Constraints on low Q^2 data for the extraction of the proton charge radius

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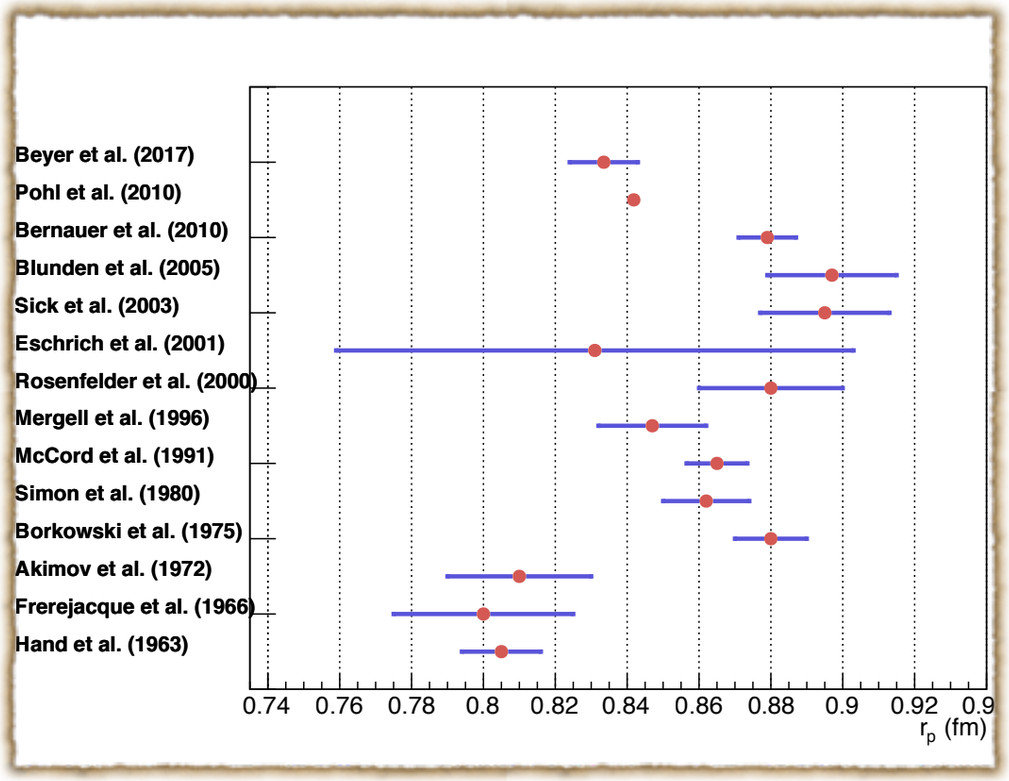
668. WE-Heraeus-Seminar, Physikzentrum, Bad Honnef, Germany 23-27 April 2018

Outline

- Some introductory slides
- What are the constraint on the data from experiments at low Q2?
- What are the possible outcome and implications?
- Bonus: How to fit the data? limits from analytical considerations

The proton radius puzzle

- Different measurements that are significantly different
- No consensus on understanding the origin of the difference
- Need for new high-precision measurements



Muonic hydrogen spectroscopy

$r_p = 0.84184 \pm 0.00067$ fm

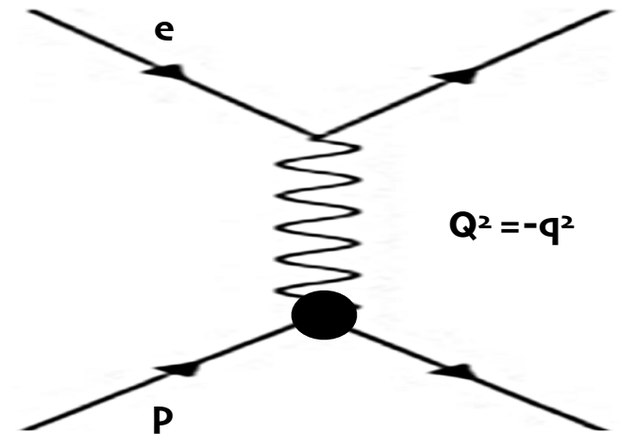
>5σ

Electron scattering experiments

$r_p = 0.87900 \pm 0.00800$ fm

Proton charge radius from e-p scattering

- Measure the e-p elastic scattering cross section (Lab frame)



$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}\Big|_p \times \left(\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau \tan^2\left(\frac{\theta}{2}\right) G_M^2(Q^2) \right)$$

- $G_E(Q^2)$ is the Fourier transform of the spatial density (in the Breit frame)

$$F(Q) \; = \; \int \rho\,(r)\; e^{iQr}\, d^3r$$

- Expanding the exponential in multi-poles and developing

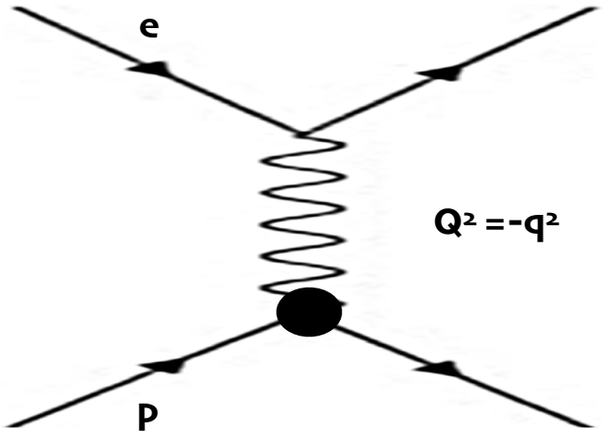
$$G_E(Q^2) = F(Q^2) \; = \; 1 \; + \; \sum_{s=1}^{\infty} \frac{(-1)^s}{(2s+1)!} \; \left(Q^2\right)^s \; \langle r^{2\,s} \rangle$$

- The radius is identified as

$$\boxed{r_p = \sqrt{< r^2 >} = \sqrt{-6 \frac{\partial G_E(Q^2)}{\partial Q^2}\Big|_{Q^2=0}}}$$

Proton charge radius from e-p scattering

- Measure the e-p elastic scattering cross section (Lab frame)



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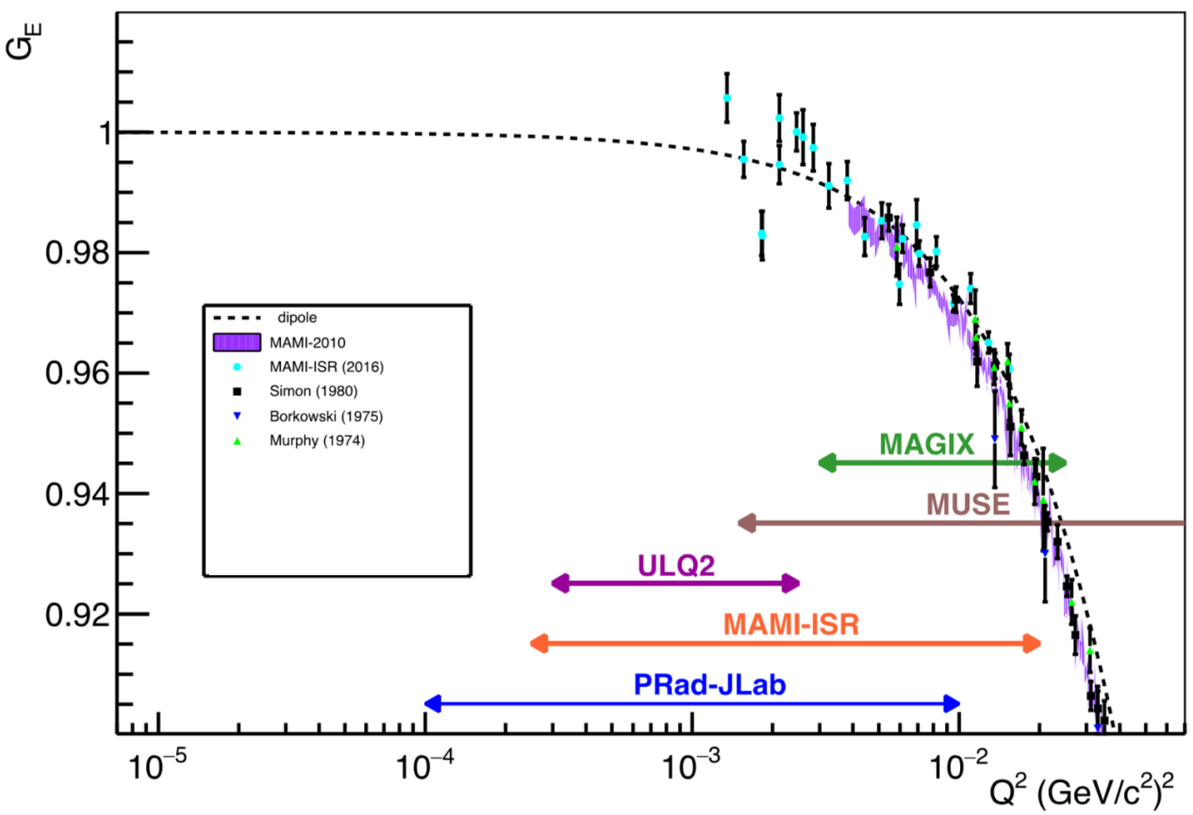
- Extract the Proton electric form factor at different values of Q^2

- Extract a value of the proton charge radius by extrapolating the form factor to $Q^2 \approx 0$

