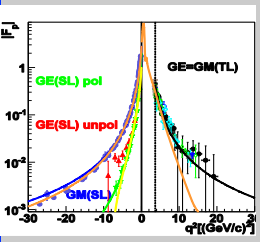
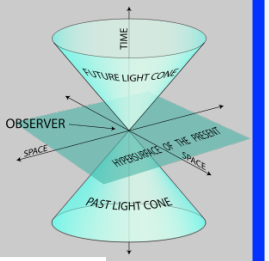
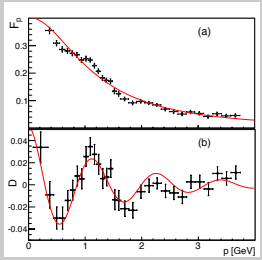
On the Physical Meaning of Time-like Electromagnetic Form Factors: The 4th Dimension of the Nucleon







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Università di Brescia & INFN Pavia

Italia

668. WE-Heraeus-Seminar on Baryon Form Factors: Where do we stand?

23-27 April 2018
Physikzentrum Bad Honnef, Germany
Europe/Berlin timezone

Supported by the WE-Heraeus Foundation

Wilhelm und Else Heraeus Stiftung

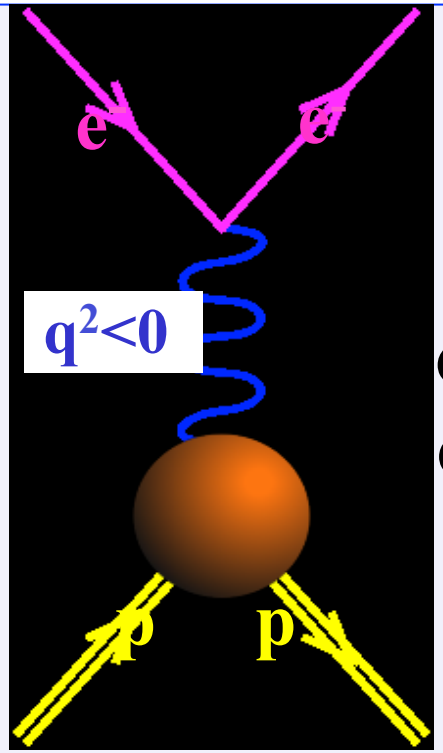
Plan

- Introduction
  - Space- and Time-like Form Factors
  - The dipole approximation
- Periodic oscillation of BaBar data
  - Data
  - Interpretation
    - Fourier transform
    - Optical model
- Modelization
  - Generalization of FF
  - The charge pair creation
  - A new picture for the nucleon structure
- Future prospects and Conclusions

Plan

- Introduction
  - Space- and Time-like Form Factors
  - The dipole approximation

Proton Charge and Magnetic Distributions



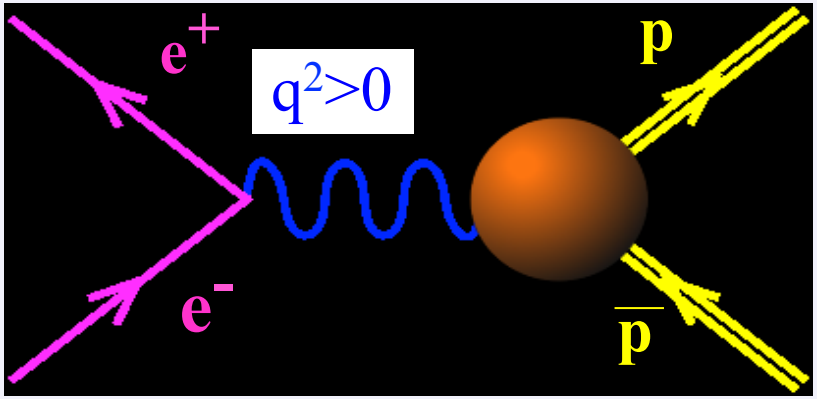
$G_E(0)=1$

$G_M(0)=\mu_p$

Asymptotics
- *QCD*
- *analyticity*

Unphysical region

$p+\bar{p} \leftrightarrow e^+ e^- + \pi^0$



Space-like
FFs are real

Time-Like
FFs are complex

$e+p \rightarrow e+p$

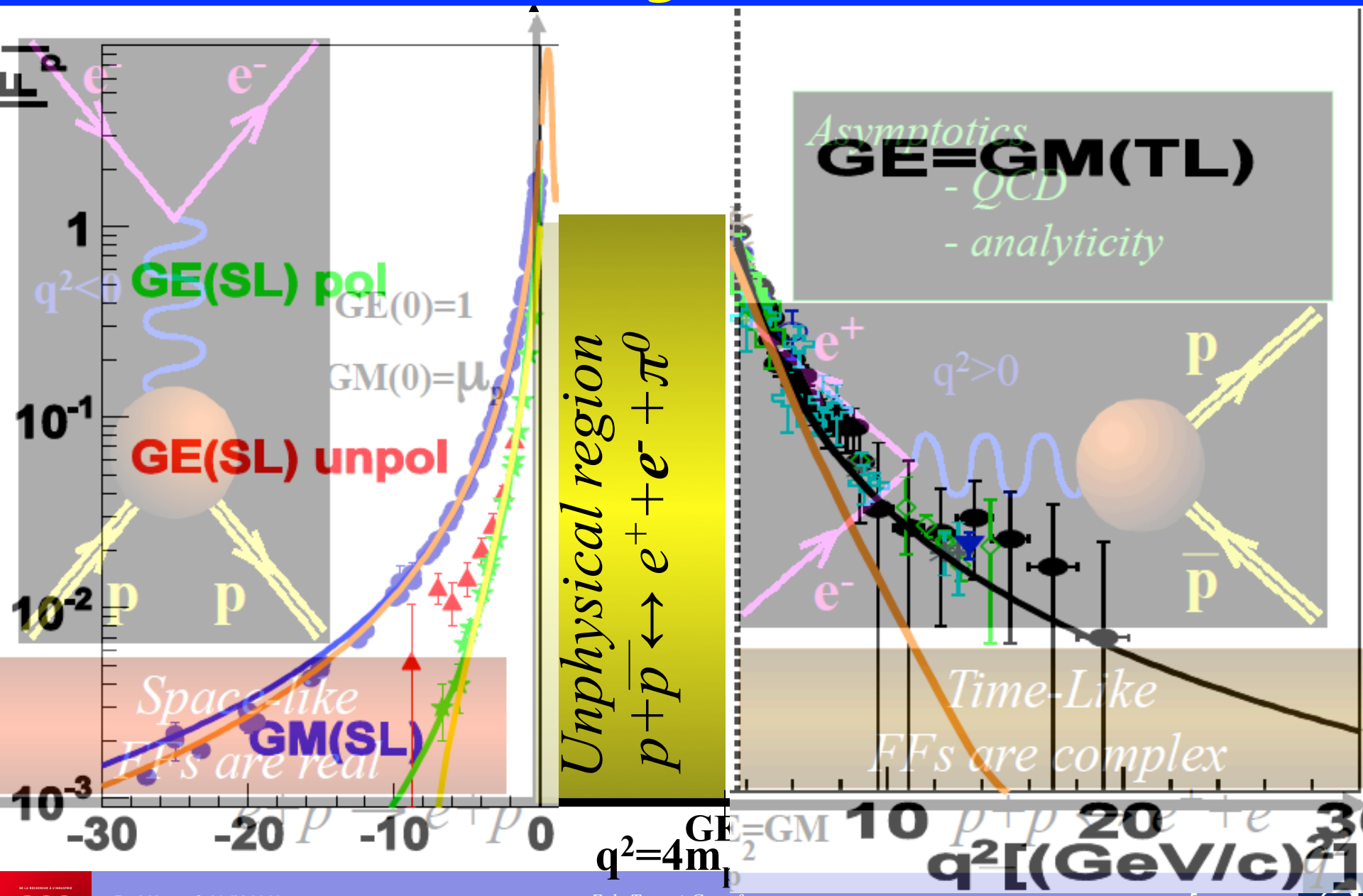
0 $q^2=4m_p^2$

GE=GM

$p+\bar{p} \leftrightarrow e^+ + e^-$

q^2

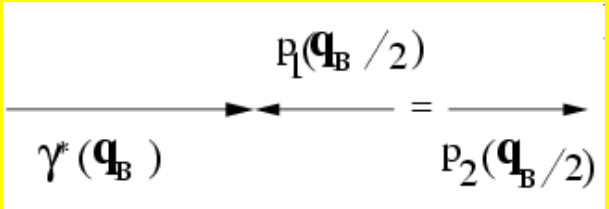
Hadron Electromagnetic Form factors



Dipole Approximation

$$G_D=(1+Q^2 / 0.71\ GeV^2)^{-2}$$

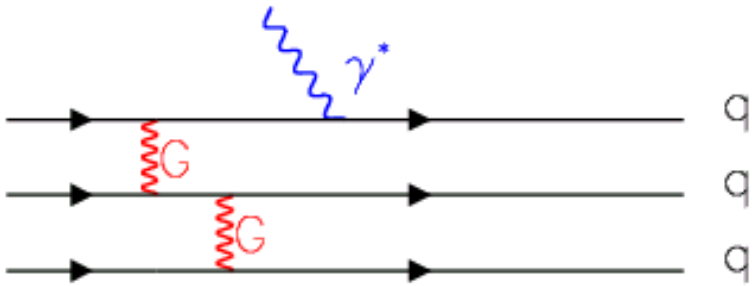
- Classical approach
  - Nucleon FF (in non relativistic approximation or in the Breit system) are Fourier transform of the charge or magnetic distribution.



