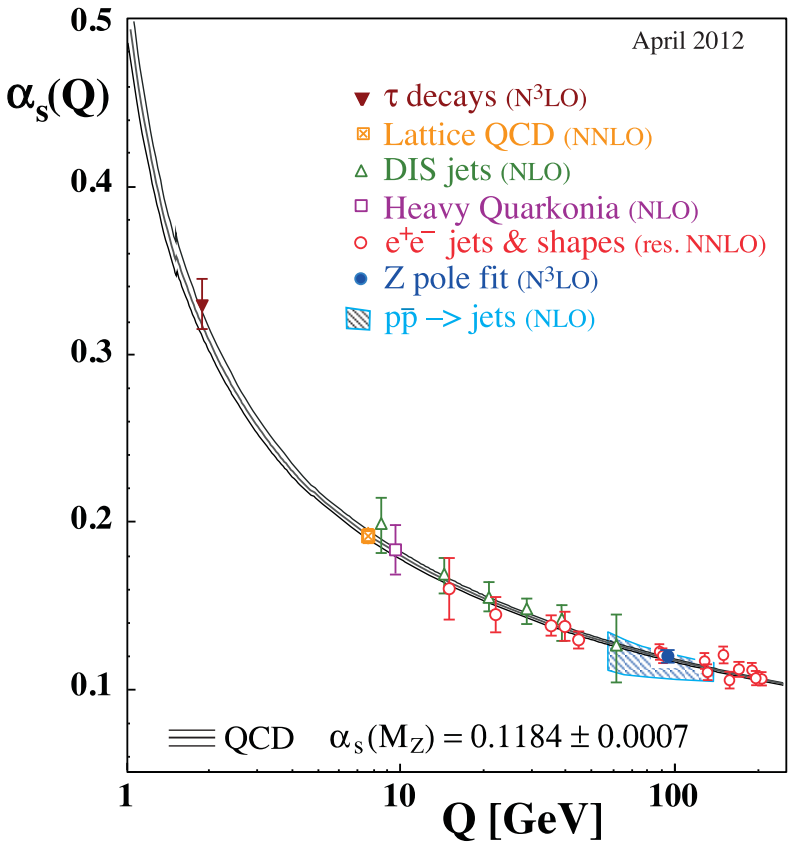
Parton distribution amplitudes and form factors at large momentum transfer

Cédric Mezrag

INFN Roma1

April 25th, 2018

Running coupling and Non-Perturbative QCD



- **Asymptotic freedom**:
  High energy $\rightarrow$ small coupling,
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  Perturbative expansion breaks down.

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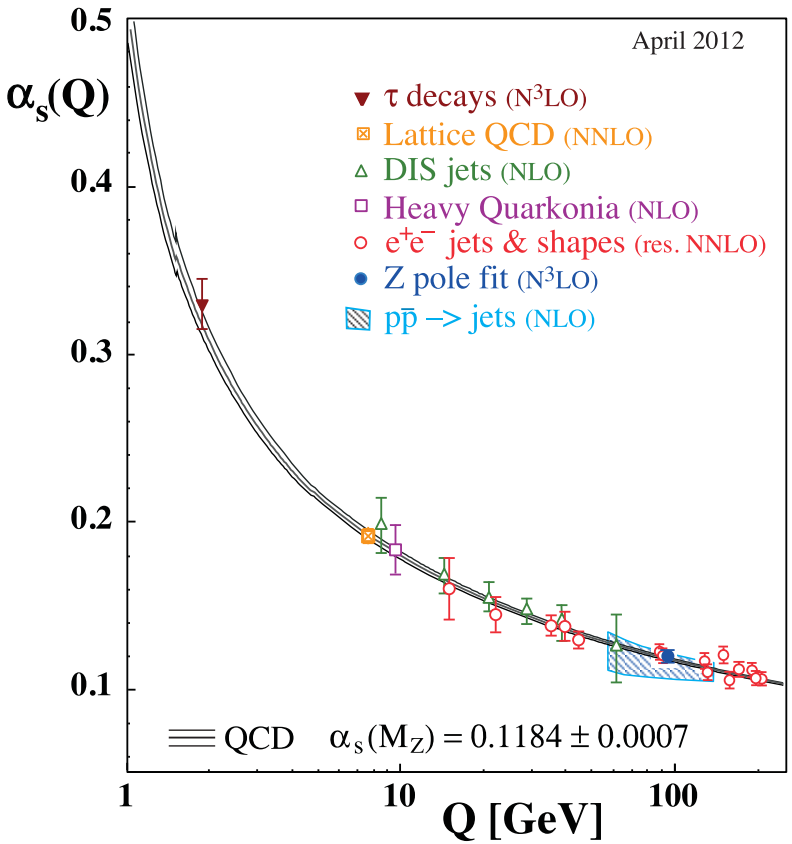

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Understanding hadron structure in terms of quarks and gluons means looking at QCD non-perturbative behaviour.

Chapter 1:
Dyson-Schwinger equations

Dyson-Schwinger Equations in a nutshell

- Dyson-Schwinger equations relate Green (Schwinger) functions among each other.
- One get in the case of QCD, an infinite system $\rightarrow$ truncations are required.
- DSEs are usually solved in Euclidean space, yielding Schwinger functions instead of Green functions.

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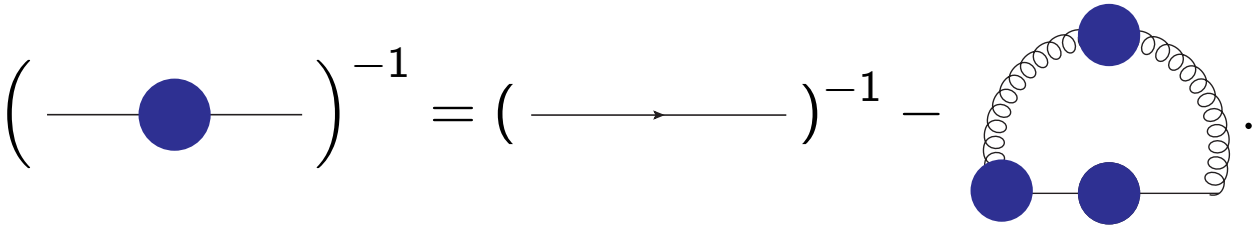
Truncating the DSEs yields an non-perturbative approximation of QCD Schwinger functions.

The Gap Equation

- The gap equation for the quark propagator $S(q)$:



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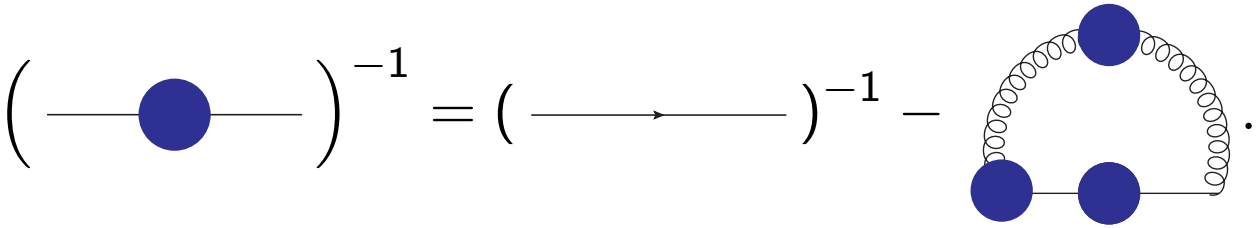
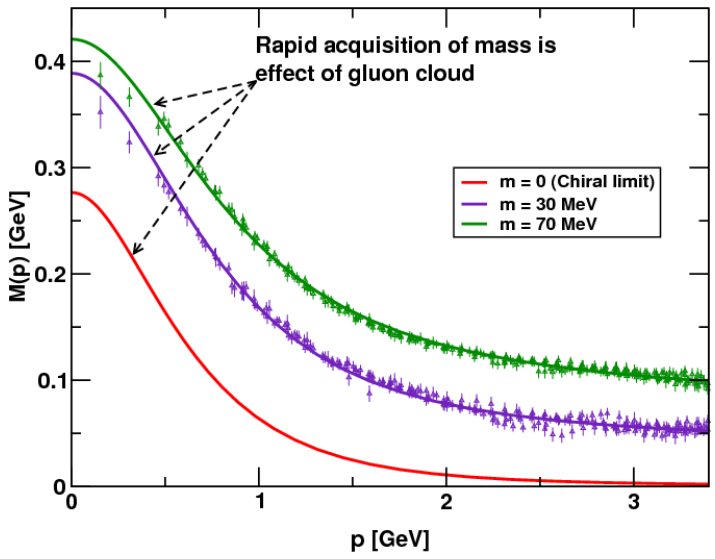


- It has successfully described the quark mass behaviour:



- $S(q)^{-1} = i\not{q}A(q^2) + B(q^2)$
- Non-perturbative description of the quark mass
- Dynamical mass generation
- Figure from Bashir *et al.* (2012)

Bound-States and the Bethe-Salpeter Equation

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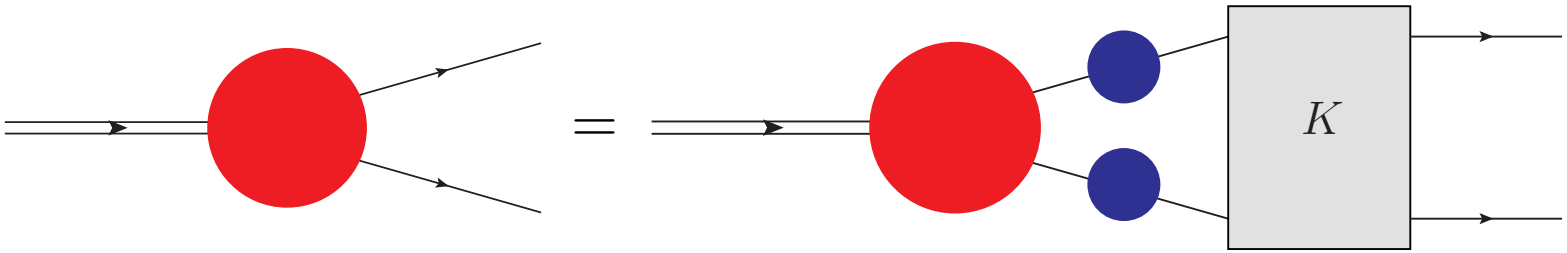
- Two-body bound states obey their own equation called the Bethe-Sapeter equation:



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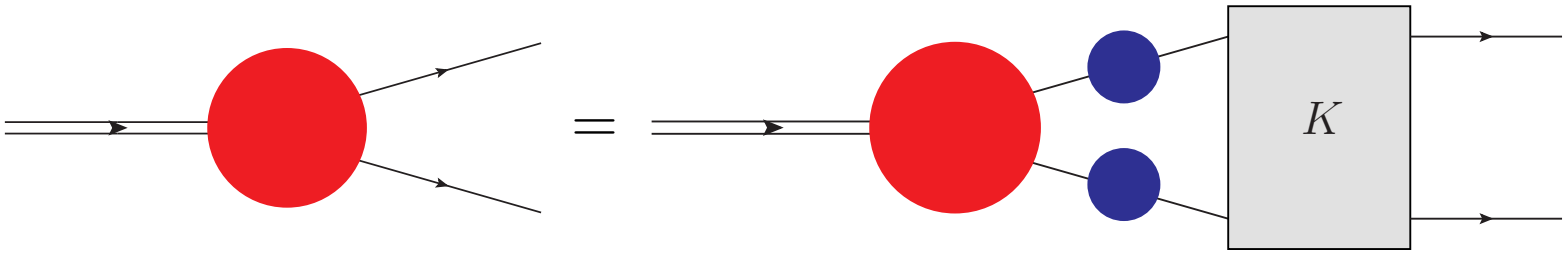

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Symmetry Considerations

This is critical: for instance we want our truncations to preserve the Ward-Takahashi Identities (WTI) and the Axial-Vector WTI.

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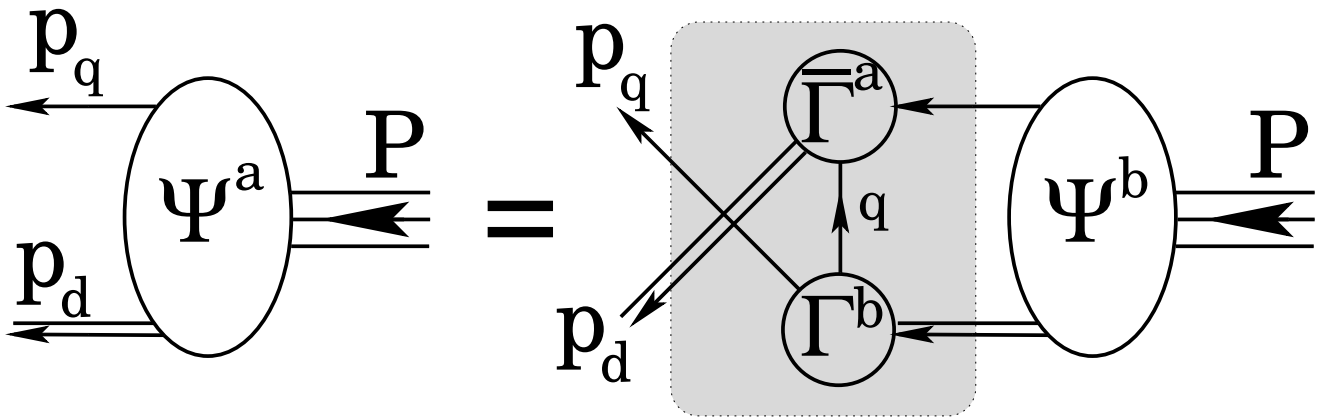
- It predicts the existence of strong diquarks correlations inside the nucleon.



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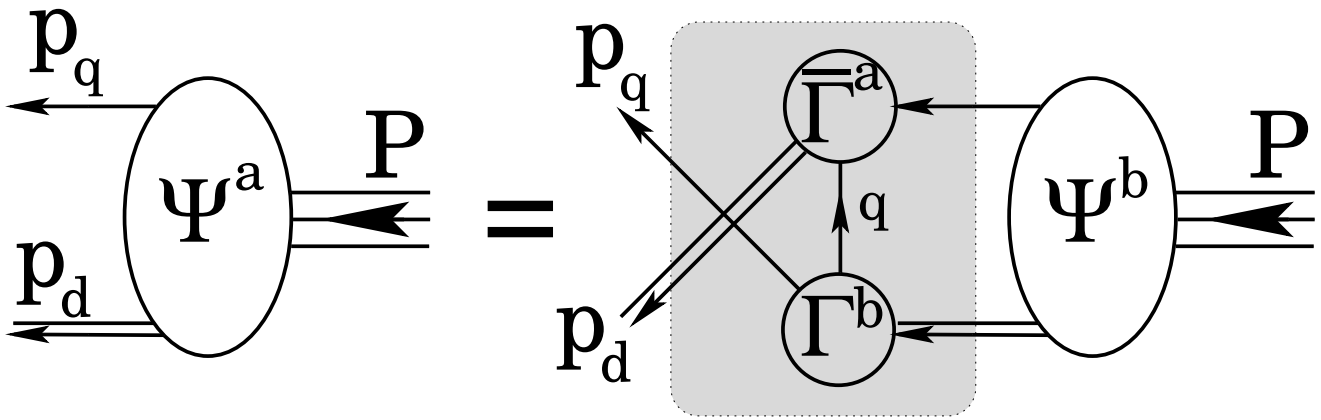

- Mostly two types of diquark are dynamically generated by the Faddeev equation:
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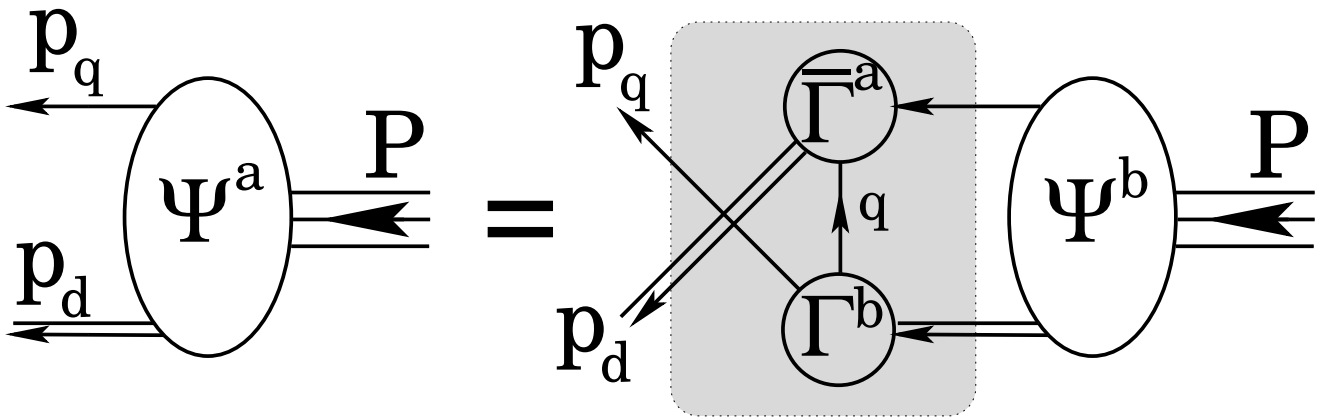

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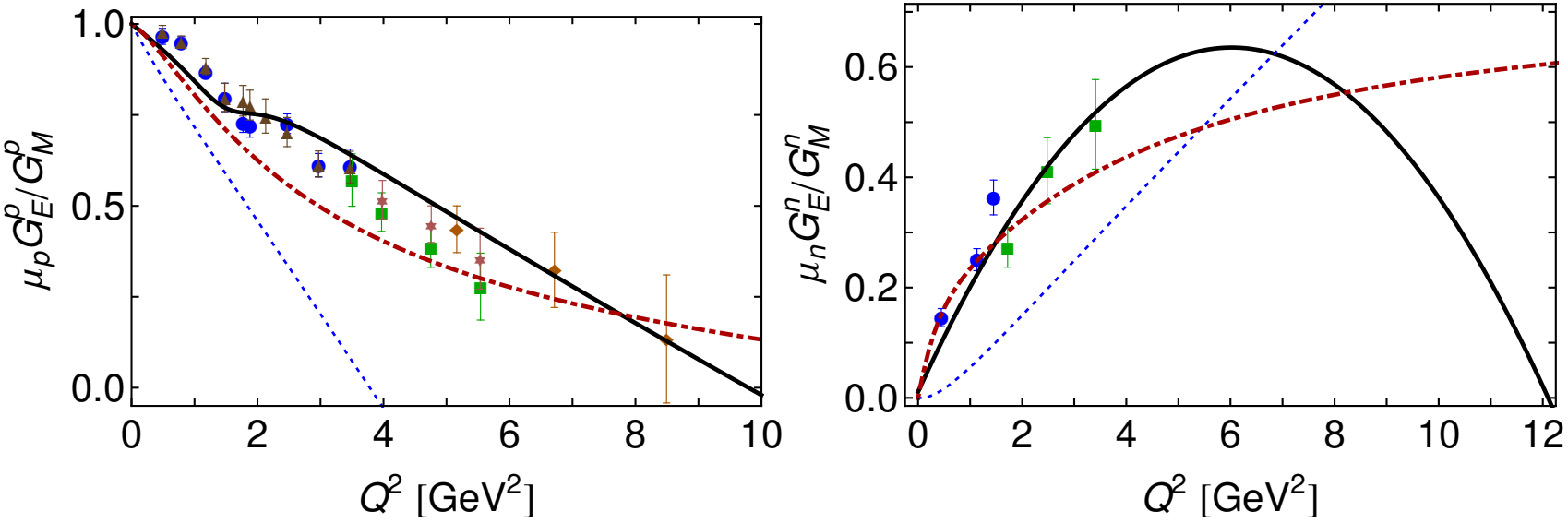
- Can we understand the nucleon structure in terms of quark-diquarks correlations?