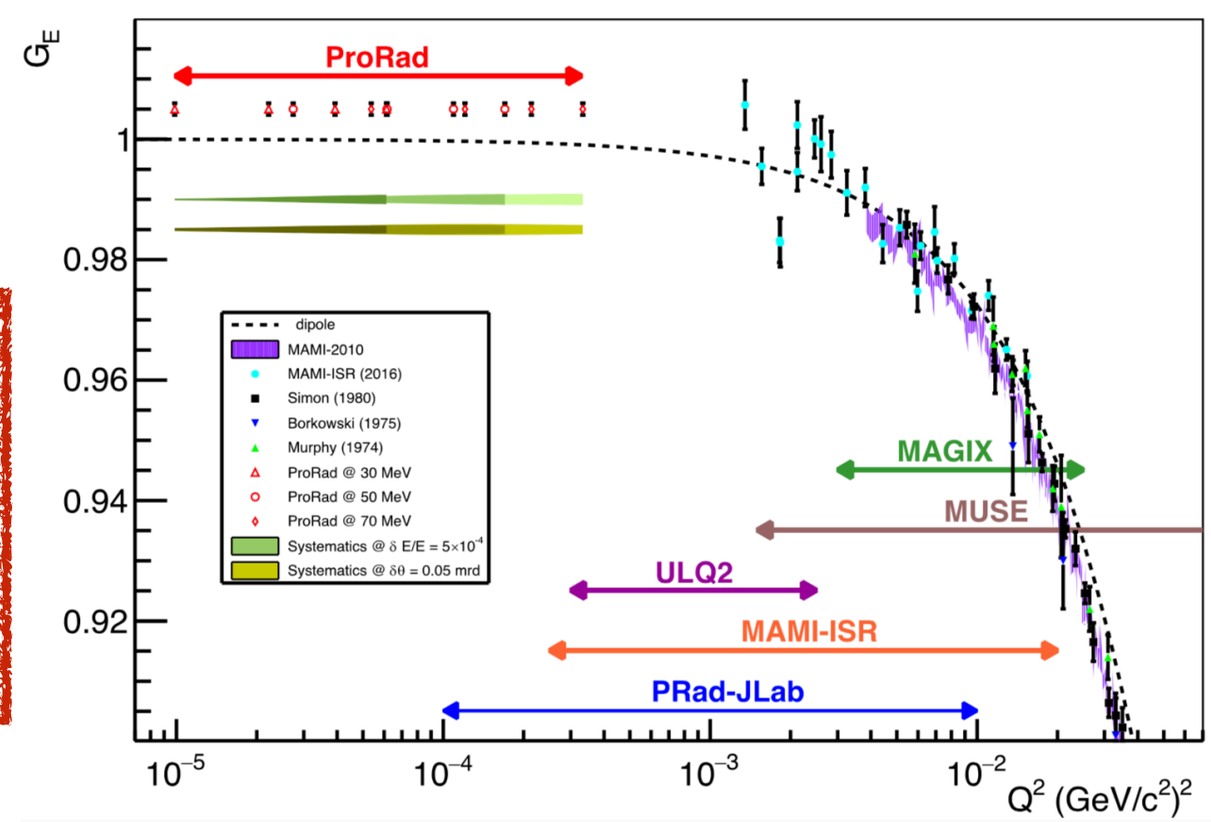
$$r_p = \sqrt{< r^2 >} = \sqrt{-6 \frac{\partial G_E(Q^2)}{\partial Q^2}\Big|_{Q^2=0}}$$



What we want to do with ProRad

- Incident electron energies varying between 30-70 MeV
- Angular range of (6°-15°) for scattered electron
- Precise measurement of the electric form factor $G_E(Q^2)$ in the momentum transfer range of 10^{-5}-10^{-4} $(GeV/c)^2$

- 0.1% precision on the measurement of the electron-proton cross section
- ProRad will provide the necessary complementary data to significantly conclude on the proton radius puzzle



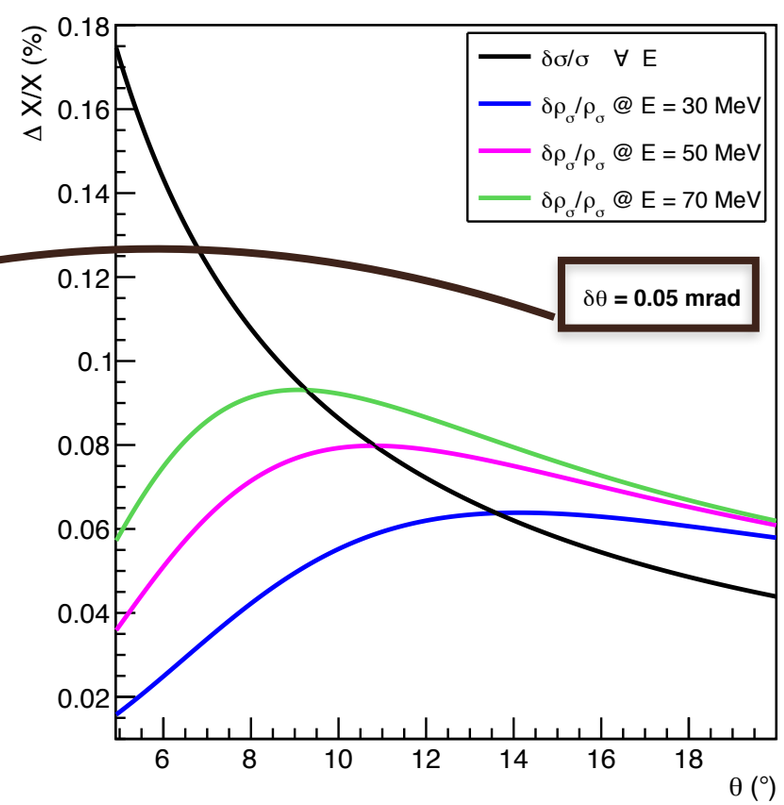
Principle of measurement

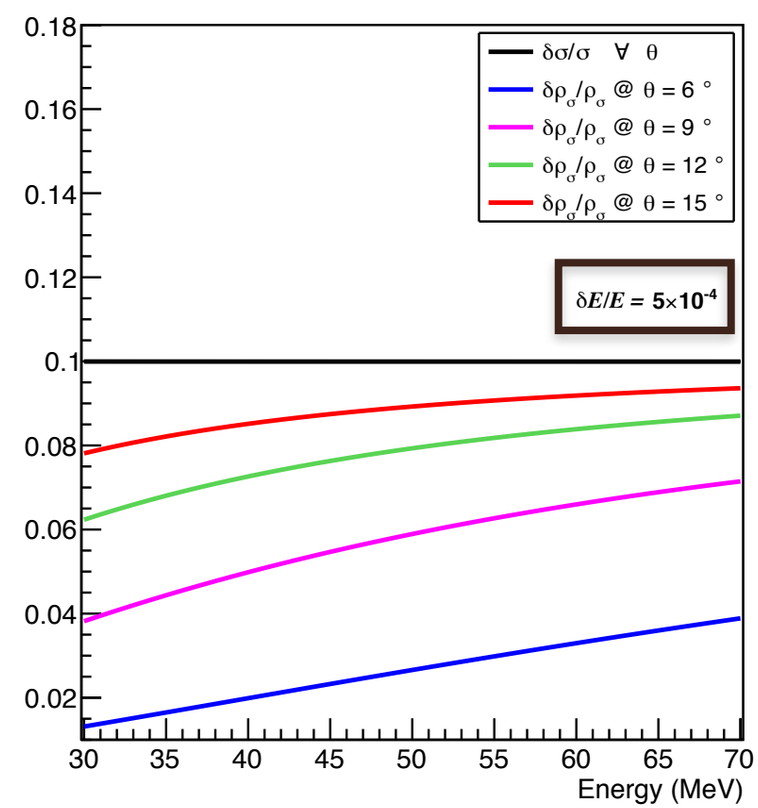
- Experimentally detect both elastic and Moeller scattered electrons
- Normalise the Elastic cross section to that of the well known Moeller cross section
- Estimate systematics on the ratio: precise control of the effects (<0.1%)

$$\rho = \frac{\int \left.\frac{d\sigma}{d\Omega}\right|_{ep}}{\int \left.\frac{d\sigma}{d\Omega}\right|_{ee}}$$

Well known theoretically

Mechanical precision of 1/10 mm (at 2 m)





Constraints on low Q^2 data

What is the required precision on low Q^2 data to asses a conclusion on the puzzle?

New e-p experiments aiming at low Q^2

Measure a radius compatible with the radius obtained from spectroscopy of muonic hydrogen

Measure a radius compatible with the radius obtained from e-p scattering (and ordinary hydrogen spectroscopy)

Measuring something different than that would create a new puzzle

Constraints on low Q^2 data

What is the required precision on low Q^2 data to asses a conclusion on the puzzle?

Measure a radius compatible with the radius obtained from spectroscopy of muonic hydrogen

- There is no longer a proton radius puzzle
- The proton radius puzzle was a mere experimental issue related to extrapolating data
- Or is it not what we expect (more on that later)

Measure a radius compatible with the radius obtained from e-p scattering (and ordinary hydrogen spectroscopy)

- Electronic hydrogen experiments are not as accurate as stated
- The two-photon exchange correction is under evaluated?

