Elastic electromagnetic form factors in the space-like region: Recent results and future experiments

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Scientific Motivation

- To understand the ground state structure of the proton and neutron as an extended system of interacting quarks and gluons
  - A challenging test for nucleon models / QCD ... important experimental constraints
  - Required for extracting information on strange quark distributions in the nucleon
- To understand nuclei
  - Nucleon form factors are a basic and essential ingredient in models of nuclei
- V. Punjabi, C.F. Perdrisat, M.K. Jones, EJB, C.E. Carlson, Eur. Phys. J. A51 (2015) 79.

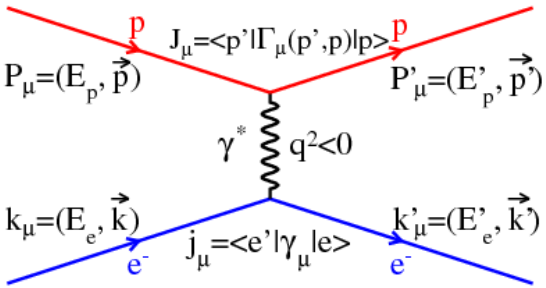
Dirac Magnetic Moments

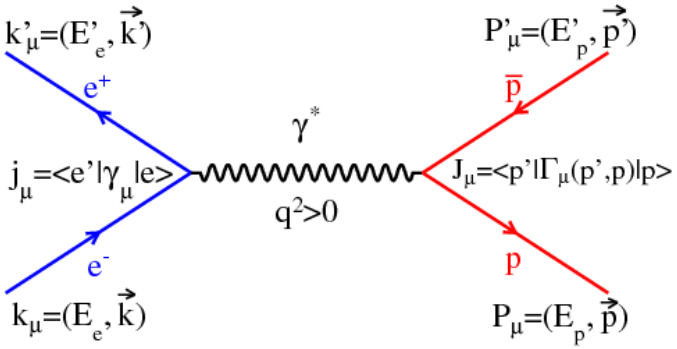
Protons and neutrons are not pointlike particles and their magnetic moments are anomalous, in the sense that they differ from the predictions of pointlike Dirac particles:

	μ (Dirac)	μ (Observed)
Proton	$\frac{q}{mc}\lvert\vec{S}\rvert = \mu_N$	$+2.79\mu_N$ (Stern, 1932)
Neutron	0	-1.91μ_N (Alvarez/Bloch, 1940)

$\longrightarrow$ Strong indication of substructure of these particles.
$\longrightarrow$ If the underlying SU(6) symmetry (expected for a nucleon composed of three valence quarks) were perfect, one would expect $\mu_p = 3\mu_N$ and $\mu_n = -2\mu_N$ ... more on this later!

Elastic Form Factors (Single Photon Exchange)





$$J_\mu = \bar{U}(p')\left\{\gamma_\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M_p}F_2(Q^2)\right\}U(p)$$

$F_1(Q^2)$: non spin-flip Dirac form factor
$F_2(Q^2)$: spin-flip Pauli form factor

$$F_1^p(0) = 1, \quad F_1^n(0) = 0$$
$$F_2^p(0) = \kappa_p, \quad F_2^n(0) = \kappa_n$$

Elastic Form Factors

- Details of nucleon substructure are in the Q^2 $(= -q^2)$ evolution of $F_1(Q^2)$ and $F_2(Q^2)$.
- A testing ground for quark/gluon theories
- Provides insight into the spatial distribution of charge and magnetization ... CAVEAT: If $Q >> m_N$, dynamical effects due to relativistic boosts are introduced, making physical interpretation more difficult.
- Wavelength of the probe can be tuned by selecting the momentum transfer, Q^2:

$< 0.1 GeV^2$	integral quantities (charge radius, ...)
$0.1 - 20 GeV^2$	internal nucleon structure
$> 20 GeV^2$	pQCD scaling?

Sachs Form Factors

In the Breit frame (infinite momentum frame), we can relate F_1 and F_2 to the charge and spatial current densities as:

$$\rho = J_0 = 2eM\left[F_1 - \tau F_2\right]$$
$$J_i = e\bar{u}(p)\gamma_i u(p)\left[F_1 + F_2\right]_{(i=1,2,3)}$$

This leads us to define the following linear combinations:

$$\text{Electric form factor: } G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2)$$
$$\text{Magnetic form factor: } G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

$$\text{where } \tau = \frac{Q^2}{4M_p^2}$$

• G_E and G_M are images of charge and current distributions inside the nucleon.

Proton Electric Form Factor

- In the IMF, G_{E_p} is the Fourier transform of the charge distribution
- Related to the charge extension: non relativistically

$$G_{E_p}(Q^2) = 1 - \frac{1}{6} r_{E_p}^2 Q^2 + ...$$

defines the proton charge radius.

- What might one expect for the charge distribution of the proton?

charge	G_{E_p}
pointlike δ	1
gaussian $e^{\left(-\frac{r^2}{a^2}\right)}$	$e^{\left(-\frac{Q^2}{a^2}\right)}$
exponential e^{-mr}	$\frac{1}{\left(1+\frac{Q^2}{m^2}\right)^2}$ (dipole)

Proton FF Data from $^1H(e,e'p)$ Cross Sections

- G_{M_p} well measured with Rosenbluth separation, but not G_{E_p}

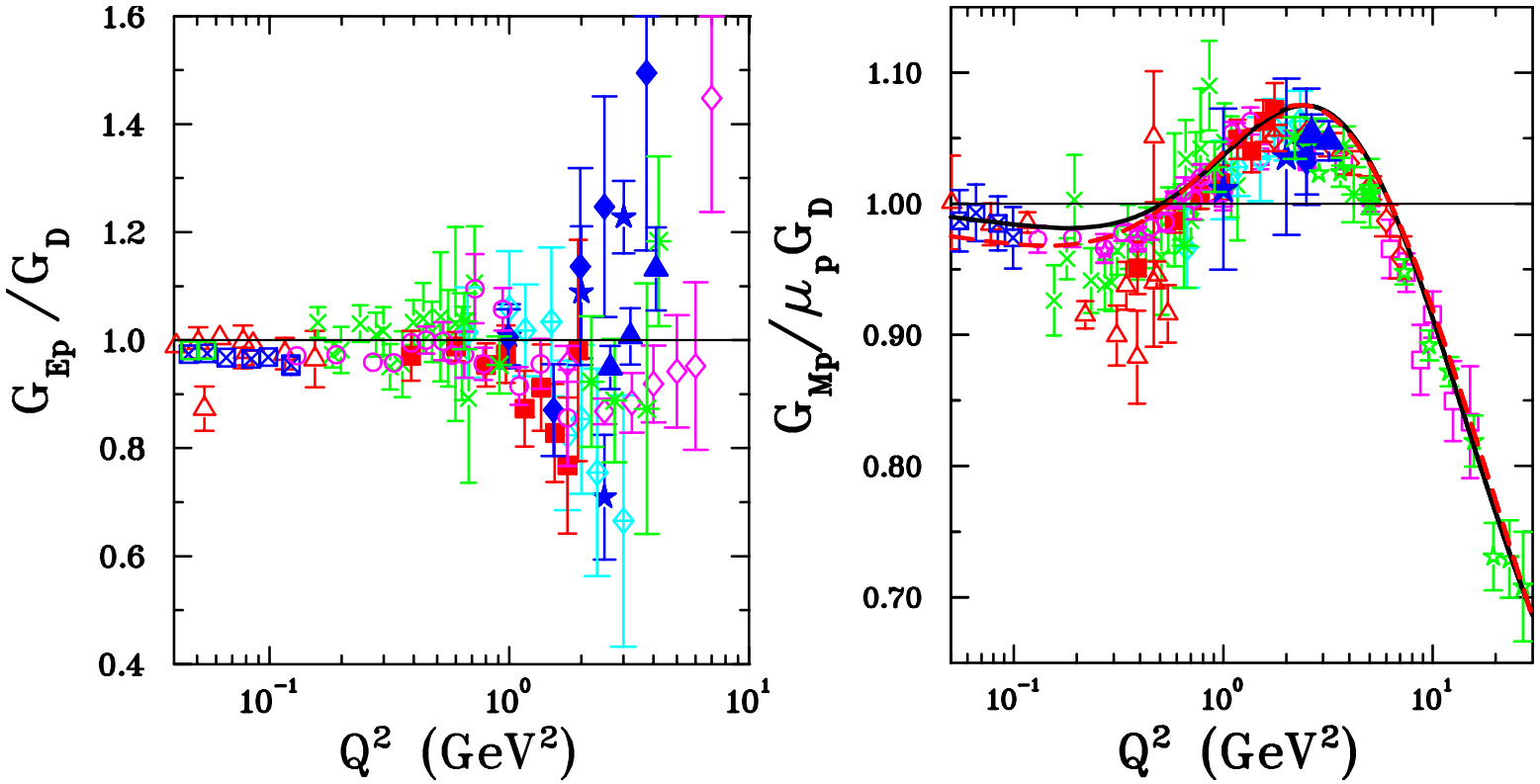
$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega_{Mott}}\left[G_E^2(Q^2) + \frac{\tau}{\epsilon}G_M^2(Q^2)\right]\frac{1}{1+\tau}$$

$$\text{with } \frac{1}{\epsilon} = 1 + 2(1+\tau)\tan^2\left(\frac{\theta_e}{2}\right)$$

- At low Q^2, found to follow the dipole approximation

$$G_{E_p} \sim \frac{G_{M_p}}{\mu_p} \sim G_d = \frac{1}{\left(1+\frac{Q^2}{0.71}\right)^2}$$

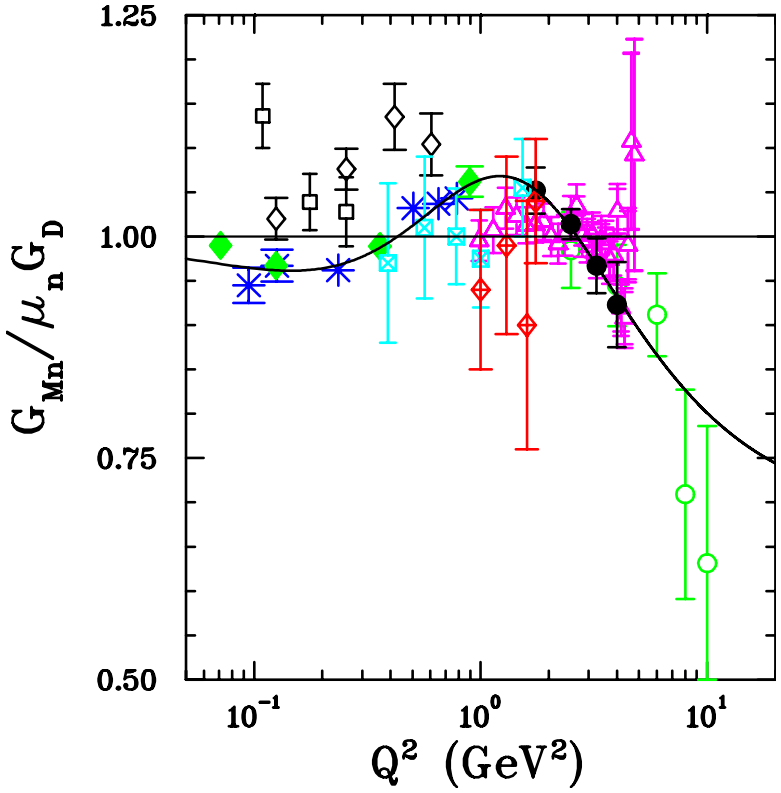
- Difficulties: radiative corrections, dominance of G_M at larger Q^2, non-single-photon-exchange and inelastic contamination, etc.



