Recent Advances in Microscopic Optical Potentials for Nuclear Reactions

25th European Conference on Few-Body Problems in Physics

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Outline

- Motivations
- Theoretical model
- Results for light nuclei
- Inclusion of the Self-Consistent Green's Function densities
- Future extensions

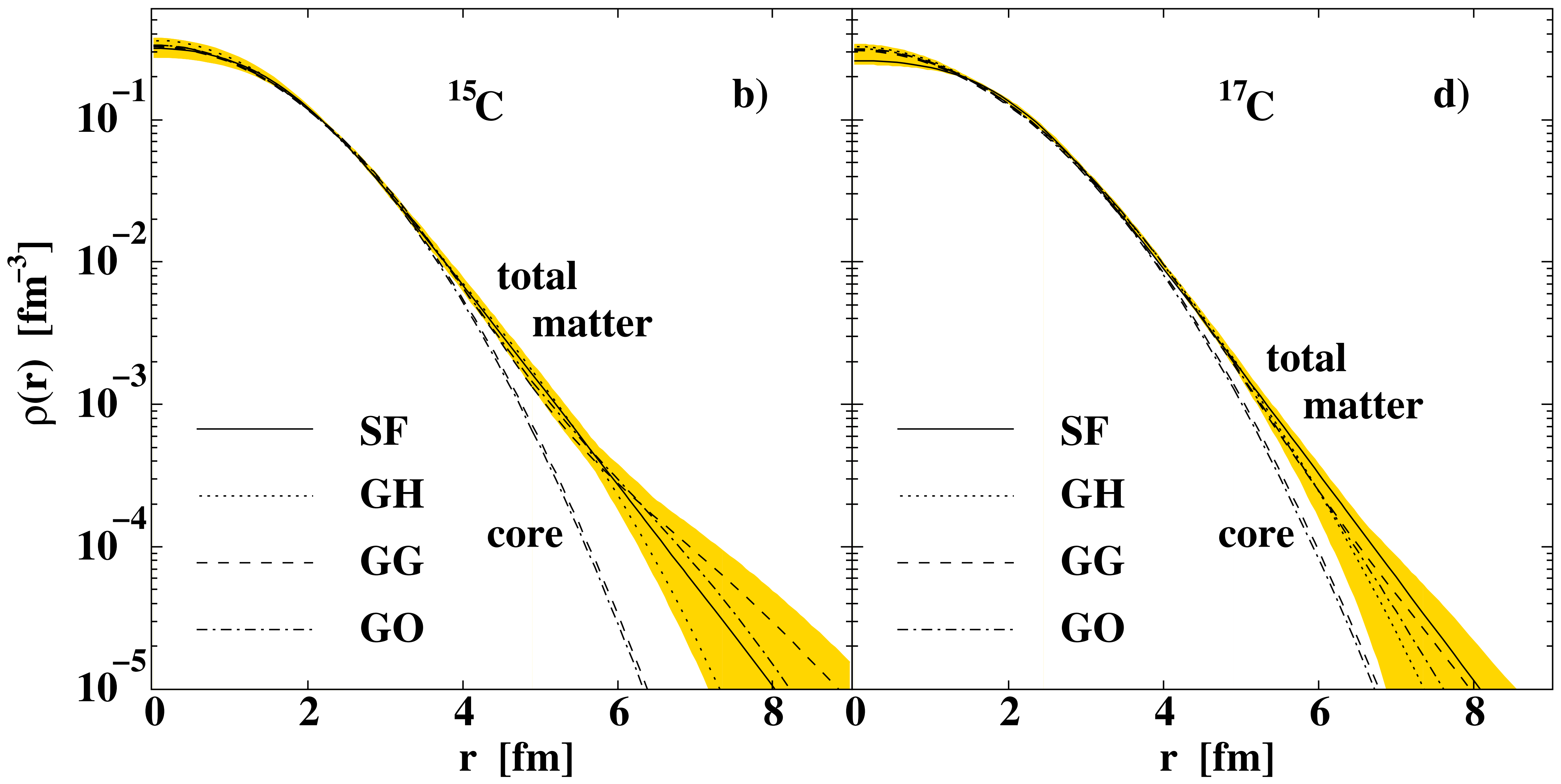
Motivations

- Increasing experimental efforts to develop the technologies necessary to study the elastic proton scattering in inverse kinematics
- Attempts to use such experiments to determine the matter distribution of nuclear systems at intermediate energies

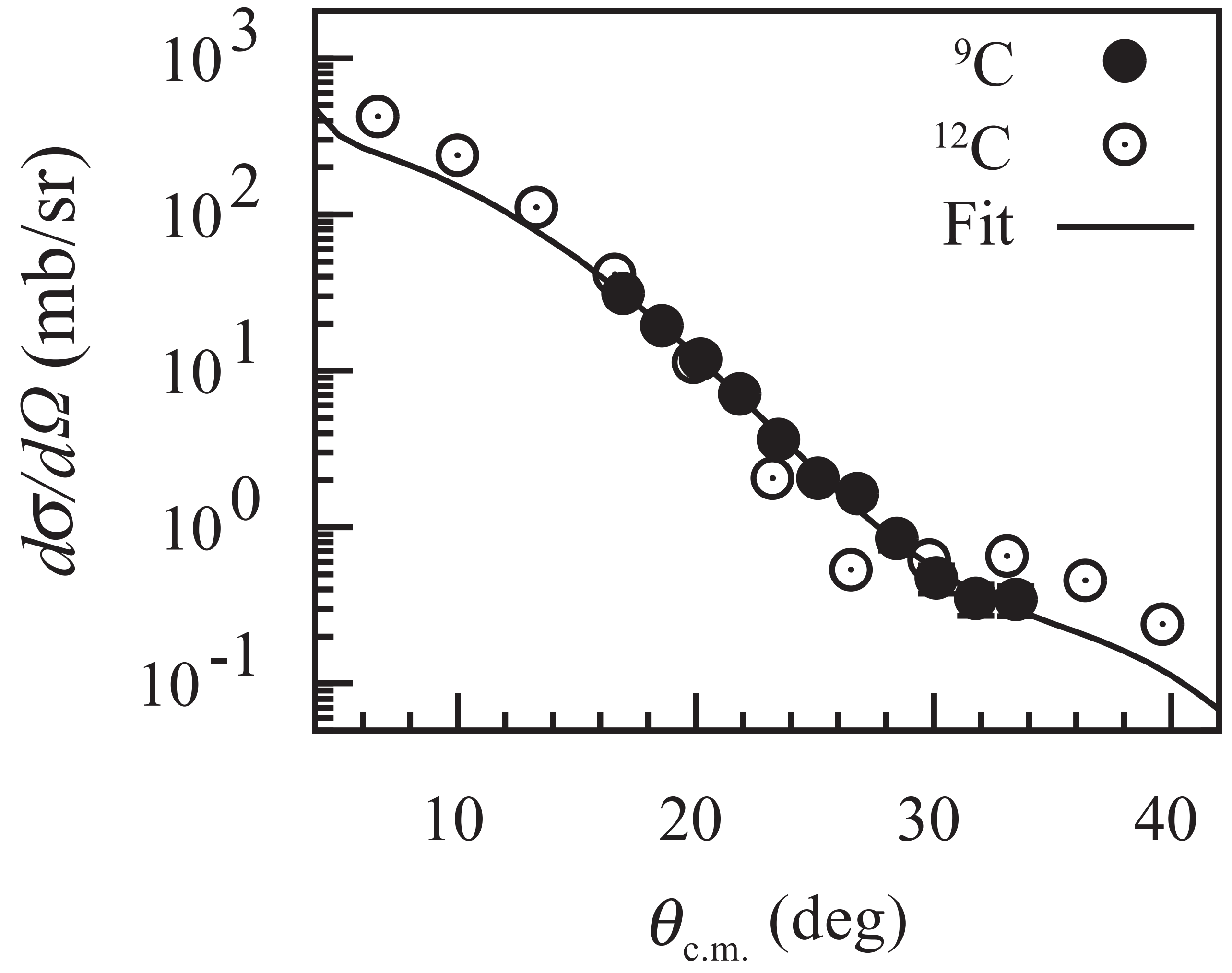
[Sakaguchi, Zenihiro, PPNP 97 (2017) 1–52]

  - Measurements are not free from sizeable uncertainties
  - The Glauber model is used to analyse the data
  - An essential step in the data analysis is the subtraction of contributions from the inelastic scattering

Develop a microscopic approach to make reliable predictions for elastic and inelastic scattering

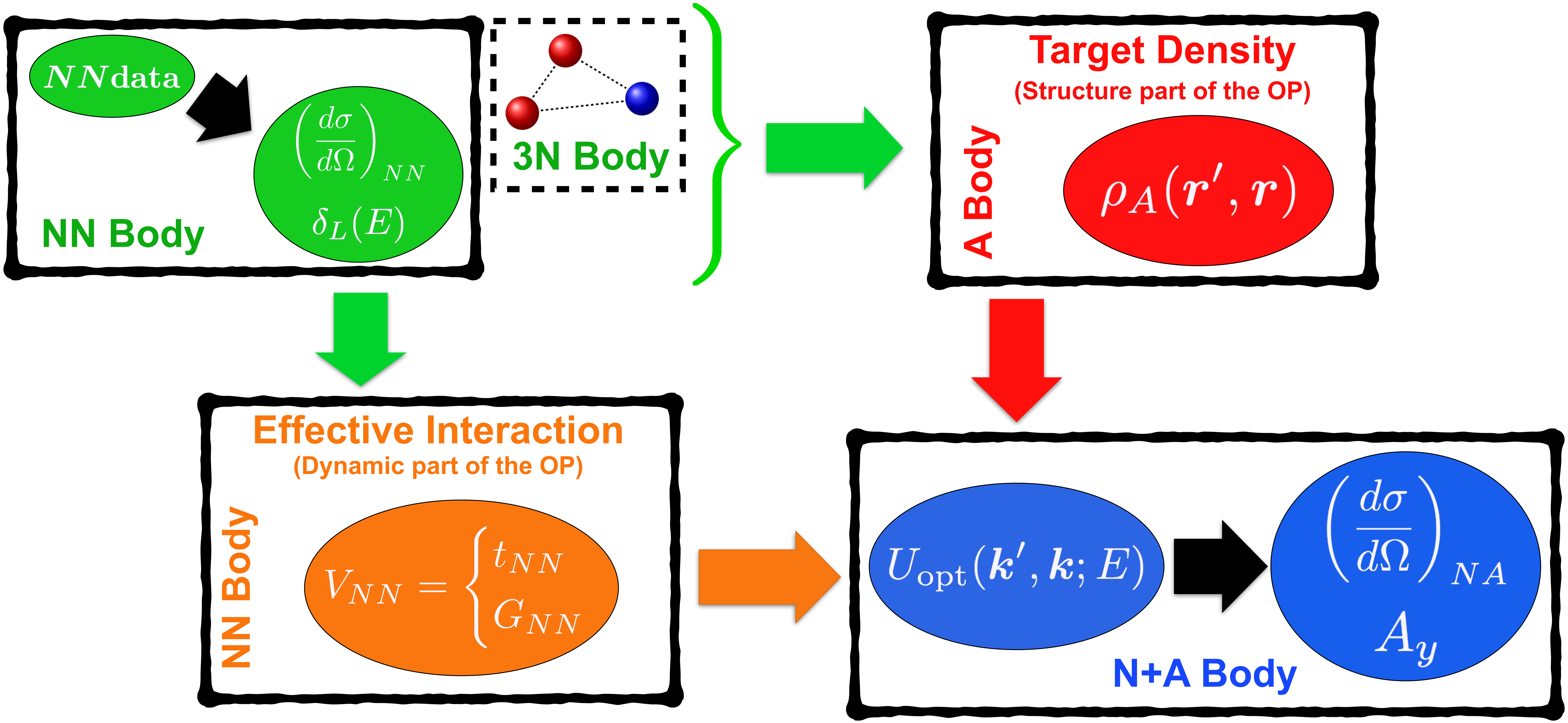


[Dobrovolsky et al., NPA 1008 (2021) 122154]



[Matsuda et al., PRC **87**, 034614 (2013)]

Road map to the optical potential



Optical potential

Phenomenological

Unfortunately, current used optical potentials for low-energy reactions are phenomenological and primarily constrained by elastic scattering data.

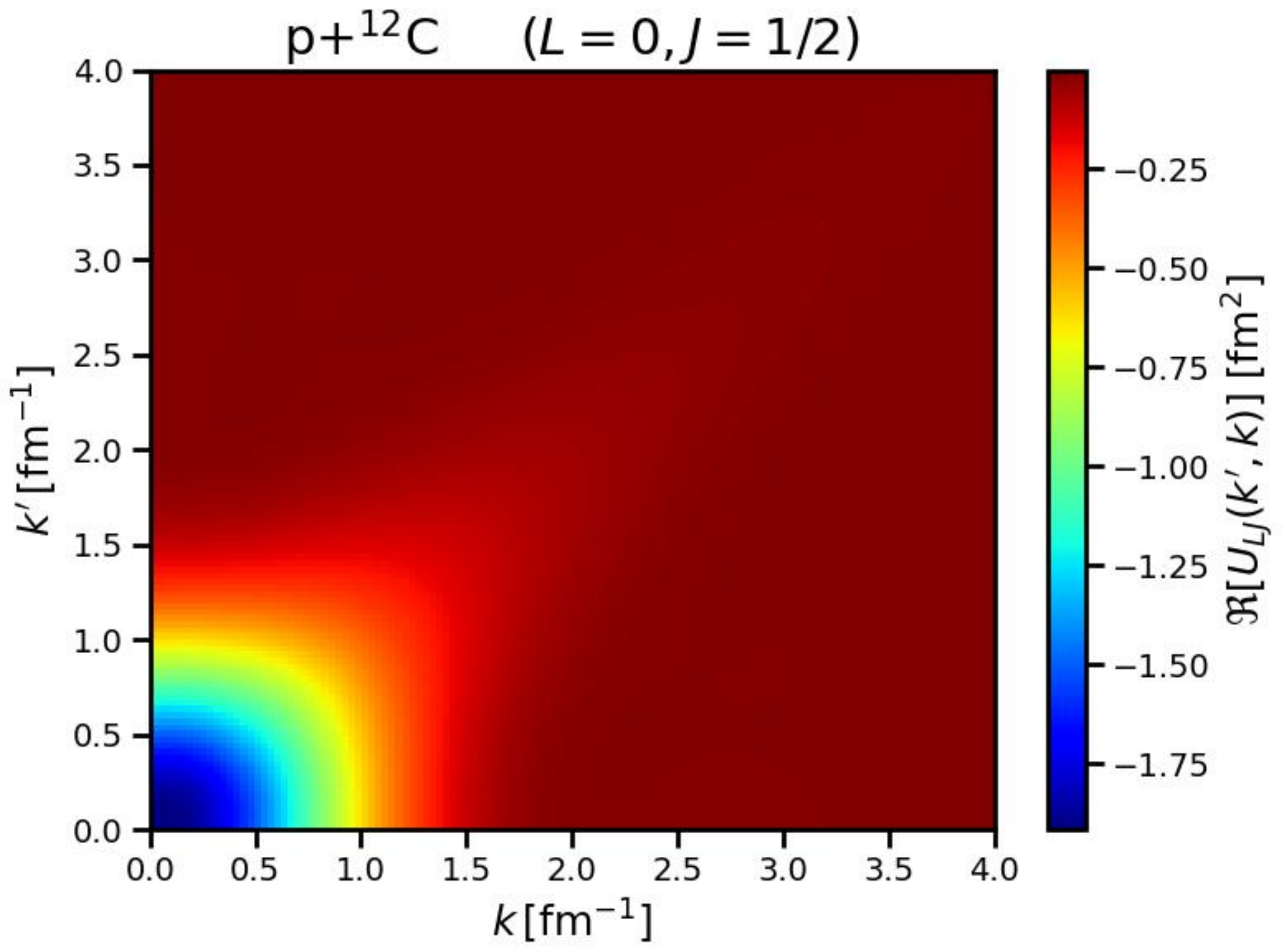
Unreliable when extrapolated beyond their fitted range in energy and nuclei