Breit system

- The dipole approximation corresponds to **exponential density distribution.**
  - $\rho = \rho_0 \exp(-r/r_0)$,
  - $r_0^2 = (0.24\ fm)^2$, $\langle r^2 \rangle \sim (0.81\ fm)^2 \leftrightarrow m^2_D = 0.71\ GeV^2$

Dipole Approximation and pQCD



Dimensional scaling

- $F_n(Q^2) = C_n\,[1/(1+Q^2/m_n)^{n-1}]$,
  - $m_n = n\beta^2$, <quark momentum squared>
  - n is the number of constituent quarks
- Setting $\beta^2 = (0.471 \pm .010)\ \mathrm{GeV}^2$ *(fitting pion data)*
  - pion: $F_\pi(Q^2) = C_\pi\,[1/(1+Q^2/0.471\ \mathrm{GeV}^2)^1]$,
  - nucleon: $F_N(Q^2) = C_N\,[1/(1+Q^2/0.71\ \mathrm{GeV}^2)^2]$,
  - deuteron: $F_d(Q^2) = C_d\,[1/(1+Q^2/1.41\ \mathrm{GeV}^2)^5]$

V. A. Matveev, R. M. Muradian, and A. N. Tavkhelidze (1973), Brodsky and Farrar (1973), Politzer (1974), Chernyak & Zhitnisky (1984), Efremov & Radyuskin (1980)…

Fourier Transform of the spatial density

$$F(q) = \frac{\int_\Omega d^3\vec{x} e^{i\vec{q}\cdot\vec{x}} \rho(\vec{x})}{\int_\Omega d^3\vec{x}\rho(\vec{x})}.$$

density $\rho(r)$	Form factor $F(q^2)$	r.m.s. $< r_c^2 >$	comments
δ	1	0	pointlike
e^{-ar}	$\frac{a^4}{(q^2+a^2)^2}$	$\frac{12}{a^2}$	dipole
$\frac{e^{-ar}}{r}$	$\frac{a^2}{q^2+a^2}$	$\frac{6}{a^2}$	monopole
$\frac{e^{-ar^2}}{r^2}$	$e^{-q^2/(4a^2)}$	$\frac{1}{2a}$	gaussian
ρ_0 for $x \leq R$ 0 for $r \geq R$	$\frac{3(\sin X - X\cos X)}{X^3}$ $X = qR$	$\frac{3}{5}R^2$	square well

Root mean square radius

$$< r_c^2 >= \frac{\int_0^\infty x^4 \rho(x) dx}{\int_0^\infty x^2 \rho(x) dx}.$$

$$F(q) \sim 1 - \frac{1}{6} q^2 < r_c^2 > + O(q^2),$$

IL NUOVO CIMENTO Vol. XXIV, N. 1 1° Aprile 1962

Proton-Antiproton Annihilation into Electrons, Muons and Vector Bosons.

A. Zichichi and S. M. Berman (*)
CERN - Geneva

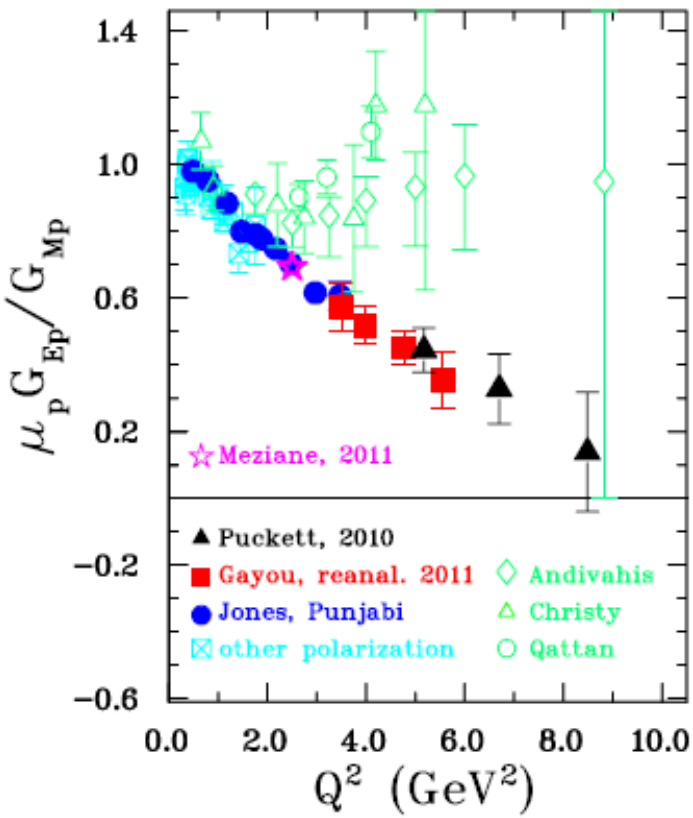
N. Cabibbo and R. Gatto
Università degli Studi - Roma e Cagliari
Laboratori Nazionali di Frascati del CNEN - Roma

Whereas in the spacelike experiments the form factors are given the physical interpretation *of the Fourier transforms of the spatial charge and magnetic structure* of the proton, *the timelike momentum transfers yield information about the frequency structure of the protons*.

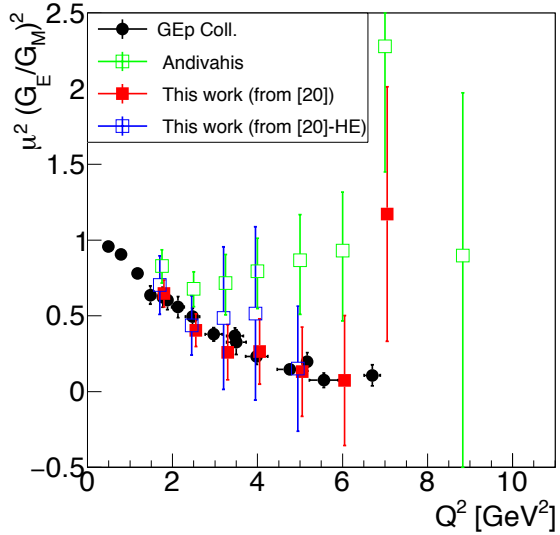
Dipole Approximation

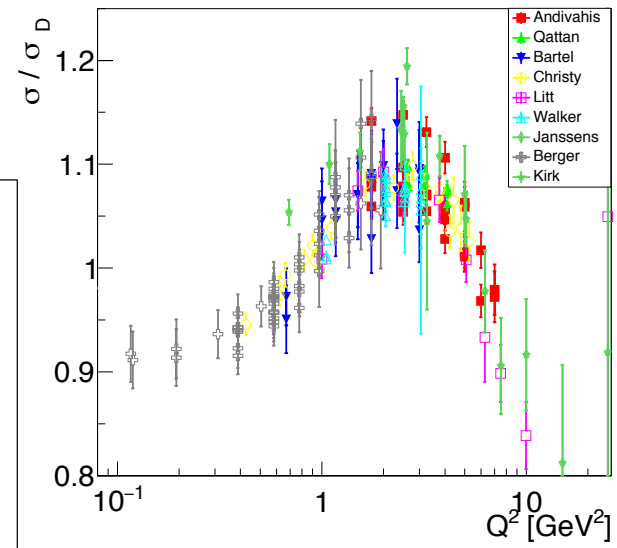
Does not hold in SL region:

neither for GE ...nor for GM



R. Taylor, SLAC, 1967
S. Pacetti, E.T-G, PRC94, 055202 (2016)





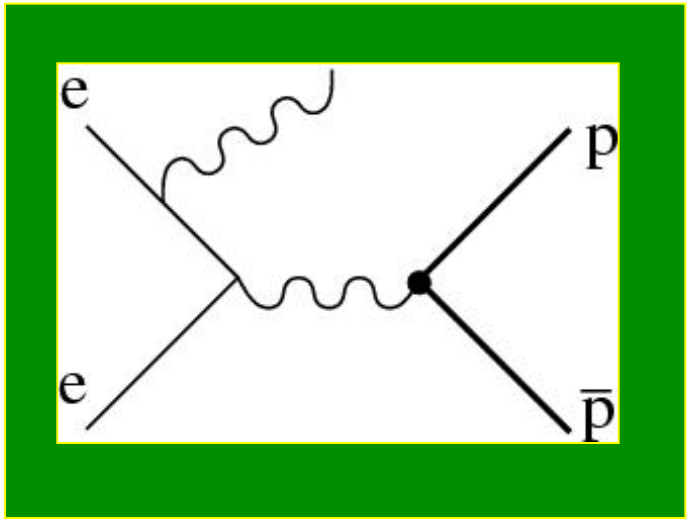
The GEP collaboration,
A.J.R. Puckett et al, PRC96(2017)055203

...and in TL ?

Plan

- Periodic oscillation of BaBar data
  - Data
  - Interpretation
    - Fourier transform
    - Optical model

Radiative Return (ISR)



$$e^+ + e^- \rightarrow p + \bar{p} + \gamma$$

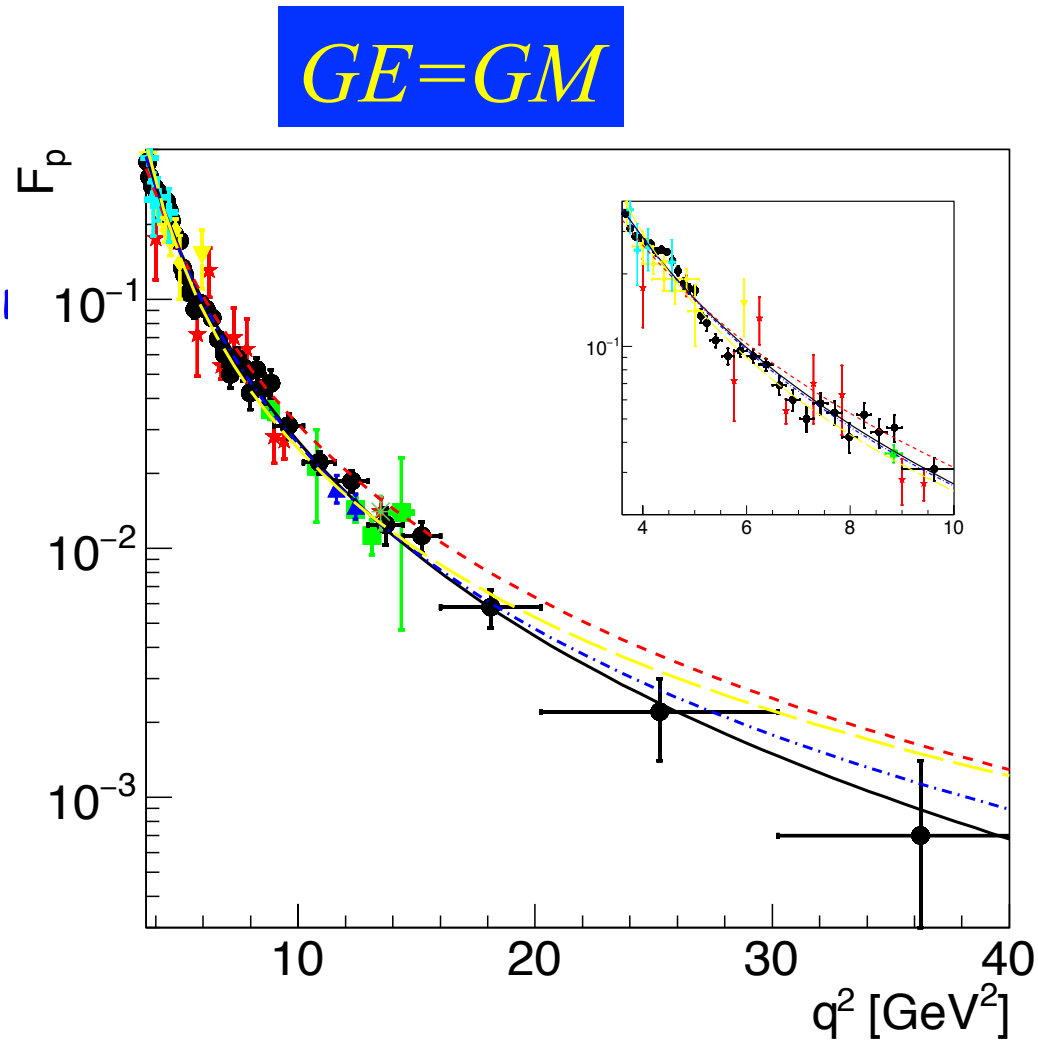
$$\frac{d\sigma(e^+e^- \rightarrow p\bar{p}\gamma)}{dm\, d\cos\theta} = \frac{2m}{s} W(s,x,\theta)\sigma(e^+e^- \rightarrow p\bar{p})(m), \quad x = \frac{2E_\gamma}{\sqrt{s}} = 1 - \frac{m^2}{s},$$

