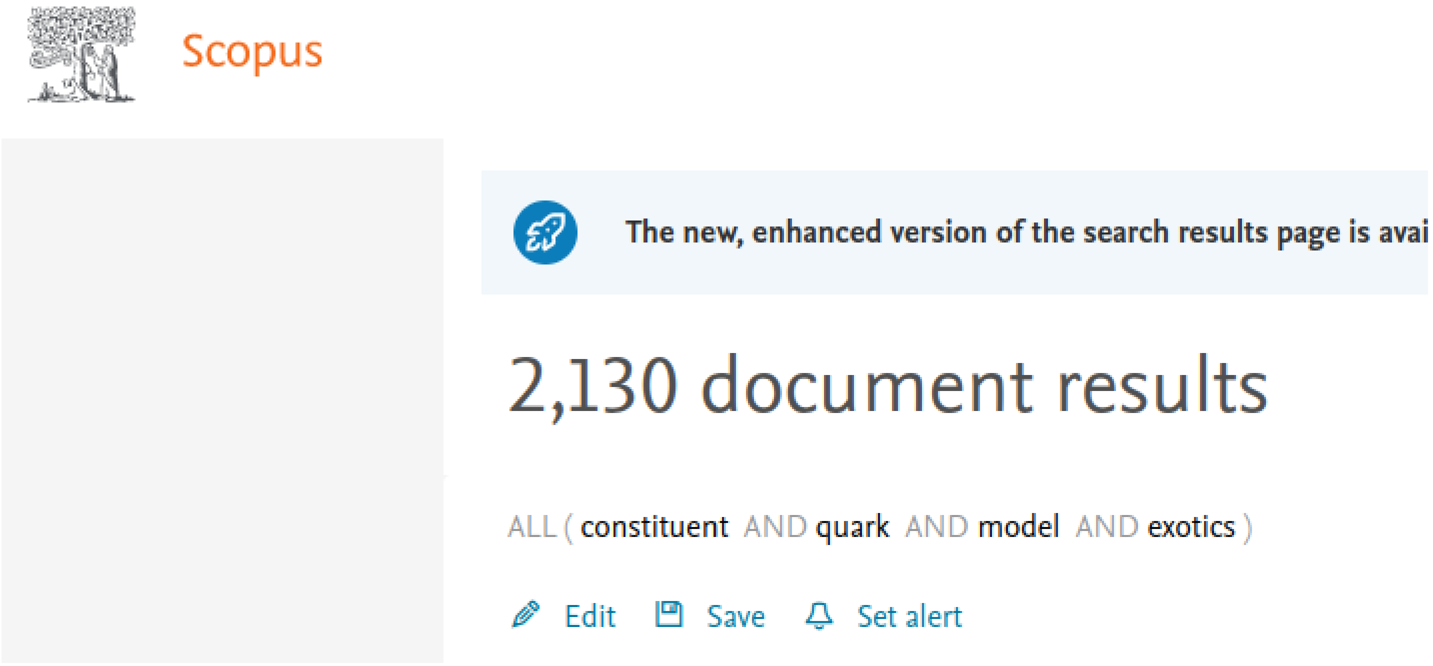
25th European Conference of Few-Body Problems in Physics

Mainz (Germany), July 30th-August 4th, 2023

Constituent quark model.
What have we learned about exotic states?

J. Vijande PhD
University of Valencia, Spain

Exotic states in the constituent quark model

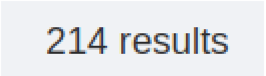


Scopus

The new, enhanced version of the search results page is avai

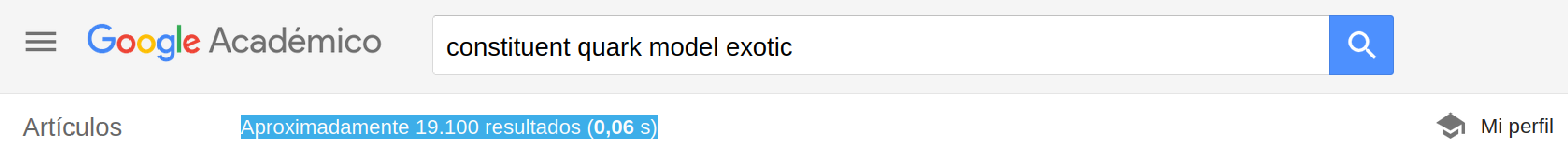
2,130 document results

ALL (constituent AND quark AND model AND exotics)



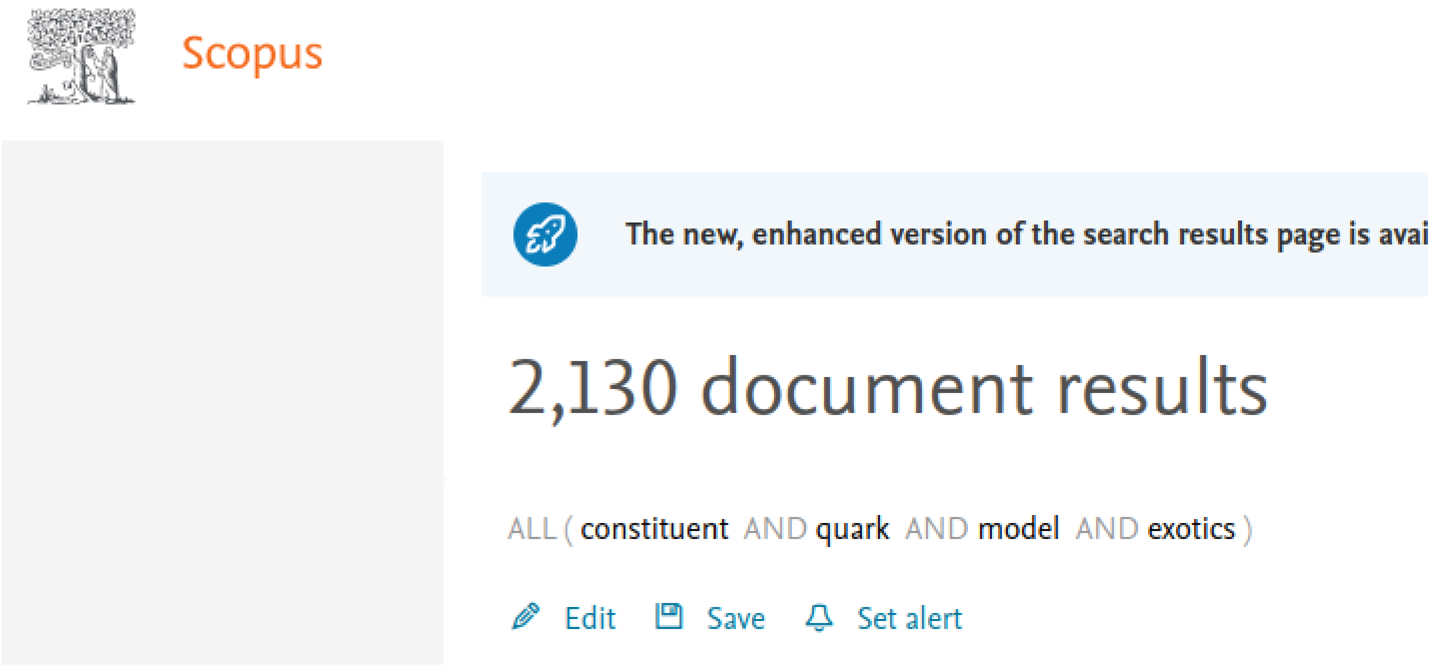
214 results

Exotic states in the constituent quark model



constituent quark model exotic

Aproximadamente 19.100 resultados (**0,06** s)



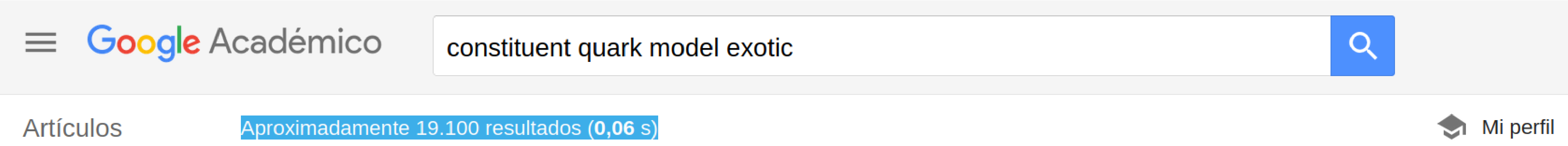
The new, enhanced version of the search results page is avai

2,130 document results

ALL (constituent AND quark AND model AND exotics)

214 results

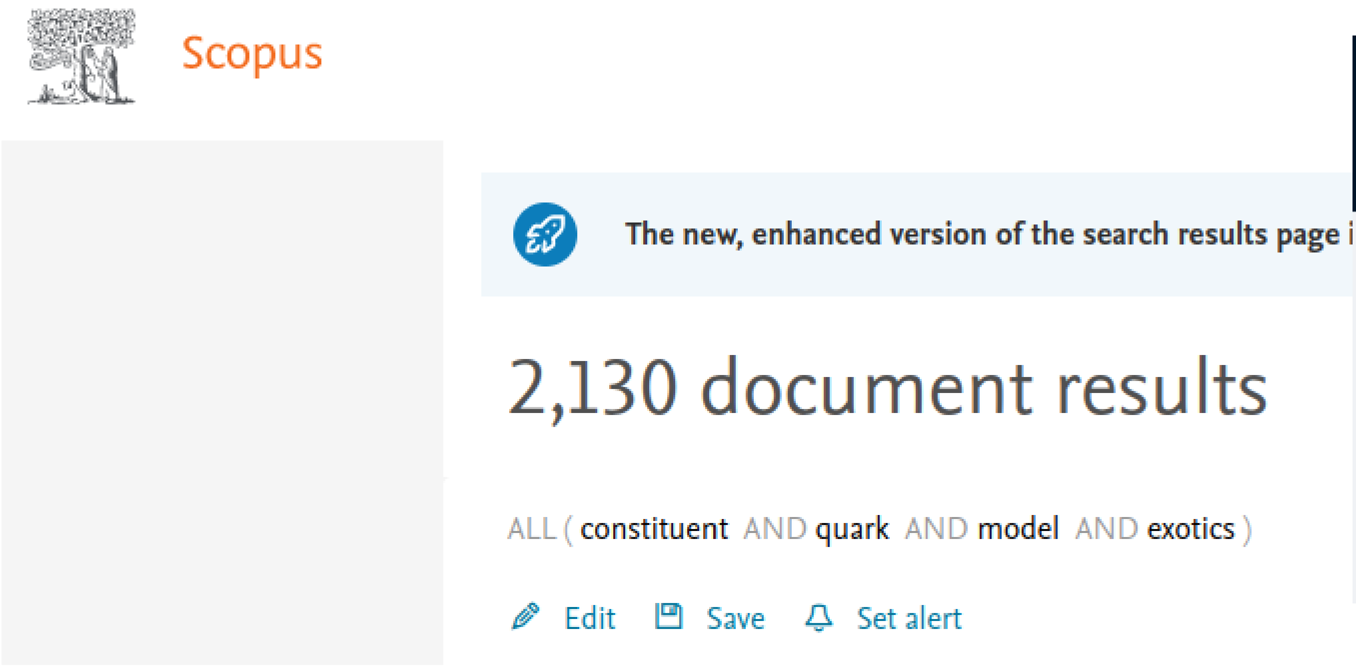
Exotic states in the constituent quark model



constituent quark model exotic

Artículos

Aproximadamente 19.100 resultados (**0,06** s)



The new, enhanced version of the search results page i

2,130 document results

ALL (constituent AND quark AND model AND exotics)

constituent quark model exotic

214 results

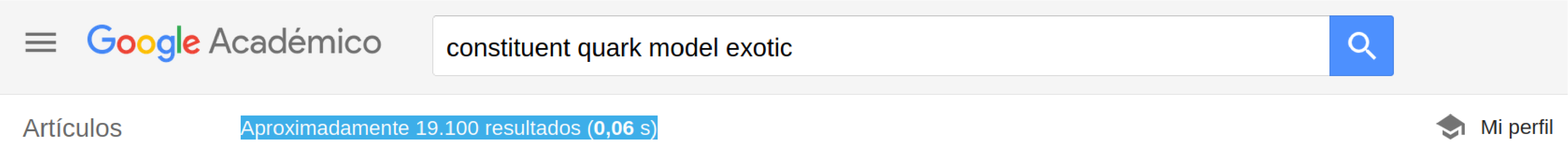
Citation Summary

☐ Exclude self-citations

Papers

Citations

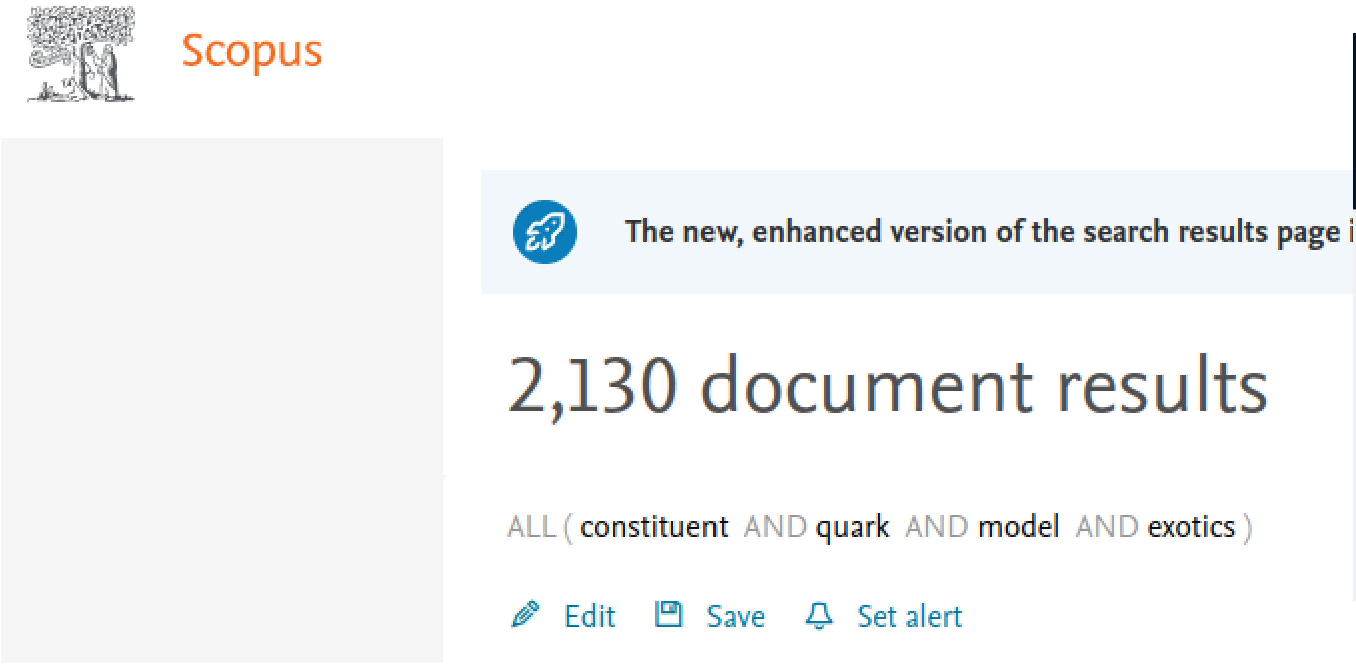
Exotic states in the constituent quark model



constituent quark model exotic

Artículos

Aproximadamente 19.100 resultados (**0,06** s)



The new, enhanced version of the search results page i

2,130 document results

ALL (constituent AND quark AND model AND exotics)

constituent quark model exotic

Date of paper

214 results

Citation Summary

☐ Exclude self-citations

Papers

Citations

This in **not** a thorough review of the field, so I probably not quote your favorite paper.

Exotic states in the constituent quark model

- And second, I will mostly focus this talk on what can we can learn from the constituent quark model approach for exotic states from the double heavy four-quark sector.

- Topics I will not cover
  - QCD sum rules.
  - Lattice QCD.
  - Dinamically generated resonances.
  - Phenomenological mass-based relations.
  - etc…

25th European Conference of
Few-Body Problems in Physics

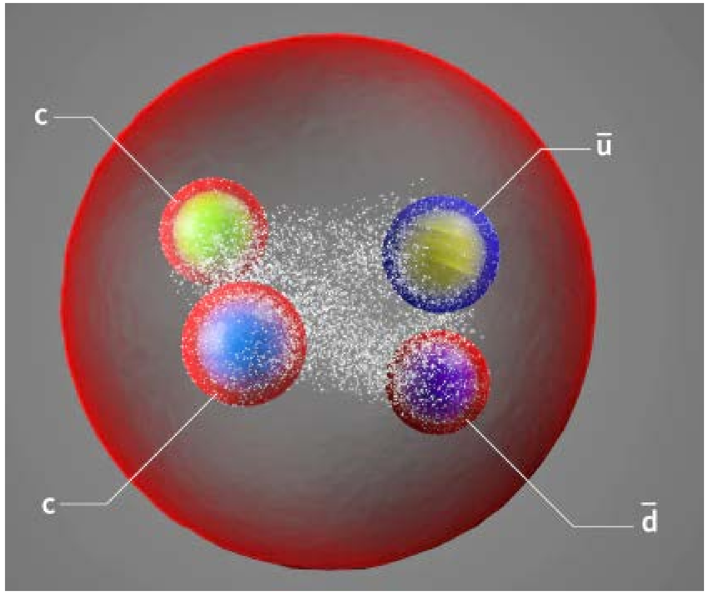
Mainz (Germany), July 30th-August 4th, 2023

25 EFB

Constituent quark model.
What I have learned from exotic systems thanks to the study of double heavy tetraquarks

J. Vijande PhD
University of Valencia, Spain

The basics

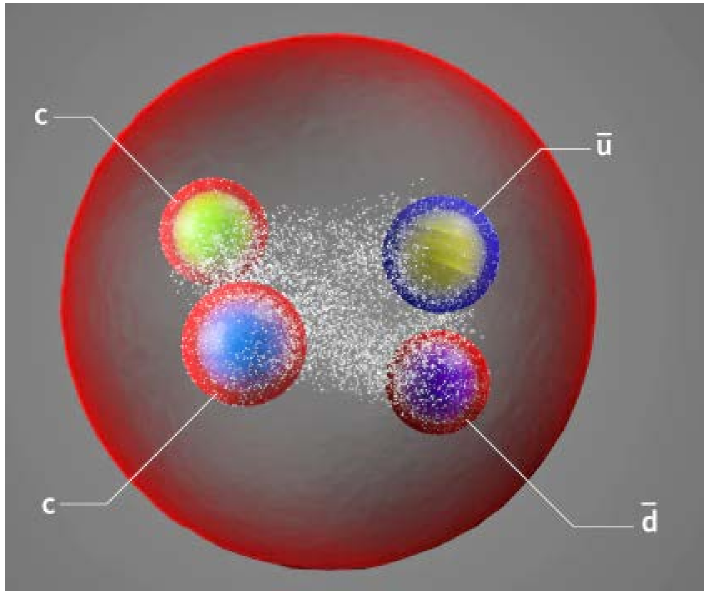


- The constituent quark model have (probably surprisingly) described rather well mesons and baryons as composite objects made of constituent valence quarks

$$m_c \approx 1.3\,GeV \qquad m_b \approx 5\,GeV \qquad m_u \approx 0.3\,GeV$$

interacting by means of a potential, normally pairwise, but not always.

The basics



- The constituent quark model have (probably surprisingly) described rather well mesons and baryons as composite objects made of constituent valence quarks

$$m_c \approx 1.3\,GeV \qquad m_b \approx 5\,GeV \qquad m_u \approx 0.3\,GeV$$

interacting by means of a potential, normally pairwise, but not always.

- A four-quark state is the simplest object with a non-trivial color structure.

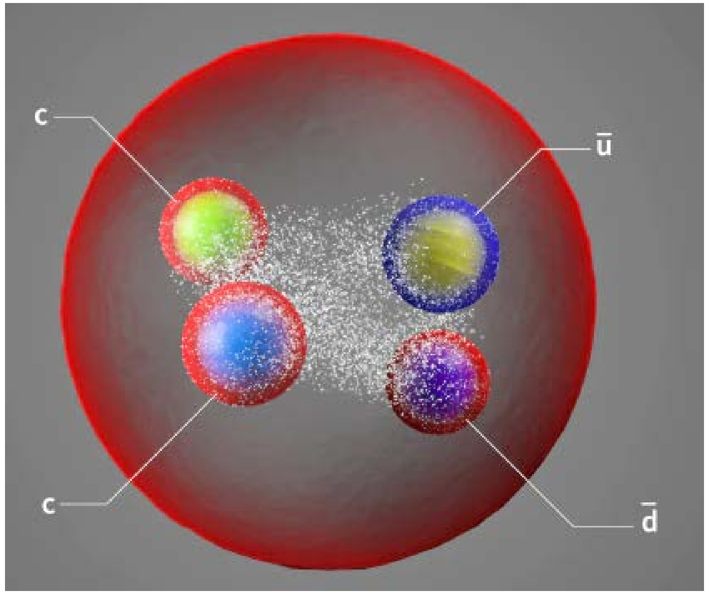
$$\boxed{3 \otimes \bar{3} = 1}$$

$$\boxed{\begin{array}{l} 3 \otimes 3 \otimes \bar{3} \otimes \bar{3} \to \\ \begin{cases} \bar{3}_{qq} \otimes 3_{\bar{q}\bar{q}} = 1 \\ 6_{qq} \otimes \bar{6}_{\bar{q}\bar{q}} = 1 \end{cases} \end{array}}$$

$$\boxed{\begin{array}{l} 3 \otimes 3 \otimes 3 \otimes \bar{3} \otimes \bar{3} \otimes \bar{3} \to \\ \begin{cases} 1_{q\bar{q}} \otimes 1_{q\bar{q}} \otimes 1_{q\bar{q}} = 1 & 1_{q\bar{q}} \otimes 8_{q\bar{q}} \otimes 8_{q\bar{q}} = 1 \\ 8_{q\bar{q}} \otimes 1_{q\bar{q}} \otimes 8_{q\bar{q}} = 1 & 8_{q\bar{q}} \otimes 8_{q\bar{q}} \otimes 1_{q\bar{q}} = 1 \\ 8_{q\bar{q}} \otimes 8_{q\bar{q}} \otimes 8_{q\bar{q}} = 1 & 8_{q\bar{q}} \otimes 8_{q\bar{q}} \otimes 8_{q\bar{q}} = 1 \end{cases} \end{array}}$$

$$|meson\rangle = \alpha_1 |q\bar{q}\rangle + \alpha_2 |qq\bar{q}\bar{q}\rangle + \alpha_3 |qqq\bar{q}\bar{q}\bar{q}\rangle + \ldots$$

The basics



- The constituent quark model have (probably surprisingly) described rather well mesons and baryons as composite objects made of constituent valence quarks

$$m_c \approx 1.3\,GeV \qquad m_b \approx 5\,GeV \qquad m_u \approx 0.3\,GeV$$

interacting by means of a potential, normally pairwise, but not always.

$$\boxed{3\otimes 3\otimes \bar{3}\otimes \bar{3}\rightarrow \begin{cases}\bar{3}_{qq}\otimes 3_{\bar{q}\bar{q}}=1\\ 6_{qq}\otimes \bar{6}_{\bar{q}\bar{q}}=1\end{cases}}$$

$$\boxed{3\otimes 3\otimes 3\otimes \bar{3}\otimes \bar{3}\otimes \bar{3}\rightarrow \begin{cases}1_{q\bar{q}}\otimes 1_{q\bar{q}}\otimes 1_{q\bar{q}}=1 & 1_{q\bar{q}}\otimes 8_{q\bar{q}}\otimes 8_{q\bar{q}}=1\\ 8_{q\bar{q}}\otimes 1_{q\bar{q}}\otimes 8_{q\bar{q}}=1 & 8_{q\bar{q}}\otimes 8_{q\bar{q}}\otimes 1_{q\bar{q}}=1\\ 8_{q\bar{q}}\otimes 8_{q\bar{q}}\otimes 8_{q\bar{q}}=1 & 8_{q\bar{q}}\otimes 8_{q\bar{q}}\otimes 8_{q\bar{q}}=1\end{cases}}$$

But for charm equal ±2 →

$$\left|T_{cc}\right\rangle = \alpha_2\left|cc\overline{qq}\right\rangle + \alpha_3\left|ccq\overline{qqq}\right\rangle + \ldots$$

We are in 2023 and…

There are many experimental results, but I will quote just one for now:

We are in 2023 and…

There are many experimental results, but I will quote just one for now:

nature communications

nature > nature communications > articles > article

Article | Open Access | Published: 16 June 2022

Study of the doubly charmed tetraquark T_{cc}^{+}

LHCb collaboration

Nature Communications **13**, Article number: 3351 (2022)

nature physics

nature > nature physics > letters > article

Letter | Open Access | Published: 16 June 2022

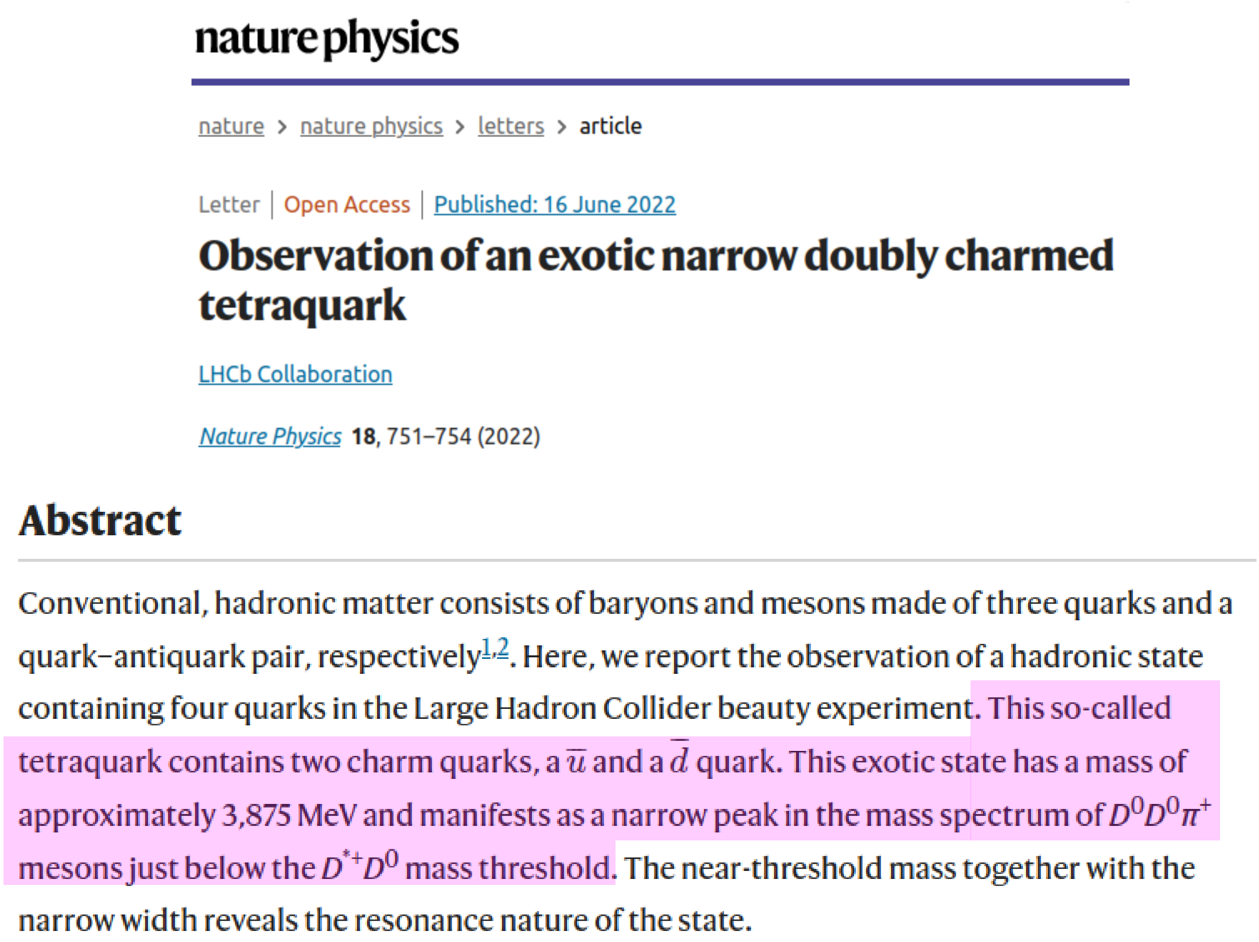
Observation of an exotic narrow doubly charmed tetraquark

LHCb Collaboration

Nature Physics **18**, 751–754 (2022)

We are in 2023 and…

There are many experimental results, but I will quote just one for now:



nature physics

nature > nature physics > letters > article

Letter | Open Access | Published: 16 June 2022

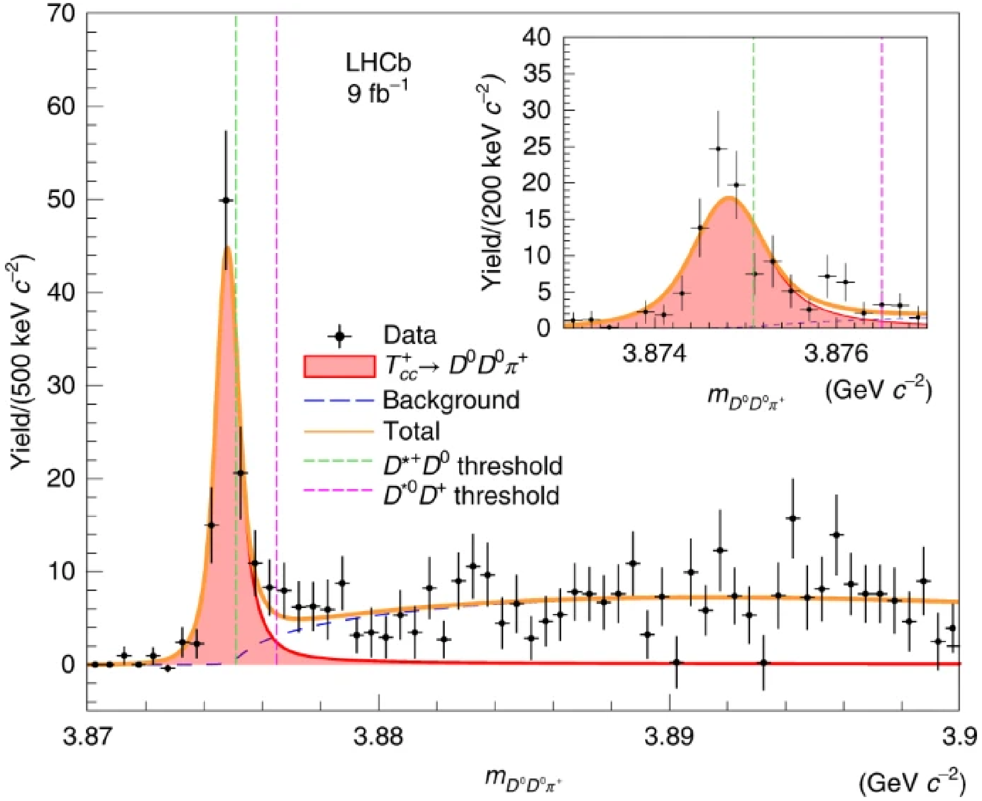
Observation of an exotic narrow doubly charmed tetraquark

LHCb Collaboration

Nature Physics **18**, 751–754 (2022)

Abstract

Conventional, hadronic matter consists of baryons and mesons made of three quarks and a quark–antiquark pair, respectively[1,2]. Here, we report the observation of a hadronic state containing four quarks in the Large Hadron Collider beauty experiment. This so-called tetraquark contains two charm quarks, a $\overline{u}$ and a $\overline{d}$ quark. This exotic state has a mass of approximately 3,875 MeV and manifests as a narrow peak in the mass spectrum of $D^0D^0\pi^+$ mesons just below the $D^{*+}D^0$ mass threshold. The near-threshold mass together with the narrow width reveals the resonance nature of the state.



In the beginning. The year 1982

In the beginning. The year 1982

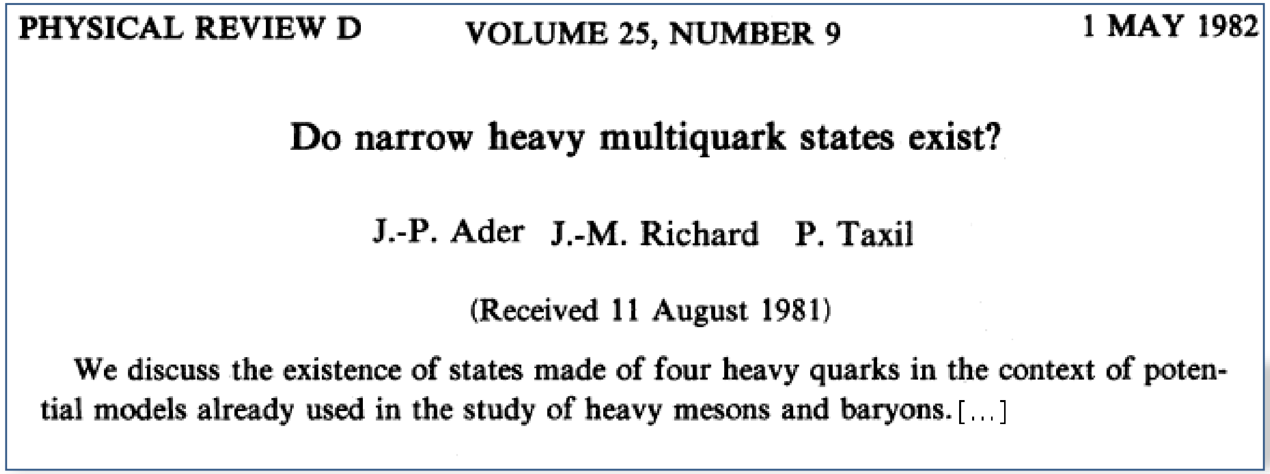
In the beginning. The year 1982

In the beginning. The year 1982

In the beginning. The year 1982

In the beginning. The year 1982

In the beginning. 40 years ago in the year 1982



PHYSICAL REVIEW D VOLUME 25, NUMBER 9 1 MAY 1982

Do narrow heavy multiquark states exist?

