Proton Structure In and Out Of Muonic Hydrogen

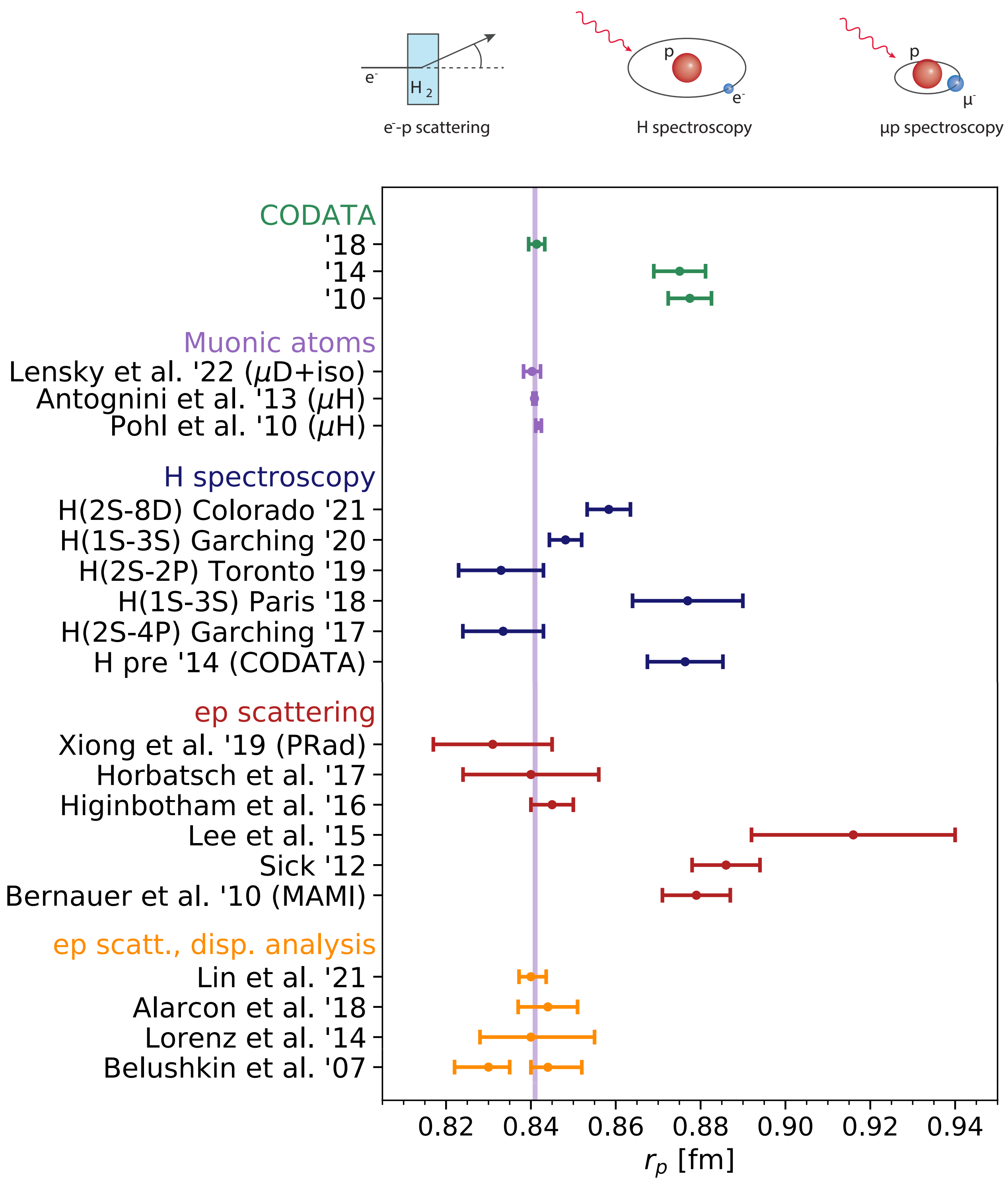
Franziska Hagelstein (JGU Mainz & PSI Villigen)

in collaboration with

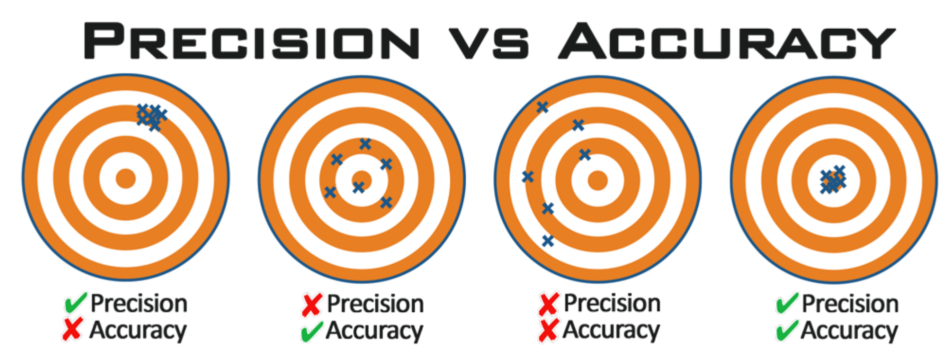
**Volodymyr Biloshytskyi, Vadim Lensky
and Vladimir Pascalutsa (JGU)**

PROTON CHARGE RADIUS

see talks by A. Antognini, H. Gao, U. Meißner



- Muonic atoms allow for PRECISE extractions of nuclear charge and Zemach radii
- CODATA since 2018 included the μH result for r_p
- Still open issues: H(2S-8D) and H(1S-3S)
- Question:



LAMB SHIFT IN MUONIC ATOMS

	THEORY $\Delta E_{TPE} \pm \delta_{theo}\,(\Delta E_{TPE})$	Ref.	**EXPERIMENT** $\delta_{exp}(\Delta_{LS})$	Ref.
μH	$33\ \mu$eV $\pm 2\ \mu$eV	Antognini et al. (2013)	$2.3\ \mu$eV	Antognini et al. (2013)
μD	$1710\ \mu$eV $\pm 15\ \mu$eV	Krauth et al. (2015)	$3.4\ \mu$eV	Pohl et al. (2016)
μ^3He$^+$	15.30 meV ± 0.52 meV	Franke et al. (2017)	0.05 meV	
μ^4He$^+$	9.34 meV ± 0.25 meV -0.15 meV ± 0.15 meV (3PE)	Diepold et al. (2018) Pachucki et al. (2018)	0.05 meV	Krauth et al. (2020)

see presentations by C. Ji, S. Li Muli, V. Lensky, T. Richardson

μH: present accuracy comparable with experimental precision

μD, μ³He⁺, μ⁴He⁺: present accuracy factor 5-10 worse than experimental precision

$r_p = 0.84087(12)_{\text{sys}}(23)_{\text{stat}}(29)_{\text{theory}}$ fm
(25) 2PE (mainly subtraction term)
(15) QED

$r_d = 2.12562(5)_{\text{sys}}(12)_{\text{stat}}(77)_{\text{theory}}$ fm
basically only nuclear 2PE

$r_\alpha = 1.67824(2)_{\text{sys}}(13)_{\text{stat}}(82)_{\text{theory}}$ fm
(70) 2PE (elastic 25, nuclear inelastic 36, nucleon inelastic 56)
(42) 3PE (inelastic contribution missing)
(4) QED

FROM PUZZLE TO PRECISION

- Several experimental activities ongoing and proposed:
  - 1S hyperfine splitting in μH and μHe (CREMA, FAMU, J-PARC)
  - Improved measurement of Lamb shift in μH, μD and μHe^+ possible ($\times 5$)
  - Medium- and high-Z muonic atoms
- **Theory support** is needed!

Muonic Atom Spectroscopy Theory Initiative

- Initials objectives:
  - Accurate theory predictions for light muonic atoms to test fundamental interactions by comparing to electronic atoms
  - Community consensus on SM predictions
  - First emphasis on the hyperfine splitting in μH

“PREN & µASTI” workshop @ JGU, 06/23

Homepage and mailing list → https://asti.uni-mainz.de

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Atomic Spectroscopy Theory Initiative

HOME SEARCH SITEMAP

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Muonic Atom Spectroscopy Theory Initiative

Inspired by the success of the Muon g-2 Theory Initiative we are launching the Muonic Atom Spectroscopy Theory Initiative (μASTI).

from PUZZLE to PRECISION

The initiative aims to support the experimental effort on the spectroscopy of light muonic atoms by improving the Standard Model theory predictions for the Lamb shift and hyperfine splitting in muonic hydrogen, deuterium, and helium, in order to match the anticipated accuracy of future measurements. An initial focus will be on the ground state hyperfine splitting in muonic hydrogen.

The **upcoming kick-off event** for the Theory Initiative is organized as a joint meeting with the Proton Radius European Network (PREN) at the Johannes Gutenberg University Mainz (June 26-30, 2023).

2212.13782

Comprehensive theory of the Lamb shift in light muonic atoms

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[1] *Faculty of Physics, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland*
[2] *Institut für Kernphysik, Johannes Gutenberg-Universität Mainz, 55128 Mainz, Germany*
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[4] *Helmholtz-Institut Mainz, Johannes Gutenberg Universität Mainz, 55099 Mainz, Germany*
[5] *Institut für Physik, Johannes Gutenberg-Universität Mainz, 55099 Mainz, Germany*

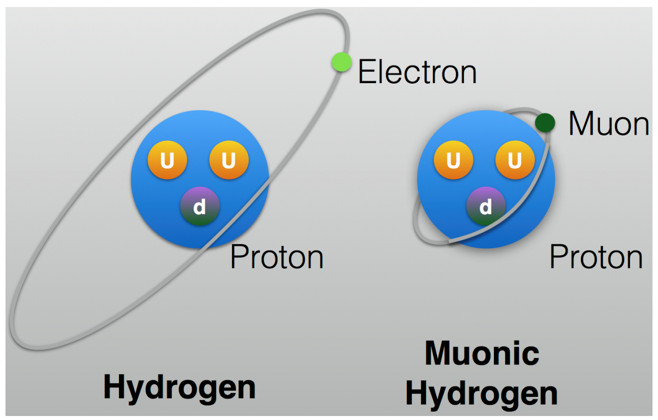
(Dated: May 19, 2023)

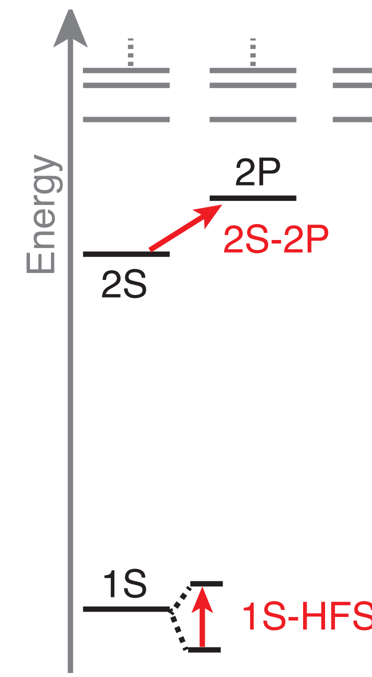
We present a comprehensive theory of the Lamb shift in light muonic atoms, such as μH, μD, μ^3He$^+$, and μ^4He$^+$, with all quantum electrodynamic corrections included at the precision level constrained by the uncertainty of nuclear structure effects. This analysis can be used in the global adjustment of fundamental constants and in the determination of nuclear charge radii. Further improvements in the understanding of electromagnetic interactions of light nuclei will allow for a promising test of fundamental interactions by comparison with "normal" atomic spectroscopy, in particular, with H-D and ^{3}He-^{4}He isotope shifts.

E_{QED}	point nucleus	206.0344(3)	228.7740(3)	1644.348(8)	1668.491(7)
$\mathcal{C}\, r_C^2$	finite size	$-5.2259\, r_p^2$	$-6.1074\, r_d^2$	$-103.383\, r_h^2$	$-106.209\, r_\alpha^2$
E_{NS}	nuclear structure	0.0289(25)	1.7503(200)	15.499(378)	9.276(433)
$E_L(\text{exp})$	experiment[a]	202.3706(23)	202.8785(34)	1258.598(48)	1378.521(48)
r_C	this work	0.84060(39)	2.12758(78)	1.97007(94)	1.6786(12)
r_C	previous[a]	0.84087(39)	2.12562(78)	1.97007(94)	1.67824(83)

NUCLEAR STRUCTURE EFFECTS

Why muonic atoms ?