Some Results on FF



figures from J. Segovia *et al.*, Few Body Sys. 55 (2014) 1185-1222

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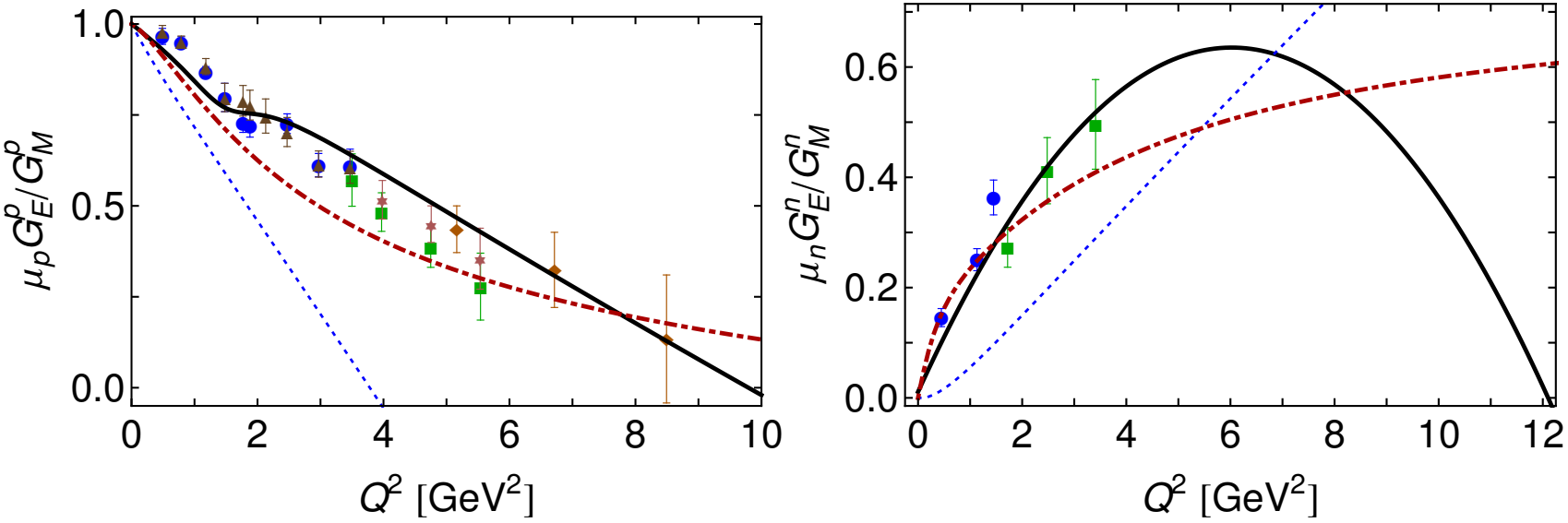

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Today I will focus on the Parton Distribution Amplitude (PDA), telling us the longitudinal structure of the nucleon and allowing us to compute the Dirac FF at large momentum transfer.

Chapter 2:
Baryon Distribution Amplitudes

Cédric Mezrag, Jorge Segovia, Lei Chang and Craig Roberts
arXiv:1711.09101

Hadrons seen as Fock States

- Lightfront quantization allows to expand hadrons on a Fock basis:

$$|P,\pi\rangle \propto \sum_{\beta} \Psi_{\beta}^{q\bar{q}} |q\bar{q}\rangle + \sum_{\beta} \Psi_{\beta}^{q\bar{q},q\bar{q}} |q\bar{q},q\bar{q}\rangle + \dots$$

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- Non-perturbative physics is contained in the N-particles Lightfront-Wave Functions (LFWF) Ψ^N
- Schematically a distribution amplitude φ is related to the LFWF through:

$$\varphi(x) \propto \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2}\Psi(x,k_\perp)$$

S. Brodsky and G. Lepage, PRD 22, (1980)

Nucleon Distribution Amplitudes

- 3 bodies matrix element:

$$\langle 0|\epsilon^{ijk} u^i_\alpha(z_1) u^j_\beta(z_2) d^k_\gamma(z_3)|P\rangle$$

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- Isospin symmetry:

$$2T(x_1,x_2,x_3) = \varphi(x_1,x_3,x_2) + \varphi(x_2,x_3,x_1)$$

Evolution and Asymptotic results

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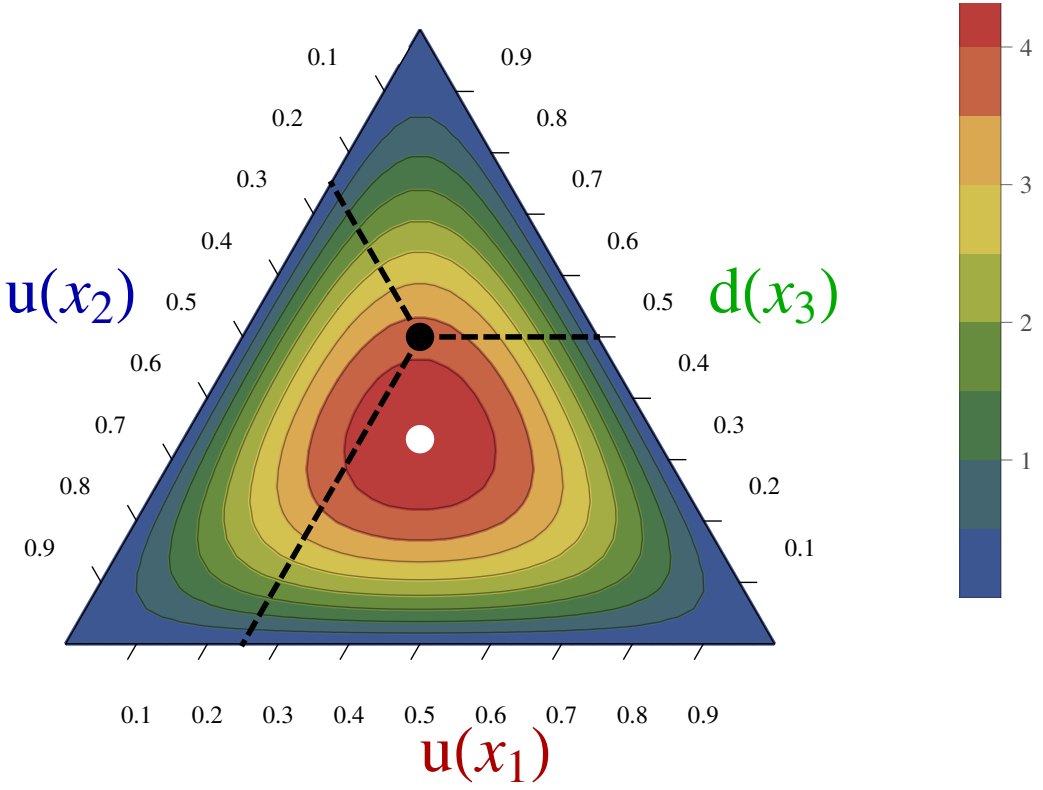
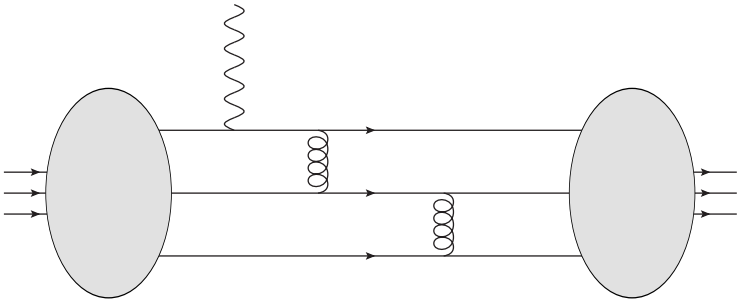


Form Factors: Nucleon case



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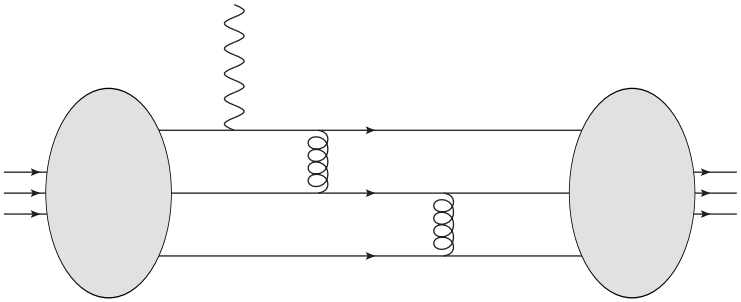

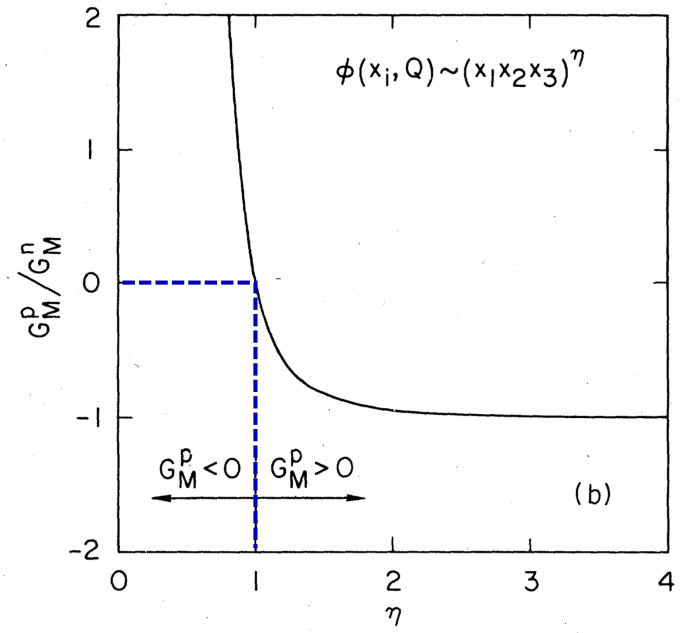

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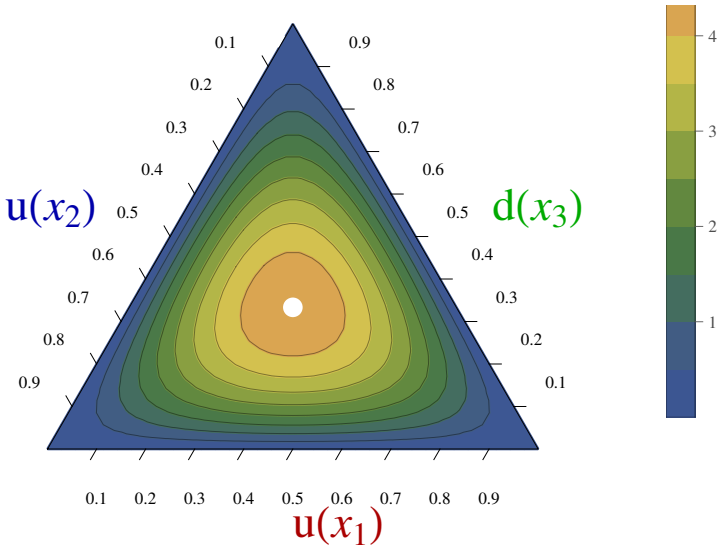

$\eta = 1$

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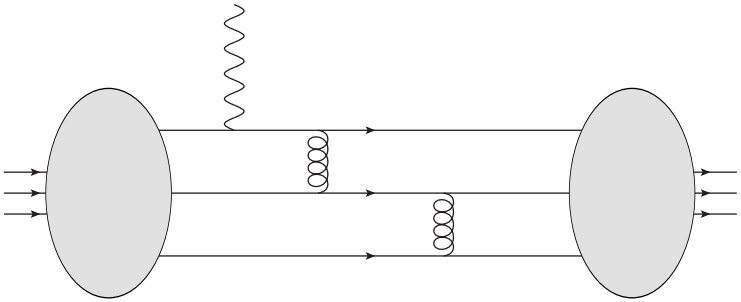

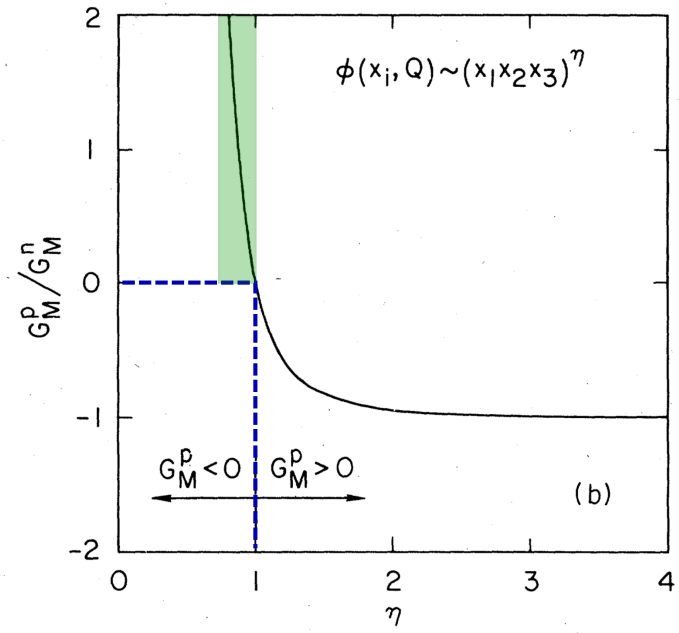

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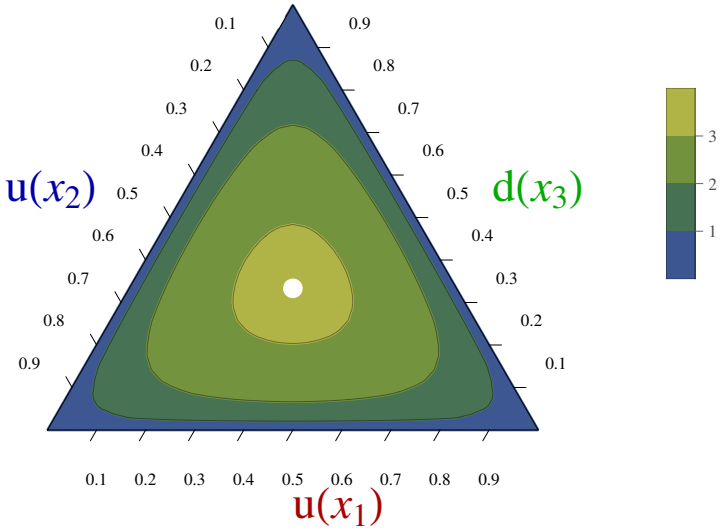

$\eta = 0.5$

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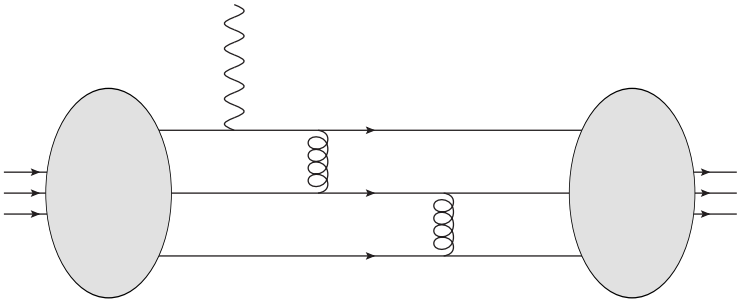