J.-P. Ader J.-M. Richard P. Taxil

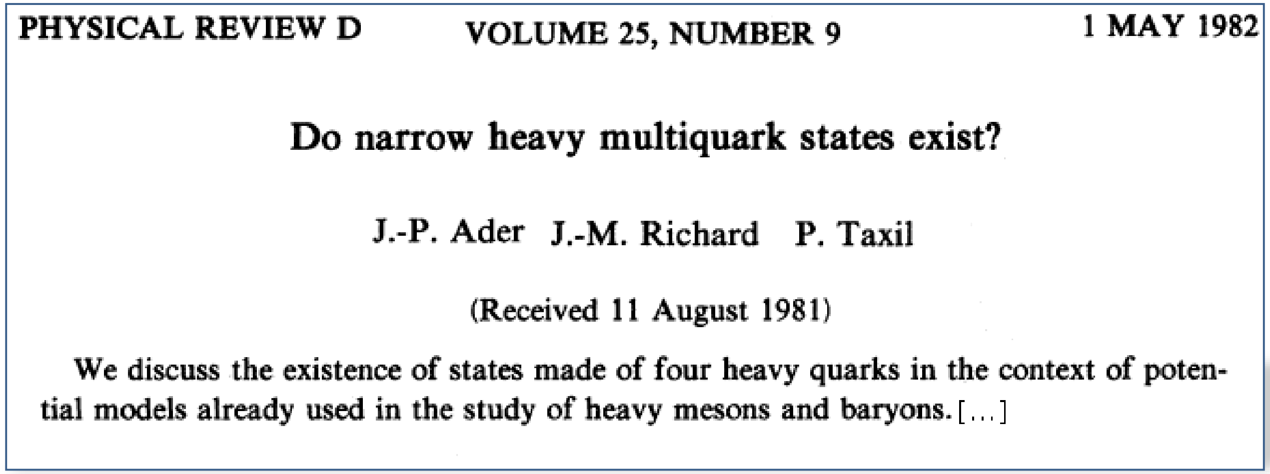
(Received 11 August 1981)

We discuss the existence of states made of four heavy quarks in the context of potential models already used in the study of heavy mesons and baryons. [...]

$cc\bar{c}\bar{c}$ $cu\bar{c}\bar{d}$

$cc\bar{u}\bar{d}$

In the beginning. 40 years ago in the year 1982



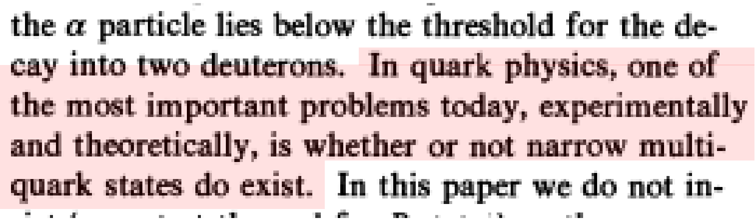
PHYSICAL REVIEW D VOLUME 25, NUMBER 9 1 MAY 1982

Do narrow heavy multiquark states exist?

J.-P. Ader J.-M. Richard P. Taxil

(Received 11 August 1981)

We discuss the existence of states made of four heavy quarks in the context of potential models already used in the study of heavy mesons and baryons. [...]

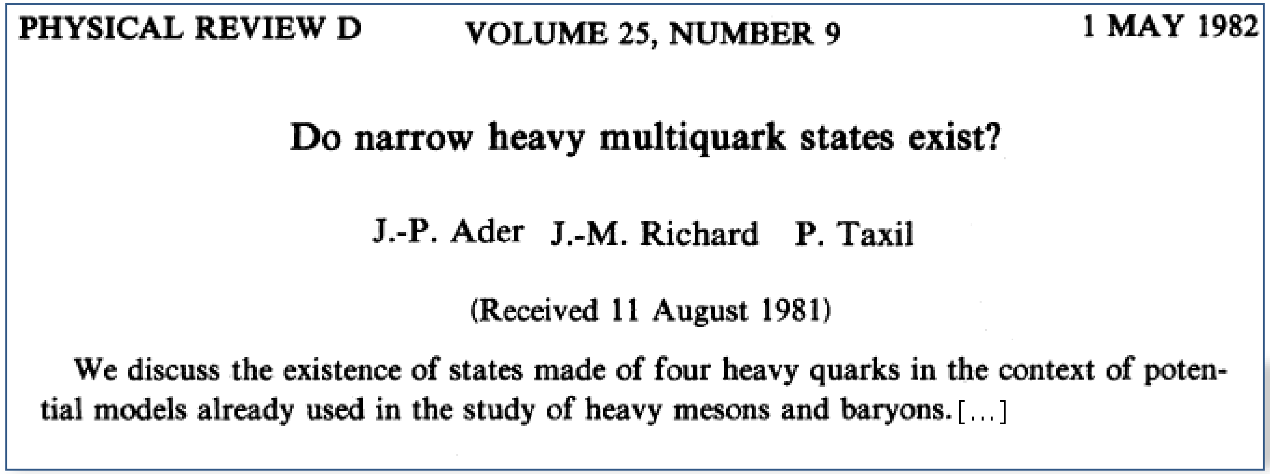


the α particle lies below the threshold for the decay into two deuterons. In quark physics, one of the most important problems today, experimentally and theoretically, is whether or not narrow multiquark states do exist. In this paper we do not in-

$cc\bar{c}\bar{c}$ $cu\bar{c}\bar{d}$

$cc\bar{u}\bar{d}$

In the beginning. 40 years ago in the year 1982



PHYSICAL REVIEW D VOLUME 25, NUMBER 9 1 MAY 1982

Do narrow heavy multiquark states exist?

J.-P. Ader J.-M. Richard P. Taxil

(Received 11 August 1981)

We discuss the existence of states made of four heavy quarks in the context of potential models already used in the study of heavy mesons and baryons. [...]

$cc\bar{c}\bar{c}$ $cu\bar{c}\bar{d}$

$cc\bar{u}\bar{d}$

The authors consider **linear+coulomb** and **power-law** potentials and a **variational approach** using a harmonic oscillator wave function .

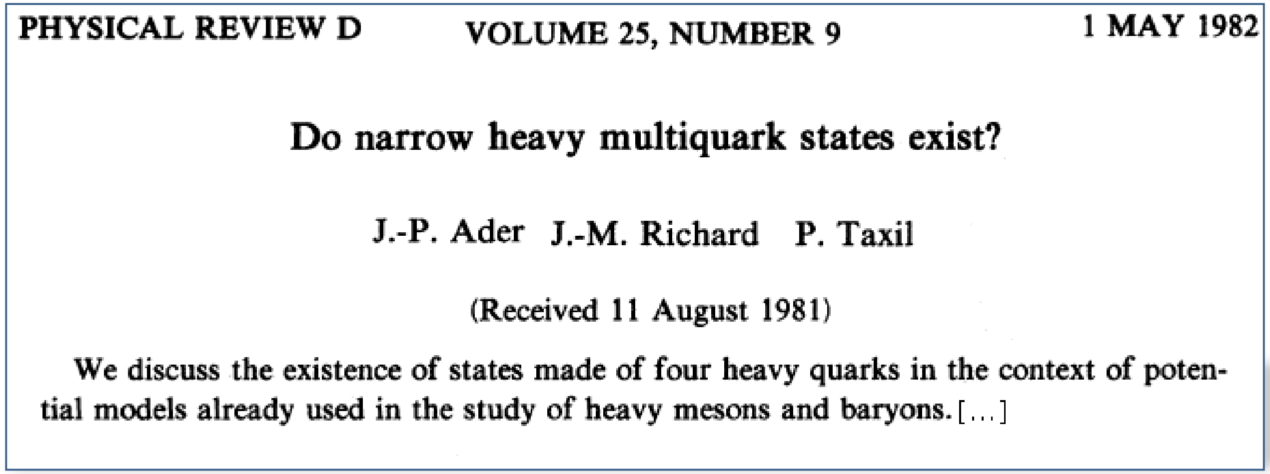
More complex options are included for the all-heavy four-quark states (chromomagnetic interaction, bag model, negative parity states, etc…)

$$V^{\mathrm{I}}_{Q\bar{Q}}(r) = -\frac{16}{3} V^{\mathrm{I}}_{8}(r)$$

$$= -\frac{4}{3}\frac{\alpha_s}{r} + \lambda r ,$$

$$V^{\mathrm{II}}_{Q\bar{Q}}(r) = -\frac{16}{3} V^{\mathrm{II}}_{8}(r) = A + Br^{\beta} .$$

In the beginning. 40 years ago in the year 1982


PHYSICAL REVIEW D VOLUME 25, NUMBER 9 1 MAY 1982

Do narrow heavy multiquark states exist?

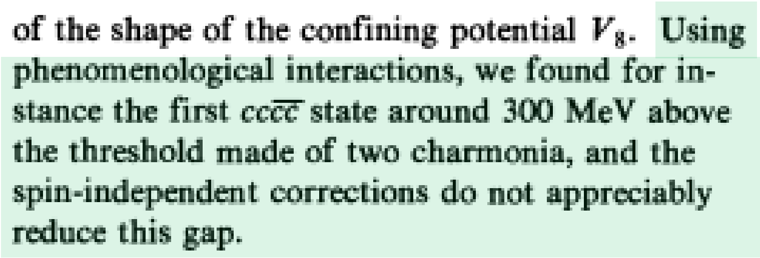
J.-P. Ader J.-M. Richard P. Taxil

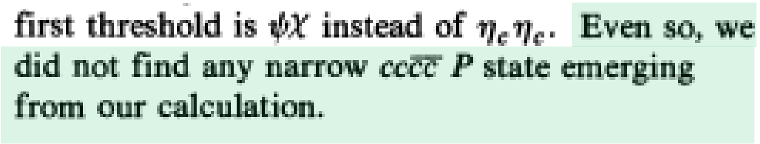
(Received 11 August 1981)

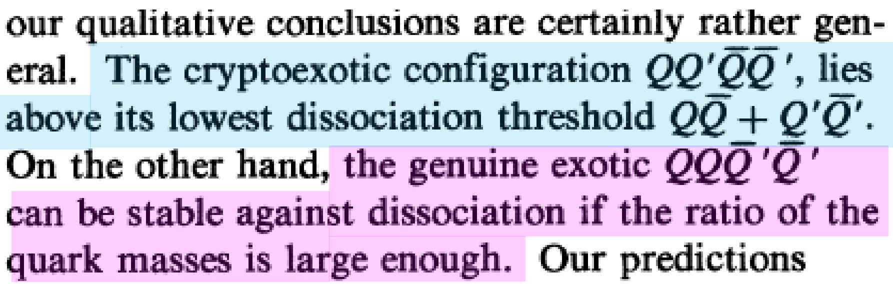
We discuss the existence of states made of four heavy quarks in the context of potential models already used in the study of heavy mesons and baryons. [...]

$\cancel{cc\bar{c}\bar{c}}$ $\cancel{cu\bar{c}\bar{d}}$

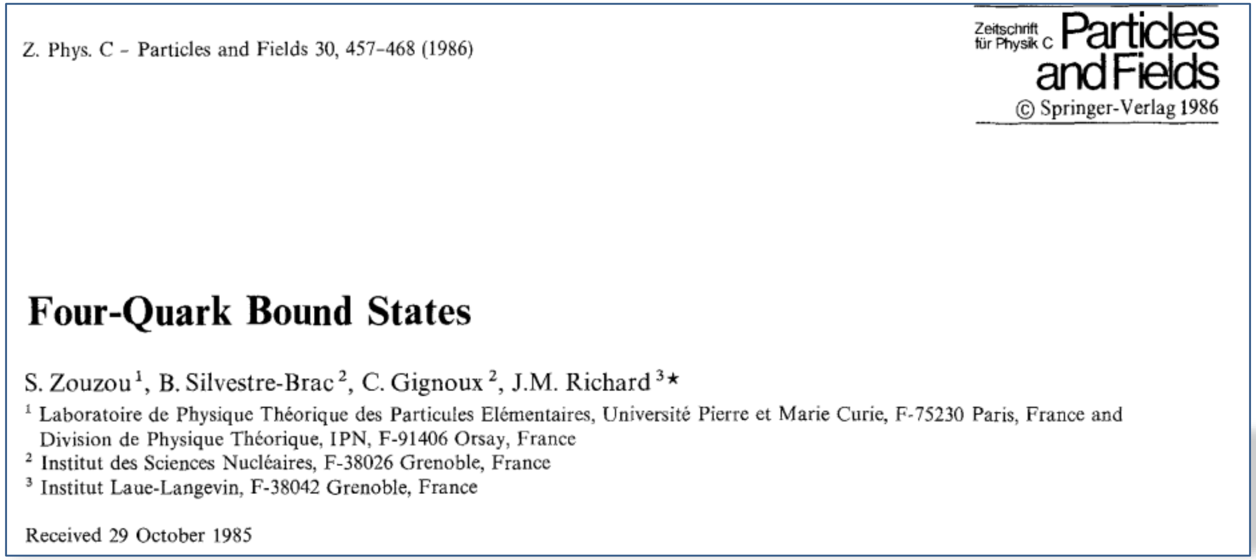
$cc\bar{u}\bar{d}$


of the shape of the confining potential V_8. Using phenomenological interactions, we found for instance the first $cc\bar{c}\bar{c}$ state around 300 MeV above the threshold made of two charmonia, and the spin-independent corrections do not appreciably reduce this gap.


first threshold is ψX instead of $\eta_c\eta_c$. Even so, we did not find any narrow $cc\bar{c}\bar{c}$ P state emerging from our calculation.


our qualitative conclusions are certainly rather general. The cryptoexotic configuration $QQ'\bar{Q}\bar{Q}'$, lies above its lowest dissociation threshold $Q\bar{Q}+Q'\bar{Q}'$. On the other hand, the genuine exotic $QQ\bar{Q}'\bar{Q}'$ can be stable against dissociation if the ratio of the quark masses is large enough. Our predictions

Exploring numerical methods



Z. Phys. C – Particles and Fields 30, 457–468 (1986)

Zeitschrift für Physik C Particles and Fields
© Springer-Verlag 1986

Four-Quark Bound States

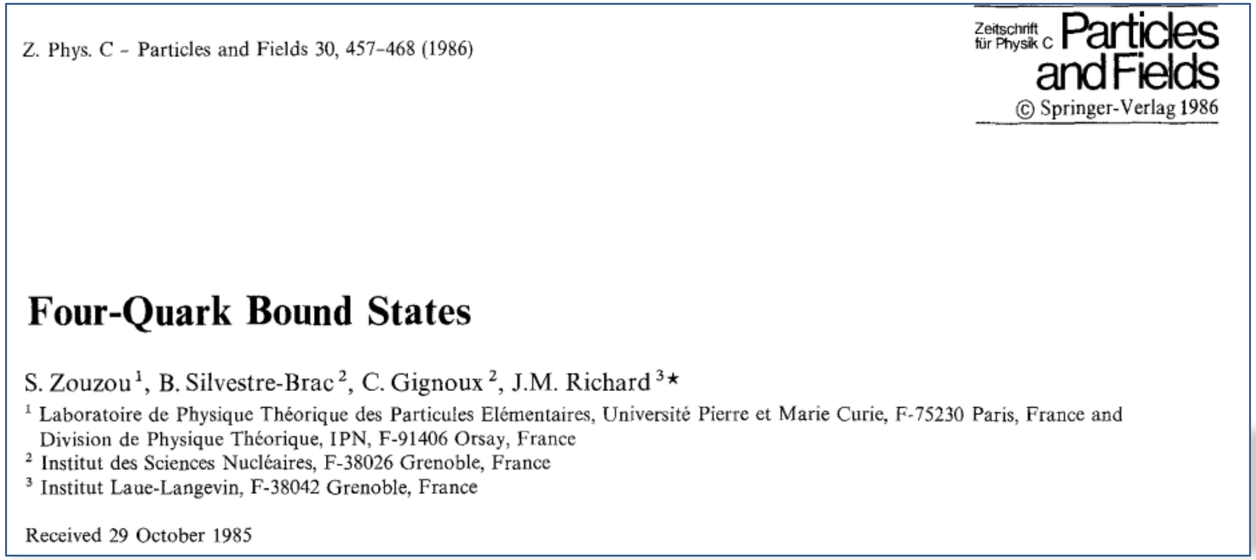
S. Zouzou[1], B. Silvestre-Brac[2], C. Gignoux[2], J.M. Richard[3]*

[1] Laboratoire de Physique Théorique des Particules Elémentaires, Université Pierre et Marie Curie, F-75230 Paris, France and Division de Physique Théorique, IPN, F-91406 Orsay, France
[2] Institut des Sciences Nucléaires, F-38026 Grenoble, France
[3] Institut Laue-Langevin, F-38042 Grenoble, France

Received 29 October 1985

The authors search bound states with central forces only, by comparing **three methods**: a gaussian parametrization of the wave-function, the harmonic oscillator expansion and the hyperspherical expansion. They include **spin-spin** terms and virtual **meson-meson configurations**.

Exploring numerical methods



Z. Phys. C – Particles and Fields 30, 457–468 (1986)

Zeitschrift für Physik C Particles and Fields

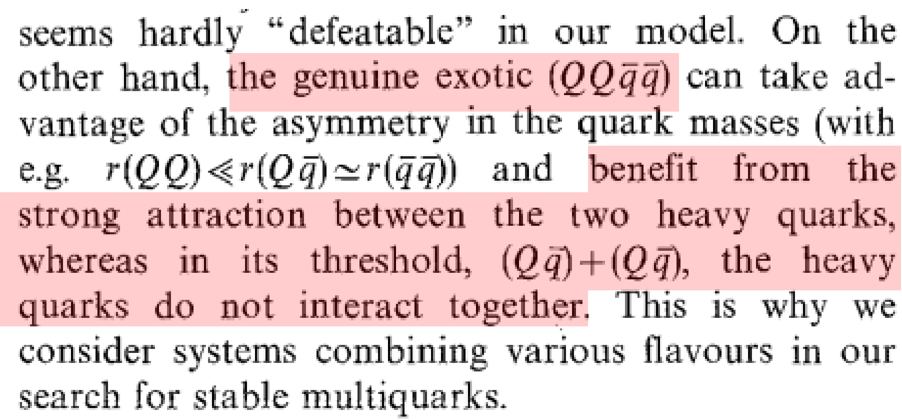
© Springer-Verlag 1986

Four-Quark Bound States

S. Zouzou[1], B. Silvestre-Brac[2], C. Gignoux[2], J.M. Richard[3]*

[1] Laboratoire de Physique Théorique des Particules Elémentaires, Université Pierre et Marie Curie, F-75230 Paris, France and Division de Physique Théorique, IPN, F-91406 Orsay, France
[2] Institut des Sciences Nucléaires, F-38026 Grenoble, France
[3] Institut Laue-Langevin, F-38042 Grenoble, France

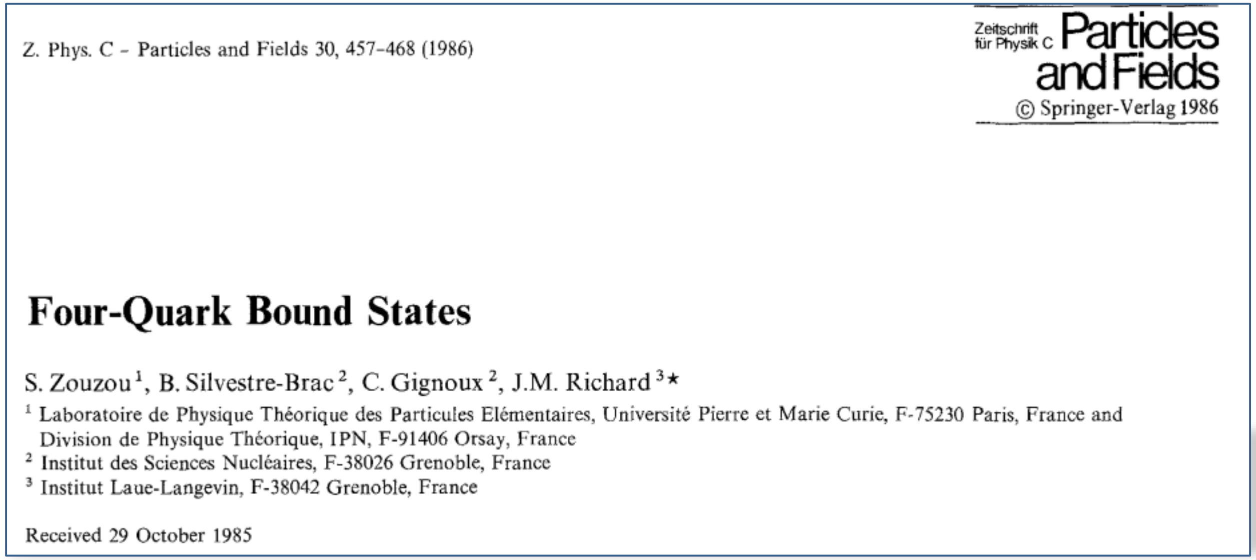
Received 29 October 1985



seems hardly "defeatable" in our model. On the other hand, the genuine exotic $(QQ\bar{q}\bar{q})$ can take advantage of the asymmetry in the quark masses (with e.g. $r(QQ) \ll r(Q\bar{q}) \simeq r(\bar{q}\bar{q})$) and benefit from the strong attraction between the two heavy quarks, whereas in its threshold, $(Q\bar{q})+(Q\bar{q})$, the heavy quarks do not interact together. This is why we consider systems combining various flavours in our search for stable multiquarks.

The authors search bound states with central forces only, by comparing **three methods**: a gaussian parametrization of the wave-function, the harmonic oscillator expansion and the hyperspherical expansion. They include **spin-spin** terms and virtual **meson-meson configurations**.

Exploring numerical methods



Z. Phys. C – Particles and Fields 30, 457–468 (1986)

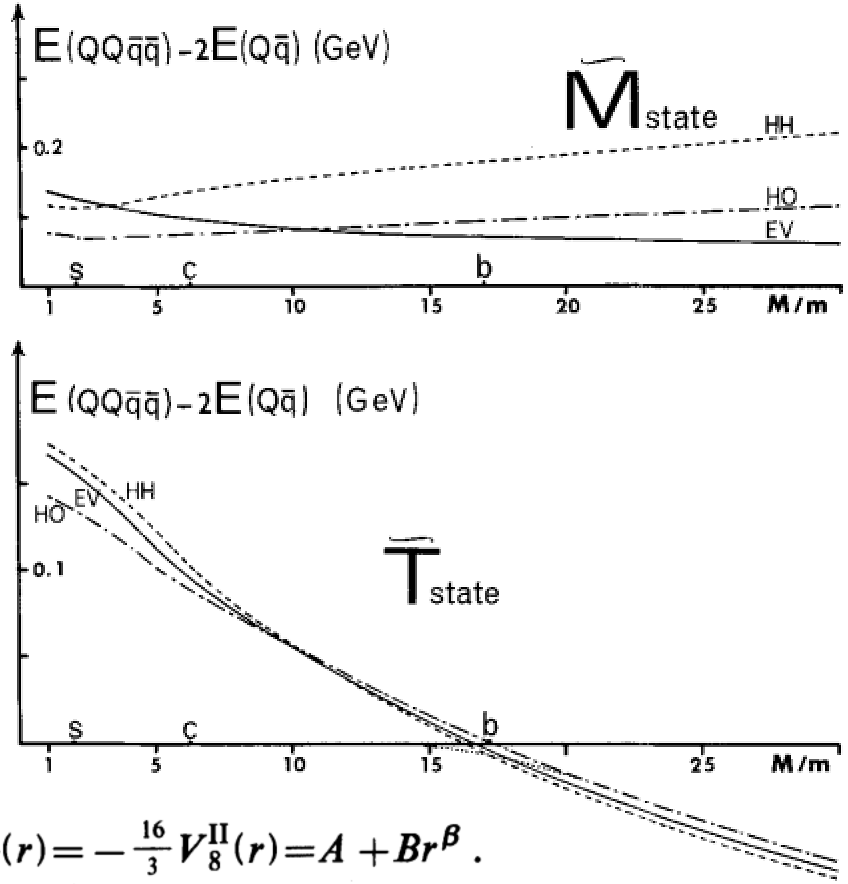
Zeitschrift für Physik C Particles and Fields
© Springer-Verlag 1986

Four-Quark Bound States

S. Zouzou[1], B. Silvestre-Brac[2], C. Gignoux[2], J.M. Richard[3]*

[1] Laboratoire de Physique Théorique des Particules Elémentaires, Université Pierre et Marie Curie, F-75230 Paris, France and Division de Physique Théorique, IPN, F-91406 Orsay, France
[2] Institut des Sciences Nucléaires, F-38026 Grenoble, France
[3] Institut Laue-Langevin, F-38042 Grenoble, France

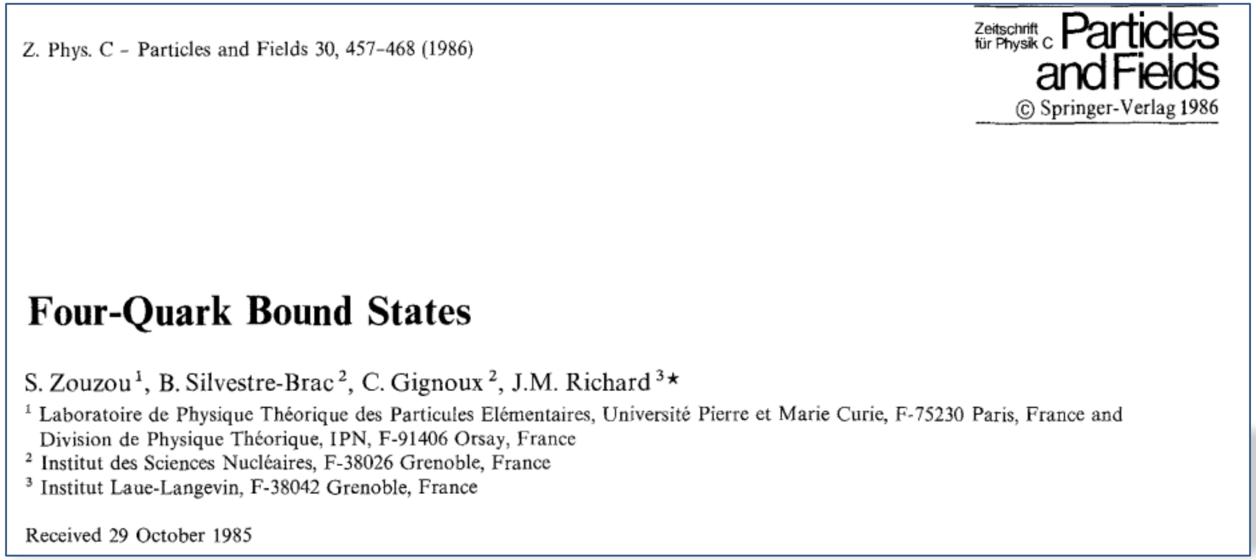
Received 29 October 1985



$$V^{\mathrm{II}}_{Q\bar{Q}}(r) = -\frac{16}{3} V^{\mathrm{II}}_{8}(r) = A + Br^{\beta} .$$

The authors search bound states with central forces only, by comparing **three methods**: a gaussian parametrization of the wave-function, the harmonic oscillator expansion and the hyperspherical expansion. They include **spin-spin** terms and virtual **meson-meson configurations**.

Exploring numerical methods



Z. Phys. C – Particles and Fields 30, 457–468 (1986)

Zeitschrift für Physik C Particles and Fields

© Springer-Verlag 1986

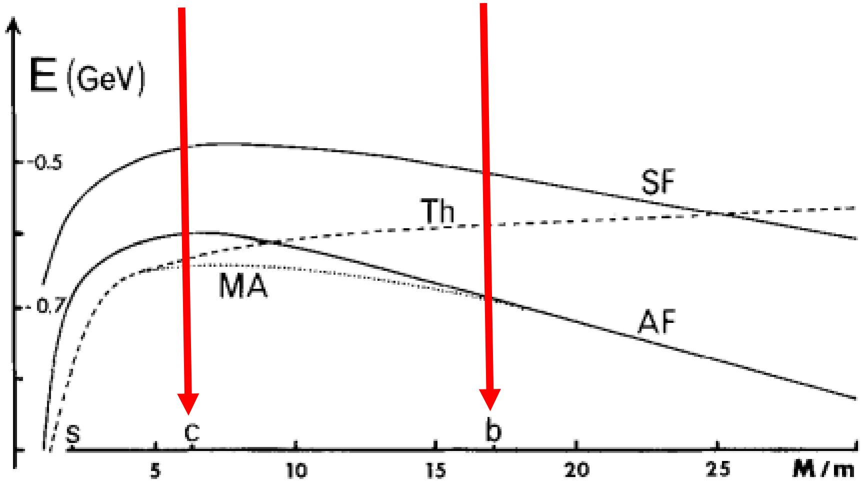
Four-Quark Bound States

S. Zouzou[1], B. Silvestre-Brac[2], C. Gignoux[2], J.M. Richard[3]*

[1] Laboratoire de Physique Théorique des Particules Elémentaires, Université Pierre et Marie Curie, F-75230 Paris, France and Division de Physique Théorique, IPN, F-91406 Orsay, France
[2] Institut des Sciences Nucléaires, F-38026 Grenoble, France
[3] Institut Laue-Langevin, F-38042 Grenoble, France

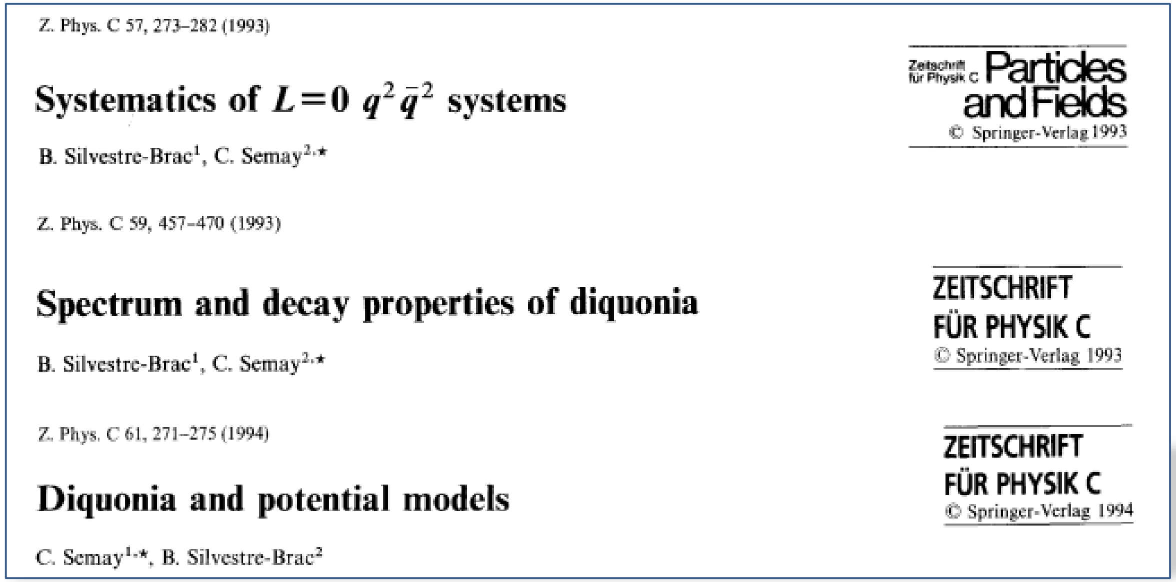
Received 29 October 1985

The authors search bound states with central forces only, by comparing **three methods**: a gaussian parametrization of the wave-function, the harmonic oscillator expansion and the hyperspherical expansion. They include **spin-spin** terms and virtual **meson-meson configurations**.



Using the Bhaduri potential they identified the ***S=1 I=0*** case as the most promising candidate for a bound state.

A systematic analysis



Z. Phys. C 57, 273–282 (1993)

Zeitschrift für Physik C Particles and Fields
© Springer-Verlag 1993

Systematics of $L=0$ $q^2\bar{q}^2$ systems

B. Silvestre-Brac[1], C. Semay[2,*]

Z. Phys. C 59, 457–470 (1993)

ZEITSCHRIFT FÜR PHYSIK C
© Springer-Verlag 1993

Spectrum and decay properties of diquonia

B. Silvestre-Brac[1], C. Semay[2,*]

Z. Phys. C 61, 271–275 (1994)

ZEITSCHRIFT FÜR PHYSIK C
© Springer-Verlag 1994

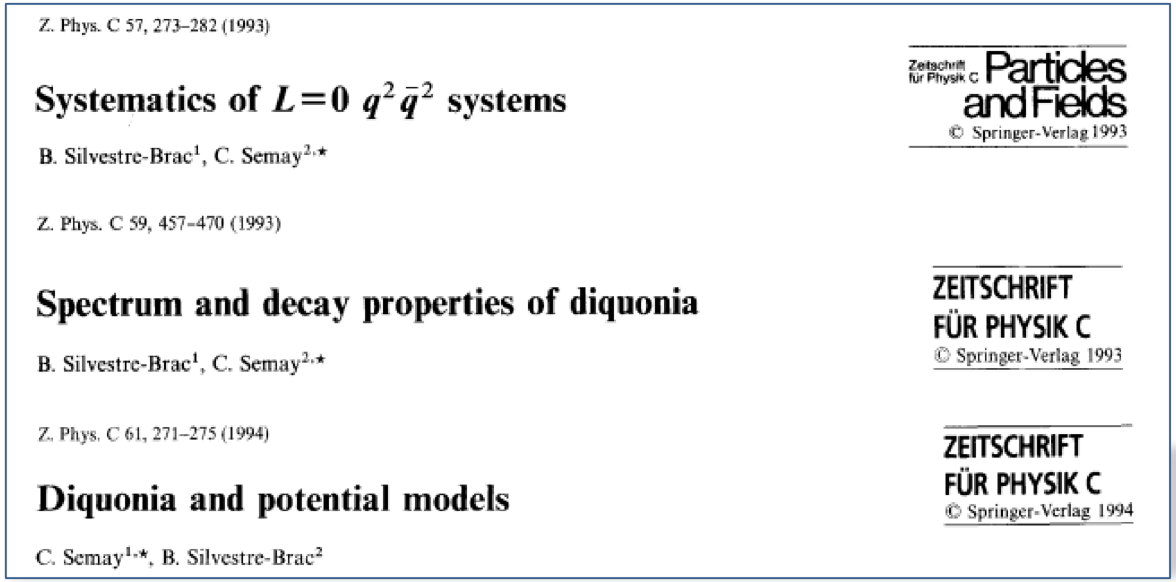
Diquonia and potential models

C. Semay[1,*], B. Silvestre-Brac[2]

Using the interquark potential due to Bhaduri *et al.*, the energies of all $L = 0,1,2,3$ four-quark states are calculated for any value of the total S and I and for q = u, d, s, c, b using a harmonic oscillator basis up to 7/8 quanta. Natural parity is considered.

This implies 924 combinations.