Lamb shift
μH, μD, μ^3He$^+$, μ^4He$^+$
⇒ Charge radii

Hyperfine splitting (HFS)
μH, μ^3He$^+$
⇒ Zemach radii, magnetic properties

- **Lamb shift:**

wave function at the origin

$$\Delta E_{nl}(\mathrm{LO{+}NLO}) = \delta_{l0}\,\frac{2\pi Z\alpha}{3}\,\frac{1}{\pi(an)^3}\left[R_E^2 - \frac{Z\alpha m_r}{2}R^3_{E(2)}\right]$$

NLO becomes appreciable in μH

- **HFS:**

$$\Delta E_{nS}(\mathrm{LO}+\mathrm{NLO}) = E_F(nS)\,[1 - 2\,Z\alpha m_r\,R_Z]$$

Fermi energy:

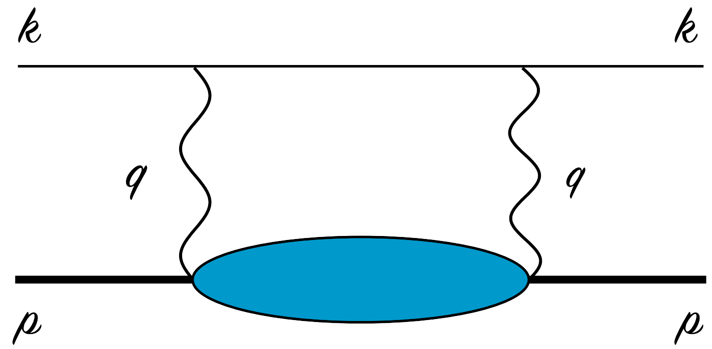
$$E_F(nS) = \frac{8}{3}\frac{Z\alpha}{a^3}\frac{1+\kappa}{mM}\frac{1}{n^3}$$

with Bohr radius $a = 1/(Z\alpha m_r)$

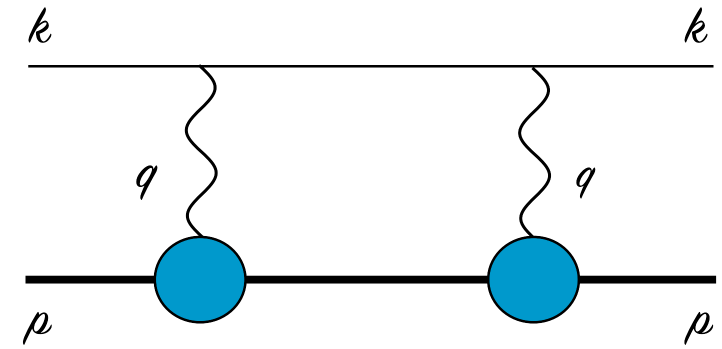
STRUCTURE EFFECTS THROUGH 2γ

- Proton-structure effects at subleading orders arise through multi-photon processes

forward two-photon exchange (2γ)



polarizability contribution
(non-Born VVCS)



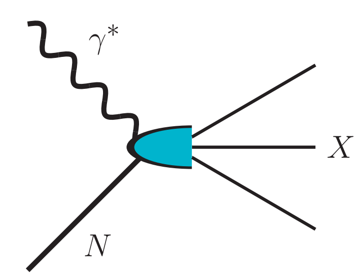
elastic contribution:
finite-size recoil,
3rd Zemach moment (Lamb shift),
Zemach radius (Hyperfine splitting)

- "Blob" corresponds to doubly-virtual Compton scattering (VVCS):

$$T^{\mu\nu}(q,p) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) \boxed{T_1(\nu,Q^2)} + \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}\,q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}\,q^\nu\right)\boxed{T_2(\nu,Q^2)}$$
$$- \frac{1}{M}\gamma^{\mu\nu\alpha} q_\alpha \boxed{S_1(\nu,Q^2)} - \frac{1}{M^2}\left(\gamma^{\mu\nu}q^2 + q^\mu\gamma^{\nu\alpha}q_\alpha - q^\nu\gamma^{\mu\alpha}q_\alpha\right)\boxed{S_2(\nu,Q^2)}$$

- Proton structure functions: $\boxed{f_1(x,Q^2),\ f_2(x,Q^2)}$, $\boxed{g_1(x,Q^2),\ g_2(x,Q^2)}$

Lamb shift — Hyperfine splitting (HFS)



2γ EFFECT IN THE LAMB SHIFT

wave function at the origin

$$\Delta E(nS) = 8\pi\alpha m\, \phi_n^2\, \frac{1}{i} \int_{-\infty}^{\infty} \frac{\mathrm{d}\nu}{2\pi} \int \frac{\mathrm{d}\mathbf{q}}{(2\pi)^3} \frac{\left(Q^2 - 2\nu^2\right) T_1(\nu, Q^2) - \left(Q^2 + \nu^2\right) T_2(\nu, Q^2)}{Q^4 (Q^4 - 4m^2\nu^2)}$$

dispersion relation & optical theorem:

$$T_1(\nu, Q^2) = \boxed{T_1(0, Q^2)} + \frac{32\pi Z^2 \alpha M \nu^2}{Q^4} \int_0^1 \mathrm{d}x\, \frac{x f_1(x, Q^2)}{1 - x^2(\nu/\nu_{\mathrm{el}})^2 - i0^+}$$

$$T_2(\nu, Q^2) = \frac{16\pi Z^2 \alpha M}{Q^2} \int_0^1 \mathrm{d}x\, \frac{f_2(x, Q^2)}{1 - x^2(\nu/\nu_{\mathrm{el}})^2 - i0^+}$$

- Caution: in the data-driven dispersive approach the $T_1(0,Q^2)$ subtraction function is modelled!

low-energy expansion:

$$\lim_{Q^2 \to 0} \overline{T}_1(0, Q^2)/Q^2 = 4\pi\beta_{M1}$$

modelled Q^2 behavior:

$$\overline{T}_1(0, Q^2) = 4\pi\beta_{M1}\, Q^2 / \left(1 + Q^2/\Lambda^2\right)^4$$

Assuming ChPT is working, it should be best applicable to atomic systems, where the energies are very small !

POLARIZABILITY EFFECT IN LAMB SHIFT

BChPT result is in good agreement with dispersive calculations !!!

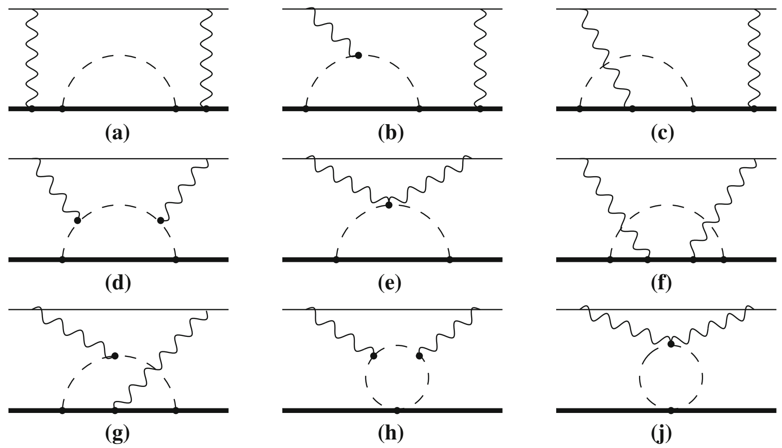
Agreement also for the contribution of the T_1 subtraction function !!!

Table 1 Forward 2γ-exchange contributions to the 2S-shift in μH, in units of μeV.

Reference	$E_{2S}^{(\mathrm{subt})}$	$E_{2S}^{(\mathrm{inel})}$	$E_{2S}^{(\mathrm{pol})}$	$E_{2S}^{(\mathrm{el})}$	$E_{2S}^{\langle 2\gamma\rangle}$
DATA-DRIVEN					
(73) Pachucki '99	1.9	−13.9	−12(2)	−23.2(1.0)	−35.2(2.2)
(74) Martynenko '06	2.3	−16.1	−13.8(2.9)		
(75) Carlson *et al.* '11	5.3(1.9)	−12.7(5)	−7.4(2.0)		
(76) Birse and McGovern '12	4.2(1.0)	−12.7(5)	−8.5(1.1)	−24.7(1.6)	−33(2)
(77) Gorchtein et al.'13 [a]	−2.3(4.6)	−13.0(6)	−15.3(4.6)	−24.5(1.2)	−39.8(4.8)
(78) Hill and Paz '16					−30(13)
(79) Tomalak'18	2.3(1.3)		−10.3(1.4)	−18.6(1.6)	−29.0(2.1)
LEADING-ORDER BχPT					
(80) Alarcòn *et al.* '14			$-9.6^{+1.4}_{-2.9}$		
(81) Lensky *et al.* '17 [b]	$3.5^{+0.5}_{-1.9}$	−12.1(1.8)	$-8.6^{+1.3}_{-5.2}$		
LATTICE QCD					
(82) Fu *et al.* '22					−37.4(4.9)

[a] Adjusted values due to a different decomposition into the elastic and polarizability contributions.
[b] Partially includes the Δ(1232)-isobar contribution.

LO BChPT prediction with pion-nucleon loop diagrams:



J. M. Alarcon, V. Lensky, V. Pascalutsa, Eur. Phys. J. C **74** (2014) 2852

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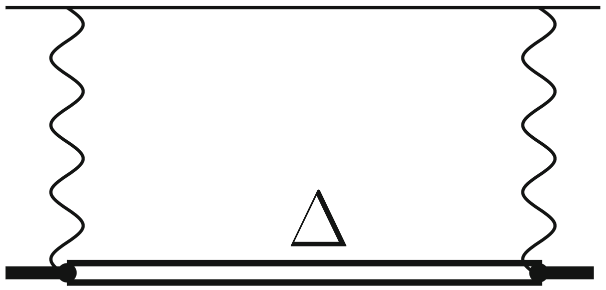
[b]Partially includes the Δ(1232)-isobar contribution.

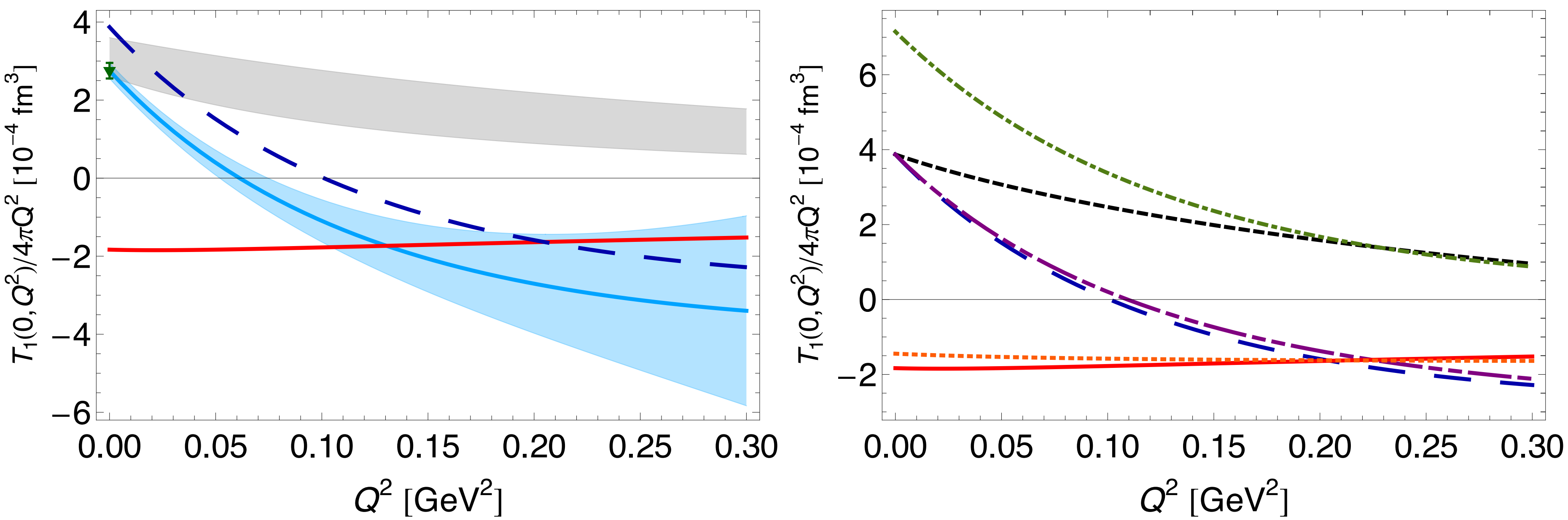
Δ prediction from Δ(1232) exchange:

- Uses large-N_c relations for the Jones-Scadron N-to-Δ transition form factors
- Small due to the suppression of β_{M1} in the Lamb shift but important for



V. Lensky, FH, V. Pascalutsa, M. Vanderhaeghen, Phys. Rev. D **97** (2018) 074012

SUBTRACTION FUNCTION



NLO BChPT δ-exp.