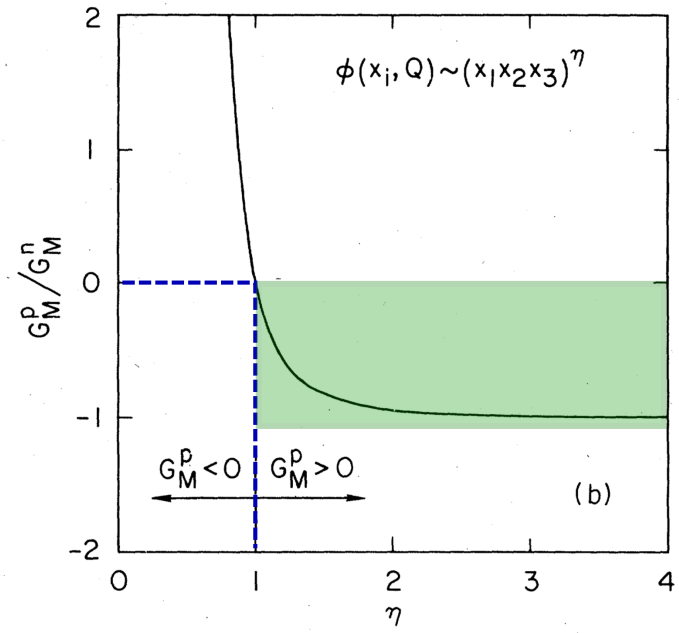

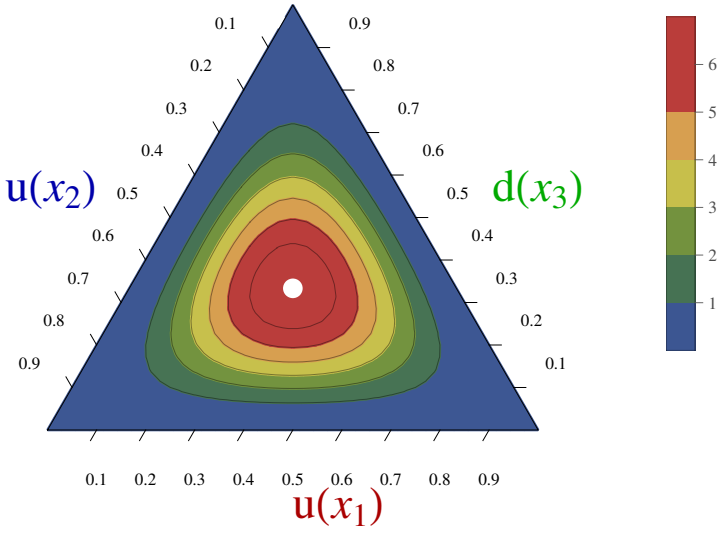

$\eta = 2$

S. Brodsky and G. Lepage, PRD 22, (1980)

Some previous studies of DA

- QCD Sum Rules
  - V. Chernyak and I. Zhitnitsky, Nucl. Phys. B 246 (1984)
- Relativistic quark model
  - Z. Dziembowski, PRD 37 (1988)
- Scalar diquark clustering
  - Z. Dziembowski and J. Franklin, PRD 42 (1990)
- Phenomenological fit
  - J. Bolz and P. Kroll, Z. Phys. A 356 (1996)
- Lightcone quark model
  - B. Pasquini *et al.*, PRD 80 (2009)
- Lightcone sum rules
  - I. Anikin *et al.*, PRD 88 (2013)
- Lattice Mellin moment computation
  - G. Bali *et al.*, JHEP 2016 02

Our Approach

- Inspired by the results obtained from DSEs and Faddeev equations.
- We do not use numerical solution of the Faddeev equation, but algebraic parametrisations based on the Nakanishi representation.
- This is an exploratory work: we want to know what we can or cannot do.
- We also assume the dynamical diquark correlations, both scalar and AV, and compare in the end with Lattice QCD one.

Nucleon DA as a Matrix Element

- Operator point of view for every DA (and at every twist):

$$\langle 0|\epsilon^{ijk}\left(u^i_\uparrow(z_1)C\not{n}u^j_\downarrow(z_2)\right)\not{n}d^k_\uparrow(z_3)|P,\lambda\rangle \to \varphi(x_i) \to O_\varphi,$$

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Braun *et al.*, Nucl.Phys. B589 (2000)

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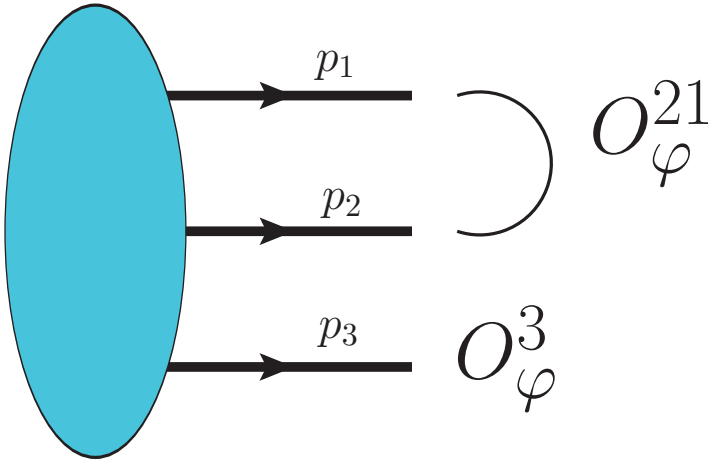

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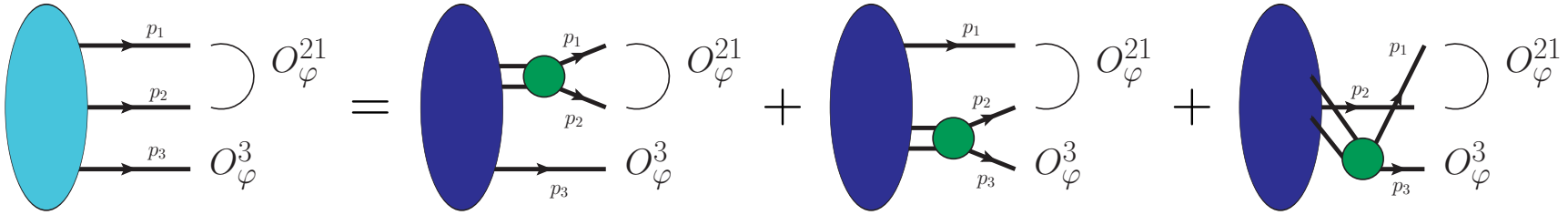

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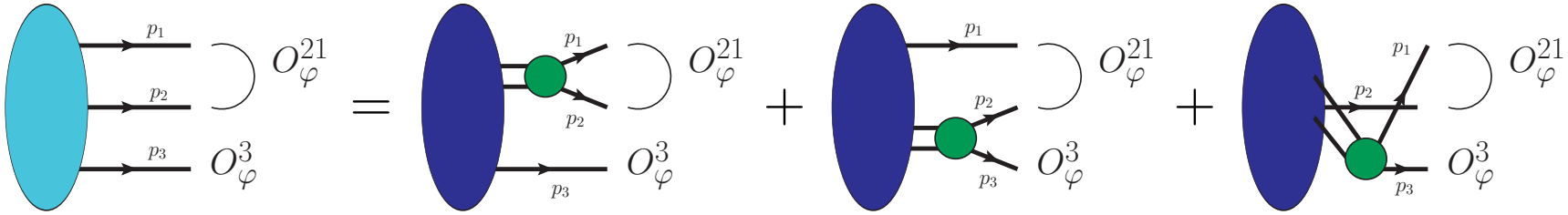

- The operator then selects the relevant component of the wave function.

Scalar diquark contribution

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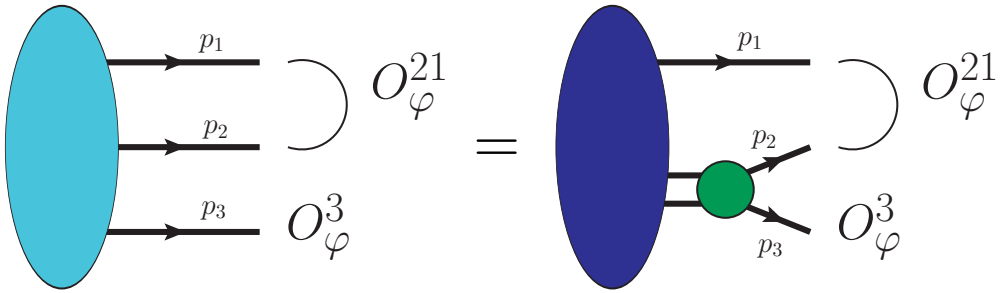

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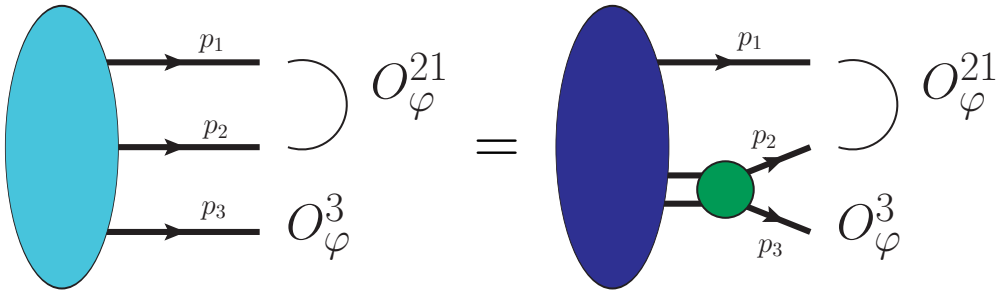

- The contraction of the Dirac indices between the single quark and the diquark makes it hard to understand.

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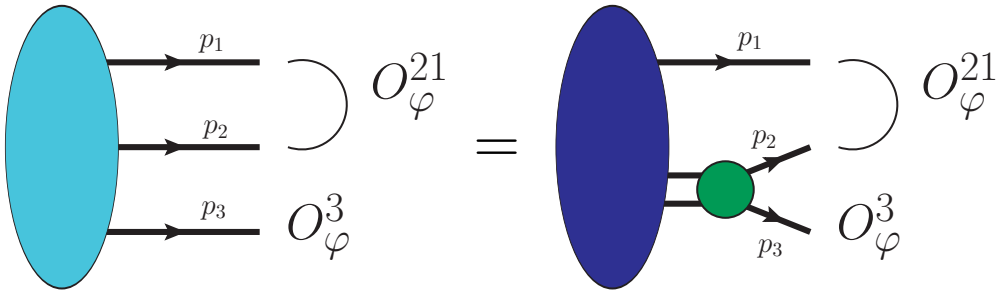
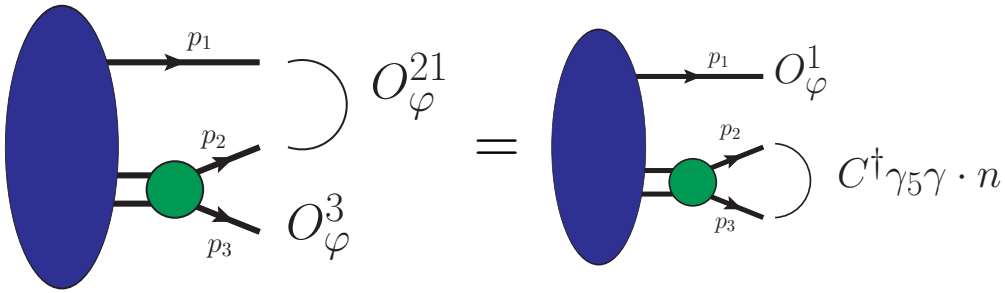


- The contraction of the Dirac indices between the single quark and the diquark makes it hard to understand.
- The way to write the nucleon Dirac structure is not unique, and can be modified (Fierz identity):



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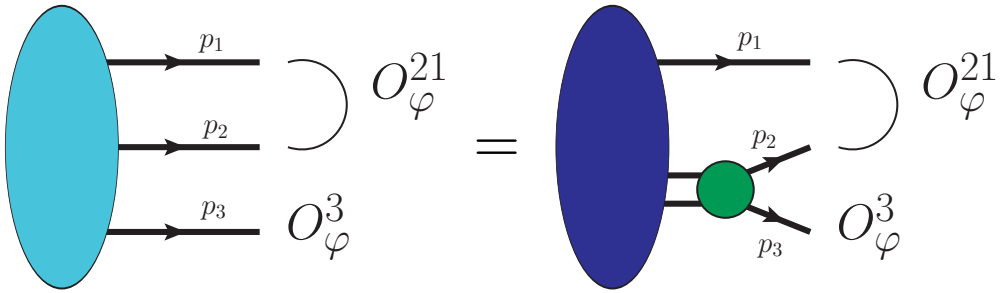

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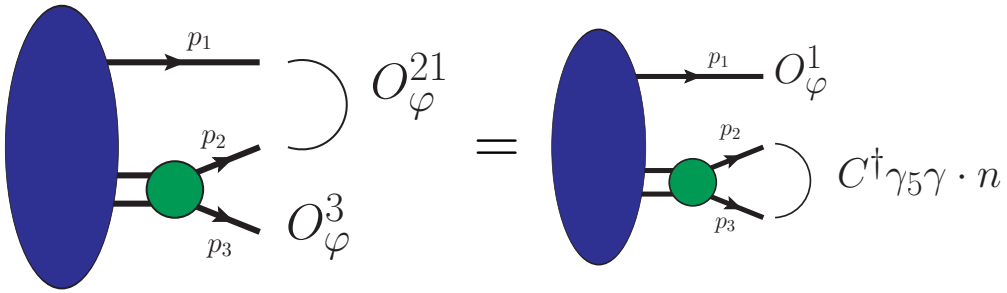

We recognise the leading twist DA of a scalar diquark

AV Contributions

$$\langle 0|\epsilon^{ijk}\left(u^i_\uparrow(z_1)C\not{n}u^j_\downarrow(z_2)\right)\not{n}d^k_\uparrow(z_3)|P,\lambda\rangle \to \varphi(x_i) \to O_\varphi,$$

$$\langle 0|\epsilon^{ijk}\left(u^i_\uparrow(z_1)Ci\sigma_{\perp\nu}n^\nu u^j_\uparrow(z_2)\right)\gamma^\perp\not{n}d^k_\uparrow(z_3)|P,\lambda\rangle \to T(x_i) \to O_T,$$

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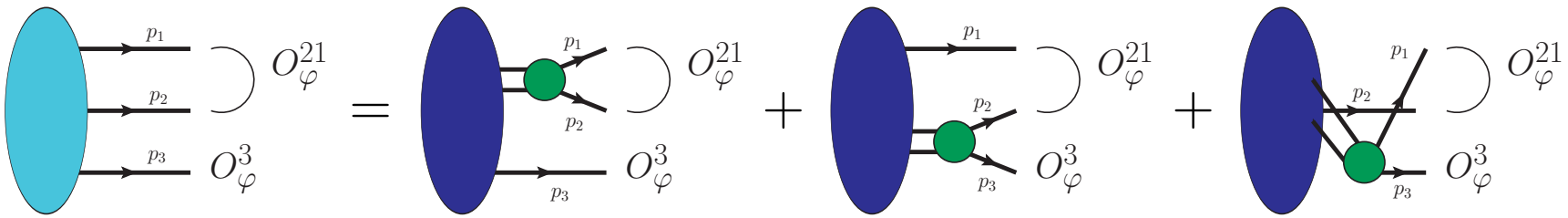

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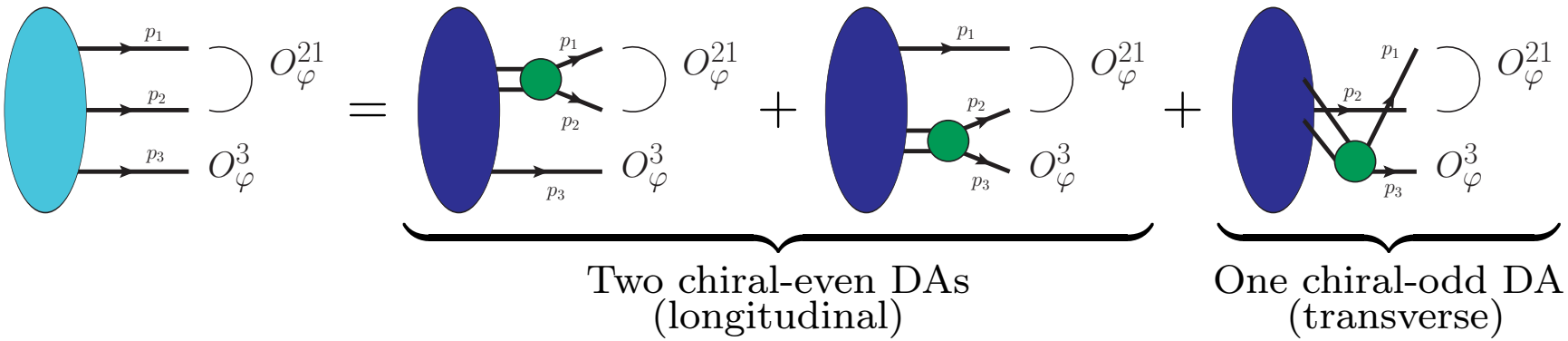

