Proton Size Puzzle: Fat or Thin?

"Fat and Thin" famous Anton Chekhov novel

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Introduction
Experimental data
Our work
Conclusions

In 2010 the CREMA (Charge Radius Experiments with Muonic Atoms) Collaboration measured very precisely the Lamb shift of muonic hydrogen. It has opened the new era of the precise investigation of the spectrum of simple atoms.

In the new experiments by this Collaboration with muonic deuterium and ions of muonic helium a charge radii of light nuclei were obtained with very high precision.

For muonic hydrogen and muonic deuterium it was shown that obtained values of the charged radii are significantly different from those which were extracted from experiments with electronic atoms and in the scattering of the electrons with nuclei and were recommended for using by CODATA, so-called, "PROTON CHARGE RADIUS PUZZLE".

Several experimental groups plan to measure the hyperfine structure of various muonic atoms with more high precision.
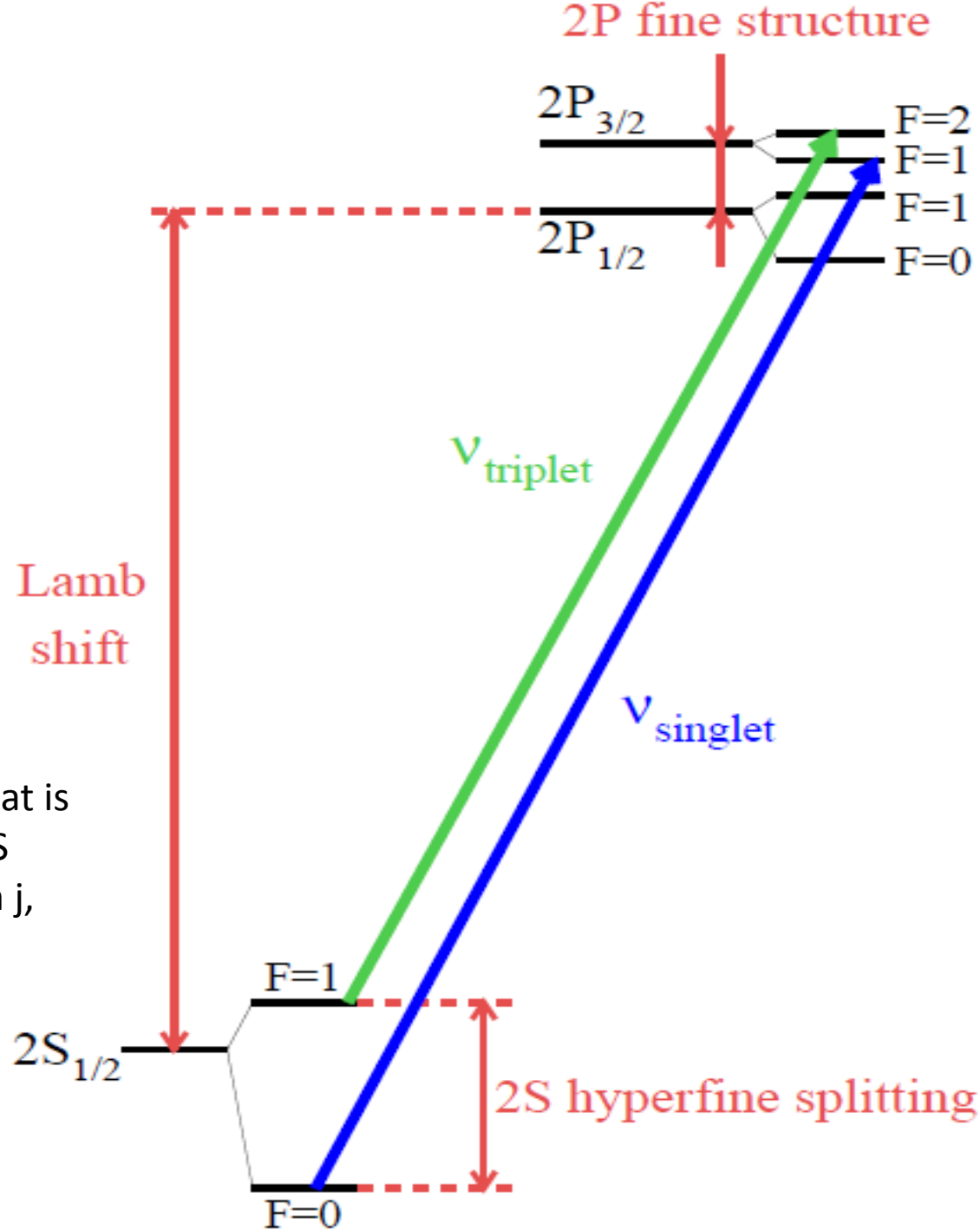
One can consider experiments with muonic atoms as a smoking gun for:

Precise measurements of the proton charge radius
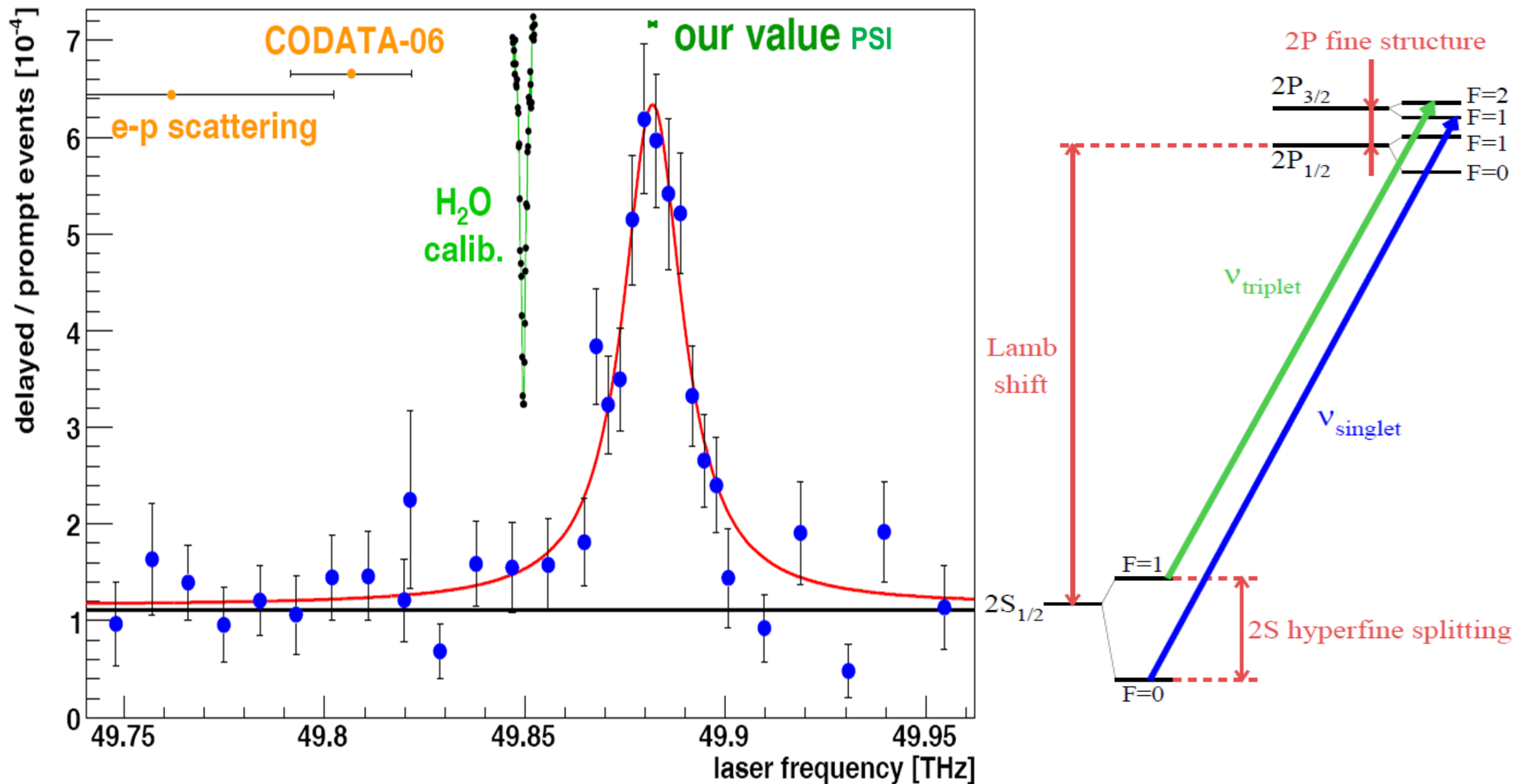
Test of the Standard Model with greater accuracy

and, possibly, to reveal the source of previously unaccounted interactions between the particles forming the bound state in QED.