total without g_M dipole

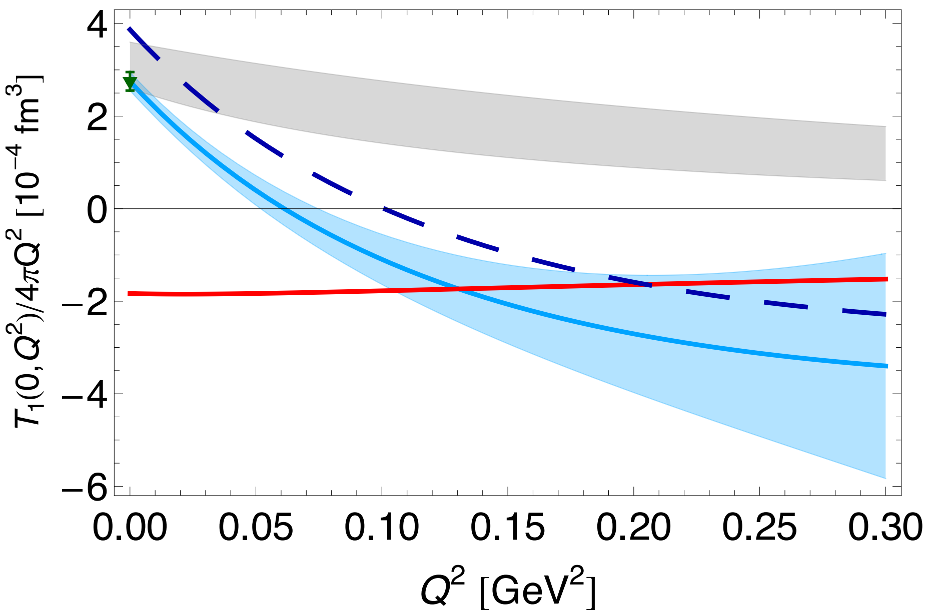
πN loops

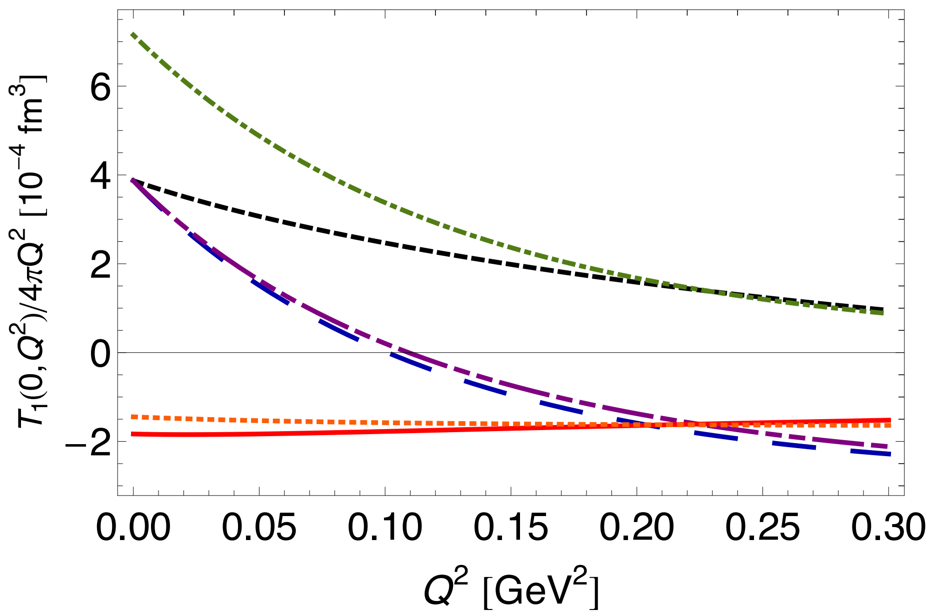
πΔ loops

Δ-exchange

J. Alarcon, FH, V. Lensky and V. Pascalutsa, Phys. Rev. D **102** (2020) 114026; ibid. **102** (2020) 114006

SUBTRACTION FUNCTION





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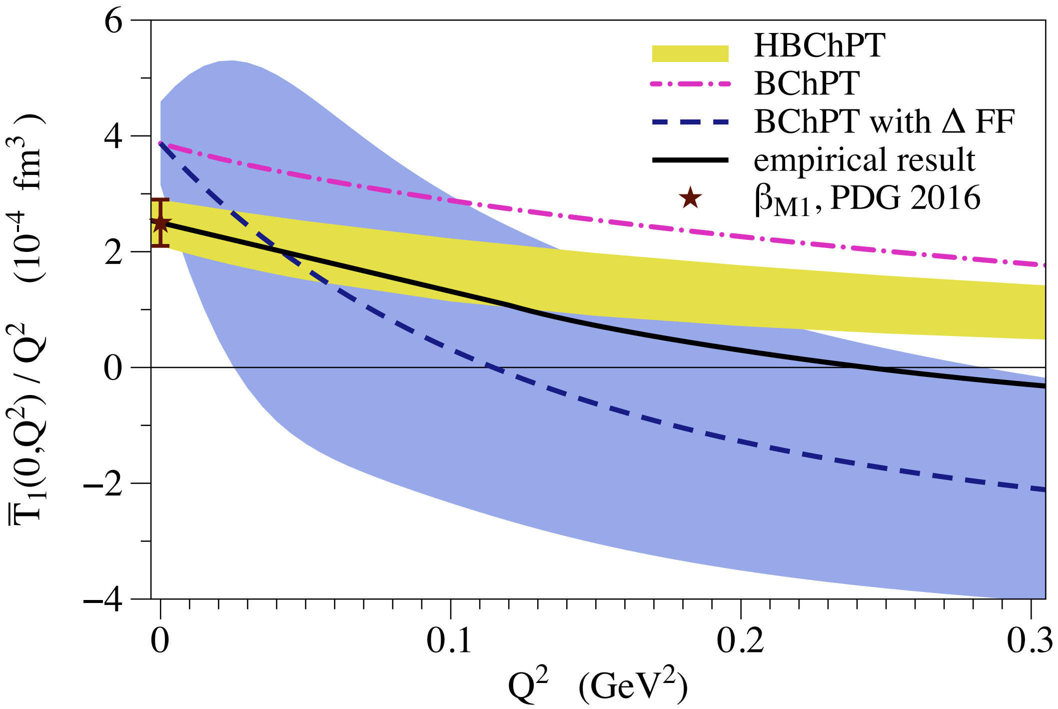

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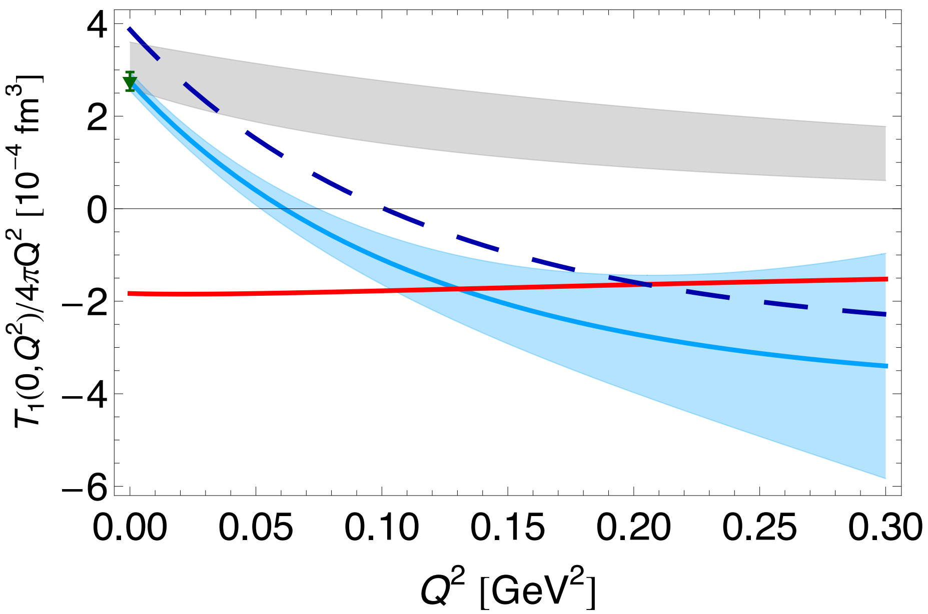

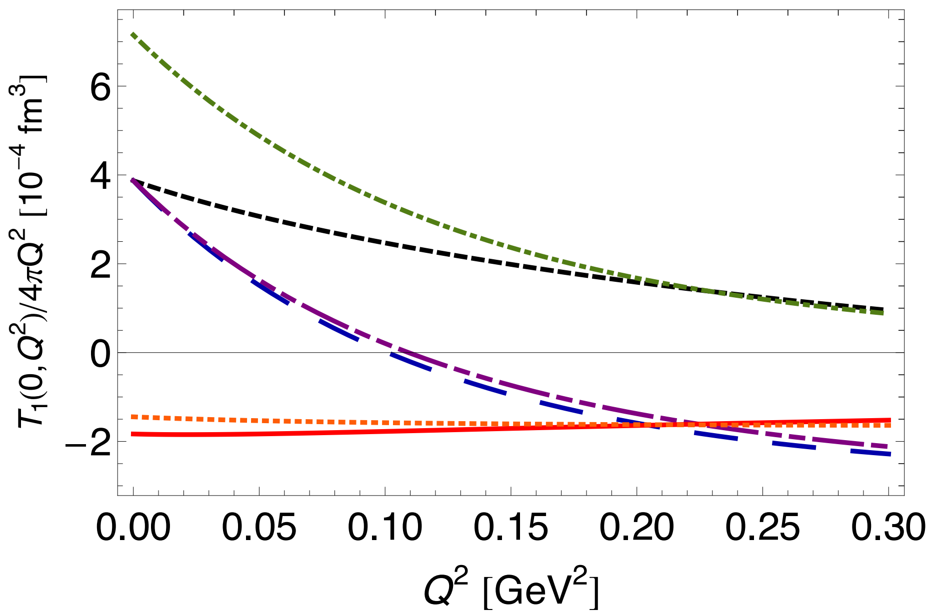

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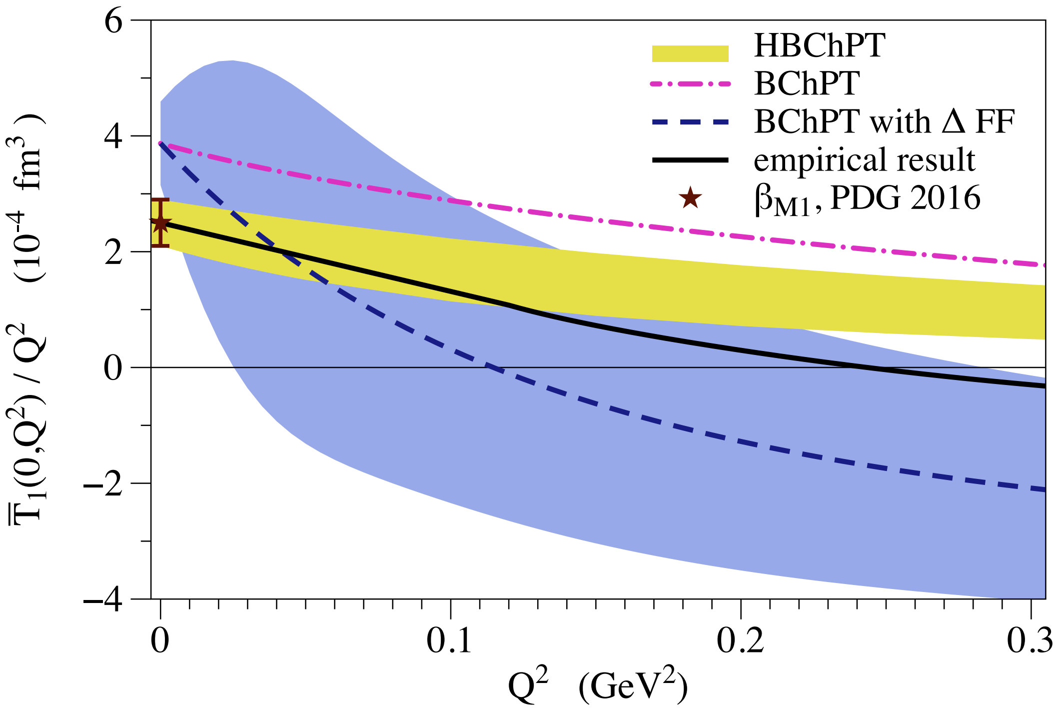
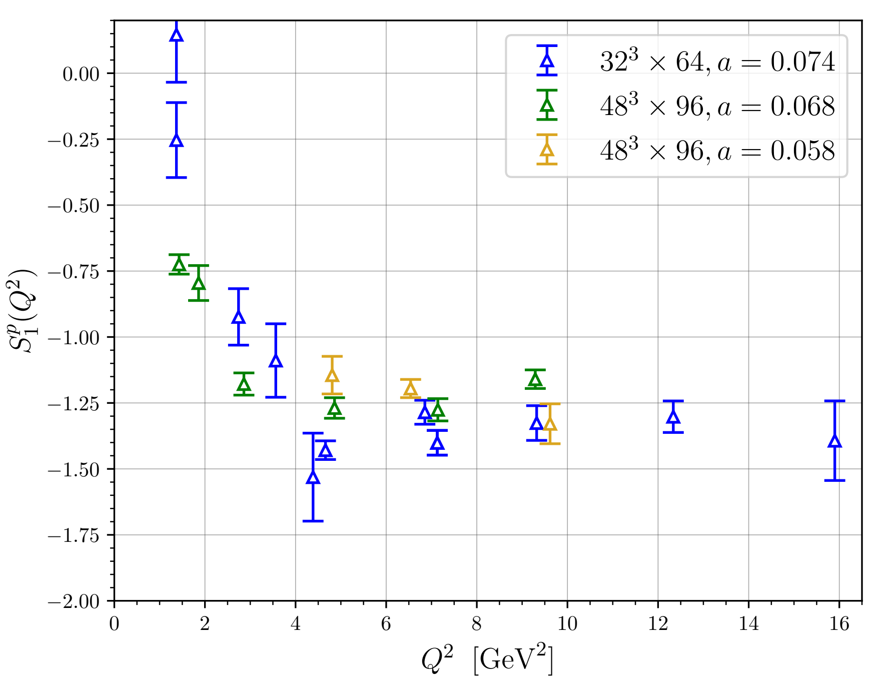


V. Lensky, FH, V. Pascalutsa and M. Vanderhaeghen Phys. Rev. D **97** (2018) 074012

First lattice results!



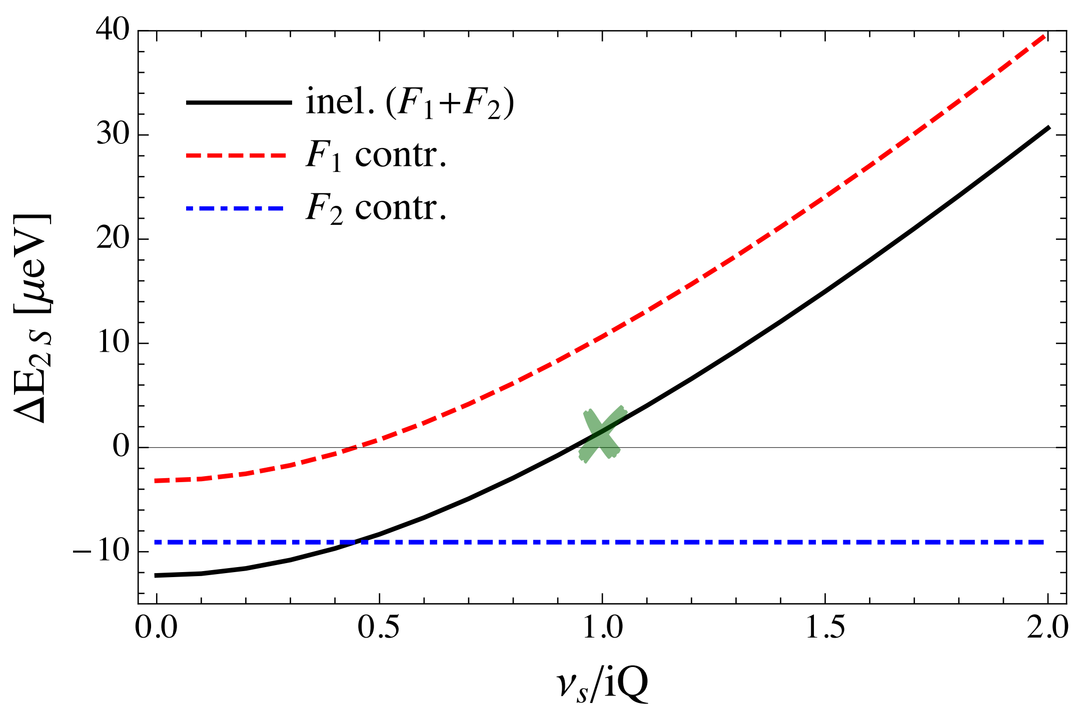
CSSM-QCDSF-UKQCD Collaboration, 2207.03040.