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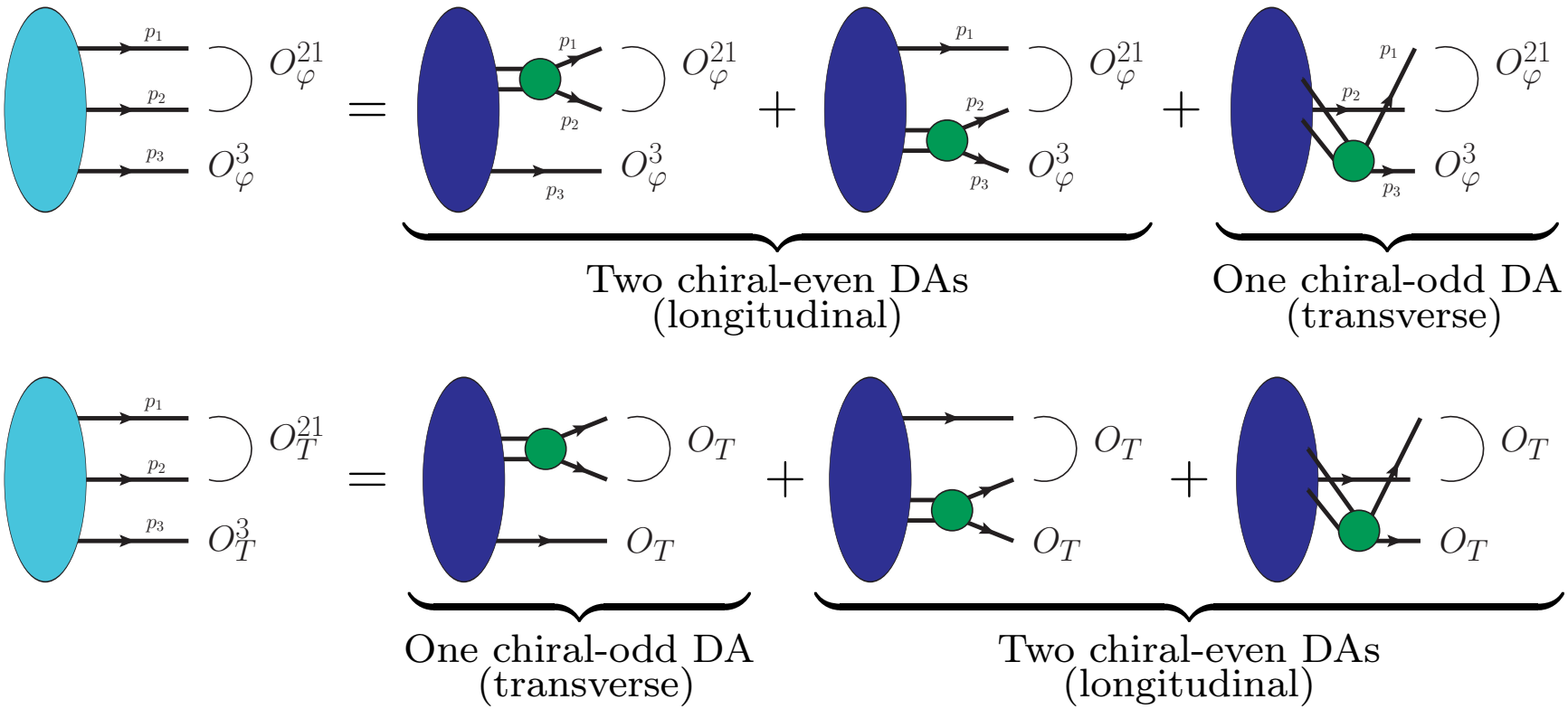

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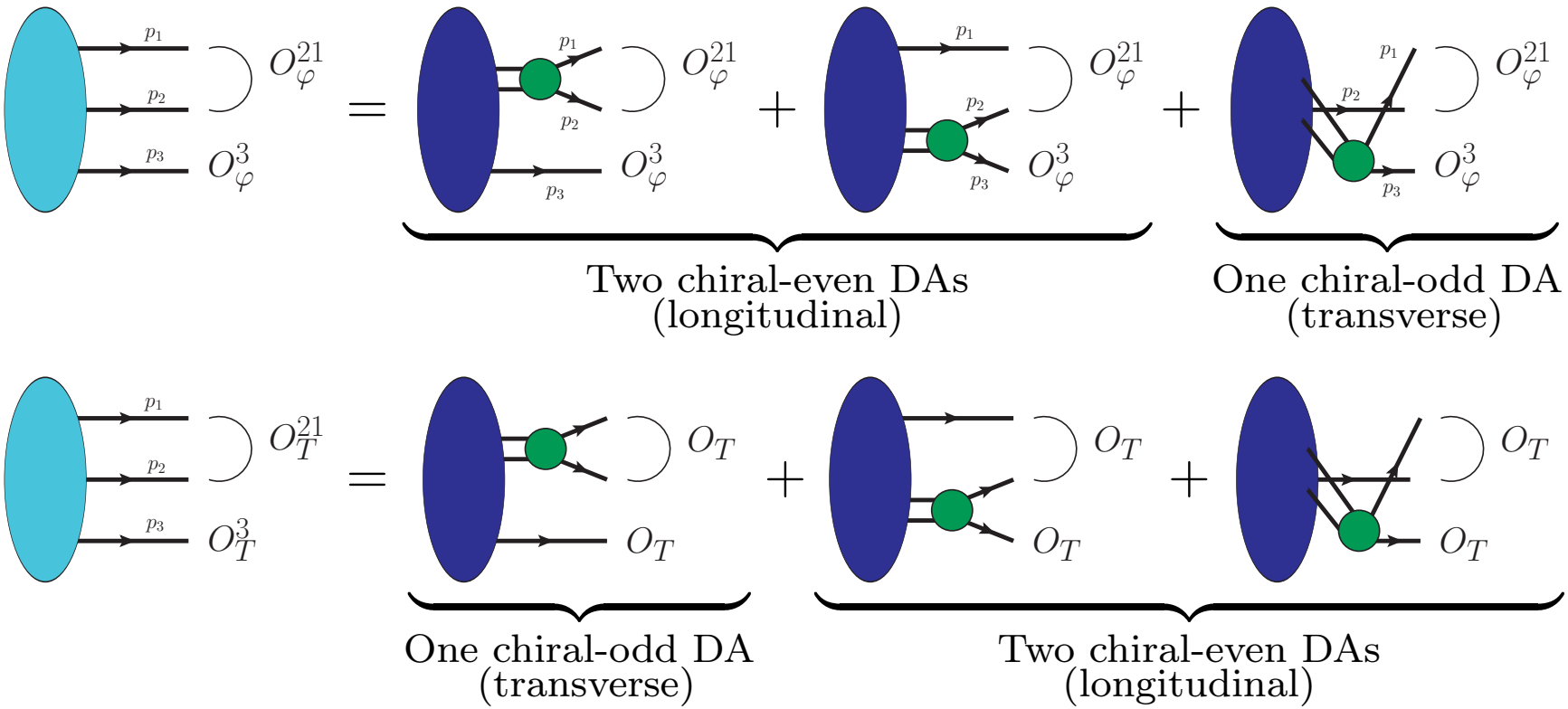

$$2T(x_1,x_2,x_3) = \varphi(x_1,x_3,x_2) + \varphi(x_2,x_3,x_1)$$

Modeling the Diquarks

Modeling hypotheses

- For the scalar diquark, we assume:
  - a point like structure
  - a extended structure based on a algebraic Nakanishi Ansatz
- For the Nakanishi Ansatz, the results are consistent with the numerical one obtained in the case of the pion, *i.e.* broader than the meson asymptotic PDA and concave.

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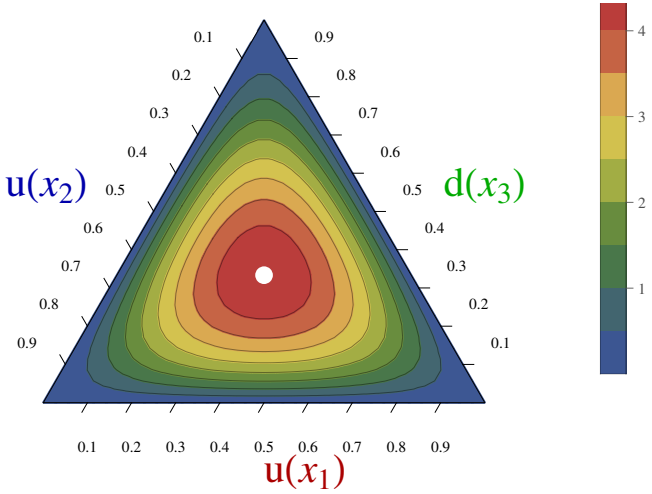
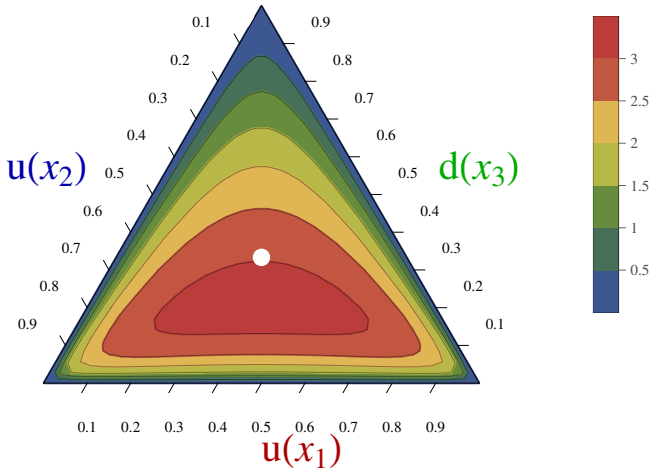
- For the Nakanishi Ansatz, the results are consistent with the numerical one obtained in the case of the pion, *i.e.* broader than the meson asymptotic PDA and concave.
- In the AV case, we looked only at an extended diquark.
- The results of the chiral-even and chiral-odd are consistent with what has been obtained on ρ meson *i.e.* broad and concave.
- But the broadness is probably to high in the chiral-odd case and too low in the chiral-even case. And that has some consequences in the following.