Or more exotically:

- Relativistic corrections?
- New Physics?:
  - Lepton universality violation
  - Heavy/Dark photons

Constraints on low Q^2 data

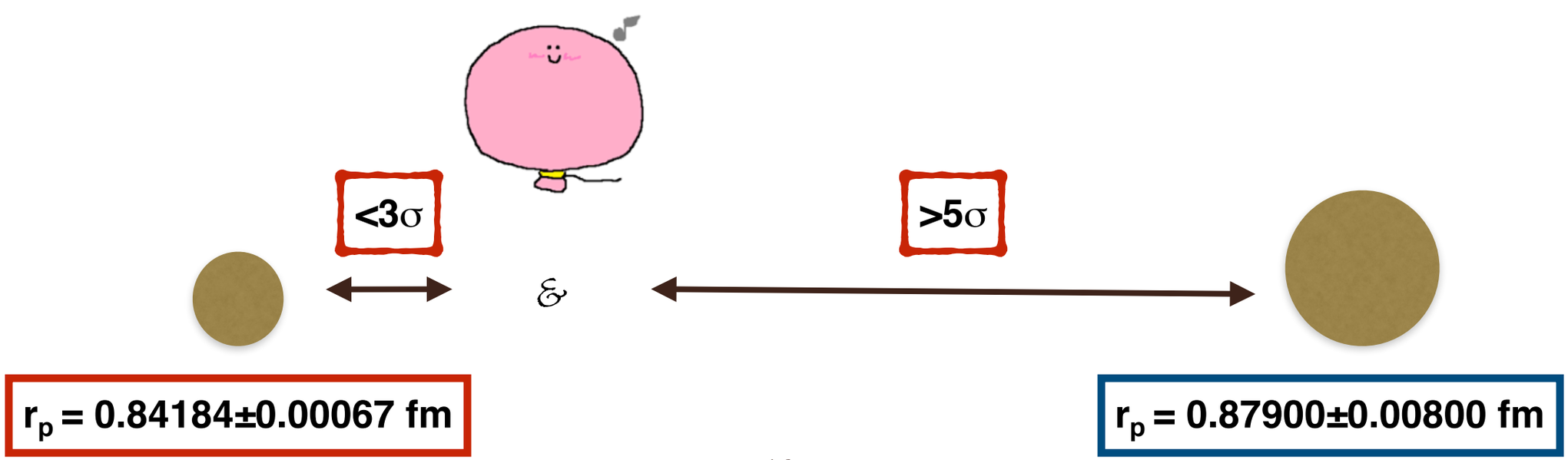
What is the required precision on low Q^2 data to asses a conclusion on the puzzle?

How to characterise the obtained result?

- In terms of compatibility between measured value and the values of the radius from muonic hydrogen spectroscopy and e-p experiments

- Two fold comparison:

$$\Delta = \frac{r_m - r_{ref}}{\sqrt{\delta^2 r_m + \delta^2 r_{ref}}}$$



Quantifying the impact of low Q^2 data

- New experimental data at low Q2 will hardly be able to extract all by itself a value of the radius that will be precise at the level of concluding on the proton radius puzzle
- The impact of the data at low Q2 will be evaluated as function of already existing data

Quantification of new data as a lever arm in the extraction of the proton charge radius

- A global fit of already existing data (mainly Mainz data) and upcoming data is necessary

- Need to fix a criteria on how to fit mainz data!!! (up to what Q2 limit to fit? what function?)

Mainz data 1420 point in Q^2

Quantifying the impact of low Q^2 data

Two possible paths to the study:

Fit to truncated Mainz data gives a radius compatible with 0.87900±0.00800 fm → Precision needed on data so that the obtained radius is consistant with 0.84184±0.0067 fm

Fit to truncated Mainz data gives a radius compatible with 0.84184±0.0067 fm → Precision needed on data so that the obtained radius is consistant with 0.84184±0.0067 fm and not with 0.87900±0.00800 fm

Truncating Mainz data gives a central value of the radius at 0.84 but with huge errors that it is compatible with any measured value

Phys. Rev. C 93, 055207 (2016)

Phys. Rev. C 93, 065207 (2016)

Is truncating data OK? arXiv:1511.00479

Quantifying the impact of low Q^2 data

This case is shown

	Up to 0.02 GeV^2	Up to 0.08 GeV^2
Proton charge	$0.99903 \pm 20.5 \times 10^{-4}$	$1.00011 \pm 5.4 \times 10^{-4}$
Radius (fm)	0.84501 ± 0.08037	0.86979 ± 0.01079
χ^2/Ndof	1.00	0.87

The proton charge is let free in the fits as we do not expect to re-measure the charge with infinite precision

- The charge is gaussian-constraint to be compatible with 1

The fit is performed with a polynomial of order 4 for Q2 =0.08 GeV^2 and 3 for Q2 = 0.02 GeV^2

Similar results can be found in Phys. Rev. C 93, 055207 (2016)
Phys. Rev. C 93, 065207 (2016)

Quantifying the impact of low Q^2 data

the G_E values. The best fit value of the parameter β gives a proton r.m.s. radius of $\langle r_E^2 \rangle^{\frac{1}{2}} = 0.84 \pm 0.02$ fm. This value is higher than the dipole value of 0.81 fm, but within the error limits it is compatible with the result (0.81 ± 0.04 fm) of a similar experiment carried out at Saskatoon [7]. The radius values reported in (I) resulted from the model-dependent extrapolation of the form factors to $q^2 = 0$, where the models demand $G_{E,M}(0) = 1$. If we include this demand in the above described fit

Nuclear physics B93 (1975) 461-478

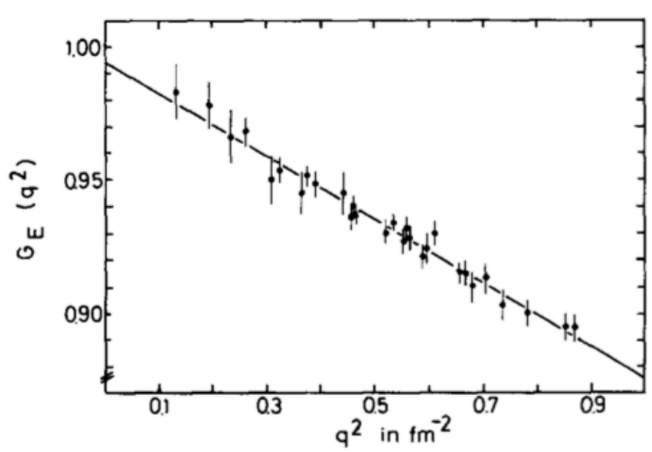
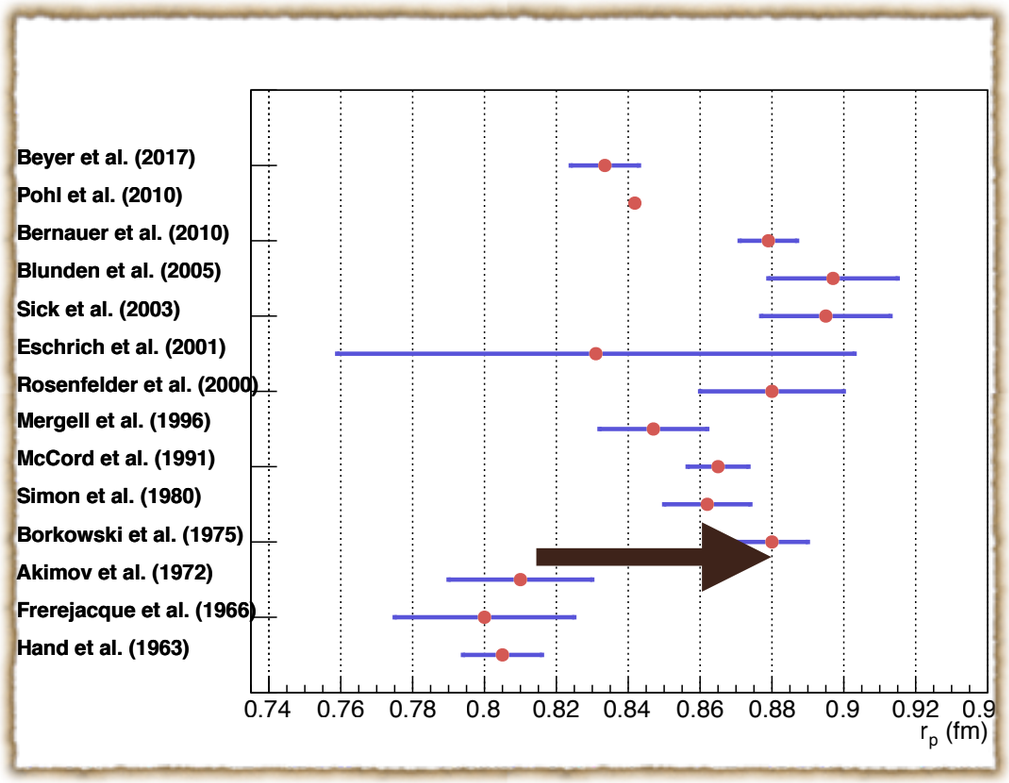


Fig. 5. Form factor values which have been extracted directly from the cross sections of (I) and of the present experiment under the assumption $G_E = G_M$. The contribution of G_M to the cross sections was less than 10%. The solid line is the best fit using the formula $G_E(q^2) = \delta + \beta q^2$.

(i.e. $\delta = 1.0$), we obtain a proton r.m.s. radius of 0.88 fm, a value which coincides with the extrapolation results.