A systematic analysis



Z. Phys. C 57, 273–282 (1993)

Zeitschrift für Physik C Particles and Fields
© Springer-Verlag 1993

Systematics of $L=0$ $q^2\bar{q}^2$ systems

B. Silvestre-Brac[1], C. Semay[2,★]

Z. Phys. C 59, 457–470 (1993)

ZEITSCHRIFT FÜR PHYSIK C
© Springer-Verlag 1993

Spectrum and decay properties of diquonia

B. Silvestre-Brac[1], C. Semay[2,★]

Z. Phys. C 61, 271–275 (1994)

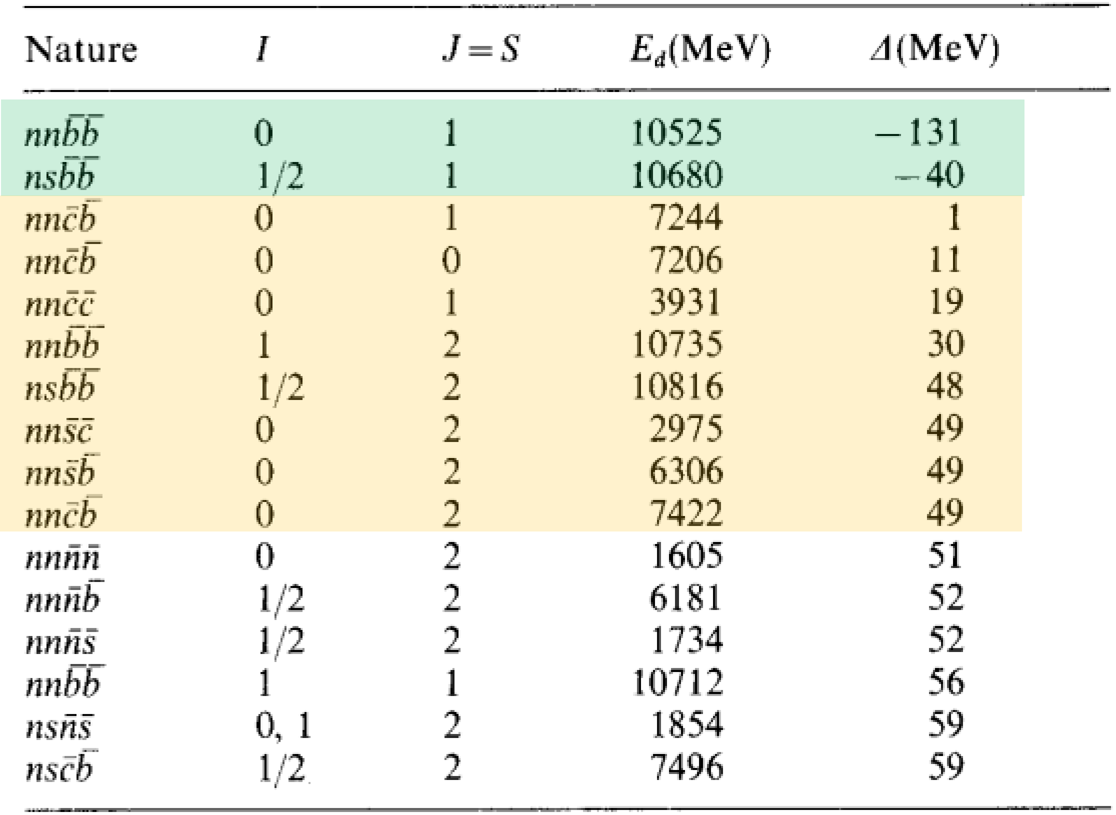
ZEITSCHRIFT FÜR PHYSIK C
© Springer-Verlag 1994

Diquonia and potential models

C. Semay[1,★], B. Silvestre-Brac[2]

Using the interquark potential due to Bhaduri *et al.*, the energies of all $L = 0,1,2,3$ four-quark states are calculated for any value of the total S and I and for q = u, d, s, c, b using a harmonic oscillator basis up to 7/8 quanta. Natural parity is considered.

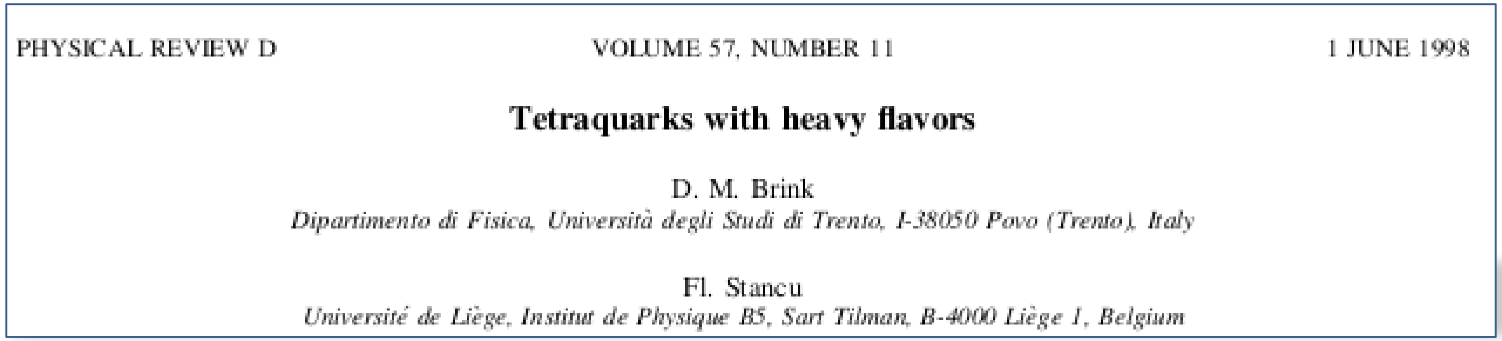
This implies 924 combinations.



Nature	I	$J=S$	E_d(MeV)	Δ(MeV)
$nn\bar{b}\bar{b}$	0	1	10525	−131
$ns\bar{b}\bar{b}$	1/2	1	10680	−40
$nn\bar{c}\bar{b}$	0	1	7244	1
$nn\bar{c}\bar{b}$	0	0	7206	11
$nn\bar{c}\bar{c}$	0	1	3931	19
$nn\bar{b}\bar{b}$	1	2	10735	30
$ns\bar{b}\bar{b}$	1/2	2	10816	48
$nn\bar{s}\bar{c}$	0	2	2975	49
$nn\bar{s}\bar{b}$	0	2	6306	49
$nn\bar{c}\bar{b}$	0	2	7422	49
$nn\bar{n}\bar{n}$	0	2	1605	51
$nn\bar{n}\bar{b}$	1/2	2	6181	52
$nn\bar{n}\bar{s}$	1/2	2	1734	52
$nn\bar{b}\bar{b}$	1	1	10712	56
$ns\bar{n}\bar{s}$	0, 1	2	1854	59
$ns\bar{c}\bar{b}$	1/2	2	7496	59

Improving the numerical methods

A different approach based on Gaussian variational wave functions including combinations of three different radial coordinates is considered.

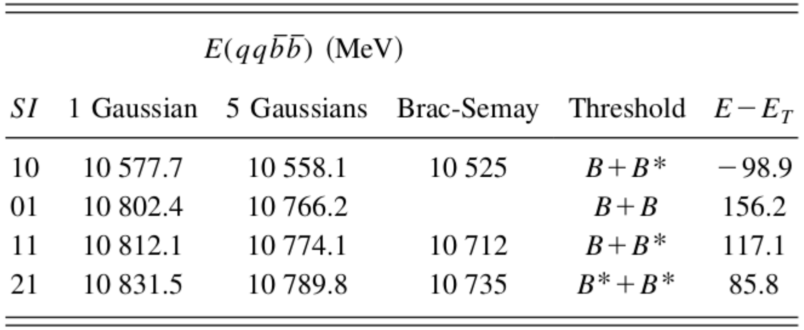


PHYSICAL REVIEW D VOLUME 57, NUMBER 11 1 JUNE 1998

Tetraquarks with heavy flavors

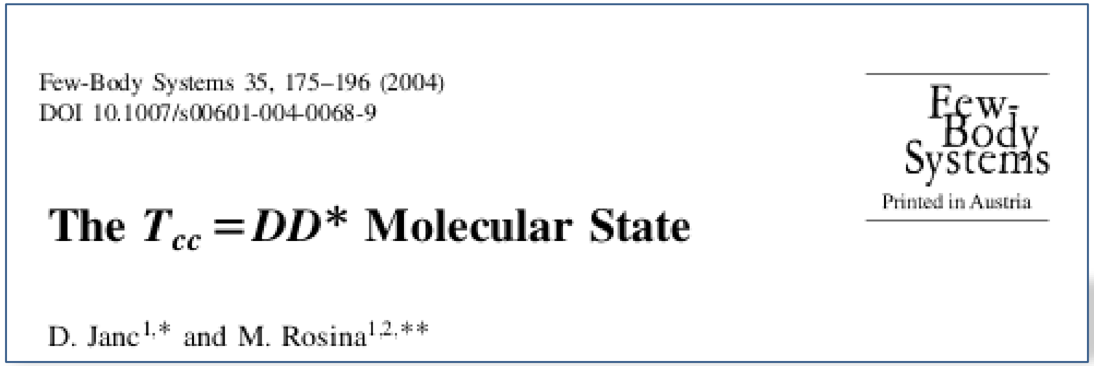
D. M. Brink
Dipartimento di Fisica, Università degli Studi di Trento, I-38050 Povo (Trento), Italy

Fl. Stancu
Université de Liège, Institut de Physique B5, Sart Tilman, B-4000 Liège 1, Belgium



$E(qq\bar{b}\bar{b})$ (MeV)

SI	1 Gaussian	5 Gaussians	Brac-Semay	Threshold	$E-E_T$
10	10 577.7	10 558.1	10 525	$B+B^*$	-98.9
01	10 802.4	10 766.2		$B+B$	156.2
11	10 812.1	10 774.1	10 712	$B+B^*$	117.1
21	10 831.5	10 789.8	10 735	B^*+B^*	85.8



Few-Body Systems 35, 175–196 (2004)
DOI 10.1007/s00601-004-0068-9

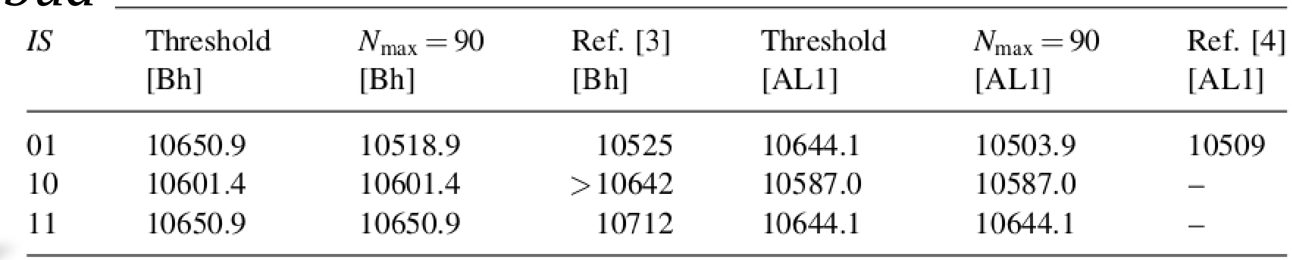
Few-Body Systems
Printed in Austria

The $T_{cc}=DD^*$ Molecular State

D. Janc[1,*] and M. Rosina[1,2,**]

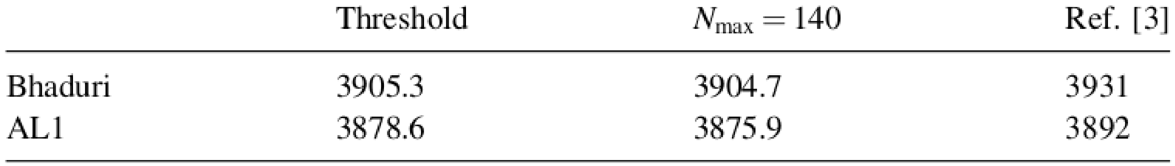
First detailed study of typical radii and radial properties.

$bb\bar{u}\bar{d}$



IS	Threshold [Bh]	$N_{\max}=90$ [Bh]	Ref. [3] [Bh]	Threshold [AL1]	$N_{\max}=90$ [AL1]	Ref. [4] [AL1]
01	10650.9	10518.9	10525	10644.1	10503.9	10509
10	10601.4	10601.4	>10642	10587.0	10587.0	–
11	10650.9	10650.9	10712	10644.1	10644.1	–

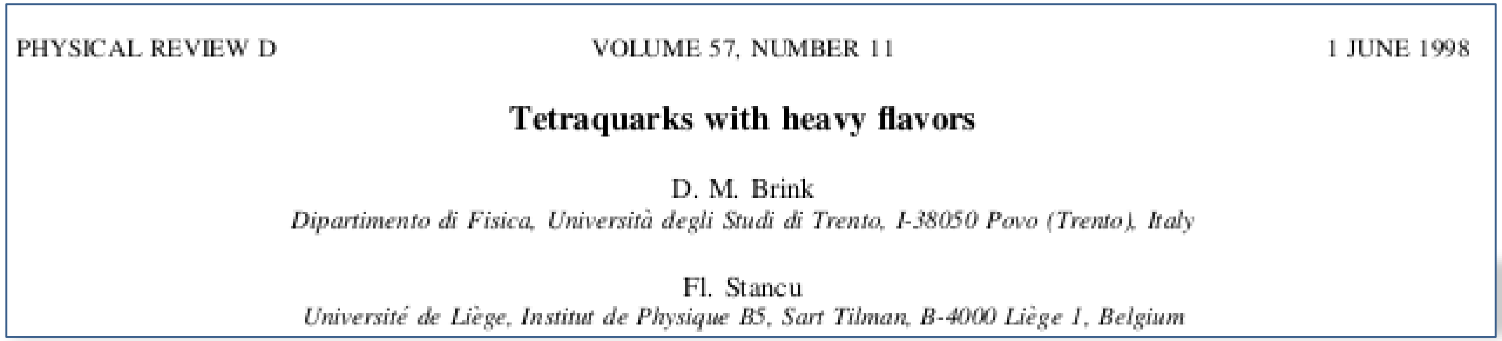
$cc\bar{u}\bar{d}$ $S=1\ I=0$



	Threshold	$N_{\max}=140$	Ref. [3]
Bhaduri	3905.3	3904.7	3931
AL1	3878.6	3875.9	3892

Improving the numerical methods

A different approach based on Gaussian variational wave functions including combinations of three different radial coordinates is considered.

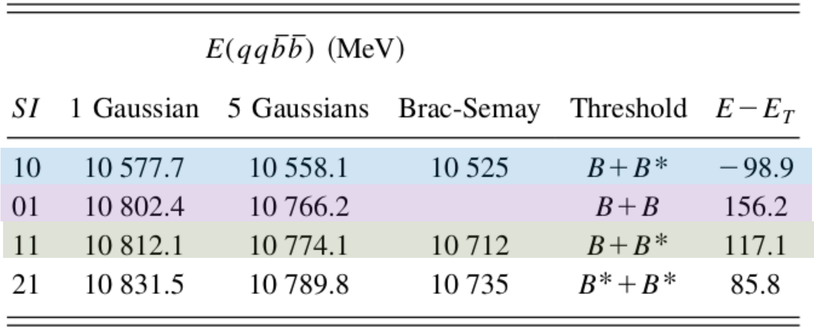


PHYSICAL REVIEW D VOLUME 57, NUMBER 11 1 JUNE 1998

Tetraquarks with heavy flavors

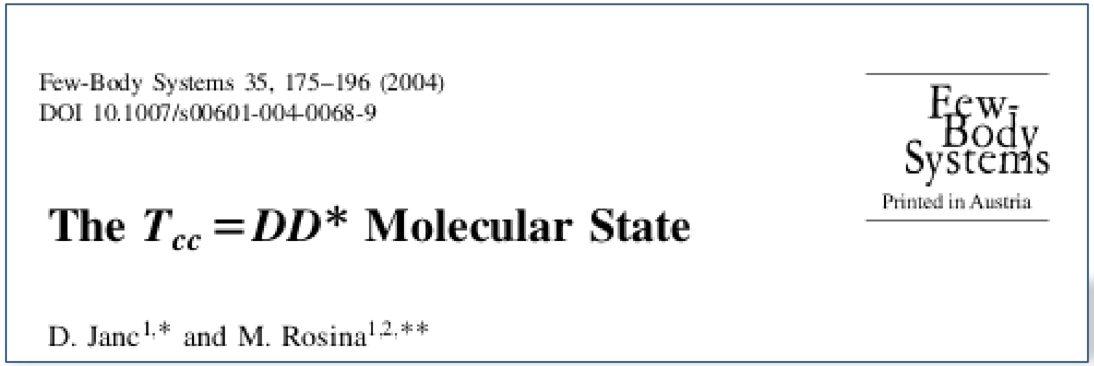
D. M. Brink
Dipartimento di Fisica, Università degli Studi di Trento, I-38050 Povo (Trento), Italy

Fl. Stancu
Université de Liège, Institut de Physique B5, Sart Tilman, B-4000 Liège 1, Belgium



$E(qq\bar{b}\bar{b})$ (MeV)

SI	1 Gaussian	5 Gaussians	Brac-Semay	Threshold	$E-E_T$
10	10 577.7	10 558.1	10 525	$B+B^*$	-98.9
01	10 802.4	10 766.2		$B+B$	156.2
11	10 812.1	10 774.1	10 712	$B+B^*$	117.1
21	10 831.5	10 789.8	10 735	B^*+B^*	85.8



Few-Body Systems 35, 175–196 (2004)
DOI 10.1007/s00601-004-0068-9

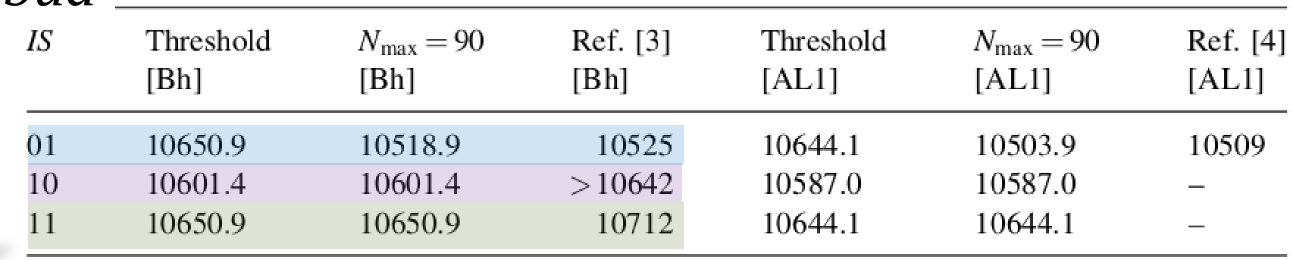
Few-Body Systems
Printed in Austria

The $T_{cc}=DD^*$ Molecular State

D. Janc[1,*] and M. Rosina[1,2,**]

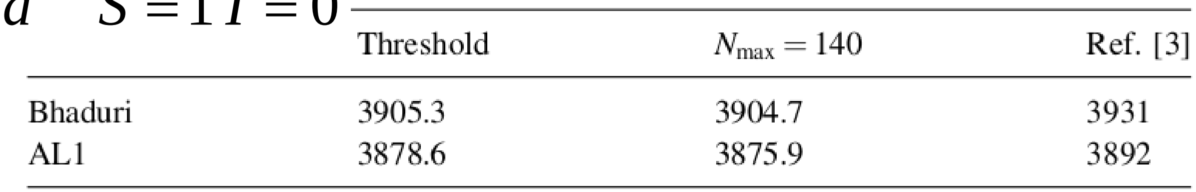
First detailed study of typical radii and radial properties.

$bb\bar{u}\bar{d}$



IS	Threshold [Bh]	$N_{\max}=90$ [Bh]	Ref. [3] [Bh]	Threshold [AL1]	$N_{\max}=90$ [AL1]	Ref. [4] [AL1]
01	10650.9	10518.9	10525	10644.1	10503.9	10509
10	10601.4	10601.4	>10642	10587.0	10587.0	–
11	10650.9	10650.9	10712	10644.1	10644.1	–

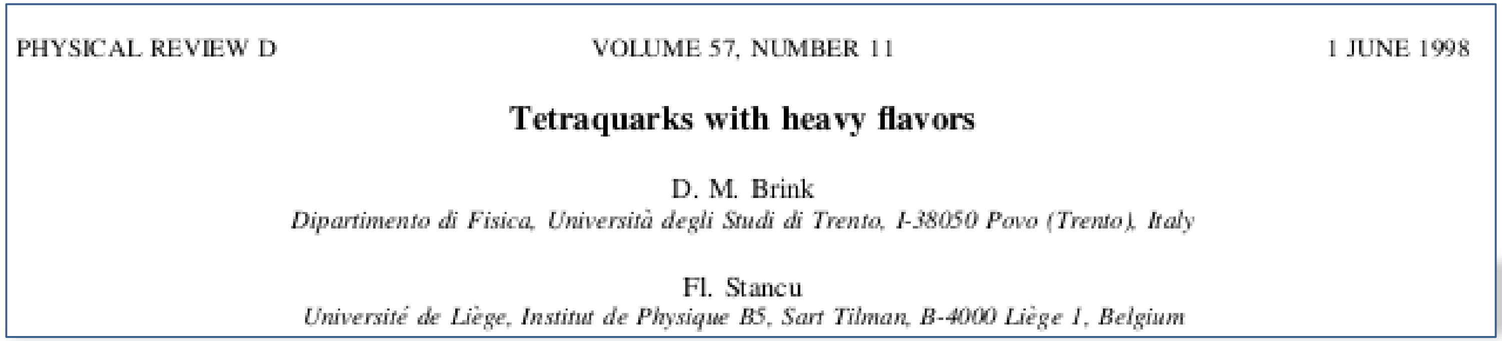
$cc\bar{u}\bar{d}$ $S=1\ I=0$



	Threshold	$N_{\max}=140$	Ref. [3]
Bhaduri	3905.3	3904.7	3931
AL1	3878.6	3875.9	3892

Improving the numerical methods

A different approach based on Gaussian variational wave functions including combinations of three different radial coordinates is considered.

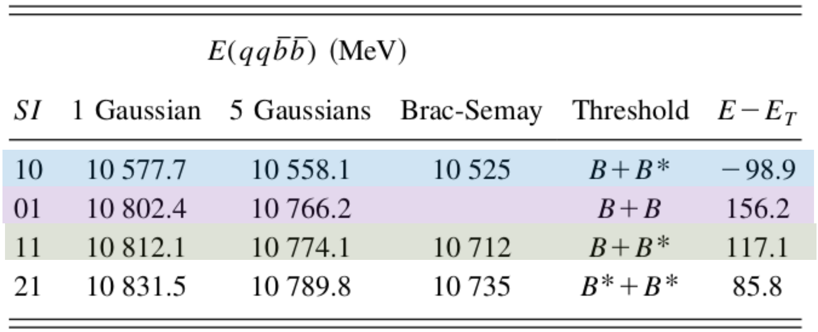


PHYSICAL REVIEW D VOLUME 57, NUMBER 11 1 JUNE 1998

Tetraquarks with heavy flavors

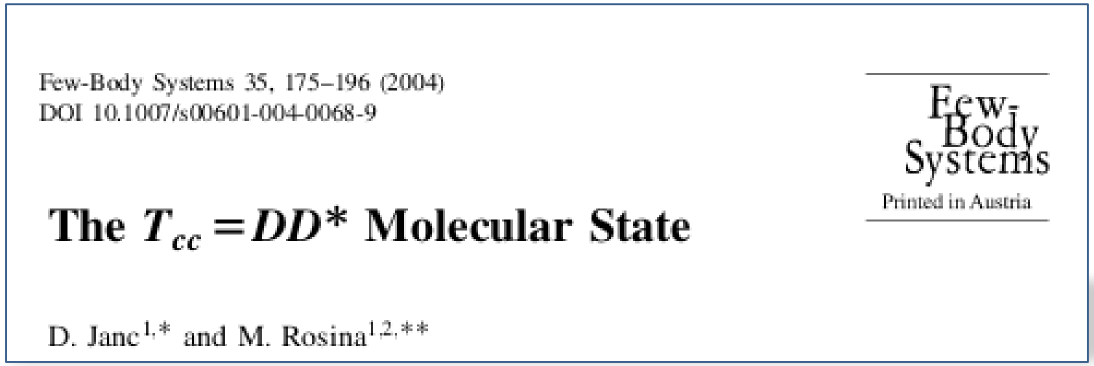
D. M. Brink
Dipartimento di Fisica, Università degli Studi di Trento, I-38050 Povo (Trento), Italy

Fl. Stancu
Université de Liège, Institut de Physique B5, Sart Tilman, B-4000 Liège 1, Belgium



$E(qq\bar{b}\bar{b})$ (MeV)

SI	1 Gaussian	5 Gaussians	Brac-Semay	Threshold	$E-E_T$
10	10 577.7	10 558.1	10 525	$B+B^*$	-98.9
01	10 802.4	10 766.2		$B+B$	156.2
11	10 812.1	10 774.1	10 712	$B+B^*$	117.1
21	10 831.5	10 789.8	10 735	B^*+B^*	85.8



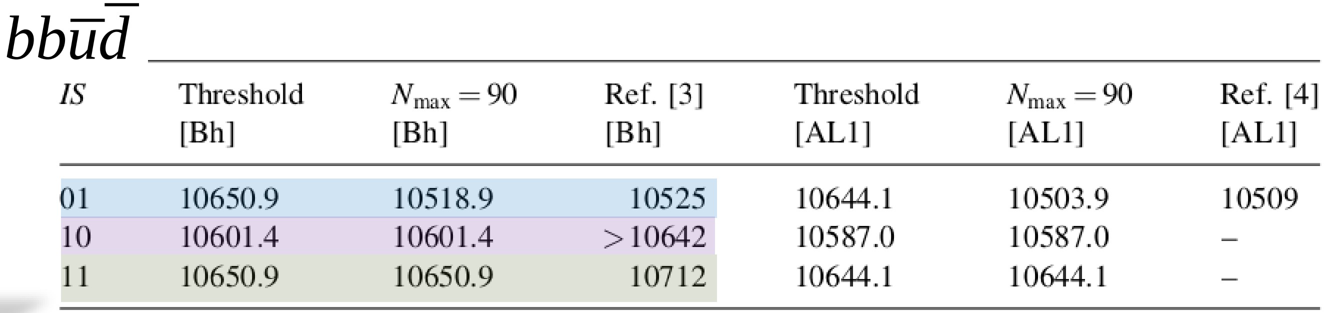
Few-Body Systems 35, 175–196 (2004)
DOI 10.1007/s00601-004-0068-9

Few-Body Systems
Printed in Austria

The $T_{cc}=DD^*$ Molecular State

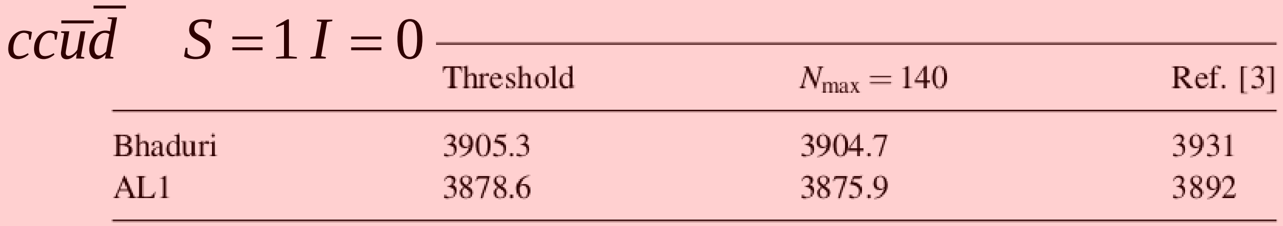
D. Janc[1,*] and M. Rosina[1,2,**]

First detailed study of typical radii and radial properties.



$bb\bar{u}\bar{d}$

IS	Threshold [Bh]	$N_{\max}=90$ [Bh]	Ref. [3] [Bh]	Threshold [AL1]	$N_{\max}=90$ [AL1]	Ref. [4] [AL1]
01	10650.9	10518.9	10525	10644.1	10503.9	10509
10	10601.4	10601.4	>10642	10587.0	10587.0	–
11	10650.9	10650.9	10712	10644.1	10644.1	–



$c c\bar{u}\bar{d}$ $S=1\ I=0$

	Threshold	$N_{\max}=140$	Ref. [3]
Bhaduri	3905.3	3904.7	3931
AL1	3878.6	3875.9	3892

Exploring constituent quark models

PHYSICAL REVIEW D **76,** 094027 (2007)

Are there compact heavy four-quark bound states?

J. Vijande,[1] E. Weissman,[2] A. Valcarce,[3] and N. Barnea[2,4]

PHYSICAL REVIEW D **79,** 074010 (2009)

Exotic meson-meson molecules and compact four-quark states

J. Vijande,[1,2] A. Valcarce,[2] and N. Barnea[3,4]

We revisit the same sector using more powerful numerical techniques:

- A hyperspherical harmonic formalism (up to $K = 30$)
- A variational method using generalized gaussians with all non-diagonal terms (relative $l \neq 0$)

$$R_s(1234) = \sum_{i=1}^{n} \beta_s^{(i)} R_s^{(i)} =$$

$$\sum_{i=1}^{n} \beta_s^{(i)} e^{-a_s^{(i)}\vec{x}^{\,2} - b_s^{(i)}\vec{y}^{\,2} - c_s^{(i)}\vec{z}^{\,2} - d_s^{(i)}\vec{x}\vec{y} - e_s^{(i)}\vec{x}\vec{z} - f_s^{(i)}\vec{y}\vec{z}}$$

Exploring constituent quark models

PHYSICAL REVIEW D **76**, 094027 (2007)

Are there compact heavy four-quark bound states?

J. Vijande,[1] E. Weissman,[2] A. Valcarce,[3] and N. Barnea[2,4]

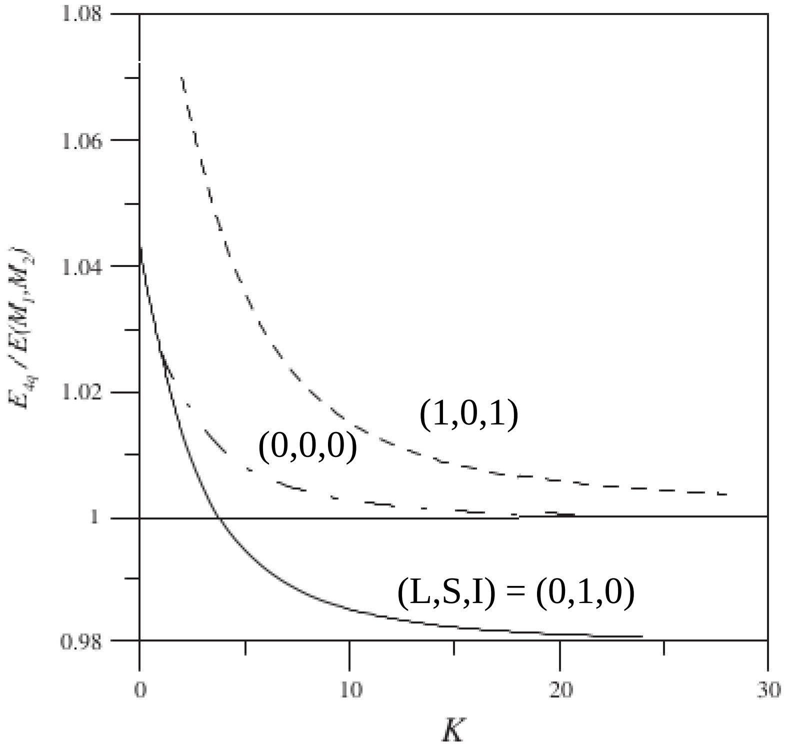
PHYSICAL REVIEW D **79**, 074010 (2009)

Exotic meson-meson molecules and compact four-quark states

J. Vijande,[1,2] A. Valcarce,[2] and N. Barnea[3,4]

We revisit the same sector using more powerful numerical techniques:

- A hyperspherical harmonic formalism (up to $K = 30$)
- A variational method using generalized gaussians with all non-diagonal terms (relative $l \neq 0$)



Exploring constituent quark models

PHYSICAL REVIEW D **76,** 094027 (2007)

Are there compact heavy four-quark bound states?

J. Vijande,[1] E. Weissman,[2] A. Valcarce,[3] and N. Barnea[2,4]

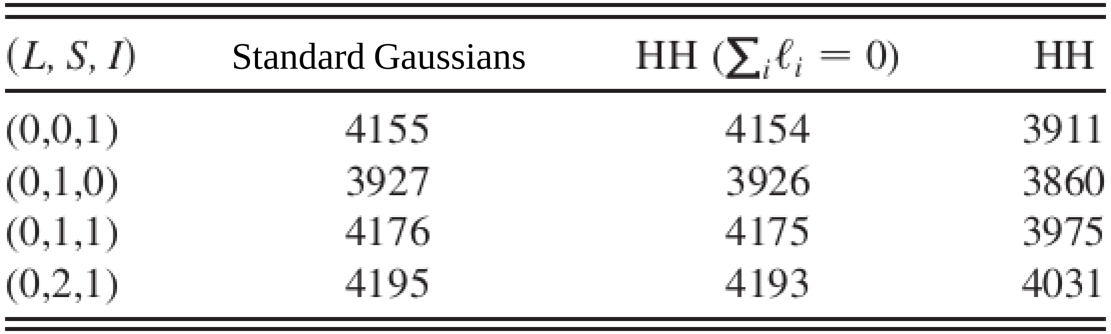
PHYSICAL REVIEW D **79,** 074010 (2009)

Exotic meson-meson molecules and compact four-quark states

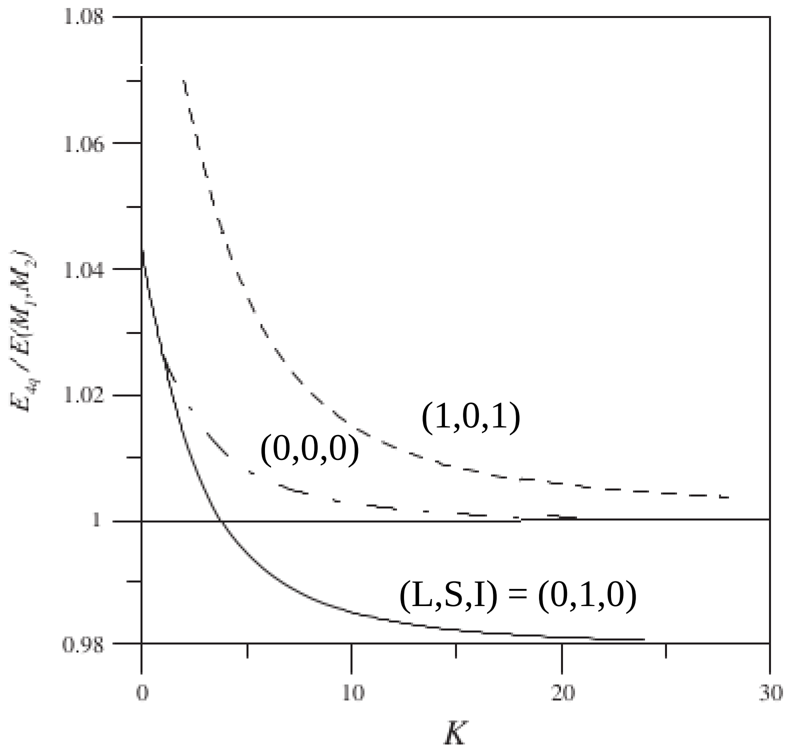
J. Vijande,[1,2] A. Valcarce,[2] and N. Barnea[3,4]

We revisit the same sector using more powerful numerical techniques:

- A hyperspherical harmonic formalism (up to $K = 30$)
- A variational method using generalized gaussians with all non-diagonal terms (relative $l \neq 0$)



(L, S, I)	Standard Gaussians	HH ($\sum_i \ell_i = 0$)	HH
(0,0,1)	4155	4154	3911
(0,1,0)	3927	3926	3860
(0,1,1)	4176	4175	3975
(0,2,1)	4195	4193	4031



Exploring constituent quark models

PHYSICAL REVIEW D **76,** 094027 (2007)

Are there compact heavy four-quark bound states?

