Higher order QED radiative corrections to elastic *ep* scattering

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Physikzentrum Bad Honnef, 23th April 2018

Outline

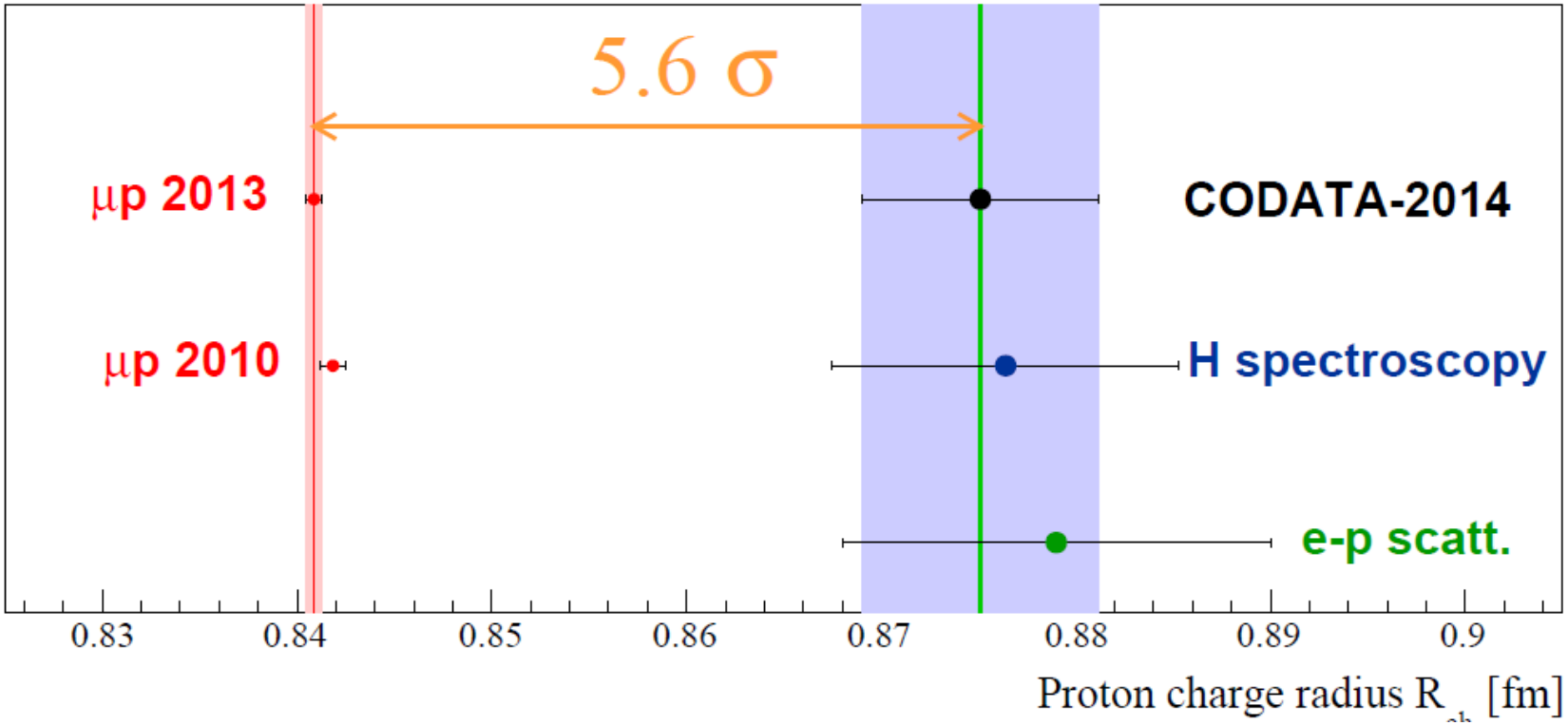
- Proton radius puzzle
- MAMI experiment
- Types of corrections to *ep* scattering
- Vacuum polarization
- Exponentiation of photonic corrections
- Light pair correction in LLA
- Complete second order NLO corrections
- Numerical results for *ep* scattering
- Open questions and discussion