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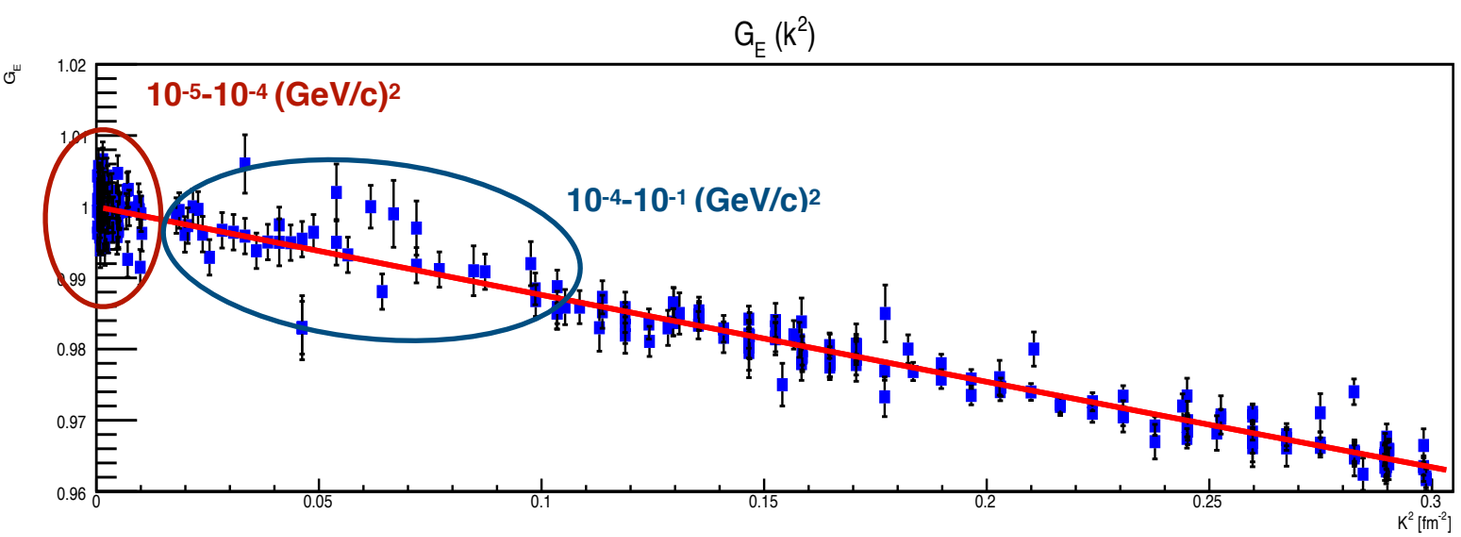
Toy studies

- Now try to answer the question: What is the required precision on low Q^2 data to asses a conclusion on the puzzle?

1. Generate toy data sets at low Q^2 featuring different Q^2 domains from upcoming experiments
2. Redistribute pseudo-data around a chosen model (chosen radius) gaussianlly supposing an error bar on each point



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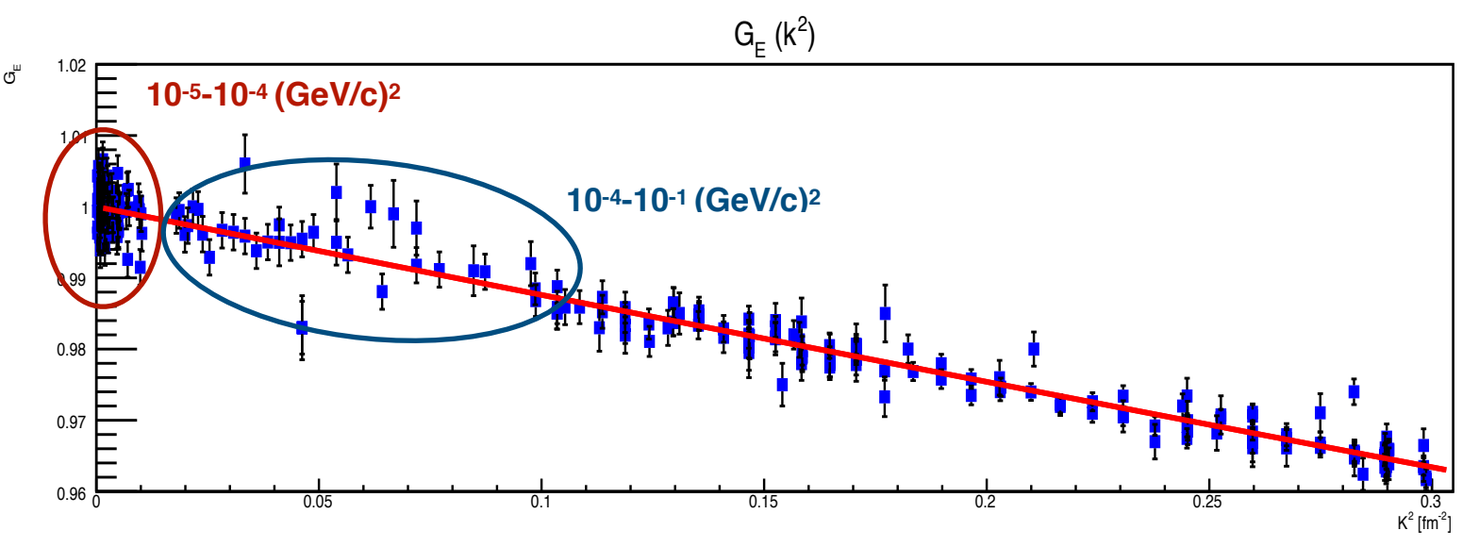
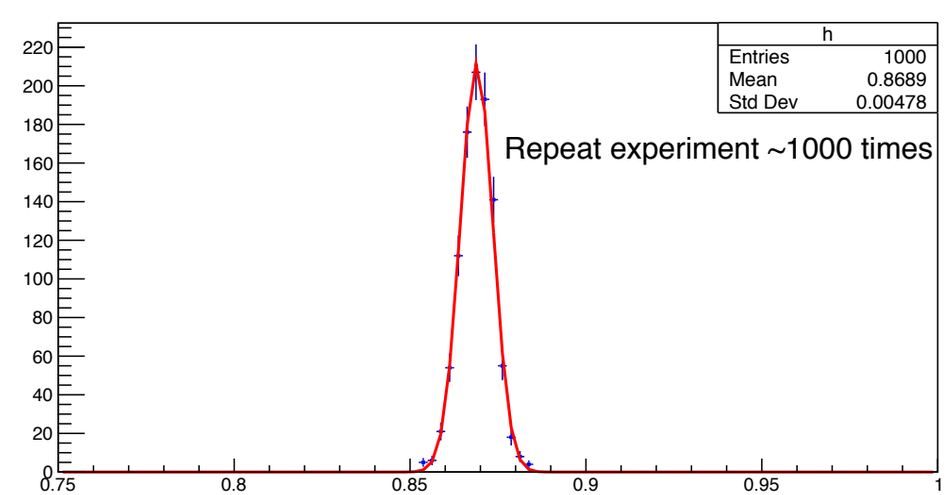
3. Repeat experiment ~1000 times and fit to extract the value of the radius





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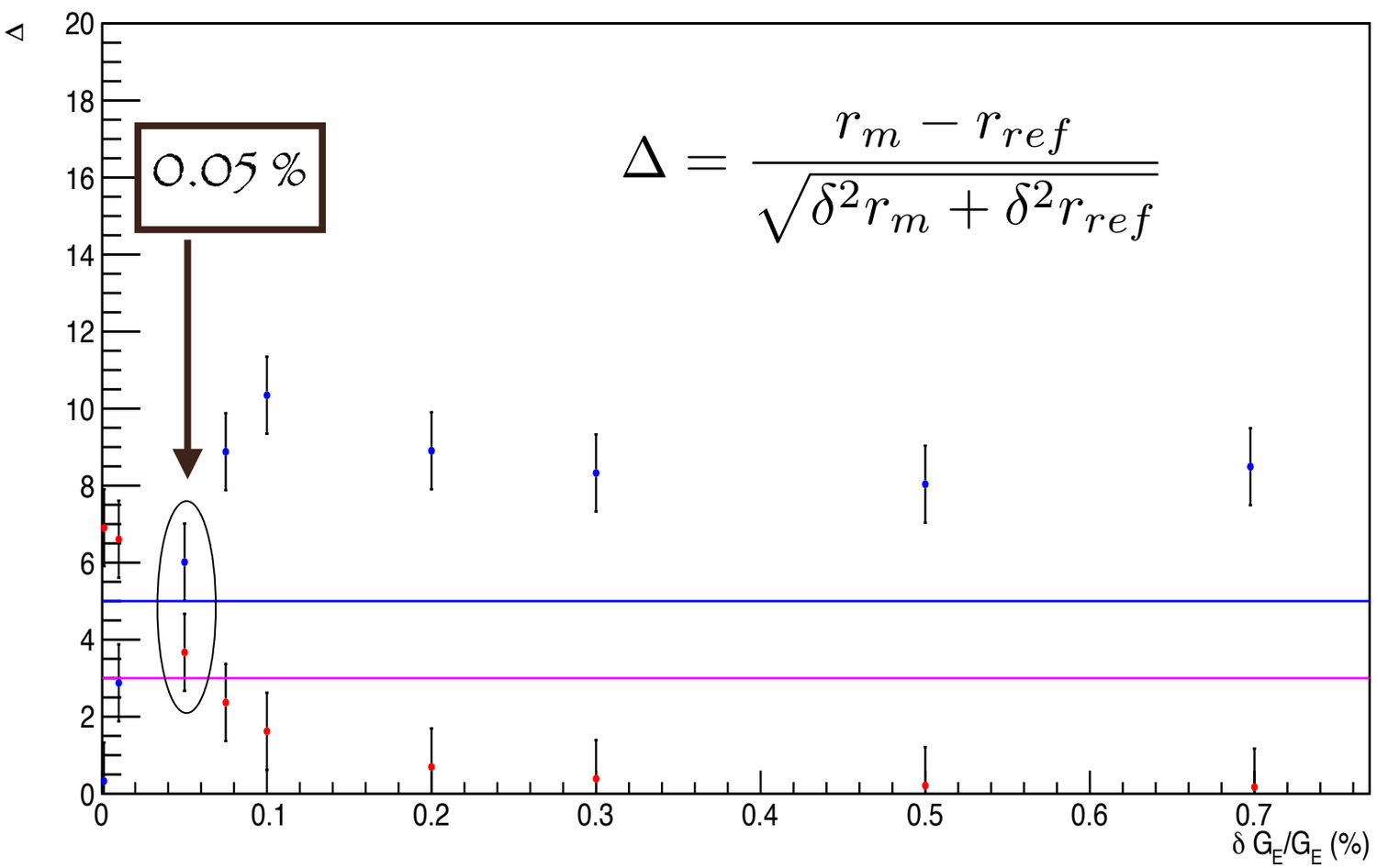
2. Redistribute pseudo-data around a chosen model (chosen radius) gaussianlly supposing an error bar on each point
3. Repeat experiment ~1000 times and fit to extract the value of the radius
4. Repeat toy-data study assuming a different error bar on pseudo-data

Ideally, you would want that the measured radius converges to the value of the radius that you used to generate the pseudo-data

Precision limits of low Q^2 data

- Small lever arm (10^{-4}-10^{-1} $(GeV/c)^2$): need 0.04% precision on ratio of cross sections (0.02% on $G_E(Q^2)$) at low Q^2 to impact the extraction of the radius

Difficult for PRad to conclude on the Proton radius puzzle alone!