- At $Q^2 = 3$ GeV^2, electric part of cross section is 5%
- At $Q^2 = 5$ GeV^2, electric part of cross section is 1%

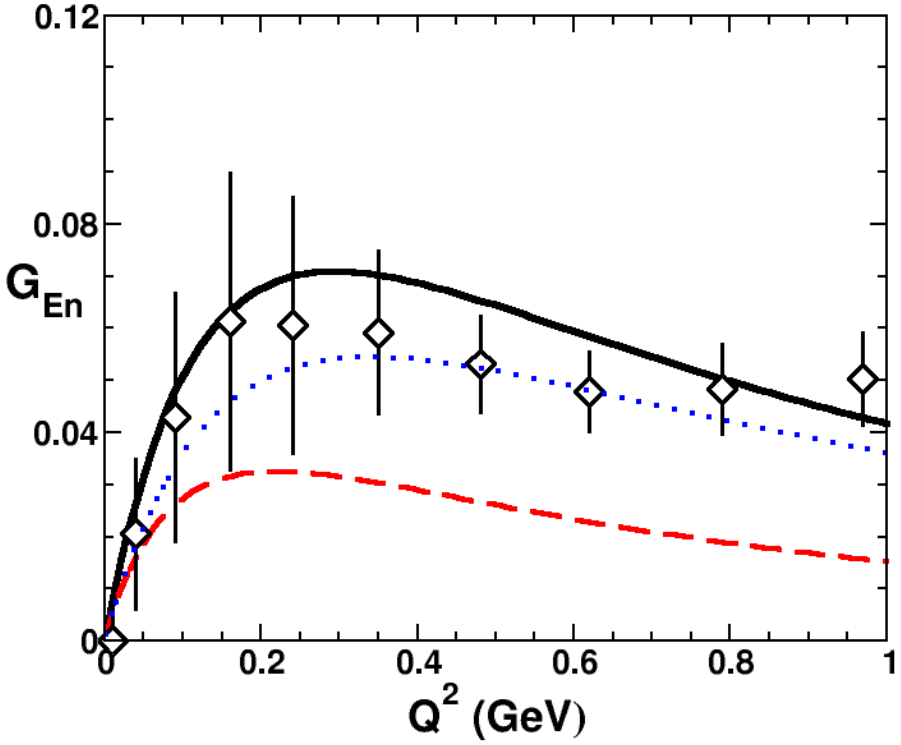
G_{Mn} Data from Cross Section Measurements

- No free neutron targets! Use deuterium target
- Combination of $d(e, e')$, $d(e, e'p)$, and $d(e, e'n)$ measurements
- Results indicate similar dipole scaling as in G_{Mp}



G_{En} Data from Cross Section Measurements

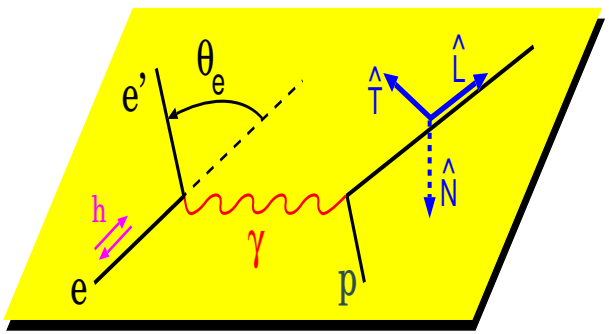
- Use *ed* elastic scattering reaction
- Much more complicated! Depends on three form factors ...
- With knowledge of the deuteron wavefunction, one can in principle extract G_{En}
- Q^2 range is quite limited due to theoretical uncertainties



Double Polarization Measurements

- Elastic $\vec{e}N \to e\vec{N}$

(A. I. Akhiezer and M. P. Rekalo, Sov. J. Part. Nuc. **3**, (1974) 277; and Arnold, Carlson and Gross, Phys. Rev. **C23** (1981) 363):



$$P_n = 0$$
$$I_0 P_t = -2\sqrt{\tau(1+\tau)}\, G_{Ep} G_{Mp} \tan\frac{\theta_e}{2}$$
$$I_0 P_\ell = \frac{1}{M_p}(E_e + E_{e'})\sqrt{\tau(1+\tau)}\, G_{Mp}^2 \tan^2\frac{\theta_e}{2}$$

$$I_0 = G_{Ep}^2 + \tau G_{Mp}^2 \left[1 + 2(1+\tau)\tan^2\frac{\theta_e}{2}\right]$$

- Similar ideas for polarized target measurements apply ...

Double Polarization Measurements

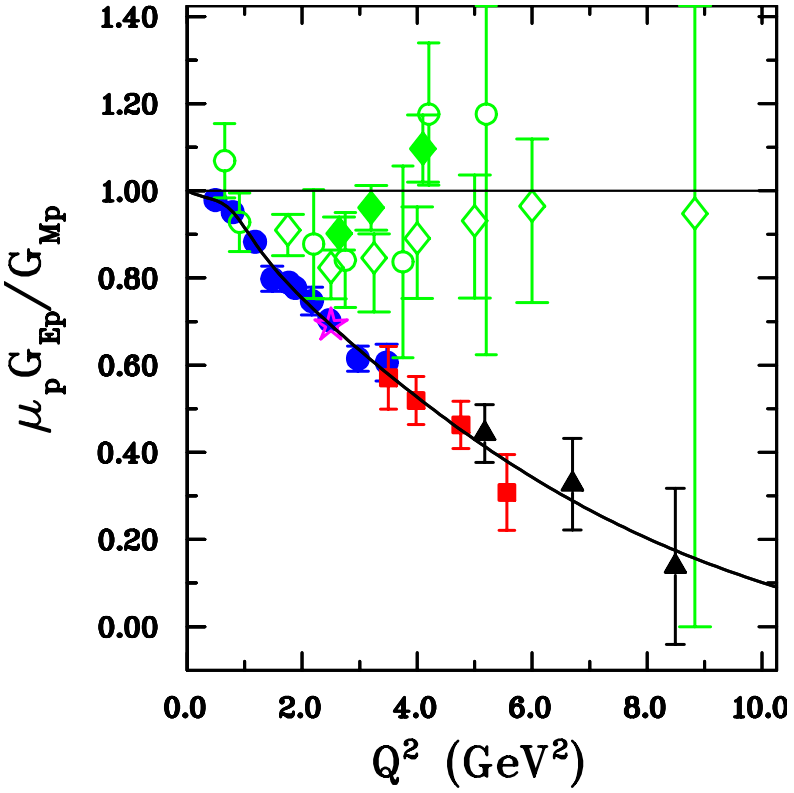
- Relative Measurement:

$$\frac{G_{Ep}}{G_{Mp}} = -\frac{P_t}{P_\ell}\frac{(E_e + E_{e'})}{2M_p}\tan\left(\frac{\theta_e}{2}\right)$$

- Small systematic uncertainties ... most troublesome factors cancel in ratio!
- Principle difficulties are understanding spin precession in the proton spectrometer, and eliminating inelastic background events ... important but very well understood.

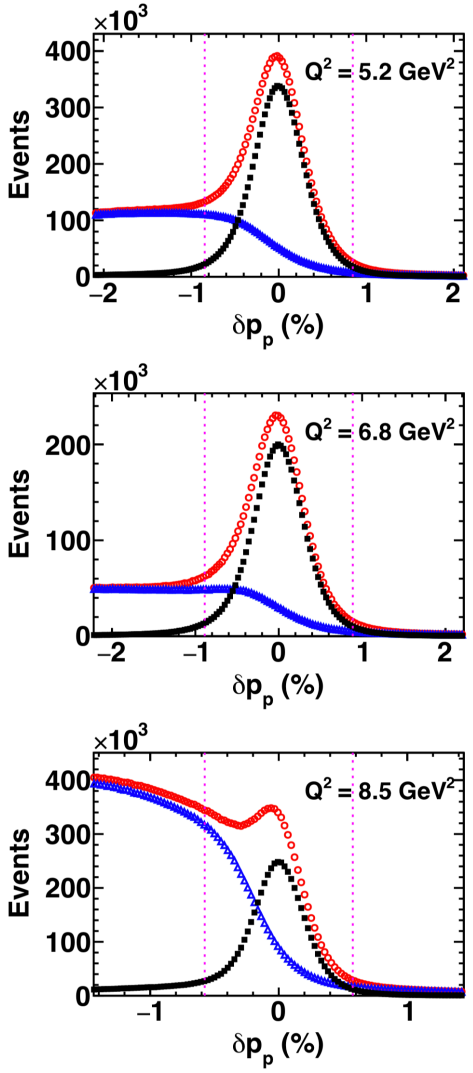
$\mu_p G_{Ep}/G_{Mp}$ from Double Polarization Measurements

- Three separate JLab experiments, each with different detectors and systematics
- Limited by statistics in all cases - systematic uncertainties smaller than stat. errors
- Results are clear - G_{Ep} deviates significantly from dipole-like behaviour for $Q^2 > 1 GeV^2$