$$W(s,x,\theta) = \frac{\alpha}{\pi x}\left(\frac{2-2x+x^2}{\sin^2\theta} - \frac{x^2}{2}\right), \quad \theta >> \frac{m_e}{\sqrt{s}}.$$

B. Aubert (BABAR Collaboration) Phys Rev. ***D73****, 012005 (2006)*

The Time-like Region

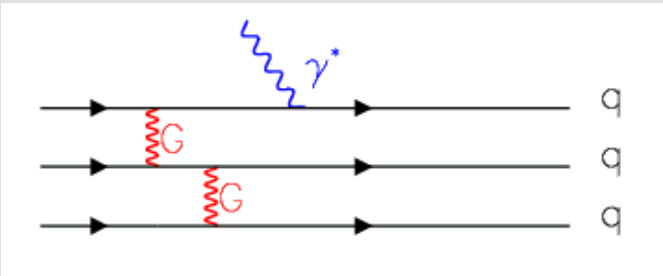
- The Experimental Status
  - No individual determinatio of GE and GM
  - TL proton FFs twice larger than in SL at the same Q^2
  - Steep behaviour at threshold
  - Babar:Structures?
    Resonances?



S. Pacetti, R. Baldini-Ferroli, E.T-G , Physics Reports, 514 (2014) 1

Panda contribution: *M.P. Rekalo, E.T-G , DAPNIA-04-01, ArXiv:0810.4245.*

The Time-like Region



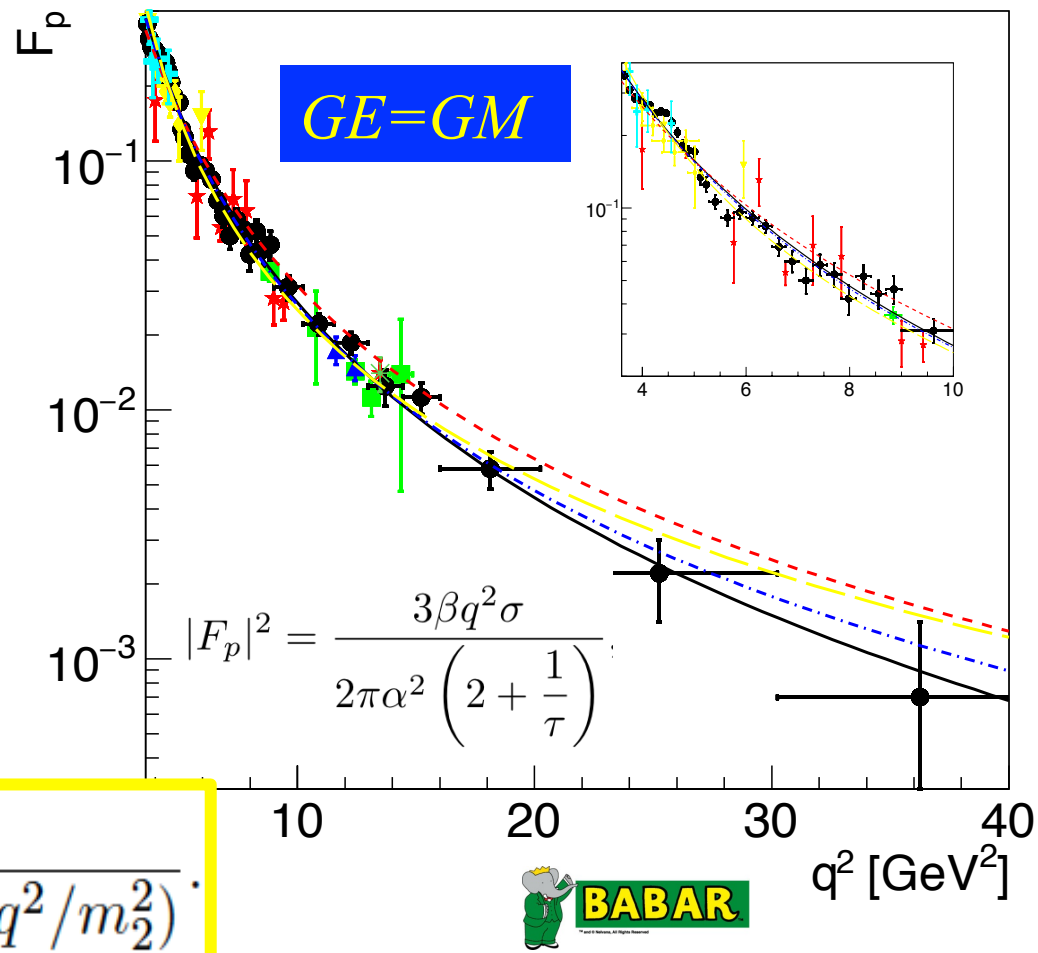
Expected QCD scaling $(q^2)^2$

$$e^+ + e^- \to \bar{p} + p\,,$$
$$\bar{p} + p \to e^+ + e^-$$

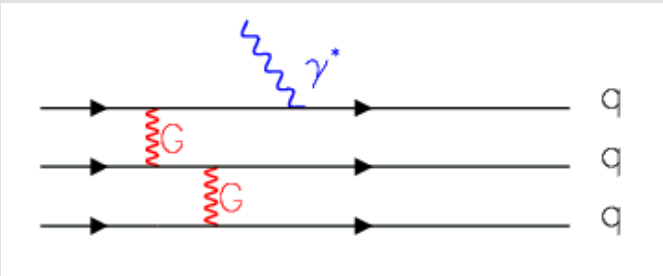
$$|F_{scaling}(q^2)| = \frac{\mathcal{A}}{(q^2)^2 \log^2(q^2/\Lambda^2)}$$