Compatibility with r_{ref} = 0.84184±0.00067 fm

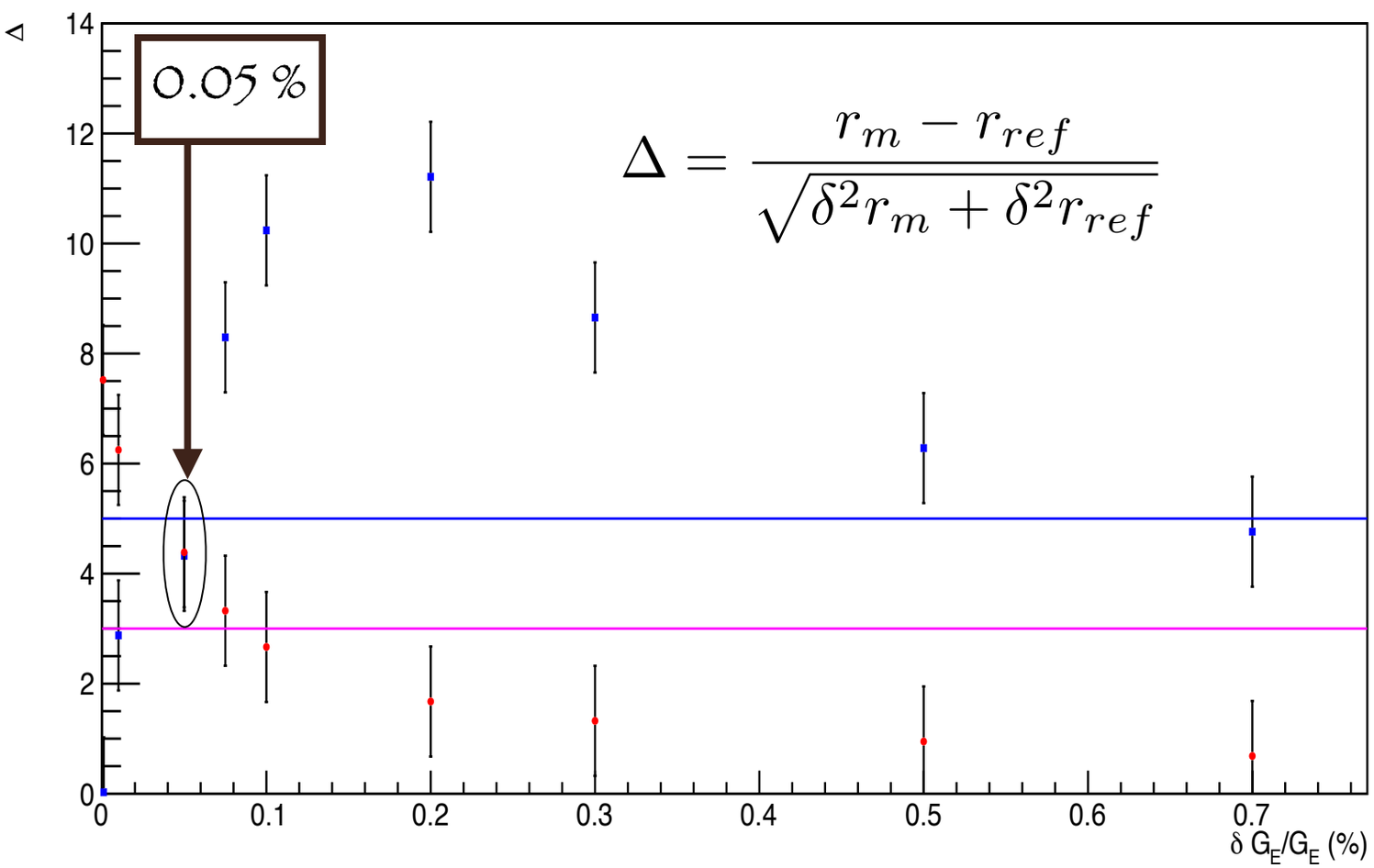
Compatibility with r_p = 0.87900±0.00800 fm

10^{-4}-10^{-1} GeV^2

Precision limits of low Q^2 data

- Large lever arm (10^{-5}-10^{-1} $(GeV/c)^2$): need 0.1% precision on ratio of cross sections (0.05% on $G_E(Q^2)$) at low Q^2 to impact the extraction of the radius

PRad will not be able to bring any new conclusion to the proton radius puzzle <u>but is a necessary measurement</u>



Compatibility with r_{ref} = 0.84184±0.00067 fm

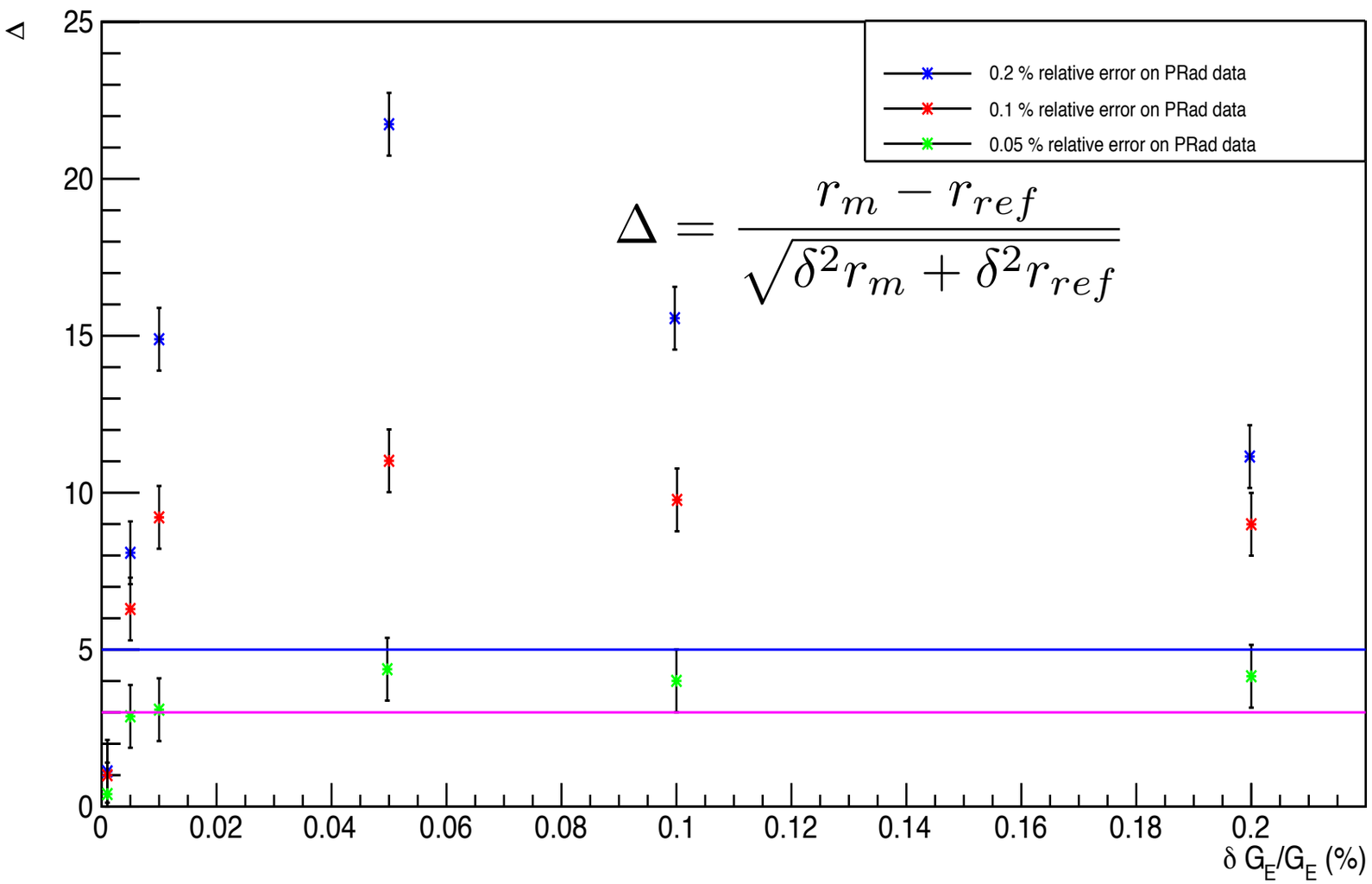
Compatibility with r_p = 0.87900±0.00800 fm

10^{-5}-10^{-1} GeV^2

Precision limits of low Q^2 data

Large versus small lever arm

It is important that all experiments aiming at low Q2 be of high precision= The Main Challenge



10^{-5}-10^{-1} GeV^2

Precision limits of low Q^2 data and radiative corrections

Radiative corrections need to be evaluated at the same level (or less) of precision as the precision expected on each Q2 point