Proton radius puzzle

The proton rms charge radius measured with

electrons: 0.8751 ± 0.0061 fm

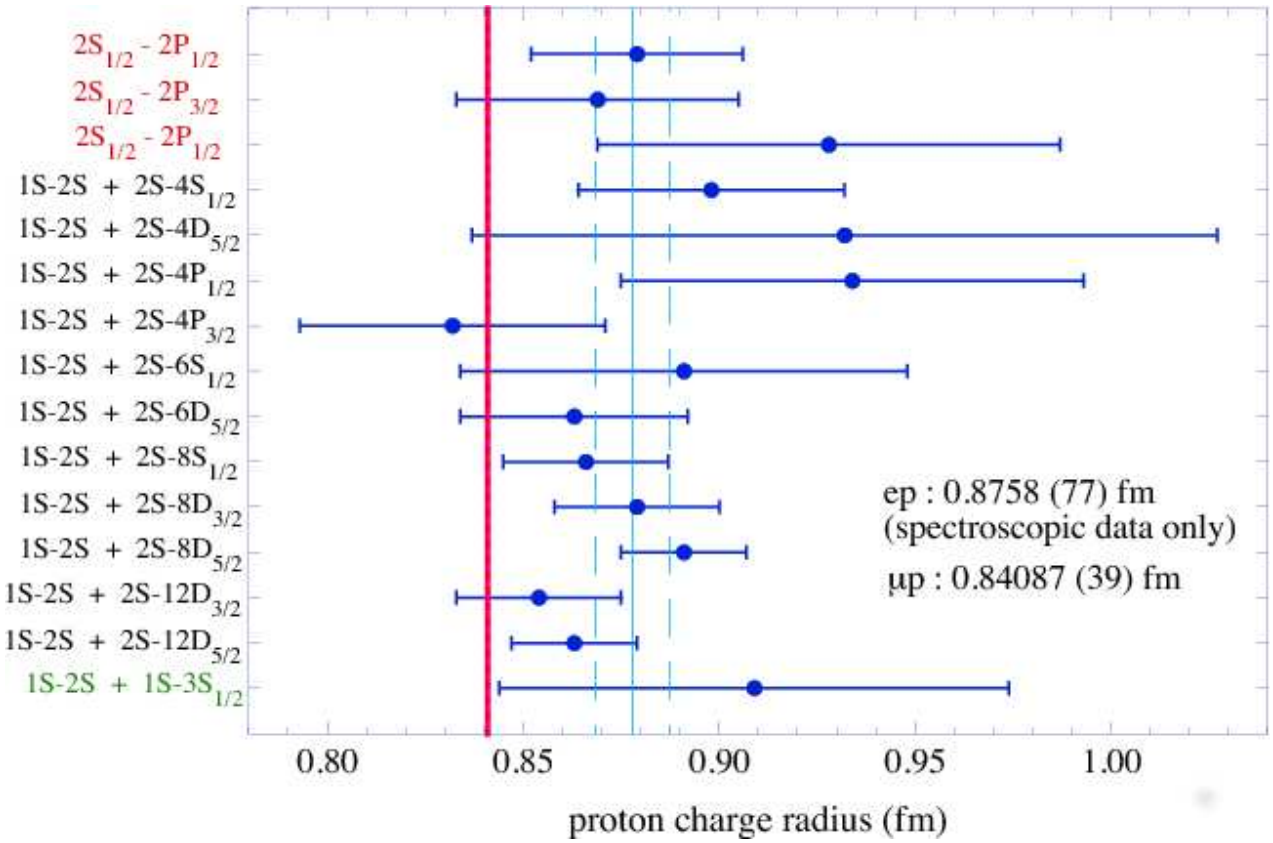
muons: 0.8409 ± 0.0004 fm



Proton radius puzzle

Proton radius puzzle (III)

Hydrogen spectroscopy data itself



Proton radius puzzle (IV)

NEW RESULTS

At the core of the proton radius puzzle is a fourstandard deviation discrepancy between the proton root-mean-square charge radii (rp) determined from the regular hydrogen (H) and the muonic hydrogen (μp) atoms. Using a cryogenic beam of H atoms, we measured the 2S-4P transition frequency in H, yielding the values of the Rydberg constant $R_\infty = 10\ 973\ 731.568\ 076(96)$ per meter and $r_p = 0.8335(95)$ femtometer. Our r_p value is 3.3 combined standard deviations smaller than the previous H world data,

but in good agreement with the μp value. We motivate an asymmetric fit function, which eliminates line shifts from quantum interference of neighboring atomic resonances.

Theodor W. Hänsch: [Beyer et al., Science 358 (2017) 79] 6 Oct. 2017

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  — look for problems in data analysis;
  — verify assumptions on charge distribution (a_1, f_1, ...);
  — look for possible new physics contributions;
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- Experiments on atomic spectra look more save, but ...
- Here: effects of radiative corrections in elastic *ep* scattering

The MAMI experiment (I)

Mainz Microtron experimental set-up:

— the electron beam energy $E_e \equiv E \lesssim 855$ MeV (1.6 GeV)

— momentum transfer range: $0.003 < Q^2 < 1$ GeV^2

— the outgoing electron energy $E_e{}' \equiv E' > E'_0 - \Delta E$

— no any other condition: neither on enrgies nor on angles

— experimental precision (point-to-point) $\simeq 0.37\%$ $\rightarrow$ 0.1% (?)

$\Rightarrow$ all effects at least of the 10^{-4} order should be taken into account. That is not a simple task in any case

N.B. $E_e^2 \gg m_e^2, \quad Q^2 \gg m_e^2, \quad (\Delta E)^2 \gg m_e^2, \quad \Delta E \ll E_e$

Ref.: J.C. Bernauer et al. [A1 Coll.] PRC 90 (2014) 015206

The MAMI experiment (III)

Born cross section via the Sachs form factors ($\tau = Q^2/(4M_p^2)$):

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_0 = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Mott}} \left[\frac{G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)}{1+\tau} + 2\tau G_M^2\left(Q^2\right)\tan^2\frac{\theta}{2}\right]$$

The proton charge radius is defined then via

$$\left\langle r^2 \right\rangle = -\frac{6}{G_E\left(0\right)} \left.\frac{\mathrm{d}G_E\left(Q^2\right)}{\mathrm{d}Q^2}\right|_{Q^2=0}$$

i.e., from the slope of the G_E form factor at $Q^2 = 0$

N.B. $\langle r^m \rangle = \int r^m \rho_p(r) \mathrm{d}^3 r$

Types of RC to elastic *ep* scattering

— Virtual (loop) and/or real emission

— QED, QCD, and (electro)weak effects

— Perturbative and/or non-perturbative contributions

— Perturbative QED effects in $\mathcal{O}(\alpha)$, $\mathcal{O}\left(\alpha^2\right)$, ...

— Leading and next-to-leading logarithmic approximations

— Corrections to the electron line, to the proton line, and their interference

— Vacuum polarization, vertex corrections, double photon exchange etc.