2P fine structure

$2P_{3/2}$ ———— F=2
———— F=1

$2P_{1/2}$ ———— F=1
———— F=0

$\nu_{triplet}$

$\nu_{singlet}$

Lamb
shift

The HFS requires the spin-spin coupling that is
the interaction between the nuclear spin S
and the electron total angular momentum j,
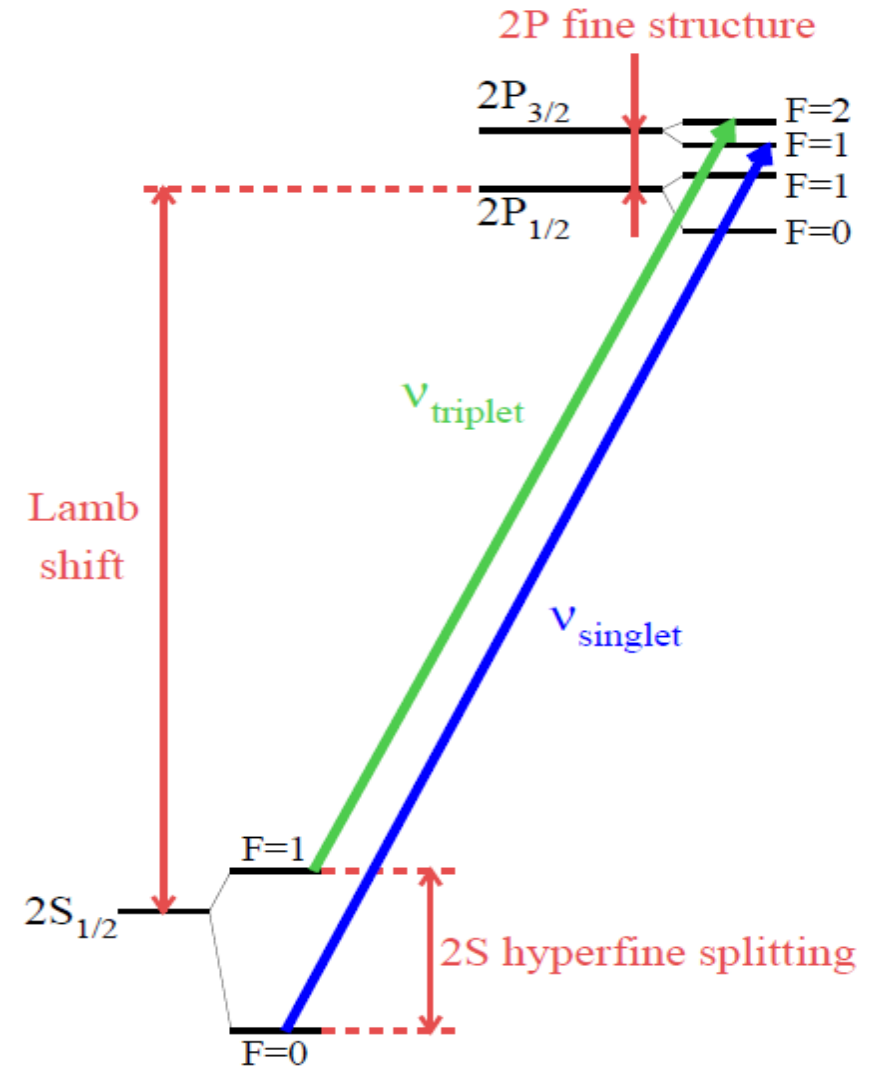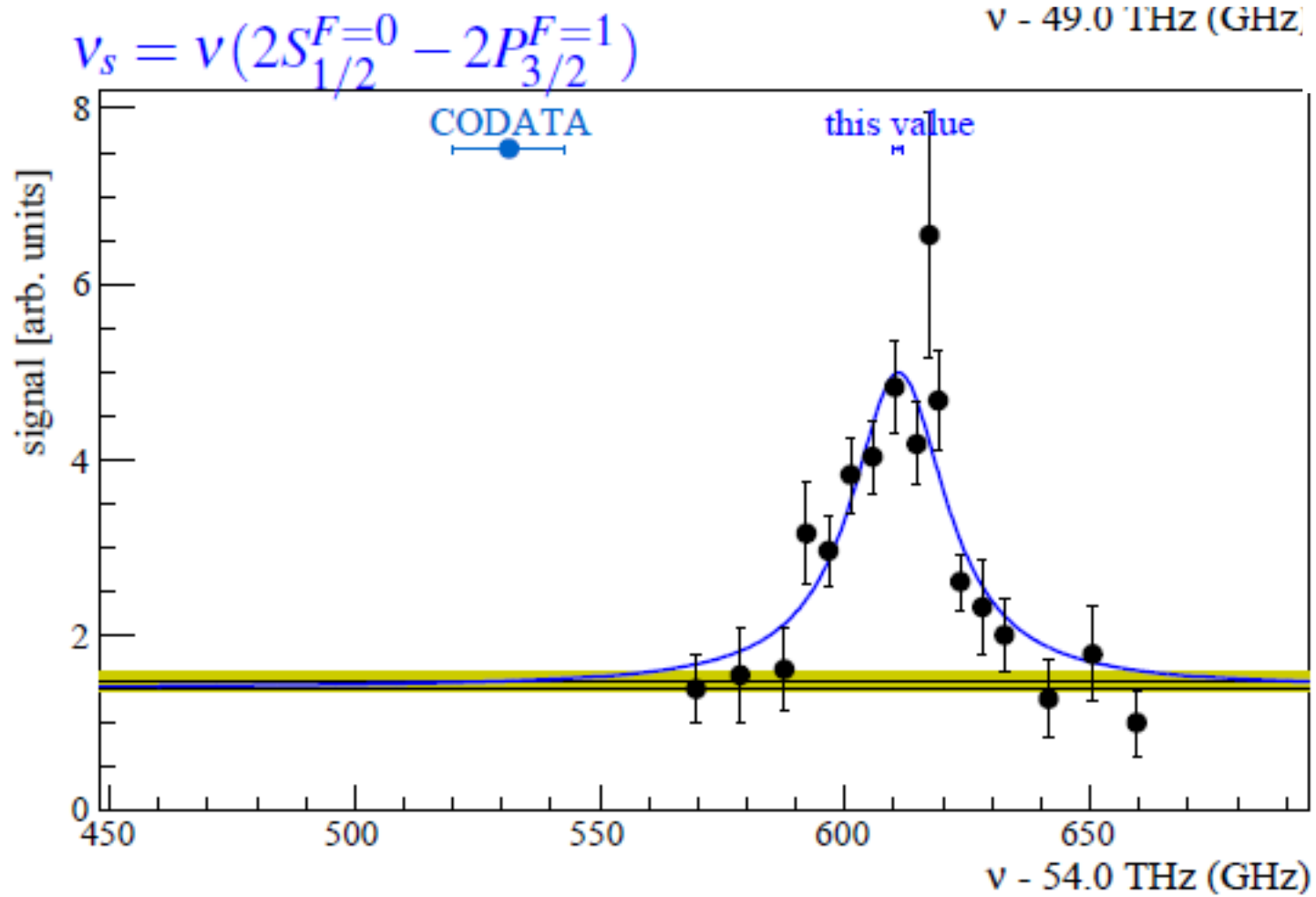where f = j + S is the atom total angular
momentum

F=1

$2S_{1/2}$

2S hyperfine splitting

F=0

$$\nu_t = \nu(2S_{1/2}^{F=1} - 2P_{3/2}^{F=2})$$

CREMA-10-13

CREMA-13

$$\nu_S = \nu(2S_{1/2}^{F=0} - 2P_{3/2}^{F=1})$$

ν - 49.0 THz (GHz)

CODATA

this value

signal [arb. units]

ν - 54.0 THz (GHz)