The lepton mass CAN NOT be neglected in the calculation of amplitudes

Collaboration with A. Afanasev and A. Koshchii

Meradgen elradgen radiative corrections for Moeller and elastic cross section tool updated with required constraint

Low Q^2 data: Alternative reality

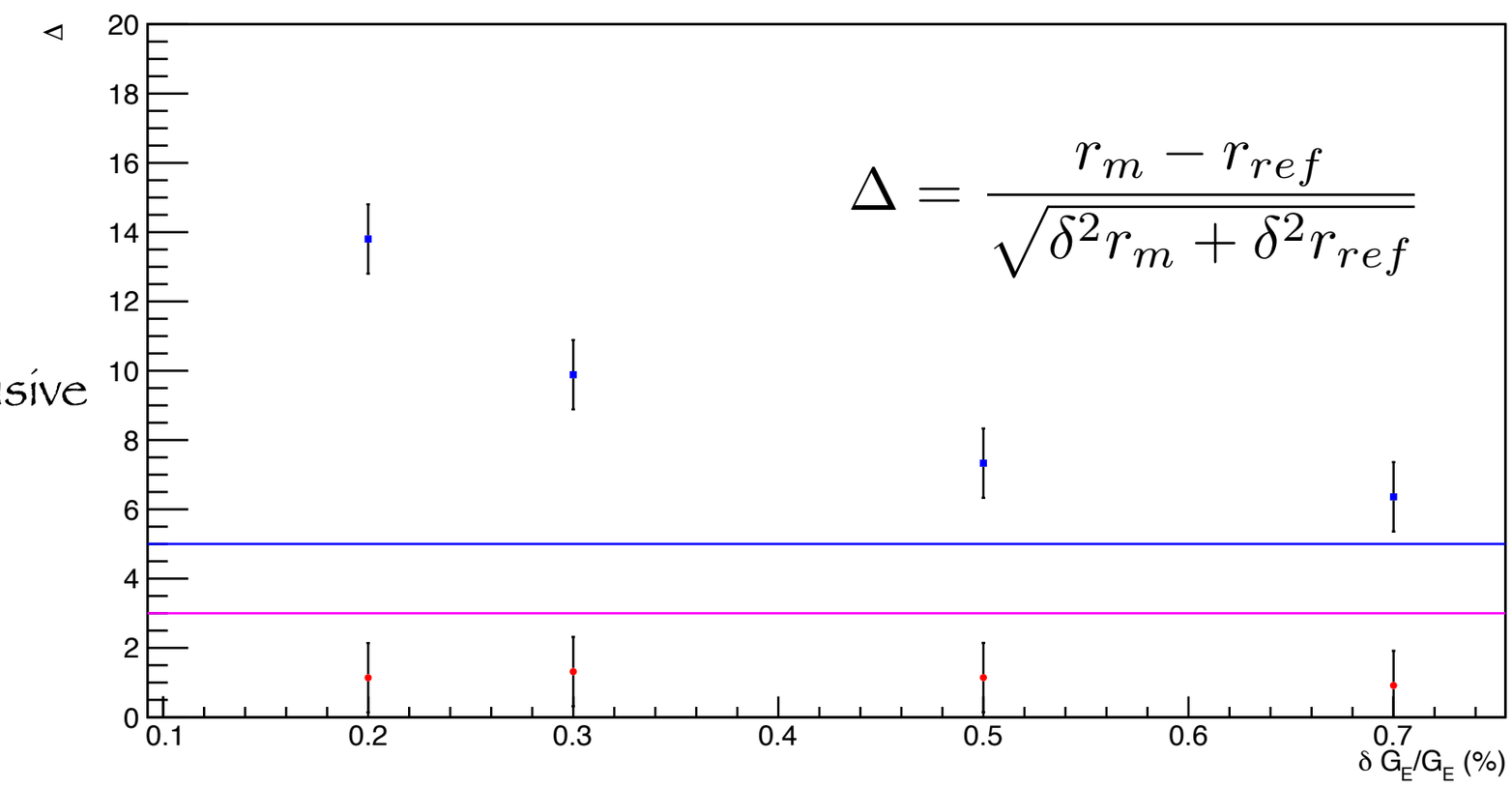
- Assume that electron scattering experiments do in reality give a different radius than muonic hydrogen spectroscopy (and ordinary hydrogen spectroscopy)

Redo toy study interchanging models

Compatibility with r_p = 0.84184±0.00067 fm

Compatibility with r_{ref} = 0.87900±0.00800 fm

Moderate accuracy, low Q^2 data is conclusive



$$\Delta = \frac{r_m - r_{ref}}{\sqrt{\delta^2 r_m + \delta^2 r_{ref}}}$$

Low Q^2 data: Alternative reality

Phys Rev D 62 071503

Measure a radius compatible with the radius obtained from e-p scattering (and ordinary hydrogen spectroscopy)

Or can not be:

contraction, are no longer negligible. What this means is that an identification of the slope of the form factor with the rms radius of a charge distribution in the rest frame is not unambiguously possible and the best one can achieve is an identification with uncertainties on the order of the Compton wavelength of the target.

~0.2% ?

- Electronic hydrogen experiments are not as accurate as stated
- The two-photon exchange correction is under evaluated?

Can not account for the difference!?

Or more exotically:

- Relativistic corrections?
- New Physics?:
  - Lepton universality violation
  - Heavy/Dark photons

What we measure is the Sach radius and not the real proton radius

Licht & Pagnamenta PRD 2, 6, 1970 (I and II)
J. J. Kelly PRC 66, 065203 (2002)
arxiv 1505.04723v2 Donnelly, Hasnel and Milner

Dark/heavy photon to solve the puzzle

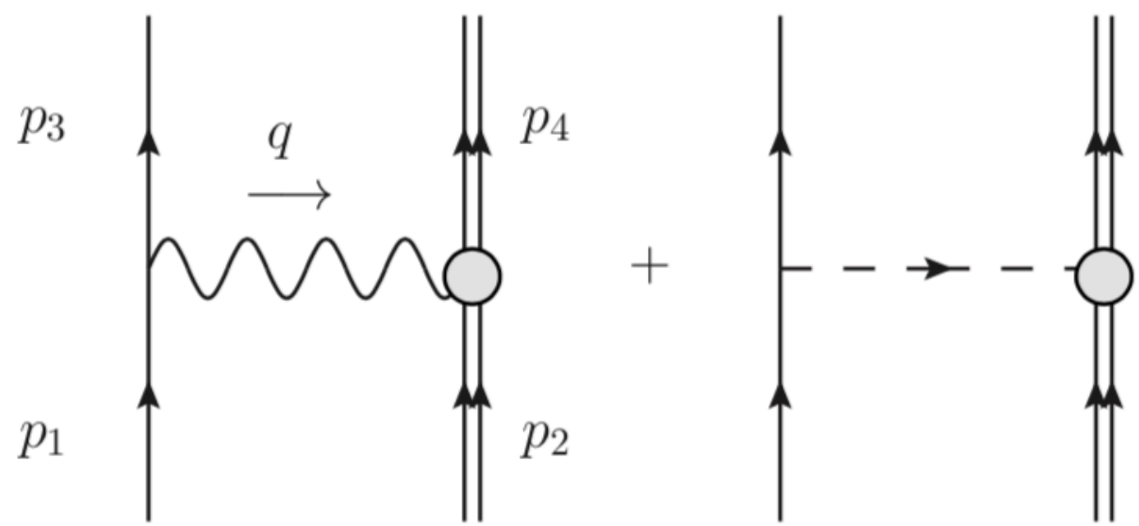
Liu, McKeen and Miller et al. Phys. Rev. Lett. 117, 101801 (2016)

Heavy Photon

Liu, McKeen and Miller et al. Phys. Rev. Lett. 117, 101801 (2016)

- A new scalar boson that can couple to leptons and nucleons

$$\mathcal{L}_\phi \supset -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m_\phi^2\phi^2 + g_l\phi\bar{\psi}_l\psi_l + g_N\phi\bar{\psi}_N\psi_N$$