EUCLIDEAN SUBTRACTION FUNCTION

- Once-subtracted dispersion relation for $\overline{T}_1(\nu, Q^2)$ with subtraction at $\nu_s = iQ$
- Dominant part of polarizability contribution:

$$\Delta E_{nS}^{\prime\,(\mathrm{subt})} = \frac{2\alpha m}{\pi}\phi_n^2 \int_0^\infty \frac{\mathrm{d}Q}{Q^3}\frac{2+v_l}{(1+v_l)^2}\,\overline{T}_1(iQ, Q^2) \text{ with } v_l = \sqrt{1+4m^2/Q^2}$$

- Inelastic contribution for $\nu_s = iQ$ is order of magnitude smaller than for $\nu_s = 0$
- Prospects for future lattice QCD and EFT calculations



FH, V. Pascalutsa, Nucl. Phys. A **1016** (2021) 122323

based on Bosted-Christy parametrization:

$$\Delta E_{2S}^{(\mathrm{inel})}(\nu_s = 0) \simeq -12.3\,\mu\mathrm{eV}$$

$$\Delta E_{2S}^{\prime\,(\mathrm{inel})}(\nu_s = iQ) \simeq 1.6\,\mu\mathrm{eV}$$

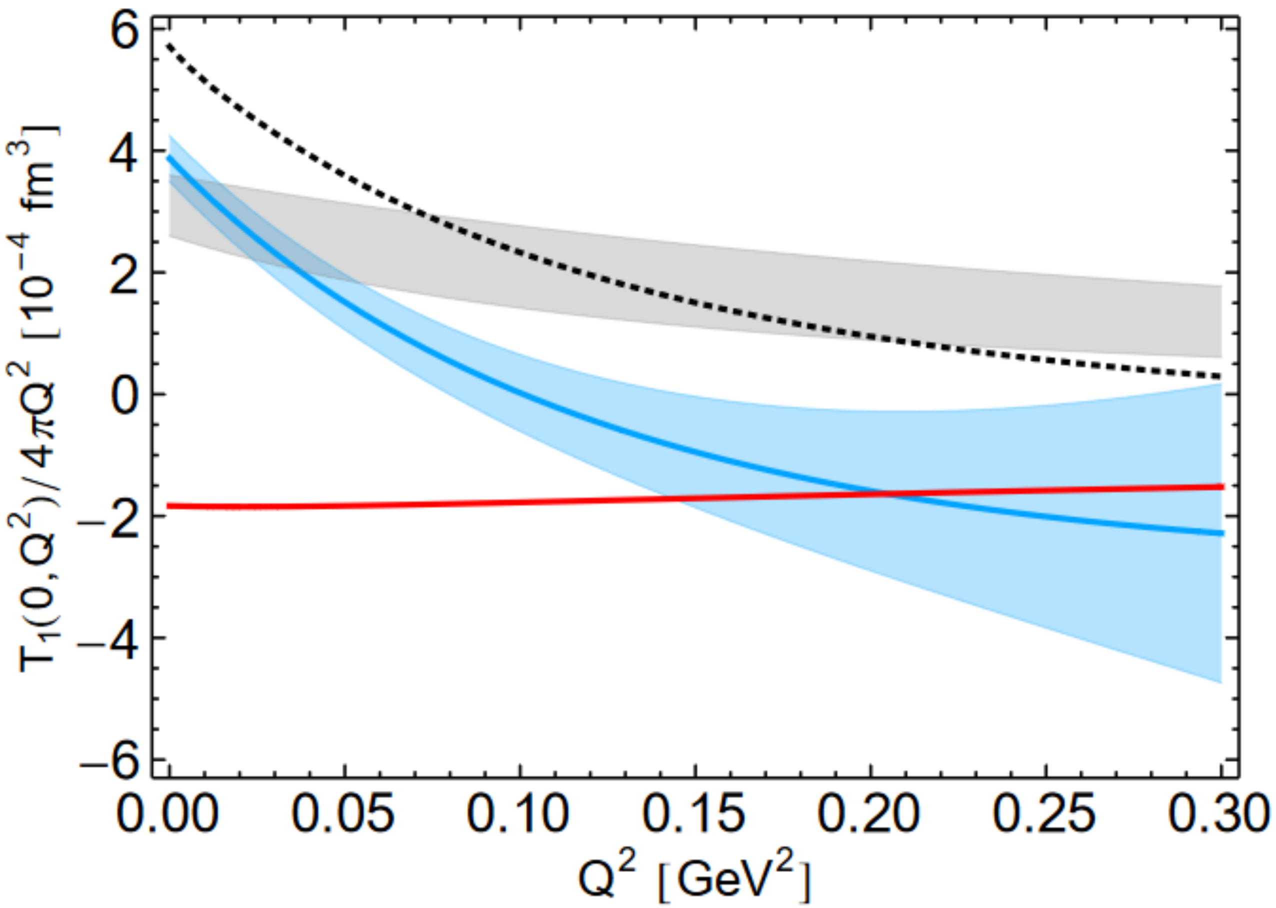
2305.08814

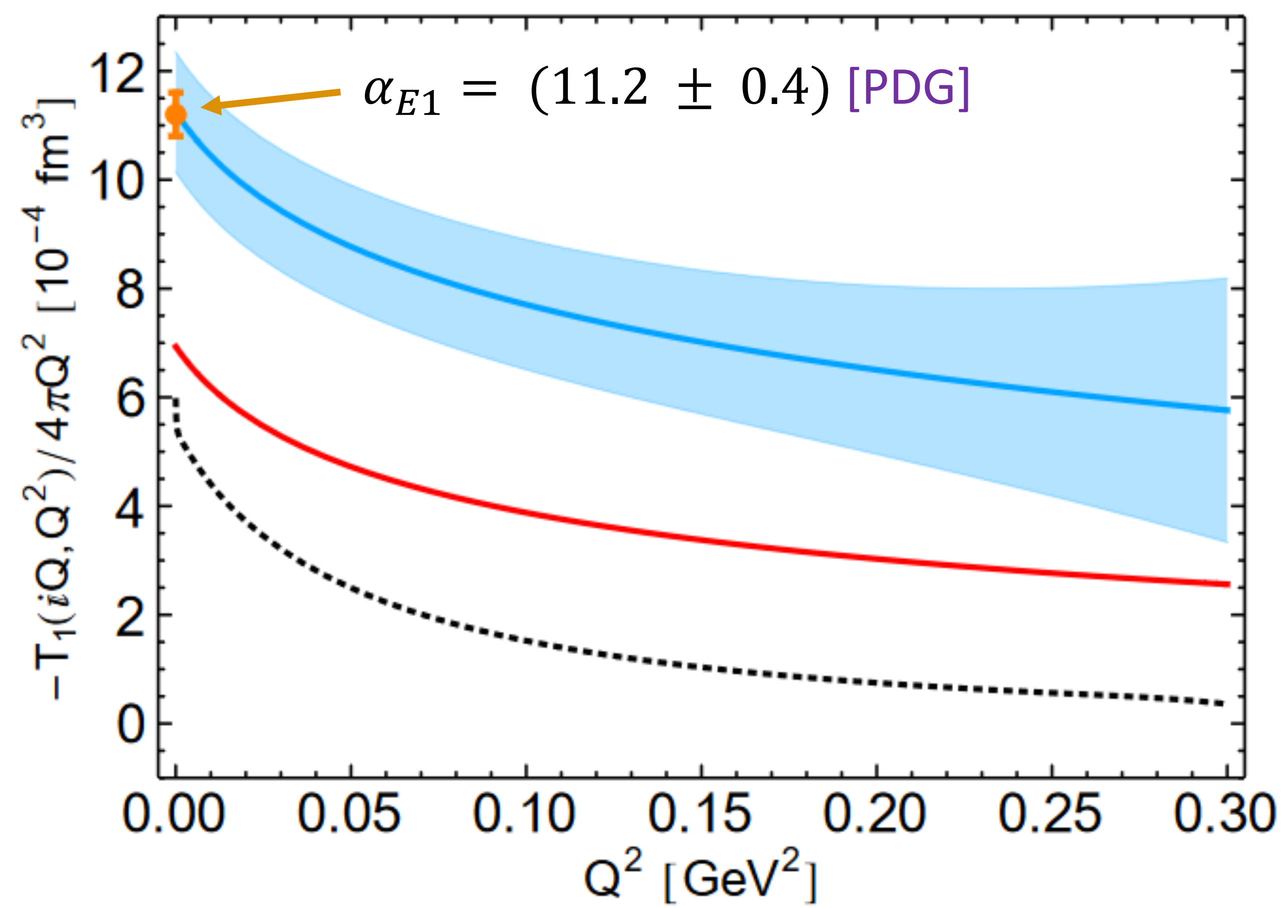
DATA-DRIVEN EVALUATION

- New integral equations for data-driven evaluation of subtraction functions
- High-quality parametrization of σ_L at $Q \to 0$ needed

$$T_1(0,Q^2) = \frac{2Q^2}{\pi}\int_{\nu_0}^{\infty} \frac{\mathrm{d}\nu}{\nu^2+Q^2}\left[\sigma_T - \frac{\nu^2}{Q^2}\sigma_L\right](\nu,Q^2)$$

$$T_L(iQ,Q^2) = \frac{2}{\pi}\int_{\nu_0}^{\infty} \mathrm{d}\nu\,\nu^2\,\frac{\sigma_L(\nu,Q^2)}{\nu^2+Q^2}$$





MAID

NLO χPT [Lensky et al., PRC (2014)] [Alarcón et al., PRD (2020)]

LO χPT: πN-loops

HBχPT [Birse and McGovern, EPJA, (2012)]

HYPERFINE SPLITTING IN μH

$$\Delta E_{\mathrm{HFS}}(nS) = [1 + \Delta_{\mathrm{QED}} + \Delta_{\mathrm{weak}} + \Delta_{\mathrm{structure}}]\, E_F(nS)$$

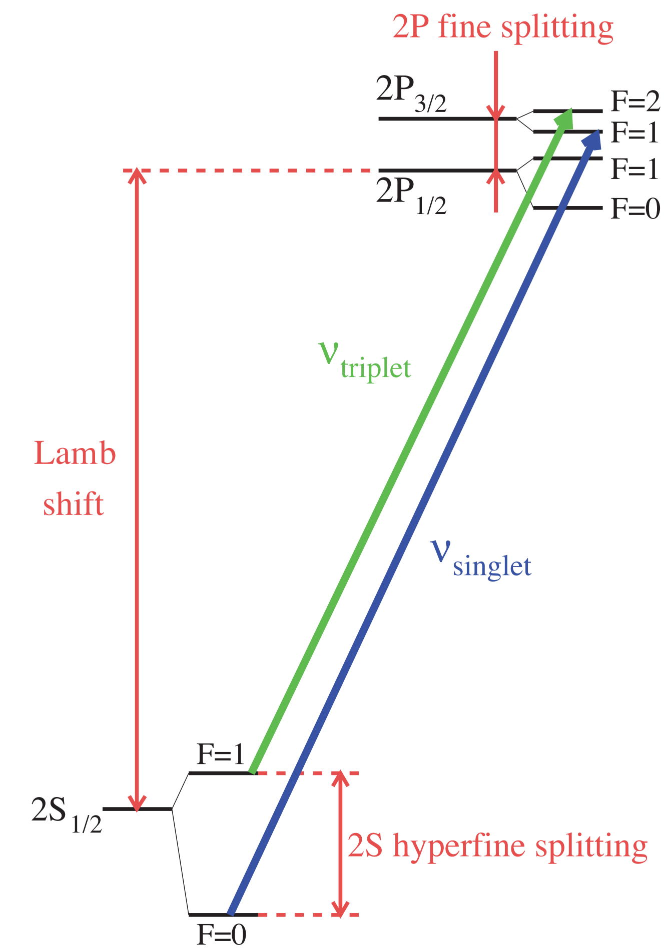
with $\Delta_{\mathrm{structure}} = \Delta_Z + \Delta_{\mathrm{recoil}} + \Delta_{\mathrm{pol}}$

Zemach radius:

$$\Delta_Z = \frac{8Z\alpha m_r}{\pi} \int_0^\infty \frac{\mathrm{d}Q}{Q^2} \left[\frac{G_E(Q^2) G_M(Q^2)}{1+\kappa} - 1 \right] \equiv -2Z\alpha m_r R_Z$$

experimental value: $R_Z = 1.082(37)\ \mathrm{fm}$

A. Antognini, et al., Science **339** (2013) 417–420



Measurements of the μH ground-state HFS planned by the CREMA, FAMU and J-PARC / Riken-RAL collaborations

- Very precise input for the 2γ effect needed to narrow down frequency search range for experiment
- Zemach radius can help to pin down the magnetic properties of the proton

HYPERFINE SPLITTING

Theory: QED, ChPT, data-driven dispersion relations, ab-initio few-nucleon theories

Experiment: HFS in μH, μHe$^+$, …

Guiding the exp.