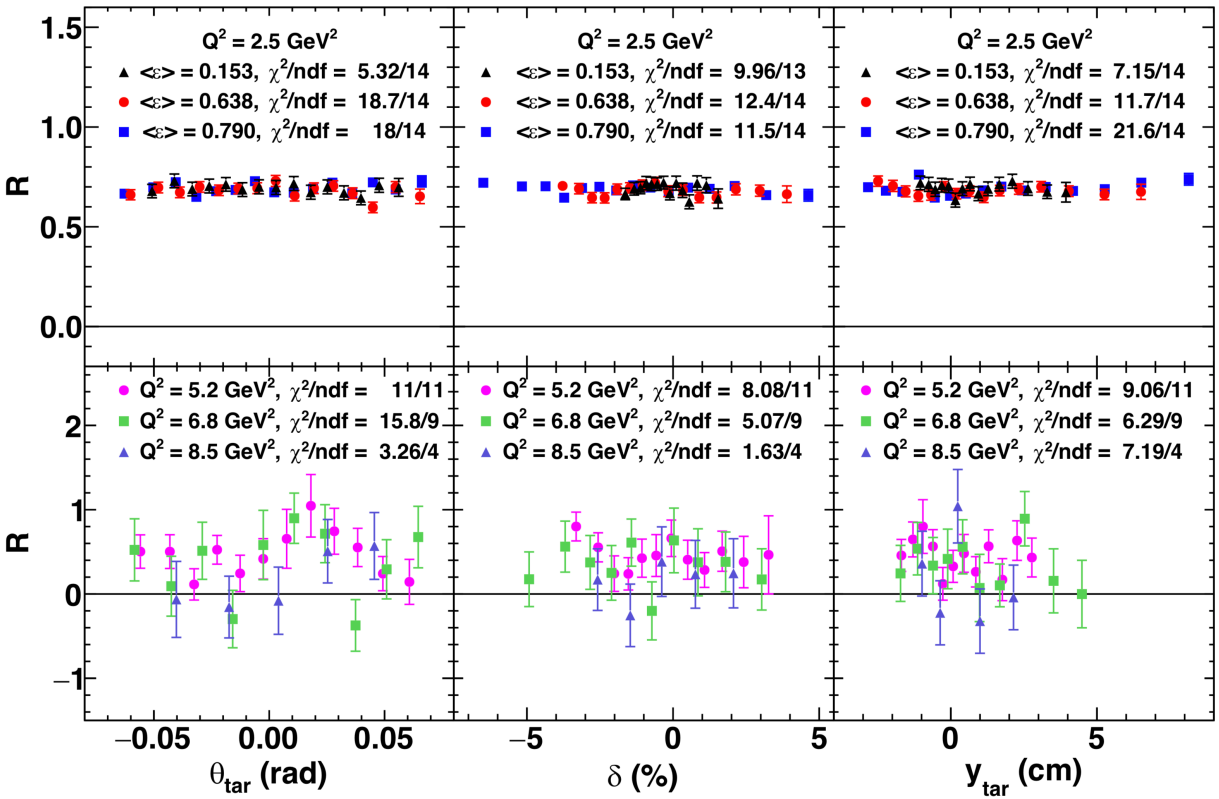
Elastic Event Selection

- In most recent experiment in Hall C at JLab, proton detected in magnetic spectrometer and electron detected in segmented Pb-glass calorimeter.
- Good angular and momentum resolution for proton, good angular resolution for electron.
- Use angular correlation of electron and proton in two-body scattering, together with momentum information to select elastic events.



Understanding Spin Precession and Other Systematics

- Polarization components measured at the focal plane of the proton spectrometer
- Spin precession depends on proton momentum and path through the spectrometer, and varies event to event
- Maximum likliehood procedure, complex but well-understood



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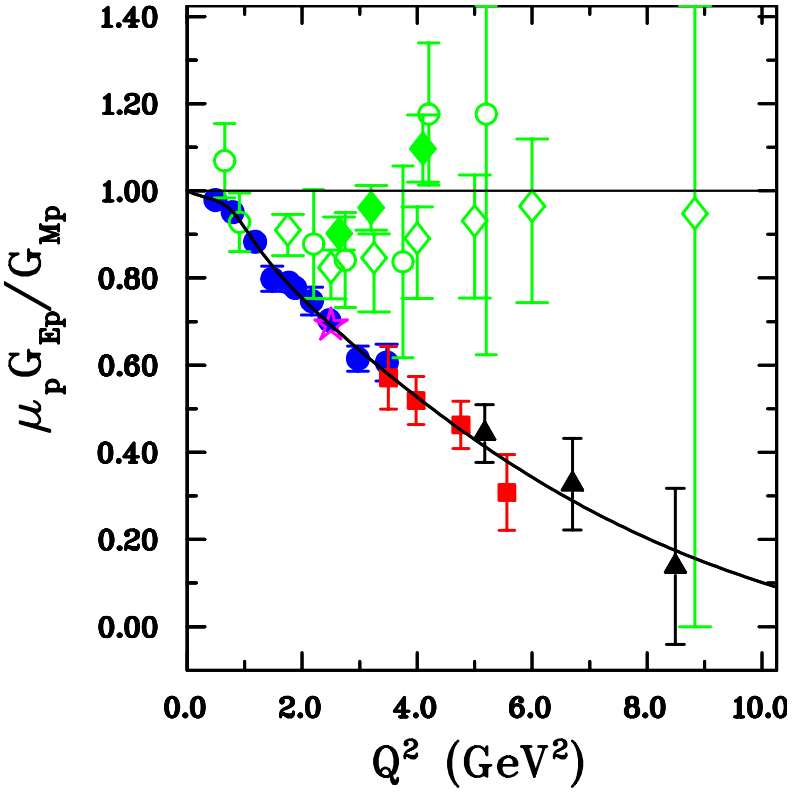
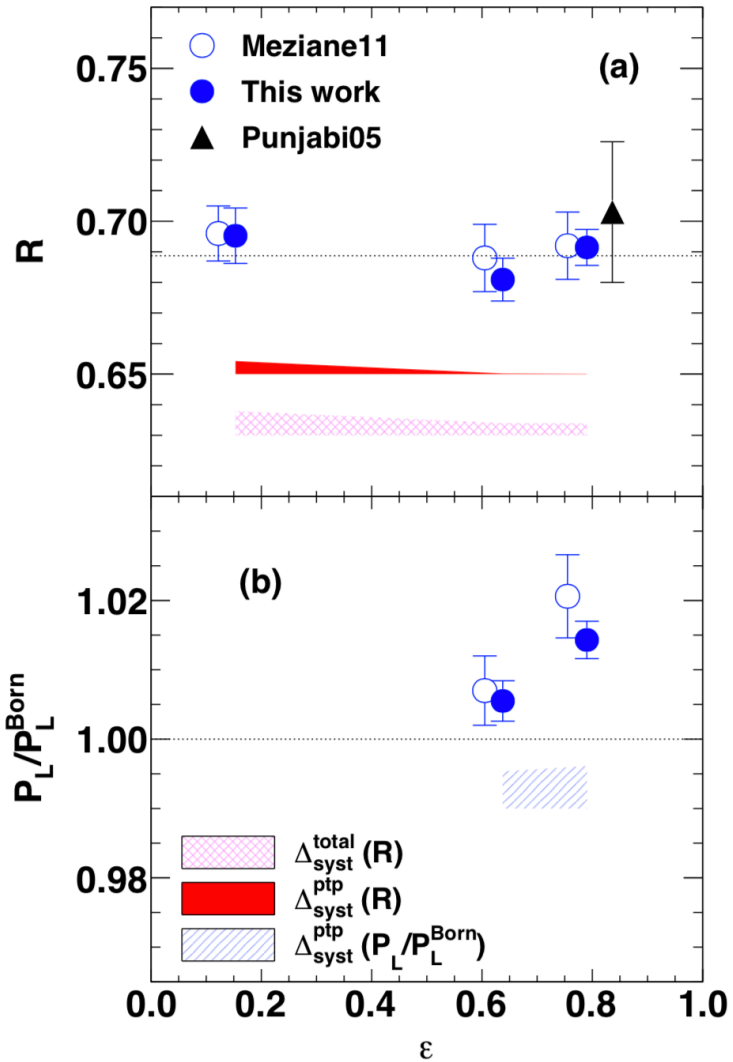


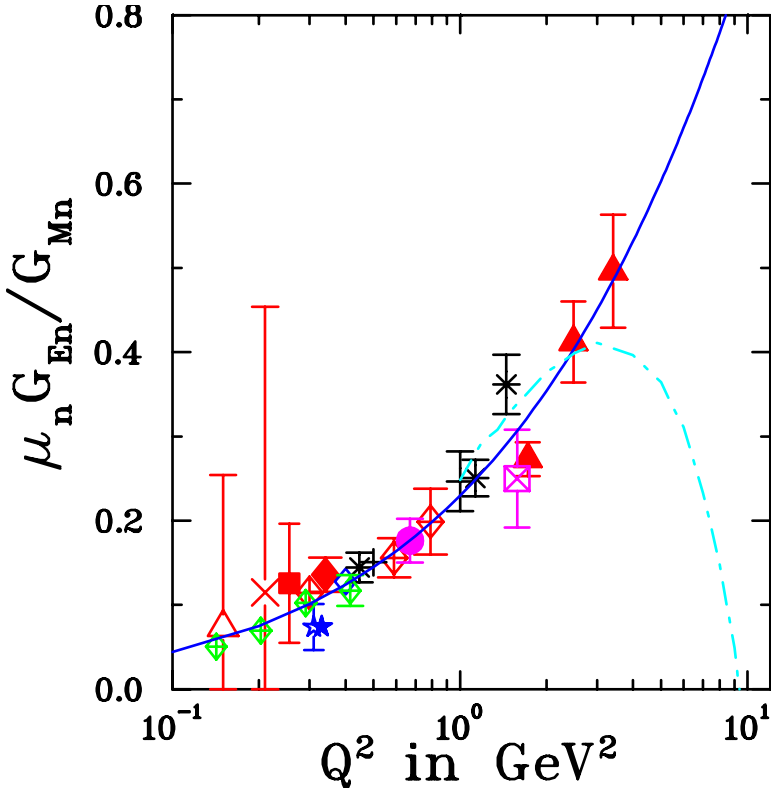
Two-photon Exchange Effects

- In one-photon-exchange, $\mu_p G_{Ep}/G_{Mp}$ should be independent of ϵ
- $G_{Ep} - 2\gamma$ - high statistics measurement at Q^2=2.5 GeV^2 for three ϵ values
- It was this analysis that required significant improvements in our understanding of the proton spectrometer optics, spin precession, and the maximum-likliehood approach.



$\mu_n G_{En}/G_{Mn}$ from Double Polarization Measurements

- Large number of experiments at different laboratories, with different detectors/spectrometers, and approaches:
- Recoil polarization with deuterium target
- Asymmetry with polarized deuterium
- Asymmetry with polarized ^3He.