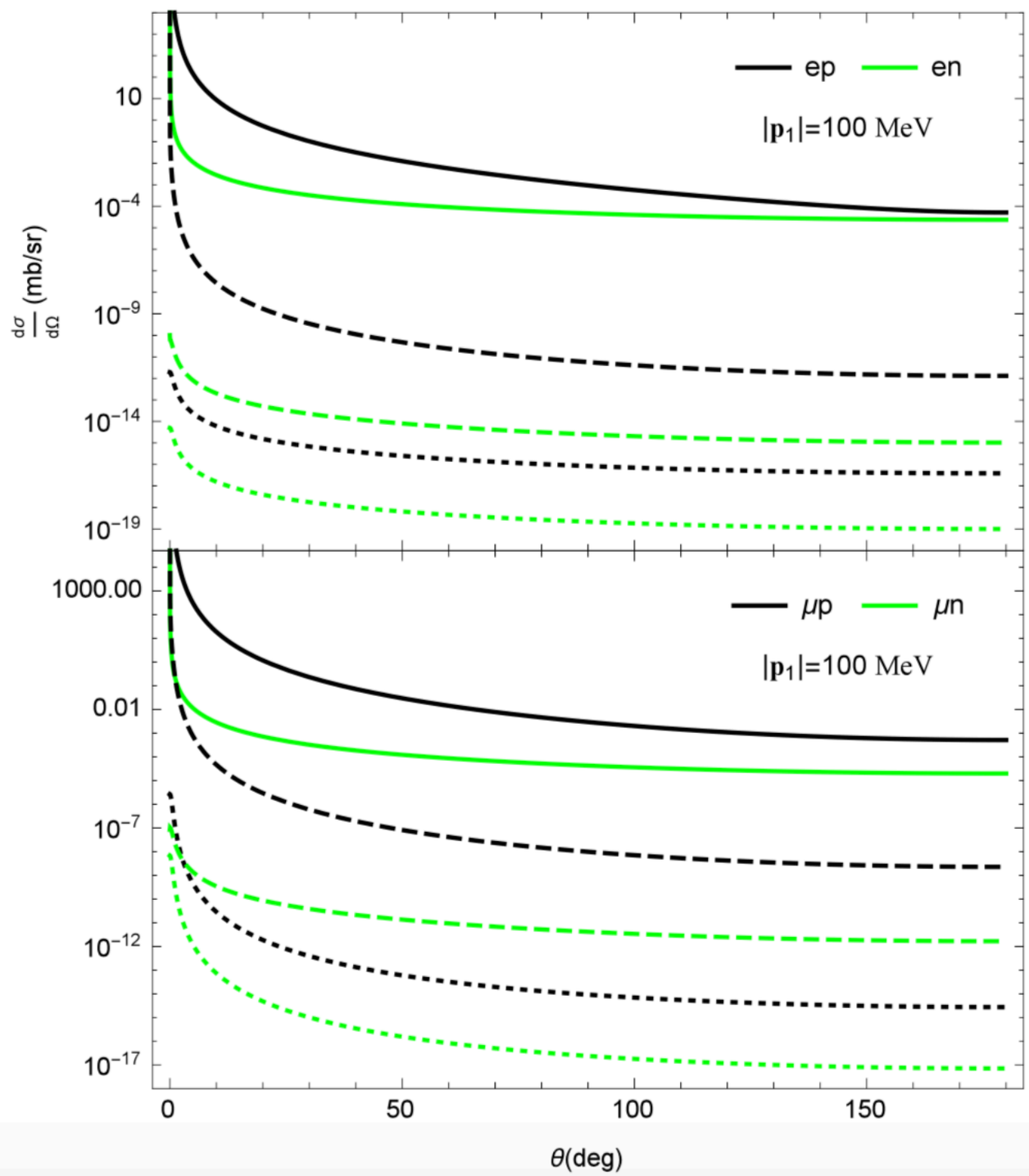


- The only way to observe such contribution is to have polarised lepton and nucleon

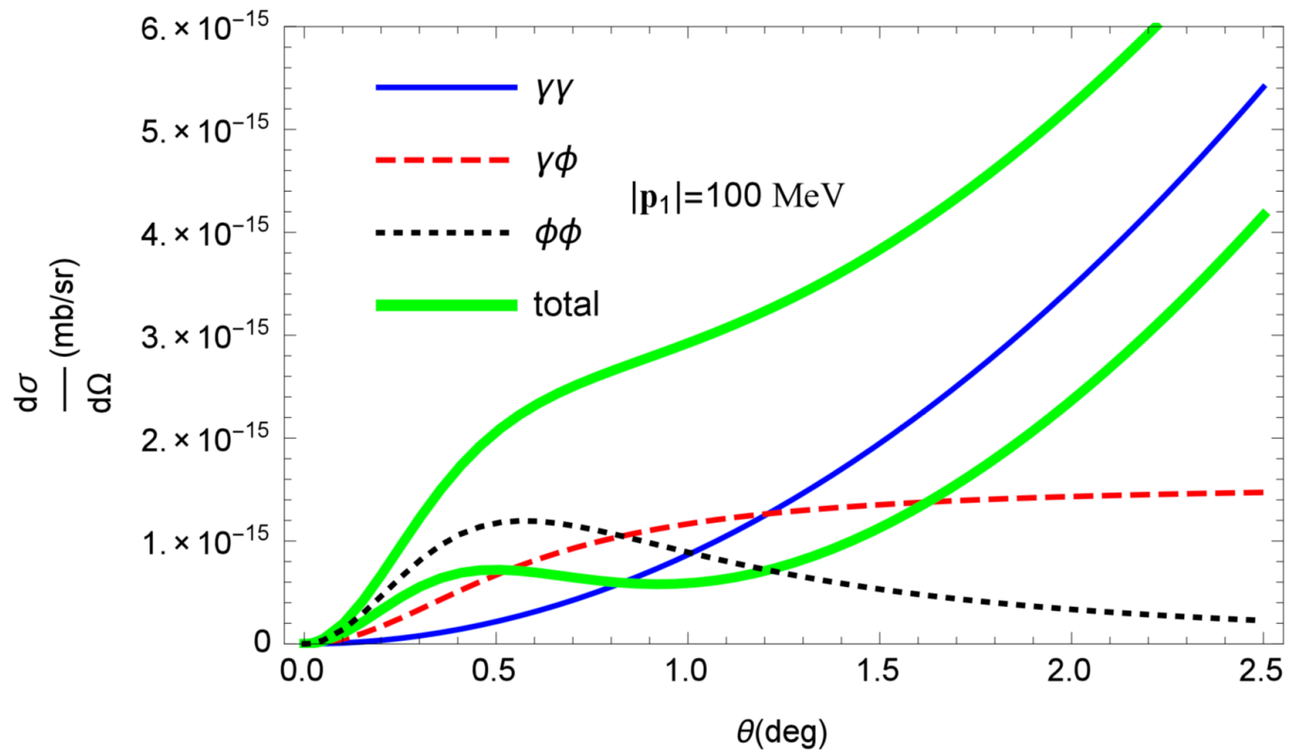
Heavy Photon

Unpolarised

Liu, McKeen and Miller et al. Phys. Rev. Lett. 117, 101801 (2016)



Polarised incoming and outgoing leptons



Relativistic effects: What are we measuring?

- The electric form is identified with the inverse Fourier transform of the spatial density in the Breit frame
- The Fourier transform in the non relativistic case

$$\frac{1}{(2\,\pi)^3}\int |\boldsymbol{k}> d^3\boldsymbol{k} <\boldsymbol{k}| = 1 \qquad\qquad <k'|k>=(2\pi)^3\delta(k'-k)$$

$$V(\boldsymbol{r}',\boldsymbol{r}) =<\boldsymbol{r}'|V|\boldsymbol{r}> = \frac{1}{(2\,\pi)^6}\int d^3\boldsymbol{k}\, d^3\boldsymbol{k}' <\boldsymbol{r}'|\boldsymbol{k}'><\boldsymbol{k}'|V|\boldsymbol{k}><\boldsymbol{k}|\boldsymbol{r}>$$

$$= \frac{1}{(2\,\pi)^6}\int d^3\boldsymbol{k}\, d^3\boldsymbol{k}'\; e^{i\,(\boldsymbol{k}'\cdot\boldsymbol{r}'-\boldsymbol{k}\cdot\boldsymbol{r})}\;\widetilde{V}(\boldsymbol{k}',\boldsymbol{k})$$

$$\rho(r) = \mathcal{N}\int d^3q\; e^{iqr}G(q)$$

What are we measuring?

- The Fourier transform in the relativistic case

$$< k'|k >= (2\pi)^3 \boxed{2\,E}\,\delta(k'-k)$$

$$\tilde{\rho}(r) = \mathcal{N}\int d^3q\; e^{iqr} G(q) \underbrace{\left(\sqrt{1+\frac{q^2}{M^2}}\right)^{-1}}_{\text{What we measure?}}$$

- The radius is obtained from the classical spatial density

$$< \tilde{r^2} > =< r^2 > + \underbrace{\mathcal{C}_{\text{relat.}}}$$

Relativistic corrections to be applied to the measured radius

Licht & Pagnamenta PRD 2, 6, 1970 (I and II)
J. J. Kelly PRC 66, 065203 (2002)

$$G_E^{nr} \equiv \sqrt{1+\tau}\,G_E^{rel}$$
$$G_M^{nr} \equiv \frac{1}{\sqrt{1+\tau}}\,G_M^{rel}$$

$$[R_p]^{nr} = (1+\tau)\,[R_p]^{rel}$$

What are we measuring?

- Would radiative corrections disappear for Low Q2 experiments?

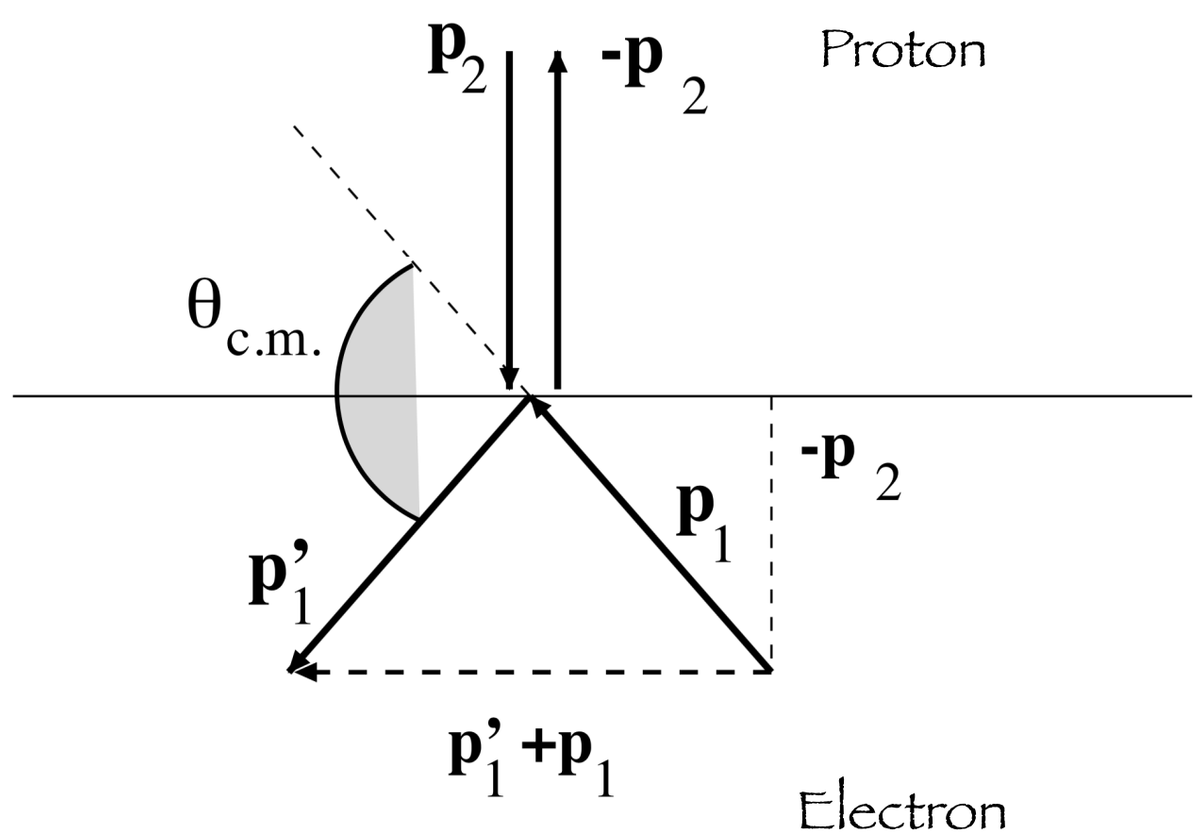
It is important to understand that these differences between relativistic and non-relativistic radii do not go away if electron scattering data are obtained at ever smaller values of the momentum transfer. As the above expressions clearly show, the relativistic boost factor arising from $(1+\tau)^{\pm 1/2}$ deviates from unity at order Q^2; however, that is the order needed to extract the charge or magnetic radii. In other words, being locked together at the same order when expanding in powers of Q^2 the effects can never be separated, no matter how small the momentum transfer becomes.