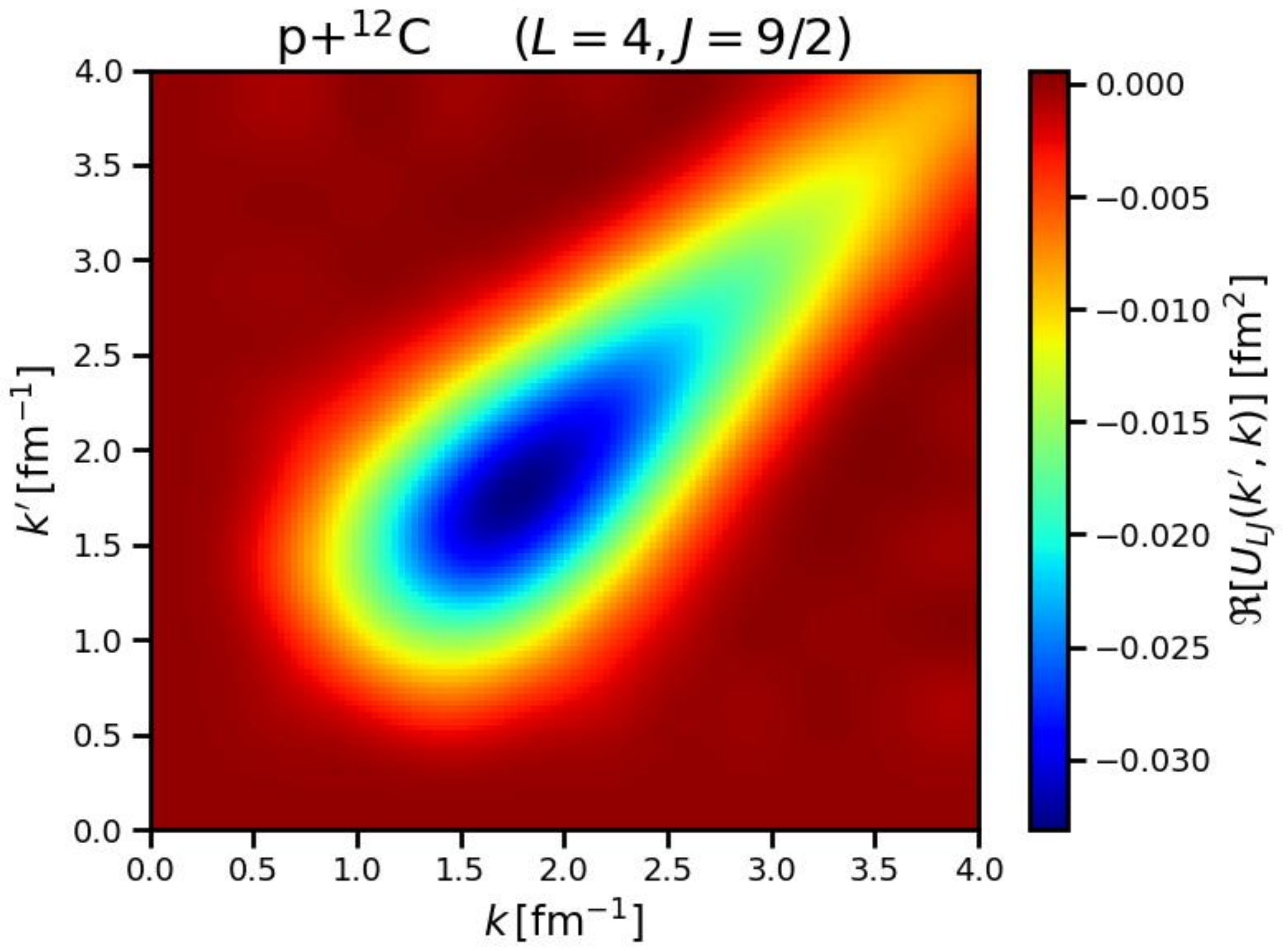
Microscopic

Existing microscopic optical potentials can be developed in a low- (Feshbach theory) or high-energy regime (Watson multiple scattering theory). Calculations are more difficult.

No fit to experimental data

$$\begin{aligned} V(r) &= -V_R f_R(r) - iW_V f_V(r) \\ &+ 4a_{VD} V_D \frac{d}{dr} f_{VD}(r) + 4ia_{WD} \frac{d}{dr} f_{WD}(r) \\ &+ \frac{\lambda_\pi^2}{r}\left[V_{SO}\frac{d}{dr} f_{VSO}(r) + iW_{SO}\frac{d}{dr} f_{WSO}(r)\right] \vec{\sigma}\cdot\vec{l} \end{aligned}$$





Theoretical framework

Lippmann-Schwinger equation for the nucleon-nucleus transition amplitude

$$T = V + V G_0(E) T$$

Projectile-target interaction

$$V = \sum_{i=1}^{A} v_{0i} + \sum_{i<j}^{A} w_{0ij}$$

Full 3N interaction
is still missing!

Many-body propagator

$$G_0(E) = (E - H_0 + i\epsilon)^{-1}$$

$$H_0 = h_0 + H_A$$

h_0 = kinetic term of the projectile

Target Hamiltonian

$$H_A |\Psi_0\rangle = E_0 |\Psi_0\rangle$$

Theoretical framework

Lippmann-Schwinger equation for the nucleon-nucleus transition amplitude

$$T = V + VG_0(E)T$$

Let's introduce the optical potential *U*

$$T = U + UG_0(E)PT$$

$$U = V + VG_0(E)QU$$

Projection operators

$$P + Q = 1$$

P space (elastic)

$$P = |\Psi_0\rangle\langle\Psi_0|$$

Q Space

$$Q = 1 - P$$

Theoretical framework

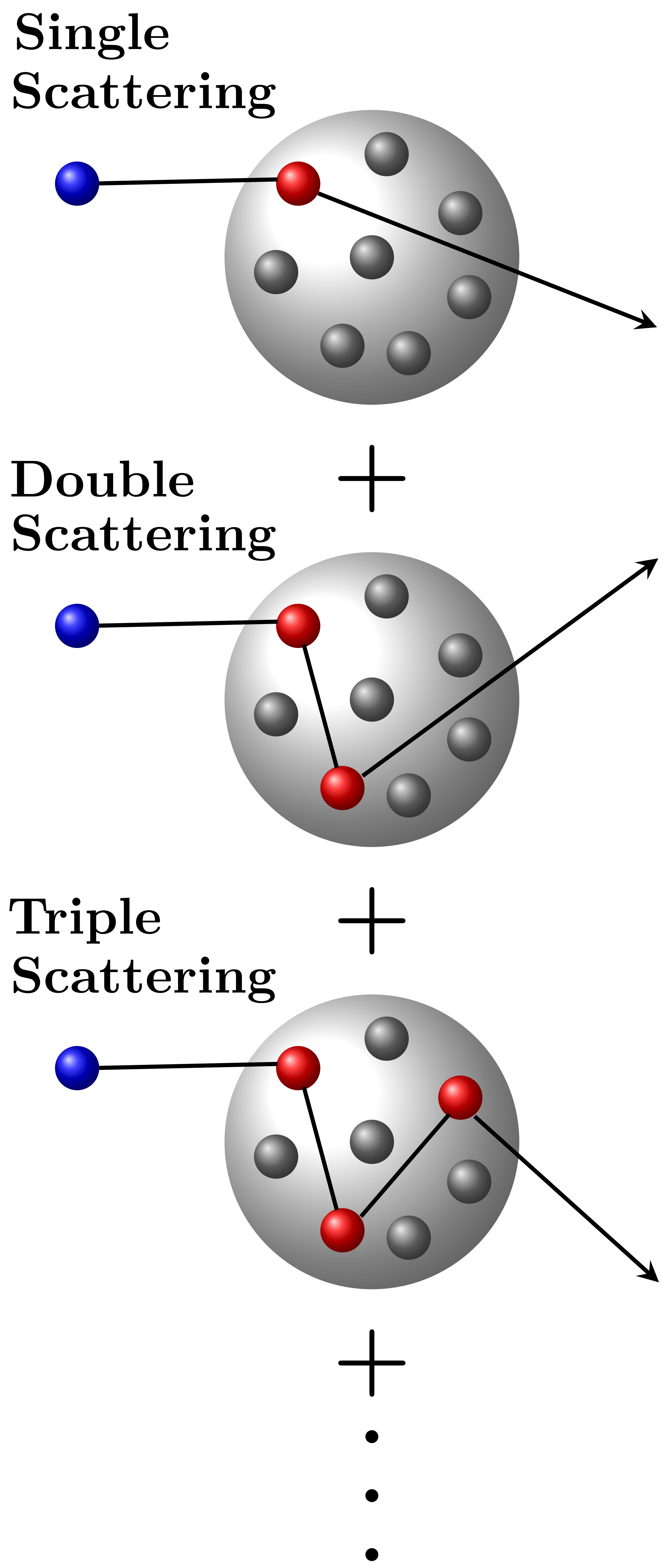
Transition amplitude for elastic scattering

$$T_{\rm el} \equiv PTP = PUP + PUPG_0(E)T_{\rm el}$$

The spectator expansion

[Chinn, Elster, Thaler, Weppner, PRC **52,** 1992 (1995)]

$$U = \underbrace{\sum_{i=1}^{A} \tau_{0i} + \sum_{i,j\neq i}^{A} \tau_{0ij} + \sum_{i,j\neq i,k\neq i,j}^{A} \tau_{0ijk} + \ldots}_{\textbf{A terms}}$$



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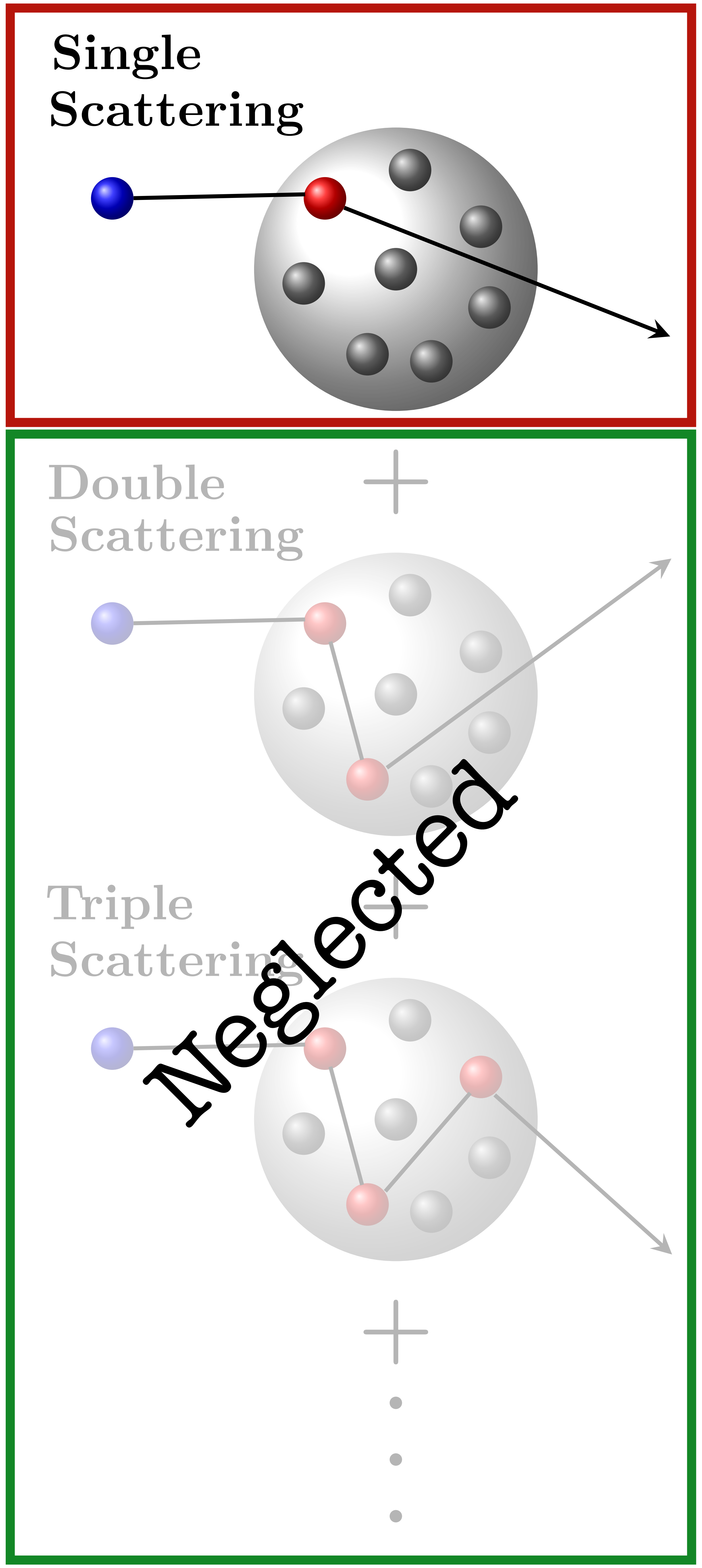
The spectator expansion

[Chinn, Elster, Thaler, Weppner, PRC **52,** 1992 (1995)]

$$U \simeq \sum_{i=1}^{A} \tau_{0i}$$

Satisfies a many-body equation and it is still difficult to compute!
Currently, it is approximated with the free NN *t* matrix.
It is a work in progress!



Theoretical framework

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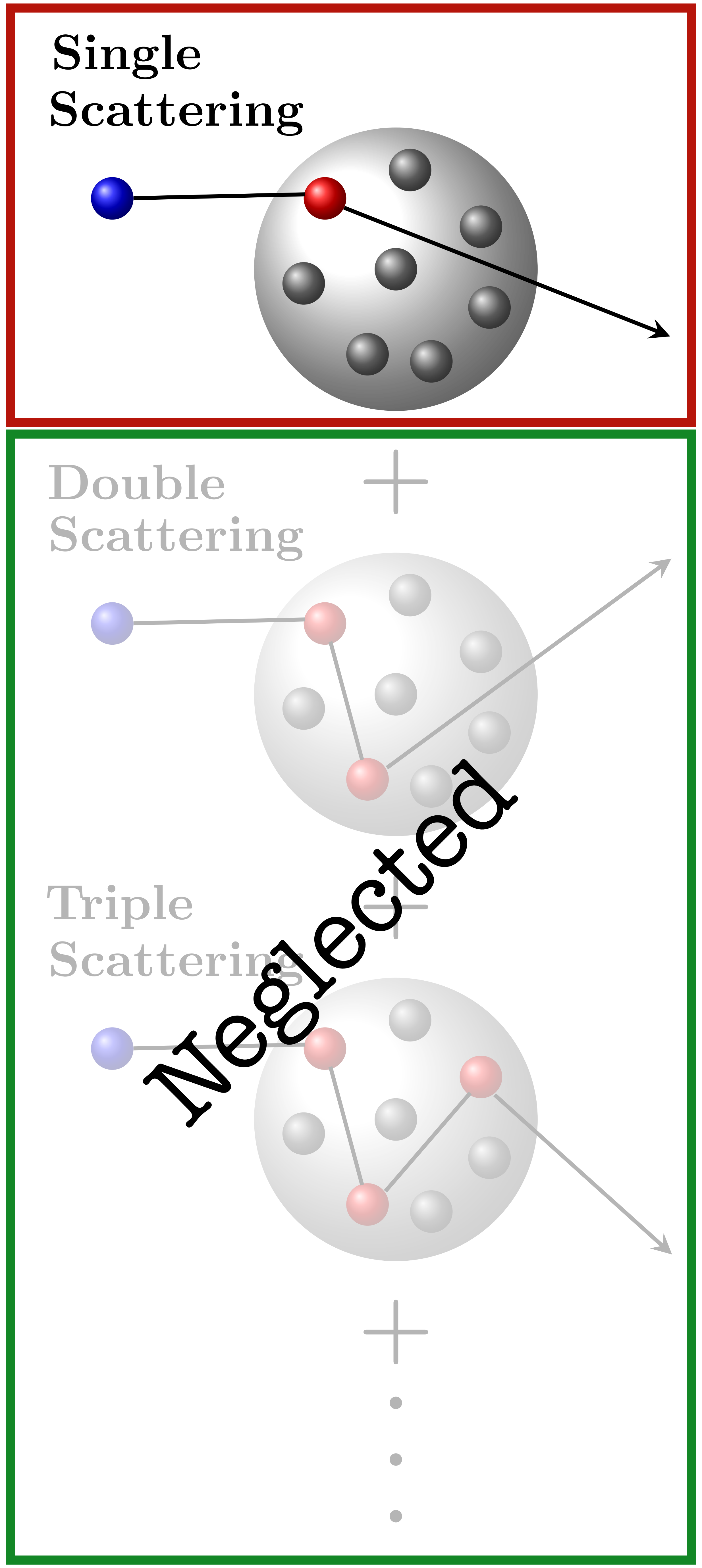
$$T_{\mathrm{el}} \equiv PTP = PUP + PUPG_0(E)T_{\mathrm{el}}$$