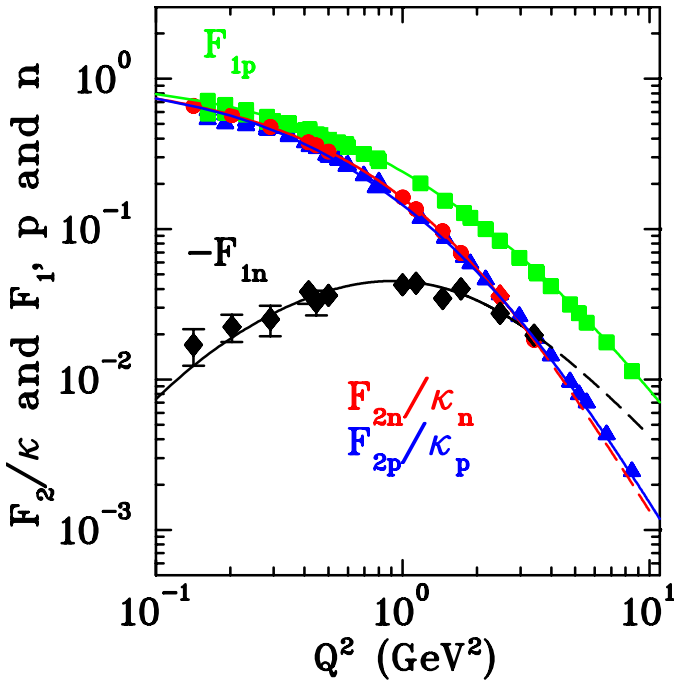
Flavor Separation of Nucleon Form Factors

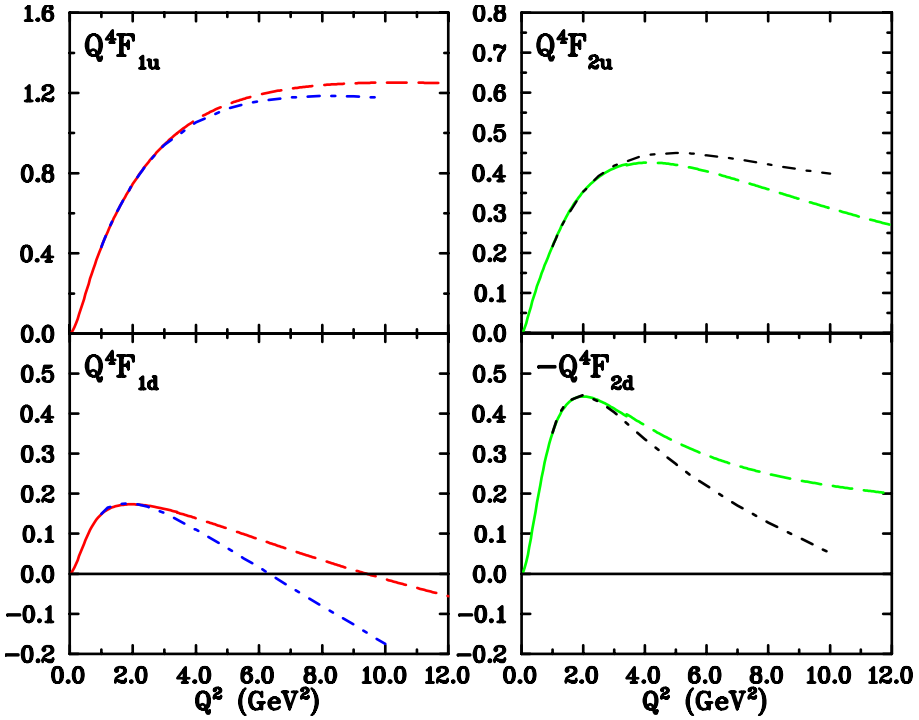
- The availability of high quality data for the proton form factors up to $Q^2 \sim 8.5\ GeV^2$ and for the neutron form factors up to $Q^2 \sim 3.4\ GeV^2$ allows for the extraction of the more fundamental F_1 and F_2 form factors.
- Charge symmetry implies that the proton and neutron wavefunctions should be identical under the exchange of up and down quark contributions (strange quark FF's are small).

$$G_{(E,M)p} = \frac{2}{3}G_{(E,M)u} - \frac{1}{3}G_{(E,M)d}$$
$$G_{(E,M)n} = \frac{2}{3}G_{(E,M)d} - \frac{1}{3}G_{(E,M)u}$$

$$F_{(1,2)u} = 2F_{(1,2)p} + F_{(1,2)n}$$
$$F_{(1,2)d} = F_{(1,2)p} + 2F_{(1,2)n}$$

Flavor Separation of the Nucleon Form Factors



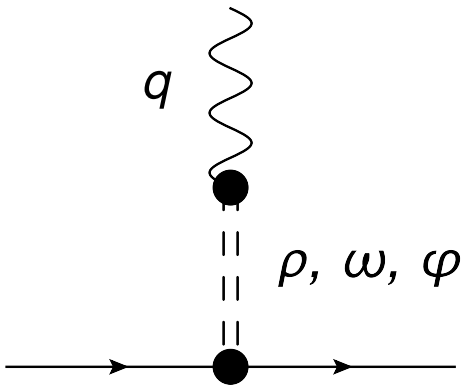


Theoretical Overview:

- Vector Meson Dominance Models
- Timelike Form Factors
- Constituent Quark Models
- Dyson-Schwinger Equations
- Links between DIS and Nucleon Form Factors
- Lattice QCD Calculations

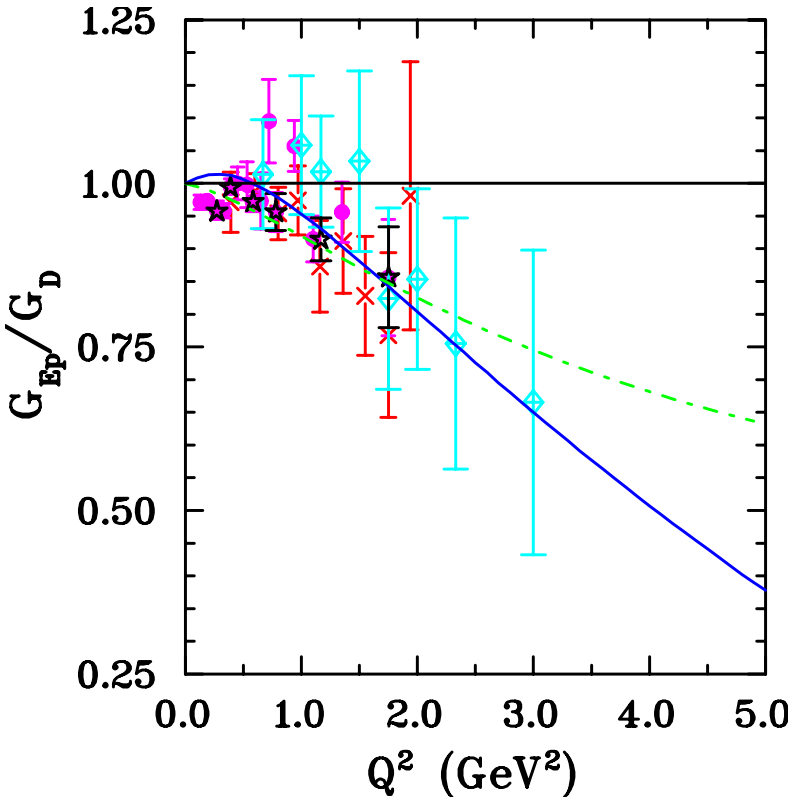
VMD History

- The photon has the same J^{PC} quantum numbers as the lowest lying vector mesons:
- $\rho(770)$, $\omega(782)$, and $\phi(1020)$
- Dominant resonances in the time-like process: $e^+e^- \to hadrons$
- Nambu (1957) suggested that the low Q^2 behavior of the proton form factor was indicative of a vector meson intermediary



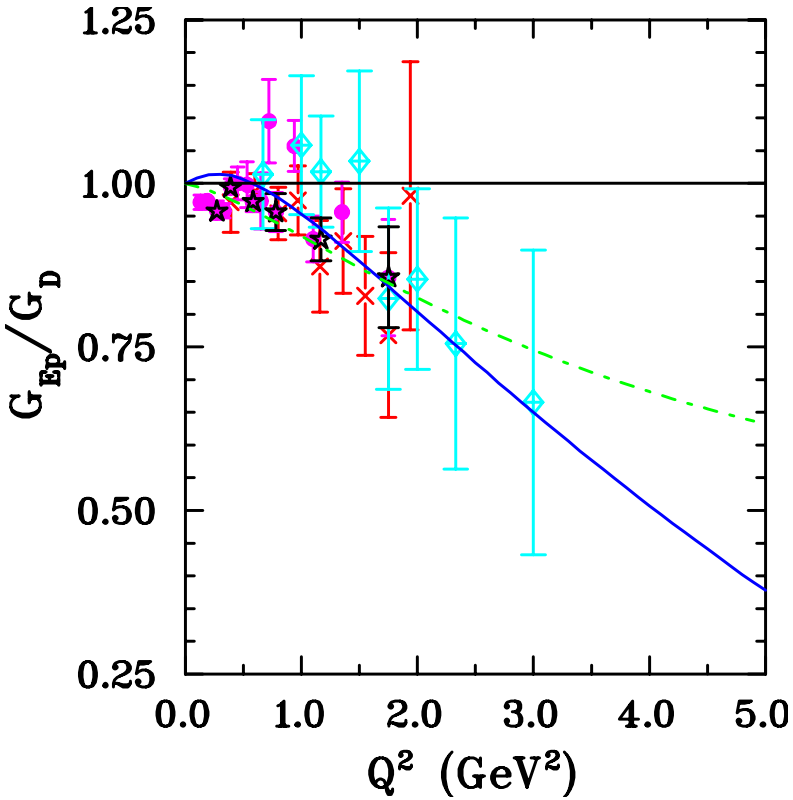
Early VMD Models

- A single vector meson exchange gives a factor: $\frac{m_V^2}{(m_V^2 - q^2)}$ from its propagator, for the falloff of the form factor
- Incorrect high Q^2 behavior ... this can be obtained through cancellations among two or more vector meson exchanges with different masses, or by giving the vector mesons themselves a form factor (e.g. ρ)