2P fine structure

2P_{3/2}   F=2
             F=1
2P_{1/2}   F=1
             F=0

Lamb shift

$\nu_{triplet}$

$\nu_{singlet}$

2S_{1/2}   F=1

2S hyperfine splitting

F=0

Two transitions measured

$$\nu_T = 49881.35(\ 65)\ \text{GHz}$$
$$\nu_S = 54611.16(1.05)\ \text{GHz}$$

From these two transition measurements, one can independently deduce both the Lamb shift $\Delta E_L = \Delta E(2P_{1/2}-2S_{1/2})$ and the 2S-HFS splitting (ΔE_{HFS}) by the linear combinations
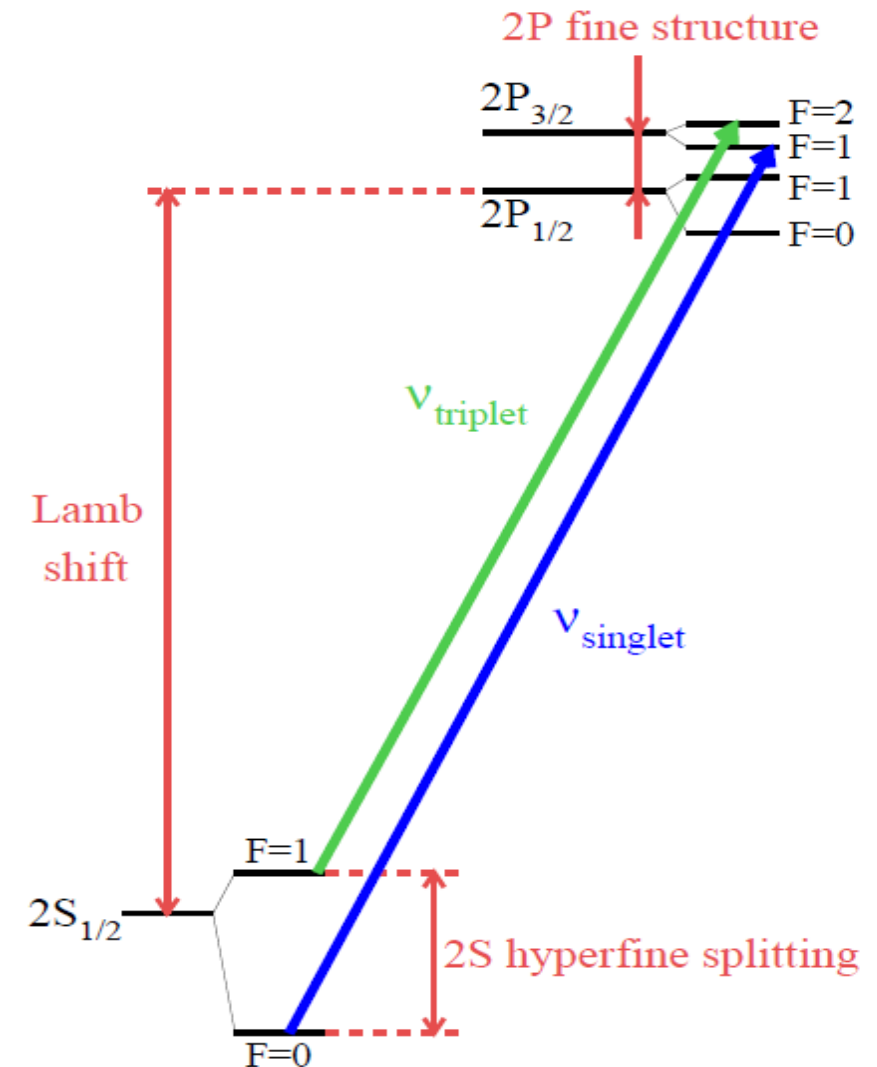
$$\frac{1}{4}h\nu_s + \frac{3}{4}h\nu_t = \Delta E_L + 8.8123(2)\text{meV}$$

$$h\nu_s - h\nu_t = \Delta E_{HFS} - 3.2480(2)\text{meV}$$

Then one get

$$\Delta E_{LS}^{\text{exp}} = 202.3706(23)\ \text{meV}$$
$$\Delta E_{HFS}^{\text{exp}} = 22.8089(51)\ \text{meV}$$

2P fine structure

$2P_{3/2}$ — F=2, F=1

$2P_{1/2}$ — F=1, F=0

$\nu_{triplet}$

Lamb shift

$\nu_{singlet}$

F=1

$2S_{1/2}$

2S hyperfine splitting

F=0

Proton Size Puzzle

$$r_p = 0.8751(61)\,\mathrm{fm}\,(\mathrm{CODATA}\text{-}2014)$$

From spectrum of electronic atoms and
the scattering of the electrons with nuclei

Lamb shift in muonic Hydrogen (μp) (a proton orbited by a negative muon)

muon mass $m_\mu \approx 200 \times m_e$

Bohr radius $r_\mu \approx 1/200 \times r_e$

μp *has much smaller Bohr radius compared to electronic hydrogen and so is much more sensitive to the finite size of the proton*

μ inside the proton: $200^3 \approx 10^7$

$$\Delta E_{\mathrm{LS}}^{\mathrm{exp}} = 202.3706(23)\ \mathrm{meV}$$

CREMA experiment **Nature 2010; Science 2013**
(0.05% precision)

$$\Delta E_{\mathrm{LS}} = 206.0668(25) - 5.2275(10)\langle r_p^2 \rangle + O\left(\langle r_p^2 \rangle^{3/2}\right)[\mathrm{meV}]$$

Theory summary: Antognini et al. AnnPhys 2013
(2% effect)

$$r_p = 0.84087(39)\,\mathrm{fm}\,(\mathrm{CREMA\ coll.\ Antognini\ et\ al.,\ 2013})$$

5.6 σ deviation!

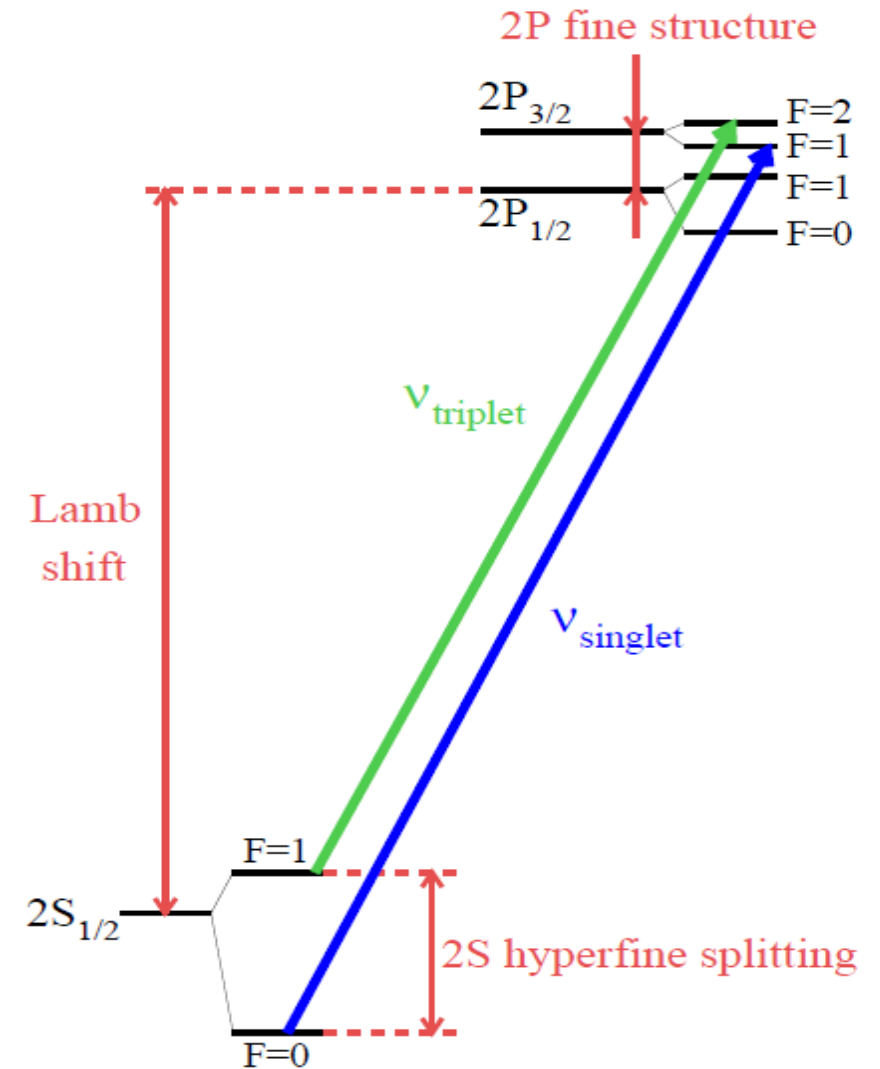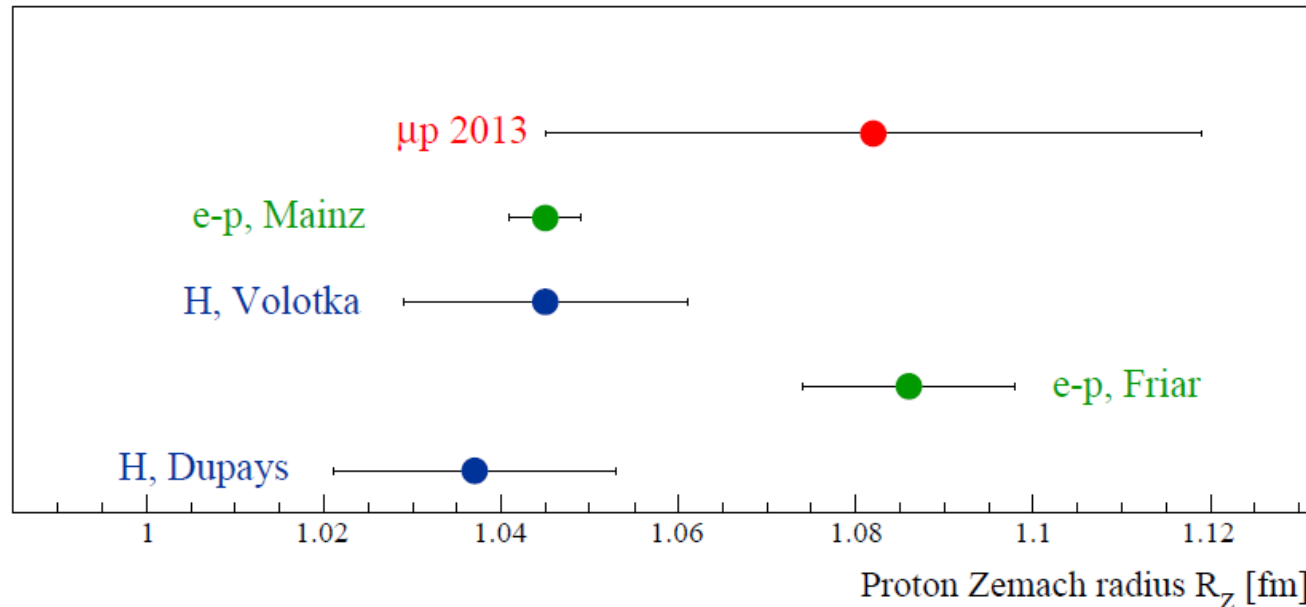$$r_{\mathrm{E}}^2 = \int d^3 r\, r^2\, \rho_E(r)$$