The spectator expansion

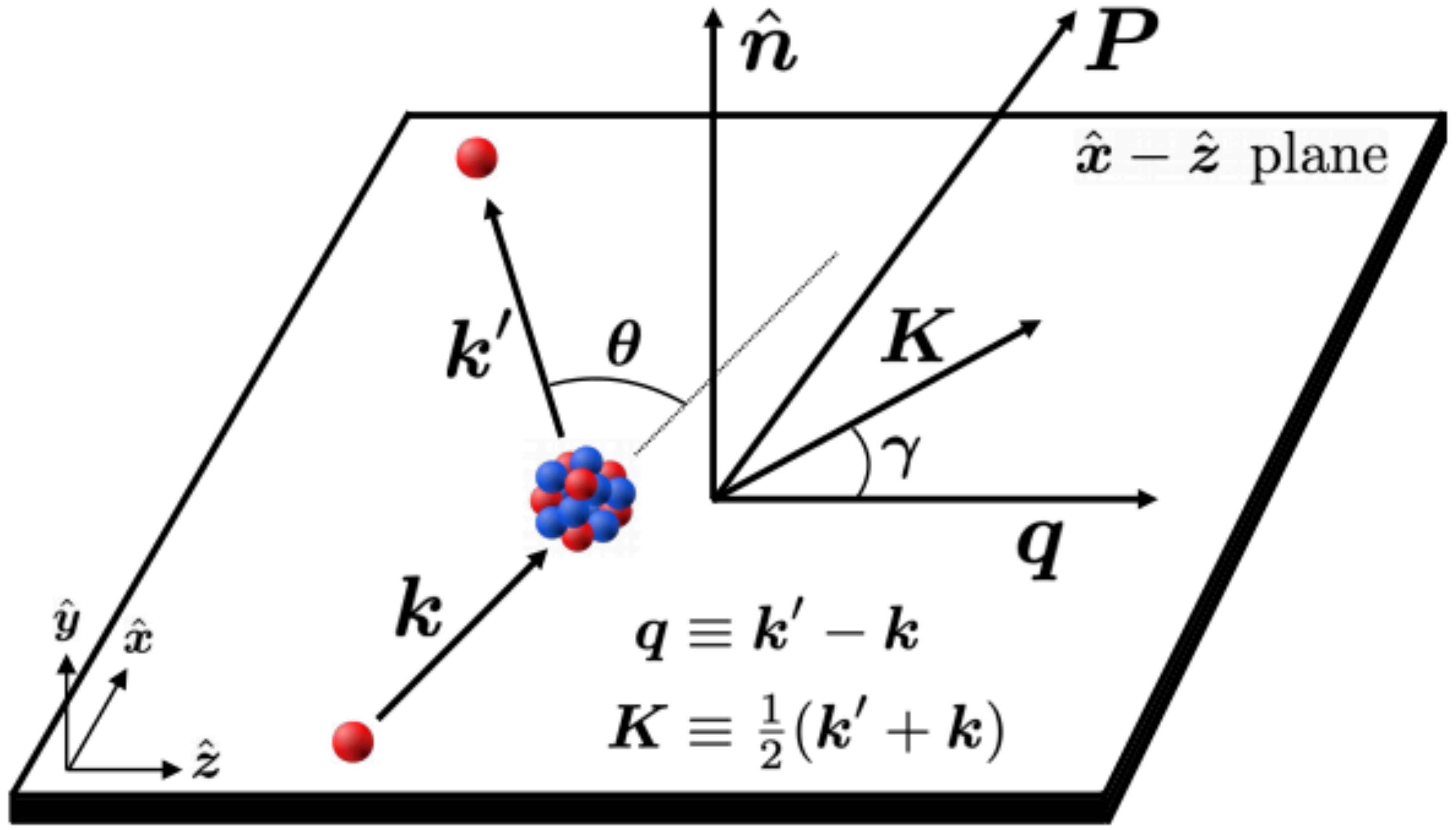
[Chinn, Elster, Thaler, Weppner, PRC **52,** 1992 (1995)]

$$U \simeq \sum_{i=1}^{A} \tau_{0i} \longrightarrow \tau_{0i} \approx t_{0i}$$

We neglect the coupling of the struck target nucleon with the residual nucleus!!!



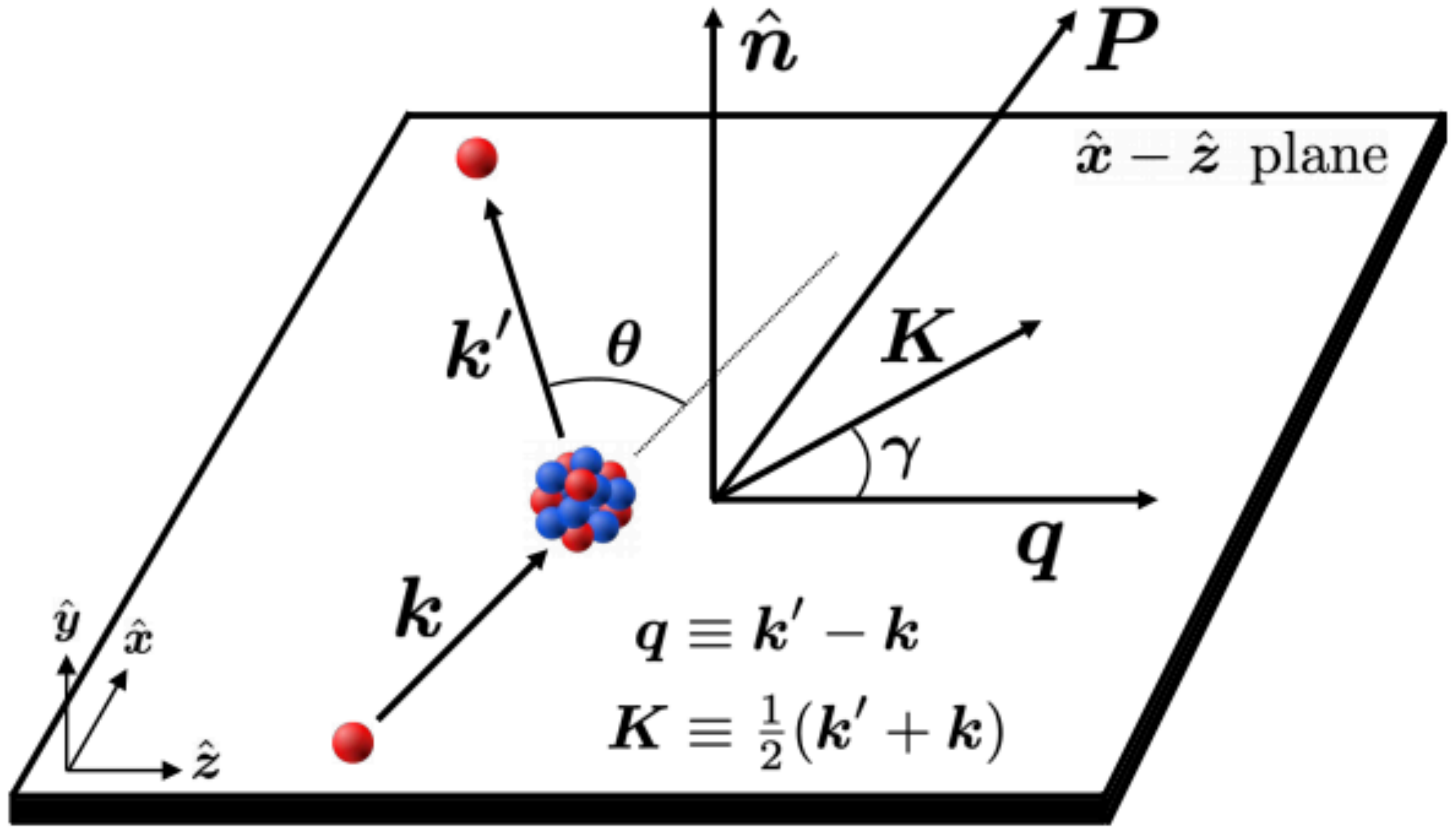
The first-order optical potential



$$U_{\mathbf{p}}(\boldsymbol{q}, \boldsymbol{K}) = \sum_{N=p,n} \int d\boldsymbol{P}\, \eta(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, t_{\mathbf{p}N}(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, \rho_N(\boldsymbol{q}, \boldsymbol{P})$$

$\mathbf{p} = (p, n, \bar{p})$

The first-order optical potential



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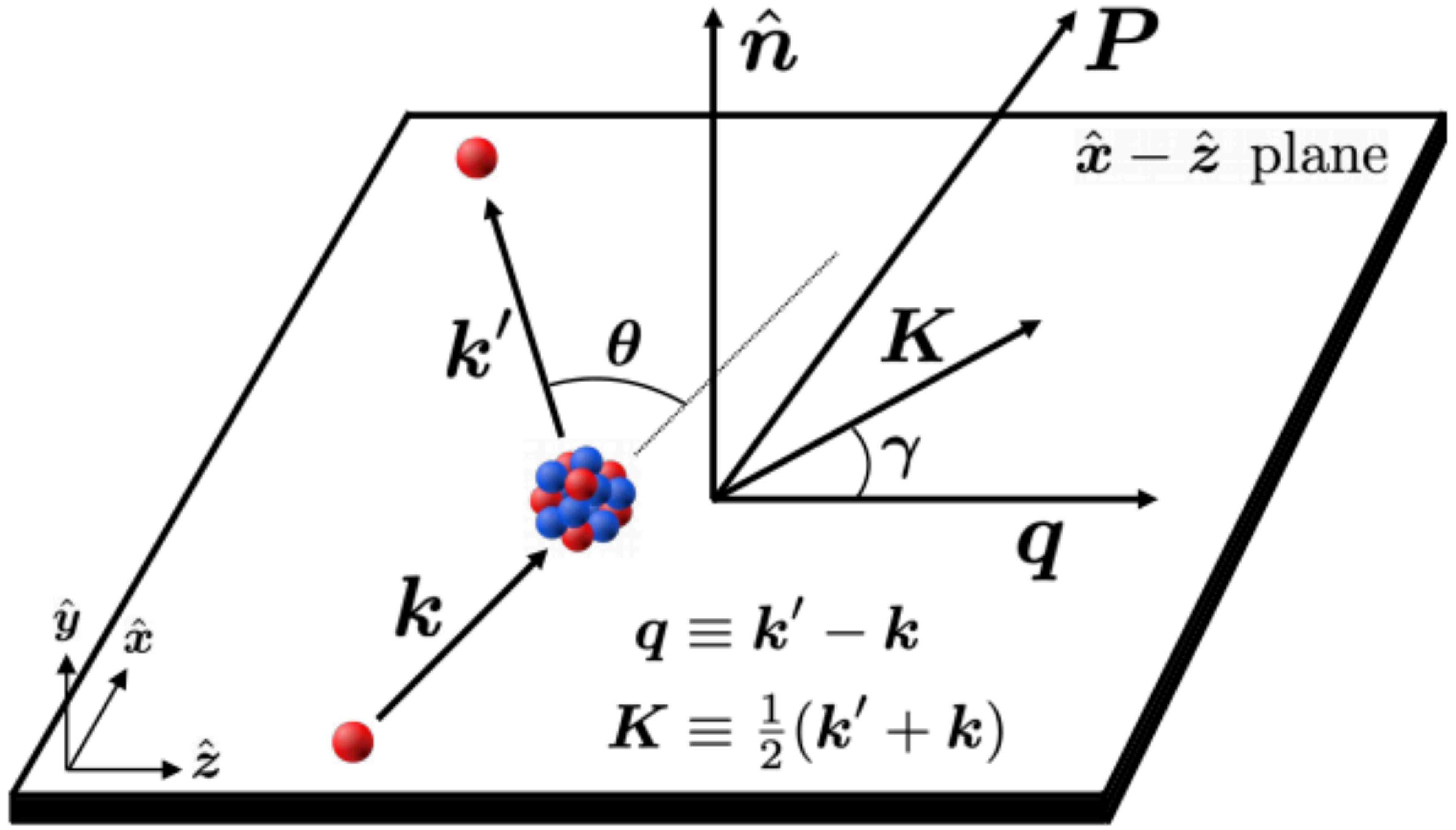
<u>Free two-body scattering matrix</u>