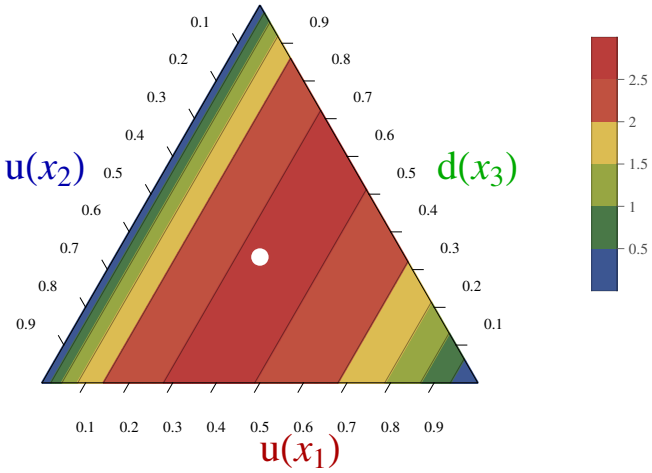
Results in the scalar channel



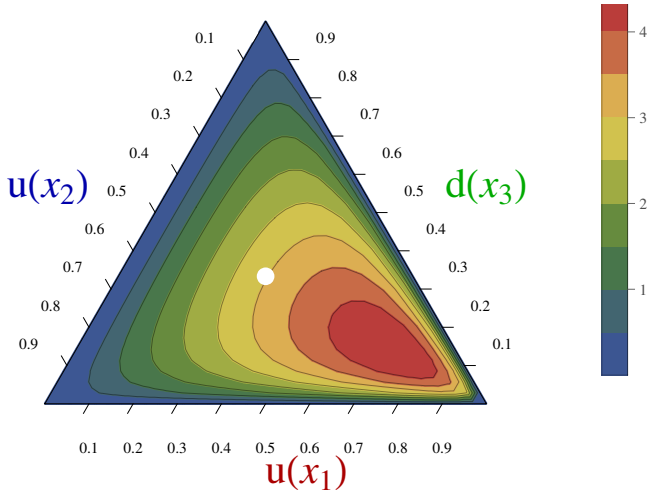
Asymptotic DA



Extended case: T

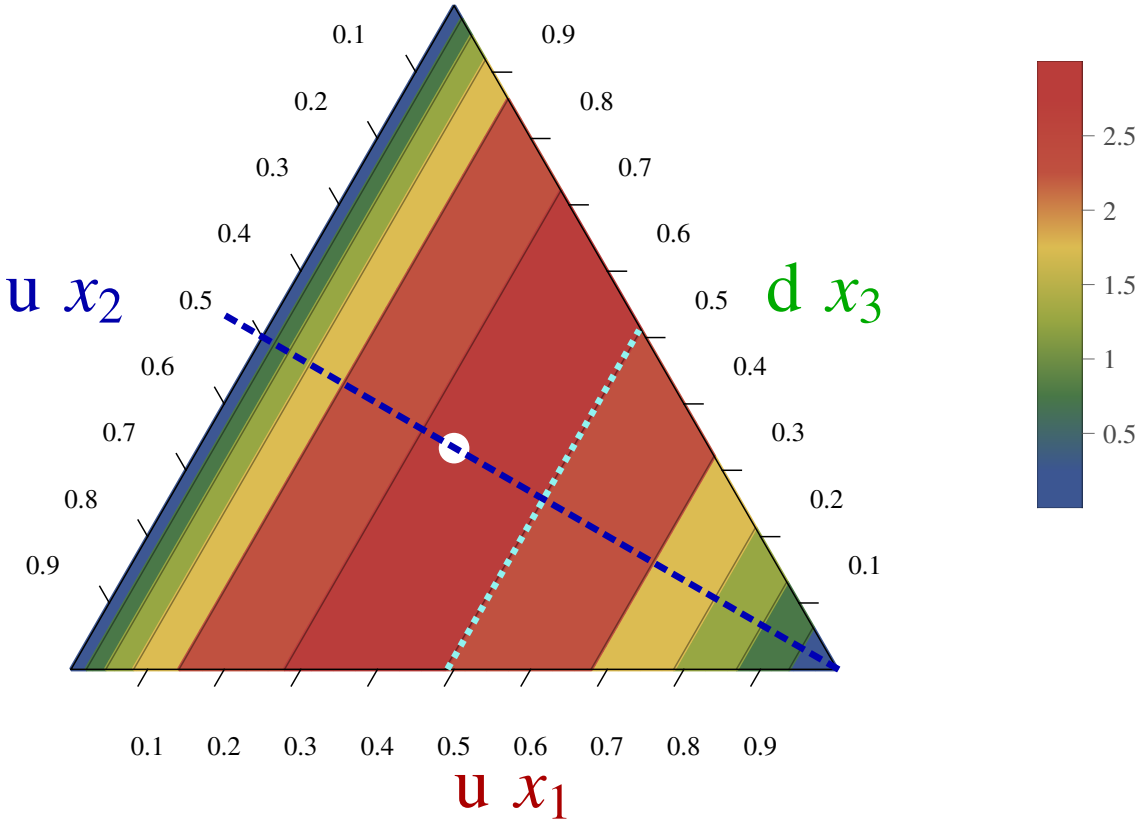


Point-like case: φ



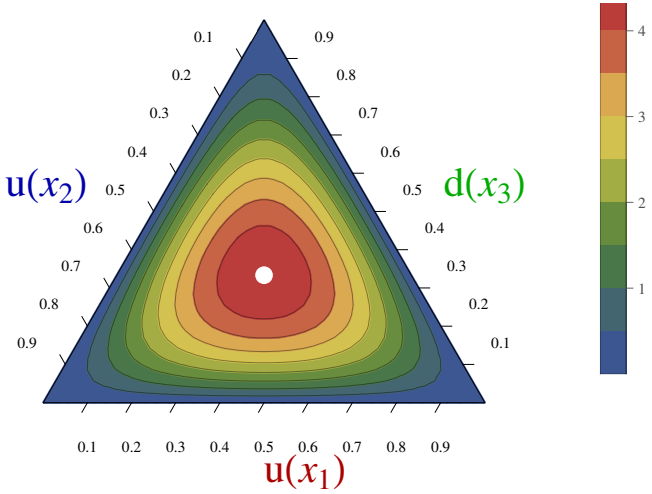
Extended case: φ

Results in the scalar channel

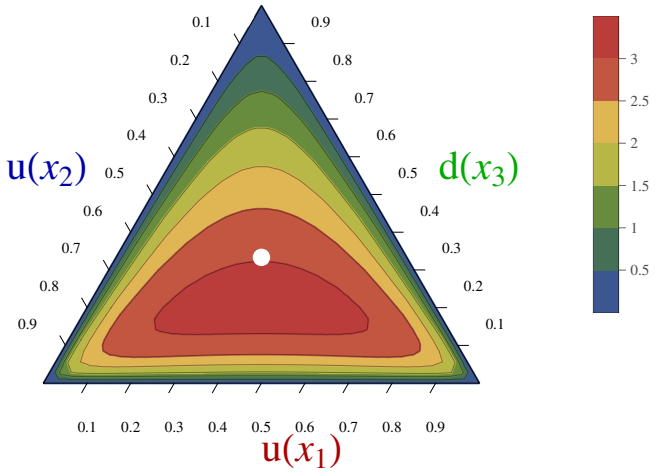


Point-like case: φ

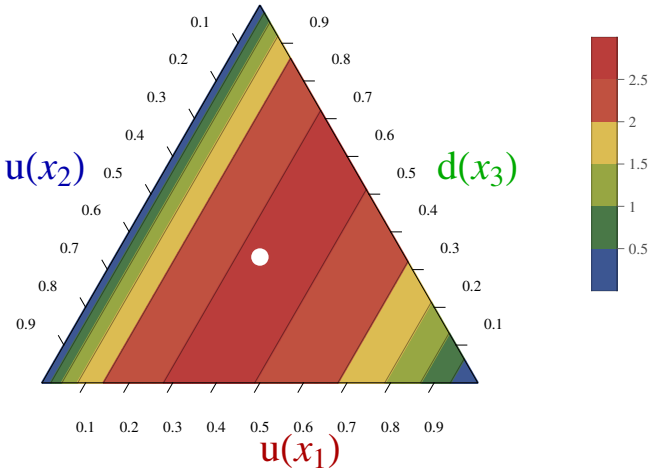
Results in the scalar channel



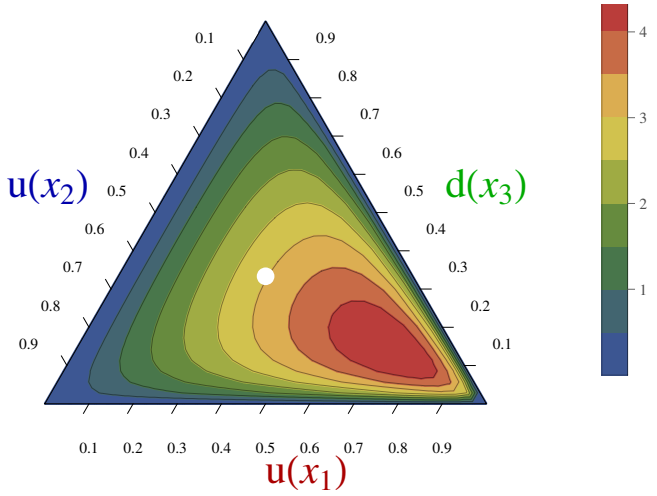
Asymptotic DA



Extended case: T



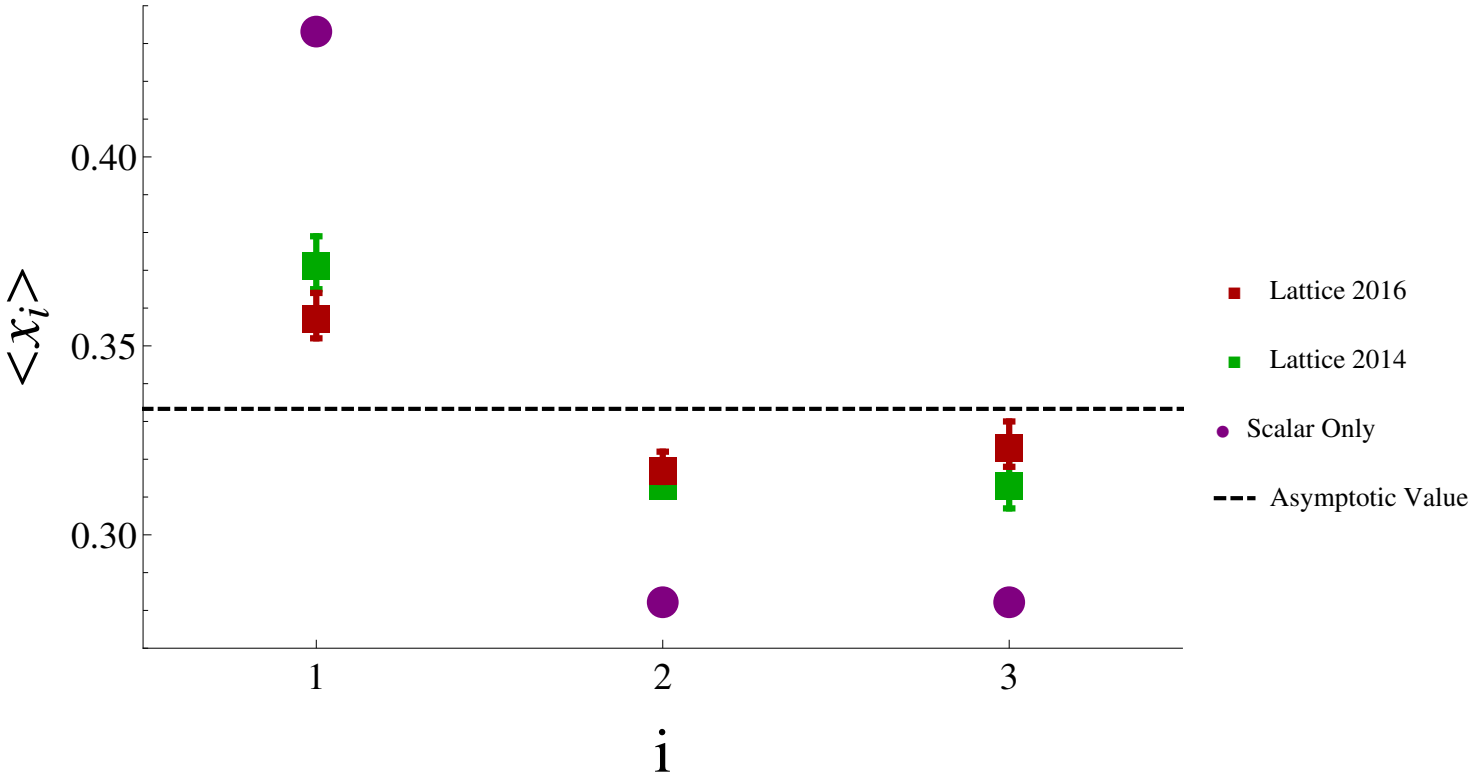
Point-like case: φ



Extended case: φ

Comparison with lattice I

$$< x_i >_\varphi = \int \mathcal{D}x \; x_i \varphi(x_1, x_2, x_3)$$

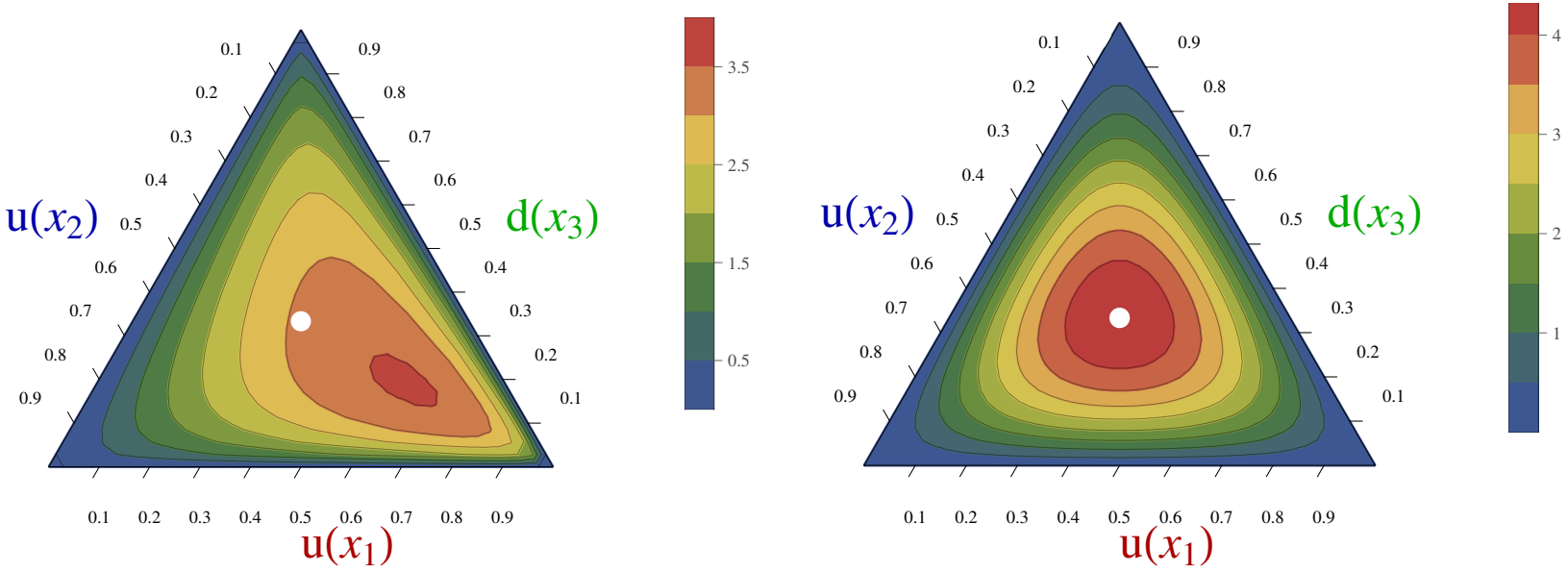


Lattice data from V.Braun *et al*, PRD 89 (2014)

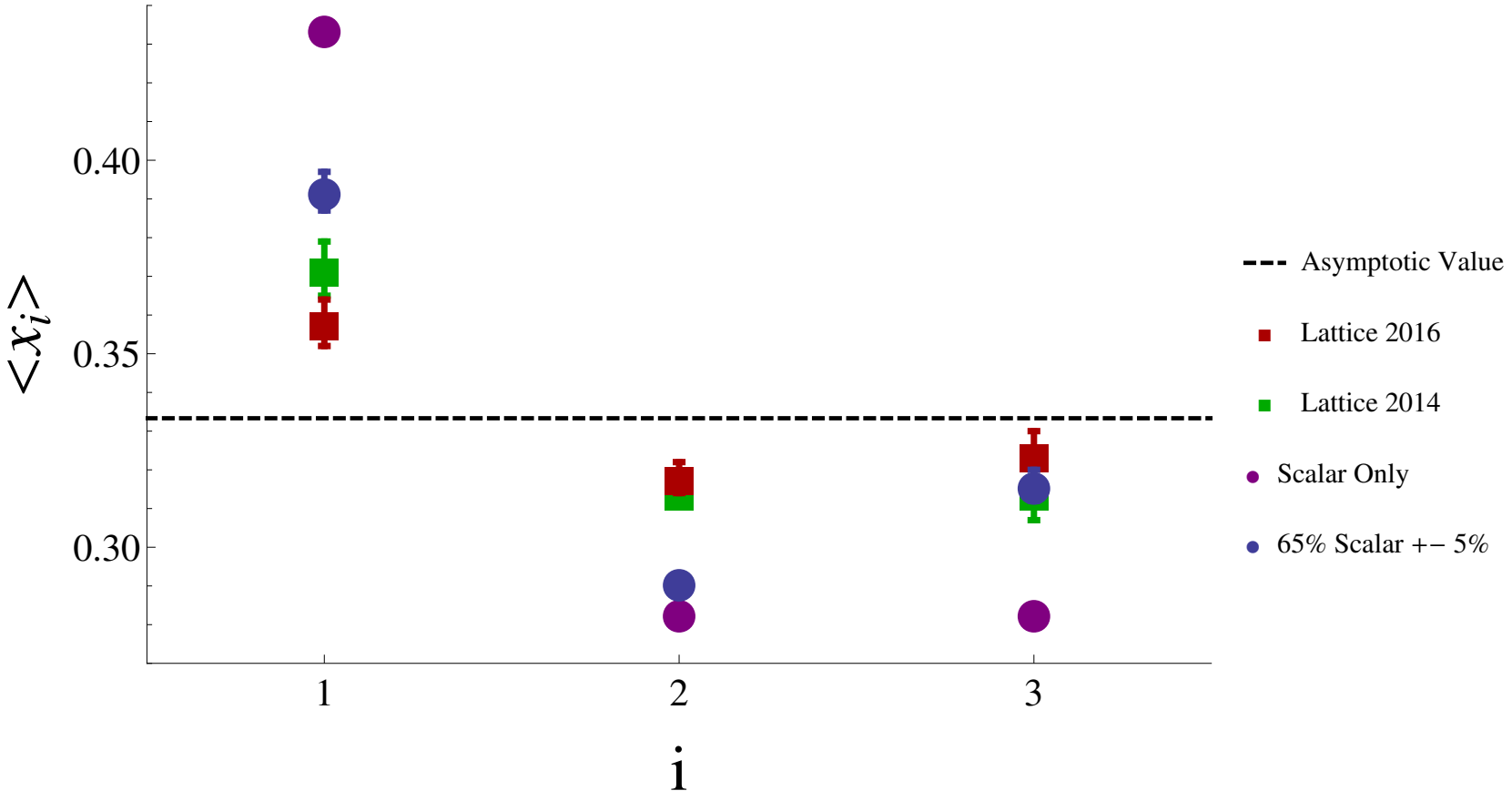
G. Bali *et al.*, JHEP 2016 02

Complete results for φ

- We use the prediction from the Faddeev equation to weight the scalar and AV contributions 65/35:



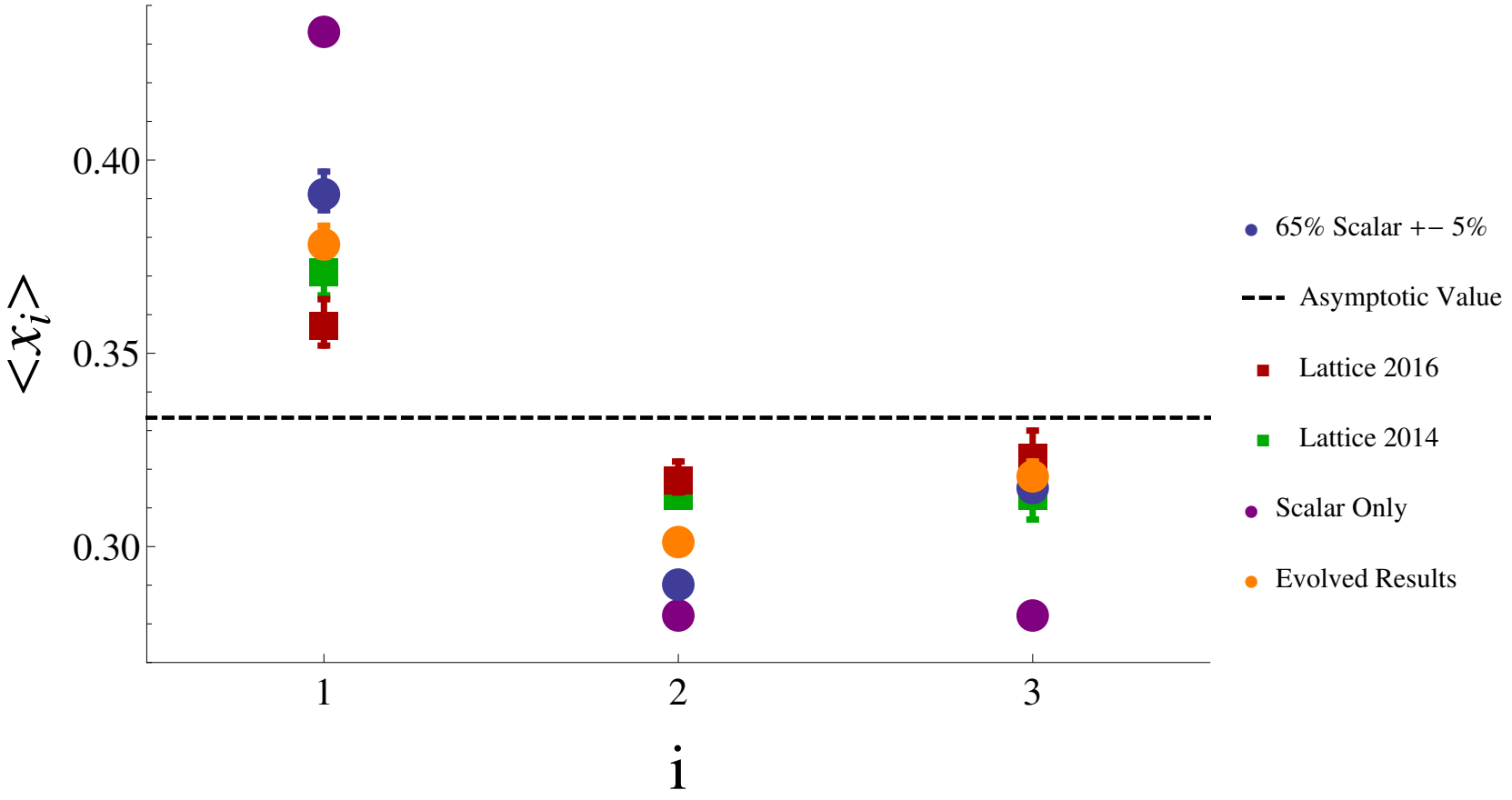
Comparison with lattice II



Lattice data from V.Braun *et al*, PRD 89 (2014)

G. Bali *et al.*, JHEP 2016 02

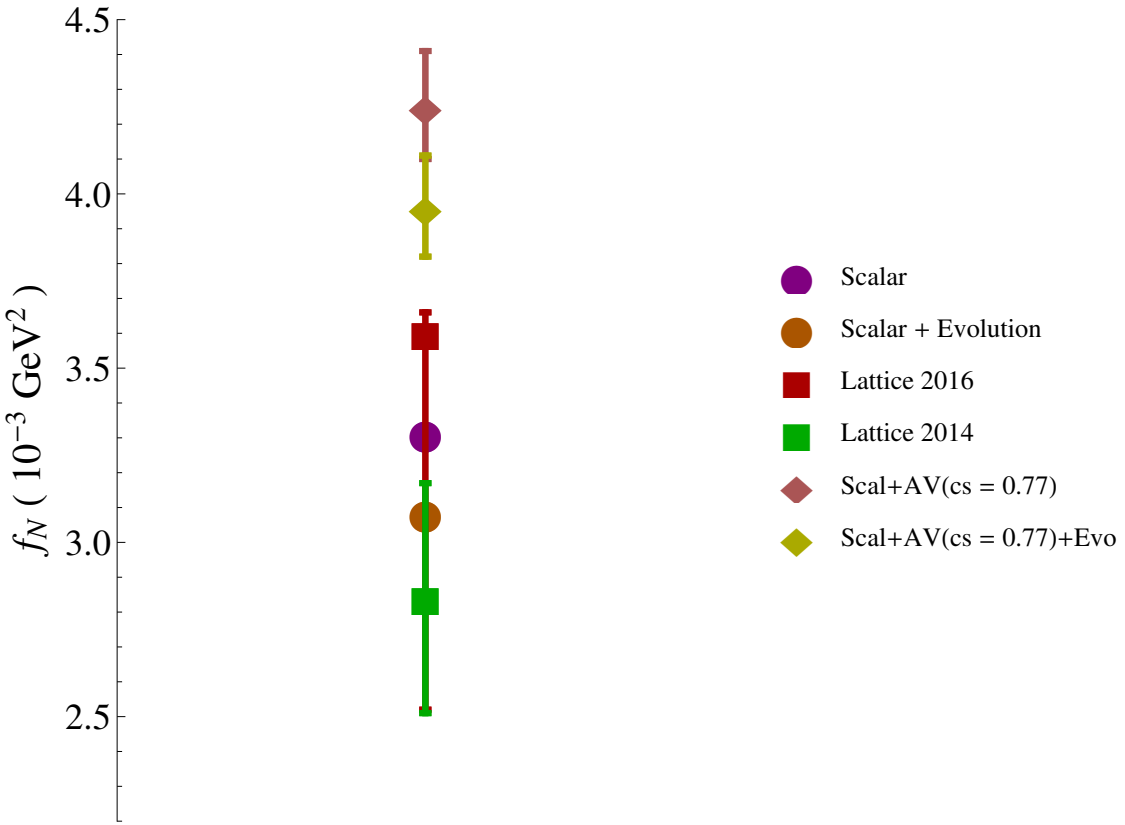
Comparison with lattice II



Lattice data from V.Braun *et al*, PRD 89 (2014)