Zemach radius

$$\Delta E_{2S\text{-HFS}}^{\text{exp}} = 22.8089(51) \ \text{meV}$$

$$\Delta E_{2S\text{-HFS}}^{\text{th}} = 22.9843(30) - 0.1621(10)r_Z \ \text{meV}$$
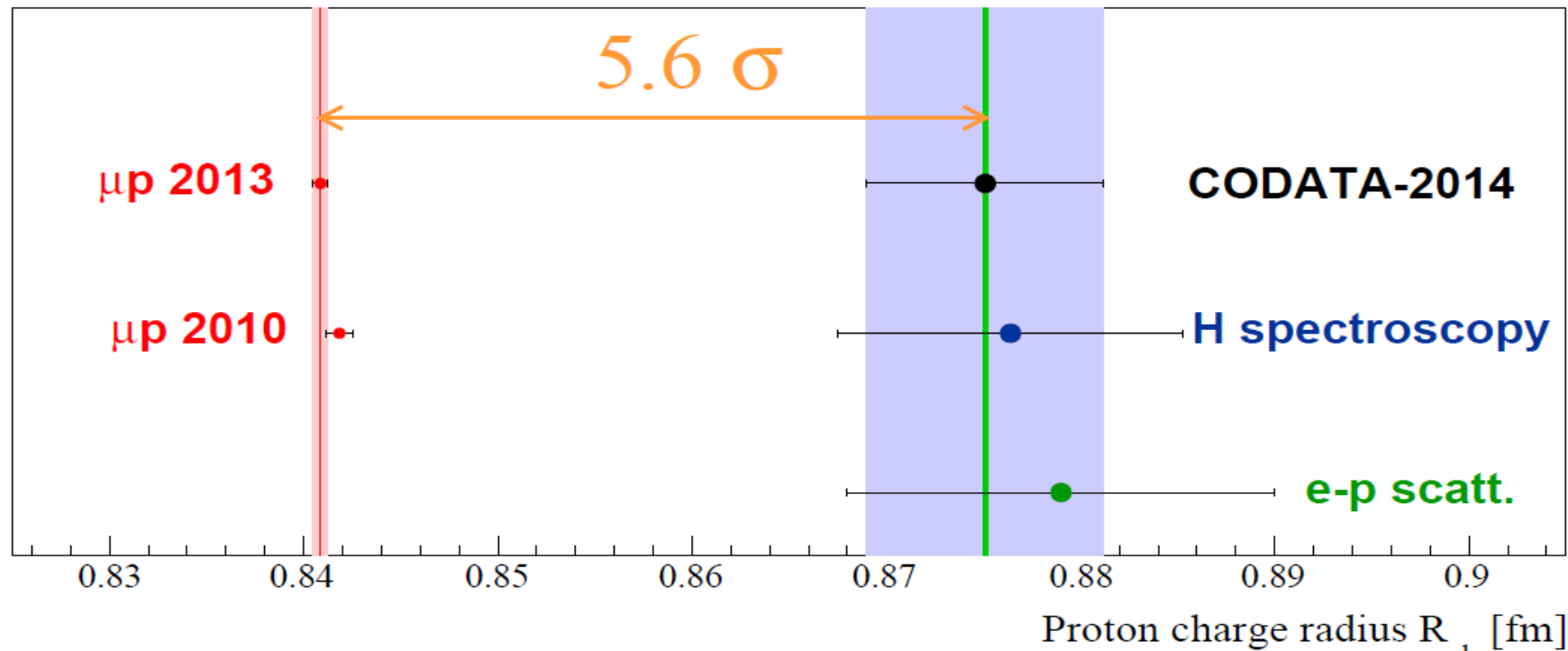
$$r_Z = \int d^3 r \int d^3 r' \ r \ \rho_E(r) \rho_M(r - r')$$

$$r_Z = 1.082(37) \ \text{fm} \ (\text{CREMA coll. Antognini et al., 2013})$$

The proton rms charge radius measured with

electrons: 0.8751 ± 0.0061 fm

muons: 0.8409 ± 0.0004 fm


r_p is 7 σ smaller than CODATA-2010

4.0 σ smaller than r_p (H spectrosopy)

The leading contribution to the hyperfine splitting (HFS) in muonic hydrogen is coming from one-photon exchange
[AP Martynenko, RN Faustov 2004]

$$\Delta V_B^{hfs} = \frac{8\pi\alpha\mu_p}{3m_\mu m_p}(\mathbf{S}_p\mathbf{S}_\mu)\delta(\mathbf{r}) - \frac{\alpha\mu_p(1+a_\mu)}{m_\mu m_p r^3}\left[(\mathbf{S}_p\mathbf{S}_\mu) - 3(\mathbf{S}_p\mathbf{n})(\mathbf{S}_p\mathbf{n})\right] +$$

$$\frac{\alpha\mu_p}{m_\mu m_p r^3}\left[1 + \frac{m_\mu}{m_p} - \frac{m_\mu}{2m_p\mu_p}\right](\mathbf{LS}_p)$$

We calculate further the contribution to HFS coming from light pseudoscalar and axial-vector meson exchanges
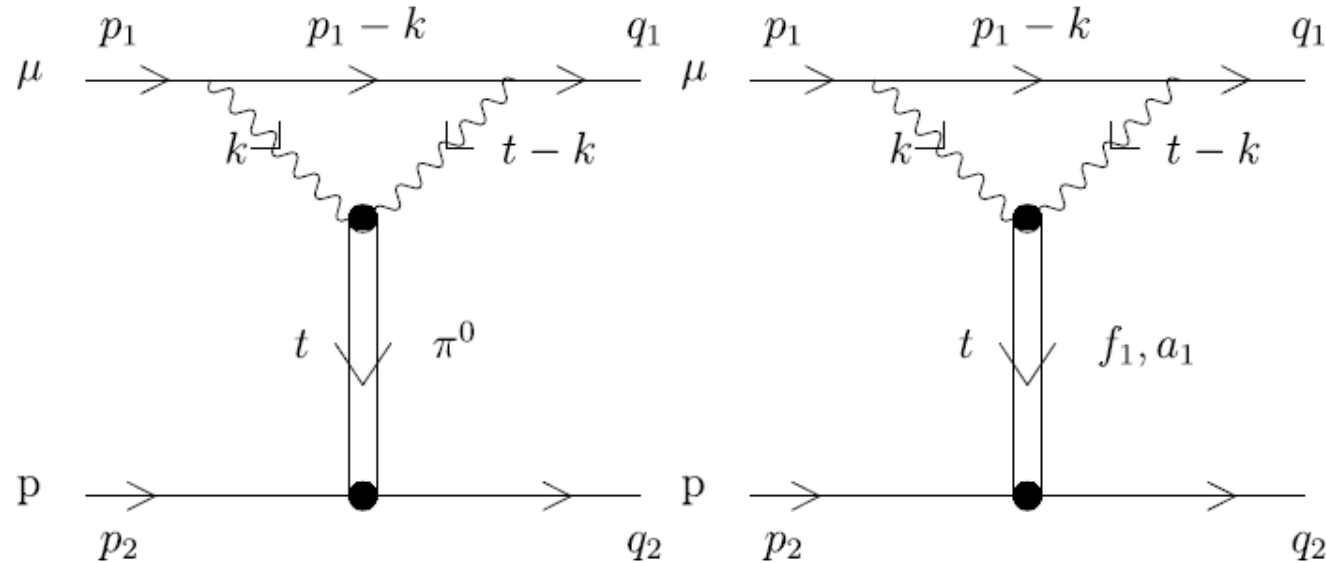
Pseudosclar mesons

