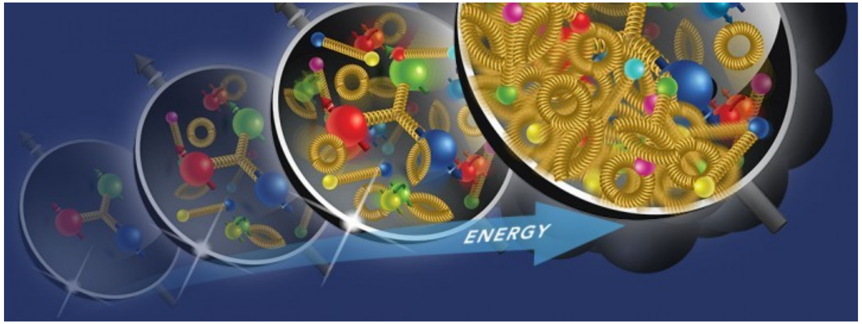
Idea. Define an **effective** coupling such that:

"All orders evolution"

Starting from fully-dressed **quasiparticles**, at ζ_H → **Sea** and **Gluon** content unveils, as prescribed by **QCD**

$$\left\{\zeta^2 \frac{d}{d\zeta^2}\int_0^1 dy\,\delta(y-x) - \boxed{\frac{\alpha(\zeta^2)}{4\pi}}\int_x^1 \frac{dy}{y}\begin{pmatrix} P_{qq}^{\mathrm{NS}}\left(\frac{x}{y}\right) & 0 \\ 0 & \mathbf{P}^{\mathrm{S}}\left(\frac{\mathbf{x}}{\mathbf{y}}\right)\end{pmatrix}\right\}\begin{pmatrix} H_\pi^{\mathrm{NS},+}(y,t;\zeta) \\ \mathbf{H}_\pi^{\mathrm{S}}(y,t;\zeta)\end{pmatrix} = 0$$

- **Not** the **LO** QCD coupling but an **effective** one.
- Making this equation **exact**.
- And connecting with the **hadron scale.**



Raya:2021zrz Cui:2020tdf

DGLAP: All orders evolution

Implication 1:

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi} S(\zeta_H,\zeta_f)\right)\langle x^n(\zeta_H)\rangle_q$$

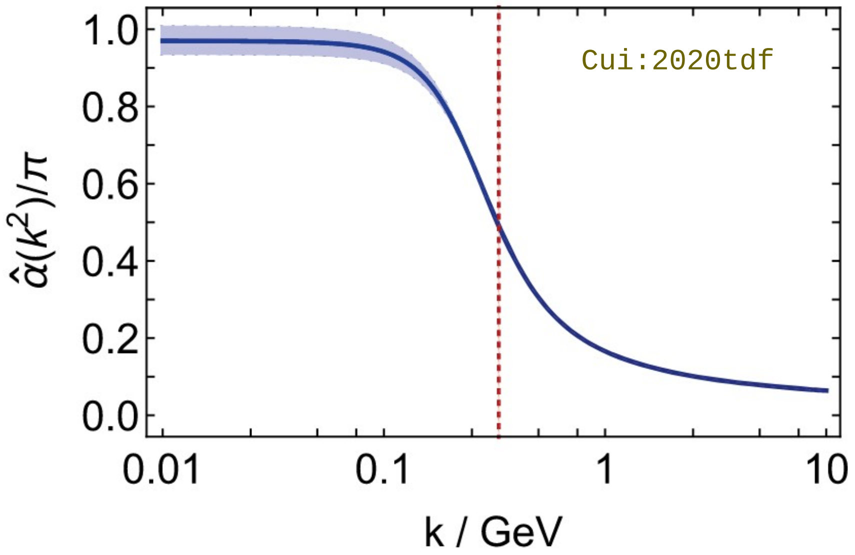
$$S(\zeta_0,\zeta_f) = \int_{2\ln(\zeta_0/\Lambda_{\rm QCD})}^{2\ln(\zeta_f/\Lambda_{\rm QCD})} dt\,\alpha(t)$$

Explicitly depending on the **effective charge**

$$\langle x^n(t;\zeta)\rangle_F = \int_0^1 dx\, x^n\, F(x,t;\zeta)$$

$$\gamma_{AB}^{(n)} = -\int_0^1 dx\, x^n P_{AB}^C(x)$$

- The **QCD PI effective charge** is our best candidate to accommodate our **all orders scheme**.

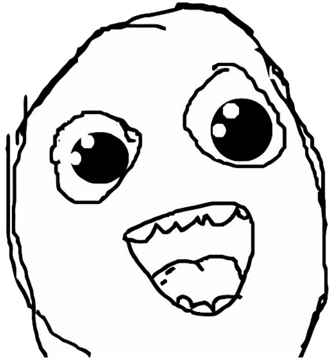


$$\hat{\alpha}(k^2) = \frac{\gamma_m \pi}{\ln\left[\frac{\mathcal{M}^2(k^2)}{\Lambda_{\rm QCD}^2}\right]} \Rightarrow \zeta_H = 0.331\ \text{GeV}$$

DGLAP: All orders evolution

Implication 1:

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi}S(\zeta_H,\zeta_f)\right)\langle x^n(\zeta_H)\rangle_q = \langle x^n(\zeta_H)\rangle_q \left(\frac{\langle x(\zeta_f)\rangle_q}{\langle x(\zeta_H)\rangle_q}\right)^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}$$



$$S(\zeta_0,\zeta_f) = \int_{2\ln(\zeta_0/\Lambda_{\rm QCD})}^{2\ln(\zeta_f/\Lambda_{\rm QCD})} dt\, \alpha(t)$$