J. Vijande,[1] E. Weissman,[2] A. Valcarce,[3] and N. Barnea[2,4]

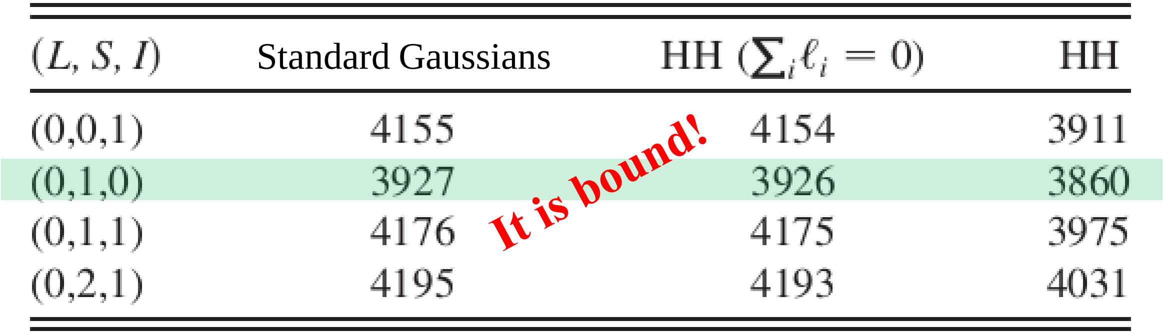
PHYSICAL REVIEW D **79,** 074010 (2009)

Exotic meson-meson molecules and compact four-quark states

J. Vijande,[1,2] A. Valcarce,[2] and N. Barnea[3,4]

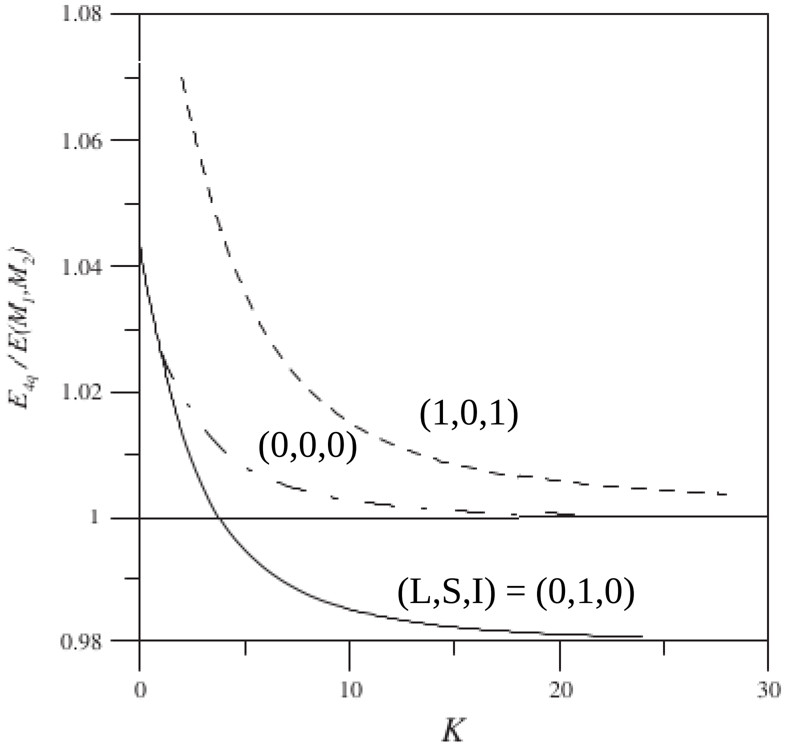
We revisit the same sector using more powerful numerical techniques:

- A hyperspherical harmonic formalism (up to *K* = *30*)
- A variational method using generalized gaussians with all non-diagonal terms (relative *l* ≠ *0*)



(L, S, I)	Standard Gaussians	HH ($\sum_i \ell_i = 0$)	HH
(0,0,1)	4155	4154	3911
(0,1,0)	3927	3926	3860
(0,1,1)	4176	4175	3975
(0,2,1)	4195	4193	4031

It is bound!



Exploring constituent quark models

PHYSICAL REVIEW D **76,** 094027 (2007)

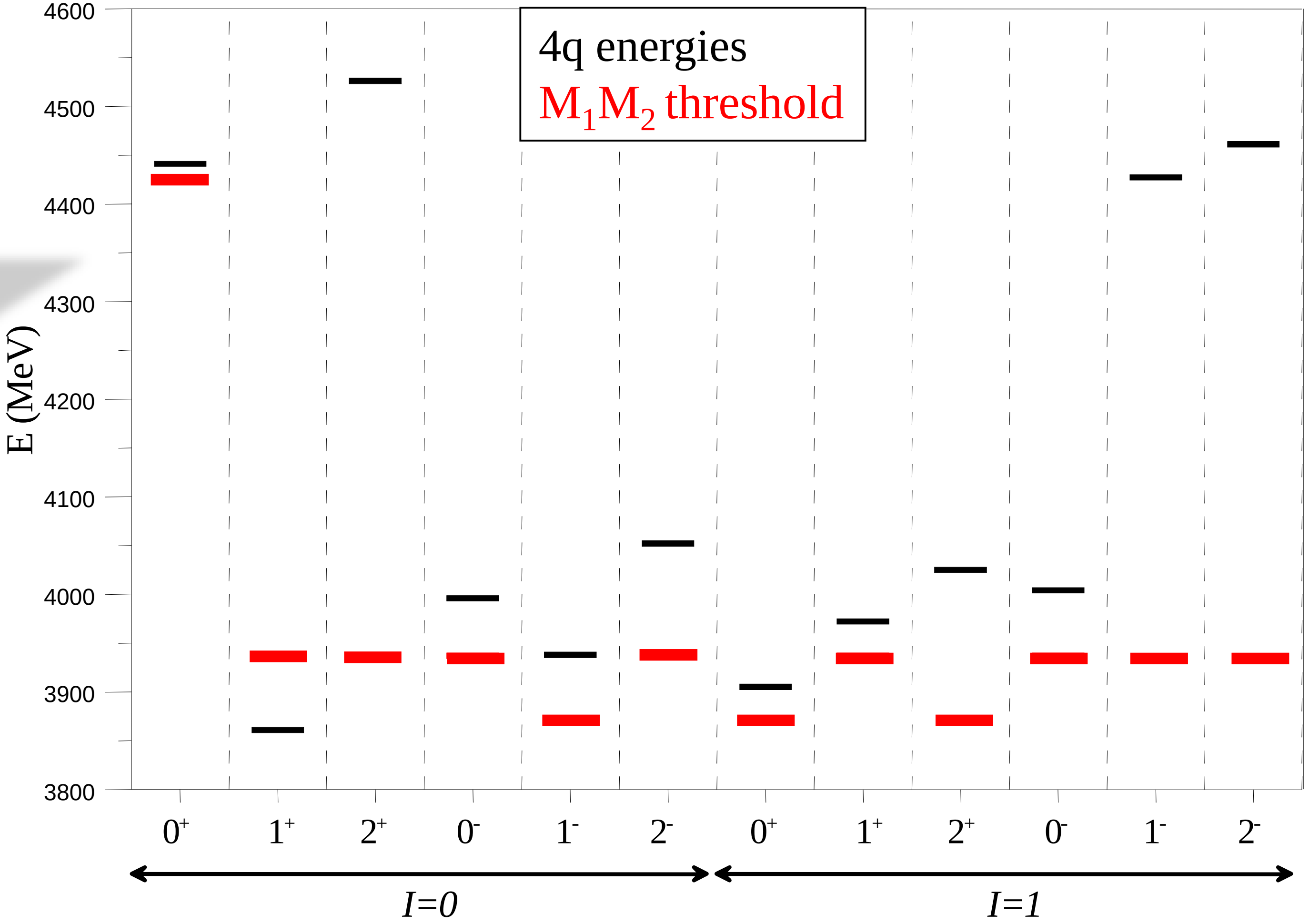
Are there compact heavy four-quark bound states?

J. Vijande,[1] E. Weissman,[2] A. Valcarce,[3] and N. Barnea[2,4]

PHYSICAL REVIEW D **79,** 074010 (2009)

Exotic meson-meson molecules and compact four-quark states

J. Vijande,[1,2] A. Valcarce,[2] and N. Barnea[3,4]



Exploring constituent quark models

PHYSICAL REVIEW D **76**, 094027 (2007)

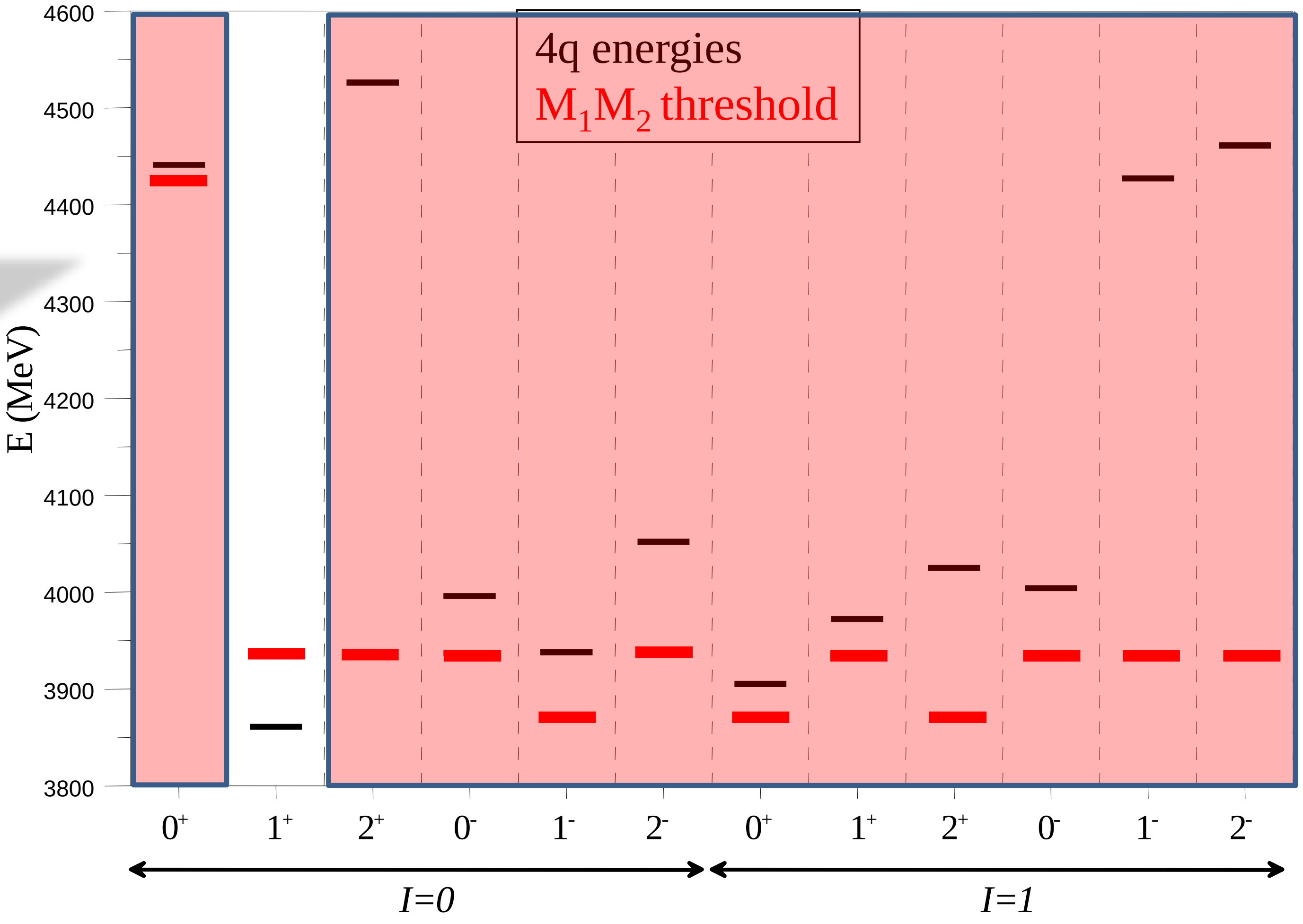
Are there compact heavy four-quark bound states?

J. Vijande,[1] E. Weissman,[2] A. Valcarce,[3] and N. Barnea[2,4]

PHYSICAL REVIEW D **79**, 074010 (2009)

Exotic meson-meson molecules and compact four-quark states

J. Vijande,[1,2] A. Valcarce,[2] and N. Barnea[3,4]



Exploring constituent quark models

PHYSICAL REVIEW D **76,** 094027 (2007)

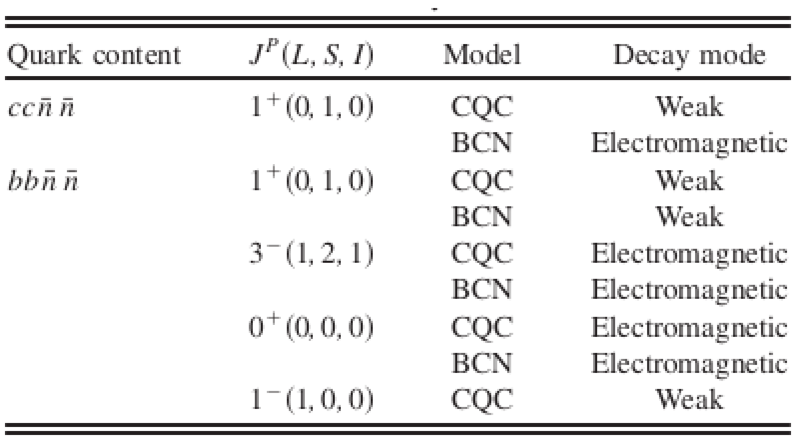
Are there compact heavy four-quark bound states?

J. Vijande,[1] E. Weissman,[2] A. Valcarce,[3] and N. Barnea[2,4]

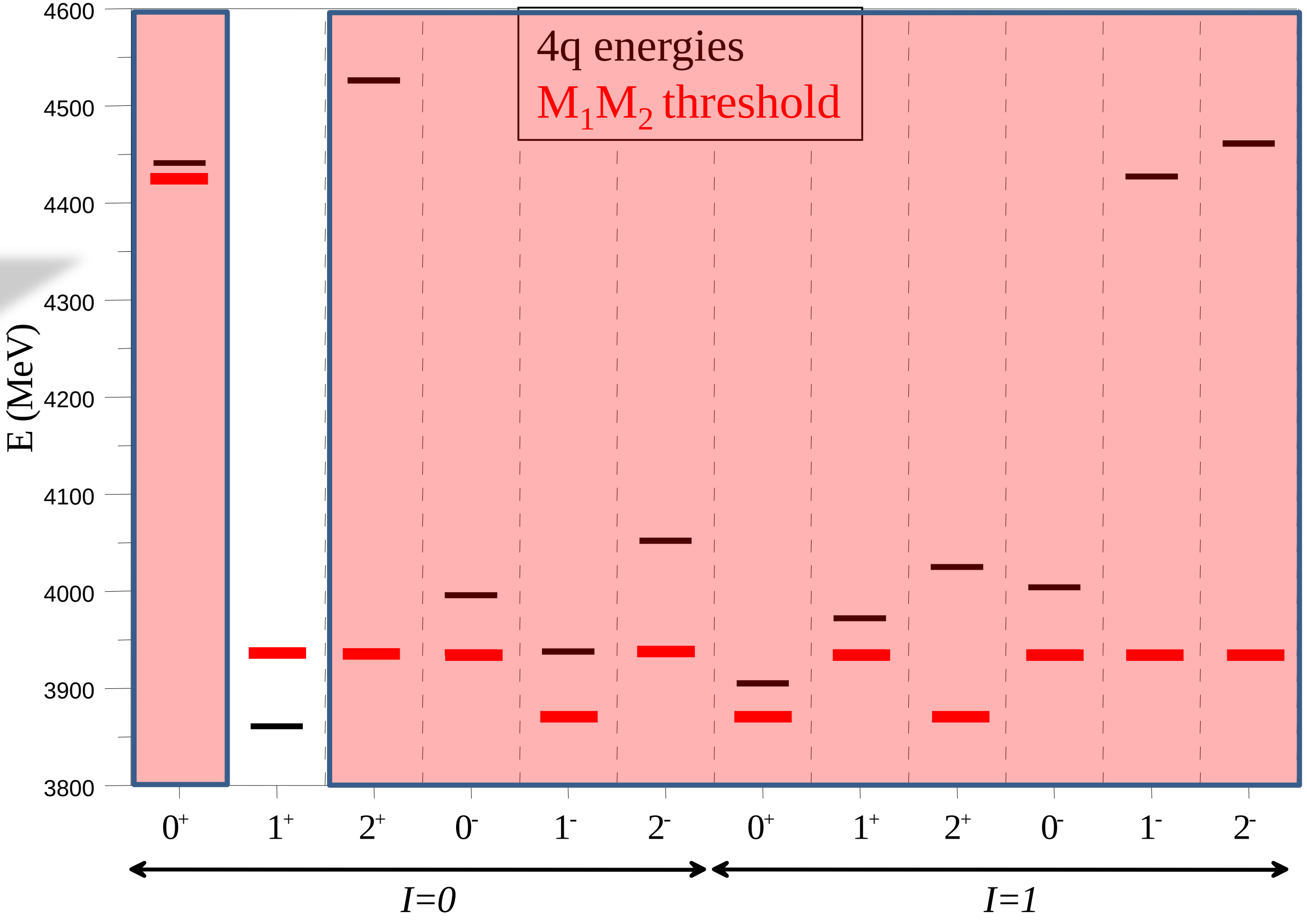
PHYSICAL REVIEW D **79,** 074010 (2009)

Exotic meson-meson molecules and compact four-quark states

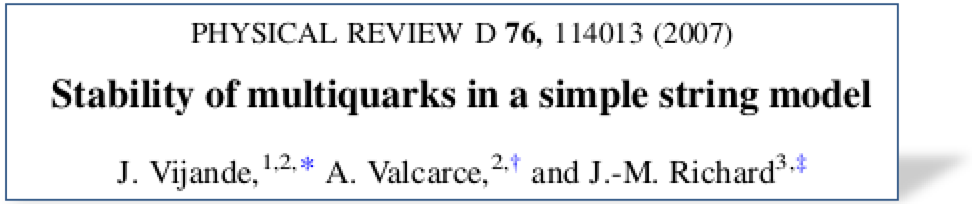
J. Vijande,[1,2] A. Valcarce,[2] and N. Barnea[3,4]



Quark content	$J^P(L,S,I)$	Model	Decay mode
$cc\bar{n}\bar{n}$	$1^+(0,1,0)$	CQC	Weak
		BCN	Electromagnetic
$bb\bar{n}\bar{n}$	$1^+(0,1,0)$	CQC	Weak
		BCN	Weak
	$3^-(1,2,1)$	CQC	Electromagnetic
		BCN	Electromagnetic
	$0^+(0,0,0)$	CQC	Electromagnetic
		BCN	Electromagnetic
	$1^-(1,0,0)$	CQC	Weak



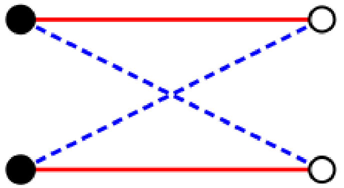
Beyond pairwise interactions



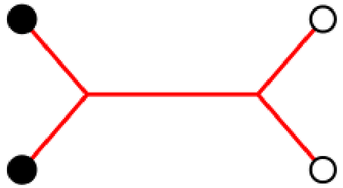
PHYSICAL REVIEW D **76**, 114013 (2007)

Stability of multiquarks in a simple string model

J. Vijande,[1,2,*] A. Valcarce,[2,†] and J.-M. Richard[3,‡]



Flip-Flop model

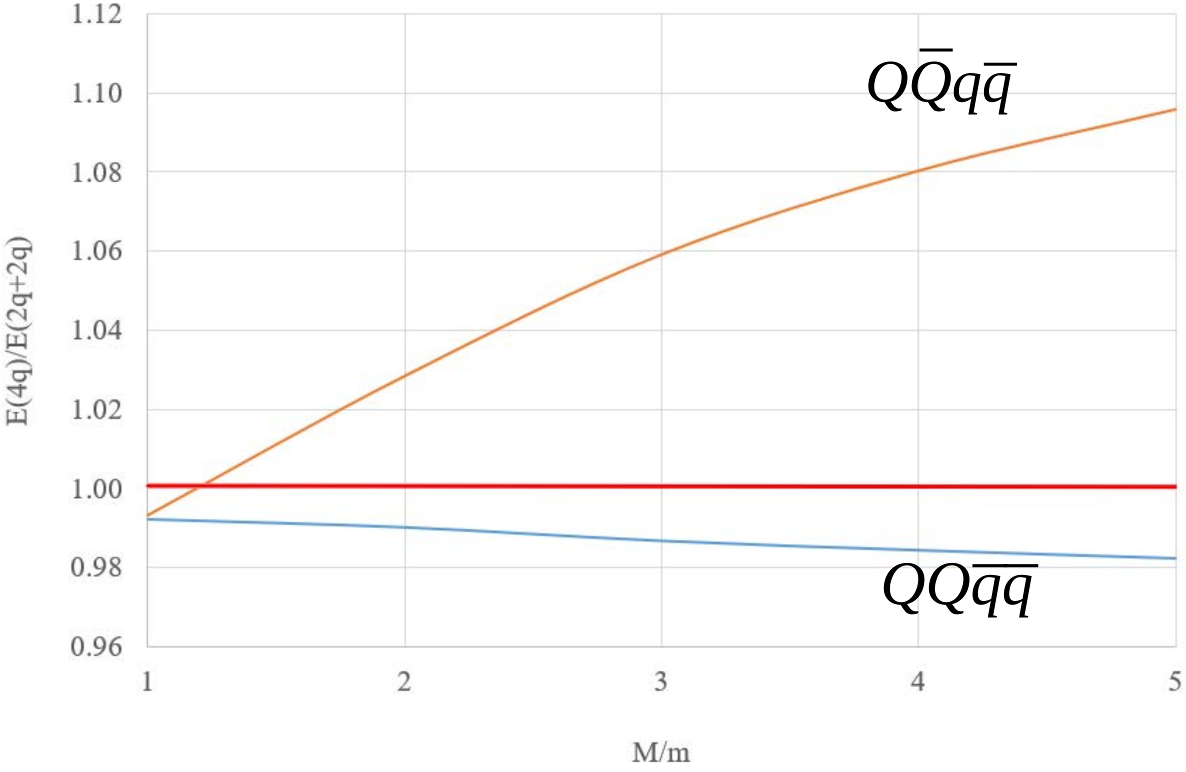


Butterfly model

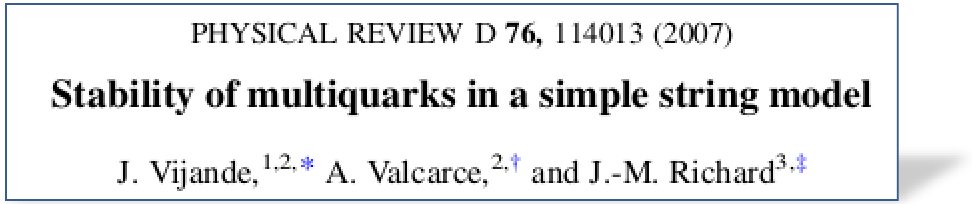
$$V_f = \lambda \min(r_{13} + r_{24}, r_{23} + r_{14})$$

$$V_s = \min(V_f, V_b).$$

$$V_b = \lambda \min_{k,\ell}(r_{1k} + r_{2k} + r_{k\ell} + r_{\ell 3} + r_{\ell 4})$$

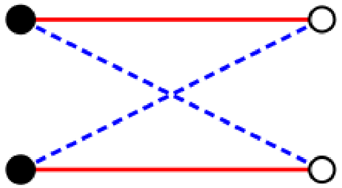


Beyond pairwise interactions


PHYSICAL REVIEW D **76**, 114013 (2007)

Stability of multiquarks in a simple string model

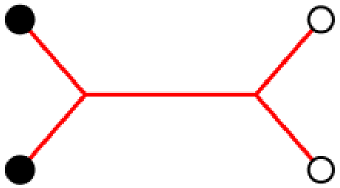
J. Vijande,[1,2,*] A. Valcarce,[2,†] and J.-M. Richard[3,‡]


Flip-Flop model

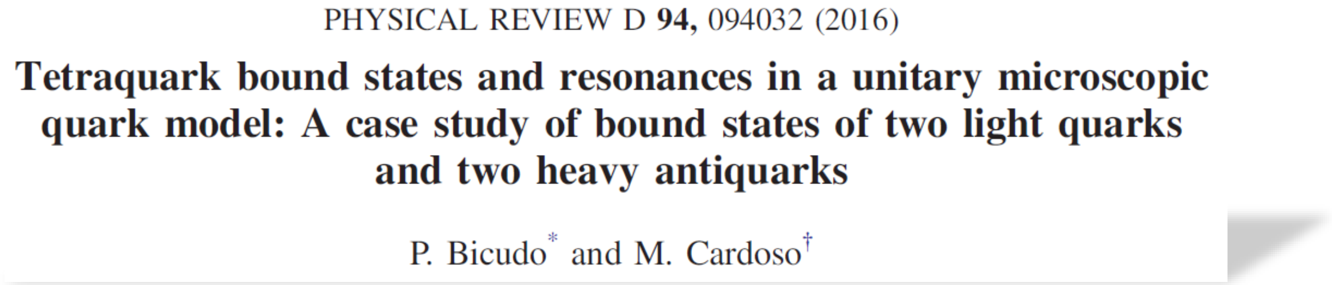
$$V_f = \lambda \min(r_{13} + r_{24}, r_{23} + r_{14})$$

$$V_s = \min(V_f, V_b).$$

$$V_b = \lambda \min_{k,\ell}(r_{1k} + r_{2k} + r_{k\ell} + r_{\ell 3} + r_{\ell 4})$$


Butterfly model

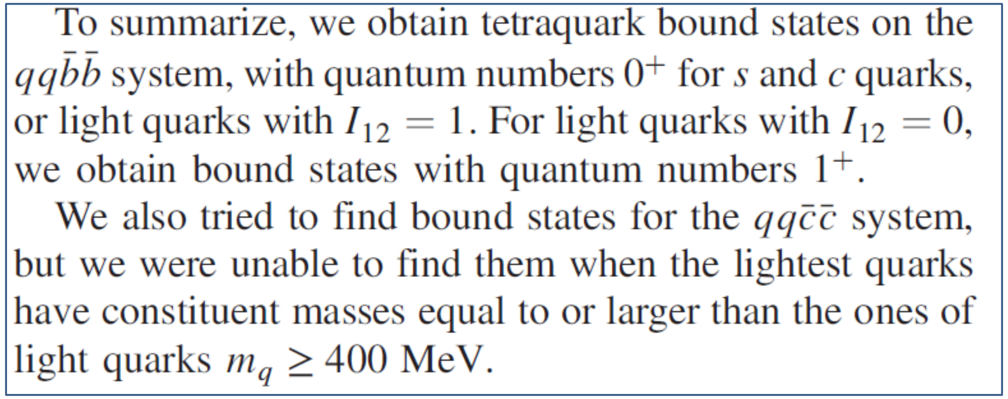
Beyond pairwise interactions



PHYSICAL REVIEW D **94,** 094032 (2016)

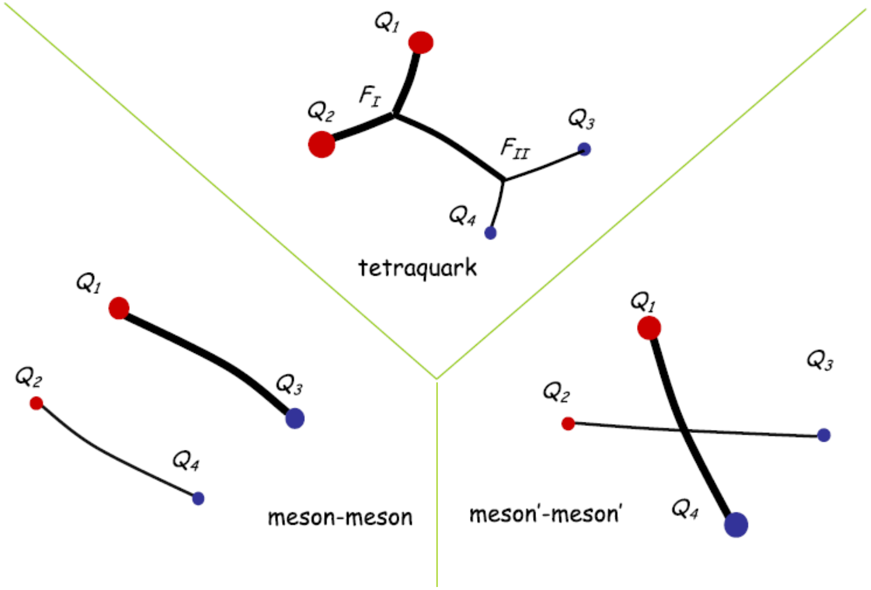
Tetraquark bound states and resonances in a unitary microscopic quark model: A case study of bound states of two light quarks and two heavy antiquarks

P. Bicudo[*] and M. Cardoso[†]



To summarize, we obtain tetraquark bound states on the $qq\bar{b}\bar{b}$ system, with quantum numbers 0^+ for s and c quarks, or light quarks with $I_{12} = 1$. For light quarks with $I_{12} = 0$, we obtain bound states with quantum numbers 1^+.

We also tried to find bound states for the $qq\bar{c}\bar{c}$ system, but we were unable to find them when the lightest quarks have constituent masses equal to or larger than the ones of light quarks $m_q \geq 400$ MeV.



The ground state potential for a system composed of two quarks and two antiquarks is well fitted by a string flip-flop potential.

$cc\bar{u}\bar{d}$

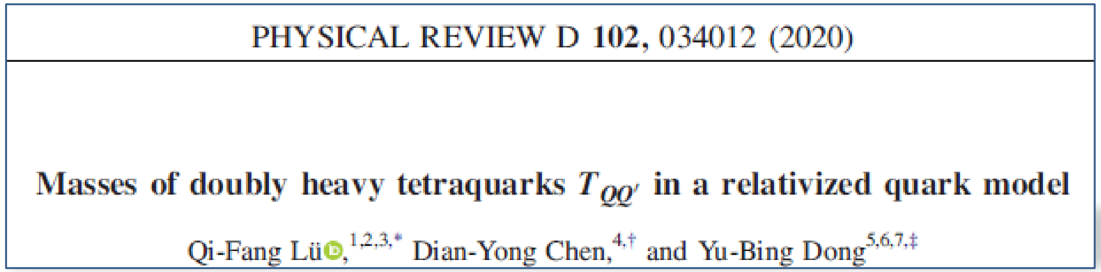
$bb\bar{u}\bar{d}$ $(S=1, I=0)$

$bb\bar{u}\bar{d}$ $(S=0, I=1)$

$bb\bar{s}\bar{s}$ $(S=0, I=0)$

$bb\bar{c}\bar{c}$ $(S=0, I=0)$

Will the relativistic kinematics increase the number of stable multiquarks?


PHYSICAL REVIEW D **102**, 034012 (2020)

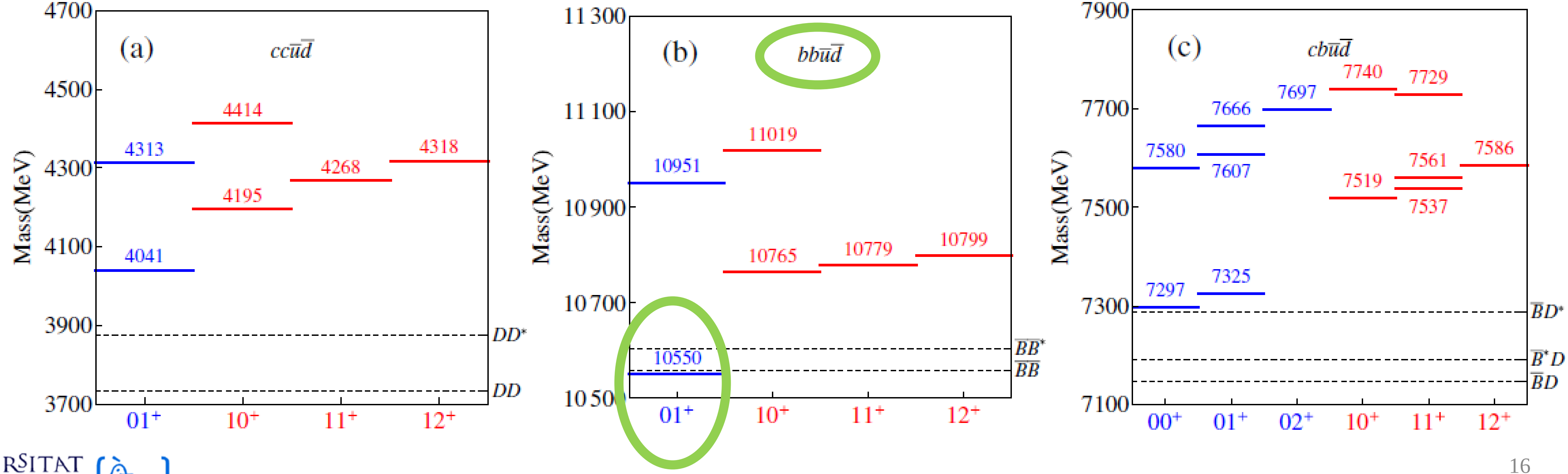
Masses of doubly heavy tetraquarks $T_{QQ'}$ in a relativized quark model

Qi-Fang Lü,[1,2,3,*] Dian-Yong Chen,[4,†] and Yu-Bing Dong[5,6,7,‡]

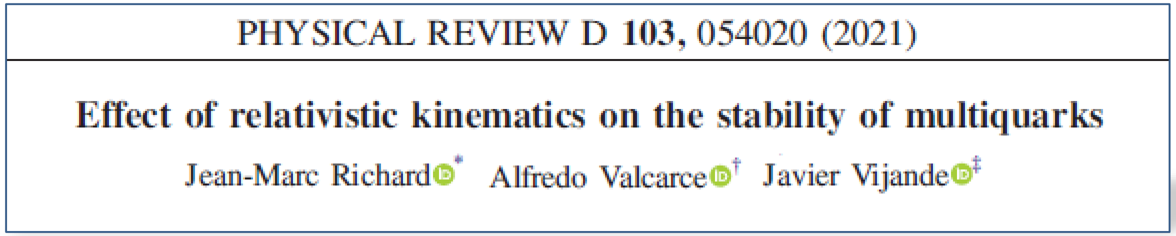
The authors investigate the mass spectra using the relativized quark model proposed by Godfrey, Capstick, and Isgur.

The spatial wave function is expanded in terms of a set of Gaussian basis functions where the Gaussian size parameters are taken in geometric progression

$QQu\bar{s}$ *and* $QQ\bar{s}\bar{s}$



Will the relativistic kinematics increase the number of stable multiquarks?