The effective vertex of the interaction of the PS mesons and virtual photons can be expressed in terms of the transition form factors
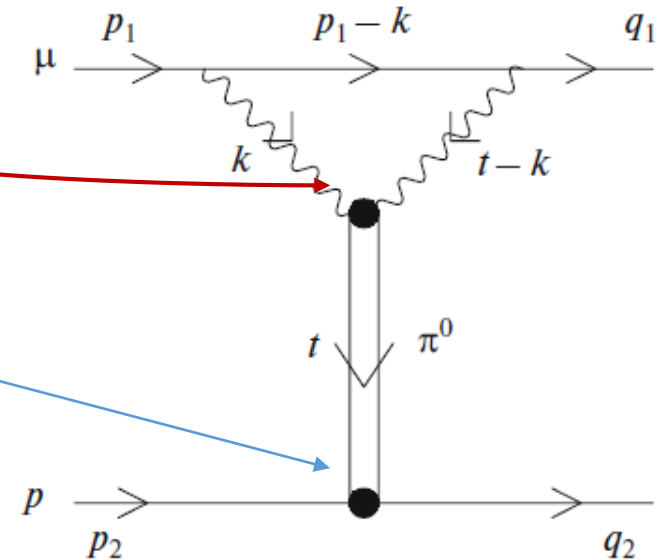
$$A\left(\gamma^*_{(q_1,\epsilon_1)}\gamma^*_{(q_2,\epsilon_2)} \to \pi_{(p)}\right) = ie^2\varepsilon_{\mu\nu\rho\sigma}\epsilon_1^\mu\epsilon_2^\nu q_1^\rho q_2^\sigma \frac{1}{4\pi^2 F_\pi} F_{\pi\gamma^*\gamma^*}\left(p^2; q_1^2, q_2^2\right)$$

As a result, the hyperfine part of the potential of the one-pion interaction of a muon and a proton in the S-state takes the form
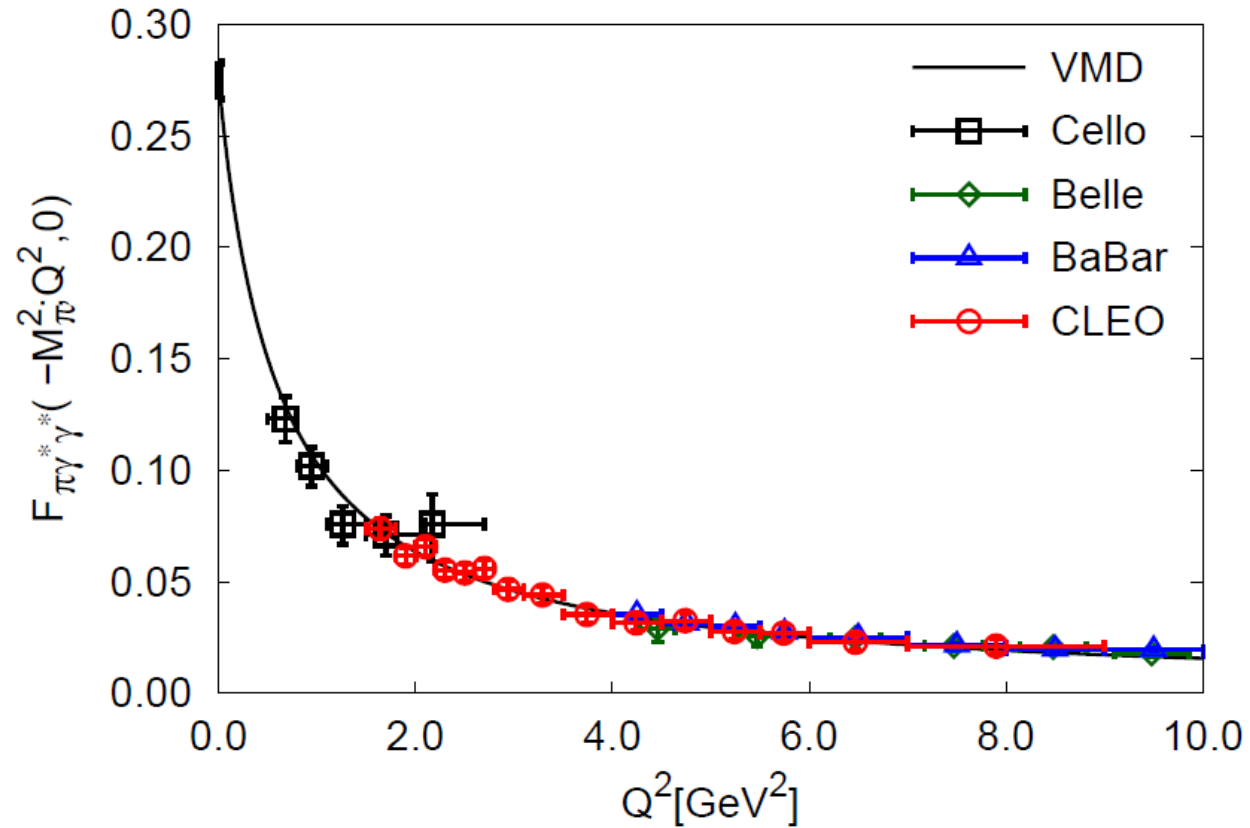
$$\Delta V^{hfs}(\mathbf{p}, \mathbf{q}) = \frac{\alpha^2}{6\pi^2} \frac{g_p}{m_p F_\pi} \frac{(\mathbf{p}-\mathbf{q})^2}{(\mathbf{p}-\mathbf{q})^2 + m_\pi^2} \mathscr{A}(t^2),$$

where

$$\mathscr{A}(t^2) = \frac{2i}{\pi^2 t^2} \int d^4k \frac{t^2 k^2 - (tk)^2}{k^2(k-t)^2(k^2-2kp_1)} F_{\pi\gamma^*\gamma^*}(k^2,(k-t)^2).$$

Upper block: two-gamma transition FF into PS meson

$$F_{\pi\gamma\gamma^*}\left(Q^2,0\right) = \frac{\Lambda_\pi^2}{\Lambda_\pi^2 + k^2}, \quad \Lambda_\pi^{CLEO} = 776 \pm 22 \text{ MeV},$$

$$F_{\pi\gamma^*\gamma^*}\left(Q^2,Q^2\right) = \frac{\overline{\Lambda}_\pi^2}{\overline{\Lambda}_\pi^2 + k^2}, \quad \overline{\Lambda}_\pi^{CLEO+QCD} = 499 \pm 50 \text{ MeV}$$

Calculating the matrix elements with wave functions of 1S , 2S and 2P $_{1/2}$ states, we obtain the corresponding contributions to the HFS spectrum (we use data from CLEO on TFF)

$$\Delta E^{hfs}(1S) = \frac{\mu^3 \alpha^5 g_A}{6F_\pi^2 \pi^3}\left\{\mathcal{A}(0)\frac{4W(1+\frac{W}{m_\pi})}{m_\pi(1+\frac{2W}{m_\pi})^2} - \frac{1}{\pi}\int_0^\infty \frac{ds}{s}Im\mathcal{A}(s)\left[1+\frac{1}{4W^2(s-m_\pi^2)}\left(\frac{m_\pi^4}{(1+\frac{m_\pi}{2W})^2}-\frac{s^2}{(1+\frac{\sqrt{s}}{2W})^2}\right)\right]\right\} = -0.0017\ meV,$$

$$\Delta E^{hfs}(2S) = \frac{\mu^3 \alpha^5 g_A}{48F_\pi^2 \pi^3}\left\{\mathcal{A}(0)\frac{W(8+11\frac{W}{m_\pi}+8\frac{W^2}{m_\pi^2}+2\frac{W^3}{m_\pi^3})}{2m_\pi(1+\frac{W}{m_\pi})^4} - \frac{1}{\pi}\int_0^\infty \frac{ds}{s}Im\mathcal{A}(s)\right.$$
$$\left.\left[1+\frac{1}{(s-m_\pi^2)}\left(\frac{m_\pi^2(2+\frac{W^2}{m_\pi^2})}{2(1+\frac{W}{m_\pi})^4}-\frac{s(2+\frac{W^2}{s})}{2(1+\frac{W}{\sqrt{s}})^4}\right)\right]\right\} = -0.0002\ meV,$$

$$\Delta E^{hfs}_{2P_{1/2}} = \frac{\alpha^7 \mu^5 g_A}{288\pi^3 F_\pi^2 m_\pi^2}\mathcal{A}(0)\frac{\left(9+8\frac{W}{m_\pi}+2\frac{W^2}{m_\pi^2}\right)}{(1+\frac{W}{m_\pi})^4} = 0.0004\ \mu eV.$$