find narrow 1S HFS transitions with the help of full theory predictions: QED, weak, finite size, polarizability

Interpreting the exp.

extract $E^{\rm TPE}$, $E^{\rm pol.}$ or R_Z

Input for data-driven evaluations

form factors, structure functions, polarizabilities

Electron and Compton Scattering

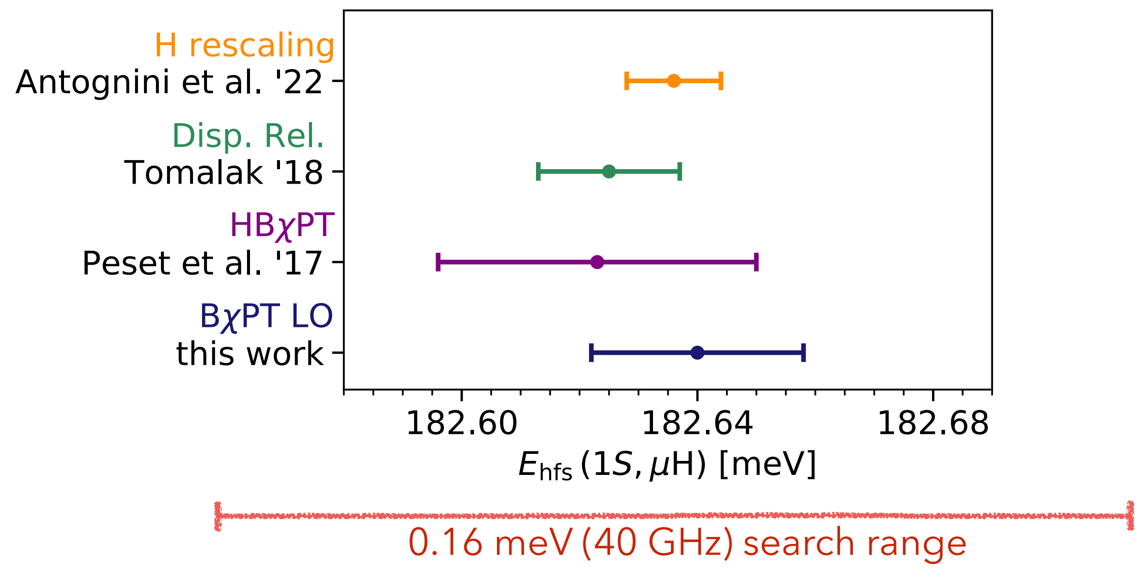
Testing the theory

- discriminate between theory predictions for polarizability effect
  - disentangle R_Z & polarizability effect by combining HFS in H & μH
- test HFS theory
  - combining HFS in H & μH with theory prediction for polarizability effect
- test nuclear theories

Spectroscopy of ordinary atoms (H, He$^+$)

Determine fundamental constants

Zemach radius R_Z



Predictions for the 1S HFS in μH are driven by the 1S HFS in H

A. Antognini, FH, V. Pascalutsa, Ann. Rev. Nucl. Part. **72** (2022)

...PLITTING

Experiment: HFS in μH, μHe$^+$, …

...esting the theory

...inate between theory predictions for polarizability effect

- disentangle R_Z & polarizability effect by combining HFS in H & μH

- test HFS theory
  - combining HFS in H & μH with theory prediction for polarizability effect
- test nuclear theories

Determine fundamental constants

Zemach radius R_Z

Interpreting the exp.

extract E^{TPE}, $E^{\text{pol.}}$ or R_Z

Guiding the exp.

find narrow 1S HFS transitions with the help of full theory predictions: QED, weak, finite size, polarizability

Input for data-driven evaluations

form factors, structure functions, polarizabilities

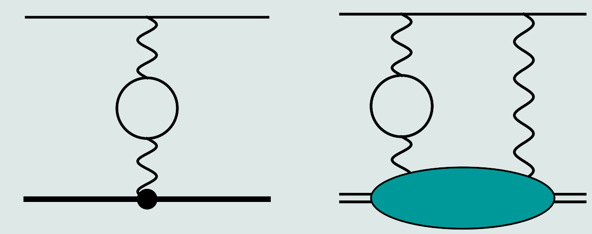
Spectroscopy of ordinary atoms (H, He$^+$)

Electron and Compton Scattering

HYPERFINE SPLITTING

The hyperfine splitting of μH (theory update):

A. Antognini, FH, V. Pascalutsa, Ann. Rev. Nucl. Part. **72** (2022)



$$E_{1S\text{-hfs}} = \Big[\underbrace{182.443}_{E_\mathrm{F}} \underbrace{+1.350(7)}_{\text{QED+weak}} \underbrace{+0.004}_{\text{hVP}} \underbrace{-1.30653(17)\left(\frac{r_{Zp}}{\text{fm}}\right) + E_\mathrm{F}\Big(1.01656(4)\,\Delta_{\text{recoil}} + 1.00402\,\Delta_{\text{pol}}\Big)}_{2\gamma\ \text{incl. radiative corr.}}\Big]\,\text{meV}$$

Guiding the exp.

find narrow 1S HFS transitions with the help of full theory predictions: QED, weak, finite size, polarizability

Interpreting the exp.

extract E^{TPE}, $E^{\mathrm{pol.}}$ or R_Z

Input for data-driven evaluations

form factors, structure functions, polarizabilities

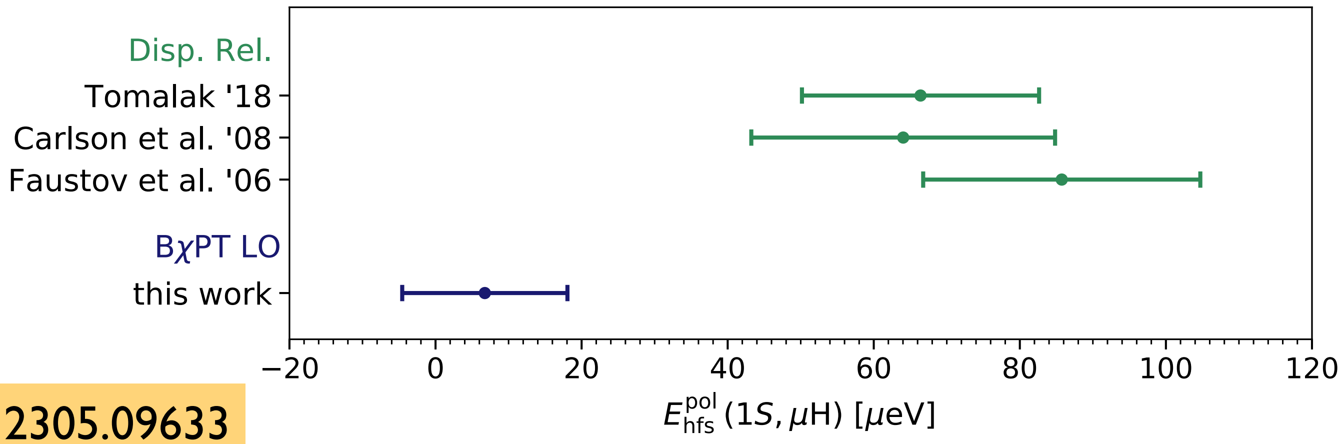
- discriminate between theory predictions for polarizability effect
  - disentangle R_Z & polarizability effect by combining HFS in H & μH
- test HFS theory
  - combining HFS in H & μH with theory prediction for polarizability effect
- test nuclear theories

Determine fundamental constants

Zemach radius R_Z

Spectroscopy of ordinary atoms (H, He^+)

Electron and Compton Scattering



2305.09633

PLITTING

Experiment: HFS in μH, μHe$^+$, …

Testing the theory

- **discriminate between theory predictions for polarizability effect**
  - disentangle R_Z & polarizability effect by combining HFS in H & μH
- test HFS theory
  - combining HFS in H & μH with theory prediction for polarizability effect
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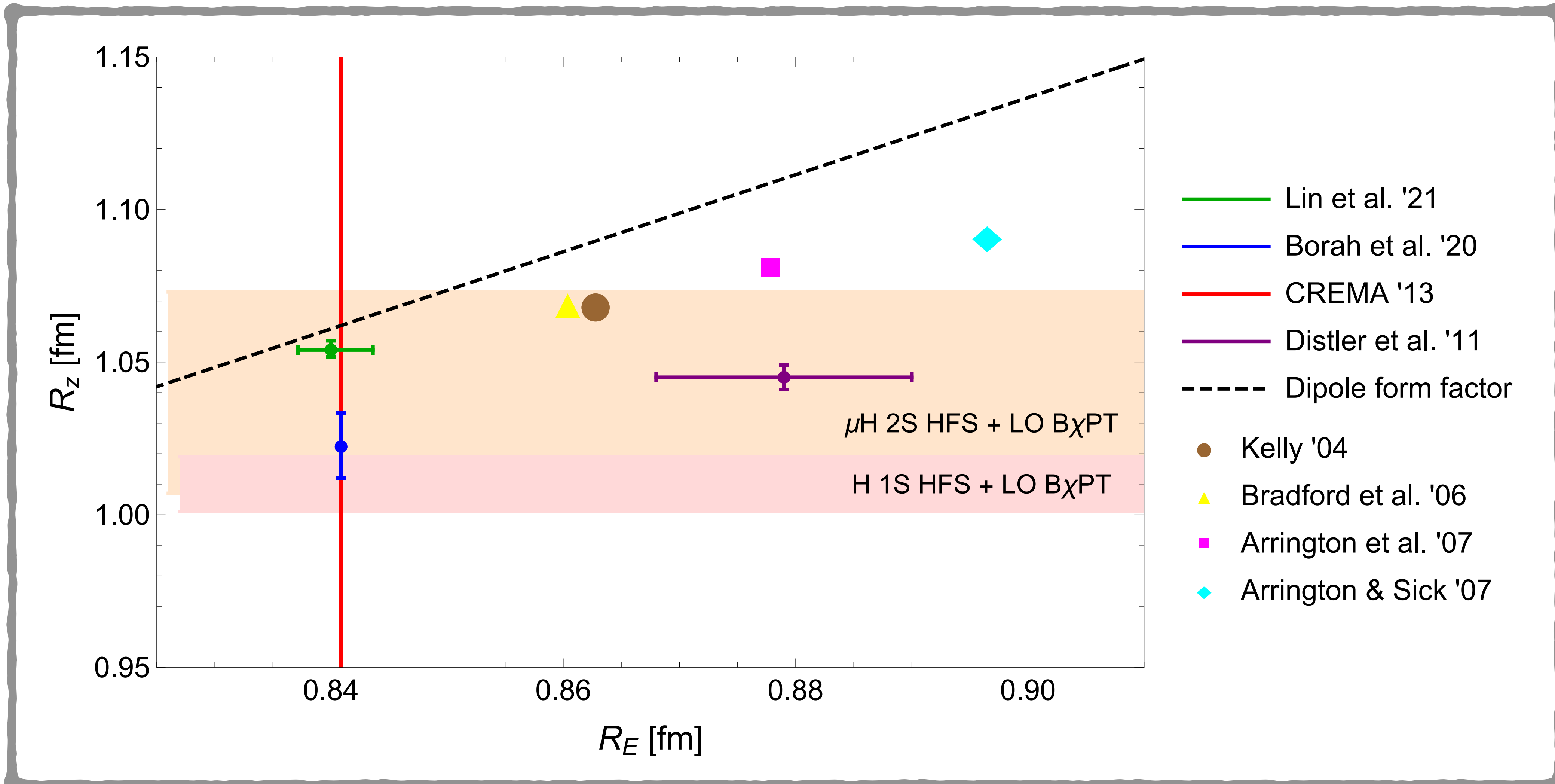


, μHe^+, …

Determine fundamental constants

Zemach radius R_Z

transitions with the help of full theory predictions: QED, weak, finite size, polarizability

Input for data-driven evaluations
form factors, structure functions, polarizabilities