First order QED RC (I)

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_1 = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_0 (1+\delta)$$

The $\mathcal{O}(\alpha)$ QED RC with point-like proton are well known:
Refs.: see e.g. L.C. Maximon & J.A. Tjon, PRC 2000

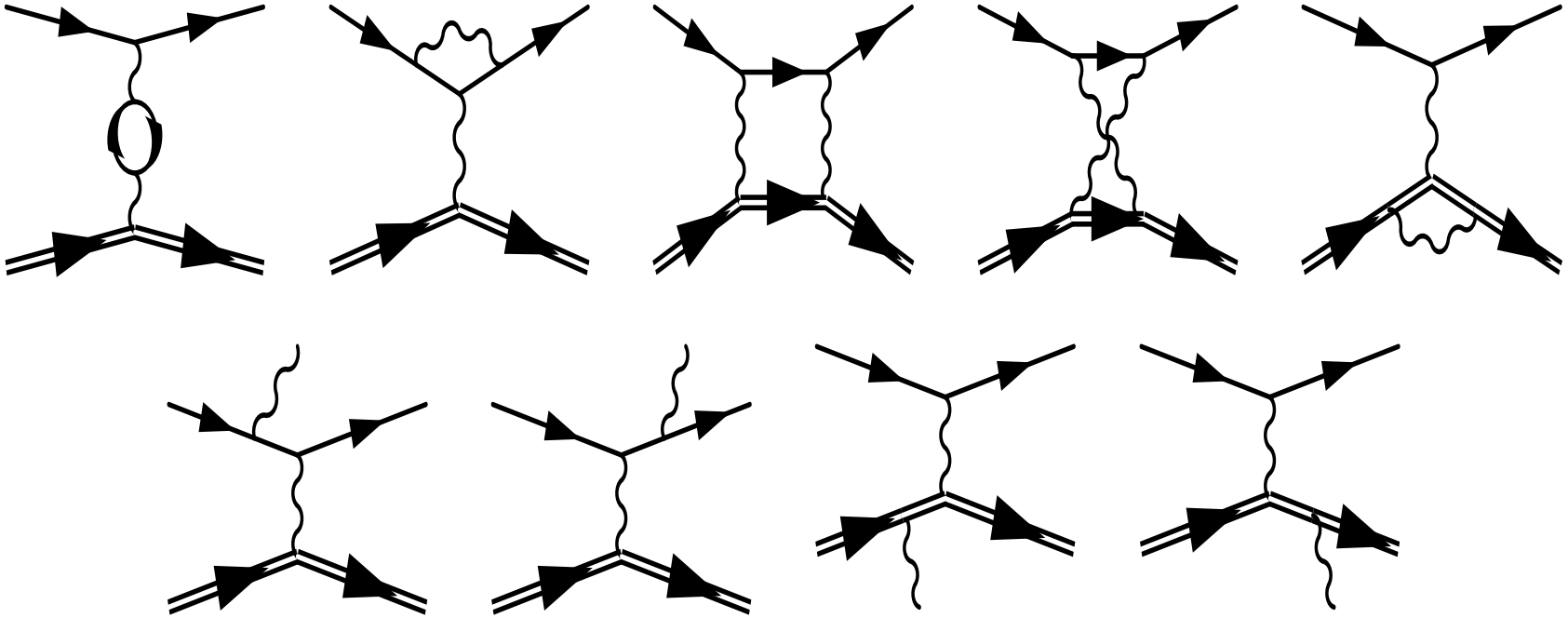
Virtual RC: Vacuum polarization, vertex, and box Feynman diagrams

Real RC: emission off the initial and final electrons and protons

N.B.1. UV divergences are regularized and renormalized

N.B.2. IR divergences cancel out in sum of virtual and real RC

First order QED RC (II)



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Magnitude of QED RC

The problem has several small and large parameters to be used in expansions:

- $\alpha/(2\pi) \approx 0.001$
- $(\alpha/(2\pi))^2 \approx 10^{-6}$
- $L \equiv \ln(Q^2/m_e^2) \approx 16$ the large log for $Q^2 = 1$ GeV^2
- $\ln(\Delta) \sim 5$, where $\Delta = \Delta E_e/E_e \ll 1$

N.B. Some $\mathcal{O}\left(\alpha^2\right)$ corrections are enhanced with 2nd, 3rd or even 4th power of large logs. So, they should be treated with care.

Vacuum polarization in one-loop

$$\delta^{(1)}_{\rm vac} = \frac{\alpha}{\pi}\frac{2}{3}\left\{\left(v^2 - \frac{8}{3}\right) + v\frac{3-v^2}{2}\ln\left(\frac{v+1}{v-1}\right)\right\}$$

$$\xrightarrow{Q^2 \gg m_l^2} \frac{\alpha}{\pi}\frac{2}{3}\left\{-\frac{5}{3} + \ln\left(\frac{Q^2}{m_l^2}\right)\right\}, \quad v = \sqrt{1 + \frac{4m_l^2}{Q^2}}, \quad l = e, \mu, \tau$$

Two ways of re-summation:

1) geometric progression

$$\Rightarrow \alpha(Q^2) = \frac{\alpha(0)}{1 - \Pi(Q^2)}, \quad \Pi(Q^2) = \frac{1}{2}\delta^{(1)}_{\rm vac} + \ldots$$

2) exponentiation

$$\alpha(Q^2) = \alpha(0)e^{\delta^{(1)}_{\rm vac}/2}$$

the latter option was used by A1 Coll.