Axial-Vector Mesons

The transition from initial state of two virtual photons with four-momenta k1, k2 to an axial vector meson A (JPC = 1++) with the mass M_A for the small values of the relative momenta of particles in the initial and final states and small value of transfer momentum t between muon and proton is

$$T^{\mu\nu\alpha} = 8\pi i\alpha\varepsilon_{\mu\nu\alpha\tau}k^\tau k^2 F^2_{AV\gamma^*\gamma^*}\left(M_A^2;0,0\right)$$

The final result for the HFS potential is

$$\Delta V_{AV}^{hfs}(\mathbf{p}-\mathbf{q}) = -\frac{32\alpha^2 g_{AVPP}}{3\pi^2(\mathbf{t}^2+M_A^2)}\int id^4k\frac{(2k^2+k_0^2)}{k^2(k^2-2m_\mu k_0)}F_{AV\gamma^*\gamma^*}(0,k^2,k^2)$$

By using L3 Collaboration data (production of $f_1(1285)$ and $f_1(1420)$), we can parameterize the transition form factor for the case of two photons with equal virtualities

$$F_{AV\gamma^*\gamma^*}\left(M_A^2;k^2,k^2\right) = F_{AV\gamma^*\gamma^*}\left(M_A^2;0,0\right)F_{AV}^2\left(k^2\right) \qquad F_{AV}\left(k^2\right) = \frac{\Lambda_A^2}{\Lambda_A^2+k^2}$$

with the normalization fixed from the decay width of axial-vector meson measured by L3 Collaboration.

$$\Lambda_{f_1(1285)}^{L3} = 1040\pm78 \text{ MeV}, \quad \Lambda_{f_1(1420)}^{L3} = 926\pm78 \text{ MeV}$$

Meson-Nucleon Couplings

Within the NJL model from Triangle diagram one has

$$F_{f_1(1285)\gamma^*\gamma^*}\left(M^2_{f_1(1285)};0,0\right)=\frac{5g_{f_1(1285)qq}}{72\pi^2 M^2_q}$$

Another couplings are related to each others by using chiral symmetry and SU(6)-model for wave function of the proton

$$g_{f_1(1260)qq}=g_{f_1(1285)qq},$$

$$g_{f_1(1285)pp}=g_{f_1(1285)qq},\quad g_{f_1(1260)pp}=\frac{5}{3}g_{f_1(1285)qq}$$

With Mq=300 MeV one gets

$$g_{f_1(1285)pp}=g_{f_1(1285)qq}=g_{f_1(1260)qq}=3.40\pm1.19,$$

$$g_{f_1(1260)pp}=5.67\pm1.98,\quad g_{f_1(1420)pp}=1.51\pm0.19$$

Our result for HFS interaction can be rewritten ($a = 2m_\mu/\Lambda_A$)

$$\Delta V_{AV}^{hfs}(\mathbf{p} - \mathbf{q}) = -\frac{32\alpha^2 g_{AVpp} F_{AV\gamma^*\gamma^*}^{(0)}(0,0,0)}{3\pi^2(\mathbf{t}^2 + M_A^2)} I\left(\frac{m_\mu}{\Lambda_A}\right)$$

$$I\left(\frac{m_\mu}{\Lambda_A}\right) = -\frac{\pi^2 \Lambda_A^2}{4(1 - a_\mu^2)^{5/2}}\left[3\sqrt{1 - a_\mu^2} - a_\mu^2(5 - 2a_\mu^2)\ln\frac{1 + \sqrt{1 - a_\mu^2}}{a_\mu}\right]$$

Making the Fourier transform and averaging the obtained expression with the wave functions for 1S and 2S states, we obtain the following contribution to hyperfine splitting

$$\Delta E_{AV}^{hfs}(1S) = \frac{32\alpha^5 \mu^3 \Lambda^2 g_{AVNN} F_{AV\gamma^*\gamma^*}^{(0)}(0,0,0)}{3M_A^2\pi^3\left(1 + \frac{2W}{M_A}\right)^2} I\left(\frac{m_\mu}{\Lambda}\right),$$

$$\Delta E_{AV}^{hfs}(2S) = \frac{2\alpha^5 \mu^3 \Lambda^2 g_{AVNN} F_{AV\gamma^*\gamma^*}^{(0)}(0,0,0)\left(2 + \frac{W^2}{M_A^2}\right)}{3M_A^2\pi^3\left(1 + \frac{W}{M_A}\right)^4} I\left(\frac{m_\mu}{\Lambda}\right)$$

where W= $\mu\,\alpha$ and μ is the reduced mass.

Axial-Vector and Pseudoscalar mesons contribution to HFS of muonic hydrogen

mesons	$I^G(J^{PC})$	Λ_A in MeV	$F^{(0)}_{AV\gamma^*\gamma^*}(0,0)$ in GeV^{-2}	$\Delta E^{hfs}(1S)$ in meV	$\Delta E^{hfs}(2S)$ in meV
$f_1(1285)$	$0^+(1^{++})$	1040	0.266	-0.0093 ± 0.0033	-0.0012 ± 0.0004
$a_1(1260)$	$1^-(1^{++})$	1040	0.591	-0.0437 ± 0.0175	-0.0055 ± 0.0022
$f_1(1420)$	$0^+(1^{++})$	926	0.193	-0.0013 ± 0.0008	-0.0002 ± 0.0001
π^0	$1^-(0^{-+})$	776		-0.0017 ± 0.0001	-0.0002 ± 0.00002
Sum				-0.0560 ± 0.0178	-0.0071 ± 0.0024

Hierarchy of contributions to Lamb shift

→ 205 meV
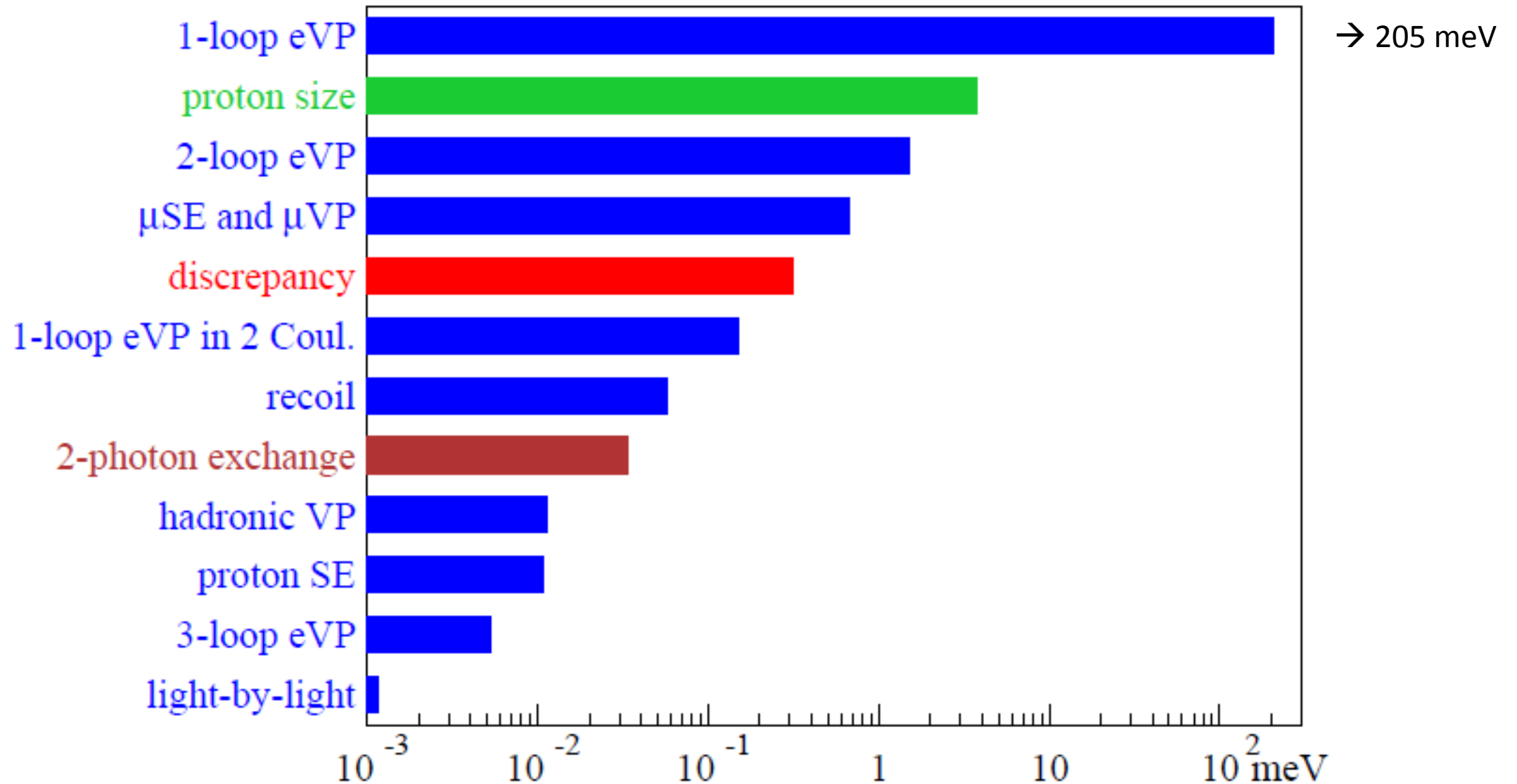
Conclusions

From CREMA experiment side:

"Proton radius puzzle" is in fact "Z=1 radius puzzle"

$$r_p = 0.84087(39)\,\text{fm} \qquad r_Z = 1.082(37)\,\text{fm}$$

Muonic helium-3 and -4 ions show (preliminary) no big discrepancy

New projects are ongoing, one of them FAMU (Fisica Atomi MUonici) with accuracy 2 ppm

In our theoretical work:

A new **large** contribution to the HFS of muonic hydrogen is discovered, that
Induced by pseudoscalar and **axial-vector** couplings to two photon state.

While results do not influence on the proton charge radius, they provide diminishing of
the Zemach radius $\boxed{r_Z = 1.040(37)\,\text{fm}}$

It should be taking into account for the interpretation of the new data on HFS in this
atom.

There are still a number of uncertainties in phenomenological input used in our
calculations and some other new effects unaccounted by us. (Work is in progress)

This will make it possible to better understand the existing "puzzle" of the proton charge radius, to check the Standard Model with greater accuracy and, possibly, to reveal the source of previously unaccounted interactions between the particles forming the bound state in QED.

One of the ways of overcoming the crisis situation arises from a deeper theoretical analysis of the fine and hyperfine structure of muonic atom spectrum, with the verification of previously calculated contributions and the more accurate construction of the particle interaction operator in quantum field theory, the calculation of new corrections whose value for muonic atoms can increase substantially in comparison with electronic atoms. The expected results will allow to get also a new very important information about the forces which are responsible for the structure of atoms.

Below, we discuss the effects of exchanges between muon and proton which can contribute to hyperfine structure of muonic hydrogen coming from the effective light meson exchange between muon and proton induced by coupling of pion to two photons. In spite that numerically such contribution was found to be rather small, its contribution will might be important for the interpretation of new data.
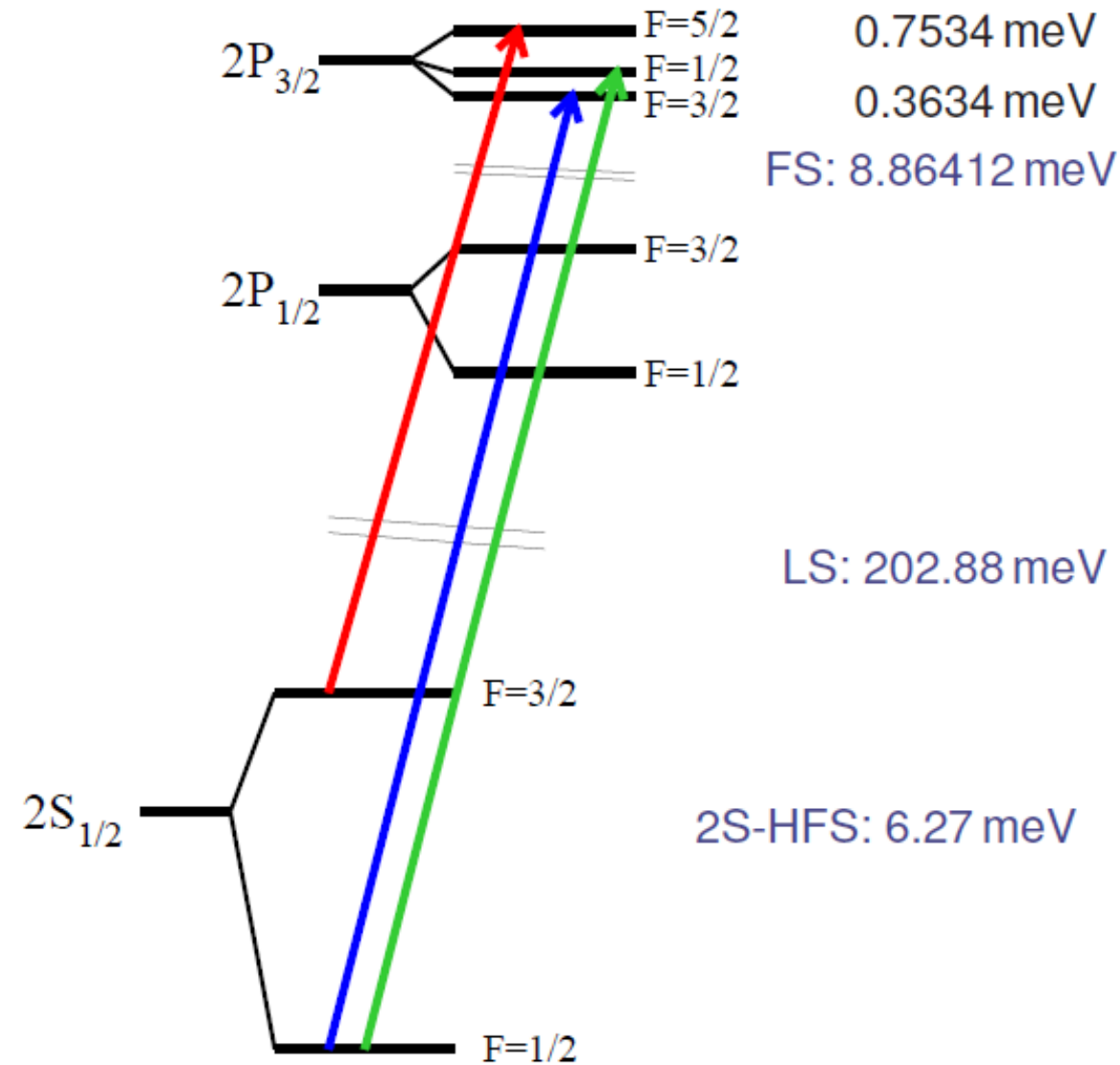
Deutron charge radius

$$\Delta E_{LS}^{exp} = 202.8785(34) \text{ meV}$$

$$\Delta E_{LS}^{th} = 228.7766(\ 10) \text{ meV (QED)}$$
$$+1.7096(200) \text{ meV (TPE)}$$
$$-6.1103(\ \ 3)r_d^2 \text{ meV/fm}^2$$

$$\mu D \Rightarrow r_d = 2.12562(78) \text{ meV (CREMA 2016)}$$

H/D isotope shift: $r_d^2 - r_p^2 = 3.82007(65) \text{ fm}^2$

r_p from μH gives $\quad r_d = 2.12771(\ 22) \text{ fm} \quad \leftarrow 5.4\sigma$ from r_p