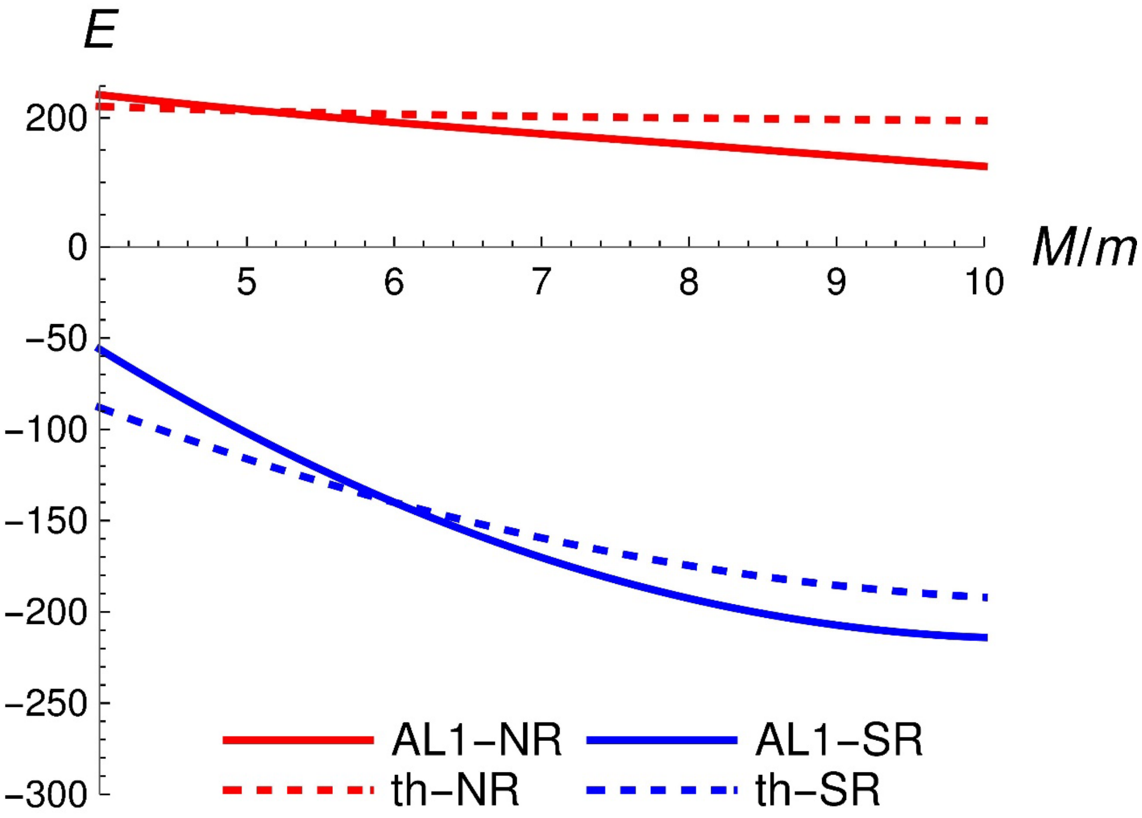
PHYSICAL REVIEW D **103**, 054020 (2021)

Effect of relativistic kinematics on the stability of multiquarks

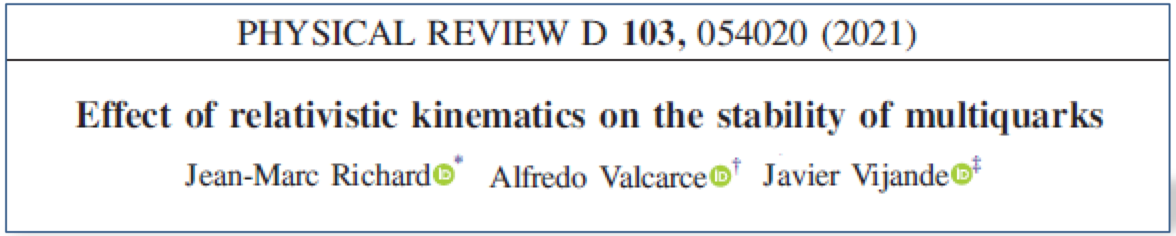
Jean-Marc Richard[*] Alfredo Valcarce[†] Javier Vijande[‡]

In this case the threshold is made of two *(qQ)* mesons while in the four-quark state there are *(qq)*, *(QQ)* and four *(qQ)* interactions. Who will benefit more from the relativistic dynamics?

We consider the AL1 potential properly re-parametrized in the SR case for keeping the description of the meson spectra.



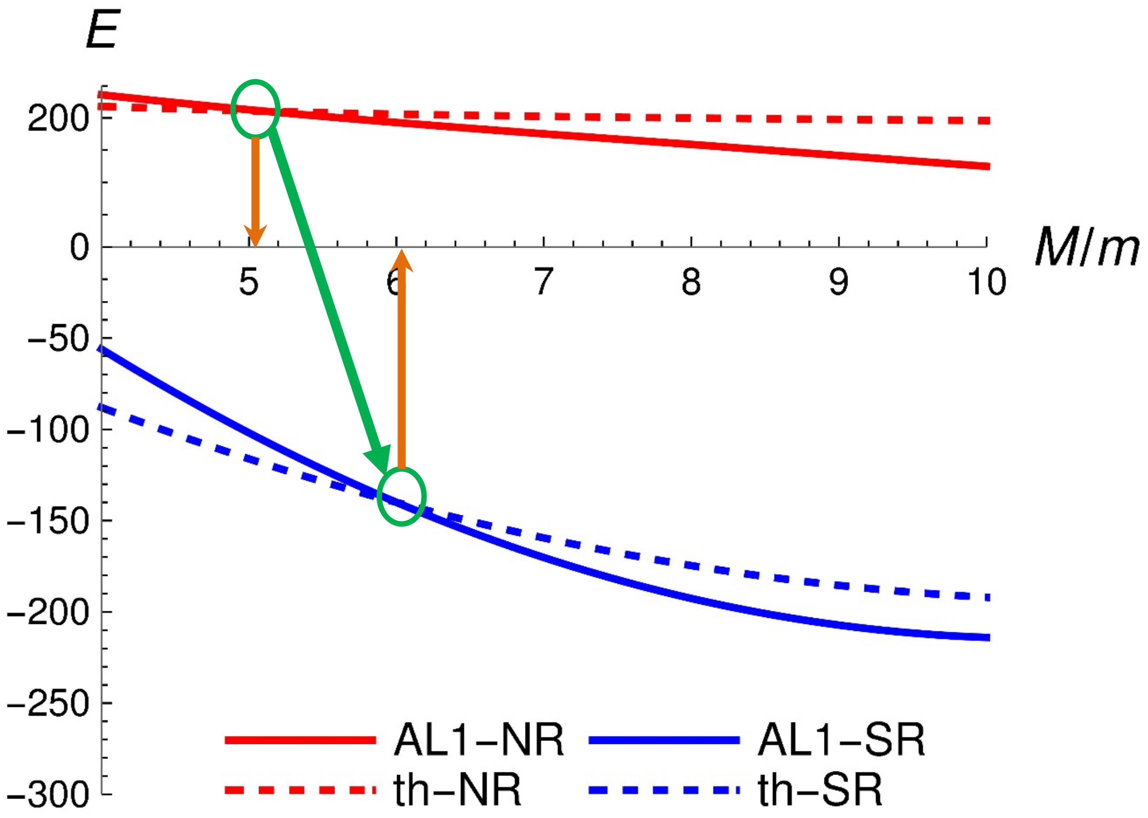
Will the relativistic kinematics increase the number of stable multiquarks?



PHYSICAL REVIEW D **103,** 054020 (2021)

Effect of relativistic kinematics on the stability of multiquarks

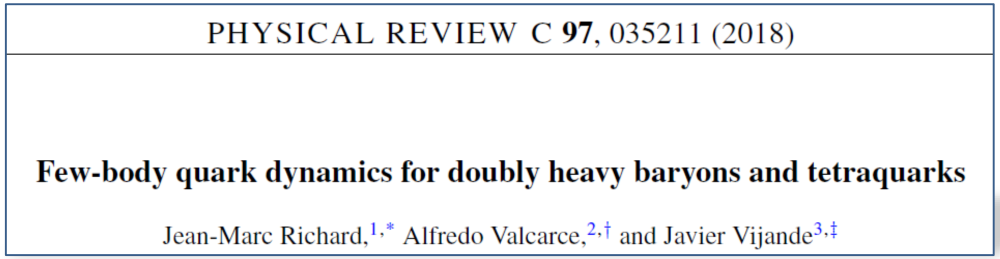
Jean-Marc Richard* Alfredo Valcarce† Javier Vijande‡



In this case the threshold is made of two *(qQ)* mesons while in the four-quark state there are *(qq)*, *(QQ)* and four *(qQ)* interactions. Who will benefit more from the relativistic dynamics?

We consider the AL1 potential properly re-parametrized in the SR case for keeping the description of the meson spectra.

Few-body dynamics

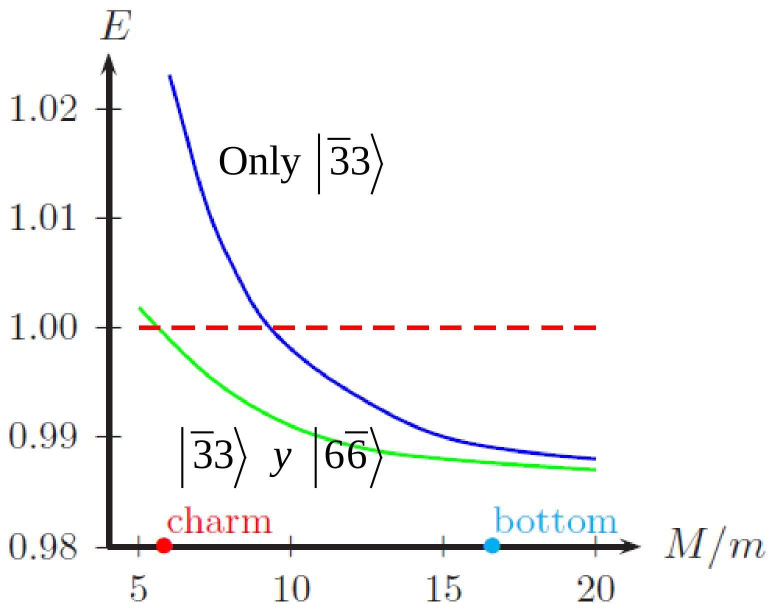


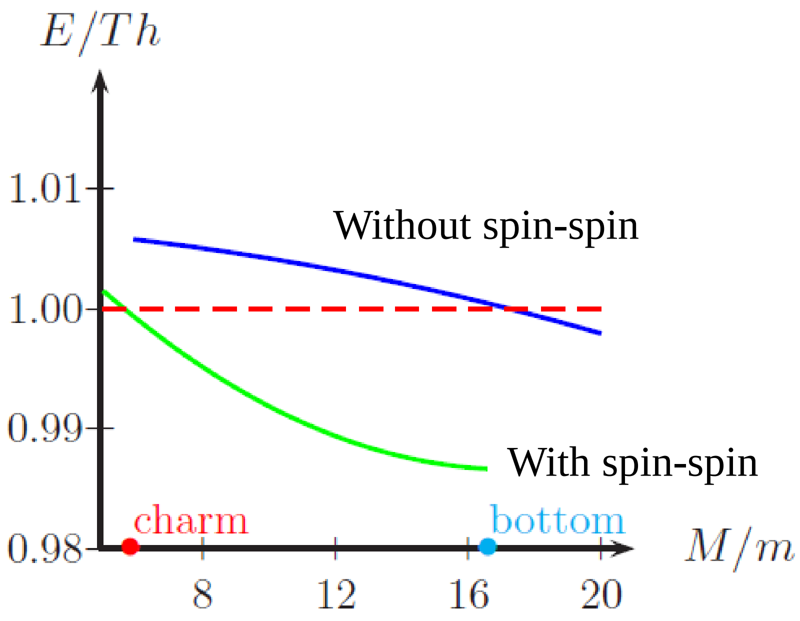
PHYSICAL REVIEW C **97**, 035211 (2018)

Few-body quark dynamics for doubly heavy baryons and tetraquarks

Jean-Marc Richard,[1,*] Alfredo Valcarce,[2,†] and Javier Vijande[3,‡]

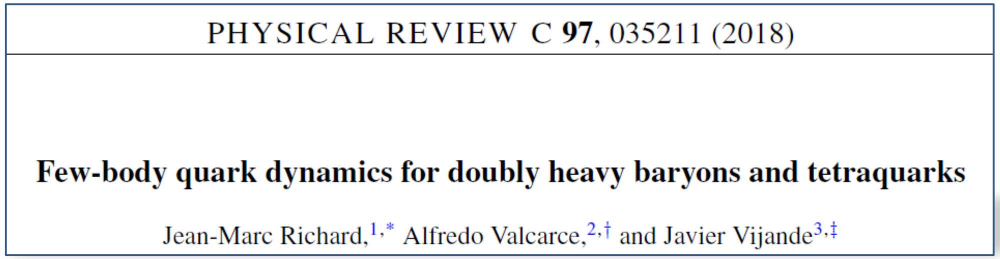
A very delicate interplay between color and spin configurations.





$QQ\overline{qq}$ using the AL1 model

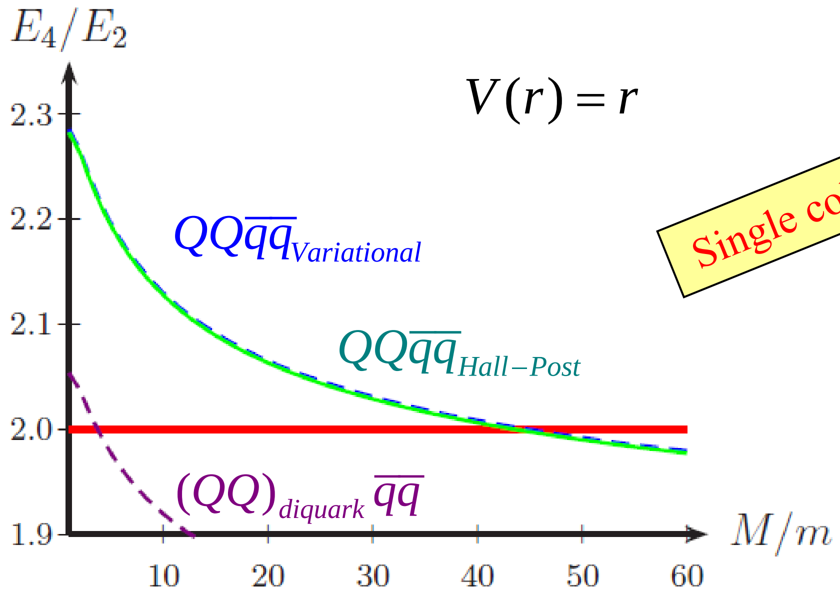
Few-body dynamics


PHYSICAL REVIEW C **97**, 035211 (2018)

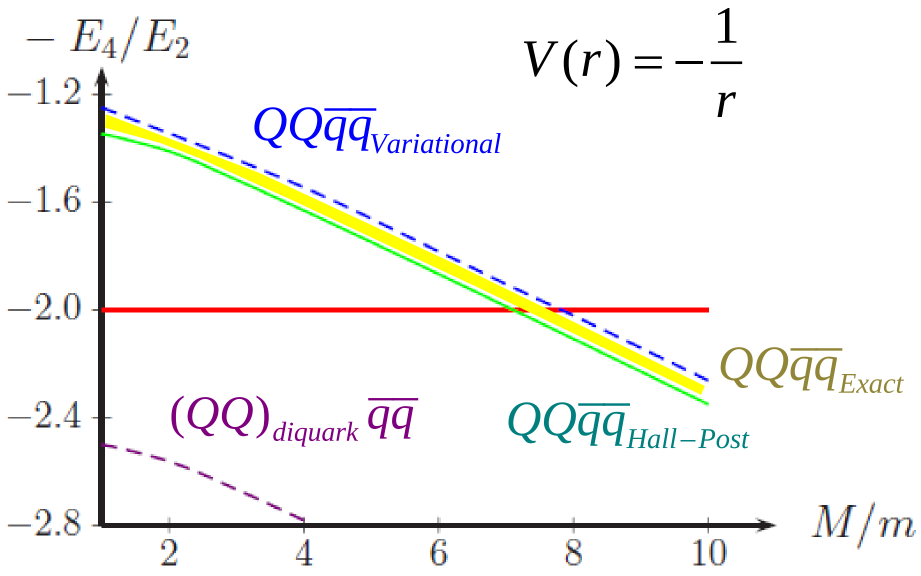
Few-body quark dynamics for doubly heavy baryons and tetraquarks

Jean-Marc Richard,[1,*] Alfredo Valcarce,[2,†] and Javier Vijande[3,‡]

The treatment of the four-body dynamics for double-charm tetraquarks is discussed. The **variational** and **Born-Oppenheimer** approximations together with the **Hall-Post** inequalities give energies very **close to the exact ones**, while the **diquark** approximation might be **more problematic**.

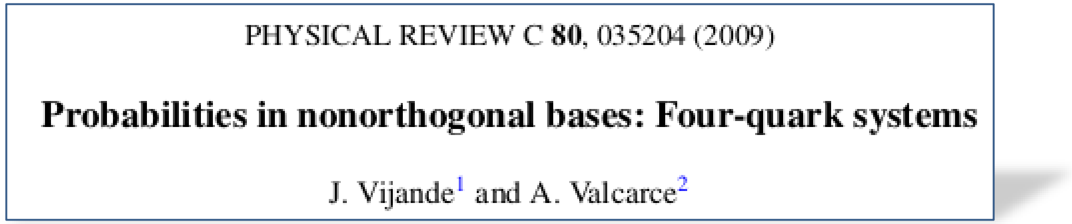


Single color channel



VNIVERSITAT DE VALÈNCIA

What about the probabilities?



PHYSICAL REVIEW C **80**, 035204 (2009)

Probabilities in nonorthogonal bases: Four-quark systems

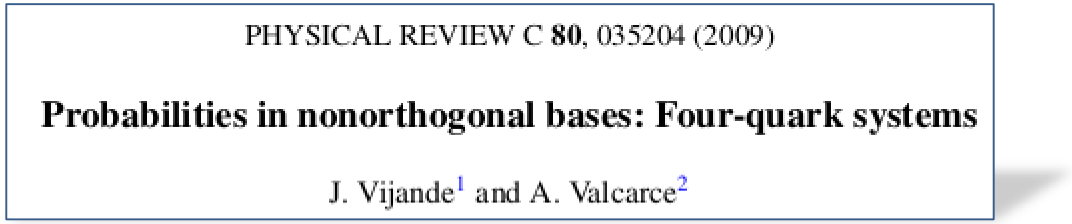
J. Vijande[1] and A. Valcarce[2]

$$\left[(q_1q_2)(\bar{q}_3\bar{q}_4)\right] \equiv \left\{ \left|\bar{3}_{12}3_{34}\right\rangle, \left|6_{12}\bar{6}_{34}\right\rangle \right\} \equiv \left\{ \left|\bar{3}3\right\rangle_c^{12}, \left|6\bar{6}\right\rangle_c^{12} \right\}$$

$$\left[(q_1\bar{q}_3)(q_2\bar{q}_4)\right] \equiv \left\{ \left|1_{13}1_{24}\right\rangle, \left|8_{13}8_{24}\right\rangle \right\} \equiv \left\{ \left|11\right\rangle_c, \left|88\right\rangle_c \right\}$$

$$\left[(q_1\bar{q}_4)(q_2\bar{q}_3)\right] \equiv \left\{ \left|1_{14}1_{23}\right\rangle, \left|8_{14}8_{23}\right\rangle \right\} \equiv \left\{ \left|1'1'\right\rangle_c, \left|8'8'\right\rangle_c \right\}$$

What about the probabilities?



PHYSICAL REVIEW C **80**, 035204 (2009)

Probabilities in nonorthogonal bases: Four-quark systems

J. Vijande[1] and A. Valcarce[2]

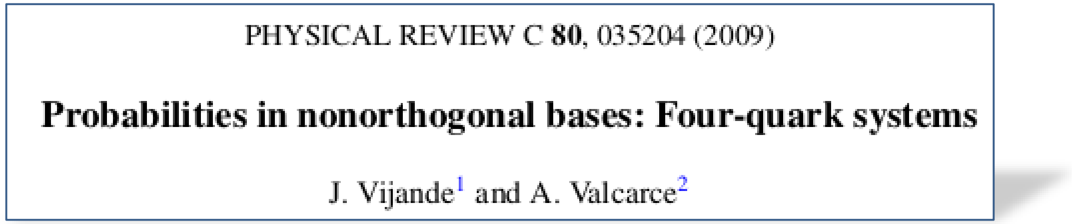
$$\left[(q_1q_2)(\bar{q}_3\bar{q}_4)\right]\equiv\left\{\left|\bar{3}_{12}3_{34}\right\rangle,\left|6_{12}\bar{6}_{34}\right\rangle\right\}\equiv\left\{\left|\bar{3}3\right\rangle_c^{12},\left|6\bar{6}\right\rangle_c^{12}\right\}$$

Calculations

$$\left[(q_1\bar{q}_3)(q_2\bar{q}_4)\right]\equiv\left\{\left|1_{13}1_{24}\right\rangle,\left|8_{13}8_{24}\right\rangle\right\}\equiv\left\{\left|11\right\rangle_c,\left|88\right\rangle_c\right\}$$

$$\left[(q_1\bar{q}_4)(q_2\bar{q}_3)\right]\equiv\left\{\left|1_{14}1_{23}\right\rangle,\left|8_{14}8_{23}\right\rangle\right\}\equiv\left\{\left|1'1'\right\rangle_c,\left|8'8'\right\rangle_c\right\}$$

What about the probabilities?



PHYSICAL REVIEW C **80**, 035204 (2009)

Probabilities in nonorthogonal bases: Four-quark systems

J. Vijande[1] and A. Valcarce[2]

$$\left[(q_1 q_2)(\bar{q}_3 \bar{q}_4)\right] \equiv \left\{ \left|\bar{3}_{12} 3_{34}\right\rangle, \left|6_{12} \bar{6}_{34}\right\rangle \right\} \equiv \left\{ \left|\bar{3}3\right\rangle_c^{12}, \left|6\bar{6}\right\rangle_c^{12} \right\}$$

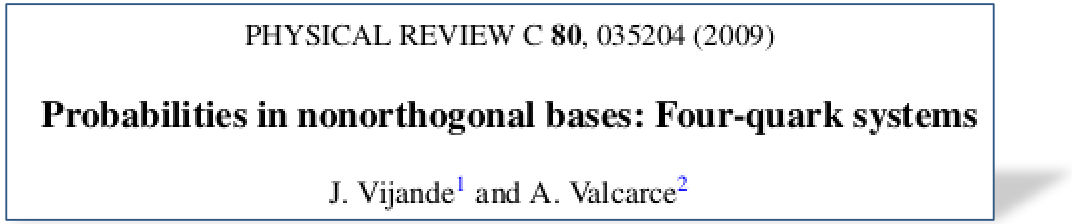
Calculations

$$\left[(q_1 \bar{q}_3)(q_2 \bar{q}_4)\right] \equiv \left\{ \left|1_{13} 1_{24}\right\rangle, \left|8_{13} 8_{24}\right\rangle \right\} \equiv \left\{ \left|11\right\rangle_c, \left|88\right\rangle_c \right\}$$

$$\left[(q_1 \bar{q}_4)(q_2 \bar{q}_3)\right] \equiv \left\{ \left|1_{14} 1_{23}\right\rangle, \left|8_{14} 8_{23}\right\rangle \right\} \equiv \left\{ \left|1'1'\right\rangle_c, \left|8'8'\right\rangle_c \right\}$$

Physical interpretation

What about the probabilities?


PHYSICAL REVIEW C **80**, 035204 (2009)

Probabilities in nonorthogonal bases: Four-quark systems

J. Vijande[1] and A. Valcarce[2]

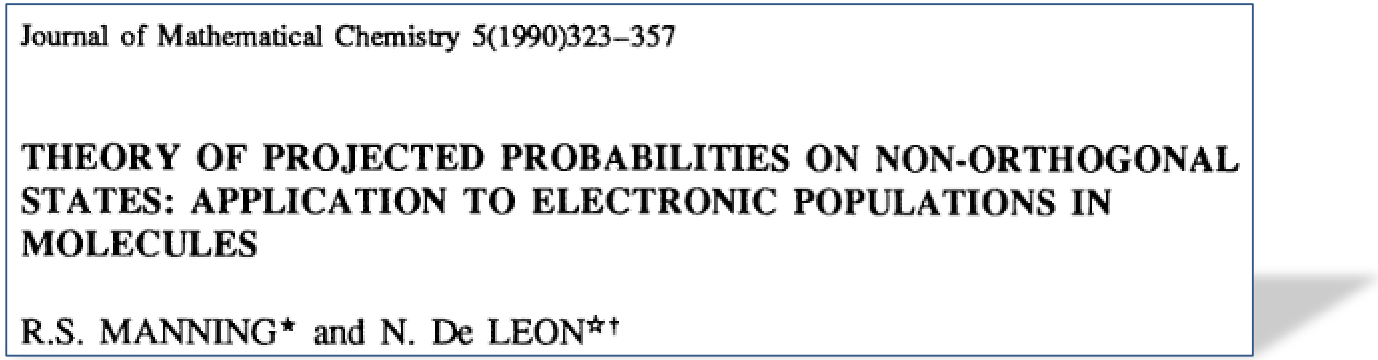
$$\left[(q_1q_2)(\bar q_3\bar q_4)\right] \equiv \left\{ |\bar 3_{12}3_{34}\rangle, |6_{12}\bar 6_{34}\rangle \right\} \equiv \left\{ |\bar 3 3\rangle_c^{12}, |6\bar 6\rangle_c^{12} \right\}$$

Calculations

$$\left[(q_1\bar q_3)(q_2\bar q_4)\right] \equiv \left\{ |1_{13}1_{24}\rangle, |8_{13}8_{24}\rangle \right\} \equiv \left\{ |11\rangle_c, |88\rangle_c \right\}$$

$$\left[(q_1\bar q_4)(q_2\bar q_3)\right] \equiv \left\{ |1_{14}1_{23}\rangle, |8_{14}8_{23}\rangle \right\} \equiv \left\{ |1'1'\rangle_c, |8'8'\rangle_c \right\}$$

Physical interpretation

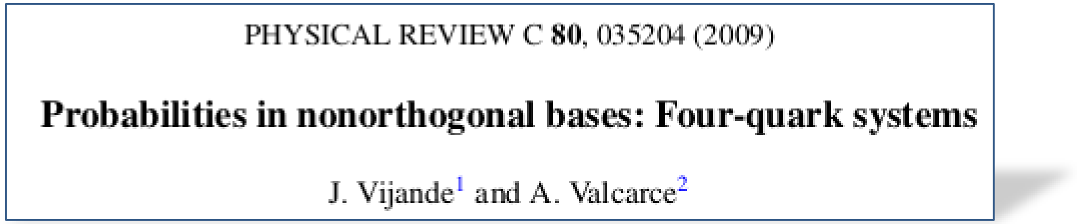

Journal of Mathematical Chemistry 5(1990)323–357

THEORY OF PROJECTED PROBABILITIES ON NON-ORTHOGONAL STATES: APPLICATION TO ELECTRONIC POPULATIONS IN MOLECULES

R.S. MANNING* and N. De LEON*†

$$\begin{aligned}
|\Psi\rangle &= \alpha|11\rangle_c + \beta|88\rangle_c \\
&= \alpha|11\rangle_c + \beta\left(\chi|1'1'\rangle_c + \delta|8'8'\rangle_c\right) \\
&= \alpha|11\rangle_c + \gamma|1'1'\rangle_c + \mu|8'8'\rangle_c \\
&= \alpha|11\rangle_c + \gamma|1'1'\rangle_c + \mu\left(\varpi|11\rangle_c + \varsigma|88\rangle_c\right) \\
&= \ldots\ldots = \wp_{|11\rangle_c}|\Psi\rangle + \wp_{|1'1'\rangle_c}|\Psi\rangle
\end{aligned}$$

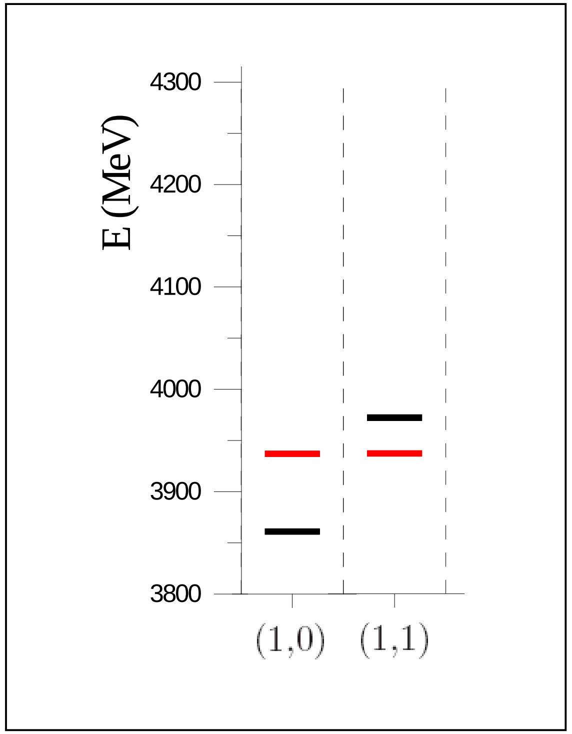
What about the probabilities?

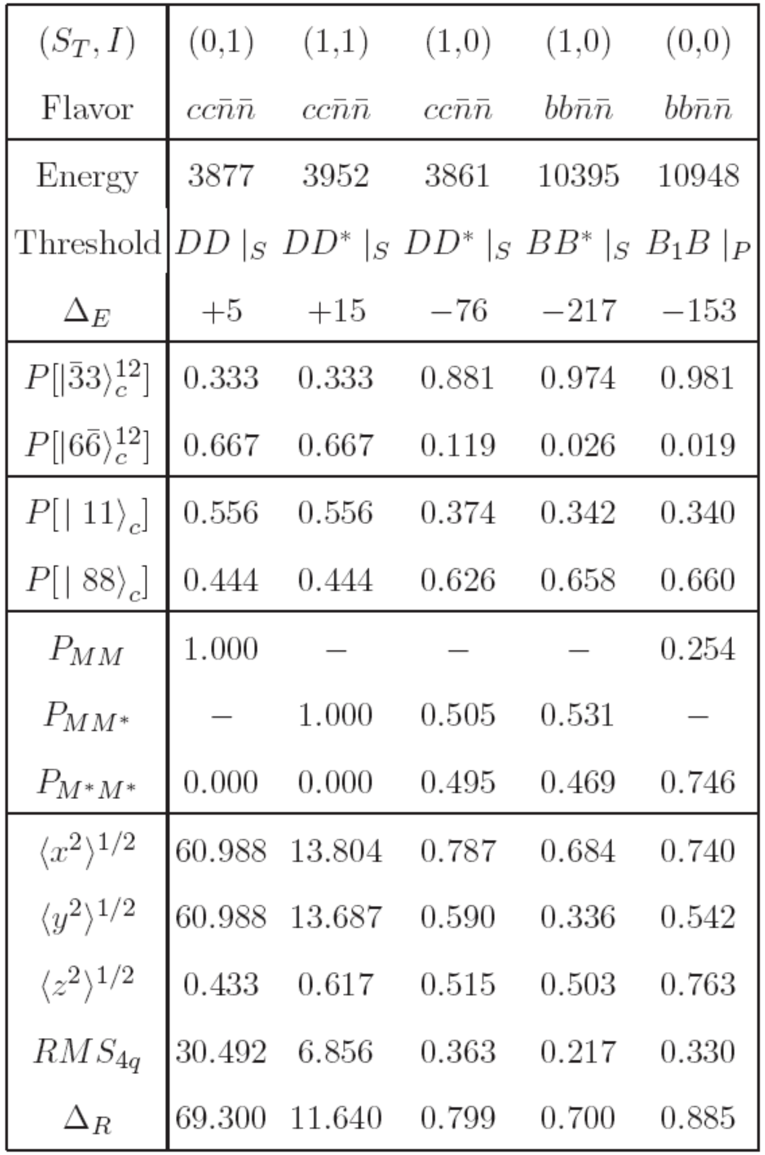


PHYSICAL REVIEW C **80**, 035204 (2009)

Probabilities in nonorthogonal bases: Four-quark systems

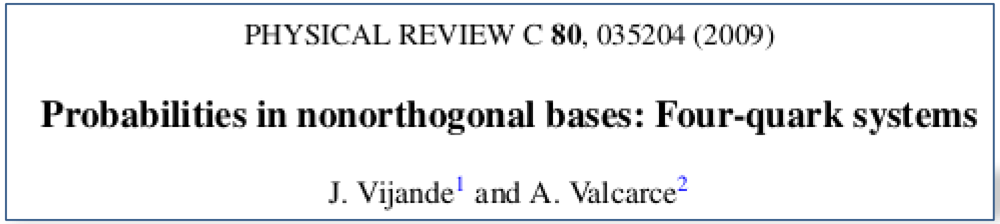
J. Vijande[1] and A. Valcarce[2]





(S_T, I)	(0,1)	(1,1)	(1,0)	(1,0)	(0,0)
Flavor	$cc\bar n\bar n$	$cc\bar n\bar n$	$cc\bar n\bar n$	$bb\bar n\bar n$	$bb\bar n\bar n$
Energy	3877	3952	3861	10395	10948
Threshold	$DD\,\|_S$	$DD^*\,\|_S$	$DD^*\,\|_S$	$BB^*\,\|_S$	$B_1B\,\|_P$
Δ_E	+5	+15	−76	−217	−153
$P[\|\bar 33\rangle_c^{12}]$	0.333	0.333	0.881	0.974	0.981
$P[\|6\bar 6\rangle_c^{12}]$	0.667	0.667	0.119	0.026	0.019
$P[\|\,11\rangle_c]$	0.556	0.556	0.374	0.342	0.340
$P[\|\,88\rangle_c]$	0.444	0.444	0.626	0.658	0.660
P_{MM}	1.000	–	–	–	0.254
P_{MM^*}	–	1.000	0.505	0.531	–
$P_{M^*M^*}$	0.000	0.000	0.495	0.469	0.746
$\langle x^2\rangle^{1/2}$	60.988	13.804	0.787	0.684	0.740
$\langle y^2\rangle^{1/2}$	60.988	13.687	0.590	0.336	0.542
$\langle z^2\rangle^{1/2}$	0.433	0.617	0.515	0.503	0.763
RMS_{4q}	30.492	6.856	0.363	0.217	0.330
Δ_R	69.300	11.640	0.799	0.700	0.885

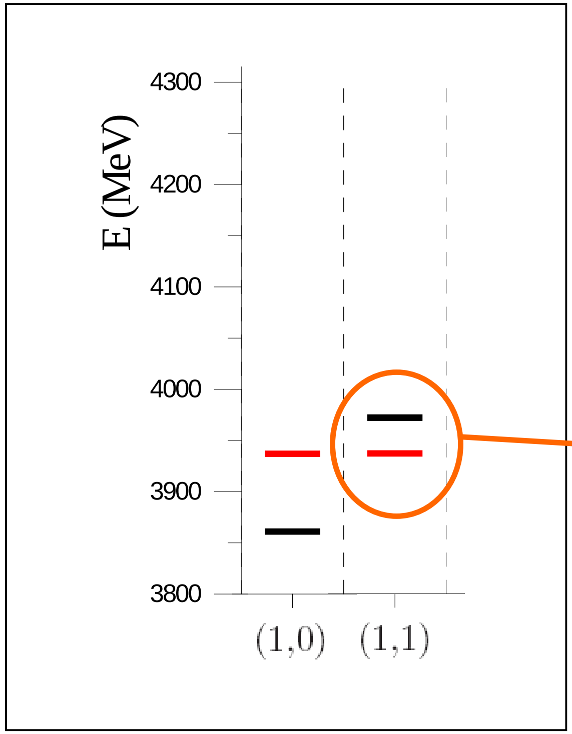
What about the probabilities?