$$t_{0i} = v_{0i} + v_{0i}\, g_{0i}\, t_{0i}$$

$$g_{0i} = (E - h_0 - h_i + i\epsilon)^{-1}$$

- Simple one-body equation
- Can be solved easily
- Only **NN** interaction

The first-order optical potential



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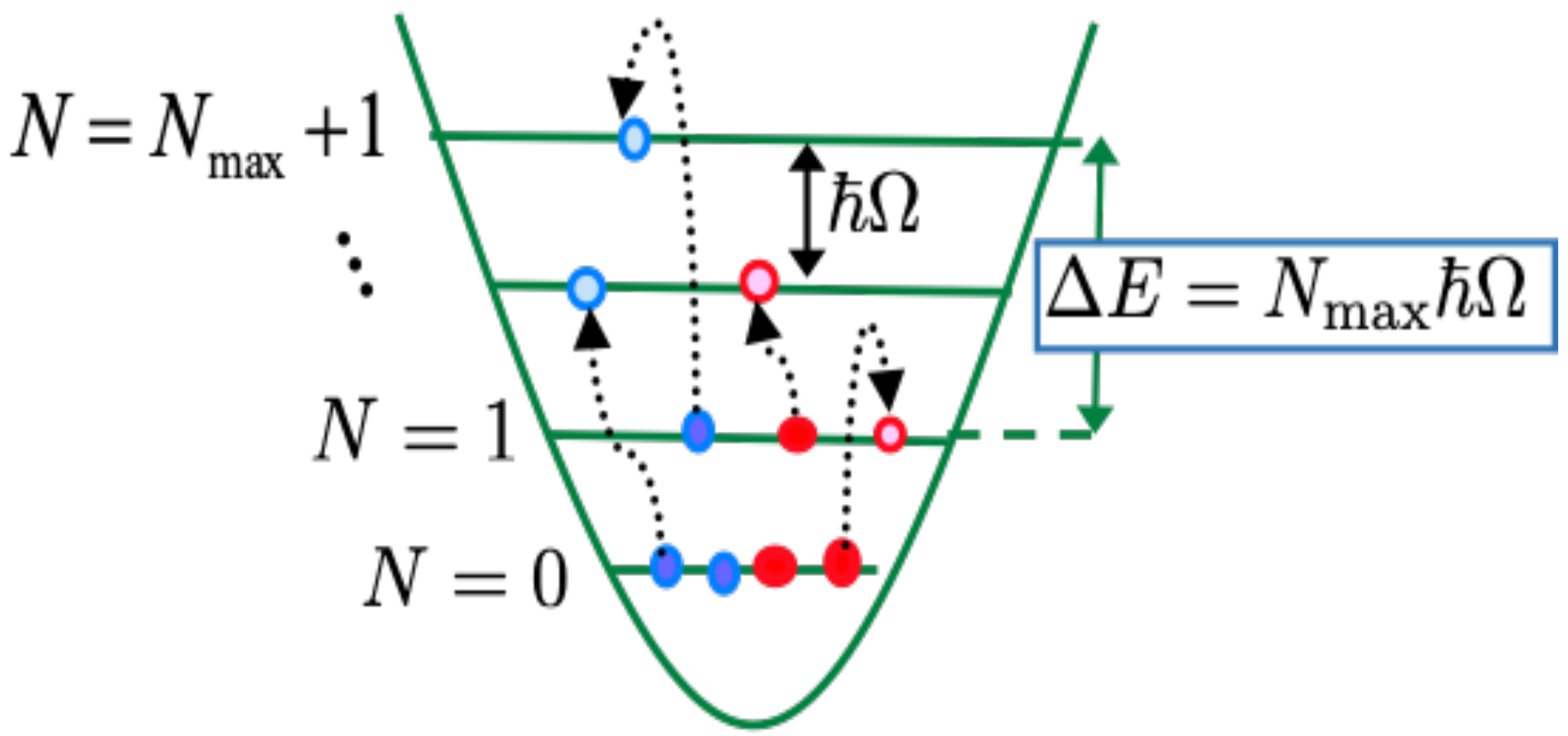
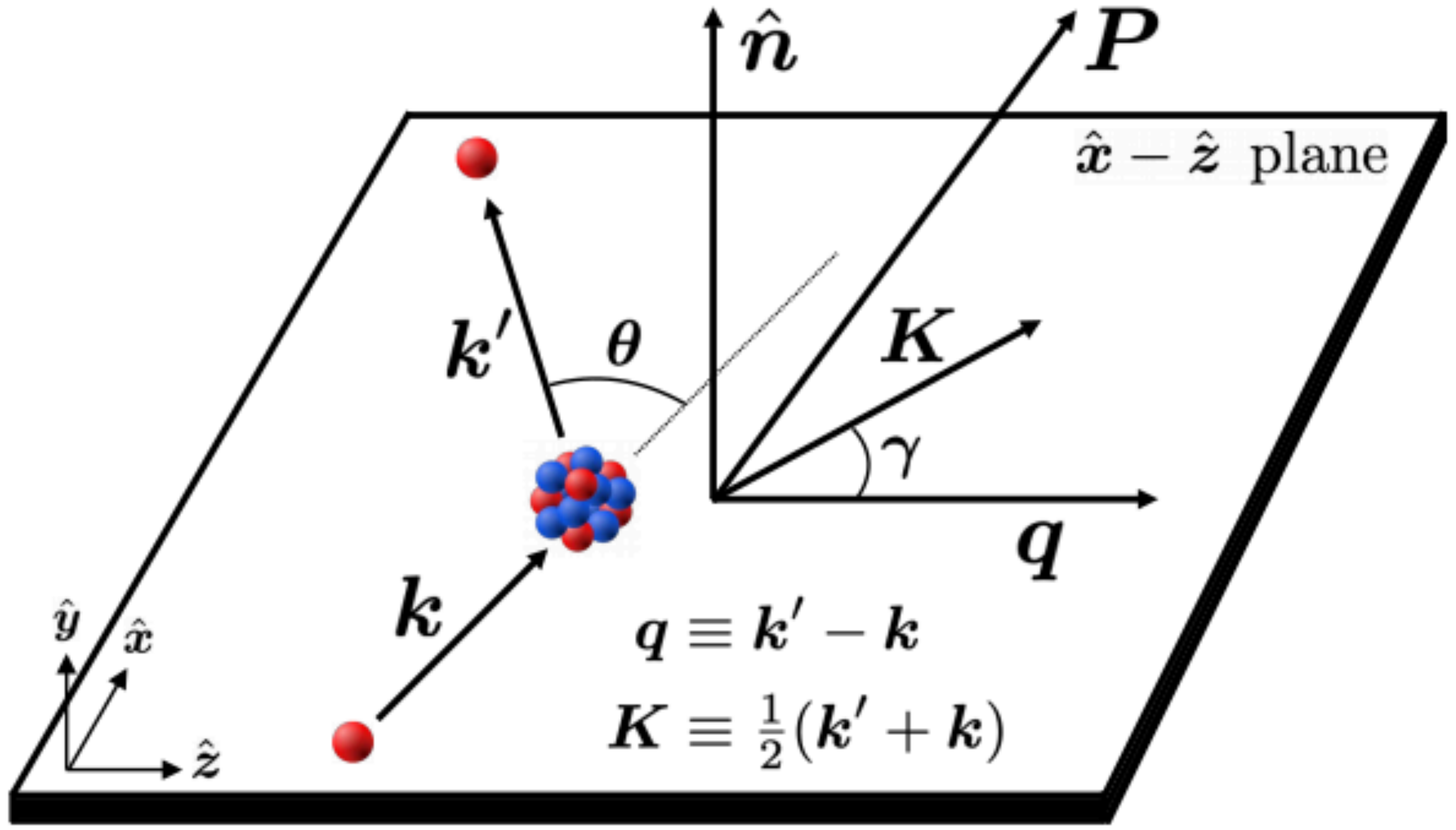
Nonlocal one-body density



- Computationally expensive
- Obtained from the No-Core Shell Model
- Calculation performed with **NN** and **3N** interaction

The first-order optical potential



Møller factor

$$t_{\mathbf{p}N}^{(NA)} = \eta \, t_{\mathbf{p}N}^{(NN)}$$

It imposes the Lorentz invariance of flux when we pass from the NA to the NN frame where the t matrices are evaluated

$$U_{\mathbf{p}}(\boldsymbol{q}, \boldsymbol{K}) = \sum_{N=p,n} \int d\boldsymbol{P} \, \eta(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P}) \, t_{\mathbf{p}N}(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P}) \, \rho_N(\boldsymbol{q}, \boldsymbol{P})$$

$\mathbf{p} = (p, n, \bar{p})$

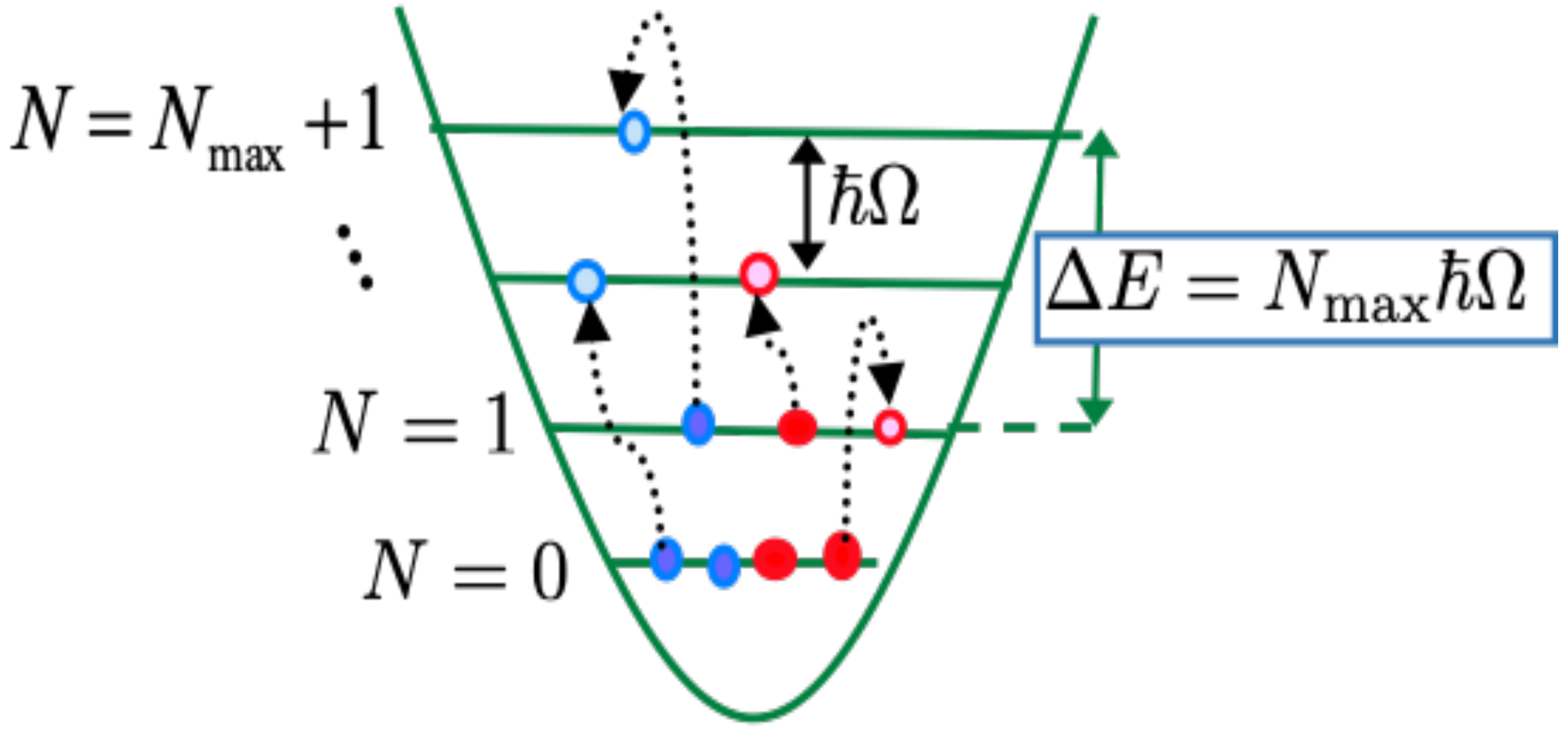
Free two-body scattering matrix

$$t_{0i} = v_{0i} + v_{0i} \, g_{0i} \, t_{0i}$$

$$g_{0i} = (E - h_0 - h_i + i\epsilon)^{-1}$$

- Simple one-body equation
- Can be solved easily
- Only **NN** interaction

Nonlocal one-body density



- Computationally expensive
- Obtained from the No-Core Shell Model
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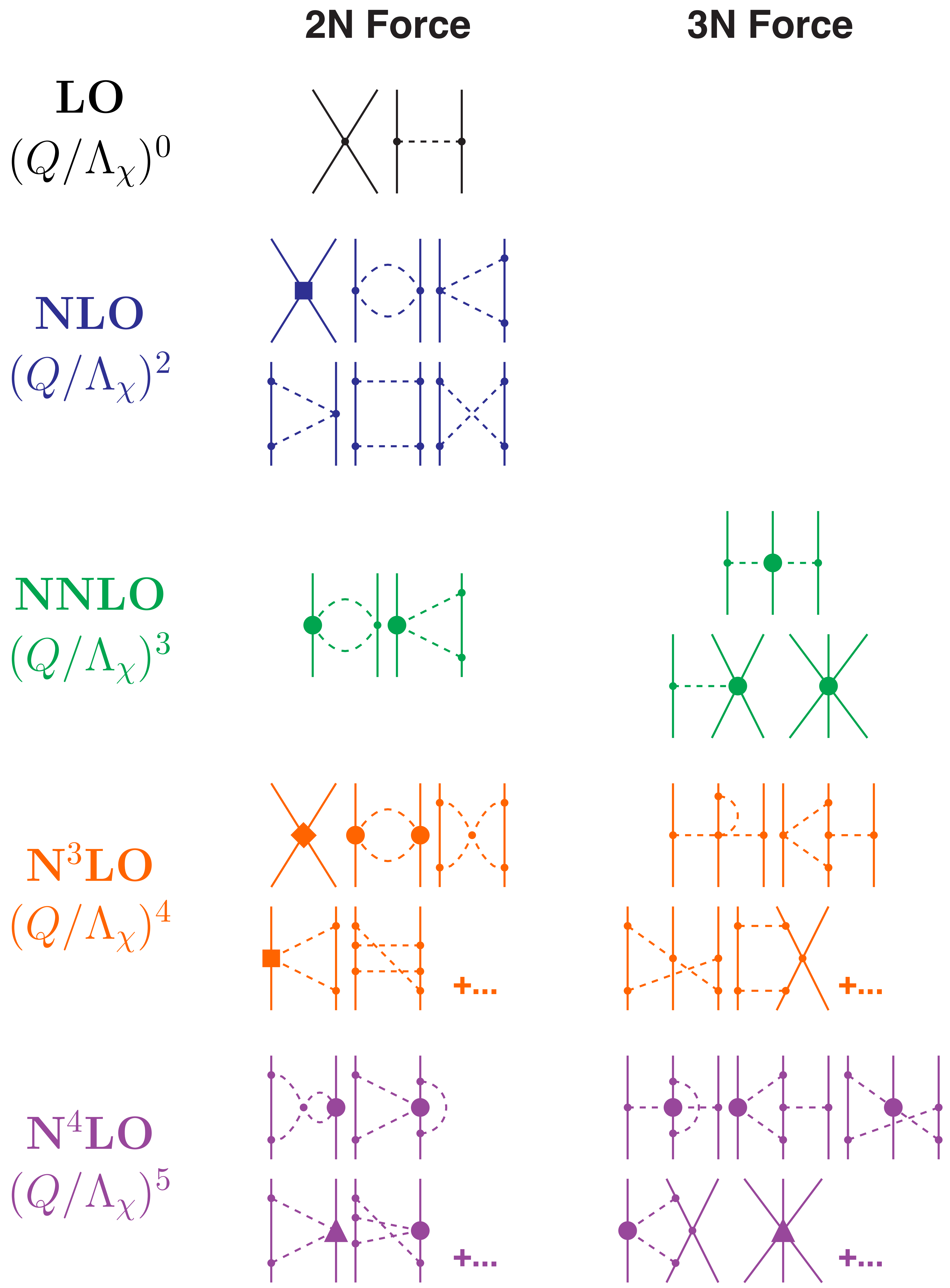
Chiral interactions

Advantages

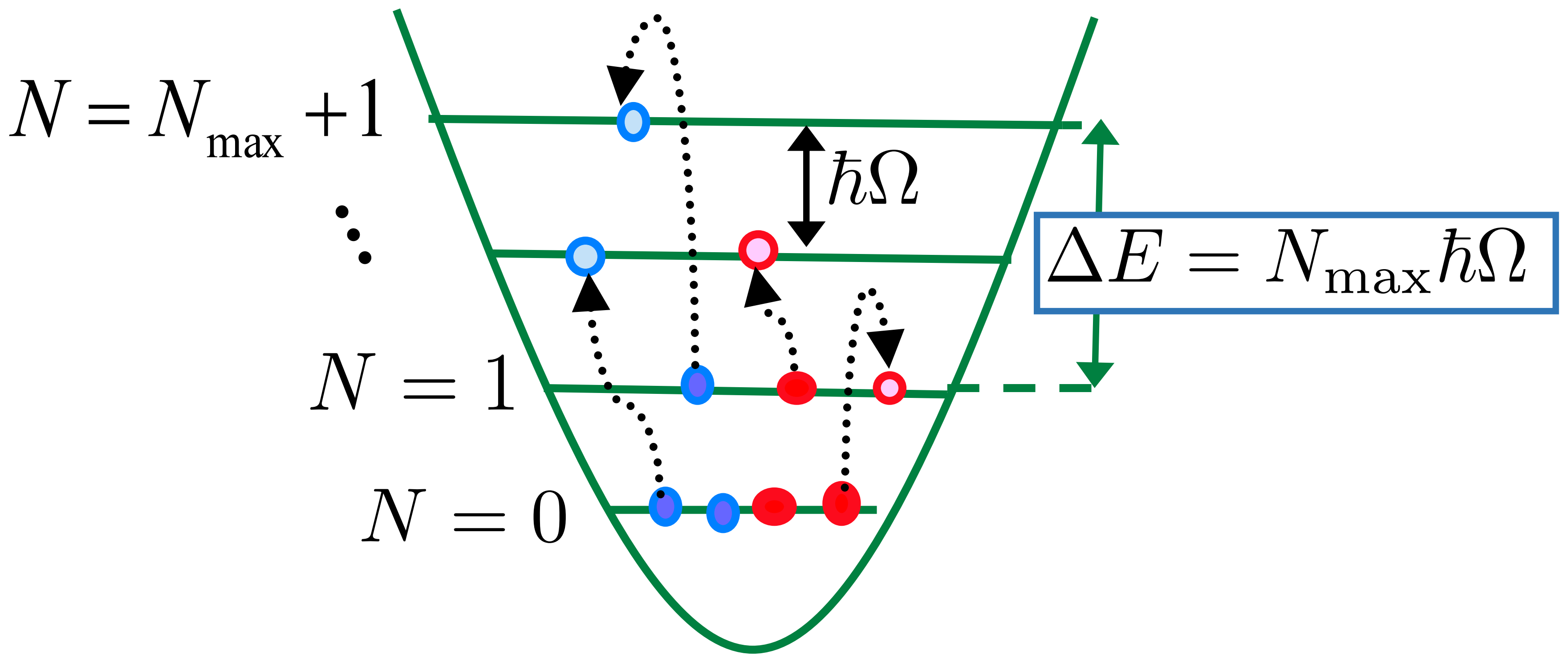
- QCD symmetries are consistently respected
- Systematic expansion (order by order we know exactly the terms to be included)
- Theoretical errors
- Two- and three-nucleon forces belong to the same framework