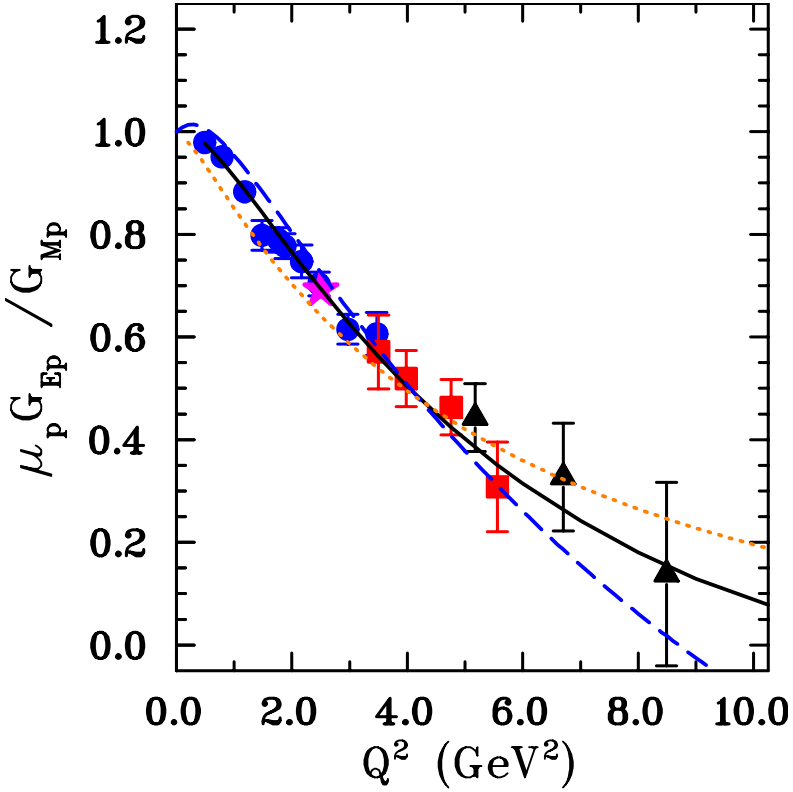
Early VMD Models

- Iachello, Jackson, and Lande (1973) were able to explain the early seen fall off of G_{Ep}!!
- Gari and Krumpelmann improved on these calculations - better match to the expected $F_1 \sim Q^{-4}$ and $F_2 \sim Q^{-6}$ behavior expected from pQCD hard-scattering models
- $\frac{F_1}{F_2} \sim Q^2$ implies $\frac{G_{Ep}}{G_{Mp}} \sim constant$



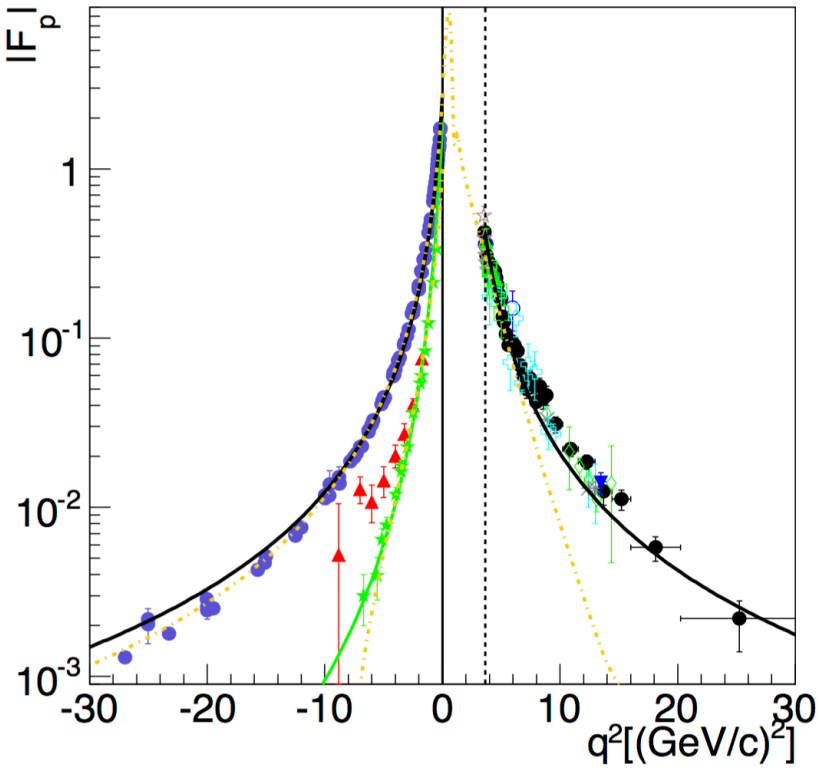
Modern VMD Models

- Iachello and a number of collaborators over the years have come up with improved models
- Lomon (up to 2006) included the $\rho'(1450)$ and the $\omega'(1419)$, and was able to achieve reasonable fits to all four nucleon form factors



Form Factors in the Time-Like Region

- Time-like region: measure differential cross section for $e^+ + e^- \leftrightarrow p + \bar{p}$
- Assume $G_E = G_M$ (true at threshold)
- The best data obtained thus far are from BABAR (Lees *et al*. 2013)
- VMD forms are straightforward to analytically continue to the time like region.



Dispersion Analyses: $G(t) = \int_{s_0}^{\infty} \frac{ImG(s)}{s-t} ds$

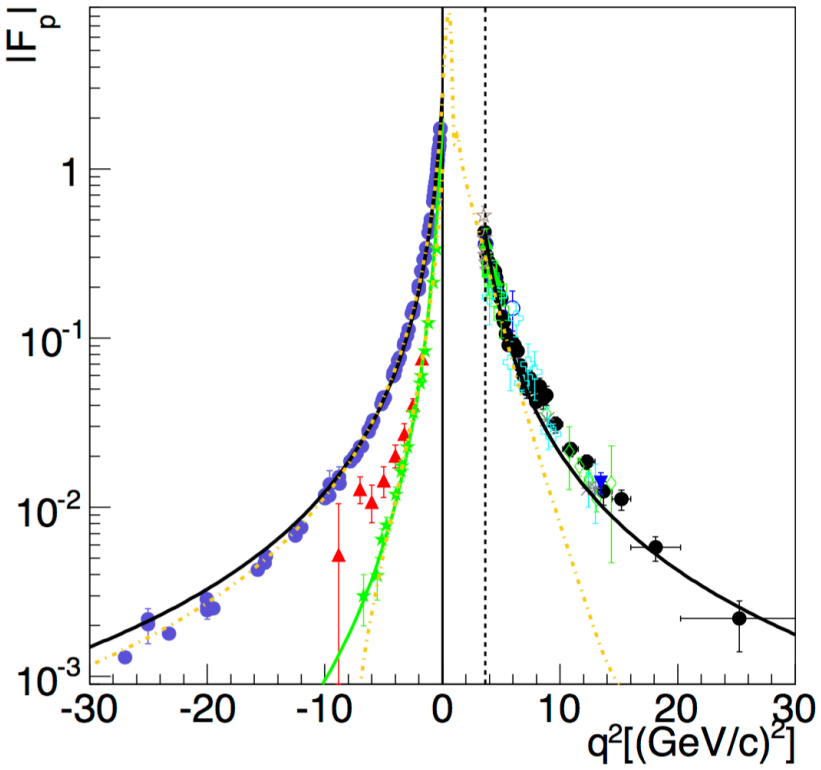
- Dispersion relations relate the form factors in the space-like and time-like regions
- In general, form factors are complex functions of q^2 that are analytic, except for known cuts
- The form factors can be calculated anywhere if one knows just their imaginary parts at these cuts
- In the time-like region, the cuts are all on the real axis, running from $q^2 = 4m_p^2$ to $q^2 = \infty$
- Lack of knowledge of these cuts in the time-like region leads to uncertainty in the space-like region, especially at high Q^2

Dispersion Analyses: $G(t) = \int_{s_0}^{\infty} \frac{ImG(s)}{s-t} ds$

- The general technique is to parametrize the imaginary part of the form factors in the time-like region, and then determine these parameters by fitting to available data in the space-like region
- Dispersive treatments lend themselves well to low Q^2 analyses $\rightarrow$ there are constraints because of the connections between the time-like and space-like regions
- This is important to consider when determining the charge and magnetic radii - it is NOT just an extrapolation to Q^2=0 ... it is in fact more of an interpolation because of the time-like constraint!

Form Factors in the Time-Like Region

- Calculations by Lorenz *et al.*, Tomasi *et al.*, and Baldini *et al.* describe well the form factors in both the TL and SL region, especially at low $|q^2|$
- More, and more precise data in the TL region could have a very positive impact on proton charge radius theoretical estimates



Constituent Quark Models

- First introduced at the point when quarks were conjectured, and predates QCD
- In the CQM, the nucleon is the QM ground state of three quarks in a confining potential
- Baryons $\rightarrow$ (uds) + SU(6) spin-flavor wavefunctions + antisymmetric color wavefunction
- Early non-relativistic models (and even some modern ones) focus on describing static properties, esp. magnetic moments
- With discovery of QCD, much work done on spectroscopy, esp. nucleon-Δ transitions, hyperfine splittings, etc.

Relativistic Constituent Quark Models

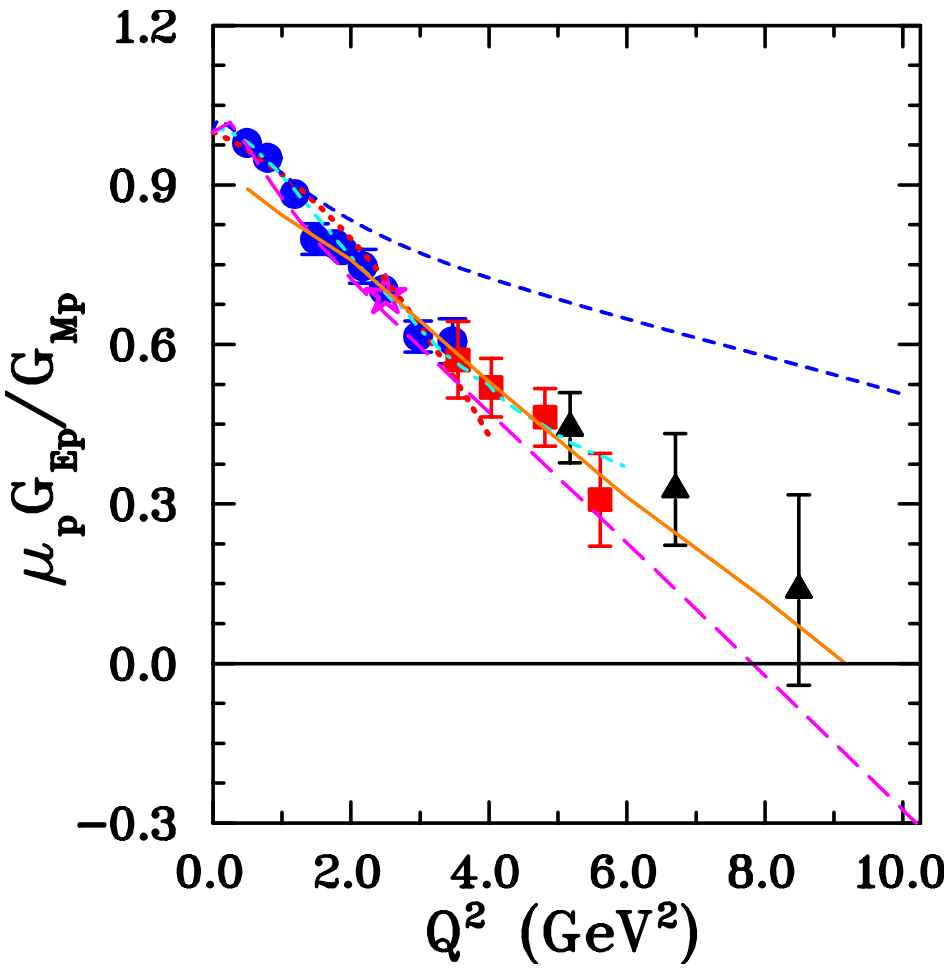
- Form factors (at moderate Q^2) require a relativistic treatment; non-relativistic calculations lead to FF's that are far too small compared to data
- How does the wavefunction, in the rest frame, transform to a moving frame? Non-trivial question!!
- The answer can, in principle, depend on the interaction binding the quarks, depending on the formalism - NOT GOOD!
- All linked to Poincaré transformations (p, rotations, boosts - laid out by Dirac)