Other $\mathcal{O}(\alpha)$ effects

$$\delta^{(1)}_{\text{vertex}} = \frac{\alpha}{\pi}\left\{\frac{3}{2}\ln\left(\frac{Q^2}{m^2}\right) - 2 - \frac{1}{2}\ln^2\left(\frac{Q^2}{m^2}\right) + \frac{\pi^2}{6}\right\}$$

$$\begin{aligned}\delta^{(1)}_{\text{real}} &= \frac{\alpha}{\pi}\left\{\ln\left(\frac{(\Delta E_s)^2}{E\cdot E'}\right)\left[\ln\left(\frac{Q^2}{m^2}\right) - 1\right] - \frac{1}{2}\ln^2\eta + \frac{1}{2}\ln^2\left(\frac{Q^2}{m^2}\right)\right.\\ &- \left.\frac{\pi^2}{3} + \mathrm{Li}_2\left(\cos^2\frac{\theta_e}{2}\right)\right\}, \quad \eta = \frac{E}{E'}, \quad \Delta E_s = \eta\cdot\Delta E'\end{aligned}$$

Interference δ_1 and radiation off proton δ_2 do not contain the large log.
A1 Coll. applied RC in the exponentiated form:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\text{exp}}(\Delta E') = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_0 e^{\delta_{\text{vac}}+\delta_{\text{vertex}}+[\delta_R+\delta_1+\delta_2](\Delta E')}$$

Higher order effects are partially taken into account by exponentiation.
Remind the Yennie-Frautschi-Suura theorem

Multiple soft photon radiation (I)

Exponentiation corresponds to independent emission of soft photons, while the cut on the total lost energy leads to sizable shifts.

For two photons:

$$\mathrm{e}^{\delta_{\mathrm{soft}}} \to \mathrm{e}^{\delta_{\mathrm{soft}}} - \left(\frac{\alpha}{\pi}\right)^2 \frac{\pi^2}{3} (L-1)^2$$

at $Q^2 = 1$ GeV^2 this gives $-3.5 \cdot 10^{-3}$

In the leading log approximation

$$\delta^{(3)}_{\mathrm{LLA}} = (L-1)^3 \left(\frac{\alpha}{\pi}\right)^3 \frac{1}{6} \left(P^{(0)} \otimes P^{(0)} \otimes P^{(0)}\right)_\Delta ,$$

$$\left(P^{(0)} \otimes P^{(0)} \otimes P^{(0)}\right)_\Delta = 8 \left(P^{(0)}_\Delta\right)^3 - 24\zeta(2) P^{(0)}_\Delta + 16\zeta(3)$$

$$\Rightarrow \quad \delta^{(3)}_{\mathrm{cut}} = (L-1)^3 \left(\frac{\alpha}{\pi}\right)^3 \left[-4\zeta(2) P^{(0)}_\Delta + \frac{8}{3}\zeta(3)\right]$$

which is not small and reaches $2 \cdot 10^{-3}$

Multiple soft photon radiation (II)

The exact LLA solution of the evolution equation for the photonic part of the non-singlet structure function in the soft limit is known

$$\mathcal{D}_\gamma^{\mathrm{N}S}(z,Q^2)\Big|_{z\to 1} = \frac{\beta}{2}\,\frac{(1-z)^{\beta/2-1}}{\Gamma(1+\beta/2)}\exp\left\{\frac{\beta}{2}\left(\frac{3}{4}-C\right)\right\}$$

where C is the Euler constant, $\beta = \frac{2\alpha}{\pi}(\ln\frac{Q^2}{m^2}-1)$

$$\int\limits_{1-\Delta}^{1} \mathrm{d}z\,\mathcal{D}_\gamma^{\mathrm{N}S}(z,Q^2) = \exp\left\{\frac{\beta}{2}\ln\Delta + \frac{3\beta}{8}\right\}\frac{\exp(-C\beta/2)}{\Gamma(1+\beta/2)},$$

$$\frac{\exp(-C\beta/2)}{\Gamma(1+\beta/2)} = 1 - \frac{1}{2}\left(\frac{\beta}{2}\right)^2\zeta(2) + \frac{1}{3}\left(\frac{\beta}{2}\right)^3\zeta(3) + \frac{1}{16}\left(\frac{\beta}{2}\right)^4\zeta(4)$$
$$+\frac{1}{5}\left(\frac{\beta}{2}\right)^5\zeta(5) - \frac{1}{6}\left(\frac{\beta}{2}\right)^5\zeta(2)\zeta(3) + \mathcal{O}(\beta^6)$$

[V. Gribov, L. Lipatov, Sov. J. Nucl. Phys. **15** (1972) 451; 675]

QED GLAPD

N.B. QED GLAPD are very useful also to estimate the leading part of one-loop and higher-order QED radiative corrections.

LLA (LO) QED GLAPD:

[1] E.A.Kuraev and V.S.Fadin, On Radiative Corrections to e+ e- Single Photon Annihilation at High-Energy, Sov. J. Nucl. Phys. **41** (1985) 466

[2] A.De Rujula, R.Petronzio, A.Savoy-Navarro, Radiative Corrections to High-Energy Neutrino Scattering, Nucl. Phys. B **154** (1979) 394

NLO QED GLAPD:

[3] F.A.Berends, W.L. van Neerven, G.J.H.Burgers, Higher Order Radiative Corrections at LEP Energies, Nucl. Phys. B **297** (1988) 429

[4] A.Arbuzov, K.Melnikov, $O(\alpha^2 \ln(m_\mu/m_e))$ corrections to electron energy spectrum in muon decay, Phys. Rev. D **66** (2002) 093003.