$$\frac{\mathcal{A}}{(1+q^2/m_a^2)\,[1-q^2/0.71]^2}\,,$$

$$|F_{T3}(q^2)| = \frac{\mathcal{A}}{(1-q^2/m_1^2)(2-q^2/m_2^2)}\,.$$



The Time-like Region



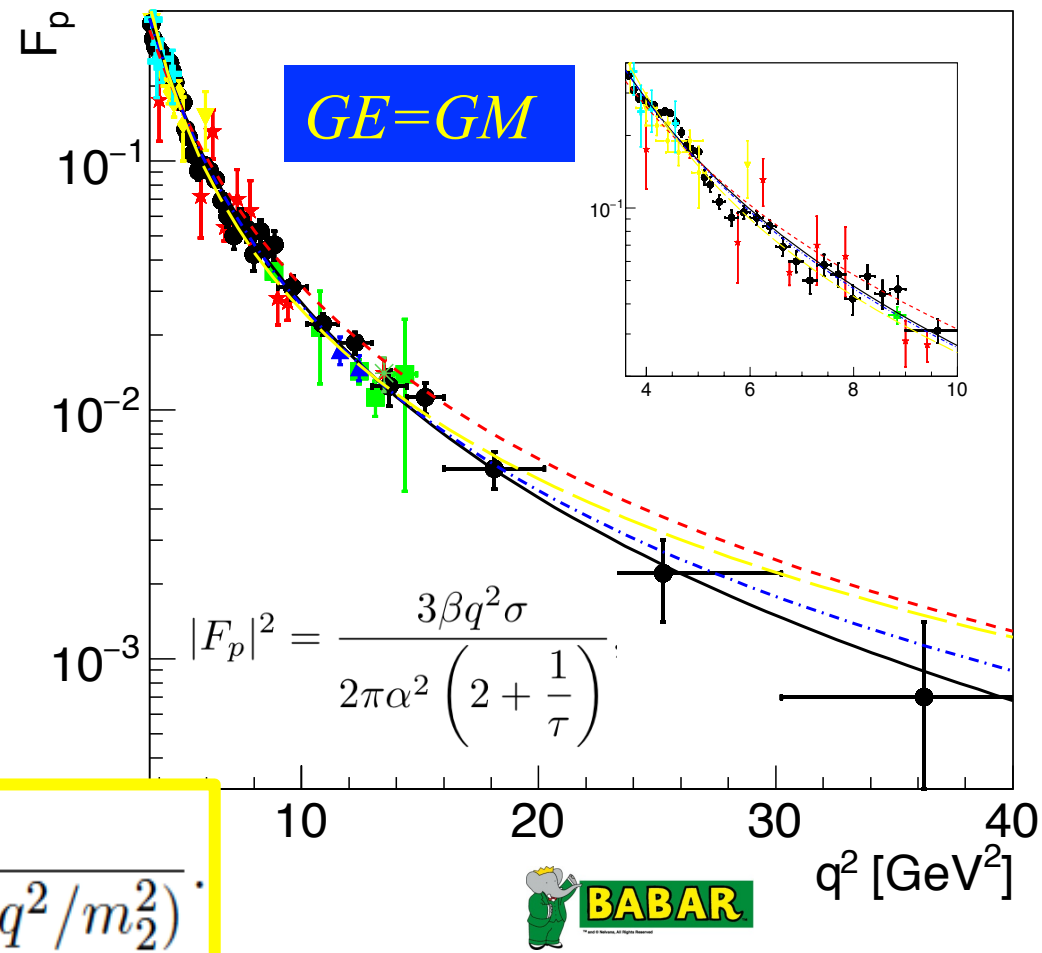
Expected QCD scaling $(q^2)^2$

$$e^+ + e^- \to \bar{p} + p\,,$$
$$\bar{p} + p \to e^+ + e^-$$

$$\frac{\mathcal{A}}{(q^2)^2\left[\log^2(q^2/\Lambda^2)+\pi^2\right]}.$$

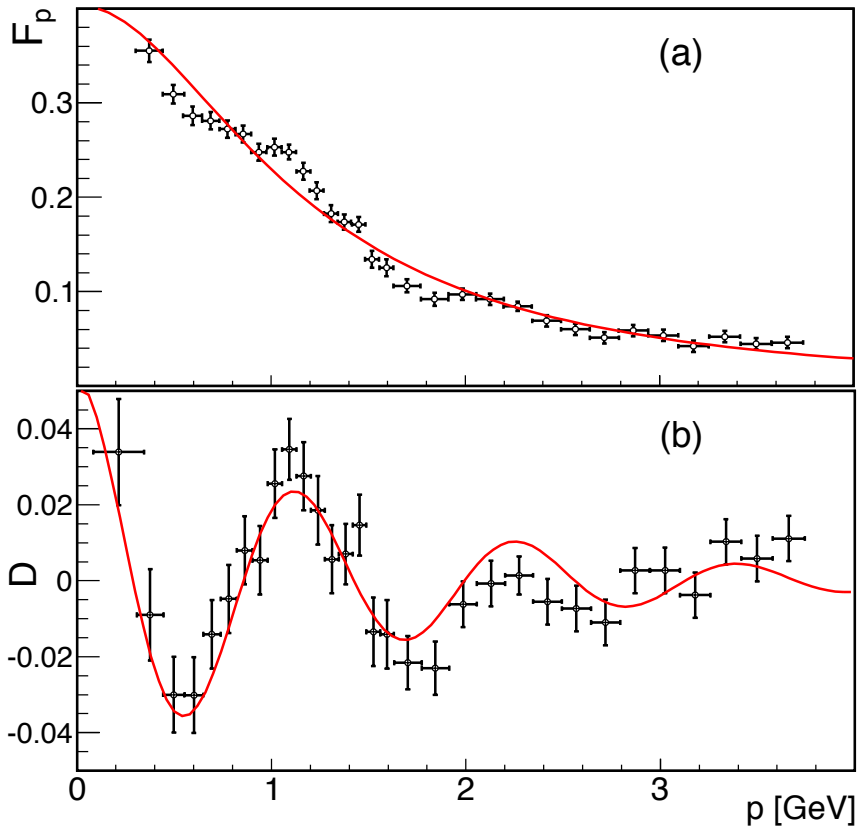
$$\frac{\mathcal{A}}{(1+q^2/m_a^2)\left[1-q^2/0.71\right]^2},$$

$$|F_{T3}(q^2)| = \frac{\mathcal{A}}{(1-q^2/m_1^2)(2-q^2/m_2^2)}.$$



Oscillations : regular pattern in P_{Lab}

The relevant variable is p_{Lab} associated to the relative motion of the final hadrons.



$$F_{osc}(p) \equiv A \exp(-Bp) \cos(Cp + D).$$

$A \pm \Delta A$	$B \pm \Delta B$ $[GeV]^{-1}$	$C \pm \Delta C$ $[GeV]^{-1}$	$D \pm \Delta D$	$\chi^2/n.d.f$
0.05 ± 0.01	0.7 ± 0.2	5.5 ± 0.2	0.03 ± 0.3	1.2

A: Small perturbation B: damping
C: r < 1fm D=0: maximum at p=0

Simple oscillatory behaviour
Small number of coherent sources

A. Bianconi, E. T-G. Phys. Rev. Lett. 114,232301 (2015)

Oscillations : regular pattern in P_{Lab}

The releva

motion of

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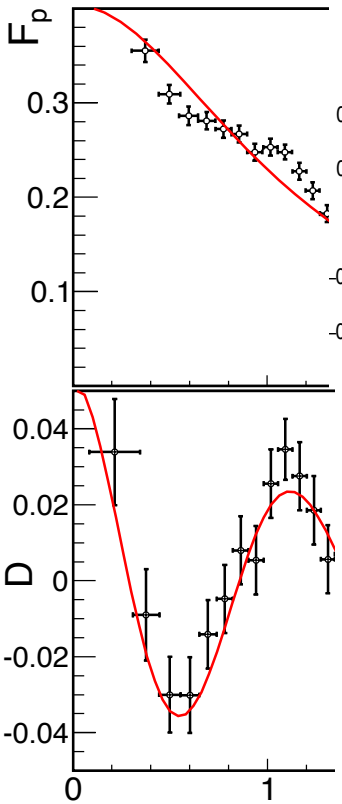
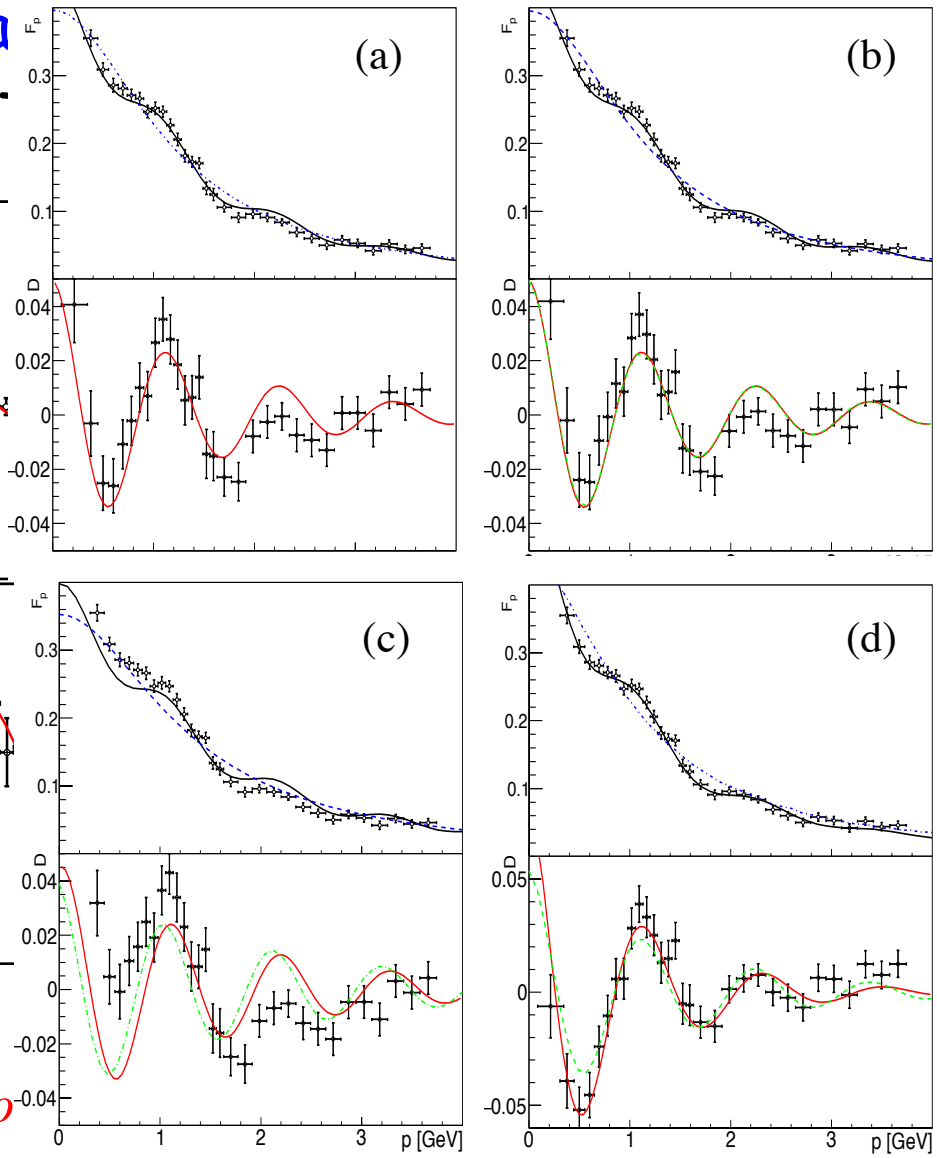




$$\exp(-Bp)\ \cos(Cp + D).$$

$C \pm \Delta C$ $[GeV]^{-1}$	$D \pm \Delta D$	$\chi^2/n.d.f$
5.5 ± 0.2	0.03 ± 0.3	1.2

on B: damping
D=0: maximum at p=0

ory behaviour
f coherent sources

A. Bianco

)

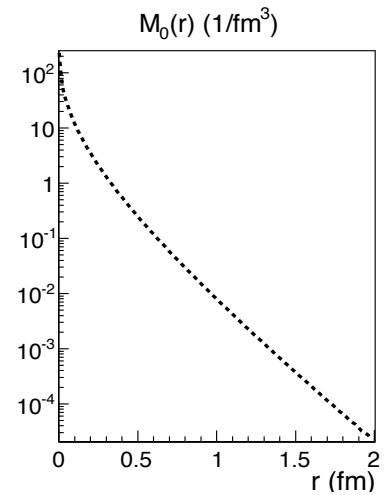
Fourier Transform

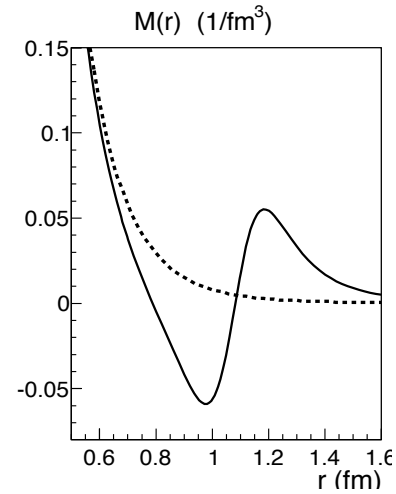
$$F_0(p) \equiv \int d^3\vec{r}\; exp(i\vec{p}\cdot\vec{r})\; M_0(r)$$

$$F(p) = F_0(p) + F_{osc}(p) \equiv \int d^3\vec{r}\; exp(i\vec{p}\cdot\vec{r}) M(r).$$

$$F_0 = \frac{\mathcal{A}}{(1+q^2/m_a^2)\,[1-q^2/0.71]^2},$$

$$F_{osc}(p) \equiv A\;\exp(-Bp)\;\cos(Cp + D).$$





- *Rescattering processes*
- *Large imaginary part*
- *Related to the time evolution of the charge density?*