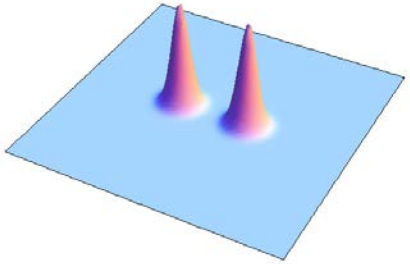
PHYSICAL REVIEW C **80**, 035204 (2009)

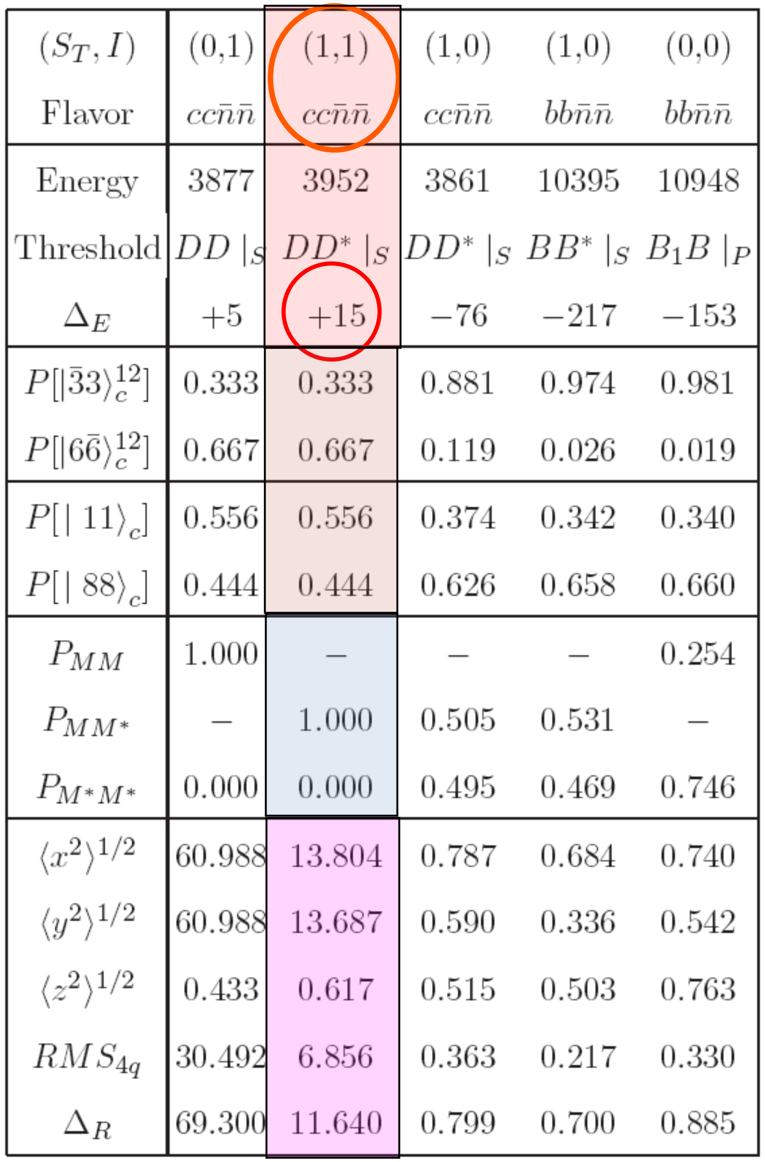
Probabilities in nonorthogonal bases: Four-quark systems

J. Vijande[1] and A. Valcarce[2]



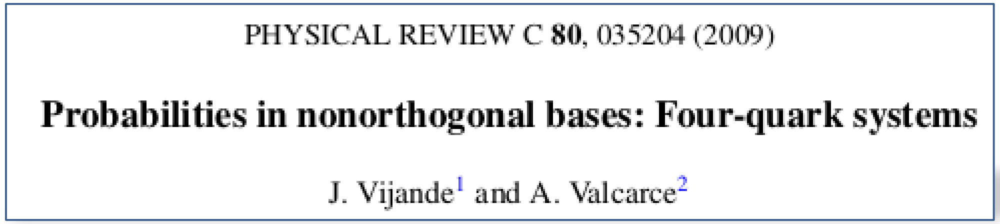
Unbound





(S_T, I)	(0,1)	(1,1)	(1,0)	(1,0)	(0,0)
Flavor	$cc\bar{n}\bar{n}$	$cc\bar{n}\bar{n}$	$cc\bar{n}\bar{n}$	$bb\bar{n}\bar{n}$	$bb\bar{n}\bar{n}$
Energy	3877	3952	3861	10395	10948
Threshold	$DD\,\|_S$	$DD^*\,\|_S$	$DD^*\,\|_S$	$BB^*\,\|_S$	$B_1B\,\|_P$
Δ_E	+5	+15	−76	−217	−153
$P[\|\bar{3}3\rangle_c^{12}]$	0.333	0.333	0.881	0.974	0.981
$P[\|6\bar{6}\rangle_c^{12}]$	0.667	0.667	0.119	0.026	0.019
$P[\|\,11\rangle_c]$	0.556	0.556	0.374	0.342	0.340
$P[\|\,88\rangle_c]$	0.444	0.444	0.626	0.658	0.660
P_{MM}	1.000	–	–	–	0.254
P_{MM^*}	–	1.000	0.505	0.531	–
$P_{M^*M^*}$	0.000	0.000	0.495	0.469	0.746
$\langle x^2\rangle^{1/2}$	60.988	13.804	0.787	0.684	0.740
$\langle y^2\rangle^{1/2}$	60.988	13.687	0.590	0.336	0.542
$\langle z^2\rangle^{1/2}$	0.433	0.617	0.515	0.503	0.763
RMS_{4q}	30.492	6.856	0.363	0.217	0.330
Δ_R	69.300	11.640	0.799	0.700	0.885

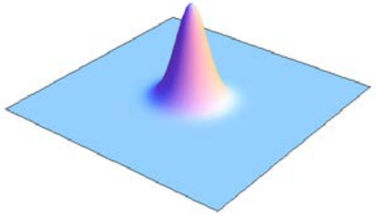
What about the probabilities?



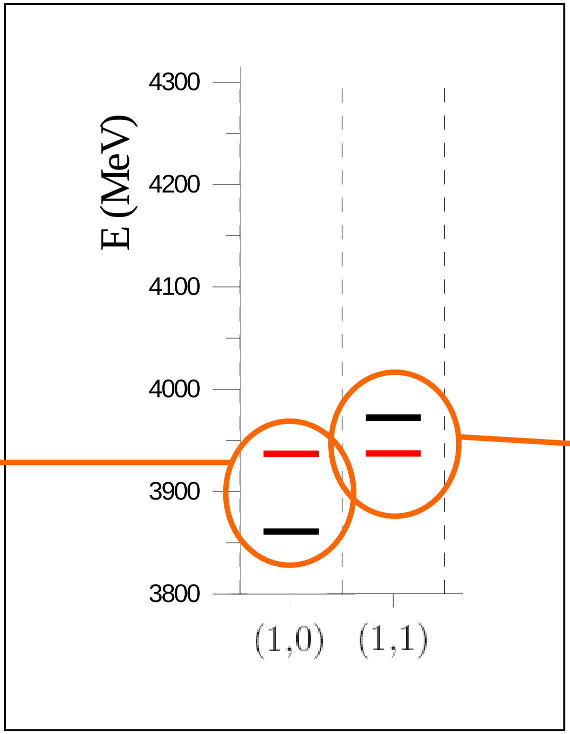
PHYSICAL REVIEW C **80**, 035204 (2009)

Probabilities in nonorthogonal bases: Four-quark systems

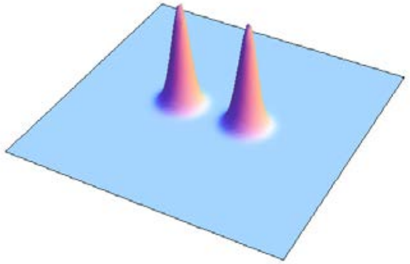
J. Vijande[1] and A. Valcarce[2]

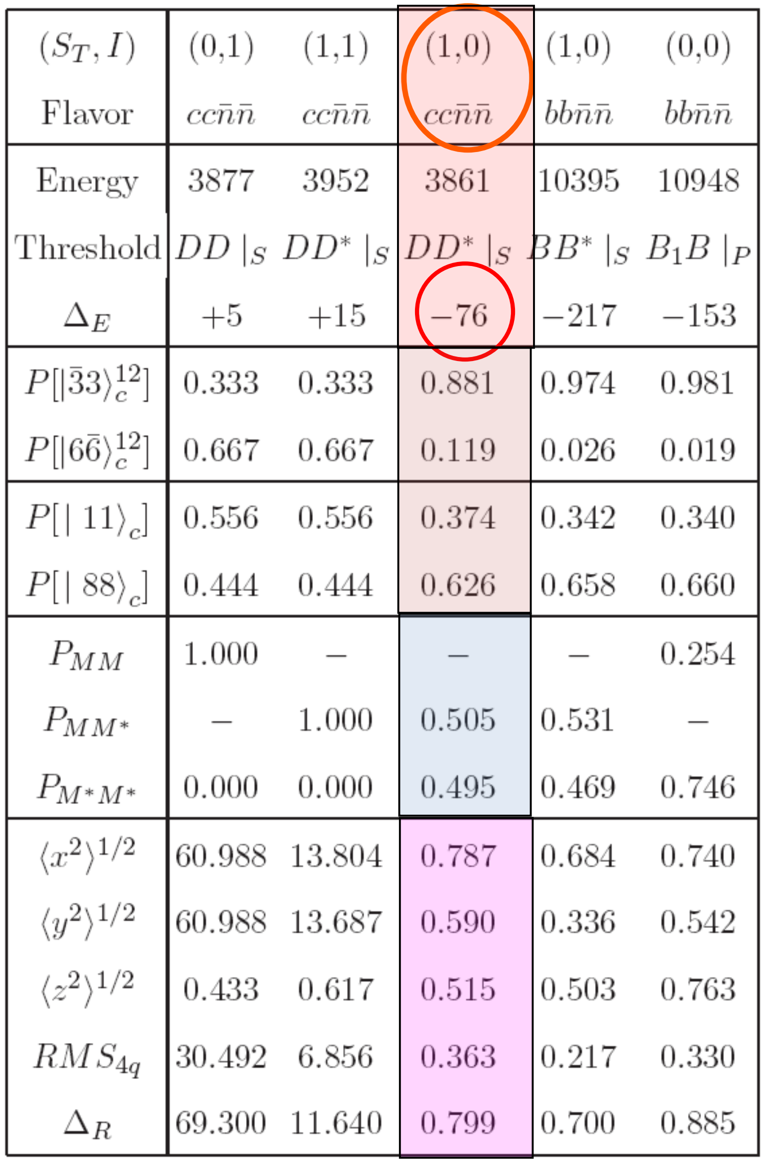


Bound



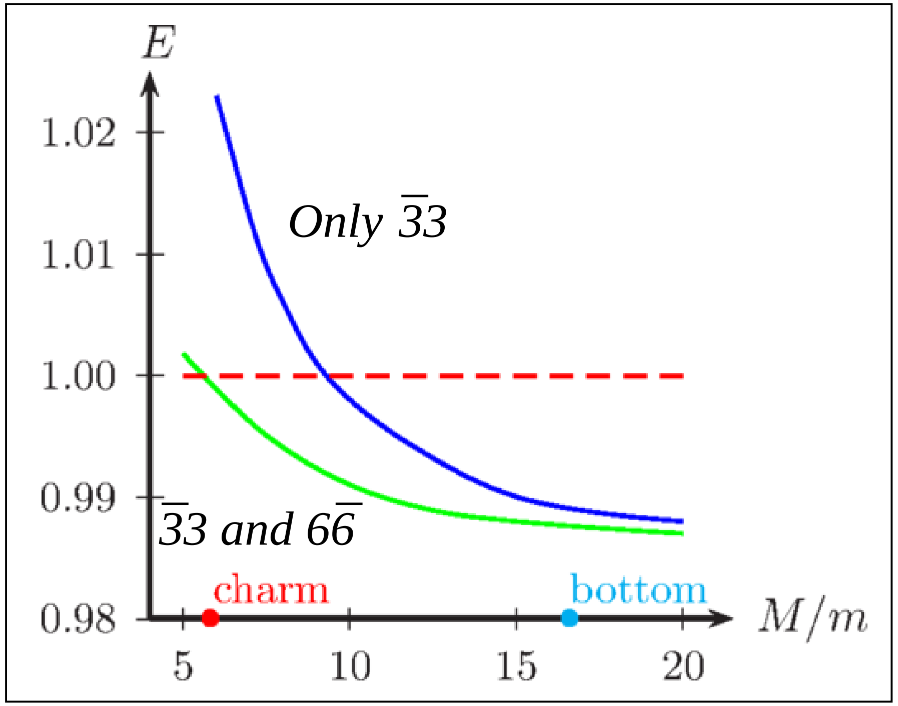
Unbound

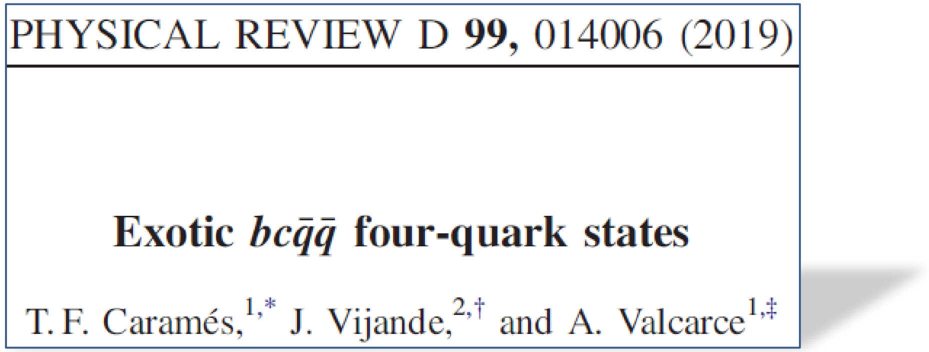




(S_T, I)	(0,1)	(1,1)	(1,0)	(1,0)	(0,0)
Flavor	$cc\bar{n}\bar{n}$	$cc\bar{n}\bar{n}$	$cc\bar{n}\bar{n}$	$bb\bar{n}\bar{n}$	$bb\bar{n}\bar{n}$
Energy	3877	3952	3861	10395	10948
Threshold	$DD\|_S$	$DD^*\|_S$	$DD^*\|_S$	$BB^*\|_S$	$B_1B\|_P$
Δ_E	$+5$	$+15$	-76	-217	-153
$P[\|\bar{3}3\rangle_c^{12}]$	0.333	0.333	0.881	0.974	0.981
$P[\|6\bar{6}\rangle_c^{12}]$	0.667	0.667	0.119	0.026	0.019
$P[\|11\rangle_c]$	0.556	0.556	0.374	0.342	0.340
$P[\|88\rangle_c]$	0.444	0.444	0.626	0.658	0.660
P_{MM}	1.000	–	–	–	0.254
P_{MM^*}	–	1.000	0.505	0.531	–
$P_{M^*M^*}$	0.000	0.000	0.495	0.469	0.746
$\langle x^2\rangle^{1/2}$	60.988	13.804	0.787	0.684	0.740
$\langle y^2\rangle^{1/2}$	60.988	13.687	0.590	0.336	0.542
$\langle z^2\rangle^{1/2}$	0.433	0.617	0.515	0.503	0.763
RMS_{4q}	30.492	6.856	0.363	0.217	0.330
Δ_R	69.300	11.640	0.799	0.700	0.885

Pauli Principle

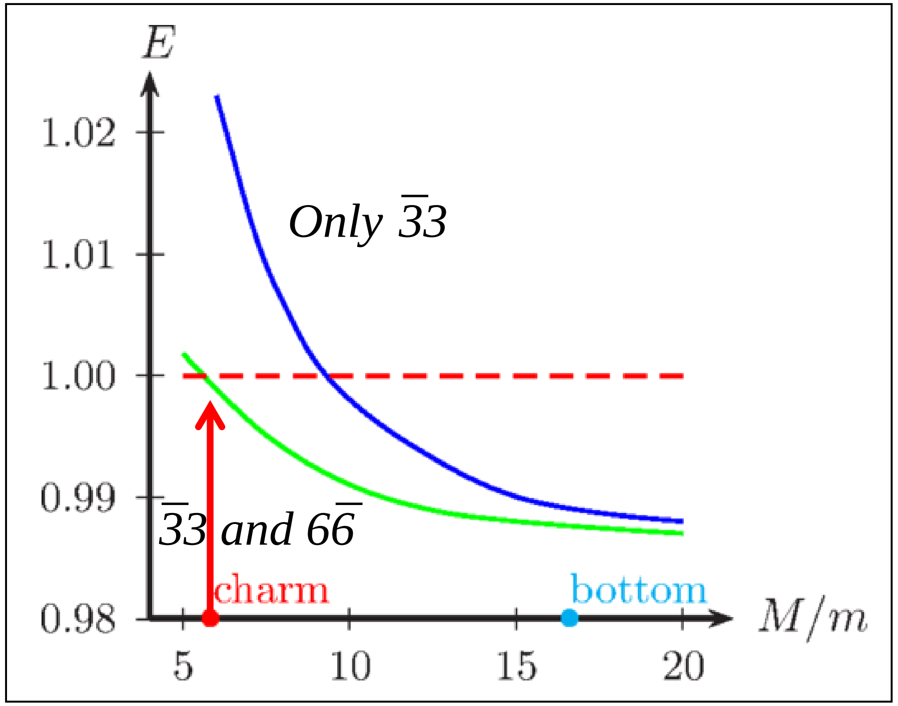


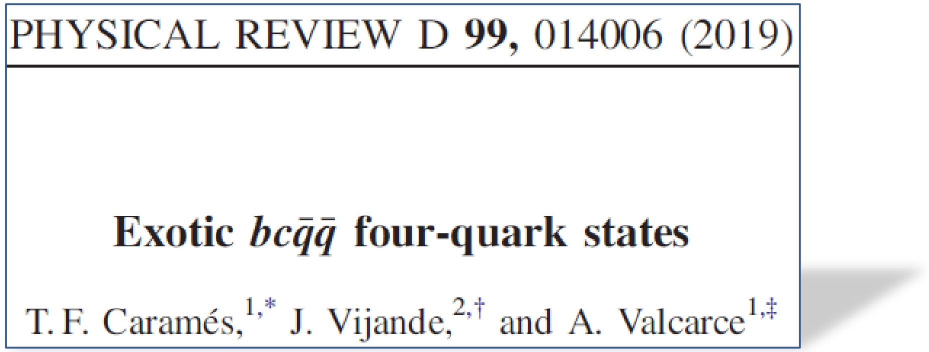

PHYSICAL REVIEW D **99,** 014006 (2019)

Exotic $bc\bar{q}\bar{q}$ four-quark states

T. F. Caramés,[1,*] J. Vijande,[2,†] and A. Valcarce[1,‡]

Pauli Principle



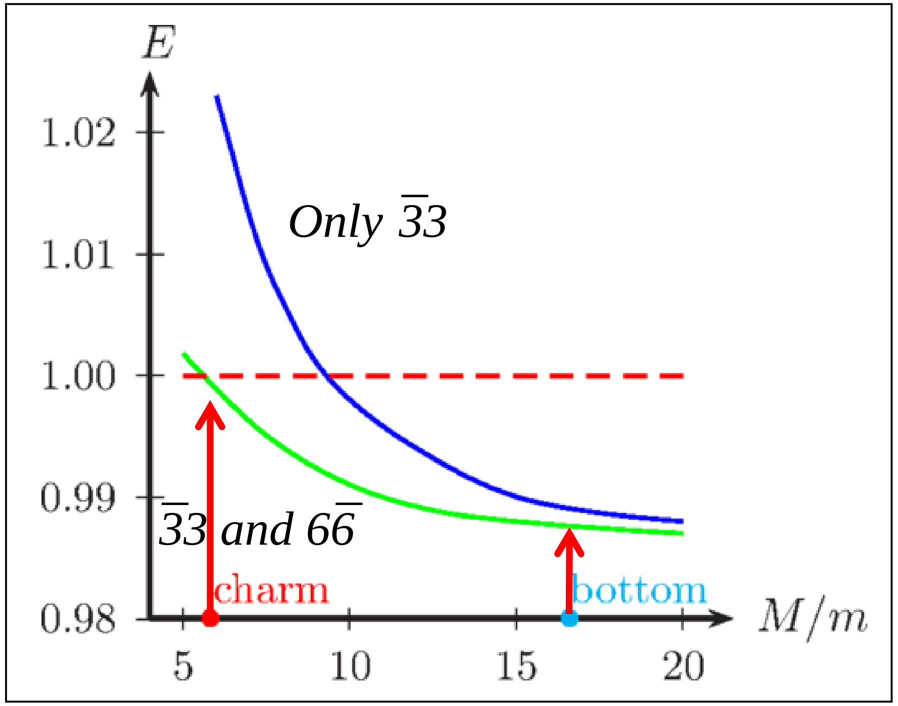

PHYSICAL REVIEW D **99**, 014006 (2019)

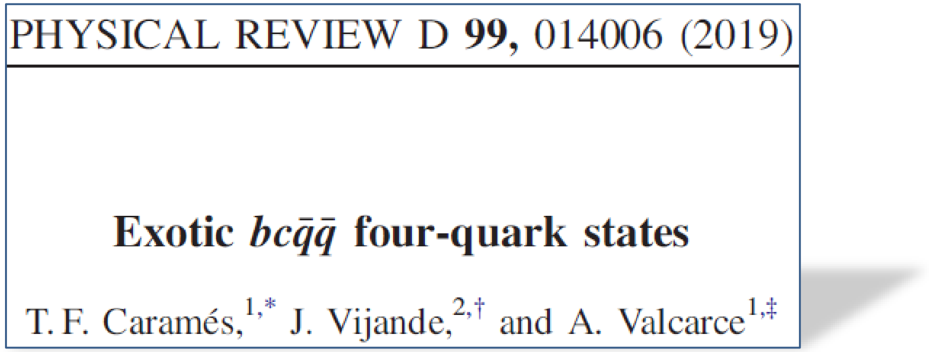
Exotic $bc\bar{q}\bar{q}$ four-quark states

T. F. Caramés,[1,*] J. Vijande,[2,†] and A. Valcarce[1,‡]

$$cc\overline{nn}$$

Pauli Principle




PHYSICAL REVIEW D **99**, 014006 (2019)

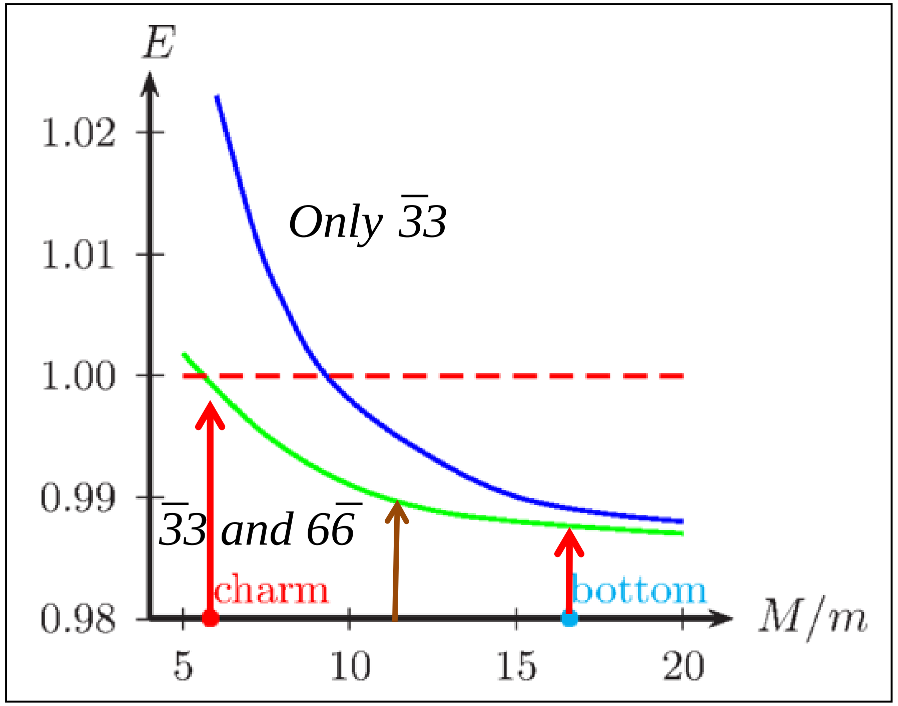
Exotic $bc\bar{q}\bar{q}$ four-quark states

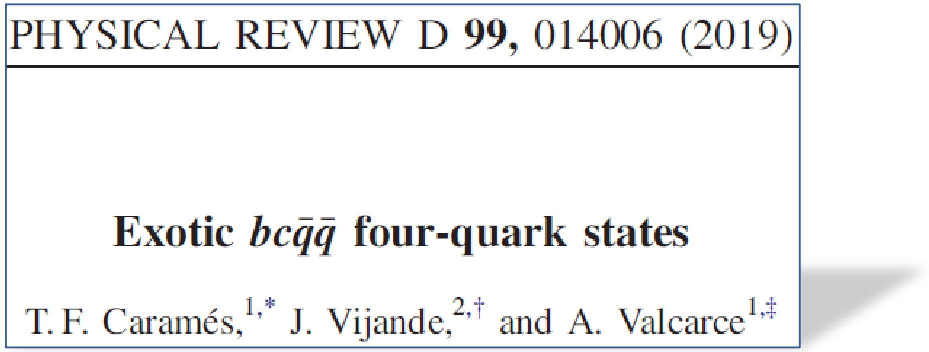
T. F. Caramés,[1,*] J. Vijande,[2,†] and A. Valcarce[1,‡]

$cc\overline{nn}$

$bb\overline{nn}$

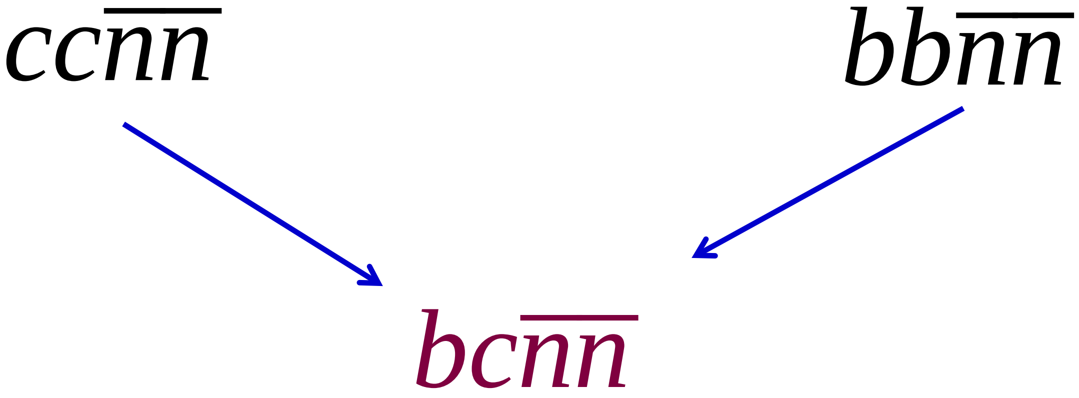
Pauli Principle




PHYSICAL REVIEW D **99,** 014006 (2019)

Exotic $bc\bar{q}\bar{q}$ four-quark states

T. F. Caramés,[1,*] J. Vijande,[2,†] and A. Valcarce[1,‡]

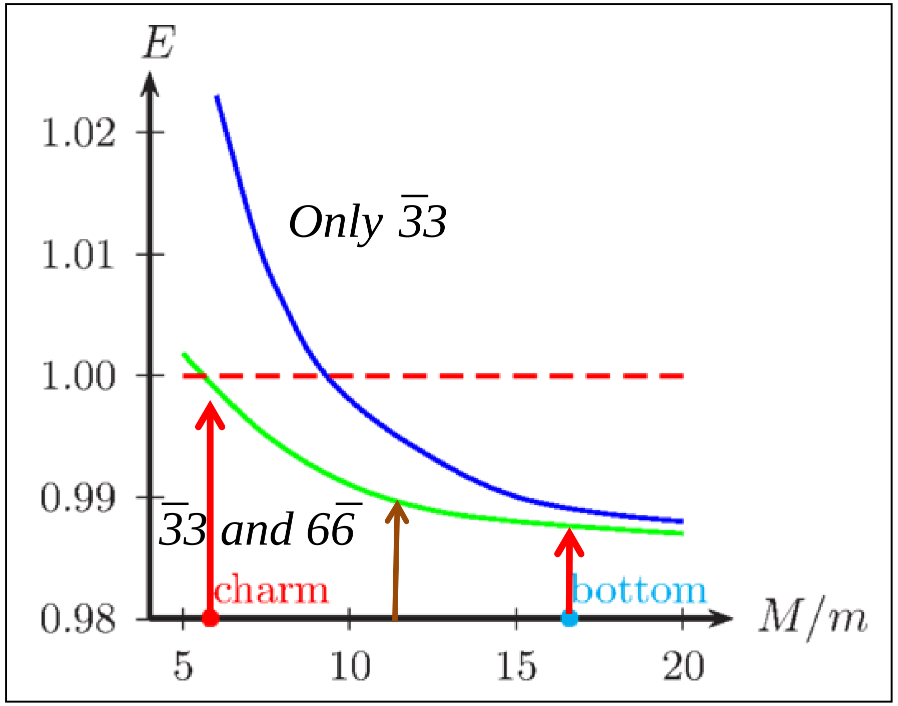

$cc\overline{nn}$

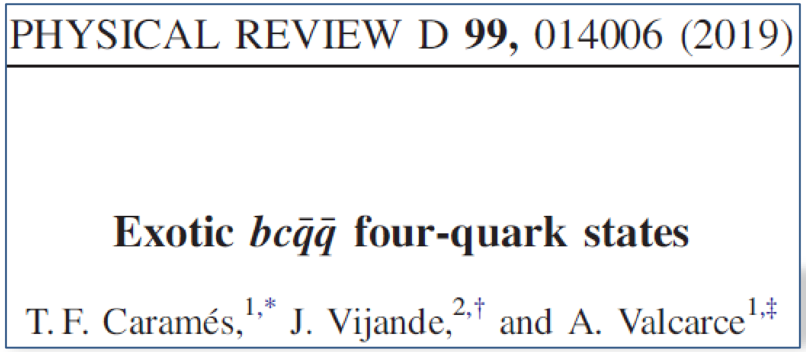
$bb\overline{nn}$

$bc\overline{nn}$

Is just an intermediate state?