Light pair corrections

A quick estimate can be done within LLA:

$$\delta^{LLA}_{\rm pair} = \frac{2}{3}\left(\frac{\alpha}{2\pi}L\right)^2 P^{(0)}_{\Delta} + \frac{4}{3}\left(\frac{\alpha}{2\pi}L\right)^3 \left\{\left(P^{(0)}\otimes P^{(0)}\right)_{\Delta} + \frac{2}{9}P^{(0)}_{\Delta}\right\} + \mathcal{O}\left(\alpha^2 L, \alpha^4 L^4\right)$$

$$P^{(0)}_{\Delta} = 2\ln\Delta + \frac{3}{2}, \qquad \left(P^{(0)}\otimes P^{(0)}\right)_{\Delta} = \left(P^{(0)}_{\Delta}\right)^2 - \frac{\pi^2}{3}$$

The energy of the emitted pair is limited by the same parameter: $E_{\rm pair} \leq \Delta E$. Both virtual and real e^+e^- pair corrections are taken into account.

Typically, $\mathcal{O}\left(\alpha^2\right)$ pair RC are a few times less than $\mathcal{O}\left(\alpha^2\right)$ photonic ones, see e.g. A.A. JHEP'2001

Complete NLLA corrections (I)

The master formula for *ep* scattering reads

$$\mathrm{d}\sigma = \int_{\bar{z}}^{1} \mathrm{d}z \mathcal{D}_{ee}^{\mathrm{str}}(z) \left(\mathrm{d}\sigma^{(0)}(z) + \mathrm{d}\bar{\sigma}^{(1)}(z) + \mathcal{O}\left(\alpha^2 L^0\right) \right) \int_{\bar{y}}^{Y} \frac{\mathrm{d}y}{Y} \mathcal{D}_{ee}^{\mathrm{frg}}\left(\frac{y}{Y}\right)$$

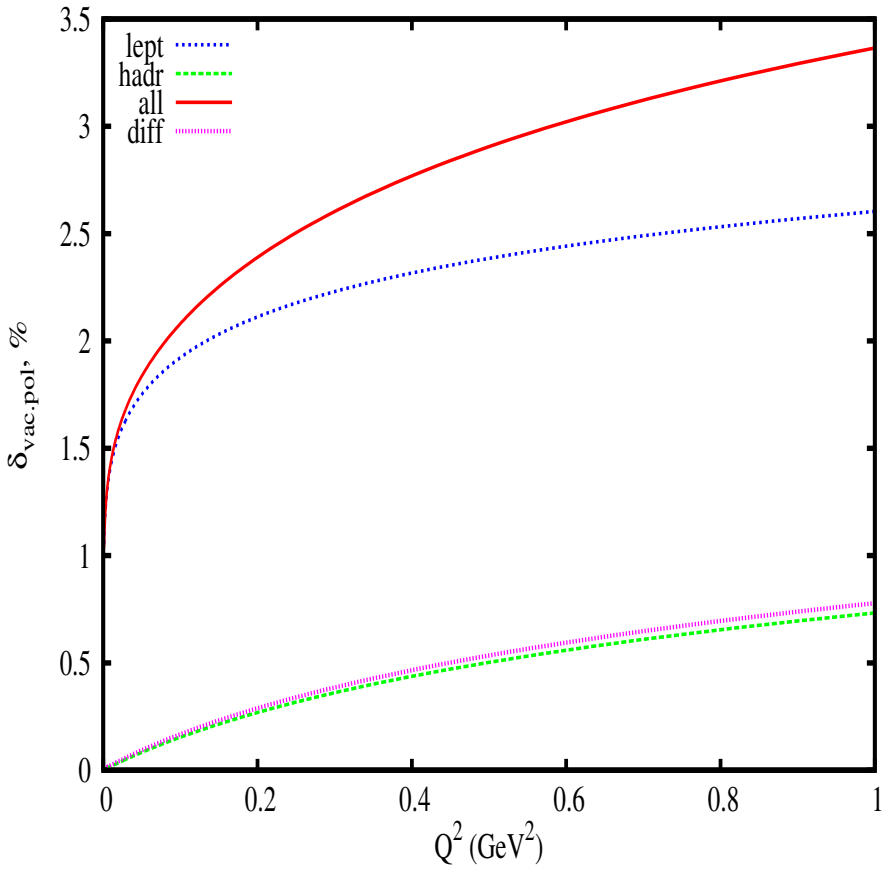
where $\mathrm{d}\bar{\sigma}^{(1)}$ is the $\mathcal{O}(\alpha)$ correction to the *ep* scattering with a "massless electron" in the $\overline{\mathrm{MS}}$ scheme