$\rightarrow$ Instant, Point, and Light-Front Forms

Relativistic Constituent Quark Models

- Light-front: 8 of 10 Poincaré group transformations are purely kinematical; two of the rotation generators are dynamical
- Begin with a wavefunction that has been developed in CQM designed to study the baryon spectrum
- Obtain the light-front form of the wavefunction through a Melosh or Wigner rotation of the Dirac spinors for each quark
- Various calculations differ most often in the original wavefunctions used, and in the details of obtaining the light-front forms of the wavefunctions
- In recent years, addition of Dirac and Pauli FF's for constituent quarks (i.e. giving them structure, corresponding to gluonic and $q\bar{q}$ degrees of freedom)

Comparison of RCQM Calculations

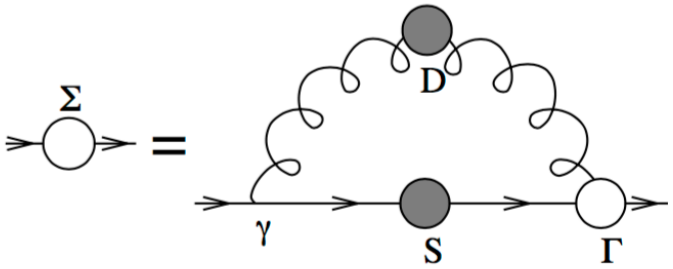


Dyson-Schwinger Equations

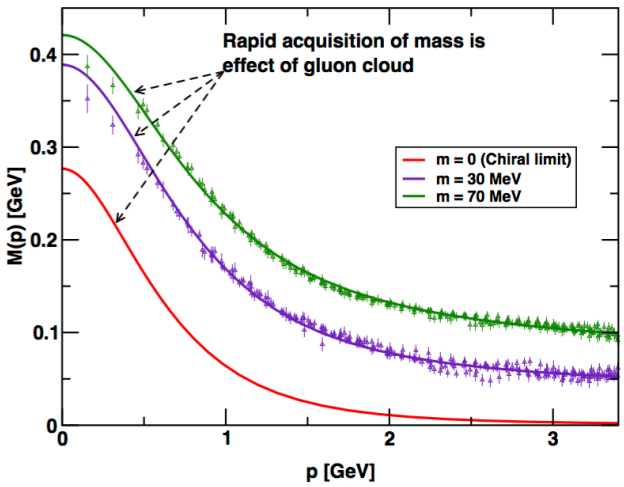
- One "problem" with VMD and RCQM calculations, from a theoretical standpoint, is the lack of a direct connection with QCD ... the Dyson-Schwinger approach attempts to address this.
- DSE's are general relations between Green functions in quantum field theories $\rightarrow$ the Langrange equations of motion of QCD
- Non-perturbative approach, thanks to Schwinger
- In princple, they are an infinite set of coupled integral equations ... must provide a symmetry preserving truncation in practice

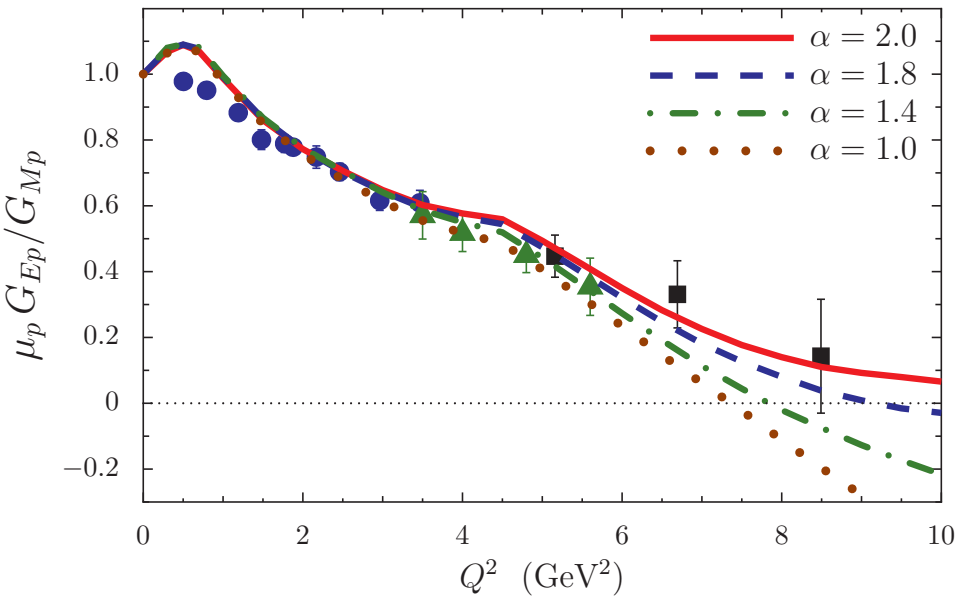
Dyson-Schwinger Equations

- In QCD, the constituent quark (or quark-parton) acquires a momentum-dependent mass function (typically 100's of times larger than the current-quark mass)
- The Dyson-Schwinger equations explain that this is primarily due to dense cloud of gluons that dress a low-momentum quark
- $S(p) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$



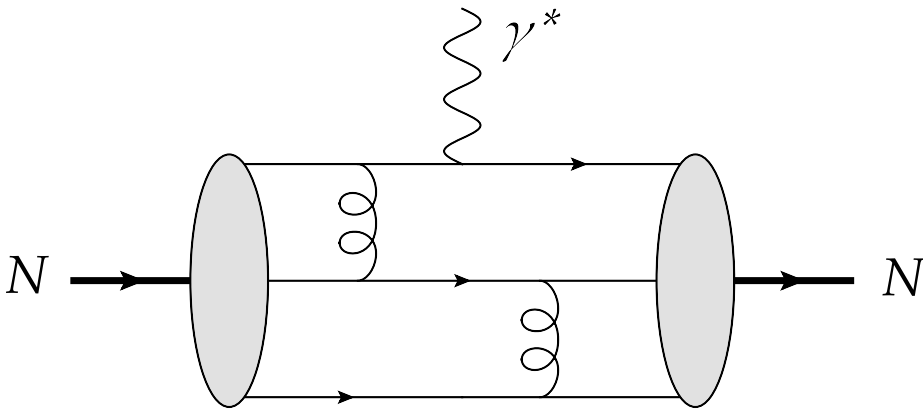
Dyson-Schwinger Equations





Perturbative QCD Inspired Models

- Perturbative QCD predicts the scaling behavior of nucleon EM FF's at high Q^2
- Three parallel quarks $\rightarrow$ three parallel quarks + two distribution amplitudes describing longitudinal momentum fraction carried by each quark
- Each gluon carries a virtuality proportional to Q^2
- $F_1 \sim \frac{1}{Q^4}$ and $F_2 \sim \frac{1}{Q^6}$ (due to helicity flip)