Pauli Principle



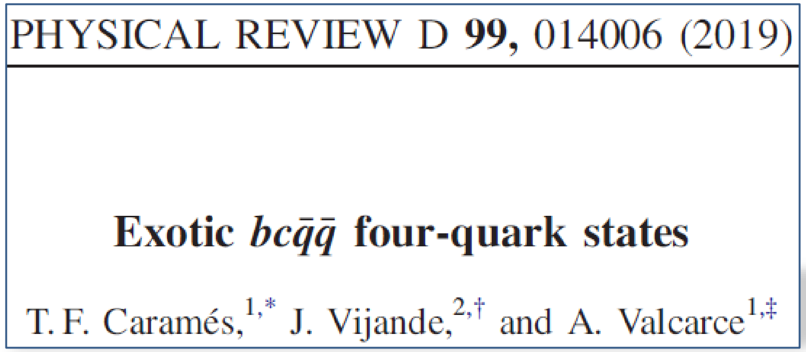


PHYSICAL REVIEW D **99**, 014006 (2019)

Exotic $bc\bar{q}\bar{q}$ four-quark states

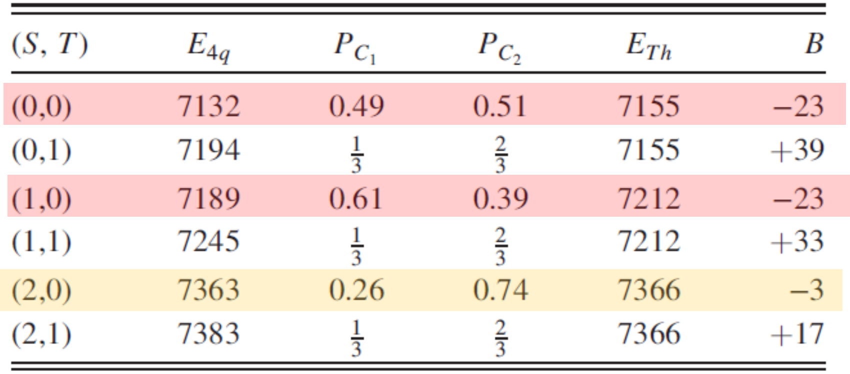
T. F. Caramés,[1,*] J. Vijande,[2,†] and A. Valcarce[1,‡]

Pauli Principle


PHYSICAL REVIEW D **99,** 014006 (2019)

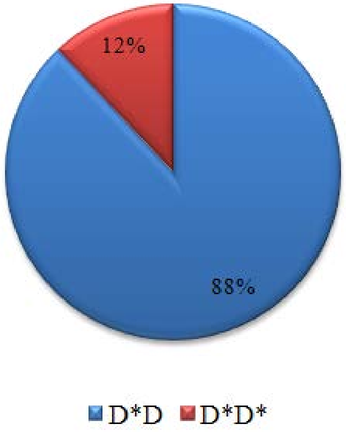
Exotic $bc\bar{q}\bar{q}$ four-quark states

T. F. Caramés,[1,*] J. Vijande,[2,†] and A. Valcarce[1,‡]

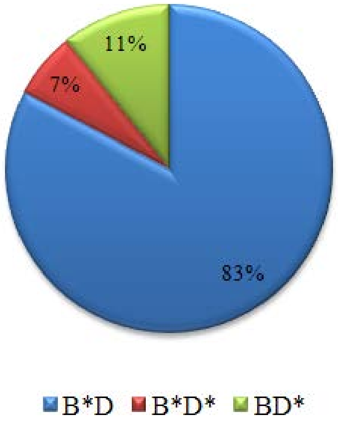


(S, T)	E_{4q}	P_{C_1}	P_{C_2}	E_{Th}	B
(0,0)	7132	0.49	0.51	7155	−23
(0,1)	7194	$\frac{1}{3}$	$\frac{2}{3}$	7155	+39
(1,0)	7189	0.61	0.39	7212	−23
(1,1)	7245	$\frac{1}{3}$	$\frac{2}{3}$	7212	+33
(2,0)	7363	0.26	0.74	7366	−3
(2,1)	7383	$\frac{1}{3}$	$\frac{2}{3}$	7366	+17

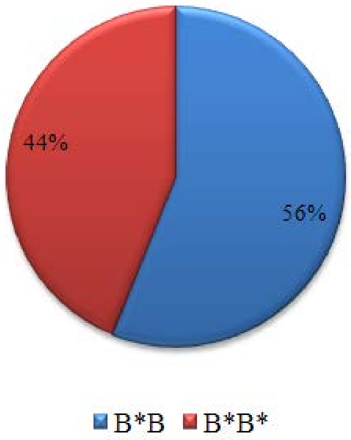
$cc\bar{n}\bar{n}$



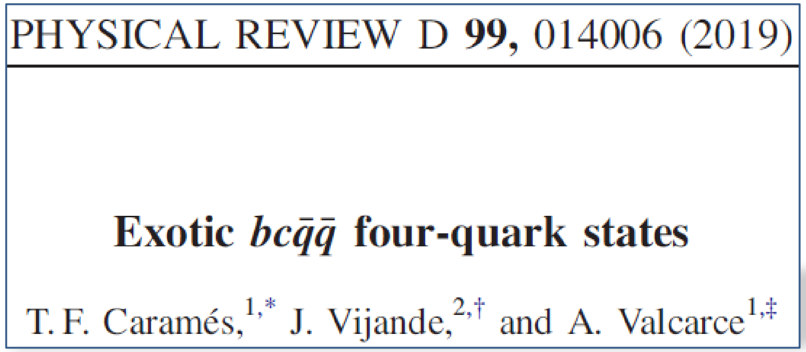
$bc\bar{n}\bar{n}$



$bb\bar{n}\bar{n}$



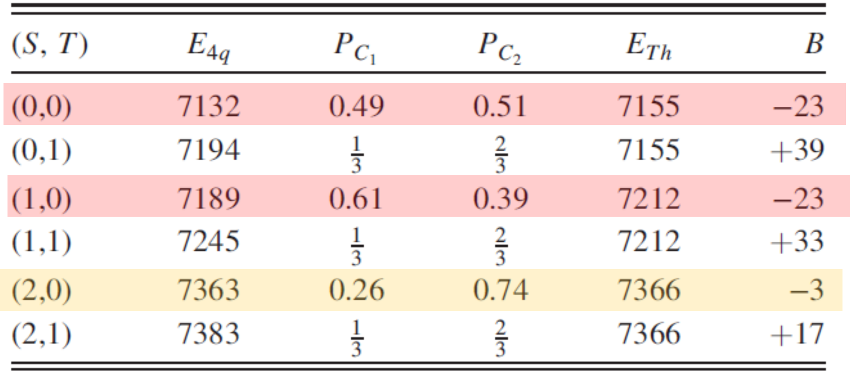
Pauli Principle



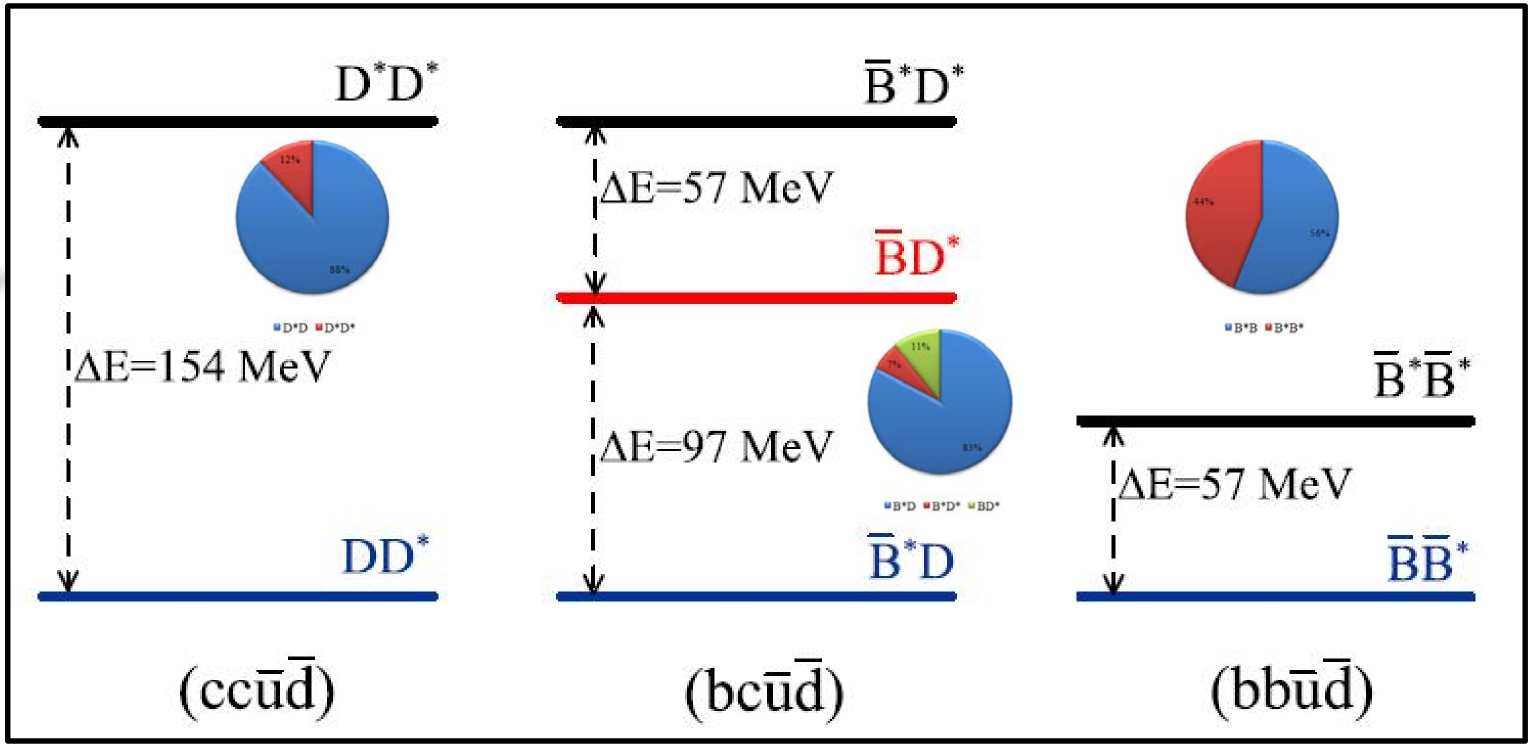
PHYSICAL REVIEW D **99,** 014006 (2019)

Exotic $bc\bar{q}\bar{q}$ four-quark states

T. F. Caramés,[1,*] J. Vijande,[2,†] and A. Valcarce[1,‡]

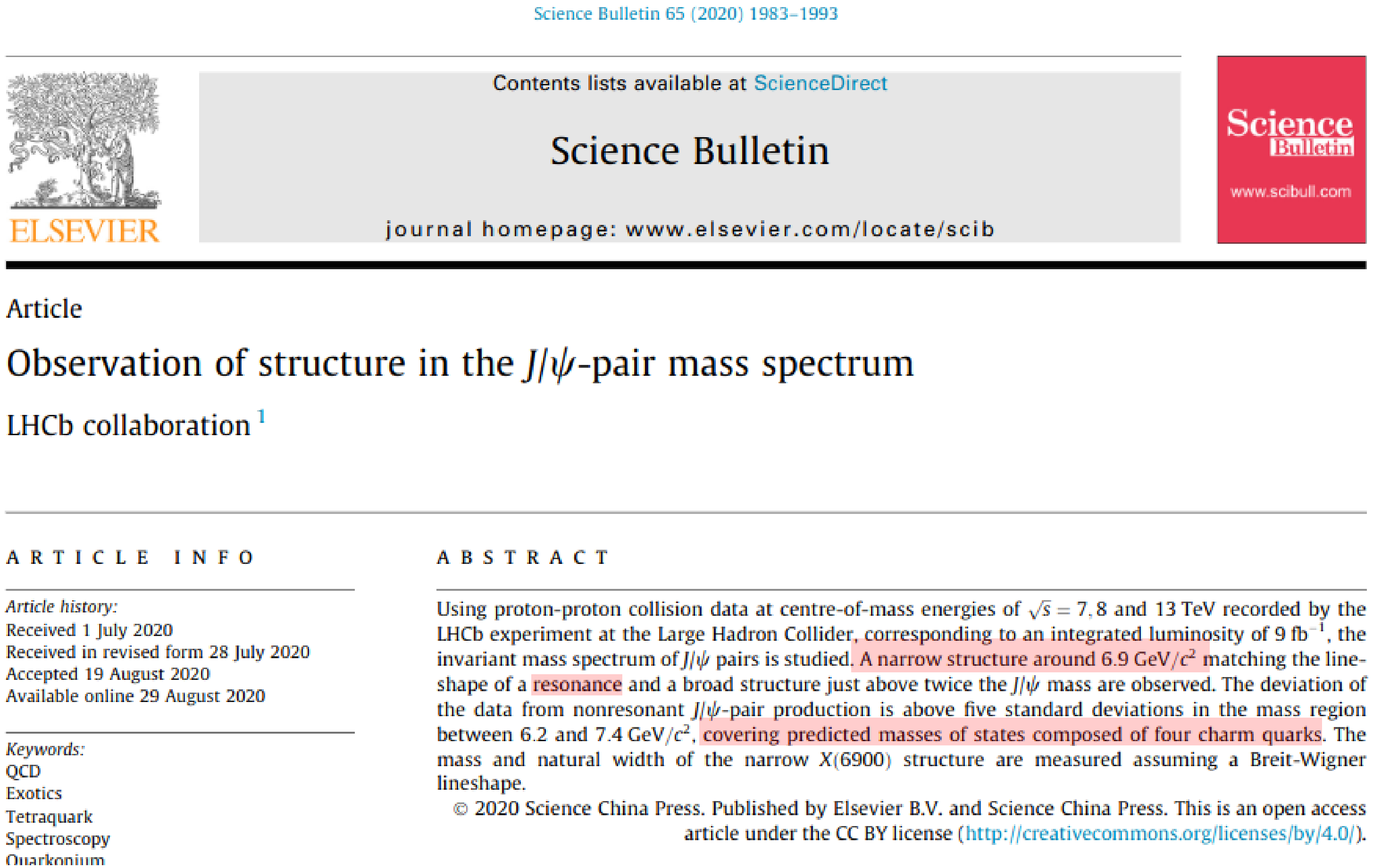


(S, T)	E_{4q}	P_{C_1}	P_{C_2}	E_{Th}	B
(0,0)	7132	0.49	0.51	7155	−23
(0,1)	7194	$\frac{1}{3}$	$\frac{2}{3}$	7155	+39
(1,0)	7189	0.61	0.39	7212	−23
(1,1)	7245	$\frac{1}{3}$	$\frac{2}{3}$	7212	+33
(2,0)	7363	0.26	0.74	7366	−3
(2,1)	7383	$\frac{1}{3}$	$\frac{2}{3}$	7366	+17



What about resonances?

- Once again, quite a few experimental data….
- ATLAS and CMS collaborations has confirmed it.



Science Bulletin 65 (2020) 1983–1993

Contents lists available at ScienceDirect

Science Bulletin

journal homepage: www.elsevier.com/locate/scib

Article

Observation of structure in the J/ψ-pair mass spectrum

LHCb collaboration [1]

ARTICLE INFO

Article history:
Received 1 July 2020
Received in revised form 28 July 2020
Accepted 19 August 2020
Available online 29 August 2020

Keywords:
QCD
Exotics
Tetraquark
Spectroscopy
Quarkonium

ABSTRACT

Using proton-proton collision data at centre-of-mass energies of $\sqrt{s} = 7, 8$ and 13 TeV recorded by the LHCb experiment at the Large Hadron Collider, corresponding to an integrated luminosity of 9 fb^{-1}, the invariant mass spectrum of J/ψ pairs is studied. A narrow structure around 6.9 GeV/c^2 matching the lineshape of a resonance and a broad structure just above twice the J/ψ mass are observed. The deviation of the data from nonresonant J/ψ-pair production is above five standard deviations in the mass region between 6.2 and 7.4 GeV/c^2, covering predicted masses of states composed of four charm quarks. The mass and natural width of the narrow $X(6900)$ structure are measured assuming a Breit-Wigner lineshape.

© 2020 Science China Press. Published by Elsevier B.V. and Science China Press. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/).

What about resonances?

- Among the various possibilities to deal with resonances in the four-quark sector scaling methods, real or complex, are being widely used nowadays.
- Numerical techniques borrowed from molecular, nuclear and/or atomic physics are being used.

Few-Body Syst (2023) 64:24
https://doi.org/10.1007/s00601-023-01808-x

Rimantas Lazauskas

Numerical Aspects of Resonant States in Quantum Mechanics

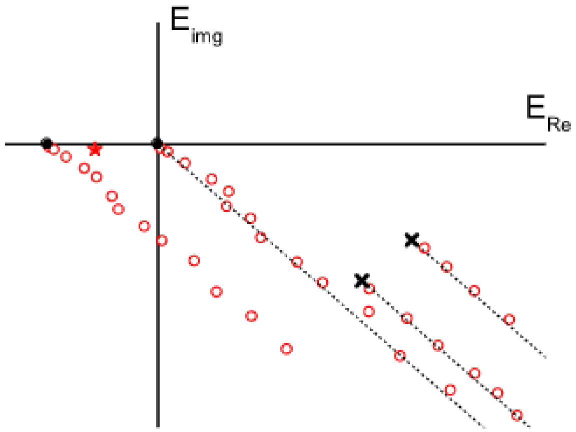
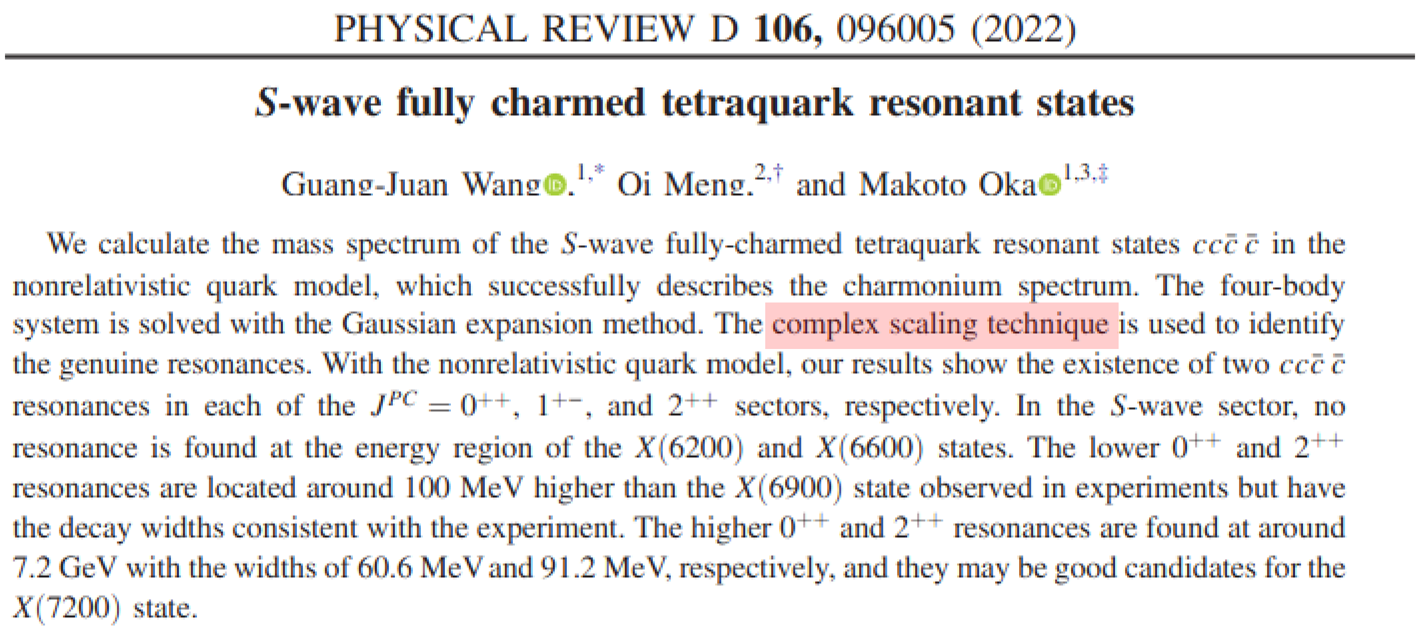


Fig. 4 Typical spectrum of the complex scaled Hamiltonian. Solid black points indicate positions of the thresholds and by crosses position of resonant states in the subsystems. From these points extends lines, rotated clockwise by angle 2θ relative to real energy axis. Complex-Hamiltonian eigenvalues corresponding to discredited continuum are scattered along these lines. Genuine resonant state is marked by a star symbol – it is separated from the rotated continuum lines

What about resonances?

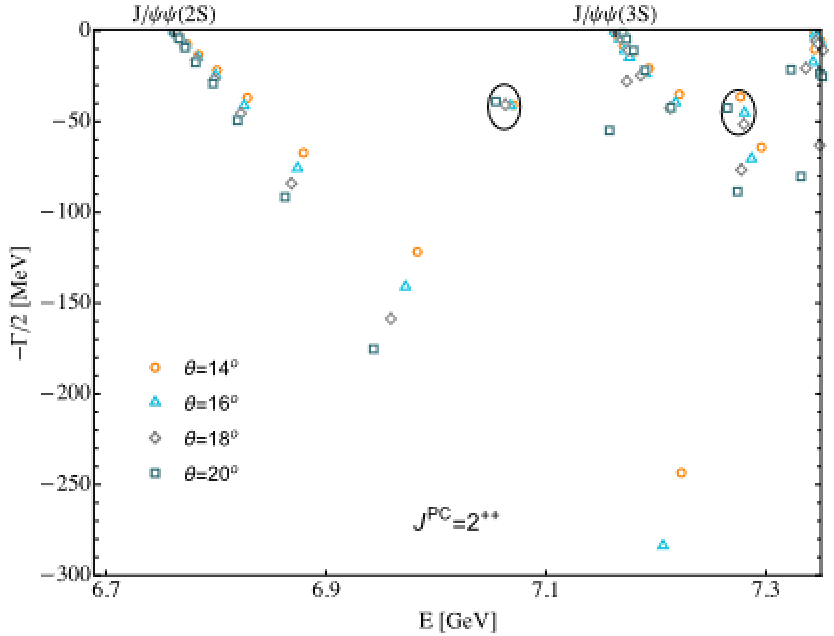


PHYSICAL REVIEW D **106,** 096005 (2022)

S-wave fully charmed tetraquark resonant states

Guang-Juan Wang,[1,*] Qi Meng,[2,†] and Makoto Oka[1,3,‡]

We calculate the mass spectrum of the *S*-wave fully-charmed tetraquark resonant states $cc\bar{c}\,\bar{c}$ in the nonrelativistic quark model, which successfully describes the charmonium spectrum. The four-body system is solved with the Gaussian expansion method. The complex scaling technique is used to identify the genuine resonances. With the nonrelativistic quark model, our results show the existence of two $cc\bar{c}\,\bar{c}$ resonances in each of the $J^{PC} = 0^{++}$, 1^{+-}, and 2^{++} sectors, respectively. In the *S*-wave sector, no resonance is found at the energy region of the $X(6200)$ and $X(6600)$ states. The lower 0^{++} and 2^{++} resonances are located around 100 MeV higher than the $X(6900)$ state observed in experiments but have the decay widths consistent with the experiment. The higher 0^{++} and 2^{++} resonances are found at around 7.2 GeV with the widths of 60.6 MeV and 91.2 MeV, respectively, and they may be good candidates for the $X(7200)$ state.



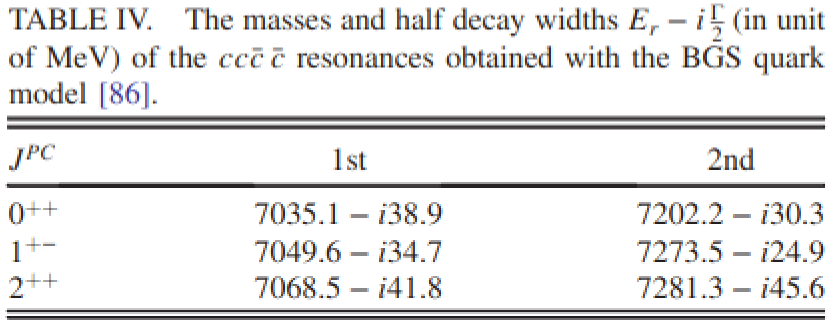
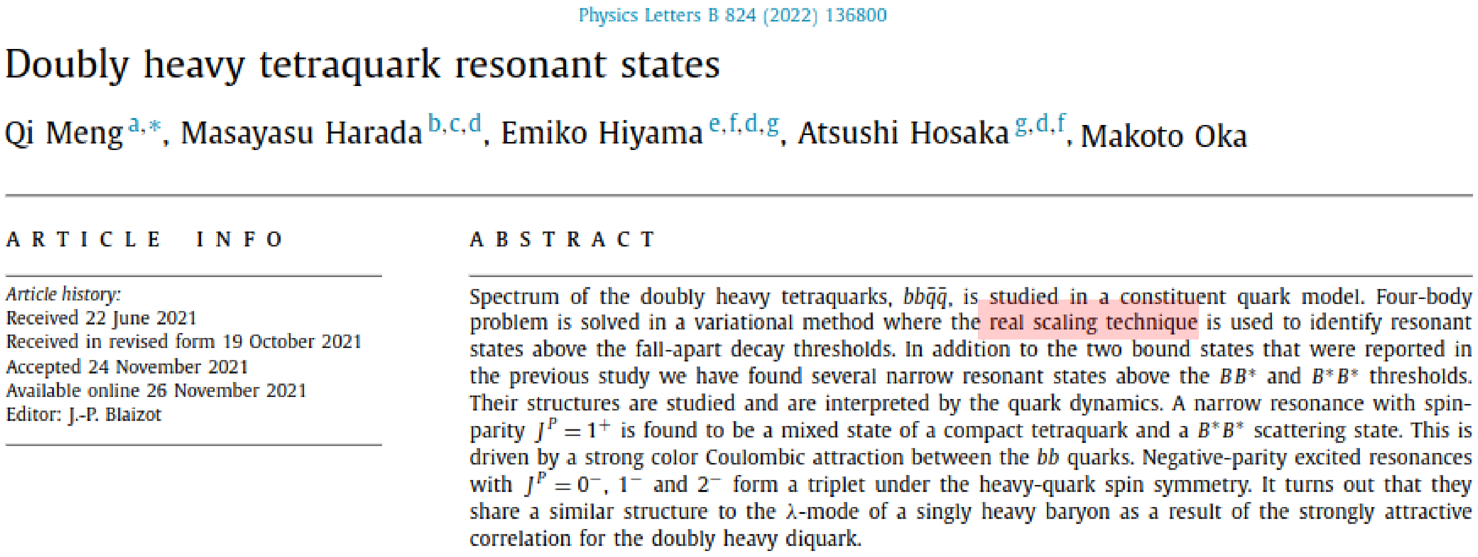


TABLE IV. The masses and half decay widths $E_r - i\frac{\Gamma}{2}$ (in unit of MeV) of the $cc\bar{c}\,\bar{c}$ resonances obtained with the BGS quark model [86].

J^{PC}	1st	2nd
0^{++}	$7035.1 - i38.9$	$7202.2 - i30.3$
1^{+-}	$7049.6 - i34.7$	$7273.5 - i24.9$
2^{++}	$7068.5 - i41.8$	$7281.3 - i45.6$

What about resonances?



Physics Letters B 824 (2022) 136800

Doubly heavy tetraquark resonant states

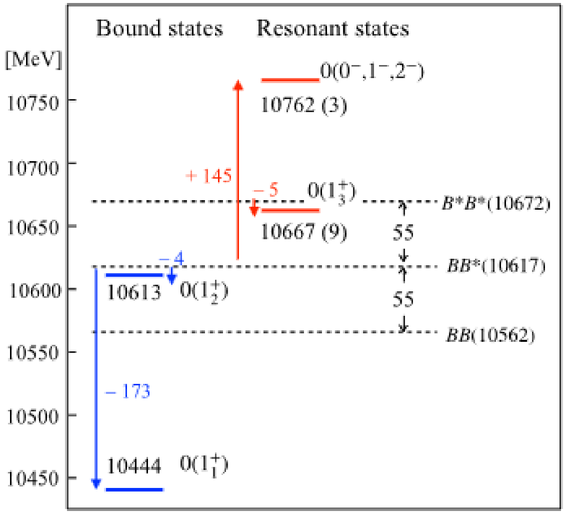
Qi Meng [a,*], Masayasu Harada [b,c,d], Emiko Hiyama [e,f,d,g], Atsushi Hosaka [g,d,f], Makoto Oka

ARTICLE INFO

Article history:
Received 22 June 2021
Received in revised form 19 October 2021
Accepted 24 November 2021
Available online 26 November 2021
Editor: J.-P. Blaizot

ABSTRACT

Spectrum of the doubly heavy tetraquarks, $bb\bar{q}\bar{q}$, is studied in a constituent quark model. Four-body problem is solved in a variational method where the real scaling technique is used to identify resonant states above the fall-apart decay thresholds. In addition to the two bound states that were reported in the previous study we have found several narrow resonant states above the BB^* and B^*B^* thresholds. Their structures are studied and are interpreted by the quark dynamics. A narrow resonance with spin-parity $J^P = 1^+$ is found to be a mixed state of a compact tetraquark and a B^*B^* scattering state. This is driven by a strong color Coulombic attraction between the bb quarks. Negative-parity excited resonances with $J^P = 0^-$, 1^- and 2^- form a triplet under the heavy-quark spin symmetry. It turns out that they share a similar structure to the λ-mode of a singly heavy baryon as a result of the strongly attractive correlation for the doubly heavy diquark.



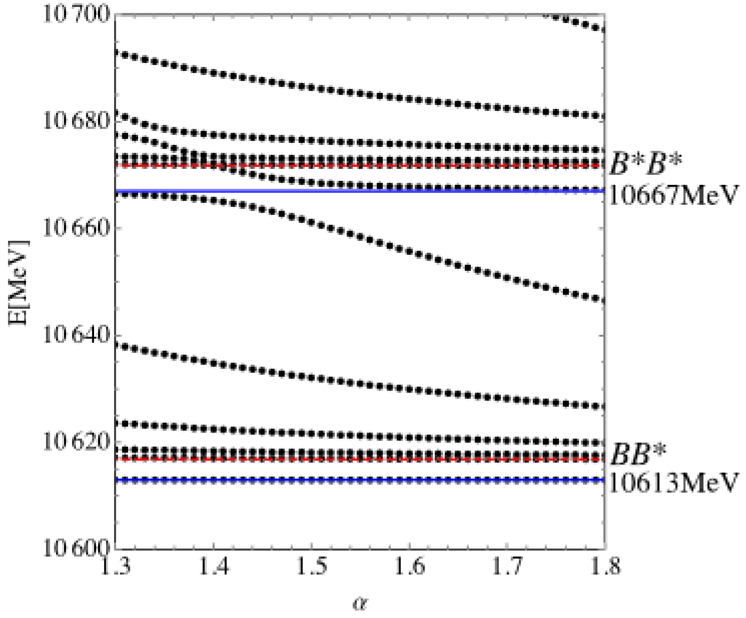


Fig. 3. The stabilization plots of the energy eigenvalues of $I(J^\pi) = 0(1^+)$ states by changing the scaling parameter α.

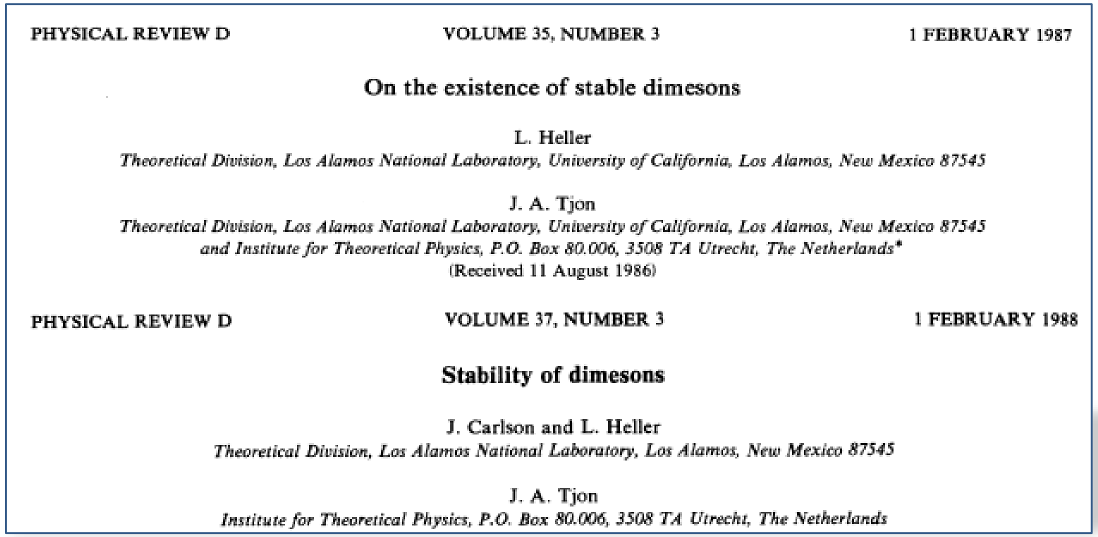
Conclusions

- The constituent quark model predicts a clear bound state, $bb\bar{u}\bar{d}$, and another one, $cc\bar{u}\bar{d}$, just below threshold with $(I)J^P=(0)1^+$. Some particular models may point to the existence of about five more bound states.
- A few resonances may exist in the heavy sector (probably too soon to say with certainty).
  - **There is not an overwhelming abundance of bound states or resonances within the constituent quark model.**

- The numerical methods required should be able to handle short- and long-range correlations, i.e. a meson-meson structures together with a more *clusterized* behaviour.

- Approximations and simplifications in the colour-spin structure should be done carefully.
  - **We should double check whether our findings are entirely due to our hypothesis and aproximations before extracting any general conclusion.**

Potentials derived from the MIT Bag model



PHYSICAL REVIEW D VOLUME 35, NUMBER 3 1 FEBRUARY 1987

On the existence of stable dimesons

L. Heller
Theoretical Division, Los Alamos National Laboratory, University of California, Los Alamos, New Mexico 87545

J. A. Tjon
Theoretical Division, Los Alamos National Laboratory, University of California, Los Alamos, New Mexico 87545
*and Institute for Theoretical Physics, P.O. Box 80.006, 3508 TA Utrecht, The Netherlands**

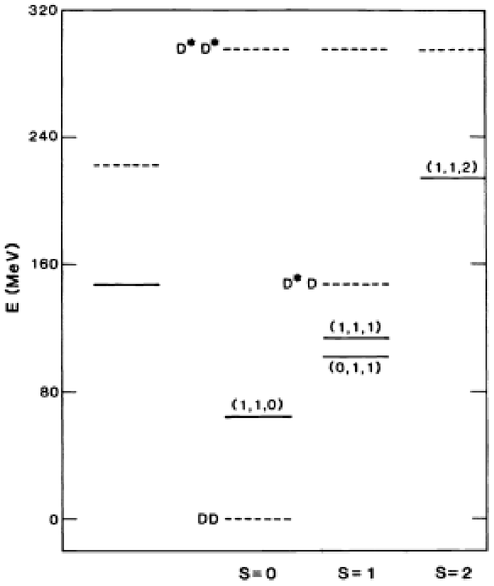
(Received 11 August 1986)

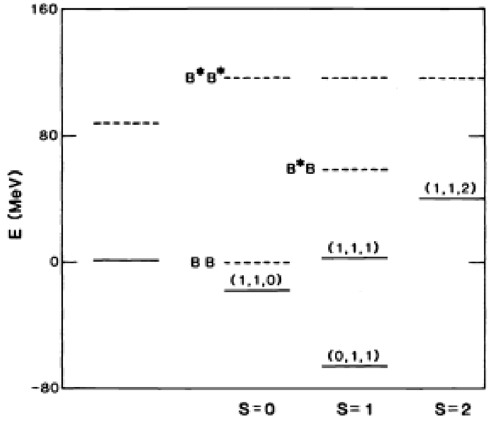
PHYSICAL REVIEW D VOLUME 37, NUMBER 3 1 FEBRUARY 1988

Stability of dimesons

J. Carlson and L. Heller
Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

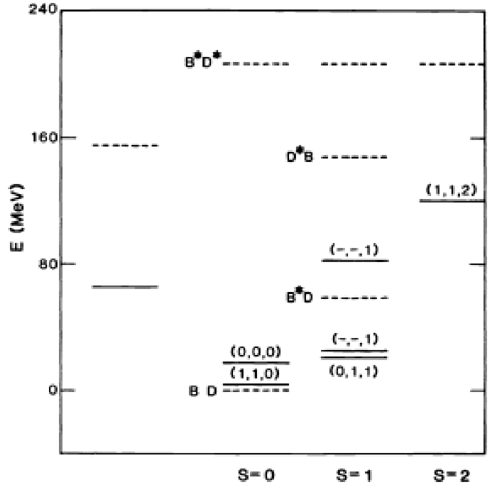
J. A. Tjon
Institute for Theoretical Physics, P.O. Box 80.006, 3508 TA Utrecht, The Netherlands



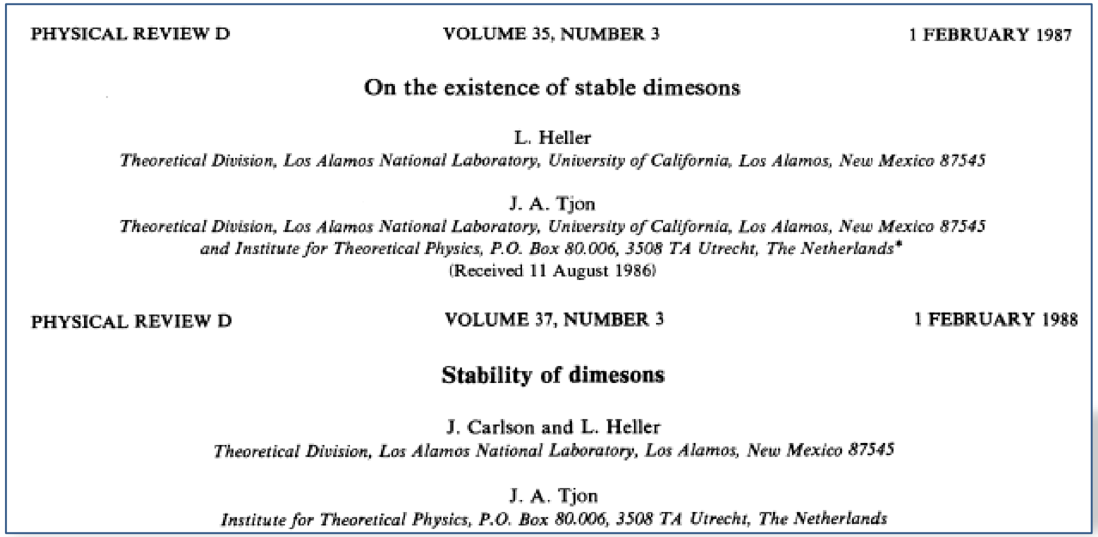


The bound-state problem of two- and four-quarks with coupled channels in color space is studied, using a potential derived from the MIT bag model.