  (E.A. Kuraev, E. T.-G., A. Dbeyssi, PLB712 (2012) 240)
- *Consequences for the SL region?*
- *Data expected at BESIII, PANDA*

Annihilation process

Fourier Transform: $$F_0(p) \equiv \int d^3\vec{r}\, \exp(i\,\vec{p}\cdot\vec{r}) M_0(r),$$

Plane wave IA: $\psi_f(\vec{r}) = \exp(i\vec{p}\cdot\vec{r})\ (PWIA).$

The matrix element:

$$F_0(p) = < \psi_f(x_1, ..., x_n)\psi_f(\vec{r})|T(r, x_1, .., x_n, x_{e^+e^-})|\psi_i(x_{e^+e^-}) >$$
$$\equiv \int d^3\vec{r}\ \psi_f(\vec{r})\ M_0(r),$$

Rescattering - Distorted wave IA:

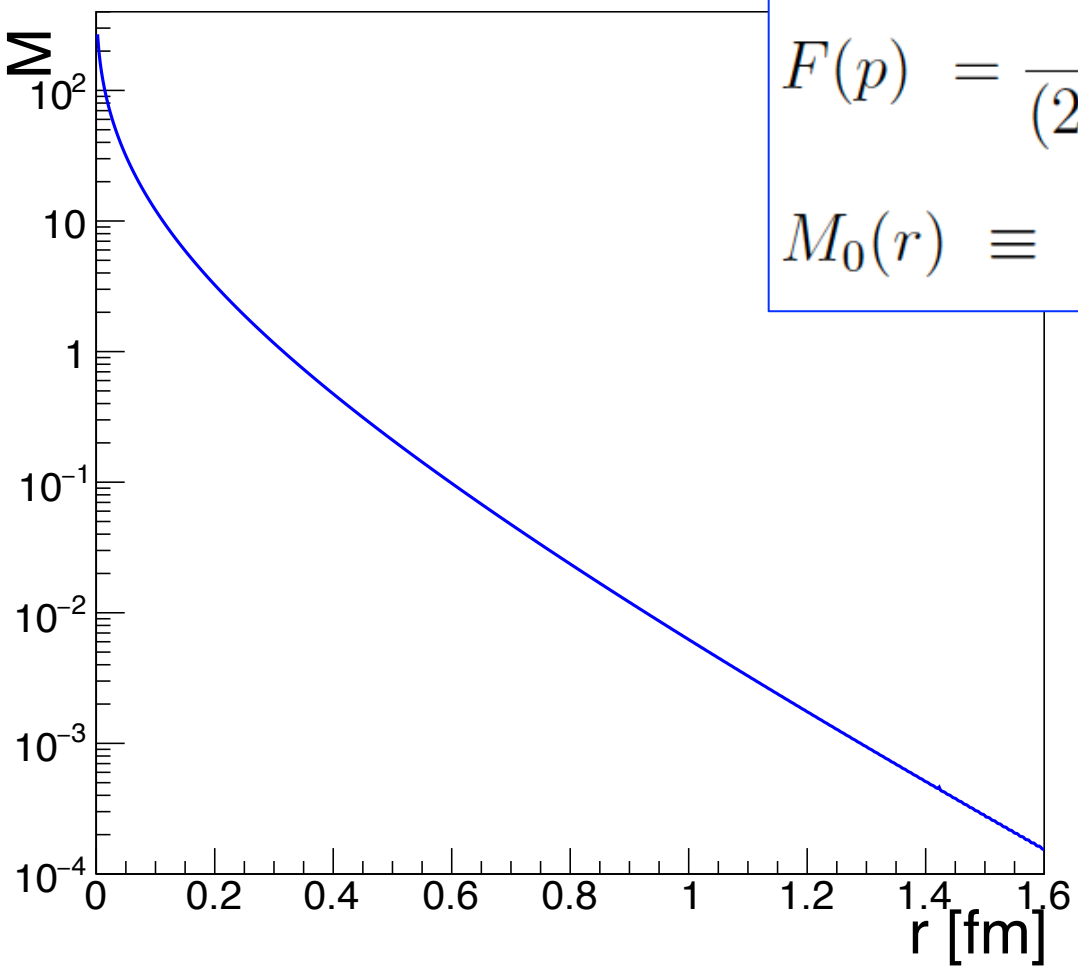
$$\psi_f(\vec{r}) = D(\vec{r})\exp(i\vec{p}\cdot\vec{r})\ (DWIA)$$

Glauber distortion factor:

$$D(x, y, z) = \exp\Big(- ib \int_z^\infty \rho(x, y, z')dz' \Big)$$

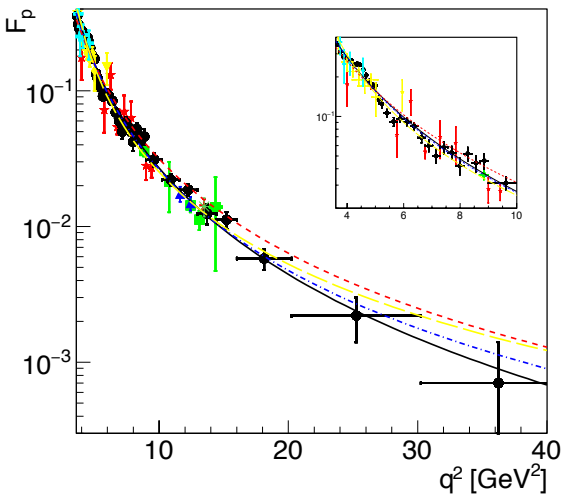
b : complex number ~ potential

Fourier Transform



$$F(p) = \frac{1}{(2\pi)^3} \int d^3\vec{r}\, e^{i\vec{p}\cdot\vec{r}} D(\vec{r}) M_0(r),$$

$$M_0(r) \equiv \int d^3\vec{p}\, e^{-i\vec{p}\cdot\vec{r}} F_0(p)$$



Potentials

Compact rescattering densities : Woods-Saxon, spherical, gaussian…:

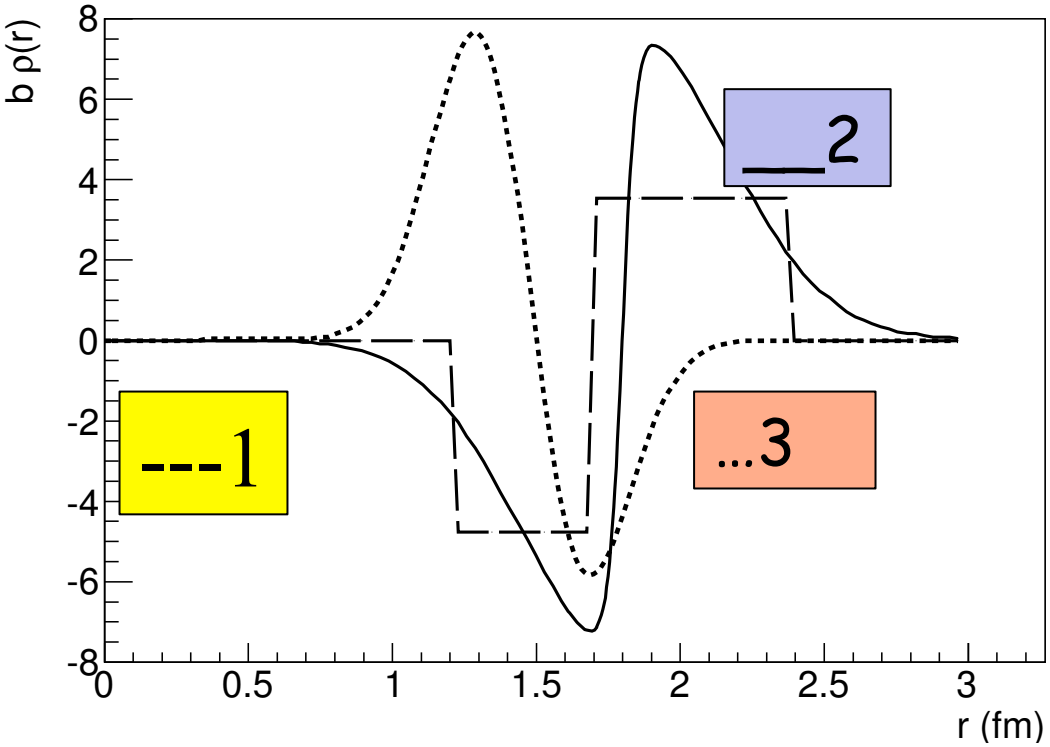
- Imaginary potentials are typical for low energy pbar-p(A) interactions
- no oscillations here -> t-channel momentum for (re)scattering versus relative momentum (s-channel)

Hollow rescattering densities: large for 0.2-2 fm, vanishing at small and large r, not changing sign.

- Real potential: peak >2 fm but M_0 <<3-4 orders of magnitude
- Imaginary potentials: need strong absorption which reduces M_0

Double layer potentials

Double layer rescattering densities : combination of two hollow potentials: one absorbing and one generating (imaginary potentials).