G. Bali *et al.*, JHEP 2016 02

Comparison with lattice III

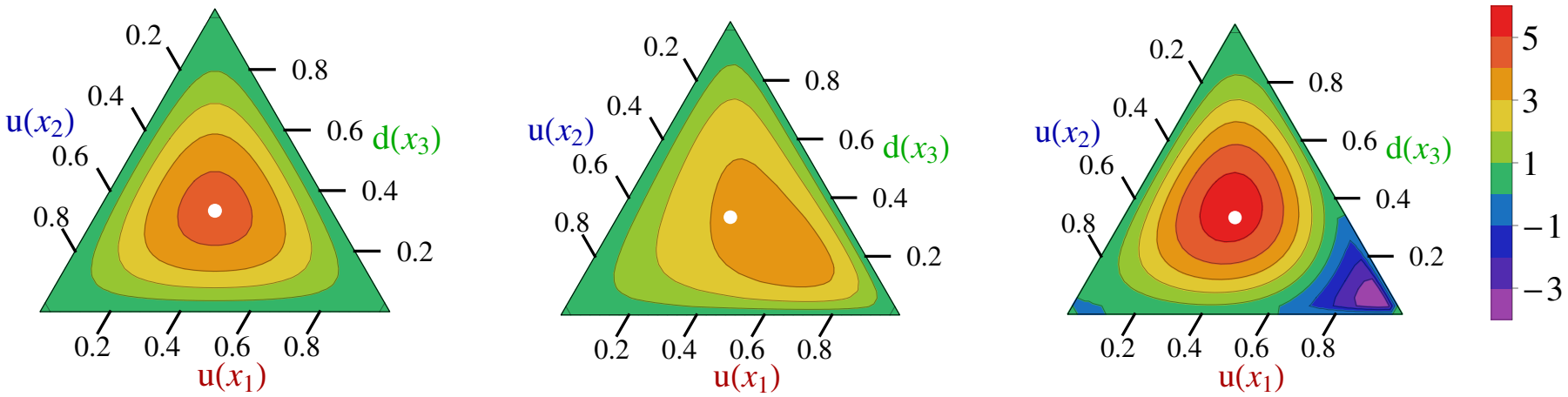


Computations done by J. Segovia

Lattice data from V.Braun *et al*, PRD 89 (2014)

G. Bali *et al.*, JHEP 2016 02

Evolved Results

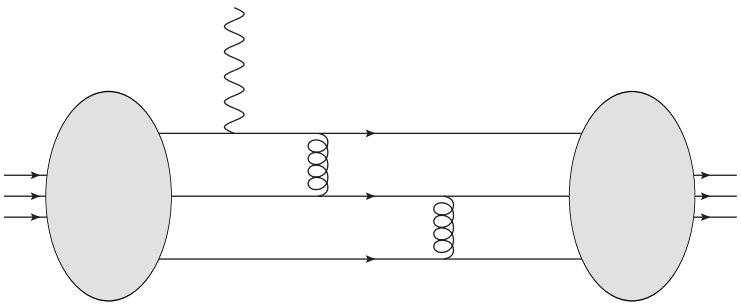


- Results for the nucleon and Roper at 2 GeV.
- The nucleon remains **broader** and the peak **shifted** toward large x_1
- The roper present a negative area consistently with our understanding of $n = 1$ excited states.
- We provide a parametrisation of our results at 2 GeV.

CM *et al.*, arXiv 1711.09101 submitted to PLB

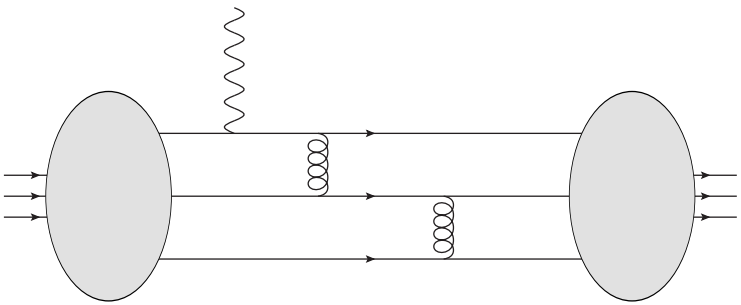
Chapter 4: Form Factors

Form Factors



$$F_1(Q^2) = \mathcal{N} \int [\mathrm{d}x_i][\mathrm{d}y_i] \left[\varphi(x_i, \zeta_x^2) H_\varphi(x_i, y_i, Q^2, \zeta_x^2, \zeta_y^2) \varphi(y_i, \zeta_y^2) + T(x_i, \zeta_x^2) H_T(x_i, y_i, Q^2, \zeta_x^2, \zeta_y^2) T(y_i, \zeta_y^2)\right]$$

Form Factors



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- LO Kernel well known since more than 30 years...
- ...but different groups have argued different choices for the treatment of scales:
  - for the DA : $\varphi(Q^2), \varphi((min(x_i) \times Q)^2)$...,
  - for the strong coupling constant : $\alpha_S(Q^2), \alpha_s(< x_i >^2 Q^2), \alpha_s^{\mathrm{reg}}(g(x_i, y_j) Q^2)$
- Use of perturbative coupling vs. effective coupling?

Digression: Pion FF

- In the pion case, the hard kernel is known at NLO allowing us to discuss more extensively the scale effects.

R Field *et al.*, NPB 186 429 (1981)
F. Dittes and A. Radyushkin, YF 34 529 (1981)
B. Melic *et al.*, PRD 60 074004 (1999)

- The UV scale dependent term behaves like:

$$f_{UV}(\mu) \propto \beta_0 \left(5/3 - \ln((1-x)(1-y)) + \ln\left(\frac{\mu^2}{Q^2}\right)\right)$$

- Here I take two examples:
  - the standard choice of $\zeta_x^2 = \zeta_y^2 = \mu^2 = Q^2/4$
  - the regularised BLM-PMC scale $\zeta_x^2 = \zeta_y^2 = \mu^2 = e^{-5/3} Q^2/4$

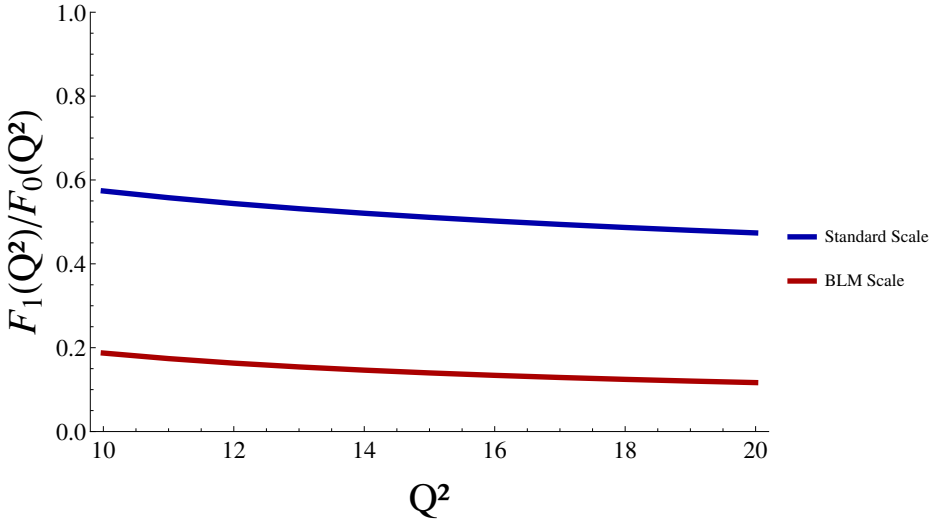
S. Brodsky *et al.*, PRD 28 228 (1983)
S. Brodsky and L. Di Giustino, PRD 86 085026 (2011)

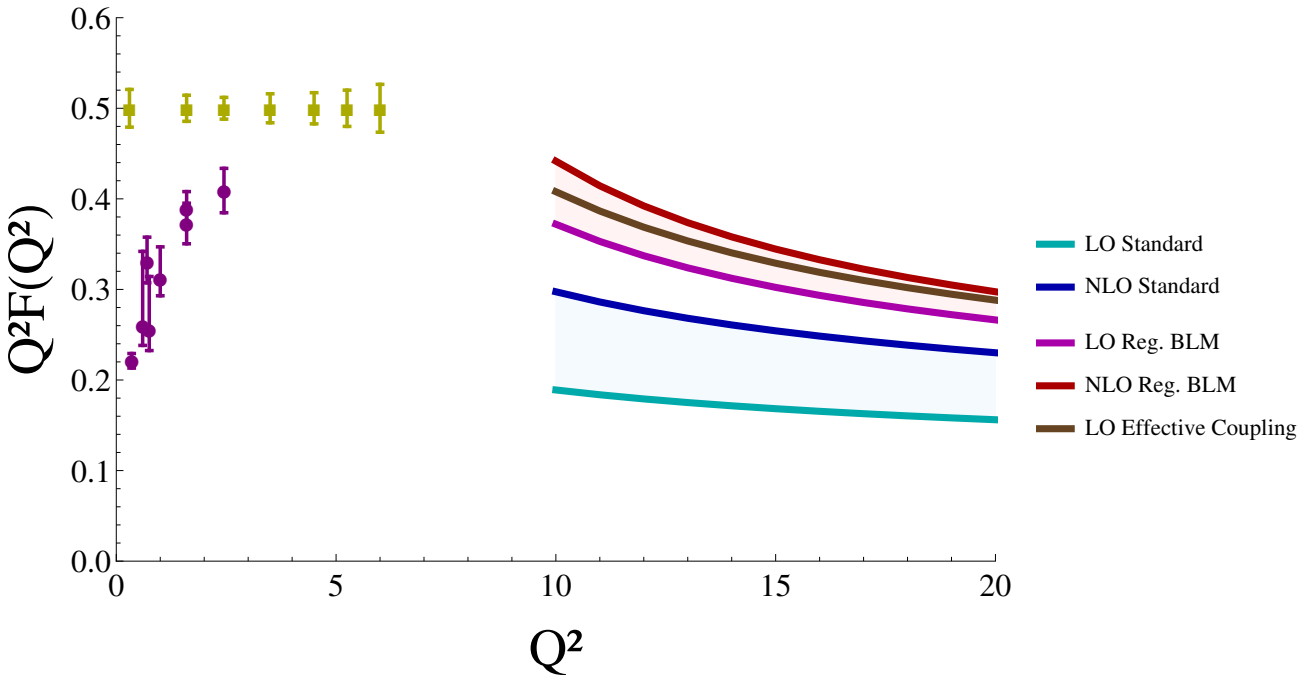
- What is the effect on our meson like DA?

$$\varphi_\pi(x, \zeta^2 = 4GeV^2) = \mathcal{N}\left(1 - \frac{\ln\left[1 + \alpha x(1-x)\right]}{\alpha x(1-x)}\right)$$

α is tuned on LQCD Mellin Moments

Digression: Pion FF





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- BLM scale reduces significantly the impact of the NLO corrections and increase dramatically the LO one.

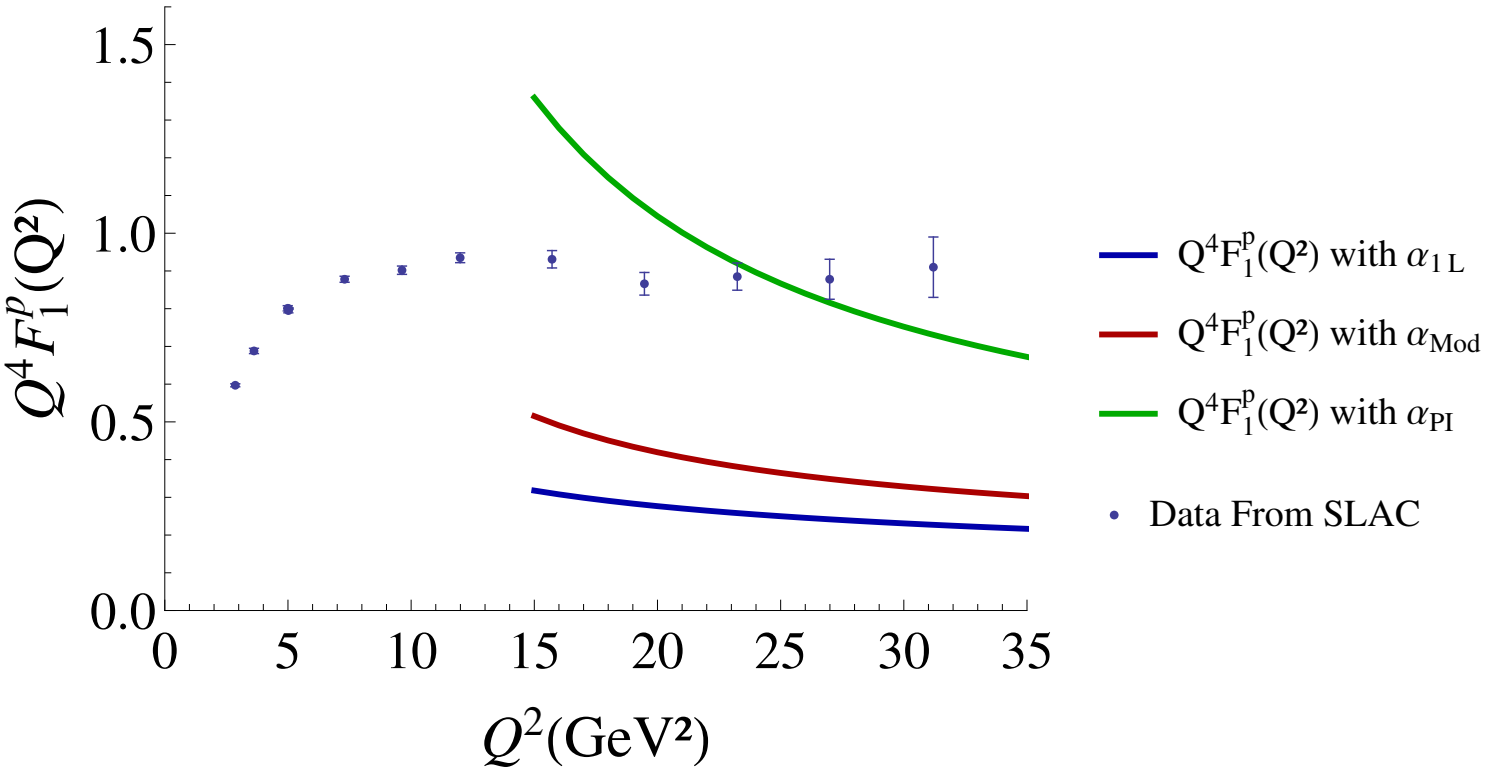
Proton case

- Unfortunately, only the LO treatment has been performed
  $\Rightarrow$ BLM scale is therefore unknown
- We use the Chernyak-Zhitnitsky formalism to compute the nucleon for factor with:
  - the CZ scale setting $\rightarrow \alpha_s(Q^2/9)\alpha_s(4Q^2/9)$
  - the pion BLM factor $\rightarrow \alpha_s(Q^2/9\, e^{-5/3})\alpha_s(4Q^2/9\, e^{-5/3})$

  and using both perturbative and effective couplings.