$cc\bar{c}\bar{c}$



Potentials derived from the MIT Bag model



PHYSICAL REVIEW D VOLUME 35, NUMBER 3 1 FEBRUARY 1987

On the existence of stable dimesons

L. Heller
Theoretical Division, Los Alamos National Laboratory, University of California, Los Alamos, New Mexico 87545

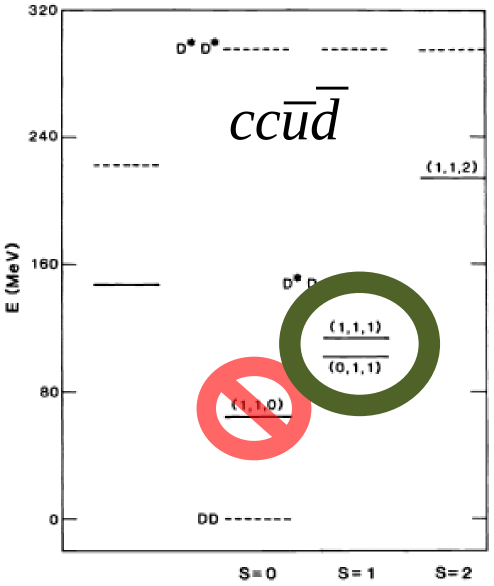
J. A. Tjon
Theoretical Division, Los Alamos National Laboratory, University of California, Los Alamos, New Mexico 87545
*and Institute for Theoretical Physics, P.O. Box 80.006, 3508 TA Utrecht, The Netherlands**
(Received 11 August 1986)

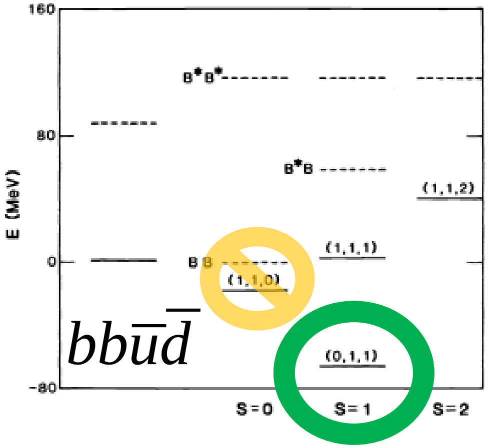
PHYSICAL REVIEW D VOLUME 37, NUMBER 3 1 FEBRUARY 1988

Stability of dimesons

J. Carlson and L. Heller
Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

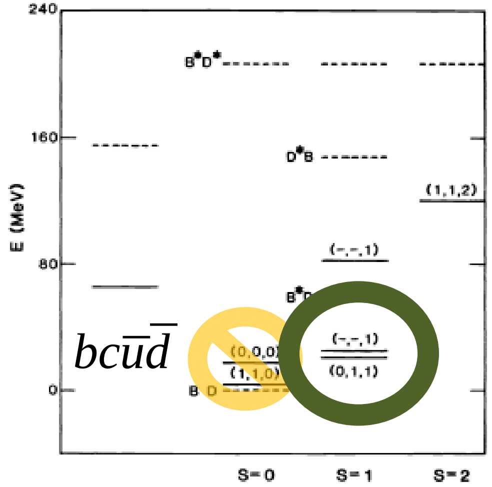
J. A. Tjon
Institute for Theoretical Physics, P.O. Box 80.006, 3508 TA Utrecht, The Netherlands



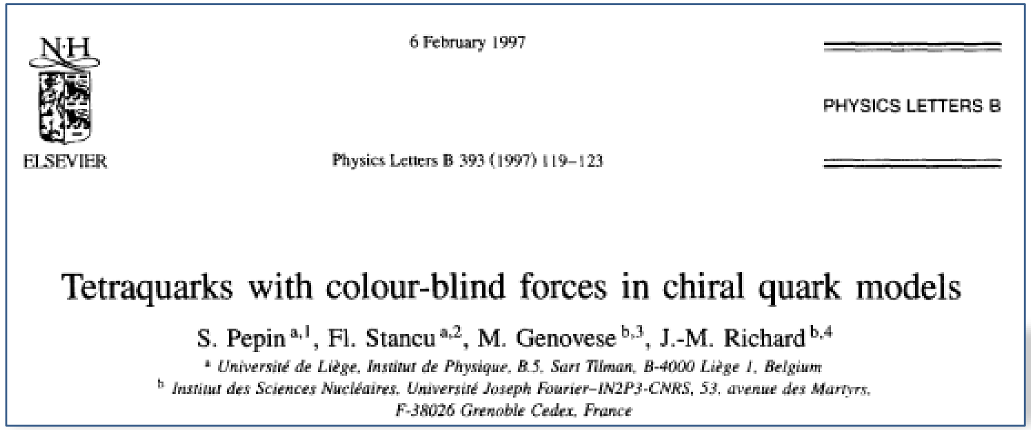


The bound-state problem of two- and four-quarks with coupled channels in color space is studied, using a potential derived from the MIT bag model.

$cc\overline{cc}$



Exploring constituent quark models



6 February 1997

PHYSICS LETTERS B

ELSEVIER

Physics Letters B 393 (1997) 119–123

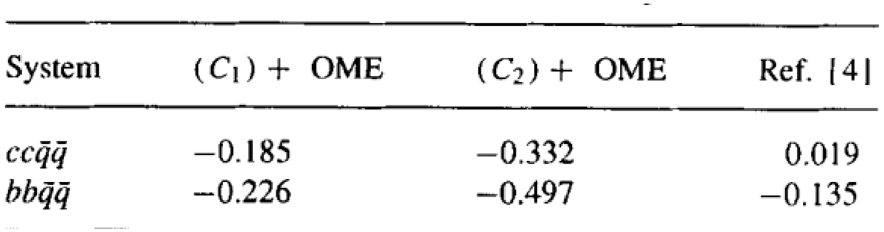
Tetraquarks with colour-blind forces in chiral quark models

S. Pepin[a,1], Fl. Stancu[a,2], M. Genovese[b,3], J.-M. Richard[b,4]

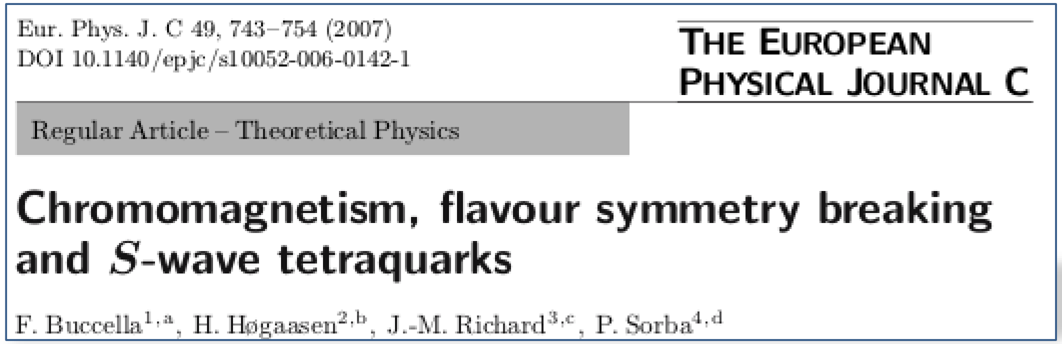
[a] Université de Liège, Institut de Physique, B.5, Sart Tilman, B-4000 Liège 1, Belgium

[b] Institut des Sciences Nucléaires, Université Joseph Fourier–IN2P3-CNRS, 53, avenue des Martyrs, F-38026 Grenoble Cedex, France

These systems were studied with a potential model fitted in the baryon spectrum that includes meson-exchange forces between quarks and entirely neglects the chromomagnetic interaction.



System	$(C_1)+$ OME	$(C_2)+$ OME	Ref. [4]
$cc\bar{q}\bar{q}$	-0.185	-0.332	0.019
$bb\bar{q}\bar{q}$	-0.226	-0.497	-0.135



Eur. Phys. J. C 49, 743–754 (2007)
DOI 10.1140/epjc/s10052-006-0142-1

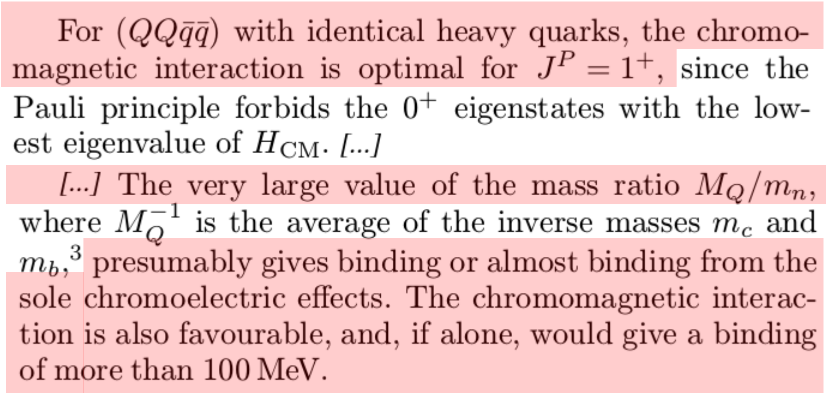
THE EUROPEAN PHYSICAL JOURNAL C

Regular Article – Theoretical Physics

Chromomagnetism, flavour symmetry breaking and S-wave tetraquarks

F. Buccella[1,a], H. Høgaasen[2,b], J.-M. Richard[3,c], P. Sorba[4,d]

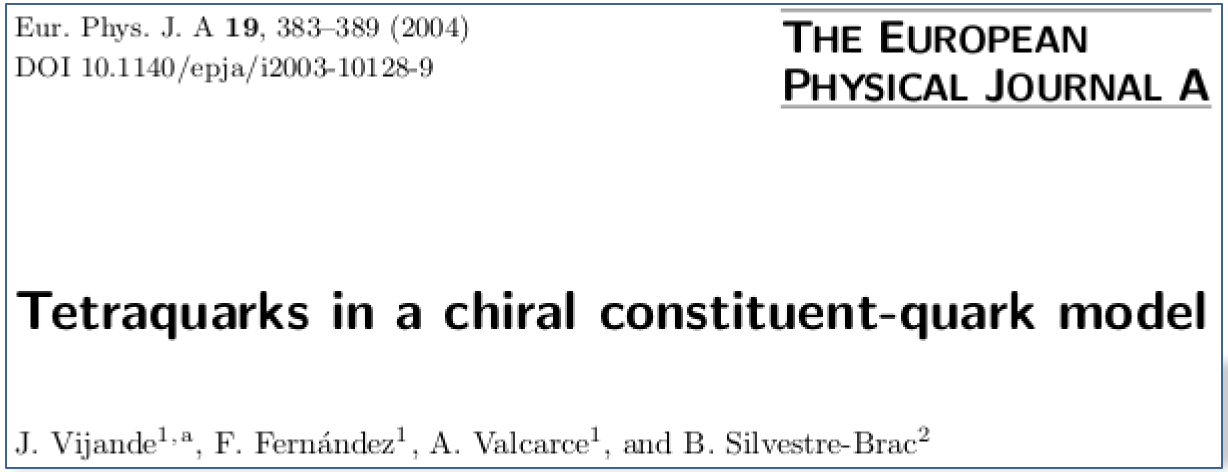
A detailed formalism is presented to fully account for flavour-symmetry breaking in the chromomagnetic interaction together with its application to four-quark systems.



For $(QQ\bar{q}\bar{q})$ with identical heavy quarks, the chromomagnetic interaction is optimal for $J^P=1^+$, since the Pauli principle forbids the 0^+ eigenstates with the lowest eigenvalue of H_{CM}. *[...]*

[...] The very large value of the mass ratio M_Q/m_n, where M_Q^{-1} is the average of the inverse masses m_c and m_b,[3] presumably gives binding or almost binding from the sole chromoelectric effects. The chromomagnetic interaction is also favourable, and, if alone, would give a binding of more than 100 MeV.

Exploring constituent quark models



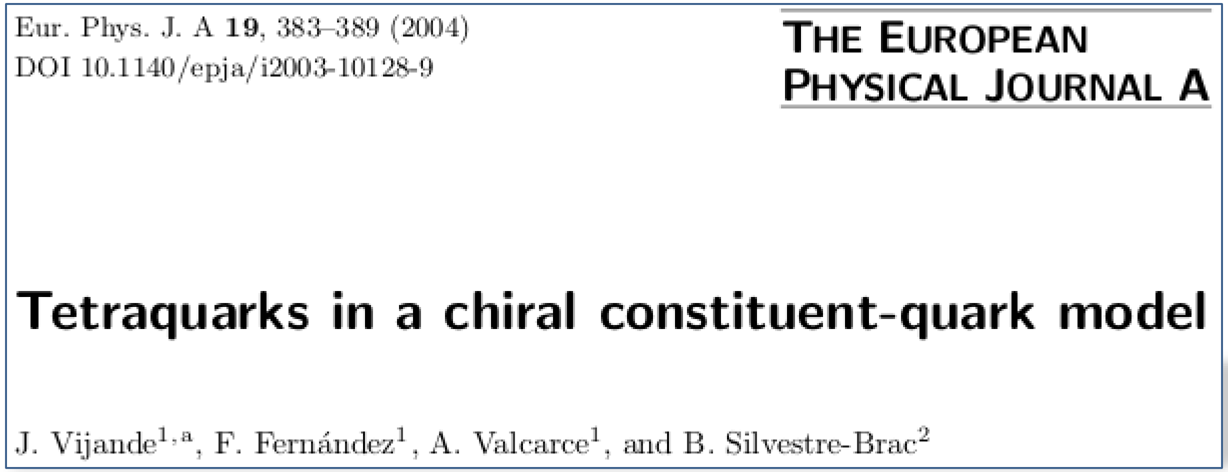
Eur. Phys. J. A **19**, 383–389 (2004)
DOI 10.1140/epja/i2003-10128-9

THE EUROPEAN
PHYSICAL JOURNAL A

Tetraquarks in a chiral constituent-quark model

J. Vijande[1,a], F. Fernández[1], A. Valcarce[1], and B. Silvestre-Brac[2]

Exploring constituent quark models

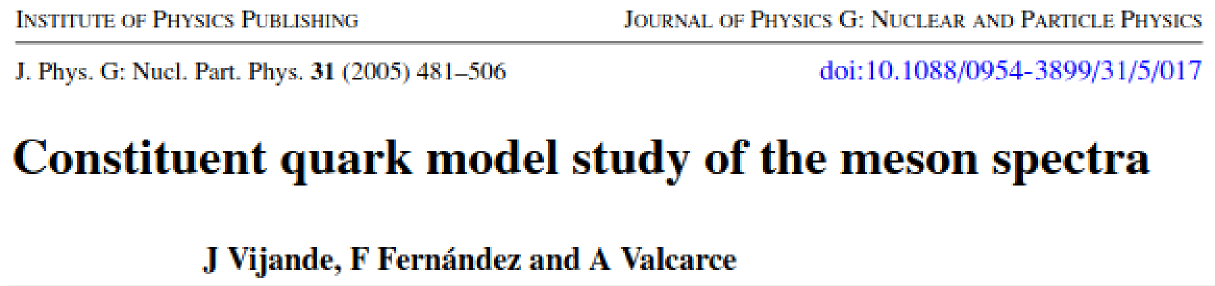


Eur. Phys. J. A **19**, 383–389 (2004)
DOI 10.1140/epja/i2003-10128-9

THE EUROPEAN PHYSICAL JOURNAL A

Tetraquarks in a chiral constituent-quark model

J. Vijande[1,a], F. Fernández[1], A. Valcarce[1], and B. Silvestre-Brac[2]



INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS G: NUCLEAR AND PARTICLE PHYSICS

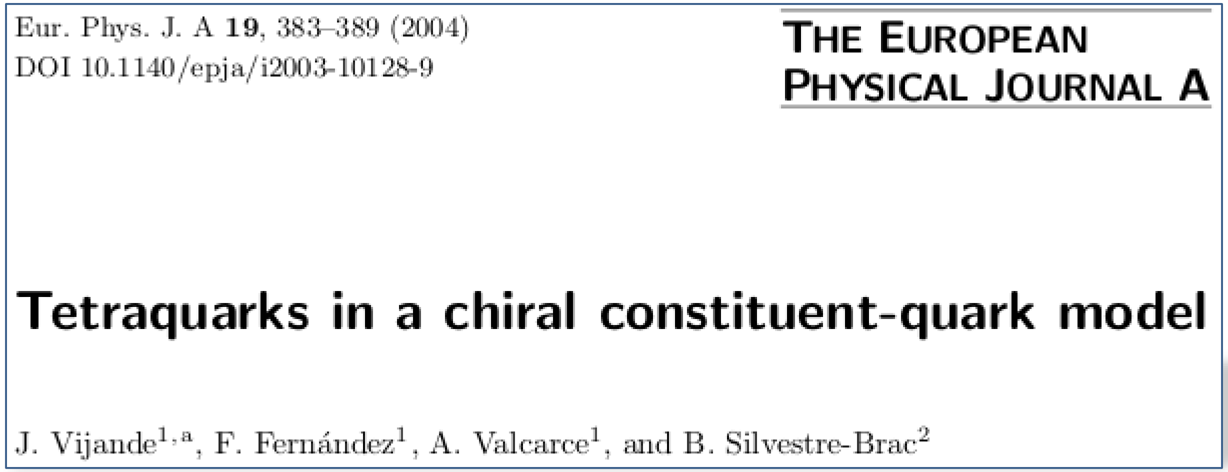
J. Phys. G: Nucl. Part. Phys. **31** (2005) 481–506 doi:10.1088/0954-3899/31/5/017

Constituent quark model study of the meson spectra

J Vijande, F Fernández and A Valcarce

A different constituent quark model incorporating meson exchanges between light quarks on top of gluon exchange was explored

Exploring constituent quark models

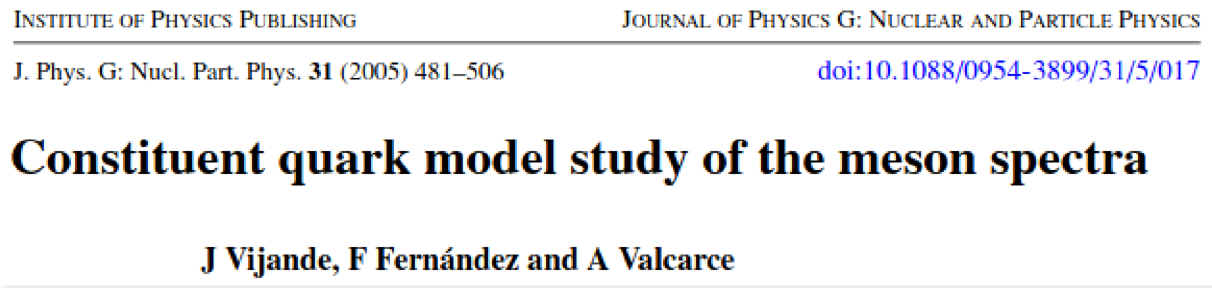


Eur. Phys. J. A **19**, 383–389 (2004)
DOI 10.1140/epja/i2003-10128-9

THE EUROPEAN
PHYSICAL JOURNAL A

Tetraquarks in a chiral constituent-quark model

J. Vijande[1,a], F. Fernández[1], A. Valcarce[1], and B. Silvestre-Brac[2]



INSTITUTE OF PHYSICS PUBLISHING
JOURNAL OF PHYSICS G: NUCLEAR AND PARTICLE PHYSICS

J. Phys. G: Nucl. Part. Phys. **31** (2005) 481–506
doi:10.1088/0954-3899/31/5/017

Constituent quark model study of the meson spectra

J Vijande, F Fernández and A Valcarce

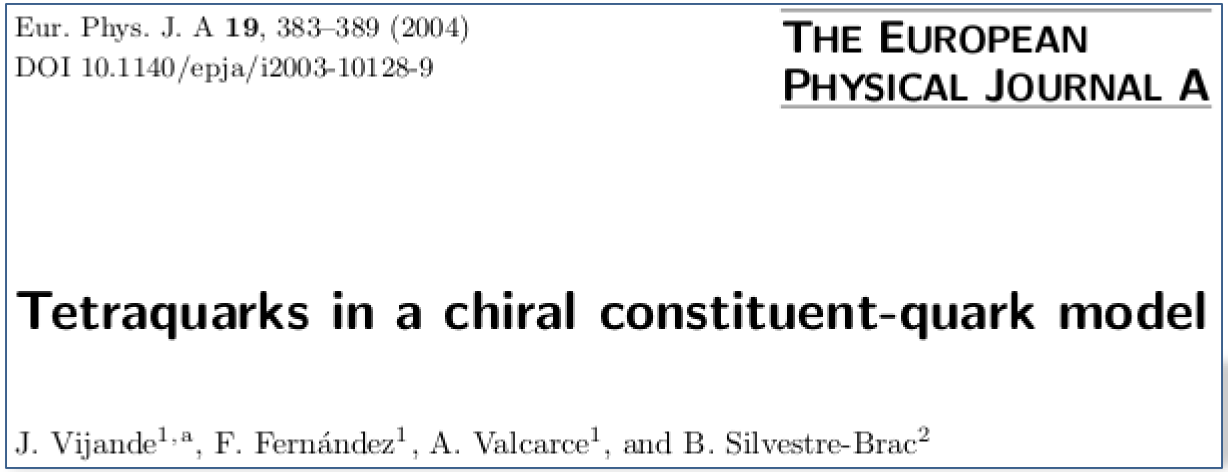
A different constituent quark model incorporating meson exchanges between light quarks on top of gluon exchange was explored

A variational method based on a Gaussian expansion was considered.

$$R_s(1234) = \sum_{i=1}^{n} \beta_s^{(i)} R_s^{(i)} =$$

$$\sum_{i=1}^{n} \beta_s^{(i)} e^{-a_s^{(i)}\vec{x}^{\,2} - b_s^{(i)}\vec{y}^{\,2} - c_s^{(i)}\vec{z}^{\,2} - d_s^{(i)}\vec{x}\vec{y} - e_s^{(i)}\vec{x}\vec{z} - f_s^{(i)}\vec{y}\vec{z}}$$

Exploring constituent quark models

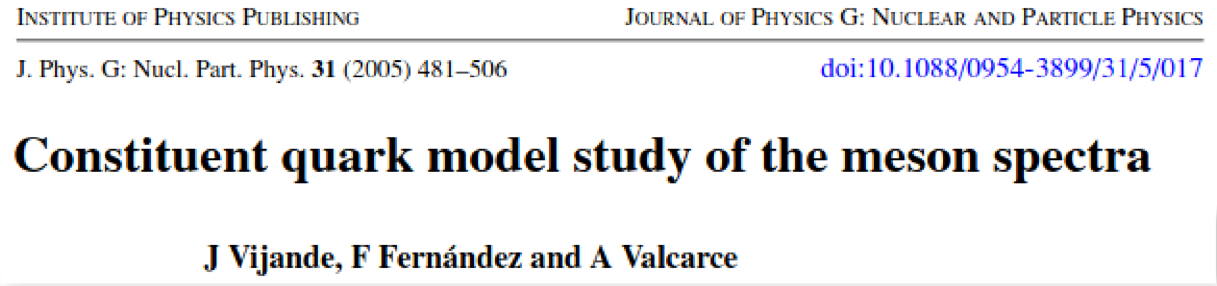


Eur. Phys. J. A **19**, 383–389 (2004)
DOI 10.1140/epja/i2003-10128-9

THE EUROPEAN
PHYSICAL JOURNAL A

Tetraquarks in a chiral constituent-quark model

J. Vijande[1,a], F. Fernández[1], A. Valcarce[1], and B. Silvestre-Brac[2]



INSTITUTE OF PHYSICS PUBLISHING — JOURNAL OF PHYSICS G: NUCLEAR AND PARTICLE PHYSICS

J. Phys. G: Nucl. Part. Phys. **31** (2005) 481–506 — doi:10.1088/0954-3899/31/5/017

Constituent quark model study of the meson spectra

J Vijande, F Fernández and A Valcarce

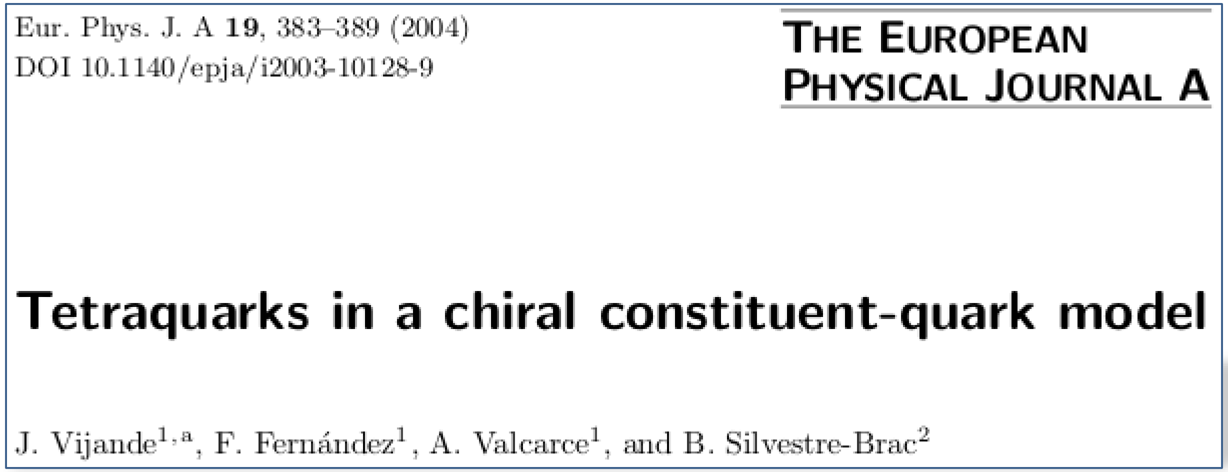
A different constituent quark model incorporating meson exchanges between light quarks on top of gluon exchange was explored

A variational method based on a Gaussian expansion was considered.

$$R_s(1234) = \sum_{i=1}^{n} \beta_s^{(i)} R_s^{(i)} =$$

$$\sum_{i=1}^{n} \beta_s^{(i)} e^{-a_s^{(i)}\vec{x}^{\,2} - b_s^{(i)}\vec{y}^{\,2} - c_s^{(i)}\vec{z}^{\,2} - \cancel{d_s^{(i)}\vec{x}\vec{y}} - \cancel{e_s^{(i)}\vec{x}\vec{z}} - \cancel{f_s^{(i)}\vec{y}\vec{z}}}$$

Exploring constituent quark models



Eur. Phys. J. A **19**, 383–389 (2004)
DOI 10.1140/epja/i2003-10128-9

THE EUROPEAN
PHYSICAL JOURNAL A

Tetraquarks in a chiral constituent-quark model

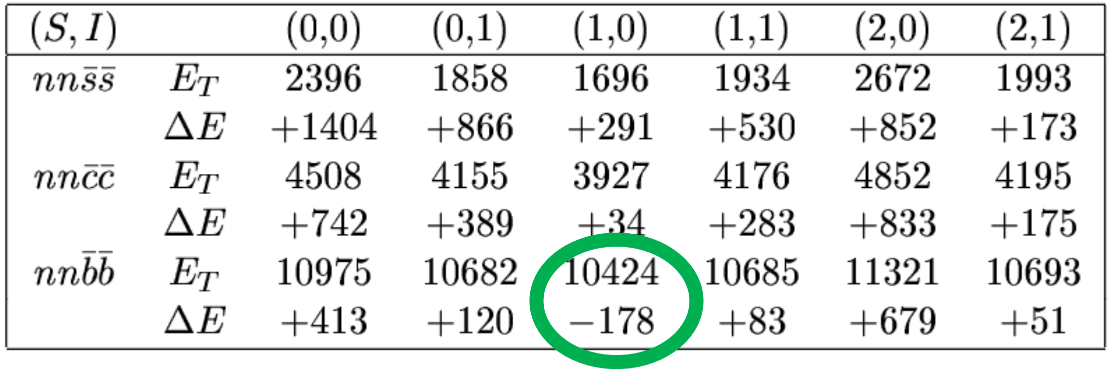
J. Vijande[1,a], F. Fernández[1], A. Valcarce[1], and B. Silvestre-Brac[2]

A different constituent quark model incorporating meson exchanges between light quarks on top of gluon exchange was explored

A variational method based on a Gaussian expansion was considered.

$$R_s(1234) = \sum_{i=1}^{n} \beta_s^{(i)} R_s^{(i)} =$$

$$\sum_{i=1}^{n} \beta_s^{(i)} e^{-a_s^{(i)}\vec{x}^{\,2} - b_s^{(i)}\vec{y}^{\,2} - c_s^{(i)}\vec{z}^{\,2} - \cancel{d_s^{(i)}\vec{x}\vec{y}} - \cancel{e_s^{(i)}\vec{x}\vec{z}} - \cancel{f_s^{(i)}\vec{y}\vec{z}}}$$



(S,I)		(0,0)	(0,1)	(1,0)	(1,1)	(2,0)	(2,1)
$nn\bar{s}\bar{s}$	E_T	2396	1858	1696	1934	2672	1993
	ΔE	+1404	+866	+291	+530	+852	+173
$nn\bar{c}\bar{c}$	E_T	4508	4155	3927	4176	4852	4195
	ΔE	+742	+389	+34	+283	+833	+175
$nn\bar{b}\bar{b}$	E_T	10975	10682	10424	10685	11321	10693
	ΔE	+413	+120	−178	+83	+679	+51

VNIVERSITAT DE VALÈNCIA

Compact or meson-meson configuration?

Physics Letters B 699 (2011) 291–295

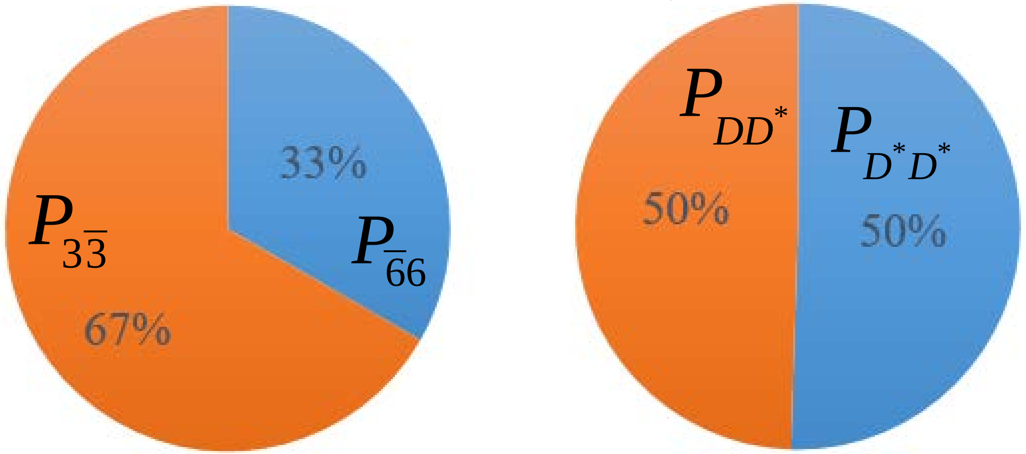
Doubly charmed exotic mesons: A gift of nature?

T.F. Caramés [a], A. Valcarce [a,*], J. Vijande [b]

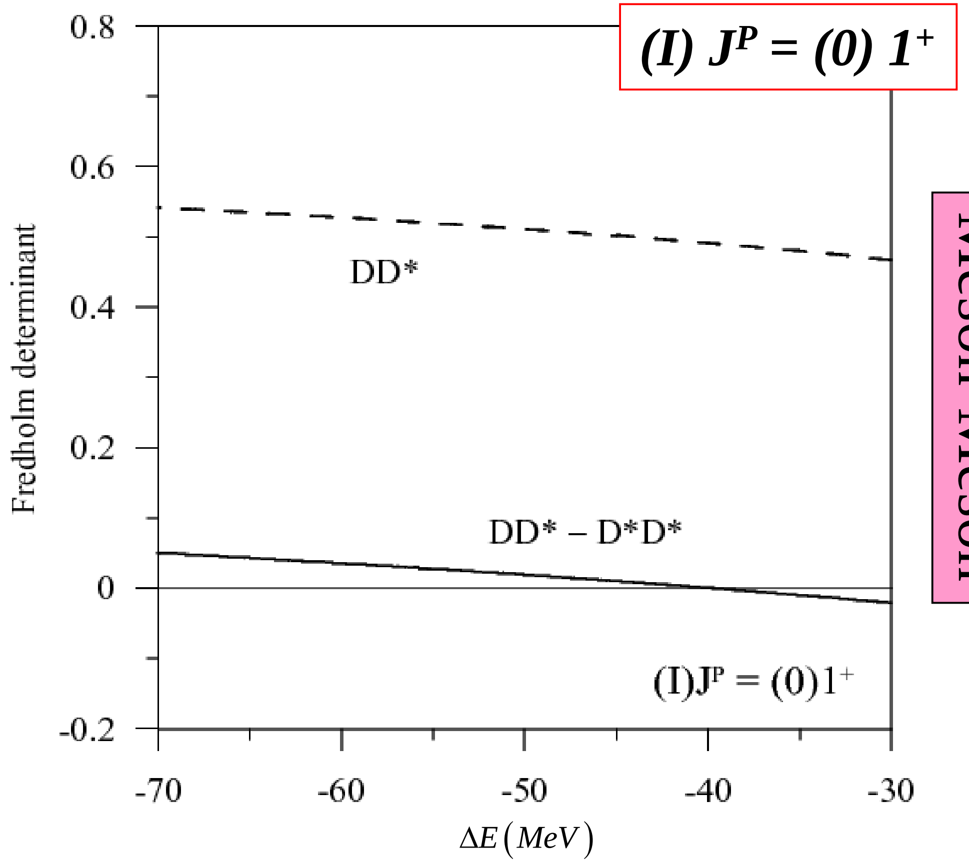
$$|cc\bar{n}\bar{n}\rangle = \alpha_1|\bar{3}3\rangle + \ldots + \alpha_2|6\bar{6}\rangle + \ldots \overset{????}{=} \alpha_1|DD^*\rangle + \alpha_2|D^*D^*\rangle + \ldots$$

In this work the meson-meson configuration is solved by means of the Lippmann-Schwinger equation using the same interaction as the four-quark problem.

$$\Delta E = -76 MeV$$



Four-quark states



Meson-Meson