1) Multiple step function

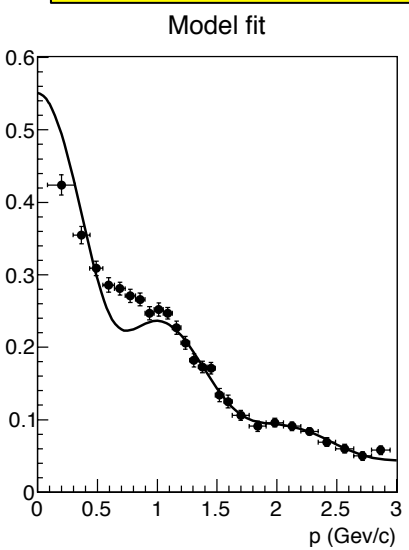
2) Soft multistep

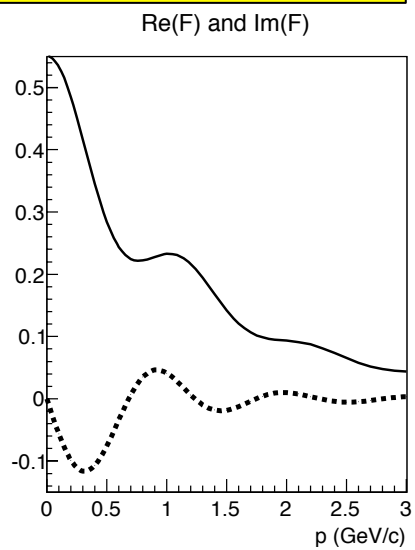
3) Two-gaussian opposite sign potential

A. Bianconi, E. T-G., PRC 93, 035201 (2016)

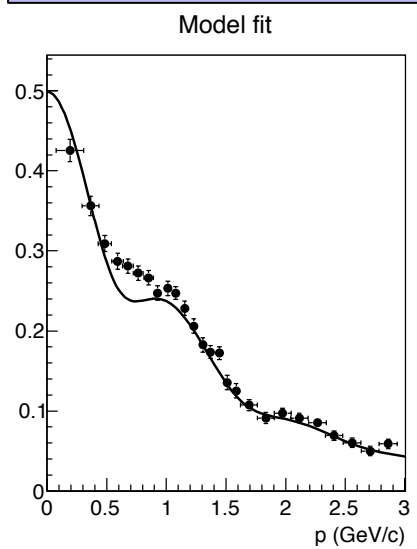
Optical model analysis

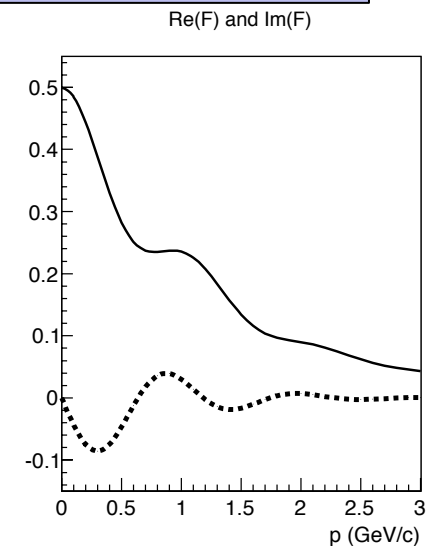
1) Multiple step function





2) Soft multistep





- At large r: purely absorptive
- At small r: the product D(r)M(r)"resonates" with the FT factor
- Importance of the steep behavior (oscillation period)
- Related to threshold enhancement

Optical model analysis

The excited vacuum created by e+e- annihilation decays in multi-quark states: pbar-p is one of them

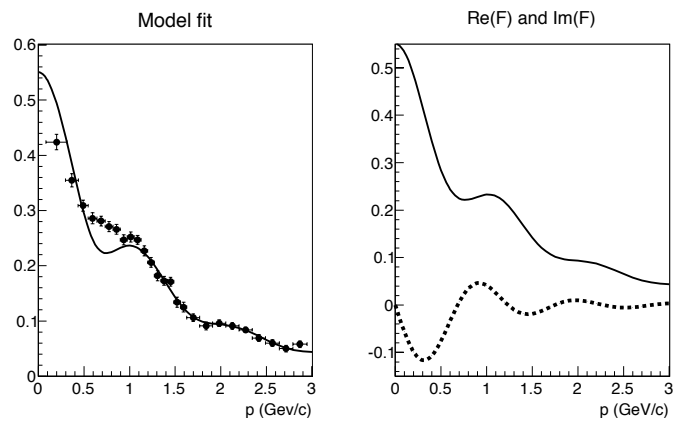
- feeding at small r by decay of higher mass states in pbar-p
- depletion at large r from pbar-p annihilation into mesons

From the pbar-p point of view, the coupling with the other channels transforms into an imaginary potential that

- destroys flux (absorption – negative potential)
- generates flux (creation – positive potential)

Optical model :

2 component imaginary potential:
absorbing outside,
regenerating inside,
with steep change of sign.



Regeneration-Absorbtion

Rescattering occurs in the spatial regions where:

- highly relativistic degrees of freedom (partons, internal properties of hadron)

and

- non- relativistic (relative motion of the two hadrons)

are both important

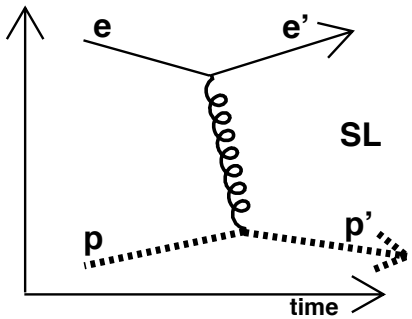
Plan

- Modelization
  - Generalization of FF
  - The charge pair creation
  - A new picture for the nucleon structure

TL-SL Generalization of Form Factors

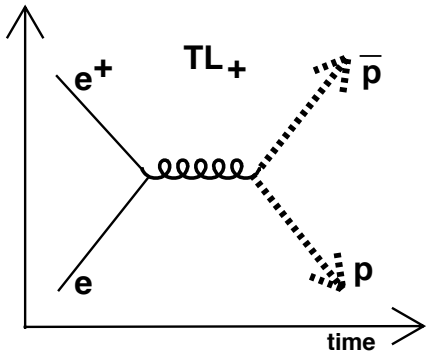
E.A. Kuraev, E. T-G, A. Dbeyssi, Phys.Lett. B712 (2012) 240
A. Bianconi, E. T-G., PRC 95, 015204 (2017)

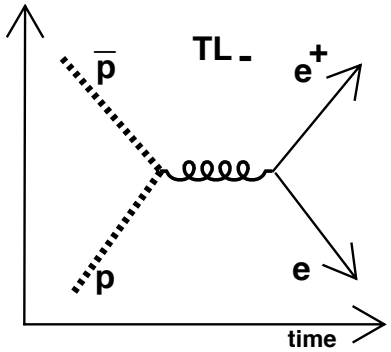
$$F(q^2) = \int_{\mathcal{D}} d^4x e^{iq_\mu x^\mu} F(x) \quad \boxed{q_\mu x^\mu = q_0 t - \vec{q}\cdot\vec{x}}$$



$F(x)=F(\vec{x},t)$ space-time distribution of the electric charge in the space-time volume $\mathcal{D}$.

$$\rho(x) = \rho(\vec{x}, t)$$





SL photon 'sees' a charge density

TL photon can NOT test a space distribution.

How to connect and understand the amplitudes?

Definition of TL-SL Form Factors

$$F(q) = \int d^4x e^{iqx} F(x).$$

$$F_{SL,Breit}(q) = \int d^3\vec{x}\, e^{-i\vec{q}\cdot\vec{x}} \int dt F(t,\vec{x}) \equiv \int d^3\vec{x}\, e^{-i\vec{q}\cdot\vec{x}} \rho(|\vec{x}|),$$

$$\rho(|\vec{x}|) = \int dt F(t,\vec{x}).$$

$$F_{TL,CM}(q) = \int dt\, e^{iqt} \int d^3\vec{x} F(t,\vec{x}) \equiv \int dt\, e^{iqt} R(t),$$

$$R(t) = \int d^3\vec{x} F(t,\vec{x}).$$

$\rho(\vec{x})$ and $R(t)$, represent projections of the same distribution in orthogonal subspaces

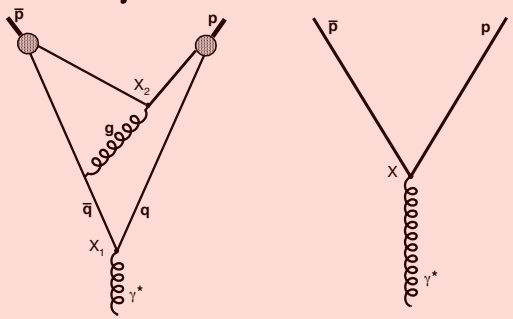
Photon-Charge coupling

$$\rho(x) = \rho(\vec{x}, t)$$

$\rho(\vec{x})$ Fourier transform of a stationary charge and current distribution

$R(t)$ Amplitude for creating charge-anticharge pairs at time t. Charge distribution => distribution in time of $\gamma^* \to charge - anticharge$ vertexes

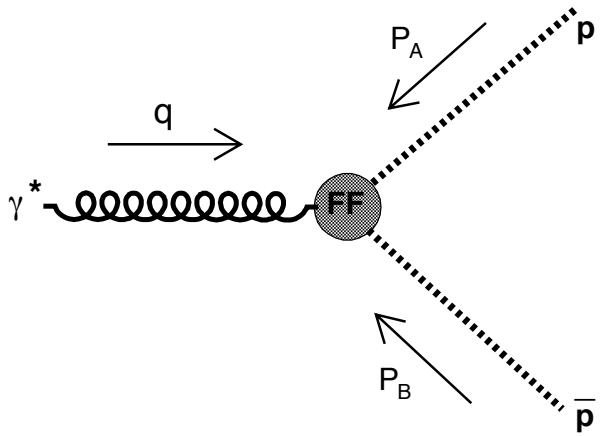
The photon



Resolved or *Unresolved*

- Both enter in the definition of a form factor
- Simplest configuration: qqbar+compact diquark

Photon-Charge coupling



$$
\begin{aligned}
SL &: \gamma^*(q_\mu) + p(p_\mu) \to p(p'_\mu) \\
TL_+ &: \gamma^*(q_\mu) \to p(p'_\mu) + \bar{p}(\bar{p}_\mu{}') \\
TL_- &: p(p_\mu) + \bar{p}(\bar{p}_\mu) \to \gamma^*(q'_\mu)
\end{aligned}
$$

q, P_A, P_B : formal arguments: scattering, hadron annihilation and production, by changing signs of the coordinates

$$
\begin{aligned}
\gamma^* + p &\to p' \quad (SL : |q_0| < |\vec{q}|), \quad P_A = p, P_B = -p', \\
\gamma^* &\to \bar{p} + p \quad (TL_+ : |q_0| > |\vec{q}|, q_0 > 0), \quad P_A = -p', P_B = -\bar{p}', \\
\bar{p} + p &\to \gamma^* \quad (TL_- : |q_0| > |\vec{q}|, q_0 < 0), \quad P_A = p, P_B = \bar{p}, q = -q',
\end{aligned}
$$