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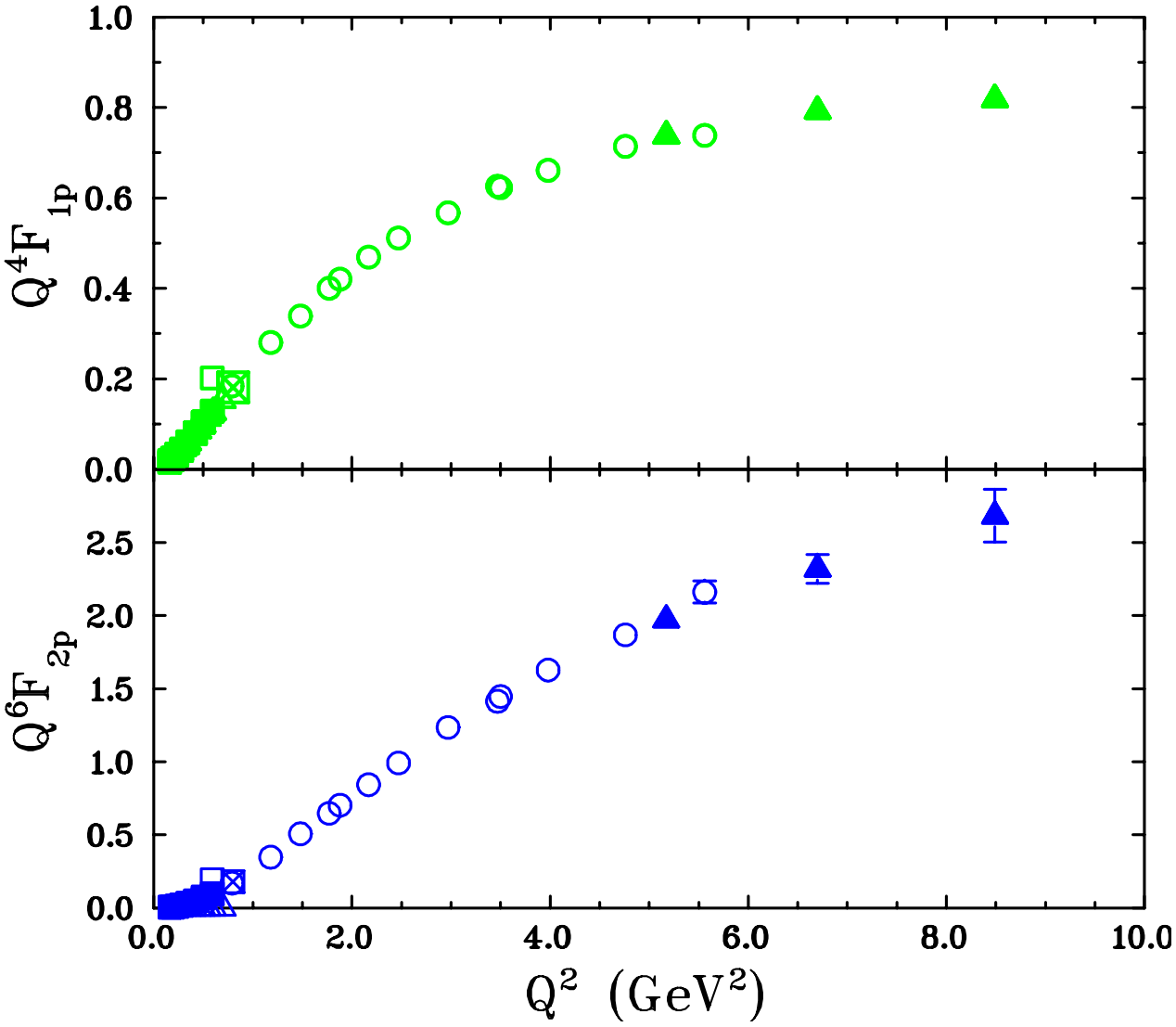
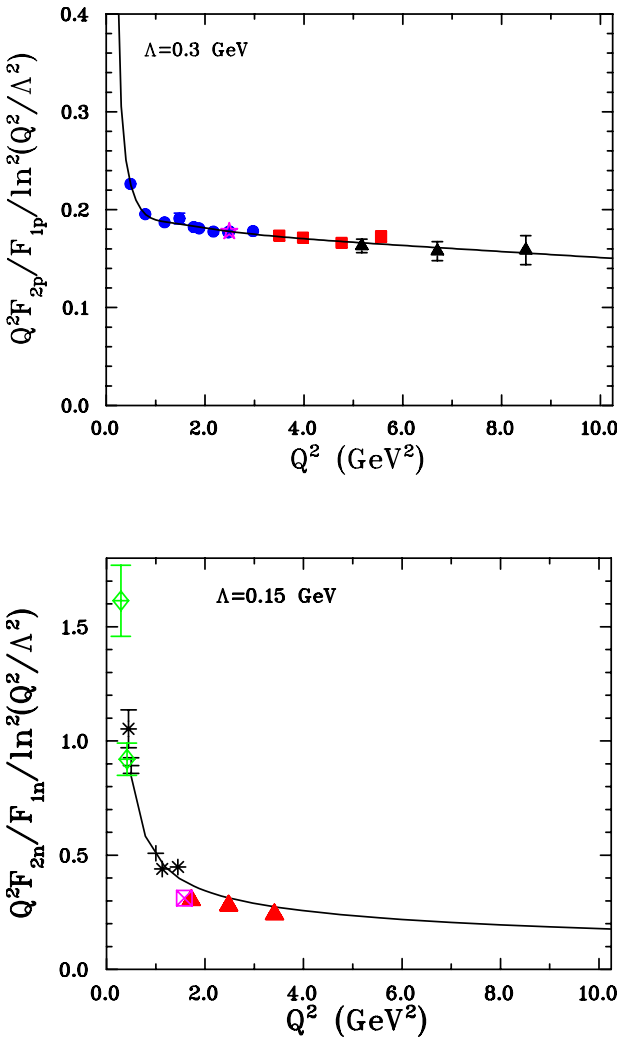


Perturbative QCD Inspired Models

- Belitsky, Ji, and Yuan investigated the assumption of quarks moving collinearly with the proton
- By including components in the nucleon light-cone wave functions with non-zero quark OAM projection, the scaling behavior changes
- $\frac{F_2}{F_1} \sim \frac{\ln^2 \frac{Q^2}{\Lambda^2}}{Q^2}$

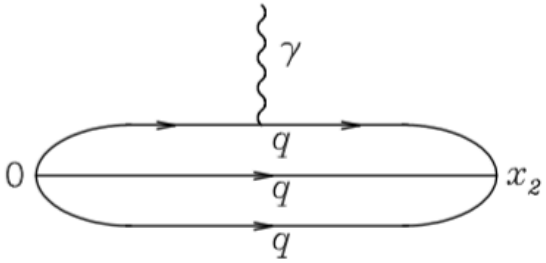


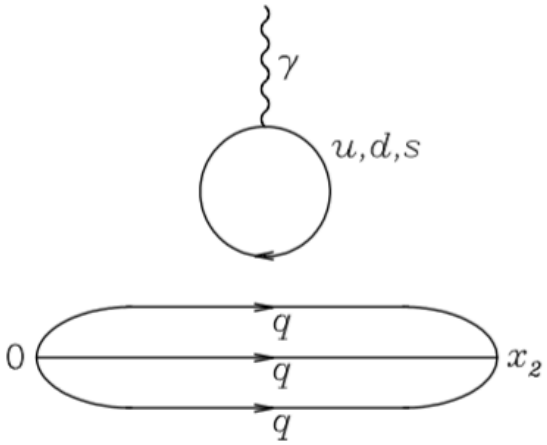
Lattice QCD

- Lattice QCD calculations provide an *ab initio* calculation of quantities such as the nucleon e.m. FF's from the underlying theory of QCD
- Only parameters are the bare quark masses and the strong coupling constant
- In practice, comparison to experimental results requires extrapolation to a) zero lattice spacing and b) realistic quark masses (i.e. experimental pion mass)
- N.B. Calculation cost $\sim (\frac{1}{m_\pi})^9$!!!

Lattice QCD

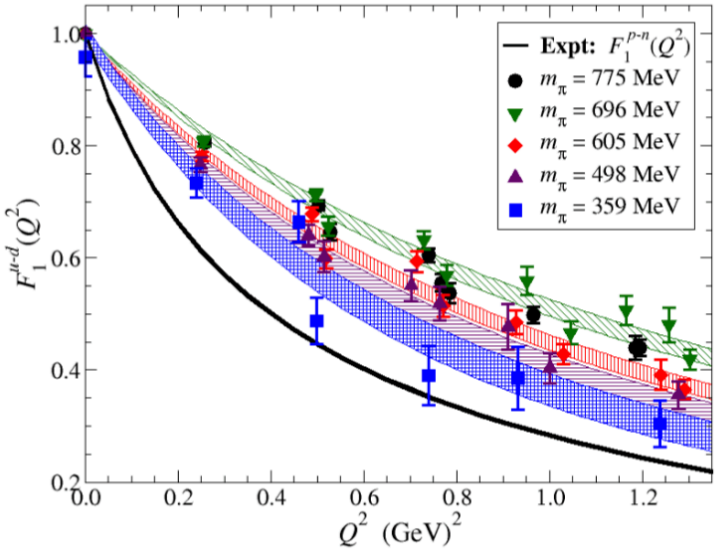
- Calculation of e.m. form factors on the lattice require the calculation of "three-point functions"
- "Quenched" approximation - ignore gluon exhange between quark lines
- Two topologically different contributions - connected and disconnected

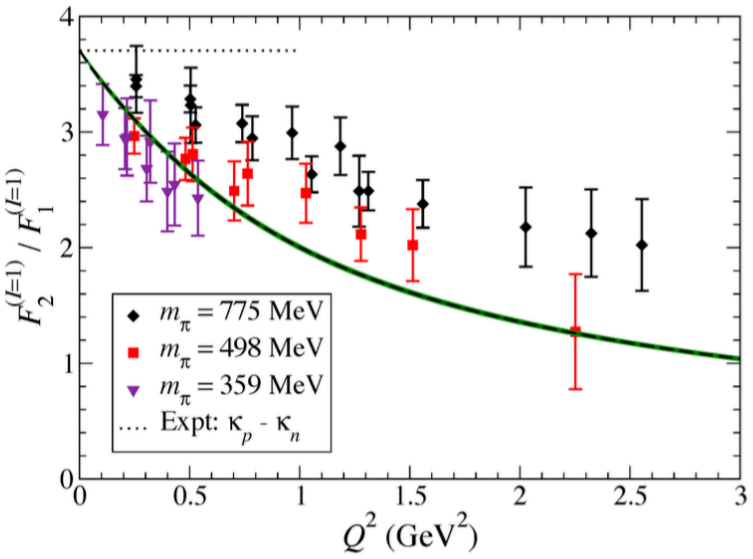




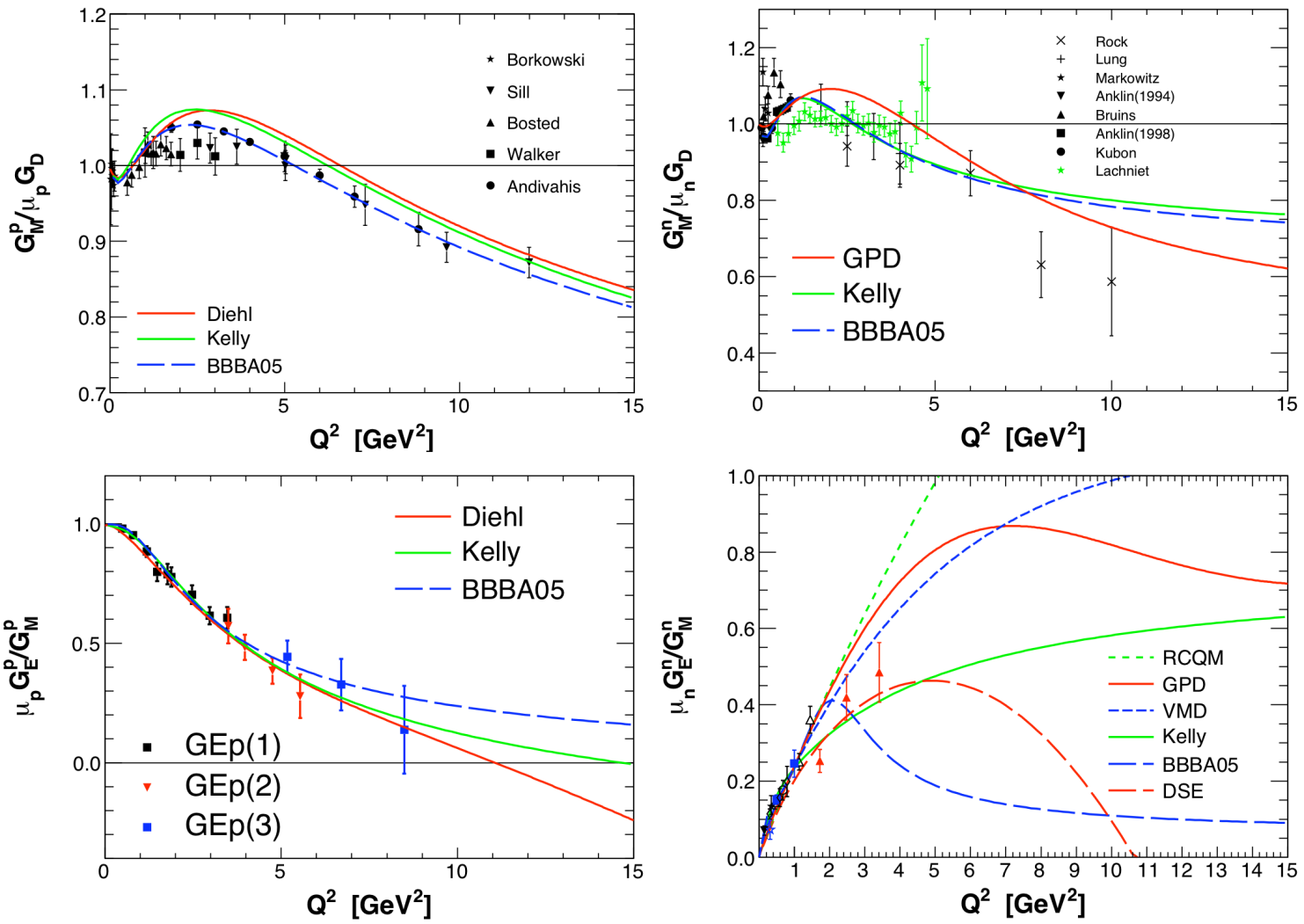
Lattice QCD

- When looking at the *difference* between proton and neutron (i.e. isovector FF's), disconnected diagrams to not contribute