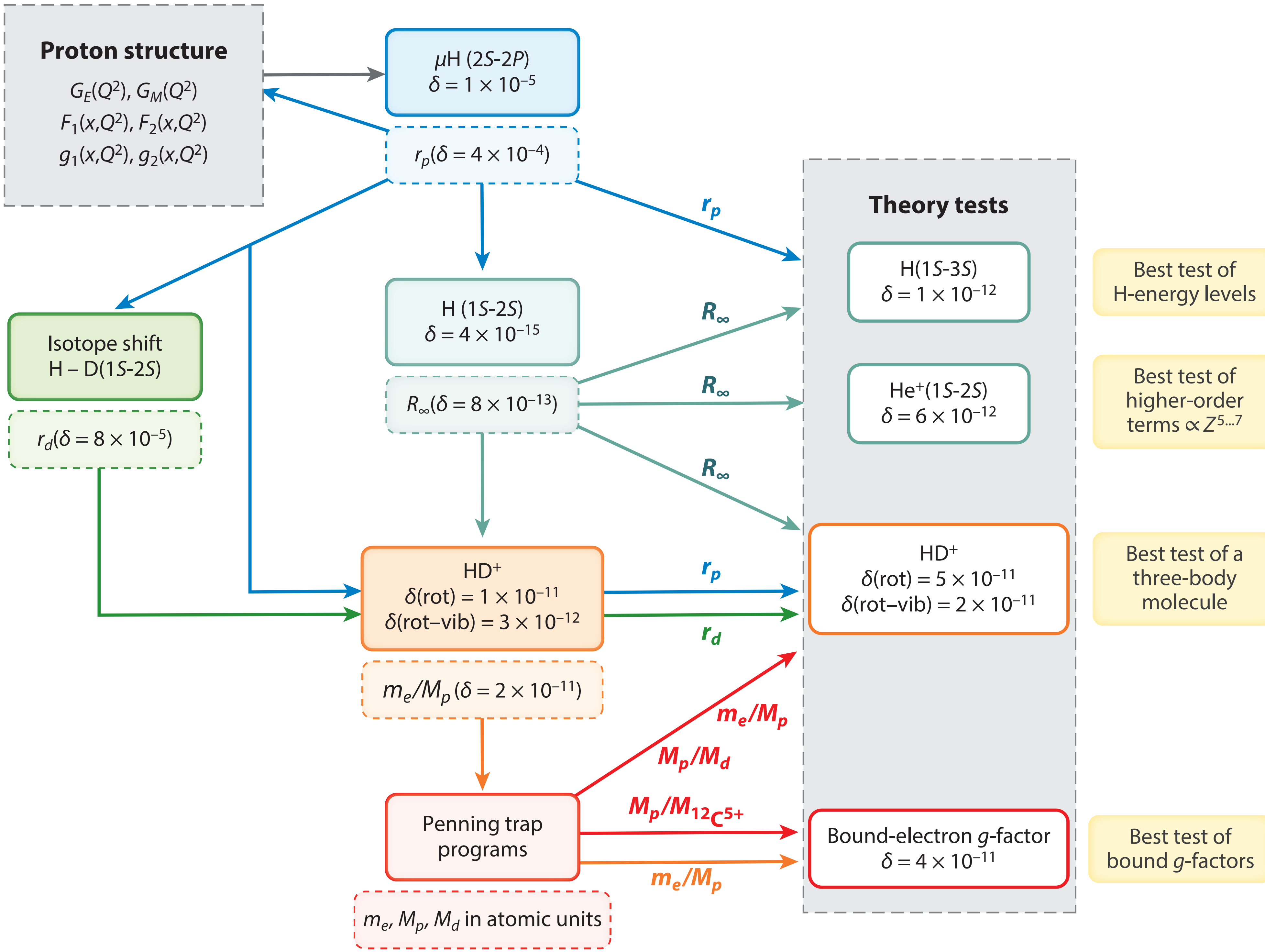
combining HFS in H & μH

- test HFS theory
  - combining HFS in H & μH with theory prediction for polarizability effect
- test nuclear theories

Spectroscopy of ordinary atoms (H, He^+)

Electron and Compton Scattering

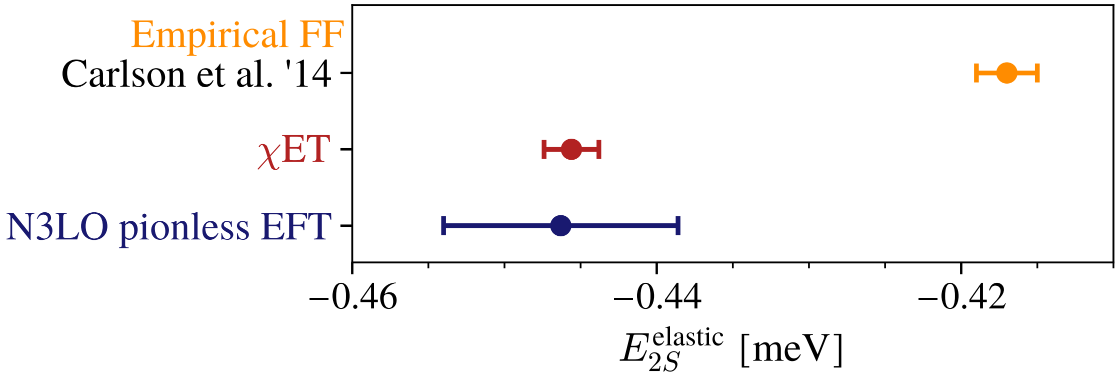
COMBINING μH, H, HE, HD+, ...

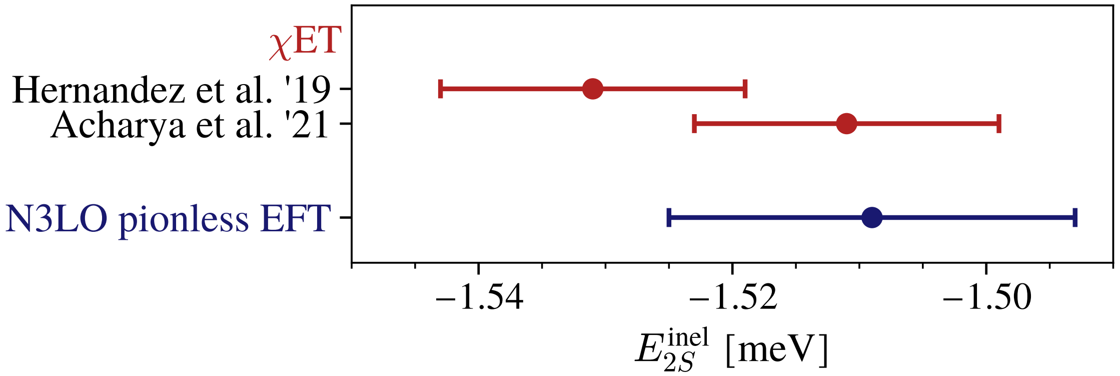


A. Antognini, FH, V. Pascalutsa, Ann. Rev. Nucl. Part. **72** (2022) 389-418
A. Antognini, FH, et al., 2210.16929 (submitted as community input for the NuPECC Long Range Plan 2024)

2γ EFFECT IN μD LAMB SHIFT

see presentations by C. Ji, S. Li Muli, V. Lensky, T. Richardson





	$E_{2S}^{2\gamma}$ [meV]
Theory prediction	
Krauth et al. '16 [5]	−1.7096(200)
Kalinowski '19 [6, Eq. (6) + (19)]	−1.740(21)
$\not\pi$EFT (this work)	−1.752(20)
Empirical (μH + iso)	
Pohl et al. '16 [3]	−1.7638(68)
This work	−1.7585(56)

V. Lensky, A. Hiller Blin, FH, V. Pascalutsa, 2203.13030
V. Lensky, FH, V. Pascalutsa, 2206.14756, 2206.14066

- N3LO pionless EFT + higher-order single-nucleon effects:

$$E_{2S}^{\text{elastic}} = -0.446(8)\,\text{meV}$$
$$E_{2S}^{\text{inel},L} = -1.509(16)\,\text{meV}$$
$$E_{2S}^{\text{inel},T} = -0.005\,\text{meV}$$
$$E_{2S}^{\text{hadr}} = -0.032(6)\,\text{meV}$$
$$E_{2S}^{\text{eVP}} = -0.027\,\text{meV}$$

- Elastic 2γ several standard deviations larger
- Inelastic 2γ consistent with other results
- Agreement with precise empirical value for the 2γ effect extracted with $r_d(\mu\text{H} + \text{iso})$

Thank you for your attention!

2γ EFFECT IN THE μH HFS

Table 1 Forward 2γ-exchange contribution to the HFS in μH.

Reference	Δ_Z [ppm]	Δ_{recoil} [ppm]	Δ_{pol} [ppm]	Δ_1 [ppm]	Δ_2 [ppm]	$E^{\langle 2\gamma\rangle}_{1S\text{-hfs}}$ [meV]
DATA-DRIVEN						
Pachucki '96 (1)	−8025	1666	0(658)			−1.160
Faustov et al. '01 (9)[a]	−7180		410(80)	468	−58	
Faustov et al. '06 (10)[b]			470(104)	518	−48	
Carlson et al. '11 (11)[c]	−7703	931	351(114)	370(112)	−19(19)	−1.171(39)
Tomalak '18 (12)[d]	−7333(48)	846(6)	364(89)	429(84)	−65(20)	−1.117(19)
HEAVY-BARYON χPT						
Peset et al. '17 (13)						−1.161(20)
LEADING-ORDER χPT						
Hagelstein et al. '16 (14)			37(95)	29(90)	9(29)	
+ Δ(1232) EXCIT.						
Hagelstein et al. '18 (15)			−13	84	−97	

[a] Adjusted values: Δ_{pol} and Δ_1 corrected by −46 ppm as described in Ref. 16.
[b] Different convention was used to calculate the Pauli form factor contribution to Δ_1, which is equivalent to the approximate formula in the limit of $m = 0$ used for H in Ref. 11.
[c] Elastic form factors from Ref. 17 and updated error analysis from Ref. 16. Note that this result already includes radiative corrections for the Zemach-radius contribution, $(1+\delta_Z^{rad})\Delta_Z$ with $\delta_Z^{rad} \sim 0.0153$ (18, 19), as well as higher-order recoil corrections with the proton anomalous magnetic moment, cf. (11, Eq. 22) and (18).
[d] Uses r_p from μH (20) as input.

POLARIZABILITY EFFECT IN THE HFS

- Polarizability effect on the HFS is completely constrained by empirical information

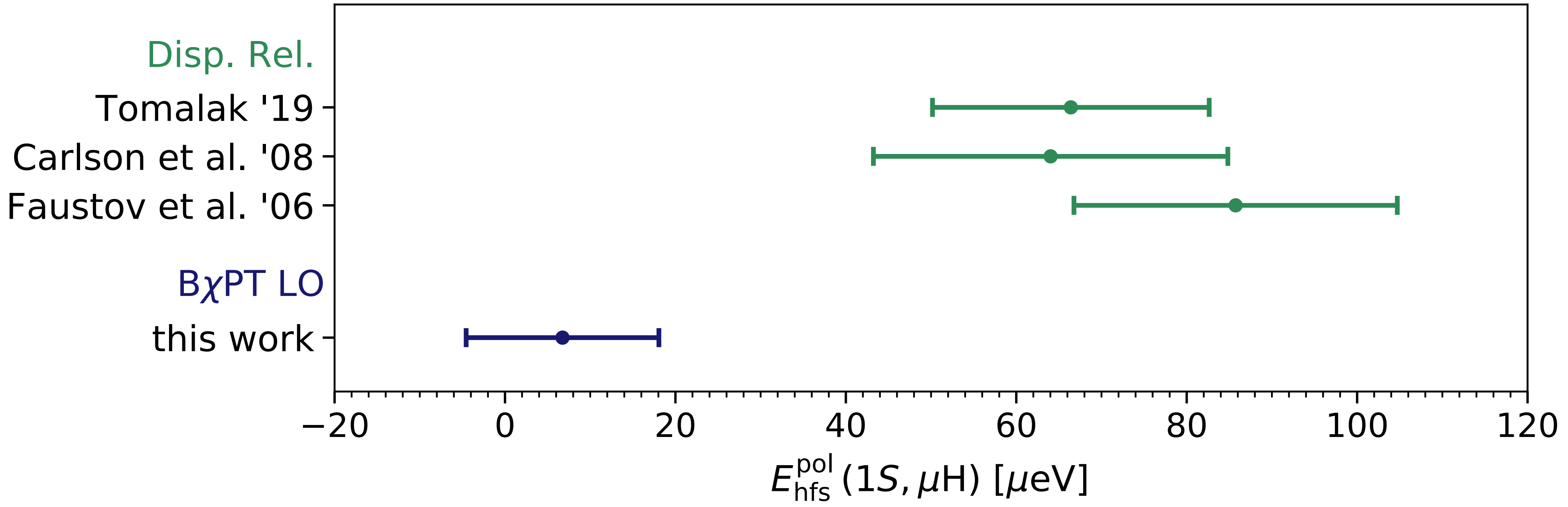
$$\Delta_{\text{pol.}} = \Delta_1 + \Delta_2 = \frac{\alpha m}{2\pi(1+\kappa)M}\left(\delta_1 + \delta_2\right)$$

$$\delta_1 = 2\int_0^\infty \frac{\mathrm{d}Q}{Q}\left\{\frac{5+4v_l}{(v_l+1)^2}\left[4I_1(Q^2) + F_2^2(Q^2)\right] - \frac{32M^4}{Q^4}\int_0^{x_0}\mathrm{d}x\, x^2 g_1(x,Q^2)\frac{1}{(v_l+v_x)(1+v_x)(1+v_l)}\left(4 + \frac{1}{1+v_x} + \frac{1}{v_l+1}\right)\right\}$$

$$\delta_2 = 96M^2\int_0^\infty \frac{\mathrm{d}Q}{Q^3}\int_0^{x_0}\mathrm{d}x\, g_2(x,Q^2)\left(\frac{1}{v_l+v_x} - \frac{1}{v_l+1}\right)$$

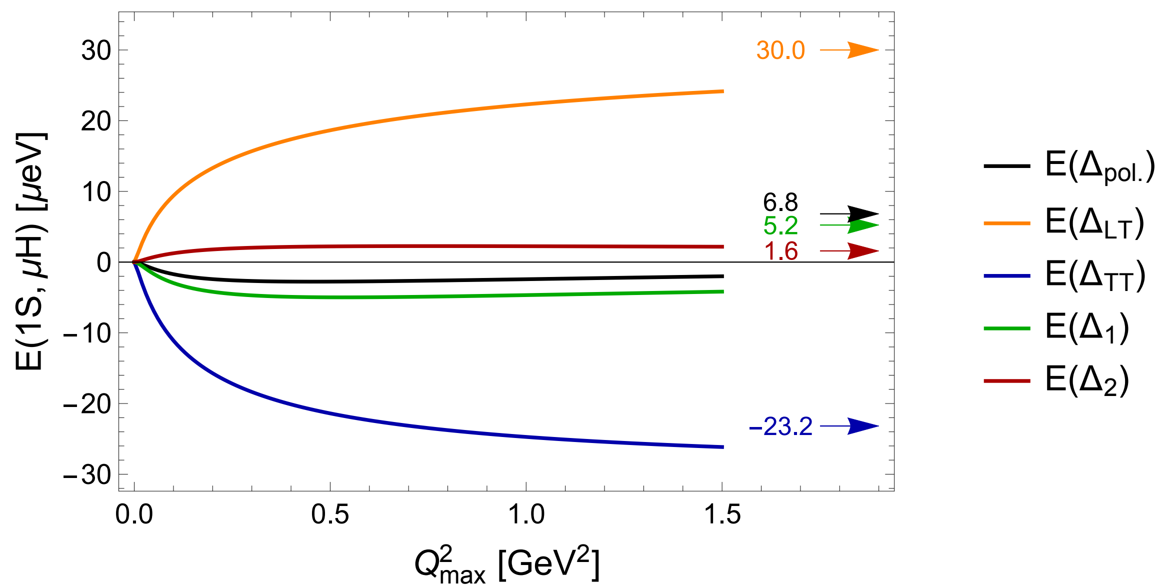
with $v_l = \sqrt{1+\frac{1}{\tau_l}}$, $v_x = \sqrt{1+x^2\tau^{-1}}$, $\tau_l = \frac{Q^2}{4m^2}$ and $\tau = \frac{Q^2}{4M^2}$