Proton case

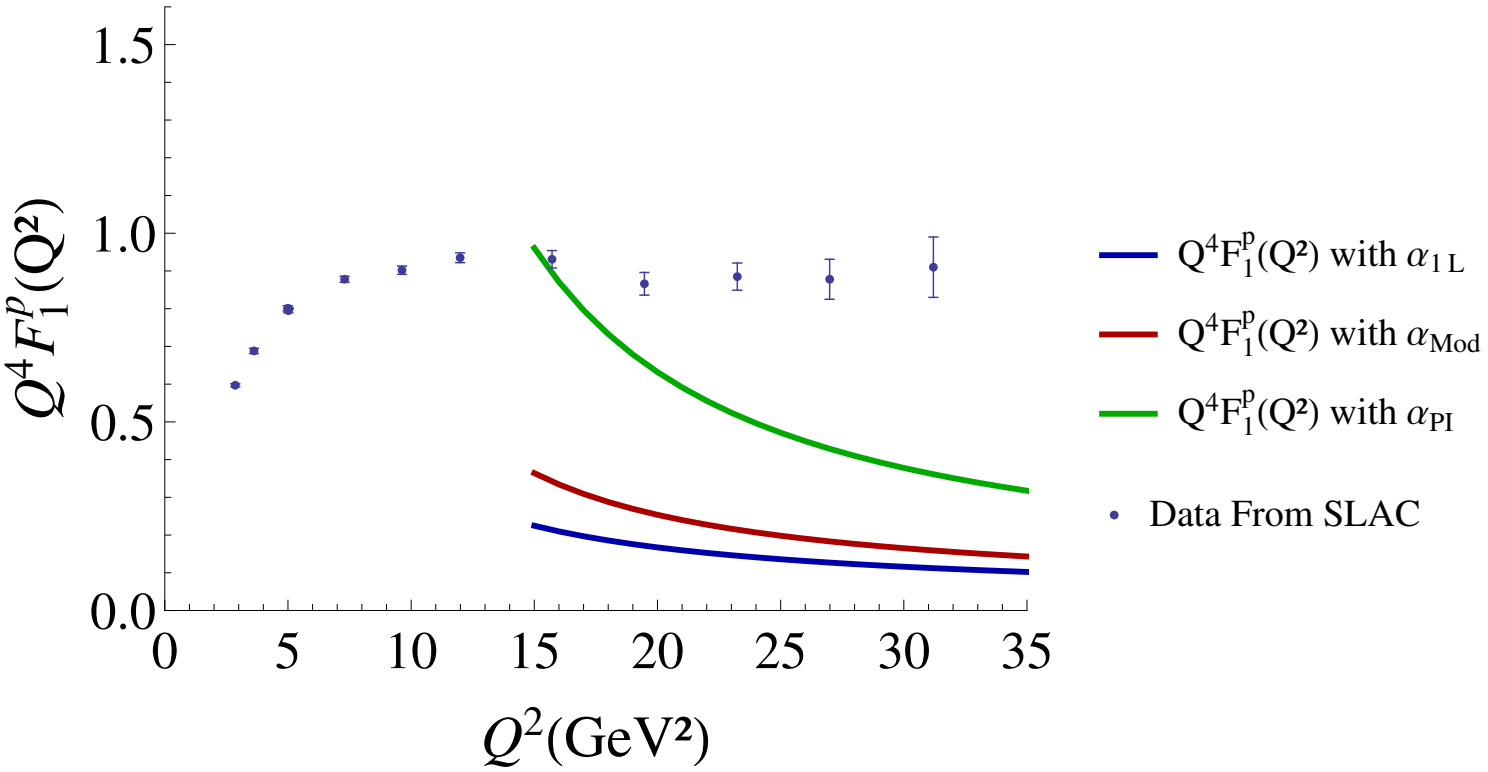
CZ scale setting with frozen PDA



Data from Arnold et al. PRL 57

Proton case

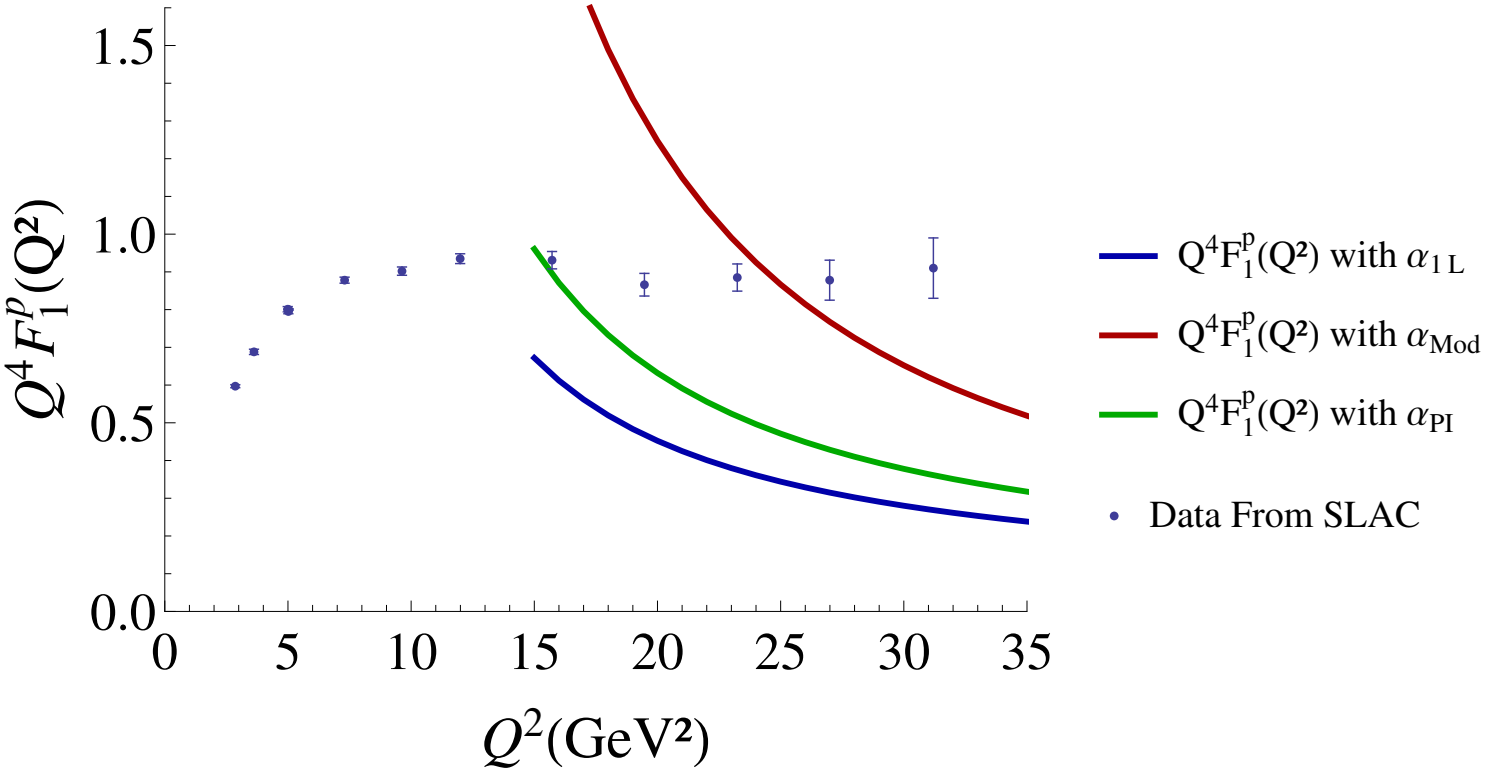
CZ scale setting + evolution



Data from Arnold et al. PRL 57

Proton case

Pion BLM Factor + evolution



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  and using both perturbative and effective couplings.
- The data remain flat while the perturbative running show a logarithmic decreasing.
- More work are required to conclude on the validity of the perturbative approach:
  - Theory side : we need NLO corrections and work with more advanced PDA models
  - Experimental side : more precise data to spot a logarithmic decreasing (or not!)

Conclusion

Summary

- Both nucleon DAs φ and T can be described using a quark-diquark approximation.
- We show how the diquark types and chiralities are selected.
- The comparison with lattice computation is very encouraging and explains how the different diquarks contribute to the total DAs, and the respective sensitivity of the latter to the AV-diquarks.
- It is possible to extend the work on the nucleon to the Roper case.
- In the Roper case, the results of individual diquarks contributions seem to be consistent with a $n = 1$ excited state.
- We now have a working evolution code.
- We have preliminary results on the nucleon Form Factors.

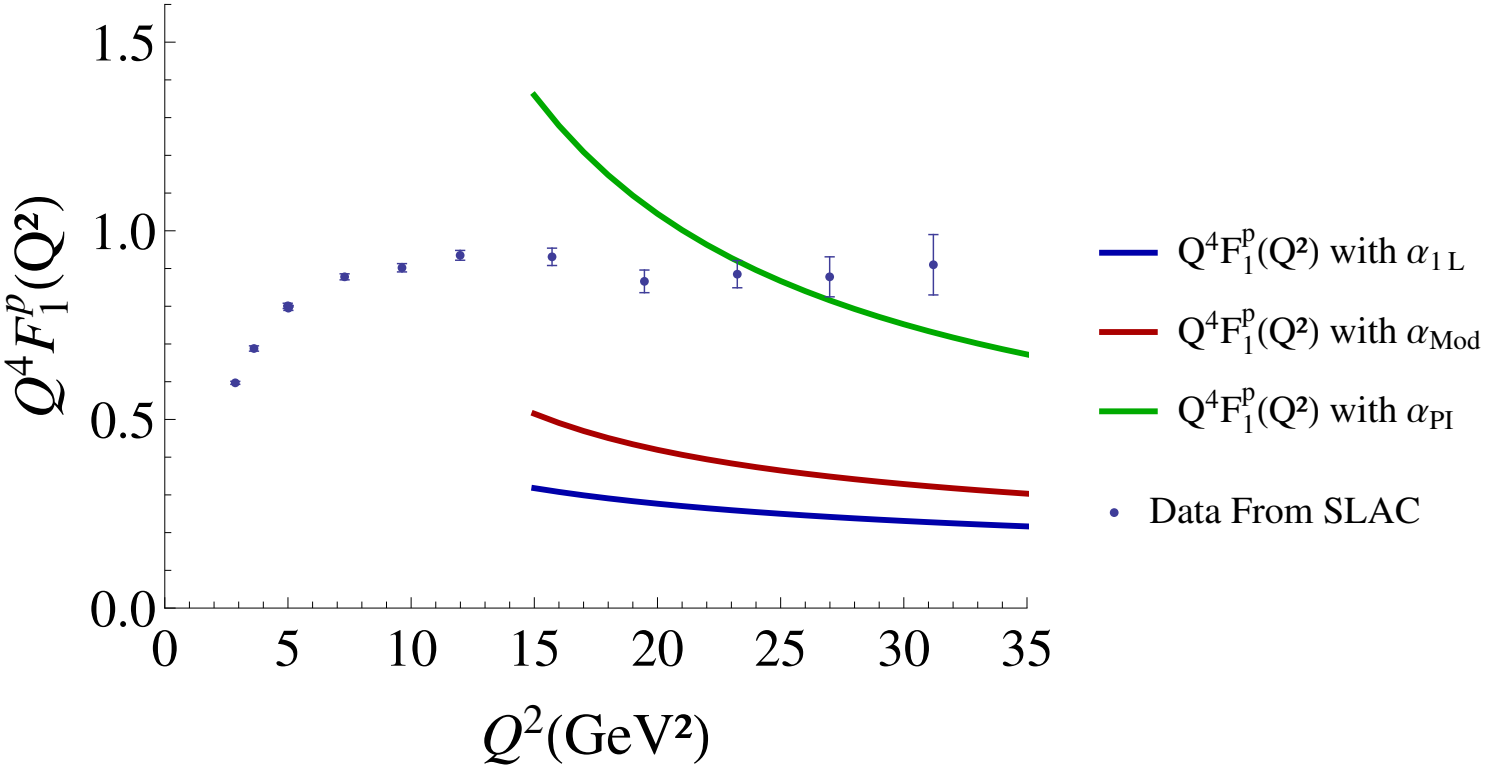
Conclusion

- Our results on the Baryon PDA are very encouraging within our simple assumptions as they almost match the lattice one. This bring us confidence in the computations of LFWFs.
- Comparison to experimental data on the nucleon form factor at large momentum transfer remains however unsatisfactory, as the experimental data suggest a flat behaviour instead of a logarithmic decreasing one.
- NLO corrections are needed, as testifies the pion case.
- We want to go beyond both the algebraic model and the PDA computations. Other matrix elements could be computed from the LFWF such as PDFs, GPDs, TMDs...

Thank you for your attention

Back up slides

Frozen IR scale

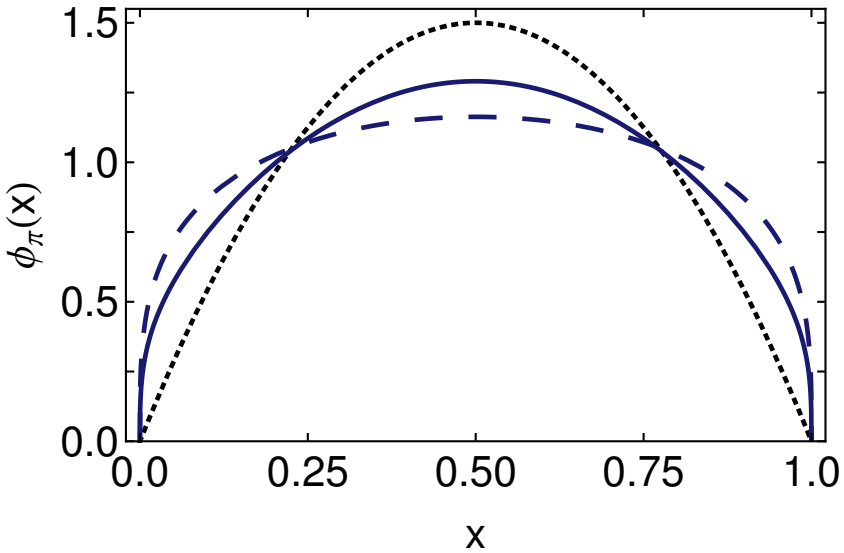


Data from Arnold et al. PRL 57

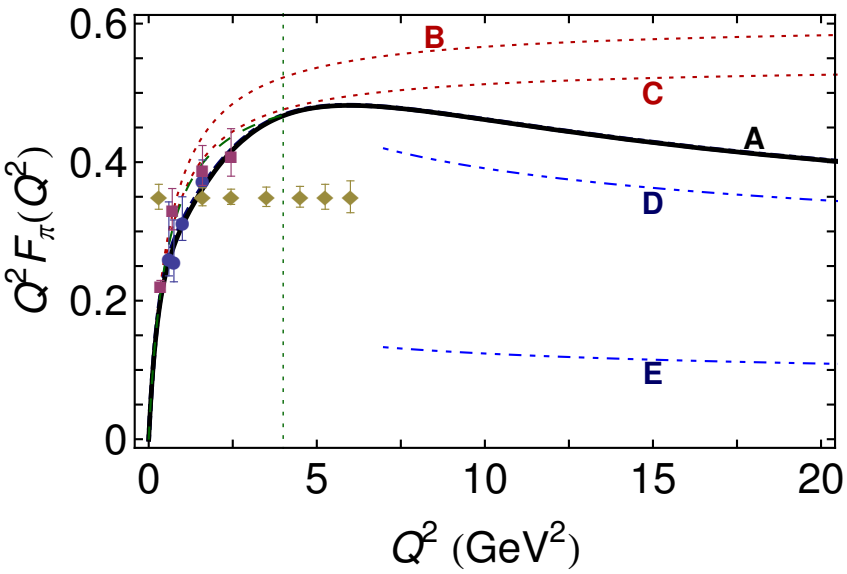
More work is required before we can conclude anything.

Pion distribution amplitude

$$\phi_{As}(x) = 6x(1-x)$$



L. Chang *et al.* (2013)



L. Chang *et al.* (2013)

- Broad DSE pion DA is much more consistent with the form factor than the asymptotic one.
- The scale when the asymptotic DA become relevant is huge.

N(1535)

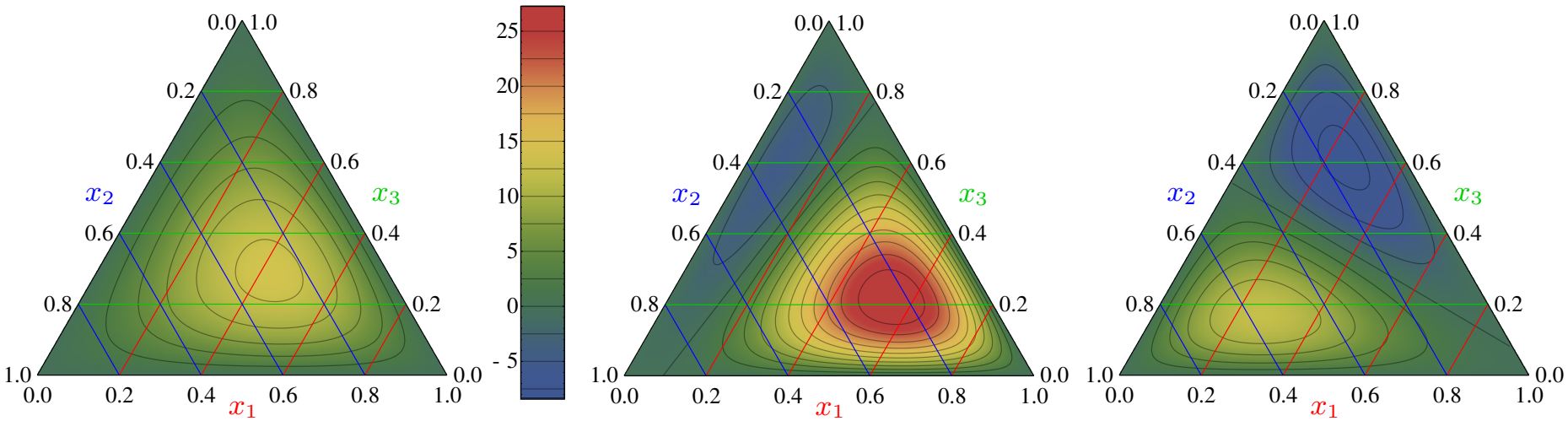
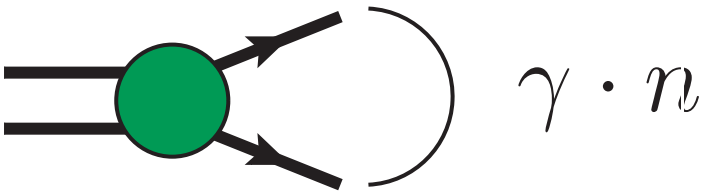


Figure from V. Braun *et la.*,Phys. Rev. **D89**, 094511 (2014)

Modeling the Diquarks

Scalar diquark I: the point-like case



- Quark propagator:

$$S(q) = \frac{-i\not{q} + M}{q^2 + M^2}$$

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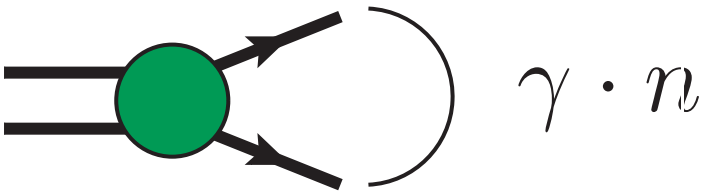

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- Bethe-Salpeter amplitude (1 out of 4 structures):

$$\Gamma^{0+}_{\rm PL}(q, K) = i\gamma_5 C \mathcal{N}^{0+}$$

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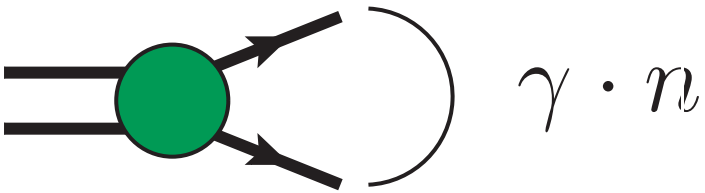