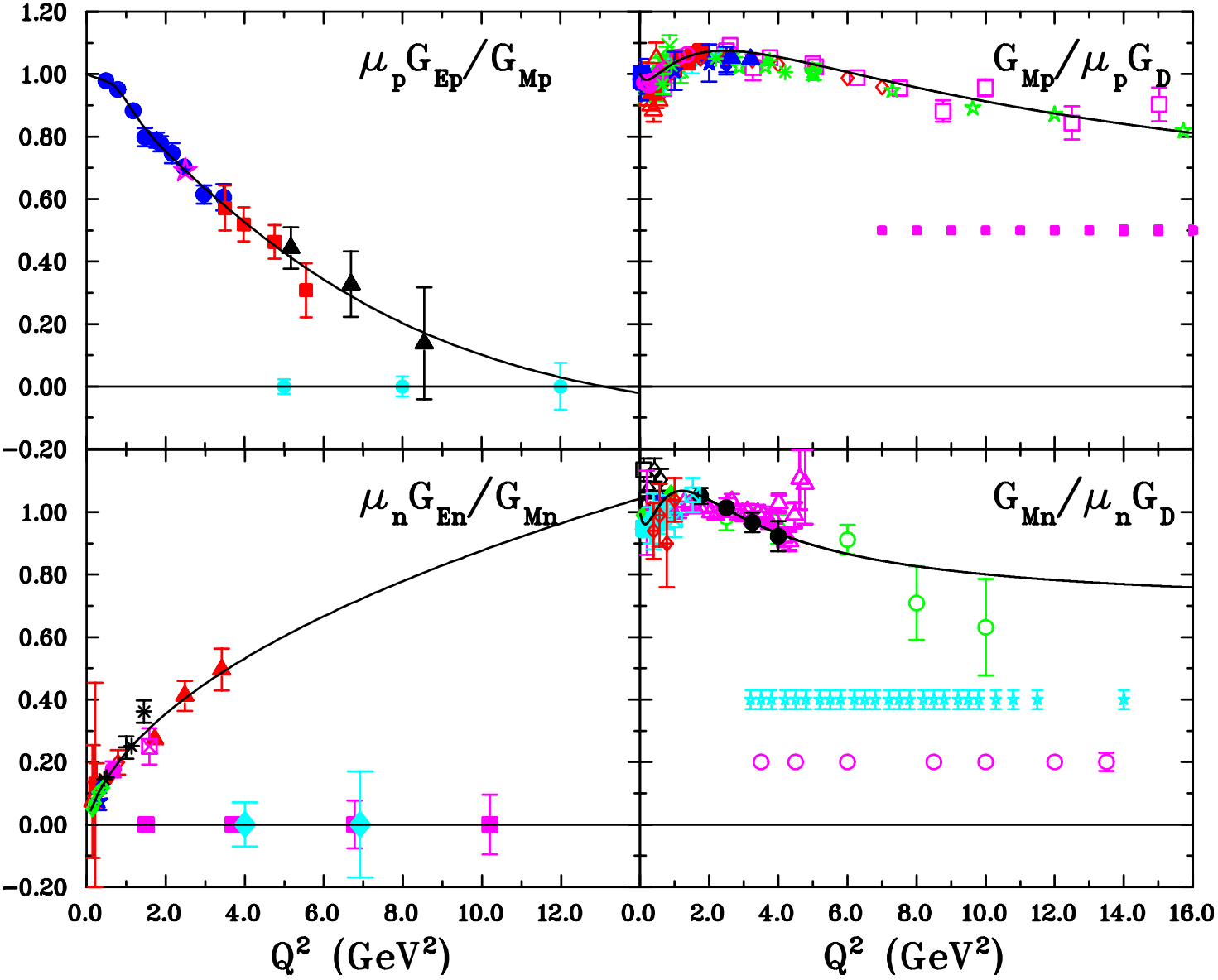




Outlook


Borkowski
Sill
Bosted
Walker
Andivahis
Diehl
Kelly
BBBA05
$G_M^p/\mu_p G_D$
Q^2 [GeV^2]
Rock
Lung
Markowitz
Anklin(1994)
Bruins
Anklin(1998)
Kubon
Lachniet
GPD
Kelly
BBBA05
$G_M^n/\mu_n G_D$
Q^2 [GeV^2]
Diehl
Kelly
BBBA05
GEp(1)
GEp(2)
GEp(3)
$\mu_p G_E^p/G_M^p$
Q^2 [GeV^2]
RCQM
GPD
VMD
Kelly
BBBA05
DSE
$\mu_n G_E^n/G_M^n$
Q^2 [GeV^2]

Outlook - Experiments in Halls A, B, and C at 12 GeV



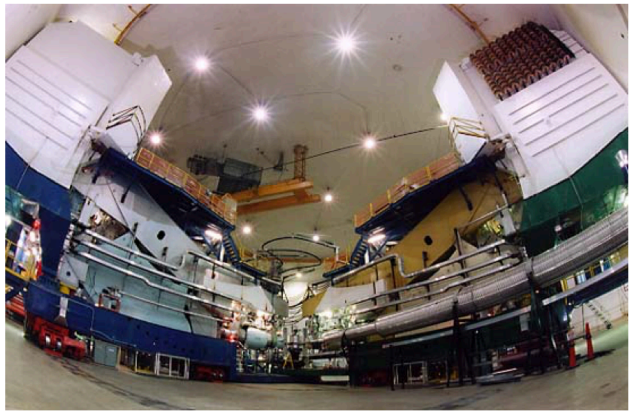
Challenges at Large Q^2

- Form factors $\propto Q^{-4}$
- Cross-section $\propto \frac{E^2}{Q^{-12}}$
- Figure of Merit $\propto \epsilon A_Y^2 \times \sigma \times \Omega$

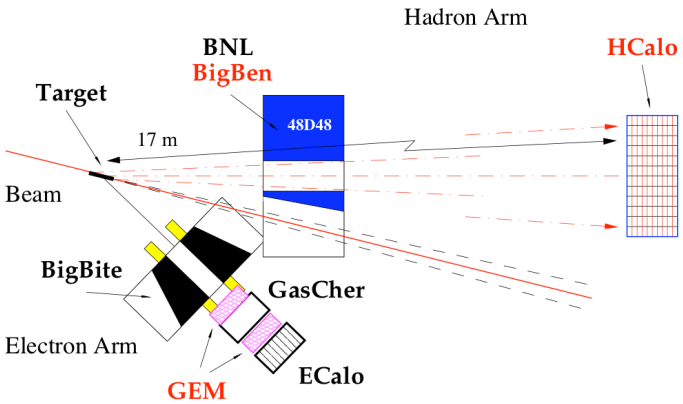
1. The need for large statistics in polarimetry experiments $\rightarrow$ high luminosity and large solid angle spectrometers
2. High luminosity and higher beam energy $\rightarrow$ large backgrounds
3. Large solid angle $\rightarrow$ small bend angle $\rightarrow$ huge backgrounds
4. Solution is a modern tracking detector based upon Gas Electron Multiplier (GEM) technologies (Sauli - 1997).

Hall A SuperBigBite Spectrometer (SBS)

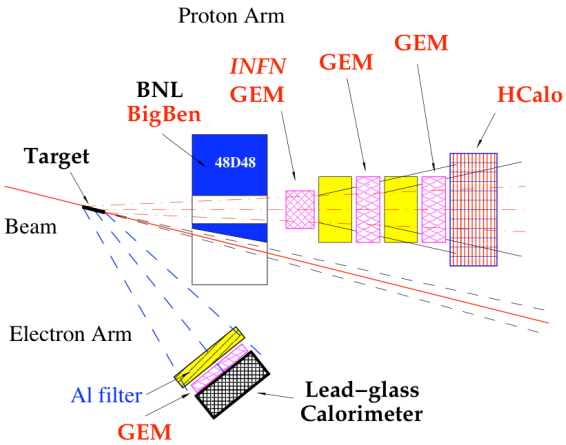
Proton magnetic form factor: E12-07-108



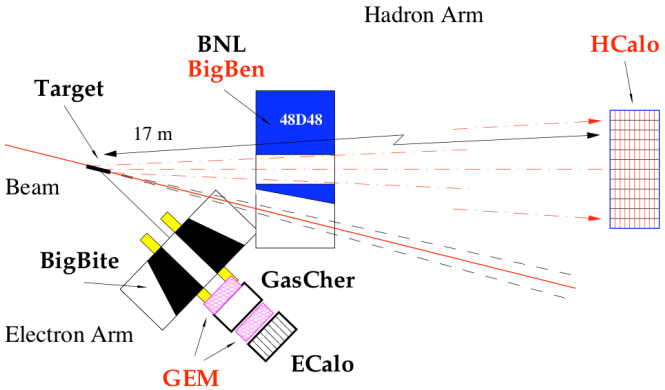
Neutron/proton form factors ratio: E12-09-019



Proton form factors ratio, GEp(5): E12–07–109



Neutron form factors ratio, GEn(2):E12-09-016



Conclusions

- High precision data for the nucleon form factors in the space-like region ($Q^2 > 0$) have provided important constraints/input for advanced calculations of nucleon structure
- More high precision data, at higher Q^2 is needed, in particular for the neutron (important for all models, in particular Lattice QCD)
- Higher precision data in the time-like region is needed (important for charge and magnetic radii)