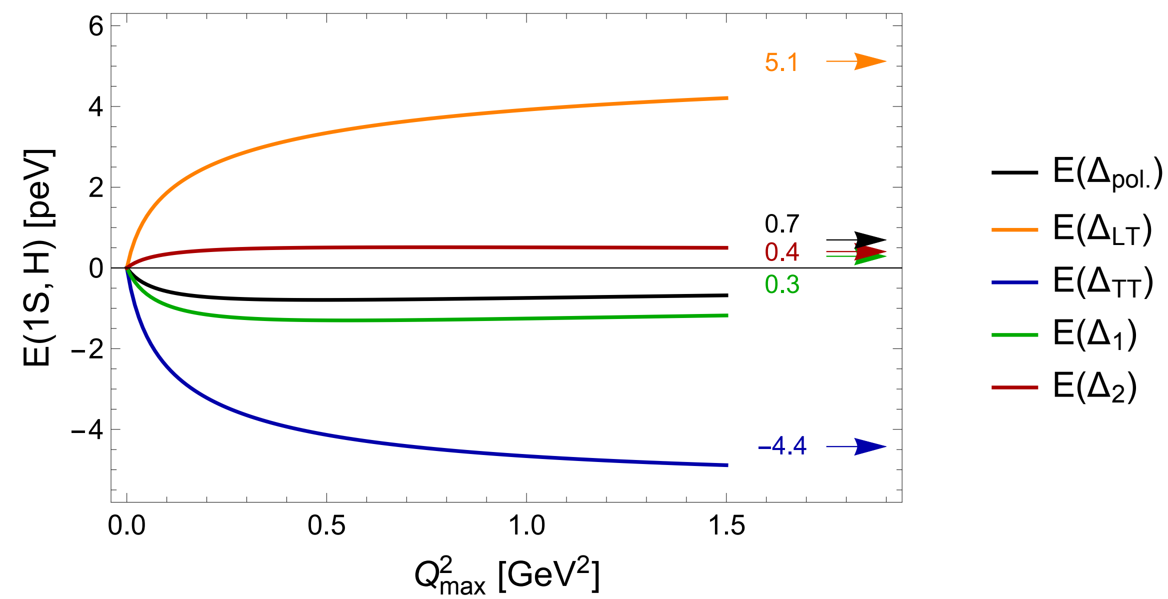
- BChPT calculation puts the reliability of dispersive calculations (and BChPT) to the test



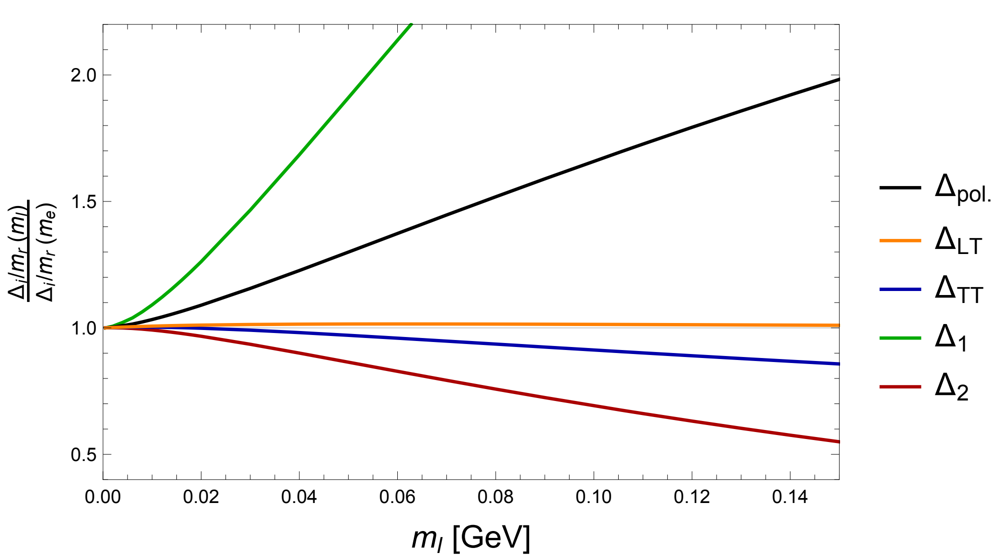
POLARIZABILITY EFFECT FROM BCHPT

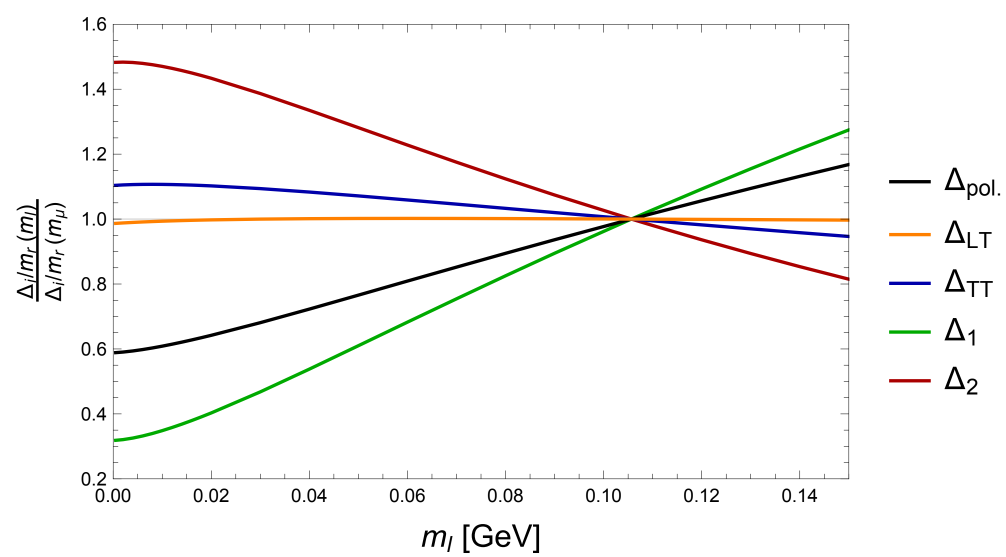
- LO BChPT result is compatible with zero
  - Contributions from σ_{LT} and σ_{TT} are sizeable and largely cancel each other





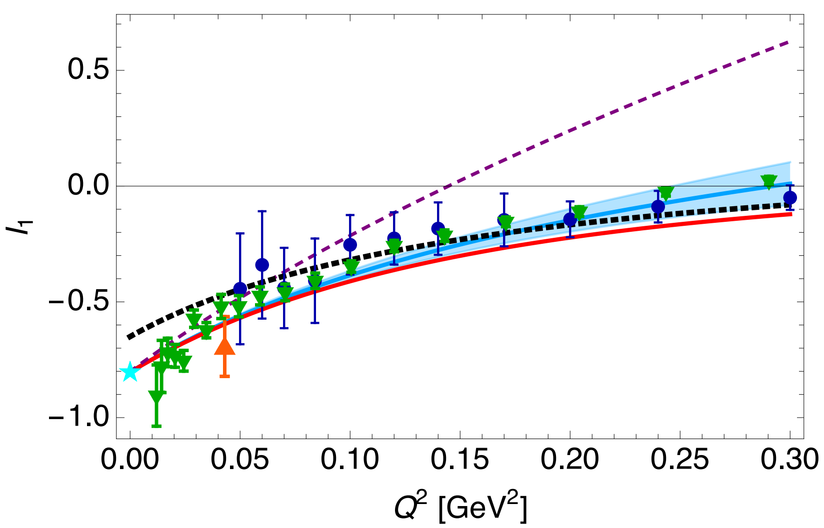
- Are the data-driven evaluations/uncertainties affected by cancelations?
- Scaling with lepton mass of the lepton-proton bound state

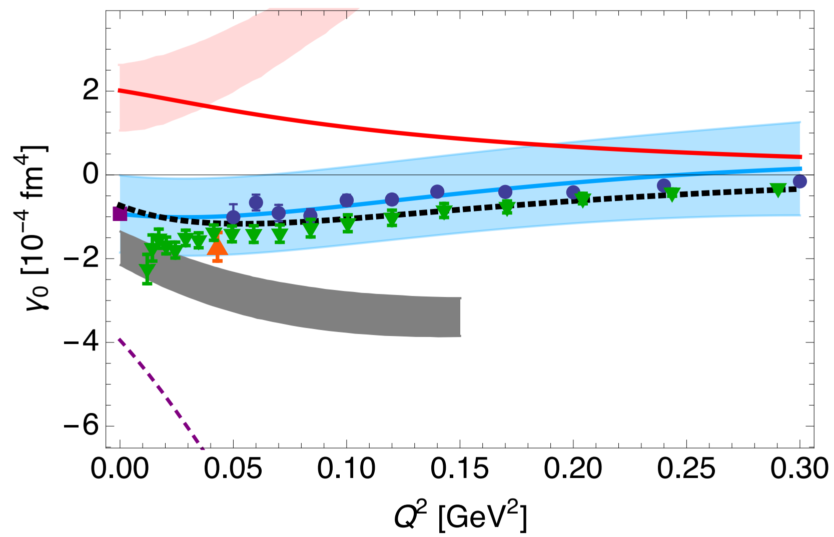


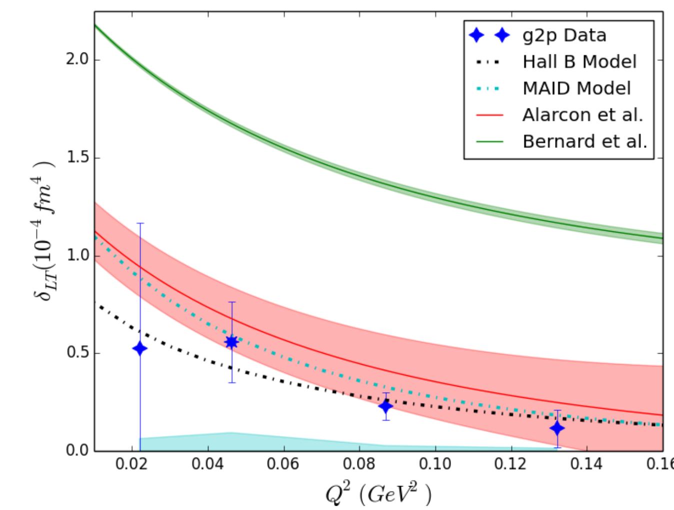


DATA-DRIVEN EVALUATION

- Empirical information on spin structure functions from JLab Spin Physics Programme



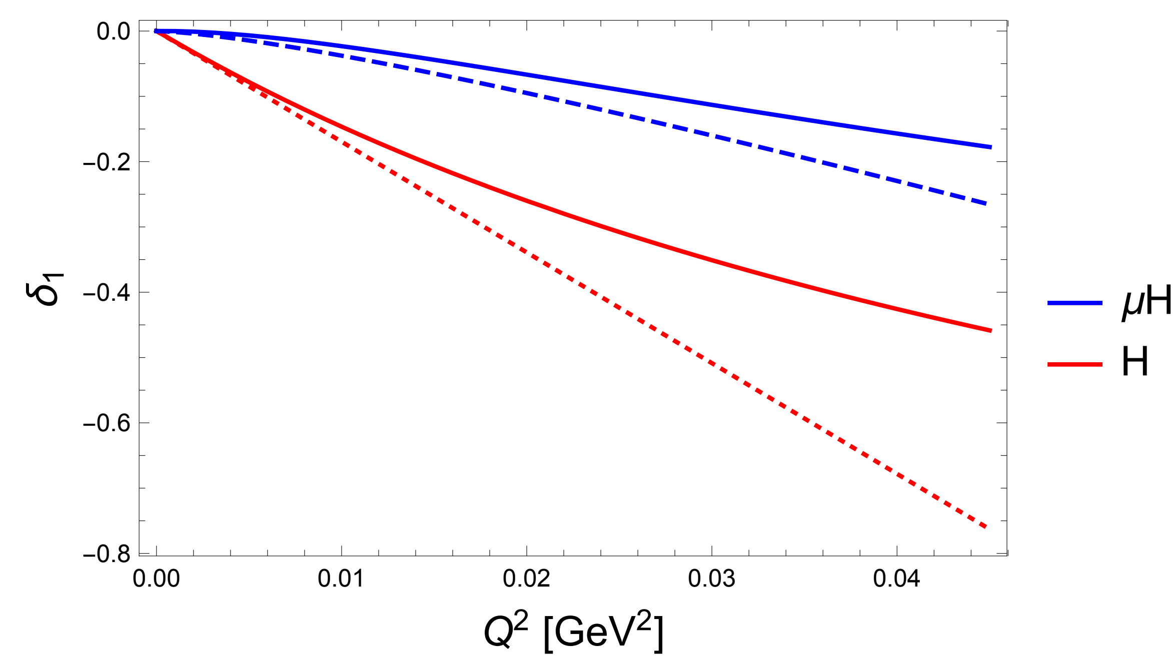




- Low-Q region is very important → cancelation between $I_1(Q^2)$ and $F_2(Q^2)$

$$\delta_1(\mathrm{H}) \sim \Bigg(\underbrace{-\frac{3}{4}\kappa^2 r_{\mathrm{Pauli}}^2}_{\to -2.19} + \underbrace{18M^2 c_{1B}}_{\to 3.54}\Bigg) Q^2_{\mathrm{max}} = 1.35(90),$$

$$\delta_1(\mu\mathrm{H}) \sim \Bigg[\underbrace{-\frac{1}{3}\kappa^2 r_{\mathrm{Pauli}}^2}_{\to -1.45} + \underbrace{8M^2 c_1}_{\to 2.13} \underbrace{-\frac{M^2}{3\alpha}\gamma_0}_{\to 0.18}\Bigg] \int_0^{Q^2_{\mathrm{max}}} \mathrm{d}Q^2 \beta_1(\tau_\mu) = 0.86(69)$$