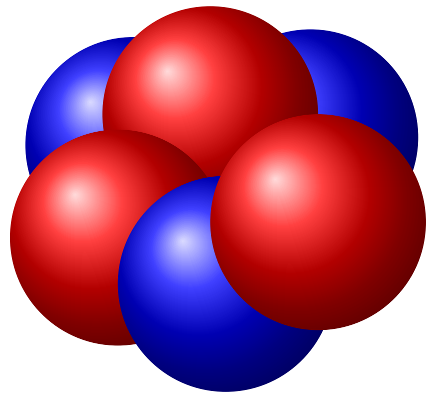
We use these interactions as the **only** input to calculate the **effective interaction** between projectile and target and the **target density**



Target description

No-Core Shell Model

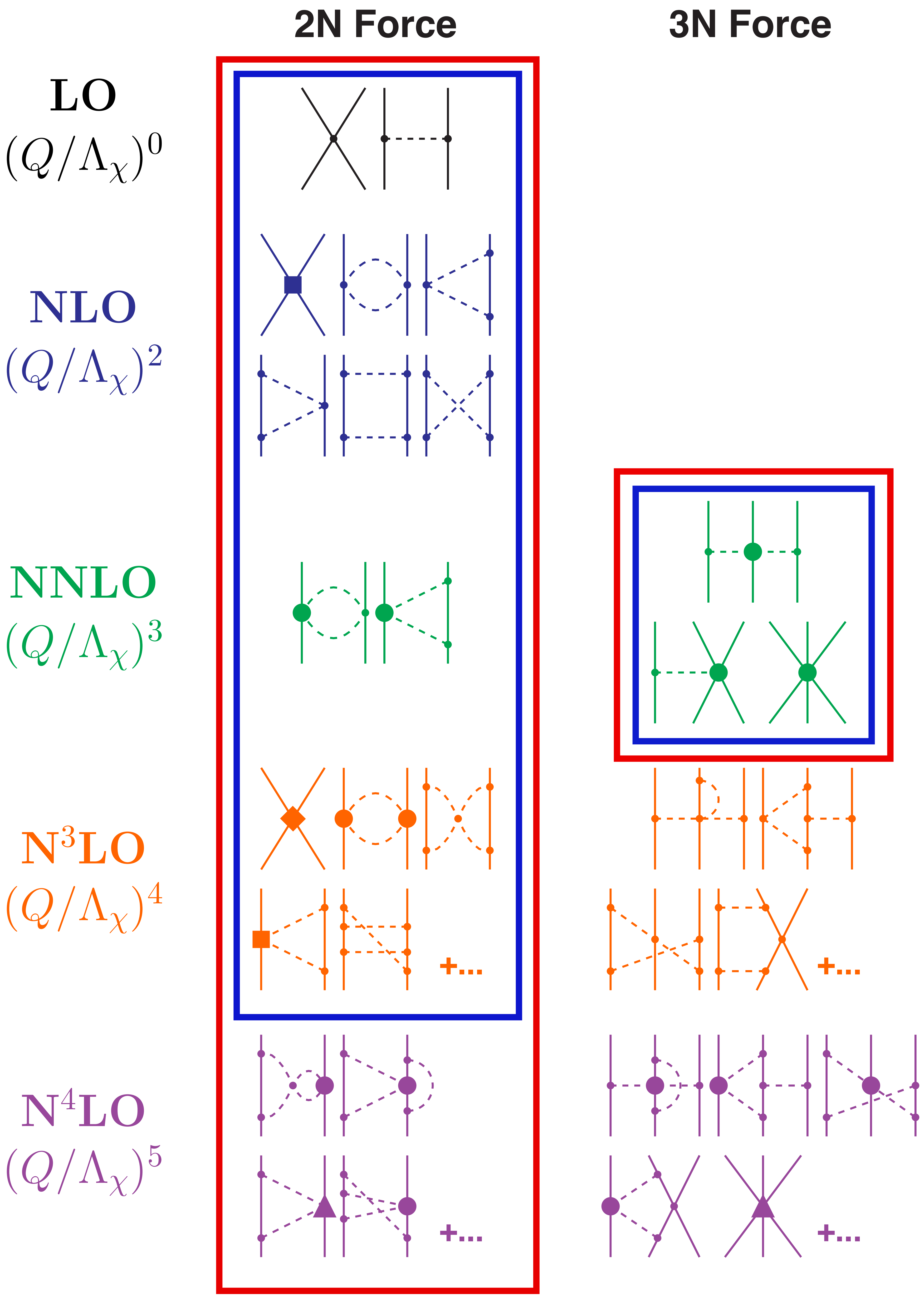




NCSM

In collaboration with P. Navrátil and M. Gennari (TRIUMF)

- **NN-N^{4}LO + 3NInl (^{12}C, ^{16}O)**
  - N^{4}LO: Entem et al., Phys. Rev. C **96**, 024004 (2017)
  - 3NInl: Navrátil, Few-Body Syst. **41**, 117 (2007)
  - c_D & c_E: Kravvaris et al., Phys. Rev. C **102**, 024616 (2020)
- **NN-N^{3}LO + 3NInl (^{9,13}C, ^{6,7}Li, ^{10}B)**
  - N^{3}LO: E&M, Phys. Rev. C **68**, 041001(R) (2003)
  - 3NInl: Navrátil, Few-Body Syst. **41**, 117 (2007)
  - c_D & c_E: Somà et al., Phys. Rev. C **101**, 014318 (2020)



Assessing the impact of the 3N interaction

General equation for the optical potential

$$U = (V_{NN} + V_{3N}) + (V_{NN} + V_{3N}) G_0(E) Q U$$

Treatment of the 3N force

[Holt et al., Phys. Rev. C **81**, 024002 (2010)]

$$V_{3N} = \frac{1}{2} \sum_{i=1}^{A} \sum_{\substack{j=1 \\ j \neq i}}^{A} w_{0ij} \approx \sum_{i=1}^{A} \langle w_{0i} \rangle$$

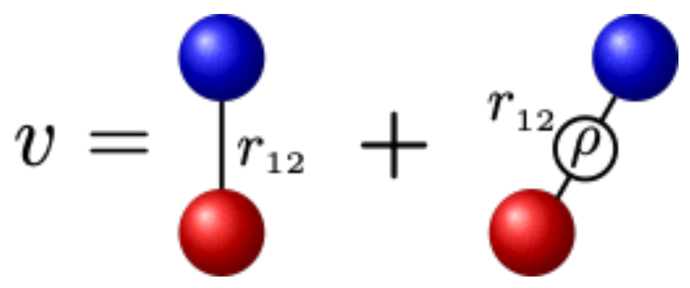
Density dependent

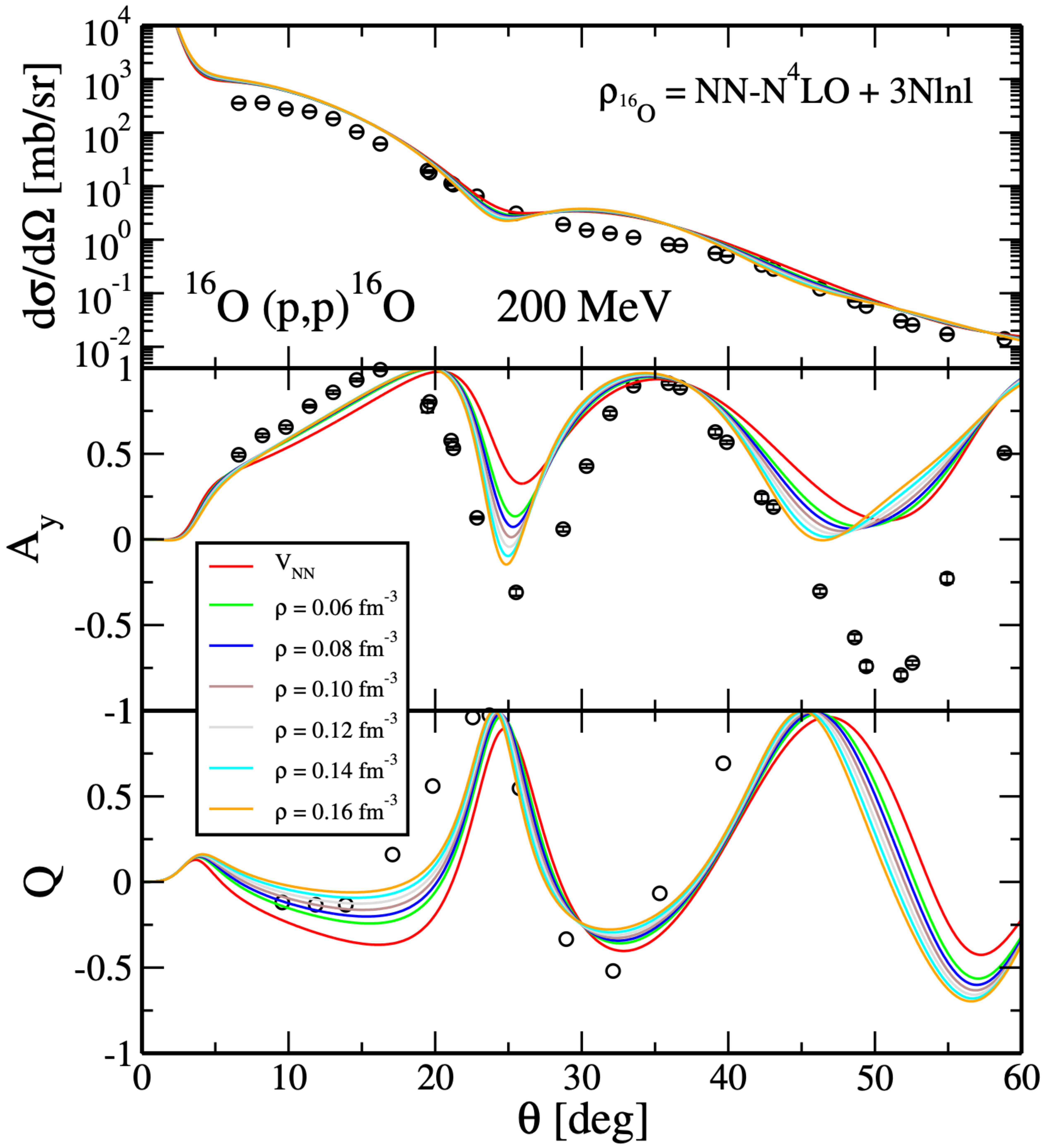
Modification of the *t* matrix

$$t_{0i} = v_{0i}^{(1)} + v_{0i}^{(2)} g_{0i} t_{0i}$$

$$v_{0i}^{(1)} = v_{0i} + \frac{1}{2} \langle w_{0i} \rangle$$

$$v_{0i}^{(2)} = v_{0i} + \langle w_{0i} \rangle$$





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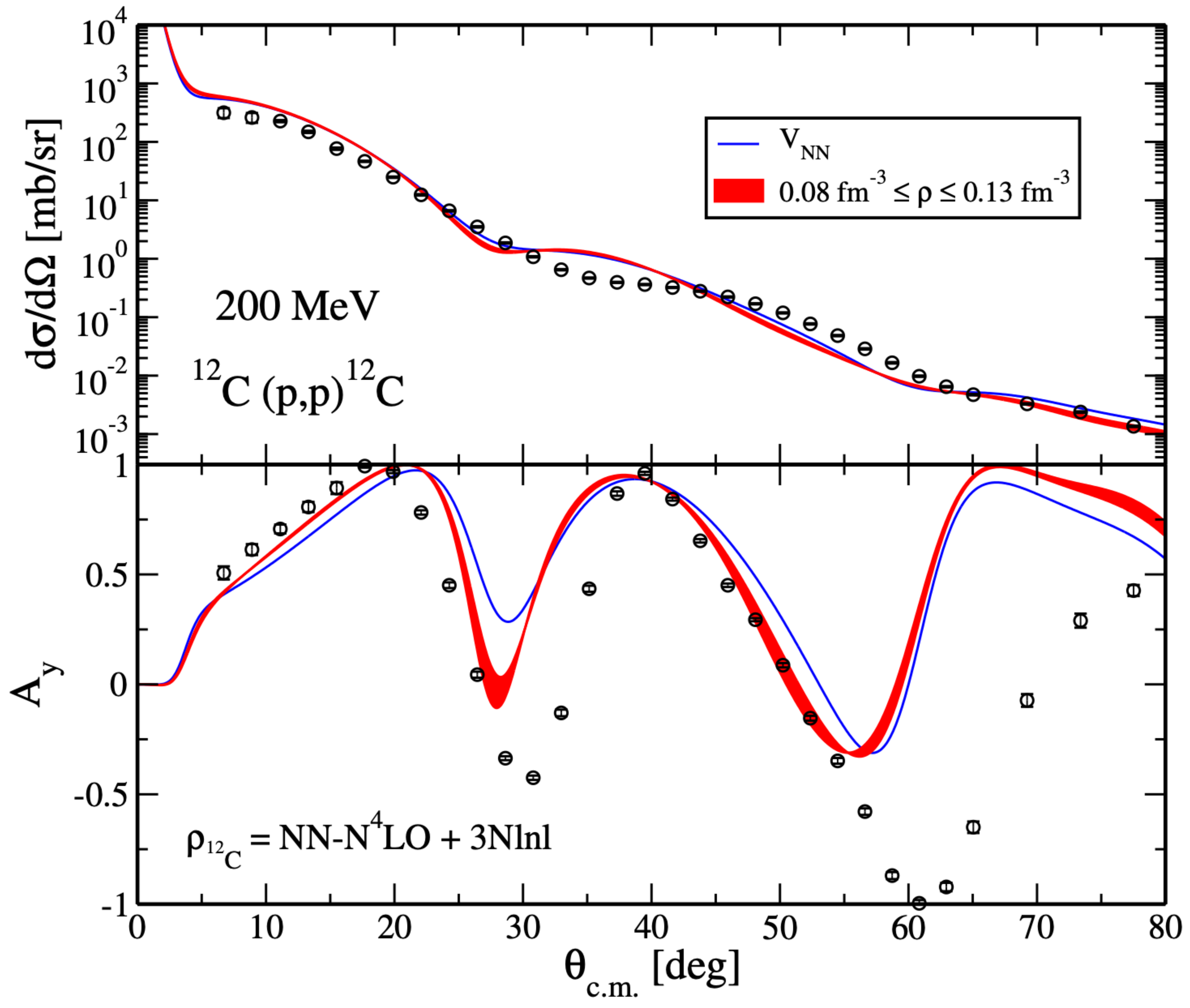
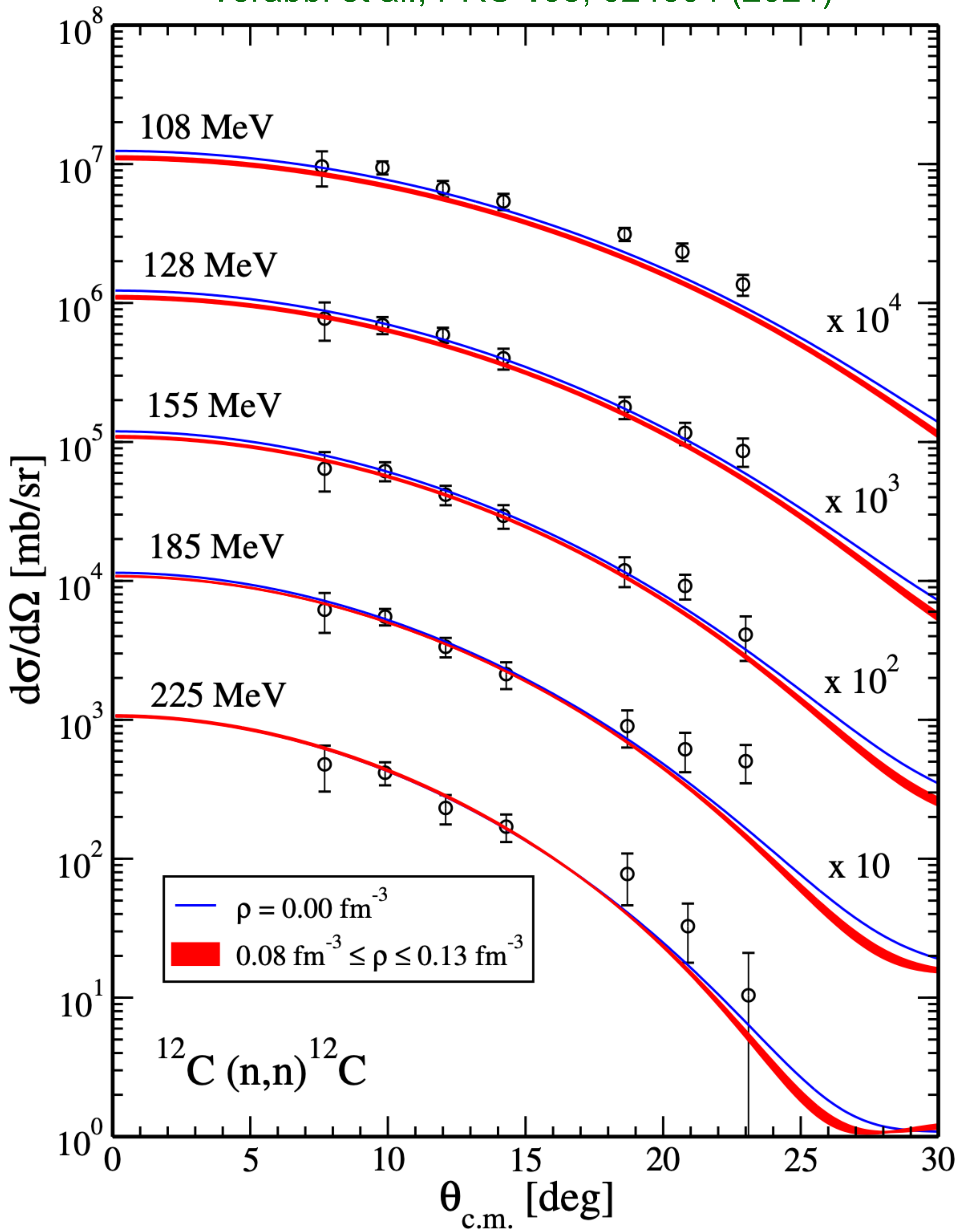