This contains, ***implicitly***, the information of the **effective charge**

- No actual ***need*** to know it. Assuming its existence is **sufficient**.
- ***Unambiguous*** definition of the **hadron scale**:

$$\langle x(\zeta_H)\rangle_q = 0.5 \;\Rightarrow\; \langle x^n(\zeta_f)\rangle_q = \langle x^n(\zeta_H)\rangle_q \left(\langle 2x(\zeta_f)\rangle_q\right)^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}$$

(flavor symmetric case)

DGLAP: All orders evolution

Implication 1:

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi}S(\zeta_H,\zeta_f)\right)\langle x^n(\zeta_H)\rangle_q = \langle x^n(\zeta_H)\rangle_q \underbrace{\left(\frac{\langle x(\zeta_f)\rangle_q}{\langle x(\zeta_H)\rangle_q}\right)^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}}$$

Information on the charge is here

- Details of the effective charge are **encoded** in the ratio of first moments.
- Natural connection with the **hadron scale**.

Implication 2:

$$\begin{aligned}
\langle 2x(\zeta_f)\rangle_q &= \exp\left(-\frac{8}{9\pi}S(\zeta_H,\zeta_f)\right), \qquad q = u,\bar{d}; \\
\langle x(\zeta_f)\rangle_{\text{sea}} &= \langle x(\zeta_f)\rangle_{\sum_q q+\bar{q}} - (\langle x(\zeta_f)\rangle_u + \langle x(\zeta_f)\rangle_{\bar{d}}), \\
&= \frac{3}{7} + \frac{4}{7}\langle 2x(\zeta_f)\rangle_u^{7/4} - \langle 2x(\zeta_f)\rangle_u \\
\langle x(\zeta_f)\rangle_g &= \frac{4}{7}\left(1 - \langle 2x(\zeta_f)\rangle_u^{7/4}\right);
\end{aligned}$$

- **Sea** and **gluon** determined from valence-quark moments

DGLAP: All orders evolution

Implication 1:

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi} S(\zeta_H,\zeta_f)\right)\langle x^n(\zeta_H)\rangle_q = \langle x^n(\zeta_H)\rangle_q \left(\frac{\langle x(\zeta_f)\rangle_q}{\langle x(\zeta_H)\rangle_q}\right)^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}$$

Information on the charge is here

- Can **jump** from one scale to another (both ways)
- Natural connection with the **hadron scale**.

Implication 2:

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- **Sea** and **gluon** determined from valence-quark moments
- **Asymptotic** (massless) limits are evident.
- And, of course, the momentum **sum rule**:

$$\langle 2x(\zeta_f)\rangle_q + \langle x(\zeta_f)\rangle_{\text{sea}} + \langle x(\zeta_f)\rangle_g = 1$$

DGLAP: All orders evolution

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Implication 3: Recurrence relation

$$\langle x^{2n+1}\rangle_{u_\pi}^{\zeta} = \frac{(\langle 2x\rangle_{u_\pi}^{\zeta})^{\gamma_0^{2n+1}/\gamma_0^1}}{2(n+1)} \times \sum_{j=0,1,\ldots}^{2n} (-)^j \binom{2(n+1)}{j} \langle x^j\rangle_{u_\pi}^{\zeta} (\langle 2x\rangle_{u_\pi}^{\zeta})^{-\gamma_0^j/\gamma_0^1} .$$

- Since **isospin symmetry** limit implies:

$$q(x;\zeta_H) = q(1-x;\zeta_H)$$

- **Odd** moments can be expressed in terms of previous **even** moments.
- Thus arriving at the recurrence **relation** on the left.

DGLAP: All orders evolution

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n	$\langle x^n\rangle^{\zeta_5}_{u_\pi}$ Lattice input	Recurrence relation
1	0.230(3)(7)	0.230
2	0.087(5)(8)	0.087
3	0.041(5)(9)	0.041
4	0.023(5)(6)	0.023
5	0.014(4)(5)	0.015
6	0.009(3)(3)	0.009
7		0.0078

Gravitational form factors

Gravitational form factors

- For a given parton class, the **spin-0** energy-momentum tensor (**EMT**) can take the following form:

$$\underbrace{\Lambda^{a}_{\mu\nu}(P,Q)}_{\langle P_f|T_{\mu\nu}(0)|P_i\rangle} = 2P_\mu P_\nu \theta^{a}_2(Q^2) + \frac{1}{2}\left(Q^2 g_{\mu\nu} - Q_\mu Q_\nu\right)\theta^{a}_1(Q^2) + 2m_\pi^2 g_{\mu\nu}\bar{c}^{a}(Q^2)$$

With: $P = [P_f + P_i]/2 \text{ and } Q = P_f - P_i$

- Such that $\theta_{1,2}(Q^2),\ \bar{c}(Q^2)$ define the so called **gravitational form factors** (**GFFs**).

(these are extracted by sensible projection operators)

$$\int d^3r\, T_q^{00}(\vec{r}) = m_\pi \Theta_{2,q}(0)$$

→ $\theta_2(Q^2)$ Is connected with the **mass** distribution inside the hadron

$$T_q^{ij}(\vec{r}) = p_q(r)\,\delta_{ij} + s_q(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right)$$
$(i,\, j = 1,\, 2,\, 3)$

→ $\theta_1(Q^2)$ Is connected with the **mechanical** properties of the hadron

p(r) : pressure
s(r) : shear forces

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- Energy-momentum **conservation** entail the following **sum rules**:

$$\sum_{q,g}\theta_2(0) = 1 \qquad \sum_{q,g}\bar{c}(t) = 0$$

- While, in the **chiral limit**, the **soft-pion theorem** constraints:

$$\sum_{q,g}\theta_1(0) = 1$$

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- At the **hadronic scale**, ζ_H, all properties of the hadron are contained within the valence quarks.

Here we shall work...