CODATA 2014 $\quad r_d = 2.14130(250) \text{ fm}$
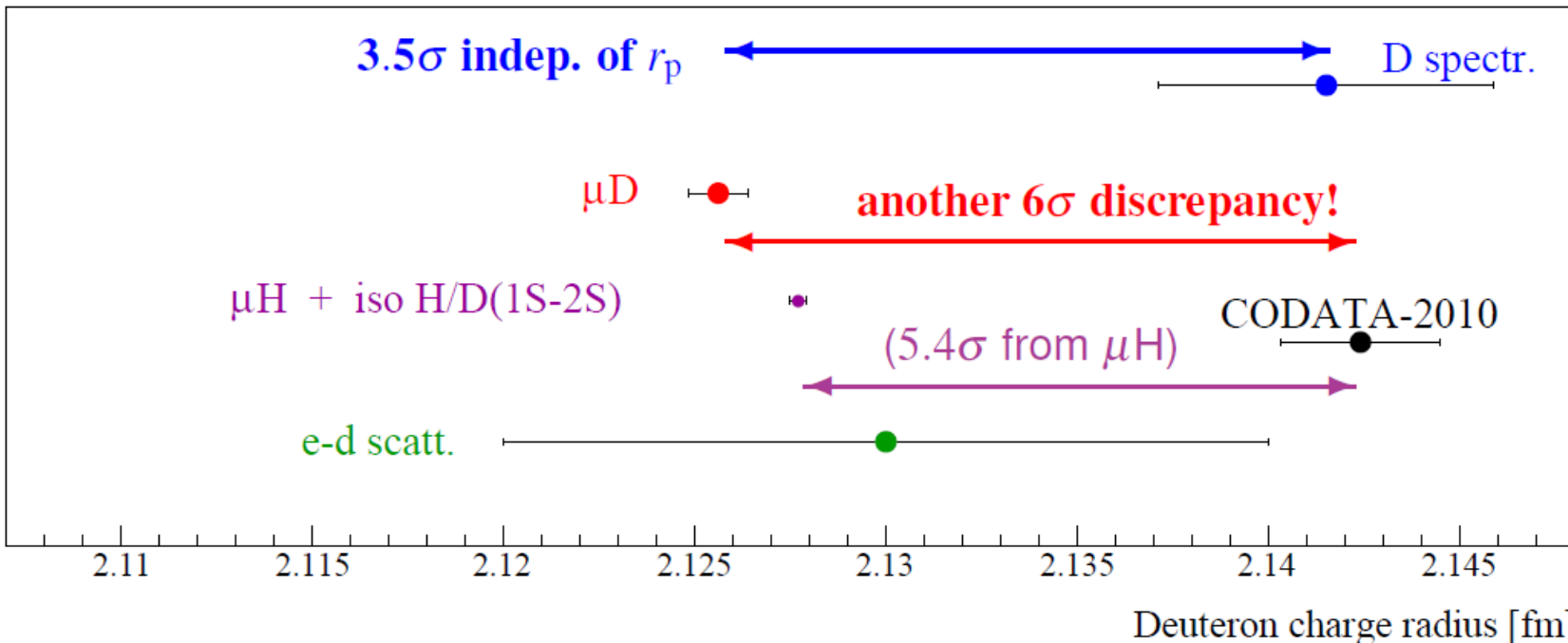

r_d is 7.5 σ smaller than CODATA-2010 (99% correlated with r_p !)

3.5 σ smaller than r_d (D spectrosopy)

- Results from muonic hydrogen and deuterium:
  - Proton charge radius: $r_{\mathrm{p}} = 0.84087\,(39)$ fm
  - Proton Zemach radius: $R_Z = 1.082\,(37)$ fm
  - Rydberg constant: $R_\infty = 3.289\,841\,960\,2495\,(10)^{r_{\mathrm{p}}}\,(25)^{\mathrm{QED}} \times 10^{15}$ Hz/c
  - Deuteron charge radius: $r_d = 2.12771\,(22)$ fm from μH + H/D(1S-2S)
  - The "Proton radius puzzle"
- muonic helium-3 and -4: charge radius 10x more precise. No big discrepancy
- H(2S-4P) gives revised Rydberg $\Rightarrow$ small r_{p} PRELIMINARY
- New projects:
  - 1S-HFS in muonic hydrogen / ^{3}He $\Leftarrow$ PSI, J-PARC, RIKEN-RAL, ...
  - LS in muonic Li, Be, B, T, ...;   muonic high-Z, ...
  - 1S-2S and 2S-$n\ell$ in Hydrogen/Deuterium/Tritium, He$^+$
  - He, H$_2$, HD$^+$,...
  - Positronium $\equiv e^+e^-$, Muonium $\equiv \mu^+e^-$
  - Electron scattering: H at lower Q^2, D, He
  - Muon scattering: MUSE @ PSI
  - ...

tates:

8S
4S
3S ——— ——— ——— 3D

2S ¦¦¦¦¦¦ ——— 2P

radial w.f.

	1S
	2S
	2P

r [Zr/a0]
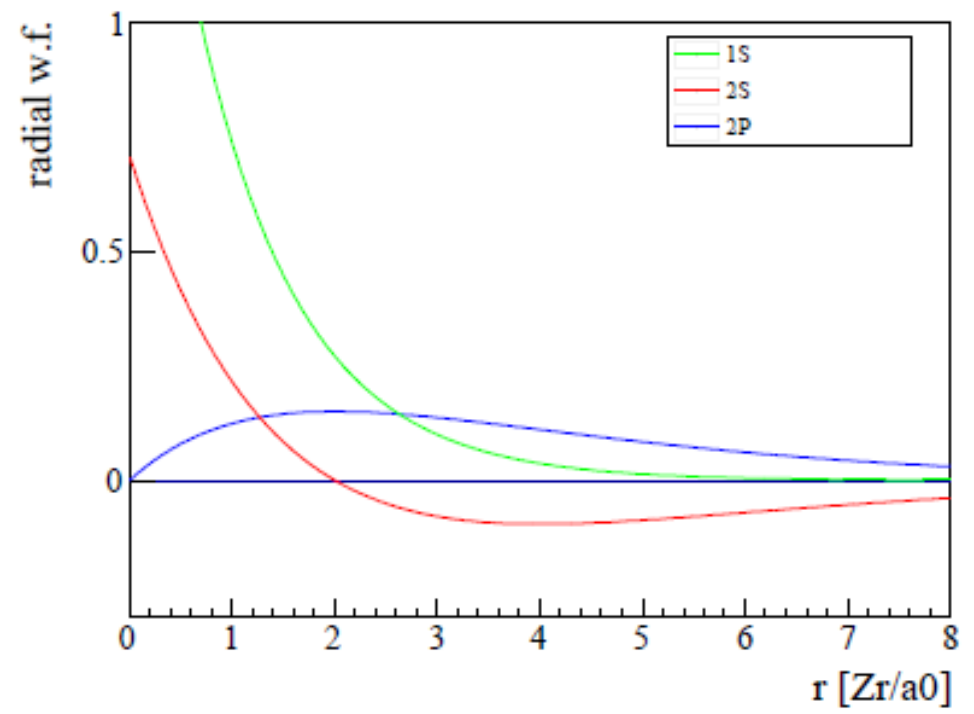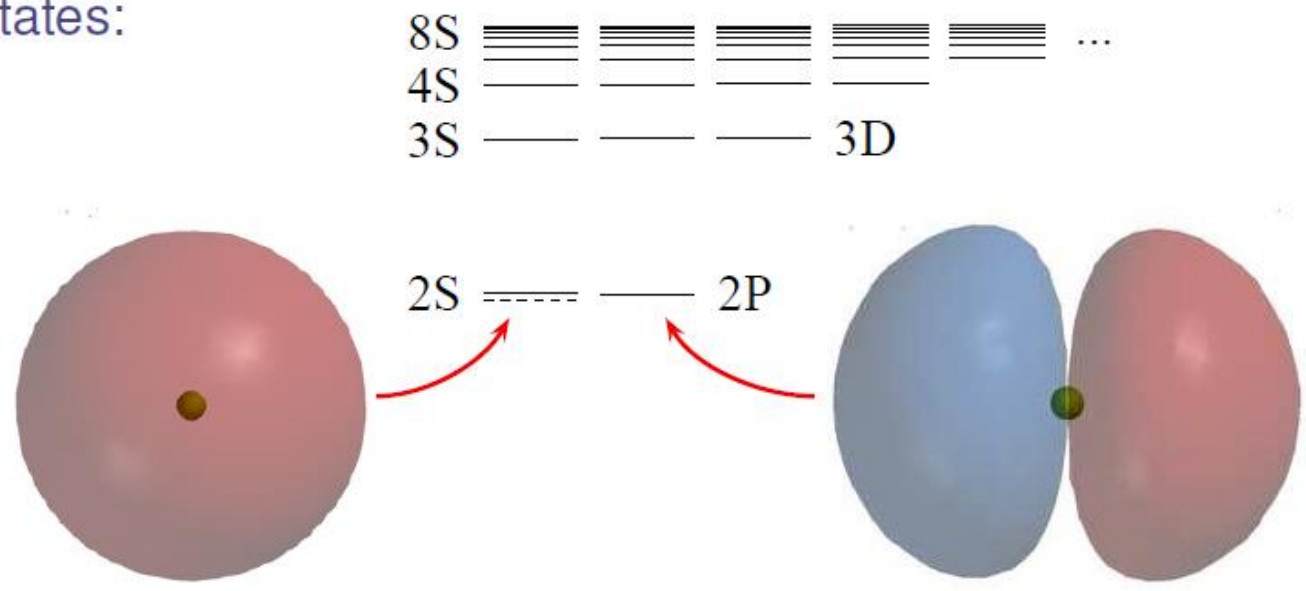
S states: max. at r=0

Electron sometimes inside the proton.

S states are shifted.

Shift ist proportional to the

size of the proton

1S ———
¦¦¦¦¦¦

P states: zero at r=0

Electron is not
inside the proton.

Future:
HFS in muonic hydrogen and helium-3
X-ray spectroscopy of muonic radium etc.
Lamb shift in muonic Li, Be, ...
1S-2S in regular tritium (triton radius)

CREMA 2013

$$\Delta\tilde{E} \equiv E(2P_{3/2}^{F=2}) - E(2S_{1/2}^{F=1}) = 206.2949 \pm 0.0032 \text{ meV}$$