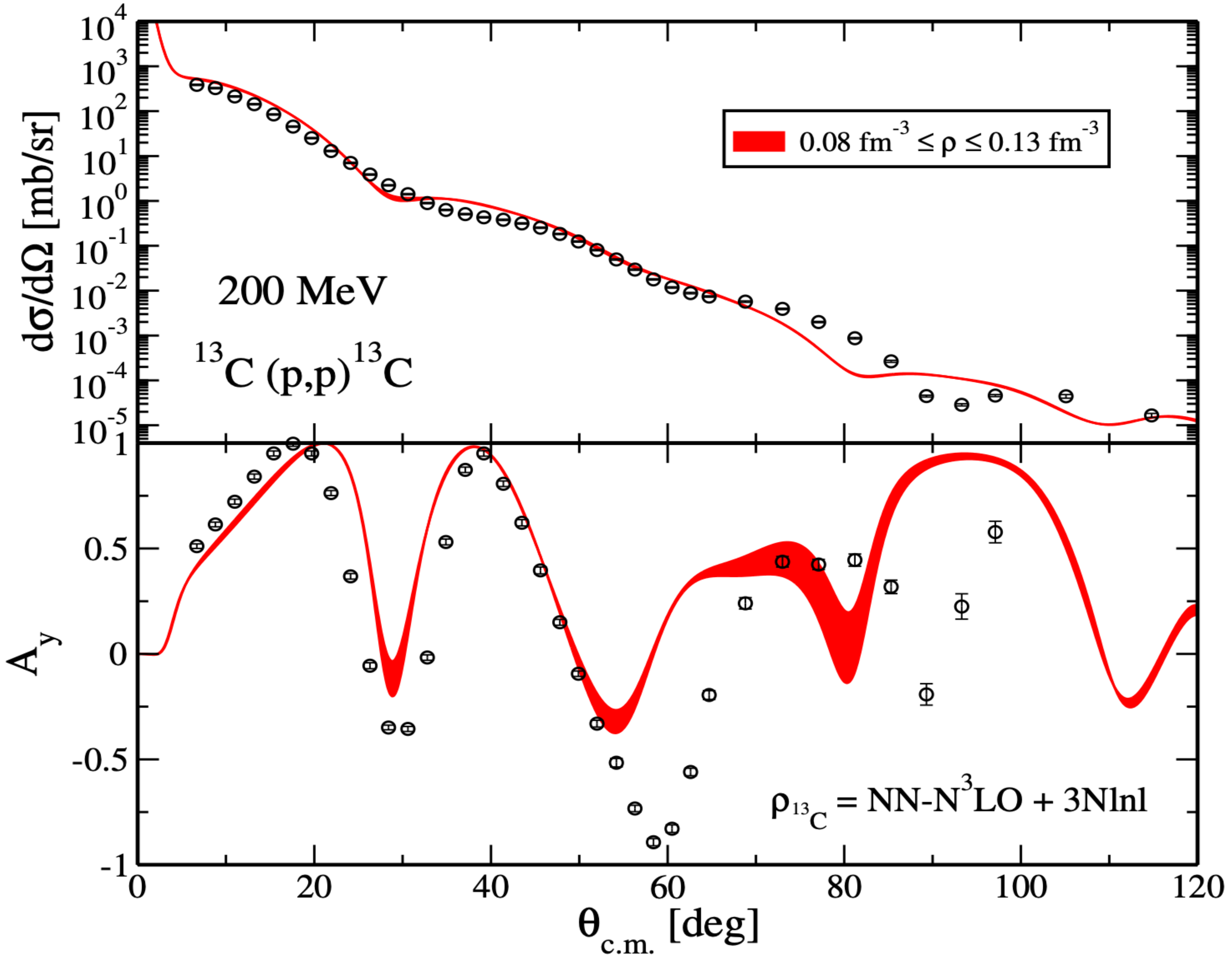
Vorabbi et al., PRC **103**, 024604 (2021)



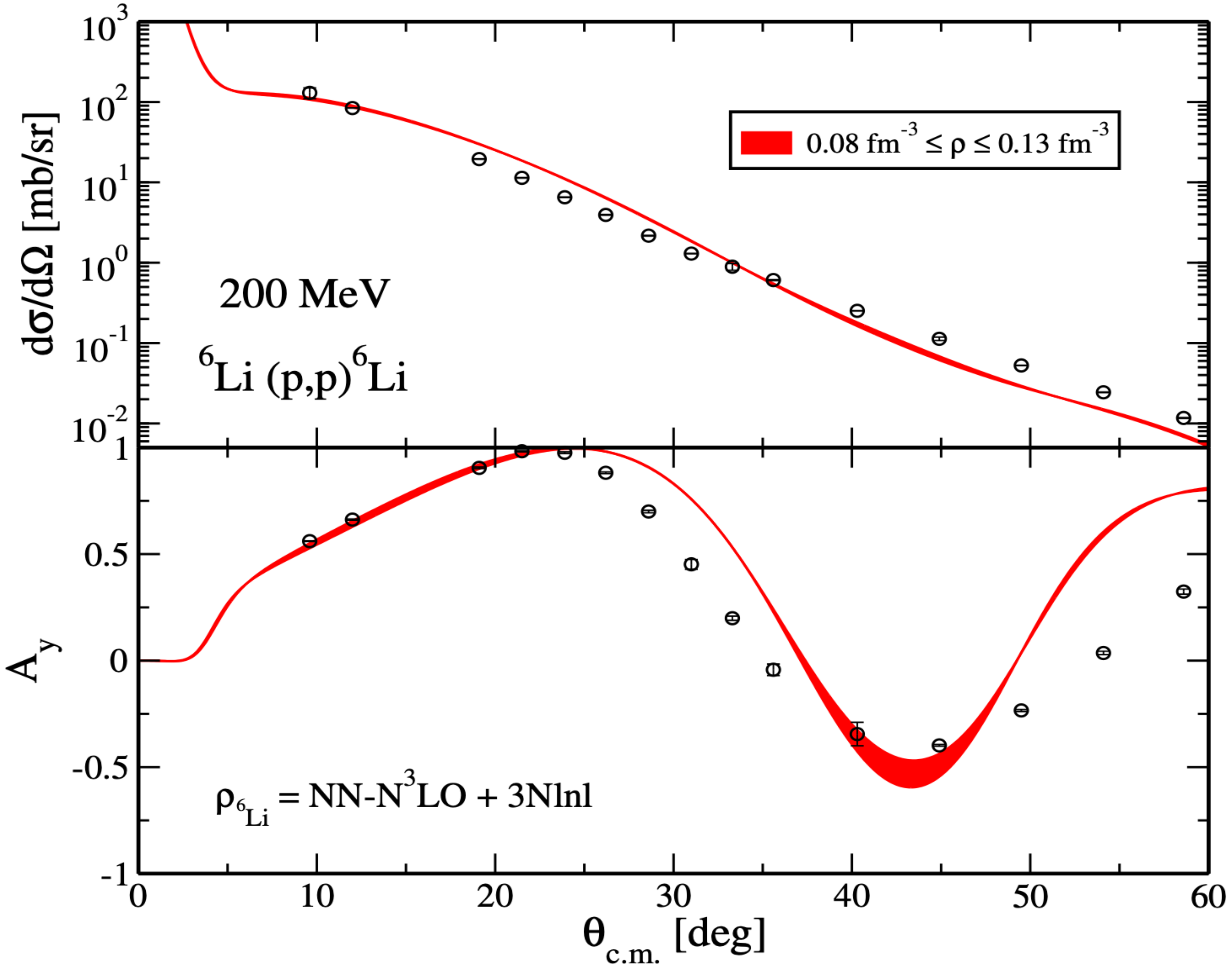
- For all nuclei we found very small contributions to the differential cross section
- The contributions to the spin observable are larger and they seem to improve the agreement with the data

Extension to non-zero spin targets

$J^\pi = 1/2^-$



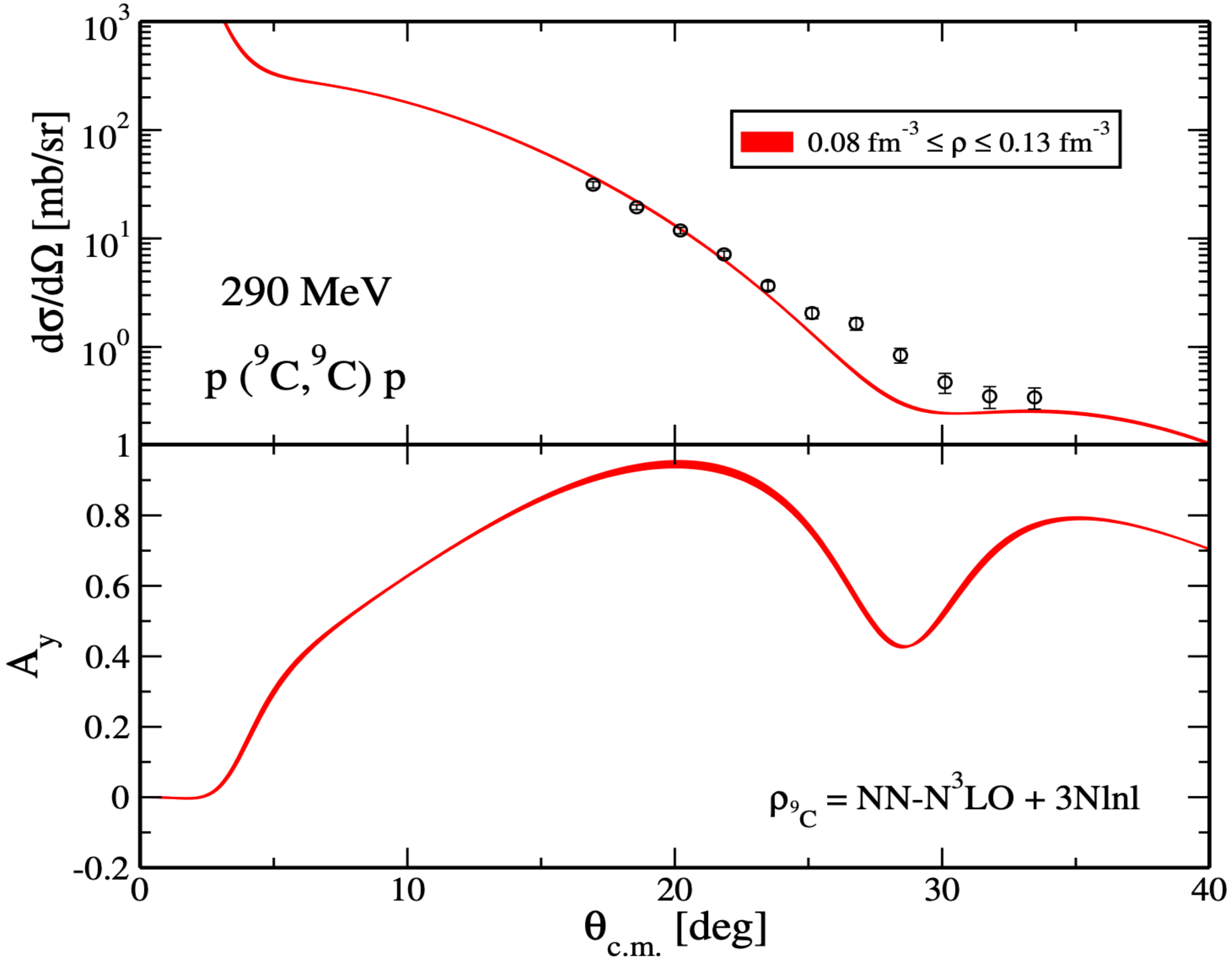
$J^\pi = 1^+$



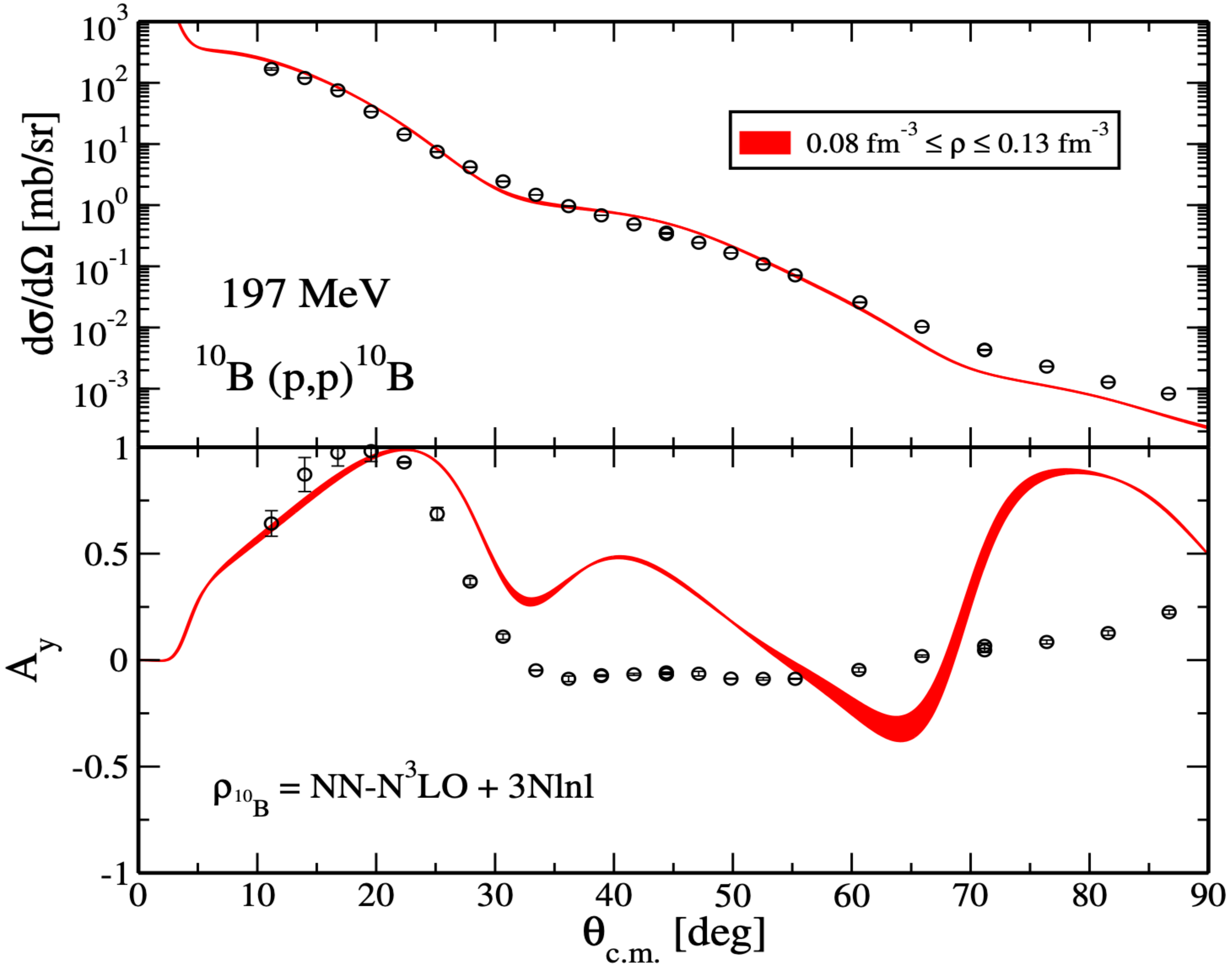
[Vorabbi et al., Phys. Rev. C **105**, 014621 (2022)]