FFs as q-dependent quantities proportional to the amplitude

Examples

Homogeneous distribution for positive times:
Assume equal probability in time to form a complet p-pbar system, inside the future light cone of the first event.
(space probability: we integrate and set to one)

$$R(t) = \theta(-t),$$

$$F(q) = \int e^{iqt}\theta(-t) = \frac{\pi}{\epsilon - iq},$$

Exponential damping (a is finite): either the spectator pair and the ppbar system are created soon or the system evolves differently (jet fragmentation…)

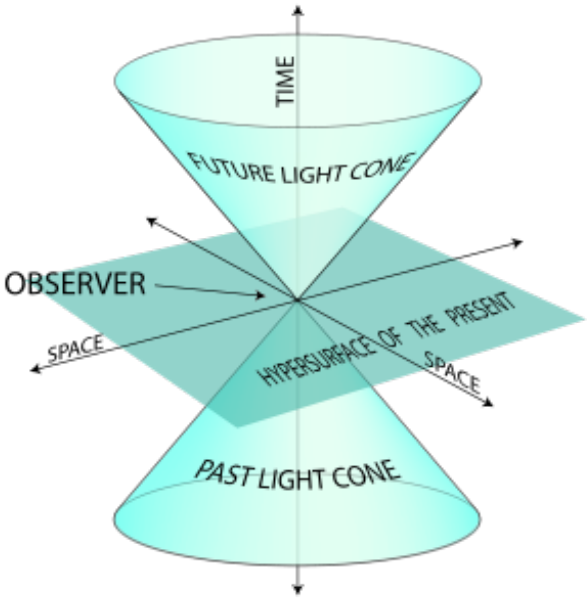
$$R(t) = \theta(-t)e^{-a|t|},$$

$$F(q) = \frac{\pi}{a - iq} = \frac{a\pi}{a^2 + q^2} + i\frac{q\pi}{a^2 + q^2},$$

Examples: Monopole-like shape

F(x) ≠ 0 in past and future LC.

- Annihilation and creation processes:
  - -time symmetric.
  - -differ by a phase.
- Summing two terms with the same phase: when t>> 1/a



$$R(t) = \theta(t)e^{-at} + \theta(-t)e^{at} = e^{-a|t|},$$

$$F(q)/\pi = \frac{1}{a-iq} + \frac{1}{a+iq} = \frac{2a}{a^2+q^2}.$$

1/a: formation time

=> zero mass resonance of width a

Examples: Lorentzian resonance

Replacing q->q-M : one obtains poles

$$F(q)/\pi = i\left(\frac{1}{q-M+ia} - \frac{1}{q-M-ia}\right) \propto \frac{1}{(q-M)^2+a^2}.$$

By Fourier transform:

$$R(t) \propto e^{iMt}\, e^{-a|t|}.$$

Response of a classical damped oscillator to an instantaneous external force $\delta(t)$

Negative energy states are allowed by particle-antiparticle symmetry.
To each pole q_0 =M+ia corresponds a pole q_0 =-(M +ia)
Positive poles => creation process
Negative poles=> annihilation process

Examples: Breit-Wigner

A Breit-Wigner probability contains all four poles:

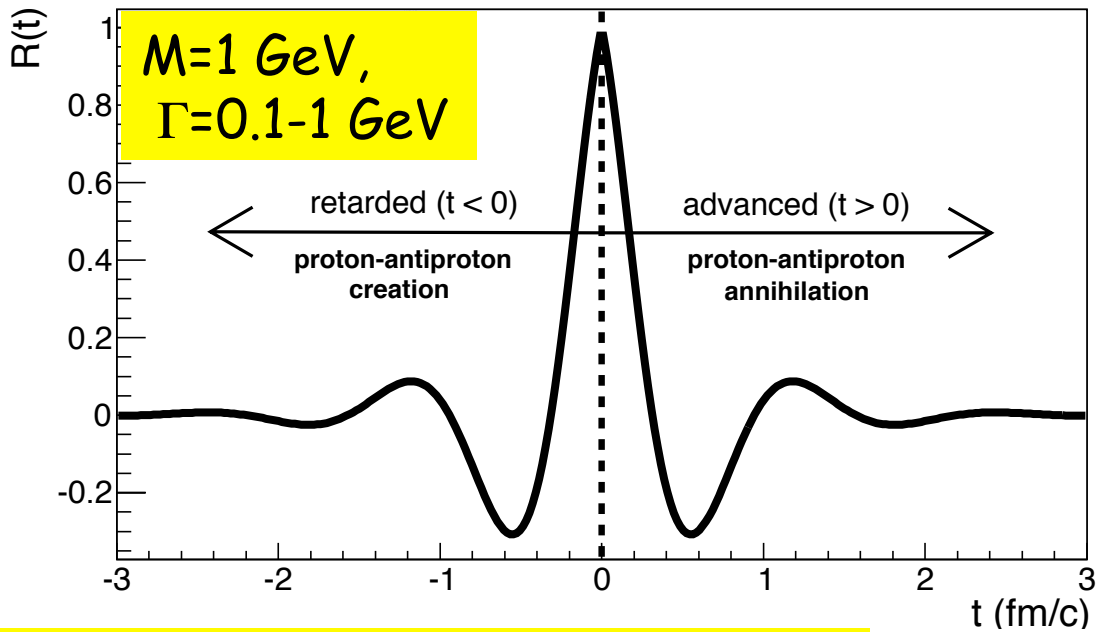
$$F_{\pm}(q) \propto \frac{1}{(q^2 - M^2) \pm iMa},$$

The combination:

$$F(q) \propto F_{+}(q) + F_{-}(q)$$

corresponds to

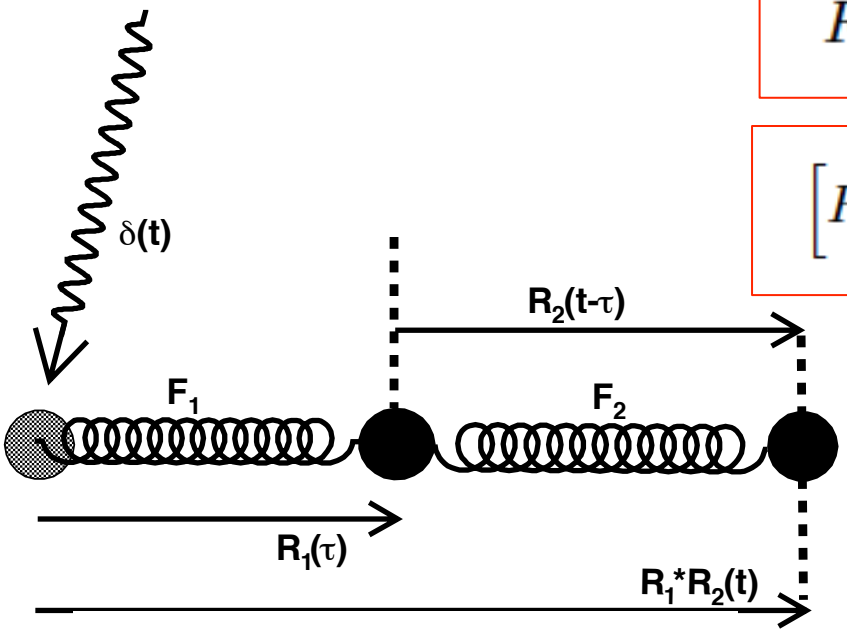
$$R(t) \propto cos(Mt)\, e^{-a|t|},$$



Retarded response of a classical bound and damped oscillator to a $\delta(t)$ external perturbation

$$R(t) \equiv R_{creation}(t)\theta(-t) + R_{ann}(t)\theta(t).$$

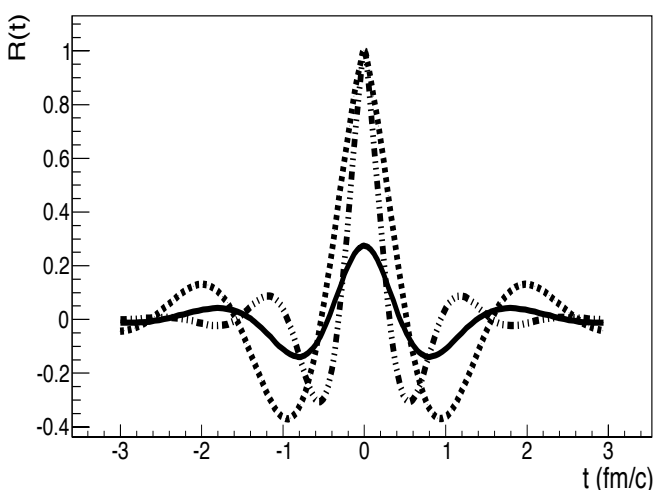
Several spectators: dipole and asymptotics



$$F_1(q)F_2(q) \;=\; F.T.\Big[R_1(t) * R_2(t)\Big],$$

$$\Big[R_1(t) * R_2(t)\Big] \;\equiv\; \int d\tau R_1(\tau)R_2(t-\tau).$$

Use FT properties of convolutions
Chain of two oscillators,
one directly connected to the photon
The second is a decaying correlation
between active quark and spectator



$$R(t) \;=\; \int d\tau\; e^{-a|t-\tau|}\; e^{-b|\tau|}.$$

$$F(q) \;\propto\; \frac{1}{(a^2+q^2)(b^2+q^2)},$$

More complicated examples

$$e^+e^- \rightarrow \bar{p}n\pi^+ \rightarrow \bar{p}p$$

Three quark-antiquarks pair in the intermediate state.