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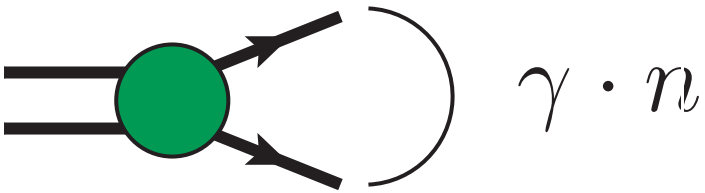
- This point-like case leads to a flat DA:

$$\phi_{\rm PL}(x) = 1$$

Scalar diquark II: the Nakanishi case



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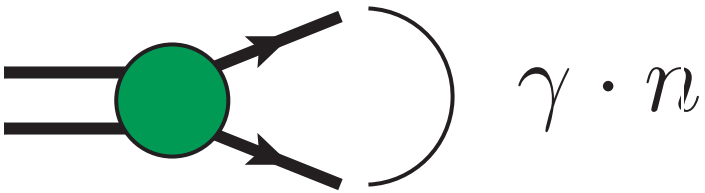

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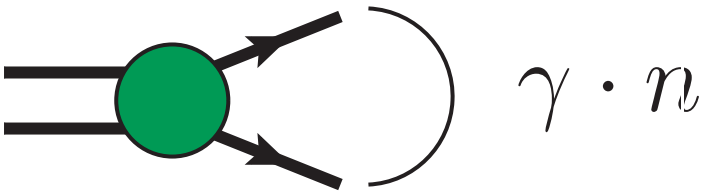

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- The Nakanishi case leads to a non trivial DA:

$$\phi(x) \propto 1 - \frac{M^2}{K^2} \frac{\ln\left[1 + \frac{K^2}{M^2}x(1-x)\right]}{x(1-x)}$$

Scalar DA behaviour

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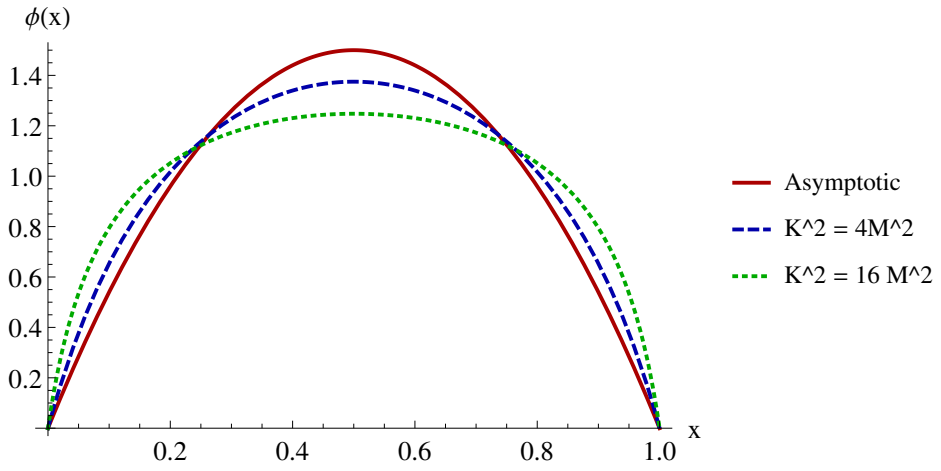
Scalar diquark



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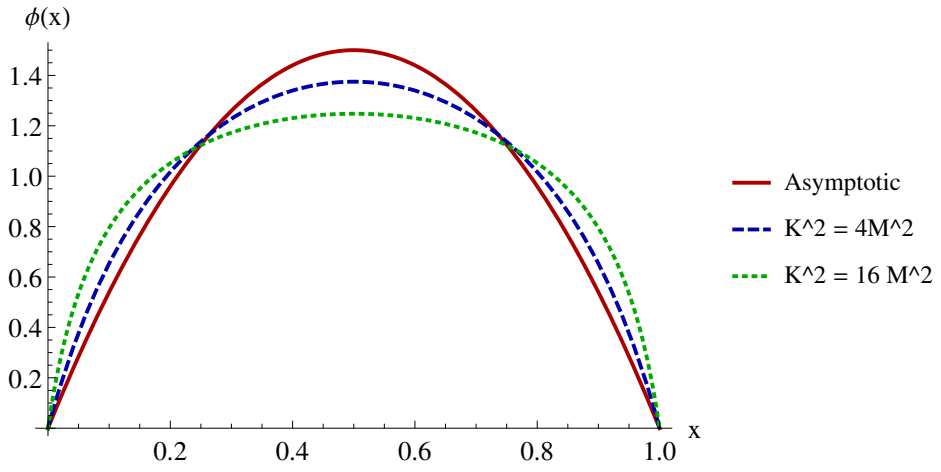
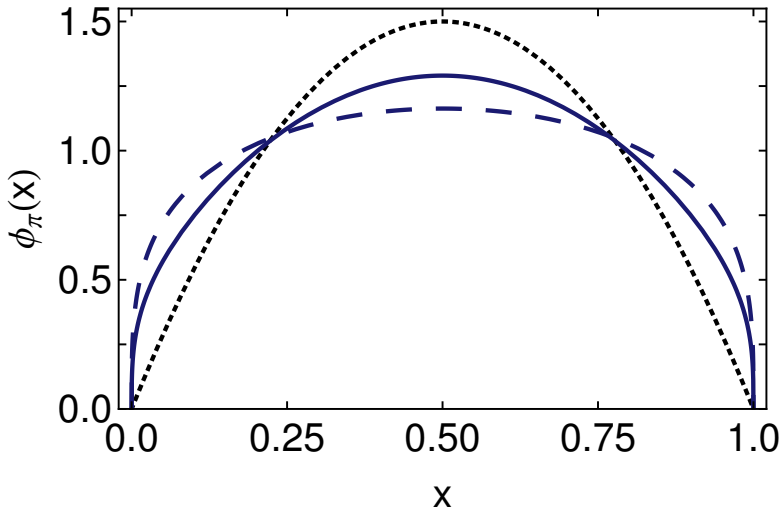


Pion



Pion figure from L. Chang *et al.*, PRL 110 (2013)

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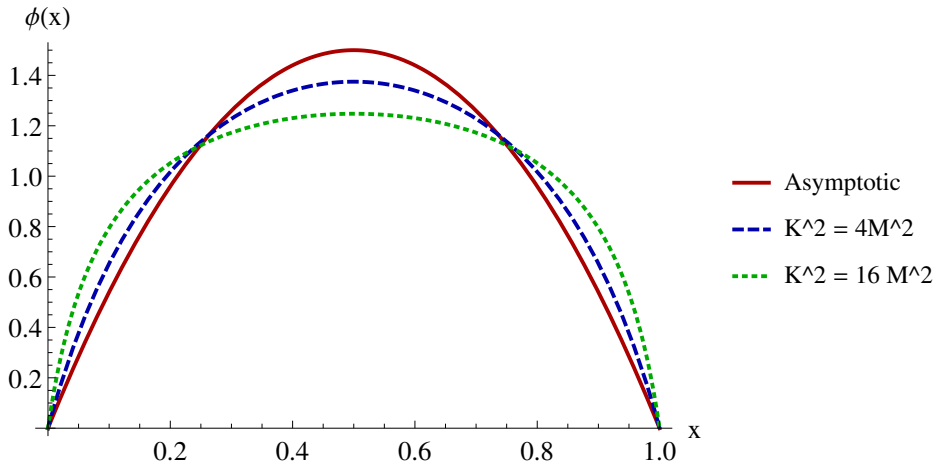

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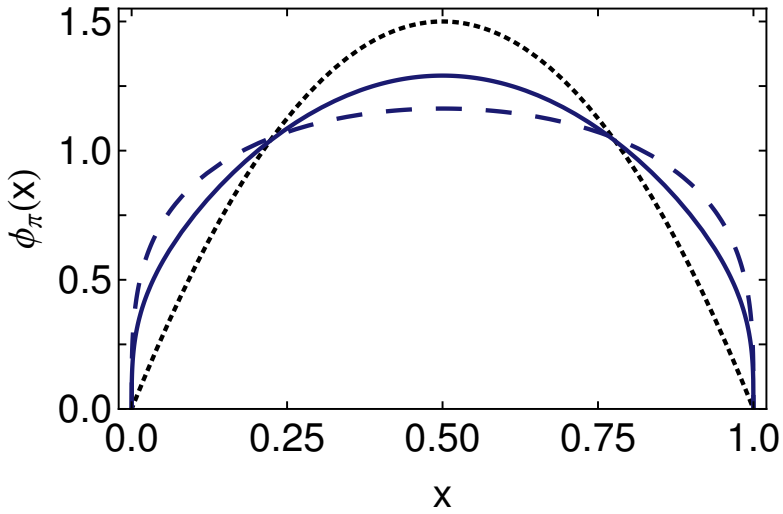
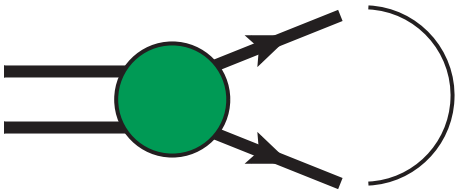


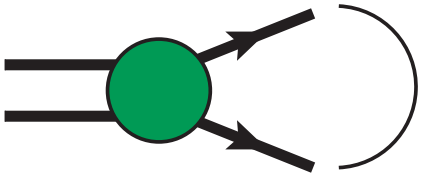
Pion figure from L. Chang *et al.*, PRL 110 (2013)

This extended version of the DA seems promising!

AV diquark DA



$\gamma \cdot n$

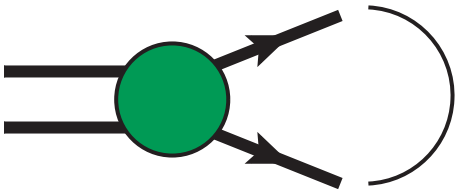


$\sigma^{\mu\nu} n_\nu$

- Quark propagator:

$$S(q) = \frac{-i\not{q} + M}{q^2 + M^2}$$

AV diquark DA



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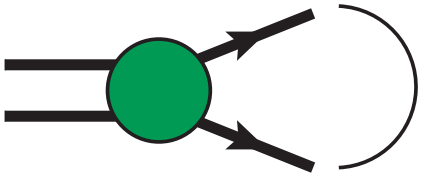


$\sigma^{\mu\nu} n_\nu$

- Quark propagator:

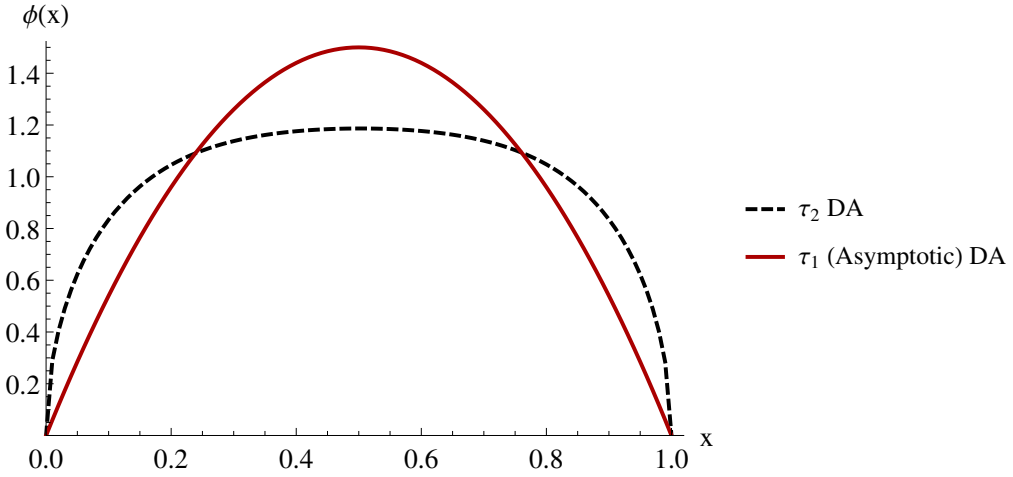
$$S(q) = \frac{-i\not{q} + M}{q^2 + M^2}$$

- Bethe-Salpeter amplitude (2 out of 8 structures):

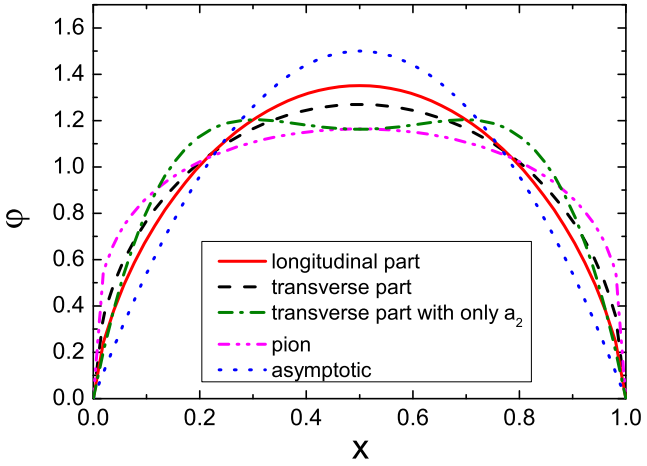
$$\begin{aligned}
\Gamma^{\mu}_{\rm PL}(q,K) &= (\mathcal{N}_1 \tau_1^\mu + \mathcal{N}_2 \tau_2^\mu)\, C \int_{-1}^{1} {\rm d}z \frac{(1-z^2)}{\left[\left(q - \frac{1-z}{2}K\right)^2 + \Lambda_q^2\right]} \\
\tau_1^\mu &= i\left(\gamma^\mu - K^\mu \frac{\not{K}}{K^2}\right) \rightarrow \text{Chiral even} \\
\tau_2^\mu &= \frac{K\cdot q}{\sqrt{q^2(K-q)^2}\sqrt{K^2}}\left(-i\tau_1^\mu \not{q} + i\not{q}\tau_1^\mu\right) \rightarrow \text{Chiral odd}
\end{aligned}$$

Comparison with the ρ meson

AV diquark



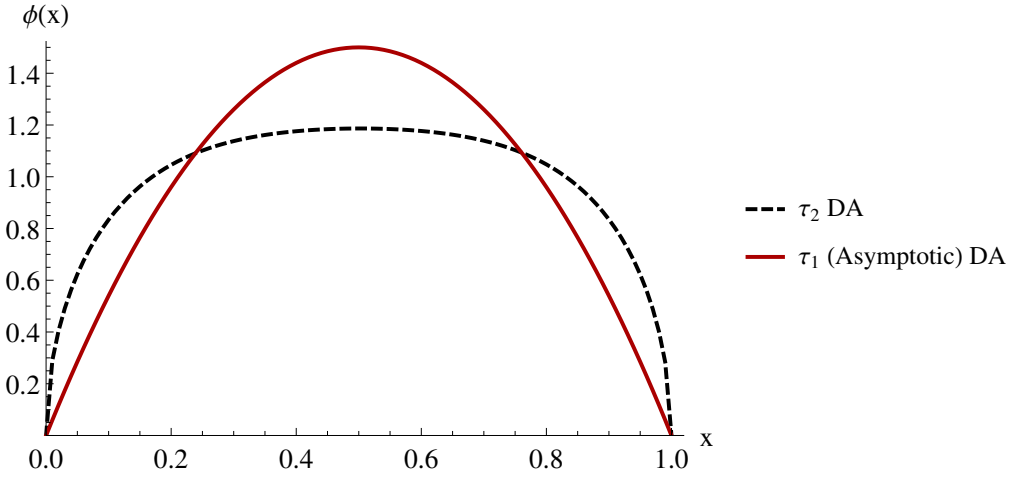
ρ meson



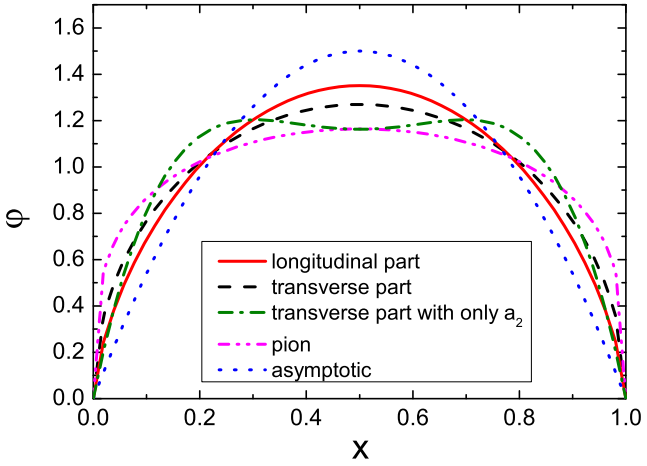
ρ figure from F. Gao *et al.*, PRD 90 (2014)

Comparison with the ρ meson

AV diquark



ρ meson

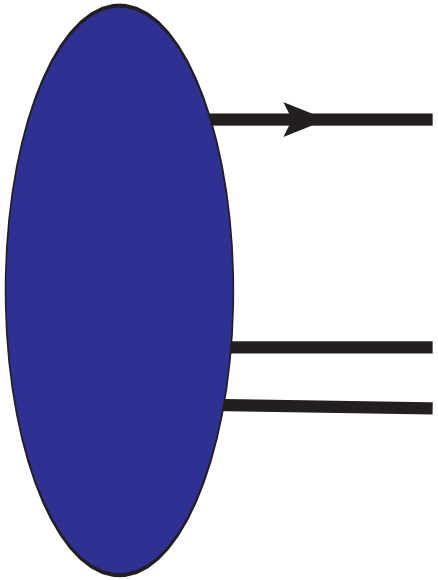


ρ figure from F. Gao *et al.*, PRD 90 (2014)

- Same "shape ordering" $\rightarrow \phi_\perp$ is flatter in both cases.
- Farther apart compared to the ρ meson case.

Modeling the Faddeev Amplitude

Faddeev Amplitude



- Scalar case:

$$s_1(K,P) \quad = \mathcal{N}_1 \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{\left[\left(K - \frac{1-z}{2}P\right)^2 + \Lambda_N^2\right]}$$

- AV case (2 out of 6 structures):

$$A^\mu(K,P) = \left(\gamma_5\gamma^\mu - i\gamma_5\hat{P}^\mu\right) \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{\left[\left(K - \frac{1-z}{2}P\right)^2 + \Lambda_N^2\right]}$$