Extension to non-zero spin targets

$J^\pi = 3/2^-$



$J^\pi = 3^+$



[Vorabbi et al., Phys. Rev. C **105**, 014621 (2022)]

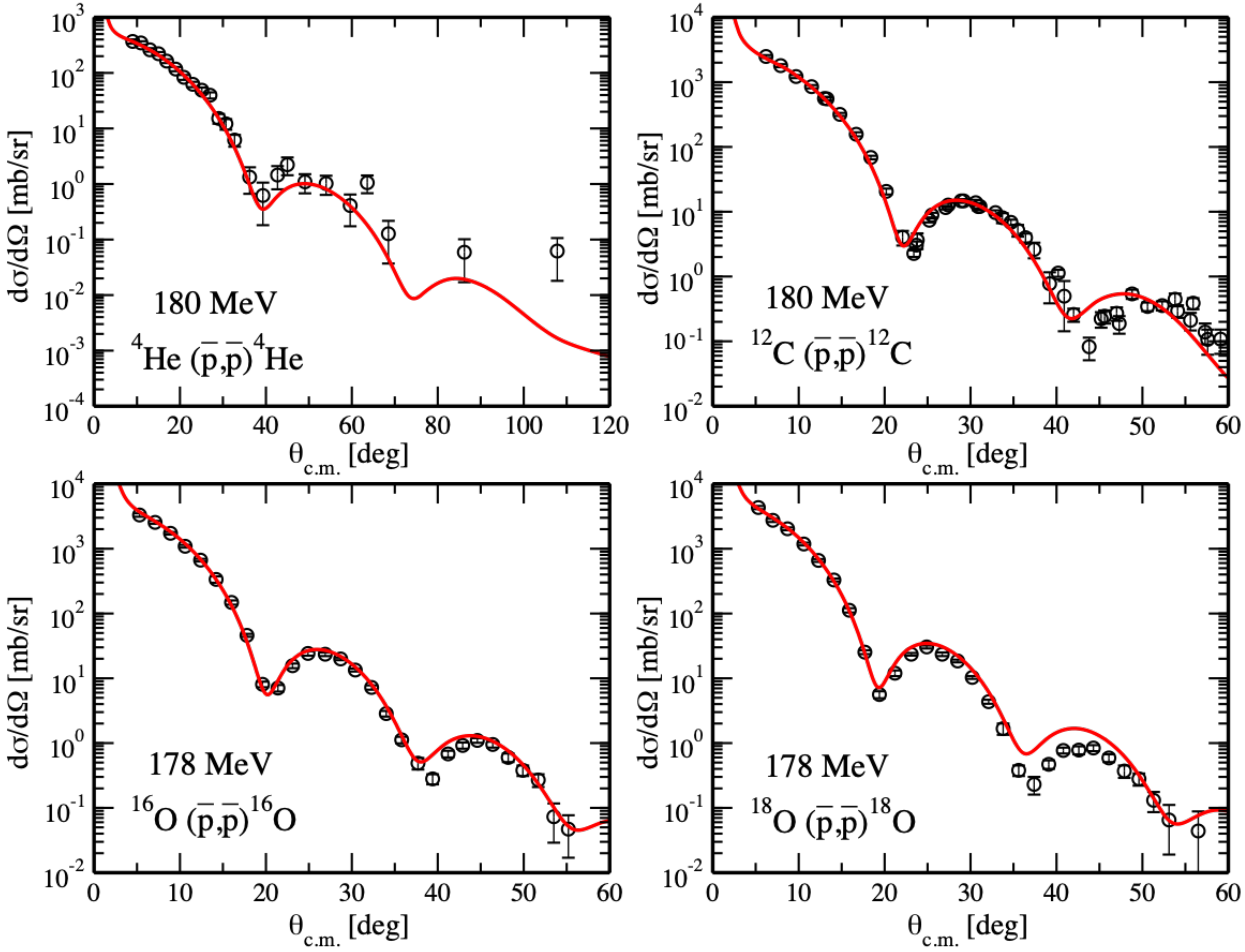
Extension to antiproton-nucleus elastic scattering

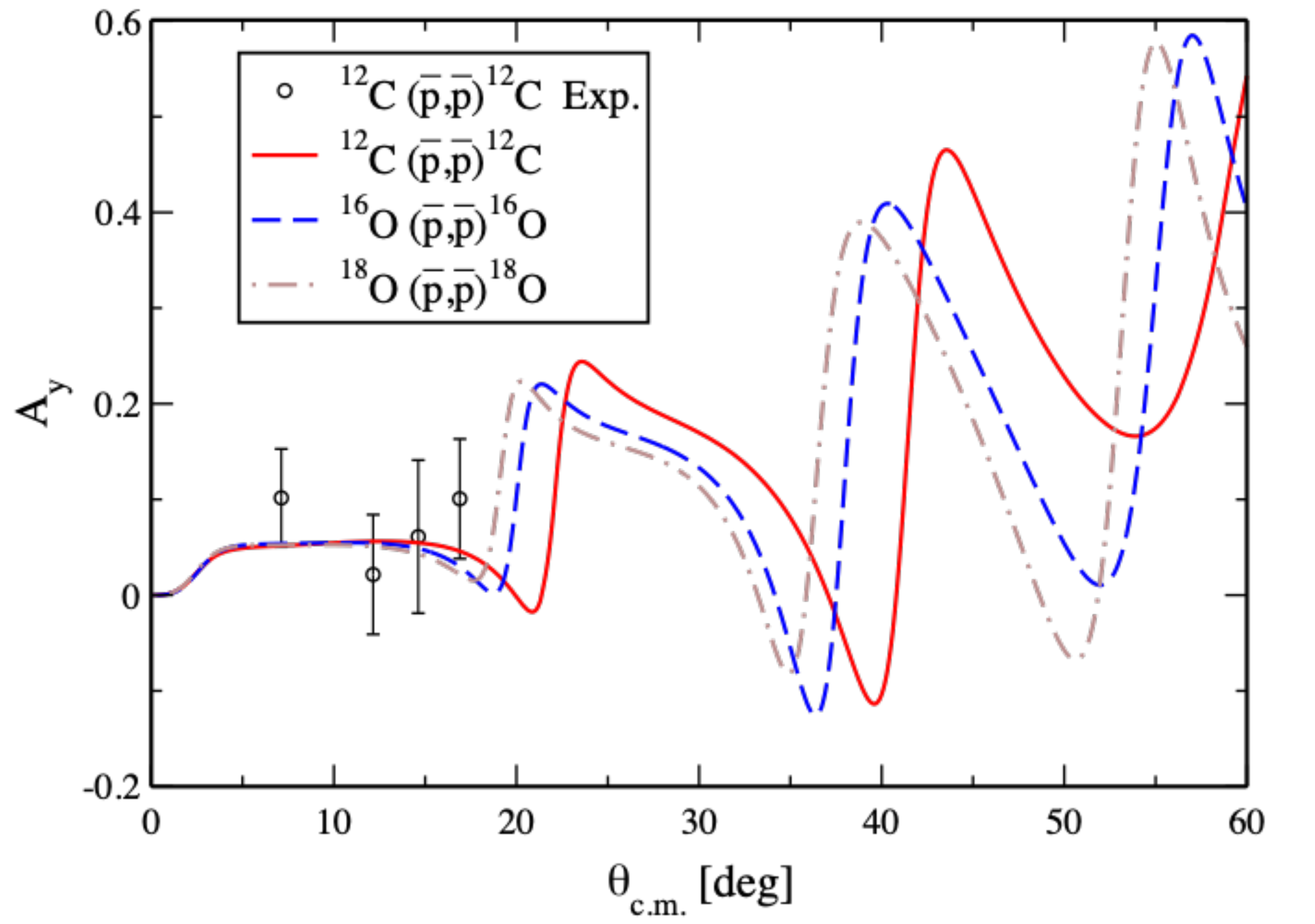
$$U_p(\boldsymbol{q},\boldsymbol{K}) = \sum_{N=p,n} \int d\boldsymbol{P}\, \eta(\boldsymbol{q},\boldsymbol{K},\boldsymbol{P})\, t_{pN}(\boldsymbol{q},\boldsymbol{K},\boldsymbol{P})\, \rho_N(\boldsymbol{q},\boldsymbol{P})$$

$$t_{pN} \Rightarrow t_{\bar{p}N}$$

- The projectile information only enters the t_{pN} matrix. For antiprotons we make the following replacement
- An antiproton-nucleon interaction is needed! $\bar{p}N$ chiral interaction derived up to N^{3}LO
  [Dai, Haidenbauer, Meißner, JHEP 07 (2017) 78]
- No projectile-target anti-symmetrisation!
- Antiprotons are mostly absorbed at the surface of the nucleus so the first-order expansion should work better in this case!

Extension to antiproton-nucleus elastic scattering





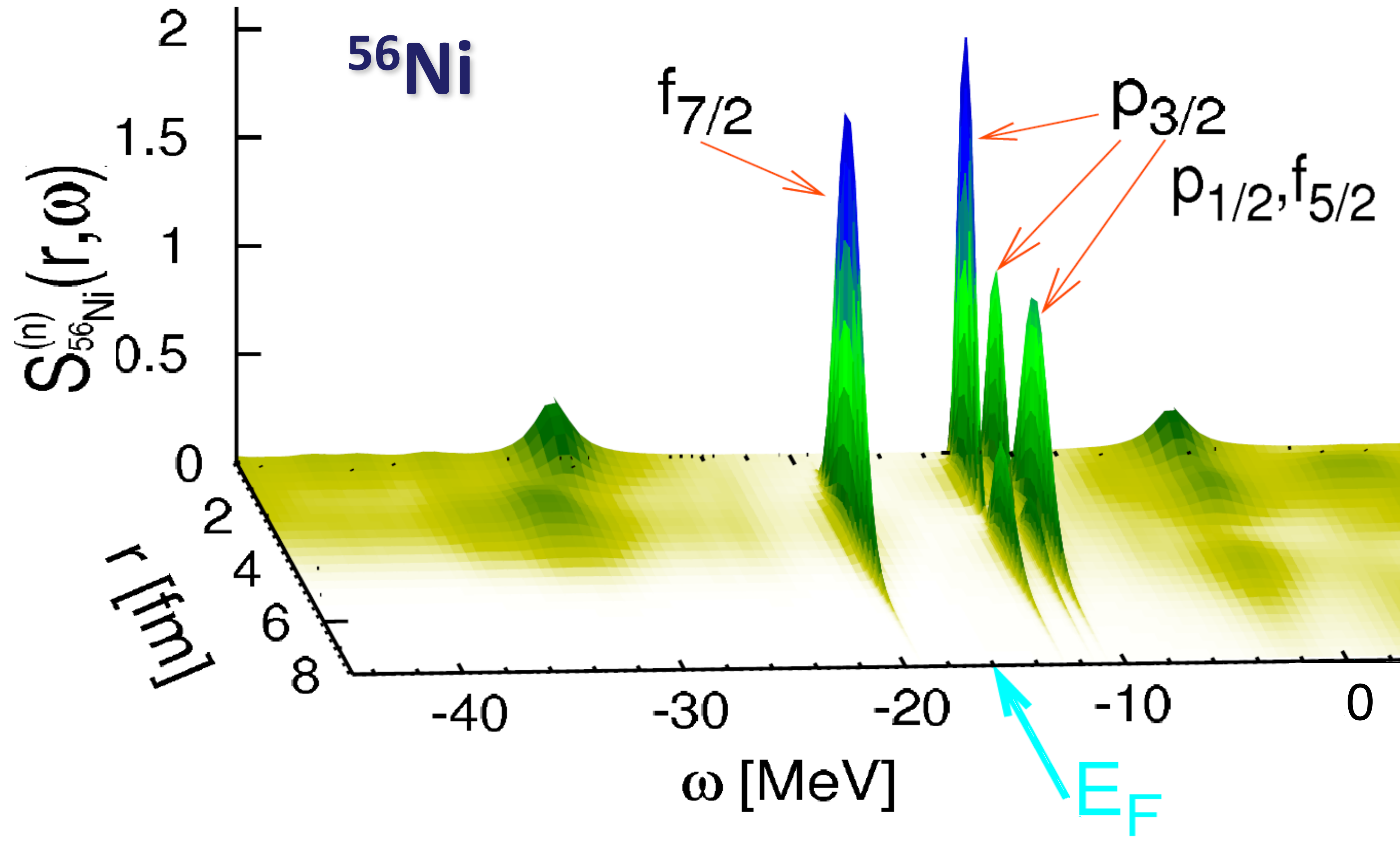
Elastic Antiproton-Nucleus Scattering from Chiral Forces

Matteo Vorabbi,[1,2] Michael Gennari,[2,3] Paolo Finelli,[4] Carlotta Giusti,[5] and Petr Navrátil[2]

PHYSICAL REVIEW LETTERS **124,** 162501 (2020)

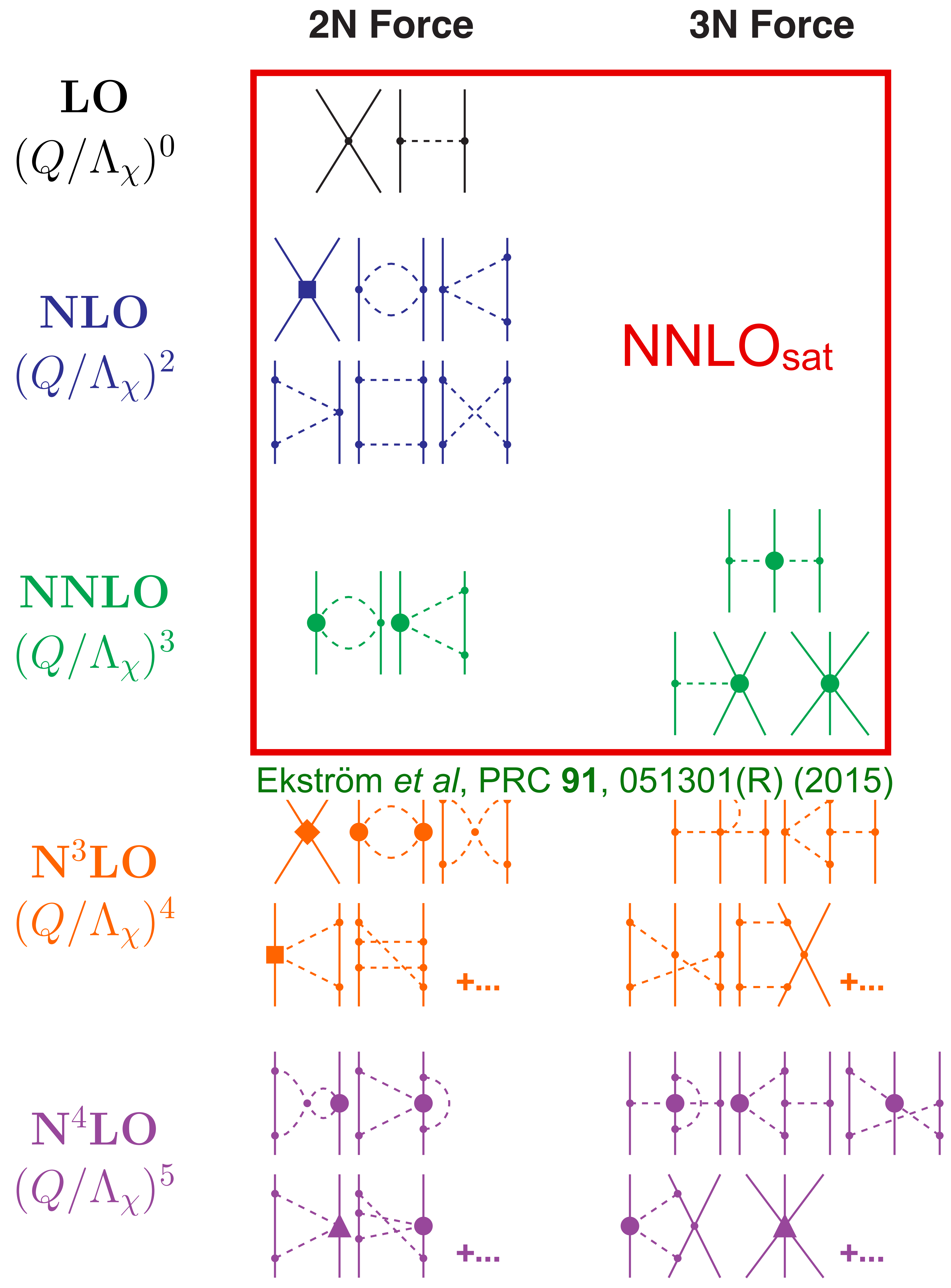
Extension to heavier nuclei

Self Consistent Green's Function (SCGF)