Complete NLLA corrections (II)

$$
\begin{aligned}
\mathrm{d}\sigma^{\mathrm{NLO}} &= \int_{1-\Delta}^{1} \mathcal{D}_{ee}^{\mathrm{str}} \otimes \mathcal{D}_{ee}^{\mathrm{frg}}(z) \left[\mathrm{d}\sigma^{(0)}(z) + \mathrm{d}\bar{\sigma}^{(1)}(z) \right] \mathrm{d}z \\
&= \mathrm{d}\sigma^{(0)}(1) \Bigg\{ 1 + 2\frac{\alpha}{2\pi} \left[L P_{\Delta}^{(0)} + (d_1)_{\Delta} \right] + 2\left(\frac{\alpha}{2\pi}\right)^2 \Bigg[L^2 \left(P^{(0)} \otimes P^{(0)} \right)_{\Delta} \\
&+ \frac{1}{3} L^2 P_{\Delta}^{(0)} + 2L(P^{(0)} \otimes d_1)_{\Delta} + L(P_{ee}^{(1,\gamma)})_{\Delta} + L(P_{ee}^{(1,\mathrm{pair})})_{\Delta} \Bigg] \Bigg\} \\
&+ \mathrm{d}\bar{\sigma}^{(1)}(1)\, 2\frac{\alpha}{2\pi} L P_{\Delta}^{(0)} + \mathcal{O}\left(\alpha^3 L^3\right) \\
(d_1)_{\Delta} &= -2\ln^2\Delta - 2\ln\Delta + 2, \quad \ldots
\end{aligned}
$$

N.B. Method gives complete $\mathcal{O}\left(\alpha^2 L\right)$ results for sufficiently inclusive observables

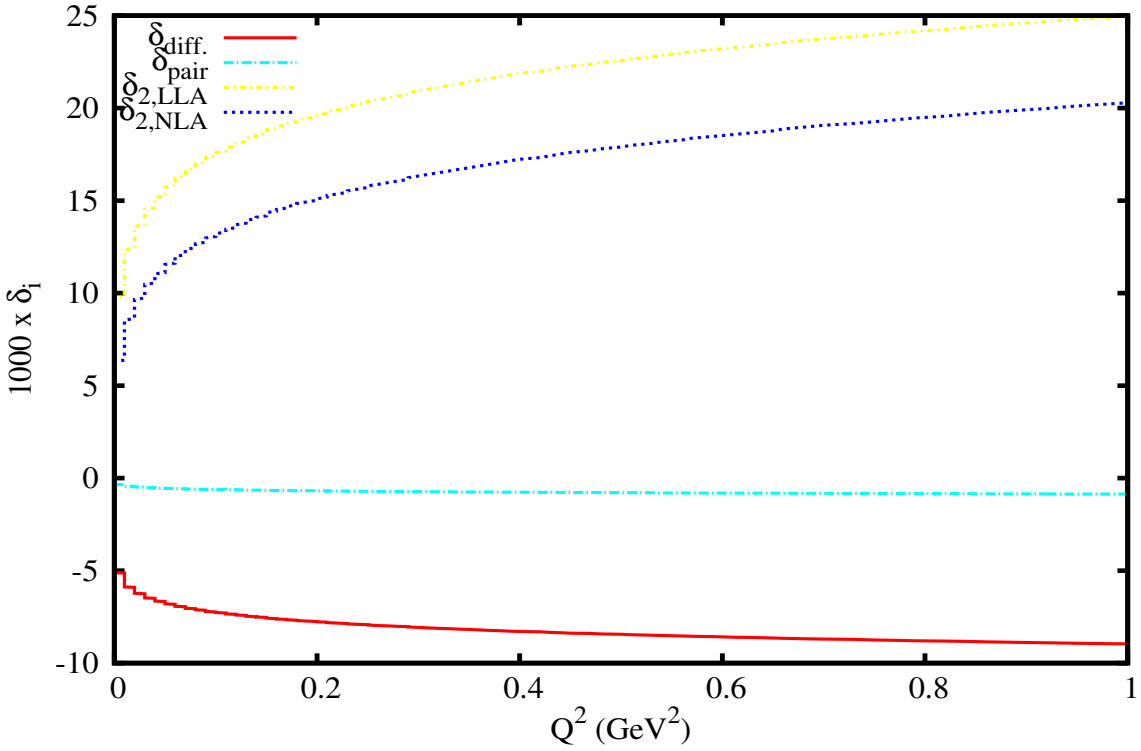
Numerical results: vacuum polarization



Vacuum polarization corrections due to hadrons (had), leptons (lept), combined effect (all) and the difference (diff=all-lept).
Program `AlphaQED` by F. Jegerlehner was used.

Higher-order corrections (1)

$E_{\rm beam} = 800$ MeV, $E_{\rm lost} \leq$ 10 MeV



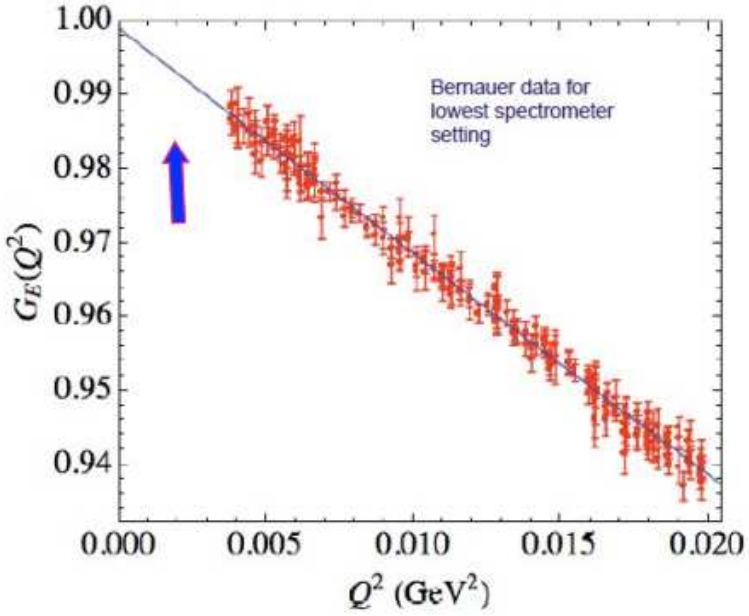
$$\delta_i = {\rm d}\sigma^{(i)}/{\rm d}\sigma^{(0)}$$

$$\delta_{\rm diff.} = \frac{{\rm d}\sigma^{\rm NLO}}{{\rm d}\sigma^{(0)}(1)} + \delta^{(3)}_{\rm LLA} + \delta^{(3)}_{\rm LLA,pair} + \delta^{(4)}_{\rm LLA} - \exp\{\delta^{(1)}\}$$