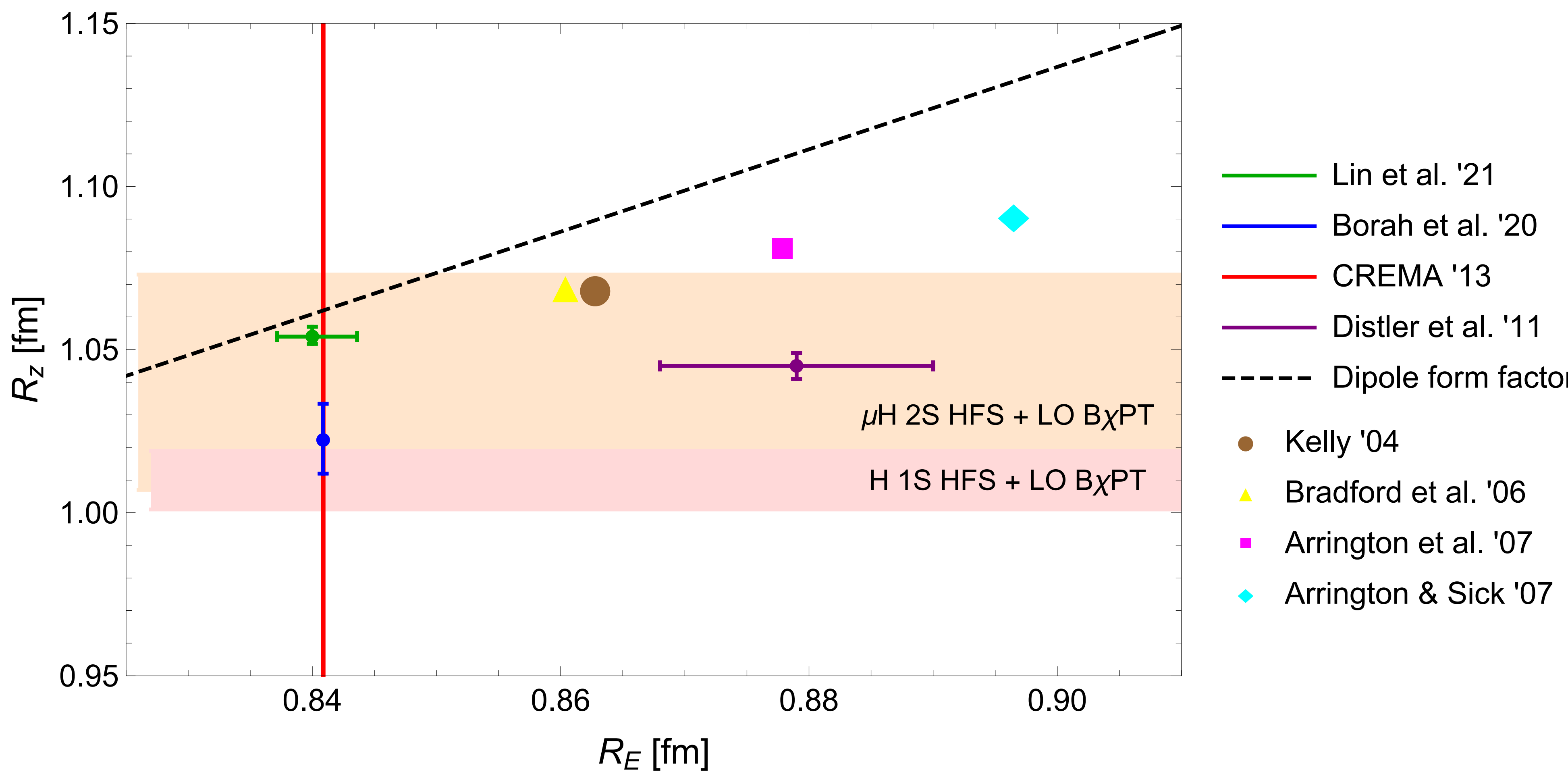


PROTON ZEMACH RADIUS

- BChPT polarizability contribution implies smaller Zemach radius (smaller, just like r_p)

TABLE I. Determinations of the proton Zemach radius R_Z, in units of fm.

ep scattering		μH 2*S* hfs		H 1*S* hfs	
Lin *et al.* '21	Borah *et al.* '20	Antognini *et al.* '13	LO BχPT	Volotka *et al.* '04	LO BχPT
$1.054^{+0.003}_{-0.002}$	1.0227(107)	1.082(37)	1.040(33)	1.045(16)	1.010(9)



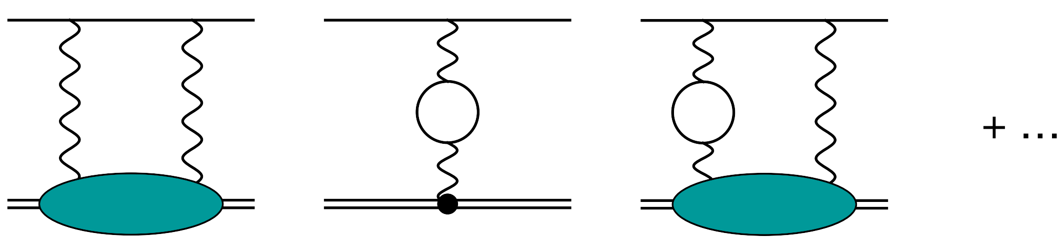
THEORY OF HYPERFINE SPLITTING

A. Antognini, FH, V. Pascalutsa, Ann. Rev. Nucl. Part. **72** (2022) 389-418

The hyperfine splitting of μH (theory update):

$$E_{1S\text{-hfs}} = \Big[\underbrace{182.443}_{E_\mathrm{F}}\ \underbrace{+1.350(7)}_{\mathrm{QED+weak}}\ \underbrace{+0.004}_{\mathrm{hVP}}\ \underbrace{-1.30653(17)\left(\frac{r_{Zp}}{\mathrm{fm}}\right) + E_\mathrm{F}\left(1.01656(4)\,\Delta_\mathrm{recoil} + 1.00402\,\Delta_\mathrm{pol}\right)}_{2\gamma\ \mathrm{incl.\ radiative\ corr.}}\Big]\,\mathrm{meV}$$

- 2γ + radiative corrections $\Longrightarrow$ differ for H vs. μH and 1S vs. 2S



The hyperfine splitting of H (theory update):

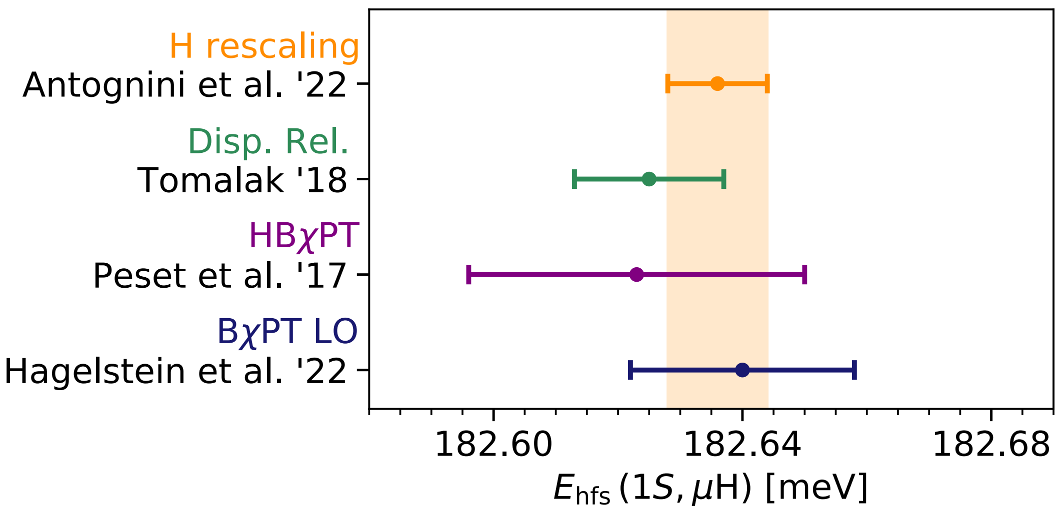
$$E_{1S\text{-hfs}}(\mathrm{H}) = \Big[\underbrace{1\,418\,840.082(9)}_{E_\mathrm{F}}\ \underbrace{+1\,612.673(3)}_{\mathrm{QED+weak}}\ \underbrace{+0.274}_{\mu\mathrm{VP}}\ \underbrace{+0.077}_{\mathrm{hVP}}$$
$$\underbrace{-54.430(7)\left(\frac{r_{Zp}}{\mathrm{fm}}\right) + E_\mathrm{F}\left(0.99807(13)\,\Delta_\mathrm{recoil} + 1.00002\,\Delta_\mathrm{pol}\right)}_{2\gamma\ \mathrm{incl.\ radiative\ corr.}}\Big]\,\mathrm{kHz}$$

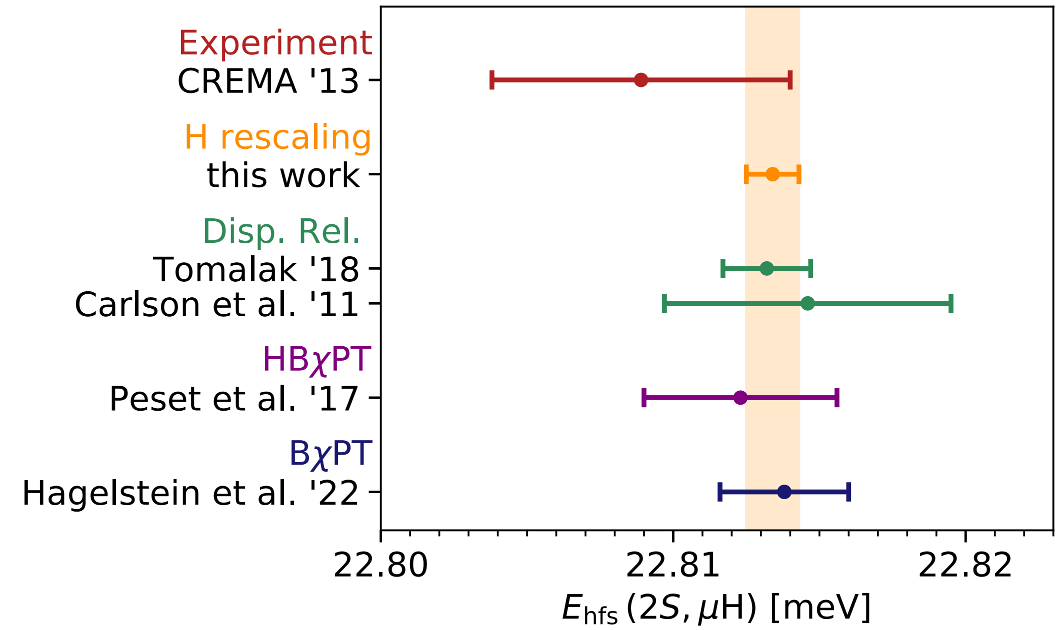
High-precision measurement of the "21cm line" in H:

$$\delta\left(E^{\mathrm{exp.}}_{1S-\mathrm{hfs}}(\mathrm{H})\right) = 10\times 10^{-13}$$

Hellwig et al., 1970

IMPACT OF H 1S HFS





- Leverage radiative corrections $E^{\rm Z+pol}_{1S-{\rm hfs}}(\mathrm{H}) = E_{\rm F}(\mathrm{H})\Big[b_{1S}(\mathrm{H})\,\Delta_{\rm Z}(\mathrm{H}) + c_{1S}(\mathrm{H})\,\Delta_{\rm pol}(\mathrm{H})\Big] = -\,54.900(71)\,\mathrm{kHz}$ and assume the non-recoil $\mathcal{O}(\alpha^5)$ effects have simple scaling $\frac{\Delta_i(\mathrm{H})}{m_r(\mathrm{H})} = \frac{\Delta_i(\mu\mathrm{H})}{m_r(\mu\mathrm{H})}, \quad i = \mathrm{Z, pol}$

1. Prediction for μH HFS from empirical 1S HFS in H

$$E^{\rm Z+pol}_{nS-{\rm hfs}}(\mu\mathrm{H}) = \frac{E_{\rm F}(\mu\mathrm{H})\,m_r(\mu\mathrm{H})\,b_{nS}(\mu\mathrm{H})}{n^3 E_{\rm F}(\mathrm{H})\,m_r(\mathrm{H})\,b_{1S}(\mathrm{H})}\,E^{\rm Z+pol}_{1S-{\rm hfs}}(\mathrm{H}) - \frac{E_{\rm F}(\mu\mathrm{H})}{n^3}\,\Delta_{\rm pol}(\mu\mathrm{H}) \underbrace{\left[c_{1S}(\mathrm{H})\frac{b_{nS}(\mu\mathrm{H})}{b_{1S}(\mathrm{H})} - c_{nS}(\mu\mathrm{H})\right]}_{=\,-\,6\times10^{-5}\ \mathrm{for\ n=1}\quad =\,-\,5\times10^{-5}\ \mathrm{for\ n=2}}$$

2. Disentangle Zemach radius and polarizability contribution
3. Testing the theory