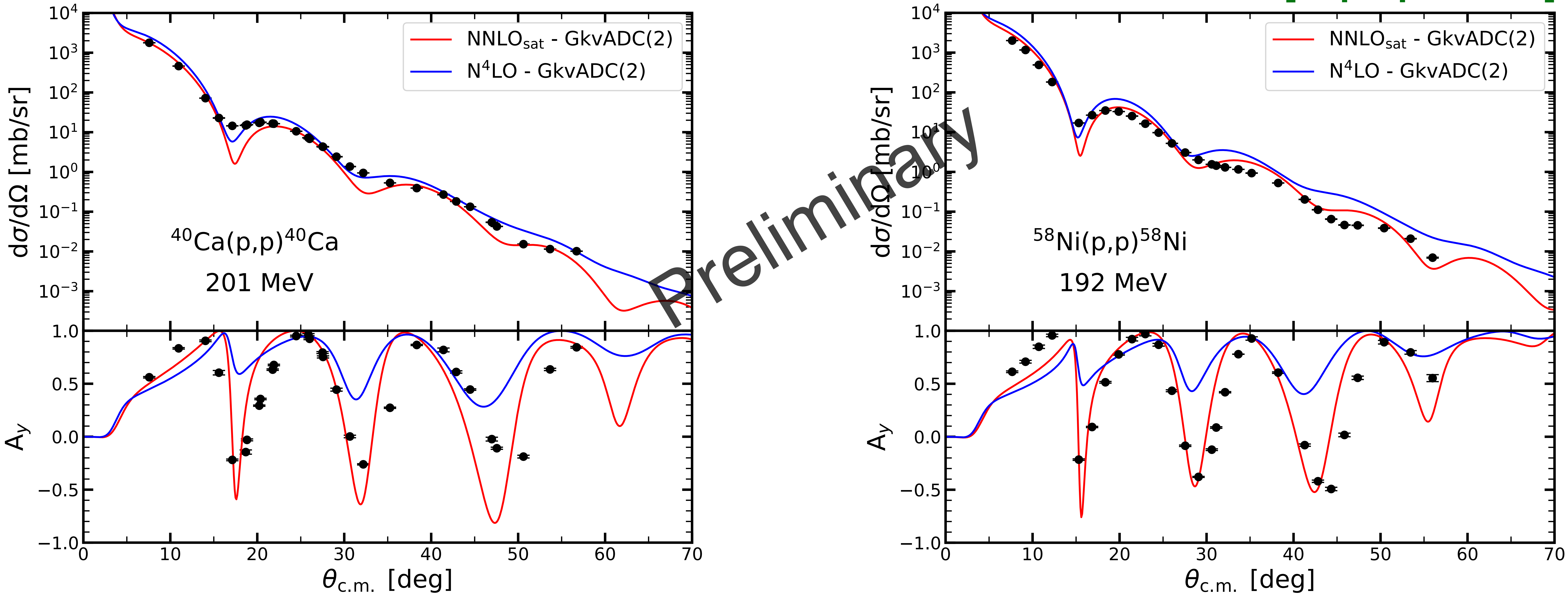
In collaboration with C. Barbieri (Milan) and V. Somà (Paris)
Somà, *SCGF Theory for Atomic Nuclei*, Frontiers **8** (2020) 340

$$U_{\mathbf{p}}(\boldsymbol{q}, \boldsymbol{K}) = \sum_{N=p,n} \int d\boldsymbol{P}\, \eta(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, t_{\mathbf{p}N}(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, \rho_N(\boldsymbol{q}, \boldsymbol{P})$$



Results with SCGF densities

[in preparation]



- First microscopic optical potential for calcium and nickel from *ab initio* densities
- Interesting results for different realistic interactions
- For this comparison the densities are always computed with the NNLO$_{sat}$

Inclusion of double scattering

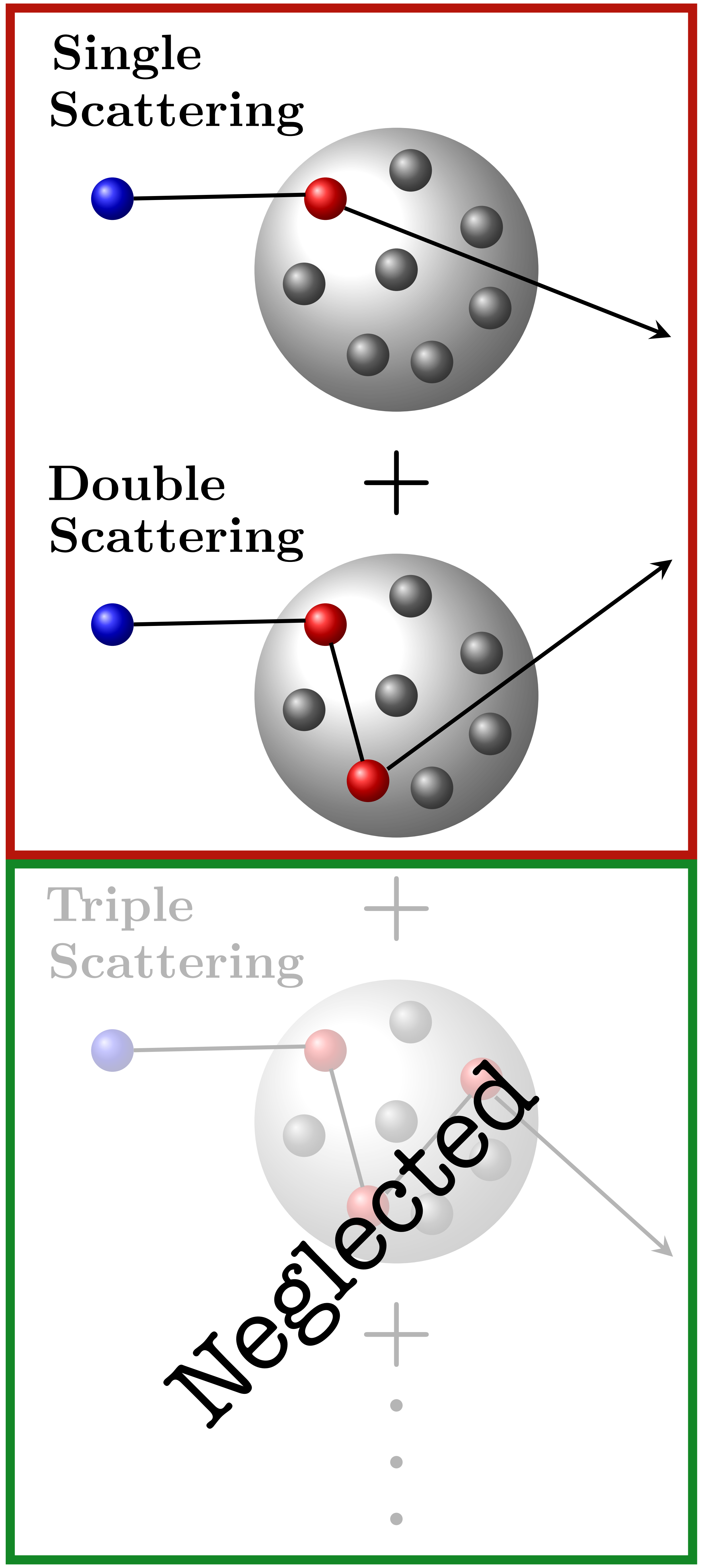
- Inclusion of the second-order term of the spectator expansion [Crespo *et al.*, PRC **46**, 279 (1992)]

$$U^{(2)} = \sum_{i=1}^{A} \tau_{0i} + \sum_{i,j \neq i}^{A} \tau_{0ij}$$

- Requires:
  1. Two-body density matrix (from NCSM)

$$\rho(\boldsymbol{r}'_1, \boldsymbol{r}'_2, \boldsymbol{r}_1, \boldsymbol{r}_2)$$

  2. Solution of the three-body scattering equation for τ_{0ij}



Distorted wave theory of inelastic scattering

[Picklesimer, Tandy, Thaler, Phys. Rev. C **25**, 1215 (1982)]
[Picklesimer, Tandy, Thaler, Phys. Rev. C **25**, 1233 (1982)]

The inelastic transition amplitude

$$T_{\text{inel}}(\boldsymbol{k}_*, \boldsymbol{k}_0) = \int d\boldsymbol{r}' \int d\boldsymbol{r}\, \psi^\dagger(\boldsymbol{k}_*, \boldsymbol{r}')\, U_{\text{tr}}(\boldsymbol{r}', \boldsymbol{r})\, \psi(\boldsymbol{k}_0, \boldsymbol{r})$$

Required potentials

$$U_{\text{ex}}(\boldsymbol{q}, \boldsymbol{K}) = \sum_{N=p,n} \int d\boldsymbol{P}\, \eta(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, t_{pN}(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, \rho_N^{(\text{ex})}(\boldsymbol{q}, \boldsymbol{P})$$

$$U_{\text{tr}}(\boldsymbol{q}, \boldsymbol{K}) = \sum_{N=p,n} \int d\boldsymbol{P}\, \eta(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, t_{pN}(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, \rho_N^{(\text{tr})}(\boldsymbol{q}, \boldsymbol{P})$$

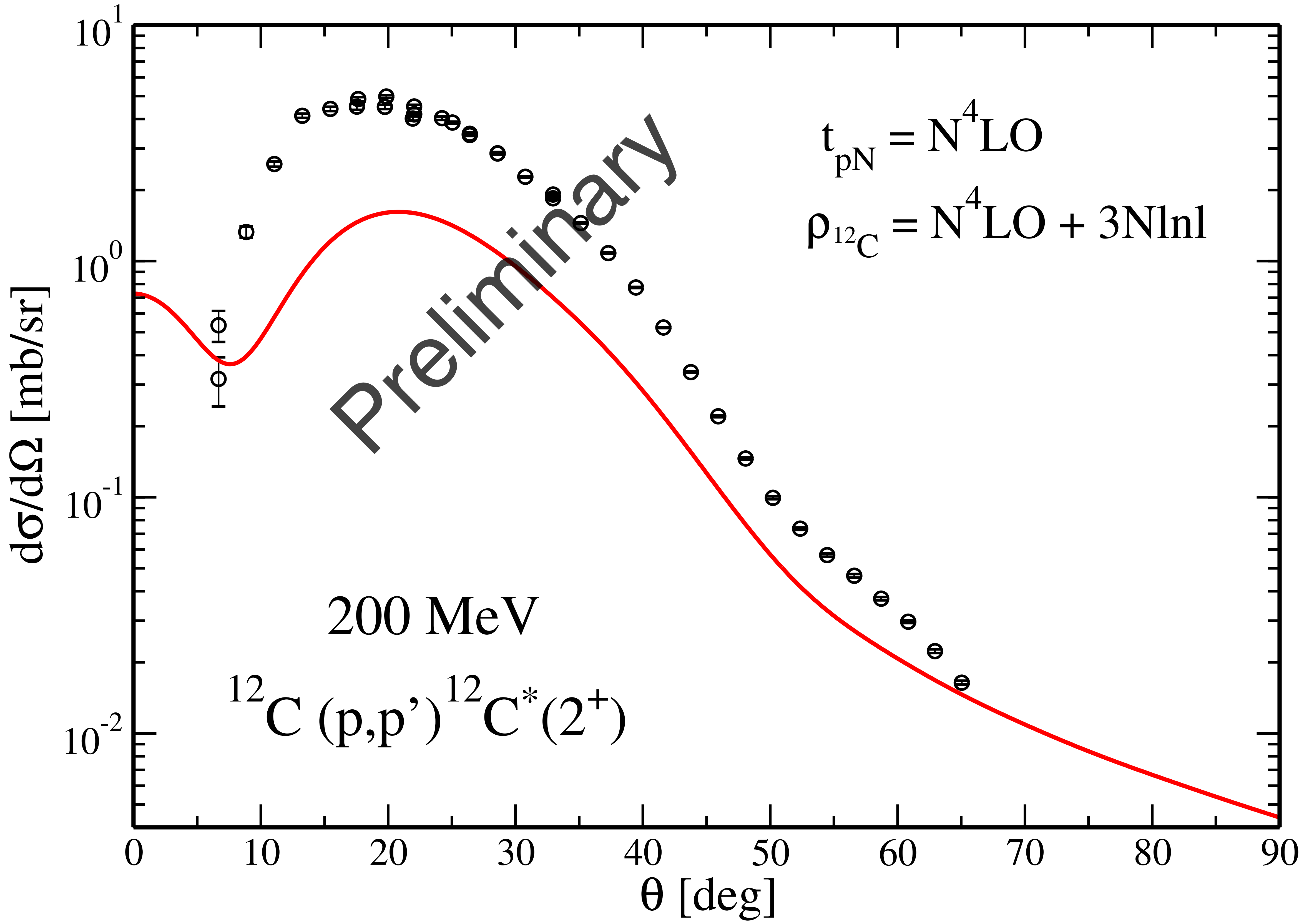
$$U_{\text{gs}}(\boldsymbol{q}, \boldsymbol{K}) = \sum_{N=p,n} \int d\boldsymbol{P}\, \eta(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, t_{pN}(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, \rho_N^{(\text{gs})}(\boldsymbol{q}, \boldsymbol{P})$$

Distorted wave theory of inelastic scattering

[Picklesimer, Tandy, Thaler, Phys. Rev. C **25**, 1215 (1982)]
[Picklesimer, Tandy, Thaler, Phys. Rev. C **25**, 1233 (1982)]

The inelastic transition amplitude

$$T_{\text{inel}}(\boldsymbol{k}_*, \boldsymbol{k}_0) = \int d\boldsymbol{r}' \int d\boldsymbol{r}\, \psi^\dagger(\boldsymbol{k}_*, \boldsymbol{r}')\, U_{\text{tr}}(\boldsymbol{r}', \boldsymbol{r})\, \psi(\boldsymbol{k}_0, \boldsymbol{r})$$



Summary & outlook

- The choice of the NN interaction is crucial to define the energy limits of applicability of the optical potential
- The 3N interaction has a sizeable effect on polarisation observables
- The extension to nonzero spin targets provides a good description of the data for stable and unstable nuclei

- Extend the high- and low-energy limits of applicability of the optical potential
- Inclusion of the second-order term of the spectator expansion
- Consistent treatment of the full 3N interaction
- Development of a coupled-channel approach
- Evaluation of theoretical uncertainties