PRad experiment at JLab

The PRad Experimental Approach

- Experimental goals:
  - reach to very low Q^2 range (~ 10^{-4} GeV/C^2)
  - reach to sub-percent precision in cross section
  - large Q^2 range in one experimental setting
- Suggested solutions:
  - ✓ use high resolution high acceptance calorimeter:
    - reach smaller scattering angles: ($\Theta = 0.7^0 - 7.0^0$)
      ($Q^2 = 1\times10^{-4} \div 6\times10^{-2}$) GeV/c^2
      large Q^2 range in one experimental setting!
      essentially, model independent r_p extraction
  - ✓ Simultaneous detection of $ee \rightarrow ee$ Moller scattering
    - (best known control of systematics)
  - ✓ Use high density windowless H_2 gas flow target:
    - beam background fully under control
    - minimize experimental background

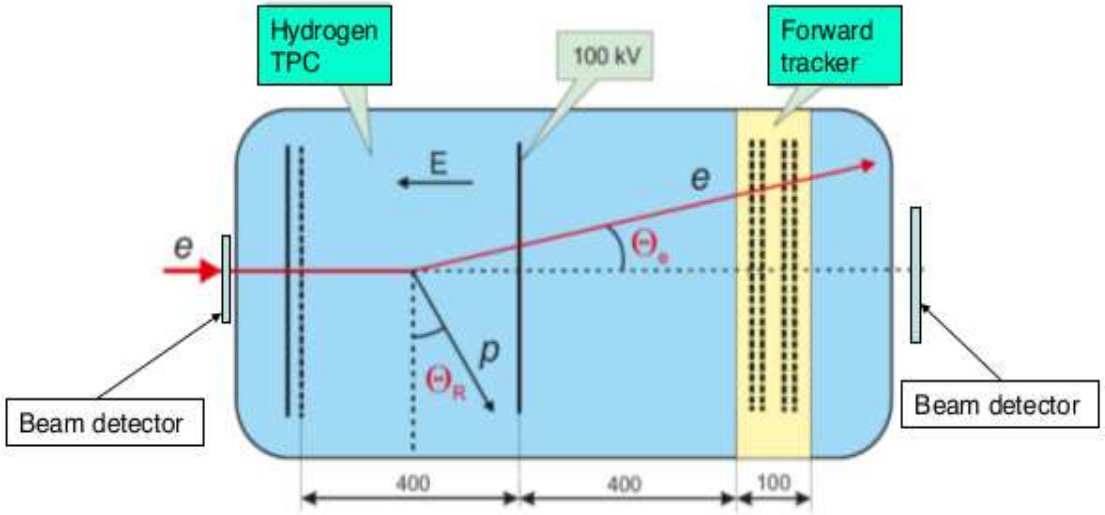


Mainz low Q^2 data set
Phys. Rev. C 93, 065207, 2016

- Two beam energies: E_0 = 1.1 GeV and 2.2 GeV to increase Q^2 range
- Will reach sub-percent precision in r_p extraction
- Approved by JLab PAC39 (June, 2012) with high "A" scientific rating

New experiment is proposed at MAMI

Proposal to perform an experiment at the A2 hall, MAMI:
High Precision Measurement of the ep elastic cross section at small Q^2
Contact persons for the Experiment:
Alexey Vorobyev, Petersburg Nuclear Physics Institute
Achim Denig, Institute for Nuclear Physics, JGU Mainz



Measured quantities:

Recoil energy T_R
Recoil angle Θ_R
Vertex Z coordinate
E scattering angle Θ_e

$$-t = \frac{4\varepsilon_e^2 \sin^2\frac{\vartheta}{2}}{1+\frac{2\varepsilon_e}{M}\sin^2\frac{\vartheta}{2}}$$

$$-t = 2MT_R$$

Kinoshita–Lee–Nauenberg theorem

The large logs are singular if $m_e \to 0$. They are so-called mass singularities. But for sufficiently inclusive observables such mass singularities should cancel out in accord with the Kinoshita–Lee–Nauenberg theorem.

In practice roughly speaking, the condition for the cancellation is the calorimetric registration of charged particles together with collinear photons which accompany them.

Another important principle for radiative corrections:

The more you cut — the more you get

i.e. corrections tend to increase for tight experimental cuts or "very exclusive" observables.