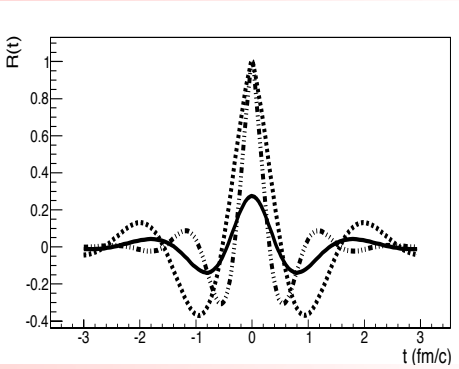
$$R(t) = \Big[[R_1(t) * R_2(t)] * R_3(t)\Big],$$

$$F(q) \propto \frac{1}{(q^2 \pm a^2)(q^2 \pm b^2)(q^2 \pm c^2)},$$

Sum of two contributions of equal shape:

$$R(t) = R_0(t) + aR_0(t-b), \; a \ll 1,$$
$$F(q) = F_0(q)[1 + ae^{ibq}],$$

periodic modulation



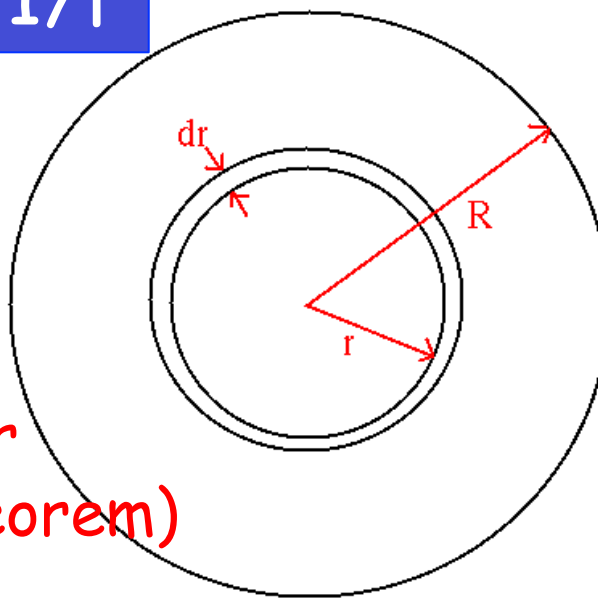
The nucleon: homogenous, symmetric sphere?

Analogy with Gravitation

Coulomb Potential ~Gravitational Potential

Mass~charge

1/r



- Spherical symmetric distributed mass density
- A point at a distance r<R from the center feels only the matter inside (Newton theorem)

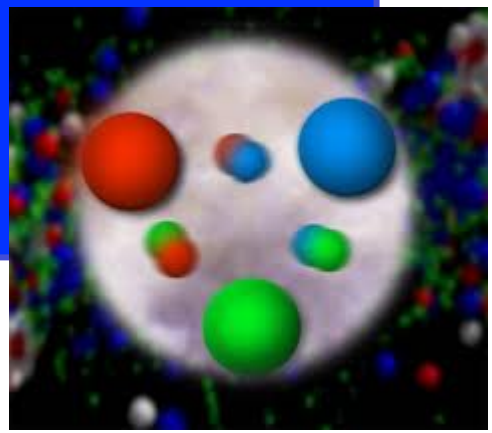
works for the SCALAR part, NOT for the VECTOR part of $A=(\Phi, \vec{A})$

$$R=\mu G_E/G_M \simeq \left(\frac{1}{rQ}\right)^3, \quad Q > \frac{1}{r}$$

r~0.7 fm → Q= 0.29 GeV

$R \sim 1/Q^3$ *is NOT the observed experimental behavior !*

The nucleon



3 valence quarks and
a neutral sea of $\overline{q}q$ pairs

antisymmetric state of colored quarks

$$|p> \sim \epsilon_{ijk}|u^i u^j d^k>$$
$$|n> \sim \epsilon_{ijk}|u^i d^j d^k>$$

Main assumption

Does not hold in the spatial center of the nucleon: the center of the nucleon *is electrically neutral,* due to strong gluonic field

E.A. Kuraev, E. T-G, A. Dbeyssi, Phys.Lett. B712 (2012) 240

The nucleon

Inner region: gluonic condensate of clusters with randomly oriented chromo-magnetic field (Vainshtein, 1982):

Intensity of the gluon field in vacuum:

$$< 0|\alpha_s/\pi(G^a_{\mu\nu})^2|0 >\sim E^2 - B^2 \sim E^2 = 0.012\ \mathrm{GeV}^{\ 4}.$$

$$G^2 \simeq 0.012\,\pi/\alpha_s GeV^4,\ \text{i.e.,}\ E \simeq\ 0.245\,GeV^2.\ \ \alpha_s/\pi\ \sim\ 0.1$$

In the internal region of strong chromo-magnetic field, the color quantum number of quarks does not play any role, due to stochastic averaging

$< G|u^i u^j|G >\sim \delta_{ij}$ proton

$d^i d^j$ neutron

Colorless quarks: Pauli principle

Model: SL and TL regions

Antisymmetric state of colored quarks

Colorless quarks: Pauli principle acts

1) uu (dd) quarks are repulsed from the inner region
2) The 3rd quark is attracted by one of the identical quarks, forming a compact di-quark
3) The color state is restored

Formation of di-quark: competition between attraction force and stochastic force of the gluon field

$$\frac{Q_q^2 e^2}{r_0^2} > e|Q_q|\,E.$$

proton: (u) Qq=-1/3

neutron: (d) Qq=2/3

attraction force > stochastic force of the gluon field

Model

Additional suppression for the scalar part due to colorless internal region: "charge screening in a plasma": the scalar part of the EM field obeys to

$$\Delta\phi = -4\pi e \sum Z_i n_i, \; n_i = n_{i0} exp\left[-\frac{Z_i e\phi}{kT}\right]$$

k:Boltzmann constant
T: temperature of the hot plasma

Neutrality condition:

$$\sum Z_i n_{i0} = 0$$

$$\Delta\phi - \chi^2\phi = 0, \; \phi = \frac{e^{-\chi r}}{r}, \; \chi^2 = \frac{4\pi e^2 Z_i^2 n_{i0}}{kT}$$

Additional suppression (Fourier transform)

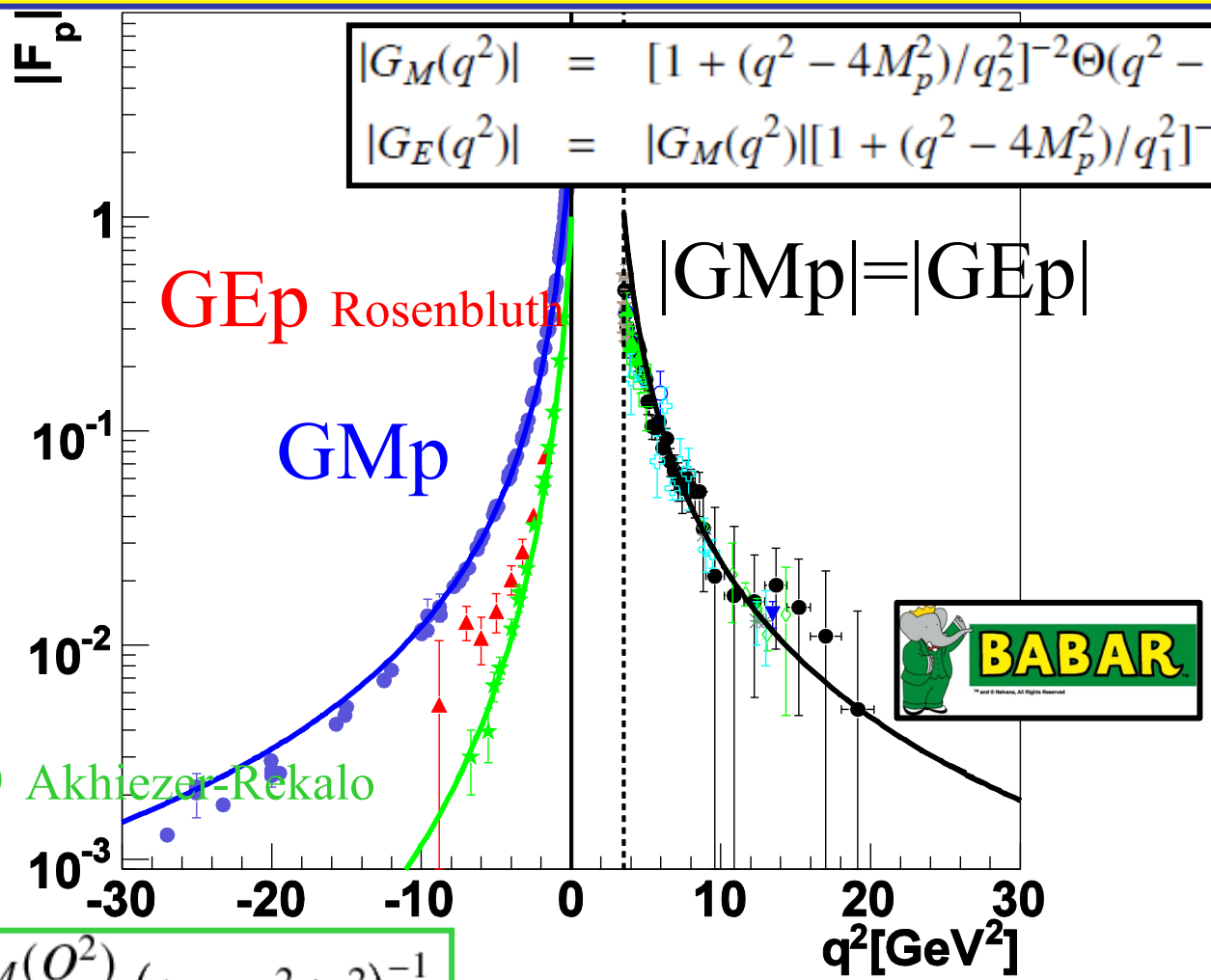
$$G_E(Q^2) = \frac{G_M(Q^2)}{\mu}\left(1 + Q^2/q_1^2\right)^{-1}$$

$q_1(\equiv \chi)$

fitting parameter

Proton Form Factors

$$|G_M(q^2)| = [1 + (q^2 - 4M_p^2)/q_2^2]^{-2}\Theta(q^2 - 4M_p^2),$$
$$|G_E(q^2)| = |G_M(q^2)|[1 + (q^2 - 4M_p^2)/q_1^2]^{-1}\Theta(q^2 - 4M_p^2),$$



$$G_E(Q^2) = \frac{G_M(Q^2)}{\mu}\left(1 + Q^2/q_1^2\right)^{-1}$$

Conclusions

- Theory: unified models in SL and TL regions:
  - *describe* proton and neutron, electric and magnetic FFs in SL and TL regions
  - *non-trivial* charge distribution at the hadron formation
  - pointlike behavior at threshold?

- New understanding of Form Factors in the Time-like region: time distribution of quark-antiquark pair creation vertices

- Experiment: measure
  - zero crossing of GE/GM in SL? 2γ? Proton radius?
  - GE and GM separately in TL
  - complex FFs in TL region: polarization!
  - new structures in TL