Leading logs in the new set-up

FSR large log corrections are cancelled out (KLN theorem)

ISR provides an effective reduction of the beam energy.
It affects the the proton Q^2 distribution rather weakly

Some PRELIMINARY results in the collinear leading log approximation were obtained for

$$E_{\rm beam} = 500\ {\rm MeV}, \qquad 0.001 < Q^2 < 0.02\ {\rm GeV}^2$$

Q^2 [GeV]	0.001	0.01	0.02
$\delta_1^{\rm LLA}$	$-2.7 \cdot 10^{-4}$	$-1.2 \cdot 10^{-3}$	$-2.0 \cdot 10^{-3}$
$\delta_2^{\rm LLA}$	$-0.7 \cdot 10^{-5}$	$-3.0 \cdot 10^{-5}$	$-4.6 \cdot 10^{-5}$
$\delta_{2+3+{\rm pairs}}^{\rm LLA}$	$-0.8 \cdot 10^{-5}$	$-3.5 \cdot 10^{-5}$	$-5.4 \cdot 10^{-5}$

$$\delta_n(Q^2) = \sigma^{\rm LLA}(Q^2)/\sigma^{\rm Born}(Q^2) - 1$$

Complete $\mathcal{O}(\alpha)$ (I)

PRELIMINARY

Correction to the total *ep* cross section for

$$0.001 \leq Q_p^2 \leq 0.04 \text{ GeV}^2$$

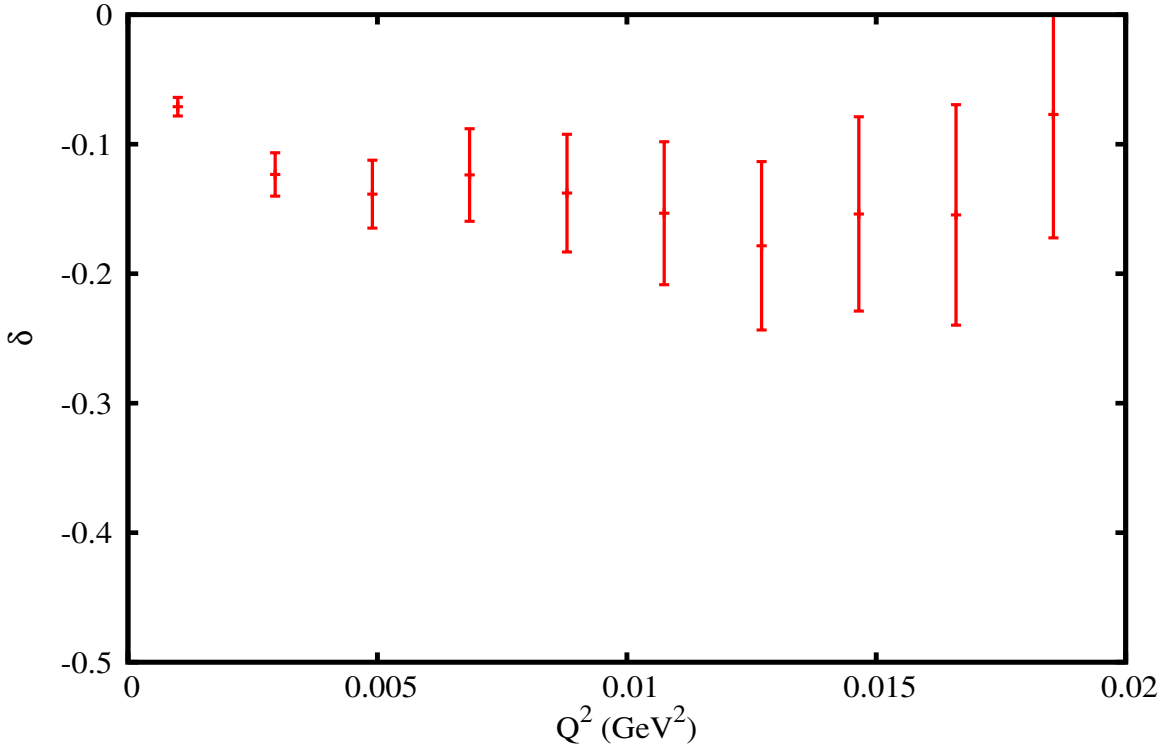
$$\begin{aligned}
\sigma_{\text{Born}} &= 252.175\ \mu\text{barn} \\
\sigma_{\text{SV}} &= -59.096 \quad (\sim\ -23\%) \\
\sigma_{\text{Hard}} &= +58.864 \quad (\sim\ +23\%) \\
\delta_{\text{tot}} &= \frac{\sigma_{\text{SV}} + \sigma_{\text{Hard}}}{\sigma_{\text{Born}}} \cdot 100\% = -0.09\%
\end{aligned}$$

N.B. Here vacuum polarization is not taken into account

Complete $\mathcal{O}(\alpha)$ (II)

PRELIMINARY

Correction to the differential *ep* cross section



Further steps

In 2014 "A new event generator for the elastic scattering of charged leptons on protons" was presented [A.V. Gramolin, V.S. Fadin, A.L. Feldman, R.E. Gerasimov, D.M. Nikolenko, I.A. Rachek, D.K. Toporkov, J.Phys.G 41 (2014) 115001]

The code already contains:
— a library of proton form factors
— vacuum polarization
— complete one-loop QED
— the dependence on m_e^2/Q^2 (not complete)
— double photon exchange treatment
— etc.

Next: update higher-order effects:
— higher order leading and next-to-leading RC
— complete $\mathcal{O}(\alpha^2)$ to electron line
— tests and tuned comparisons

Conclusions

1. Application of RC in the analysis of A1 and JLab data is criticized

2. An advanced treatment of higher order QED RC is given

3. In particular, effects due to multiple radiation and pair emission in the LLA and NLLA are calculated

4. Vacuum polarization by hadrons should be taken into account

5. The size of the higher order effects make them relevant for the high-precision *ep* scattering experiments

6. Higher order RC to the electron line should be combined with an advanced treatment of two-photon exchange and other relevant effects

7. Radiative corrections for the new proposed experimental set-ups have to be re-considered

8. Treatment of EM form factors should be coordinated with application of QED corrections