arxiv 1505.04723v2
Donnelly Hasnel and Milner

The correction is of the same order as the precision needed for the radius extraction

The Breit frame

- It might mean that what we are measuring is not actually what we think it is
- Maybe we should work in the Breit frame (cross sections, ...)



Writing the cross section in the Breit frame

$$\frac{d\sigma}{[d\Omega_{\ell'}]_B} = \frac{(\hbar\, c)^2\ \alpha_{em}^2}{4\, M_p}\ \frac{[\mathrm{p}_\ell]_B}{[\mathrm{p}_\ell]_L}\ \frac{1}{\sqrt{M_p^2 + [\mathrm{p}_\ell^2]_B\ \sin^2[\theta_{\ell\ell'/2}]_B}}$$

$$\left\{\ G_M^2([\boldsymbol{q}^2]_B)\left[\ \widetilde{Z}_B(s, [\ \boldsymbol{q}^2]_B)\frac{\tau}{1+\tau}\ - \frac{[\boldsymbol{q}^2]_B}{2}\left(M_\ell^2 - \frac{[\boldsymbol{q}^2]_B}{2}\right)\ \right]\right.$$
$$\left. + \frac{G_E^2([\boldsymbol{q}^2]_B)}{1+\tau}\ \widetilde{Z}_B(s, [\ \boldsymbol{q}^2]_B)\ \ \right\}$$

$$\widetilde{Z}_B(s, [\ \boldsymbol{q}^2]_B) \equiv \frac{\left[s - (M_\ell^2 + M_p^2)\right]^2}{2} - [\ \boldsymbol{q}^2]_B\left[\frac{s - M_\ell^2}{2}\right]$$

$$q^2 = [q^2]_B = -\ [\boldsymbol{q}^2]_B \qquad \tau = \frac{[\boldsymbol{q}^2]_B}{4\, M_p^2}$$

- In this frame, the electric (and magnetic) form factors appear naturally and is a function of the three-vector q

Writing the cross section in the Breit frame

- The Fourier transform of of G(q-vector) is unambiguously defined

3-vector

Lab frame

4-vector

$$\rho(r) = \mathcal{N} \int d^3q \; e^{iqr} G(q)$$

3-vector

Breit frame

- Moreover, relativistic corrections are not the same

$$\tilde{\rho}(r) = \mathcal{N} \int d^3q \; e^{iqr} G(q) \left(\sqrt{1 + \frac{q^2}{M^2}} \right)^{-1}$$

Lab frame

$$\tilde{\rho}(r) = \mathcal{N} \int d^3q \; e^{iqr} G(q) \left(1 + \frac{q^2}{M^2} \right)^{-1}$$

Briet frame

Summary

- Precision on all aspects is a key point for all Low Q2 experiments to reach their goals
- ProRad will contribute significantly to the investigation of the proton radius puzzle
- New physics to solve the puzzle?
- Relativistic effects:corrections to the proton radius
- From the lab to the Breit frame

Conclusions: what I wanted to shed light on

- The concept of an observable quantity in quantum mechanics is ill-defined
- The principle of induction in logical thinking works fine but is not rational

Bonus

Functional form, the radius and convergence radii

- To extract the radius, either G(Q) or ρ(r) should be known and integrated over the full range
- But this is not the case: We have to model our form factor
- And we are interested by how the form factor behaves at low Q2: series development of G(Q)

$$G(k^2) = \sum_{n=0} C_n (k^2)^n = 1 + C_1 k^2 \left(1 + \sum_{n>1} \frac{C_n}{C_1} (k^2)^{n-1} \right)$$

- Higher order terms will correlate to the radius and modify it
- The expansion can be stopped only when the correction from higher order terms to the the first term become <u>negligible</u>

Functional form, the radius and convergence radii

- Consider a spatial probability density of the form

$$\rho(r) = \mathcal{N}\, r^{\alpha} exp^{-\lambda r}$$

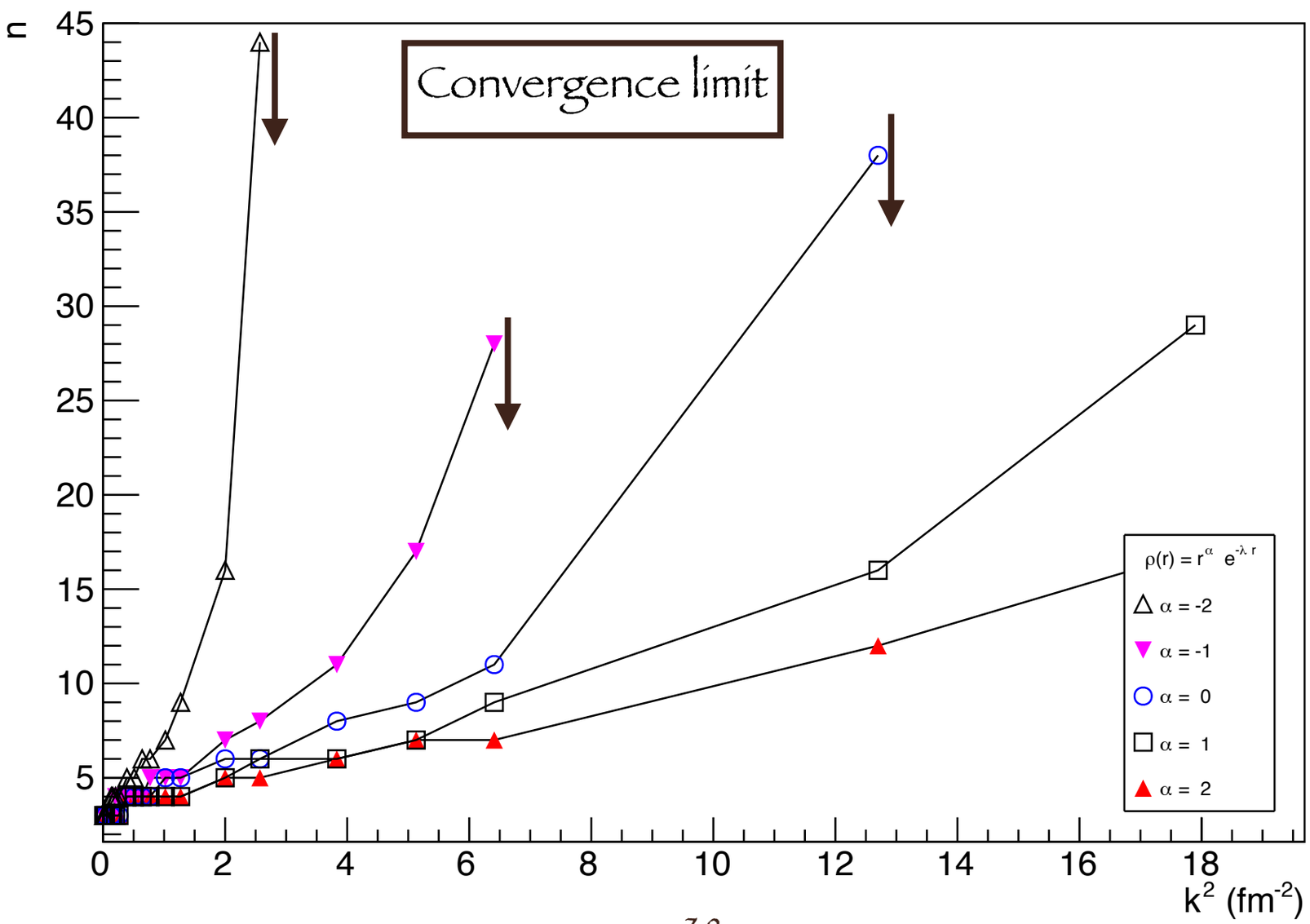
- Some models are not adapted to describe the form factor over a wide Q2 range
- Need to check convergence radii for each model

$$\lim_{n\to\infty} \frac{|C_{n+1}\,(k^2)^{n+1}|}{|C_n\,(k^2)^n|} = \frac{|k^2|}{\lambda^2} < 1$$

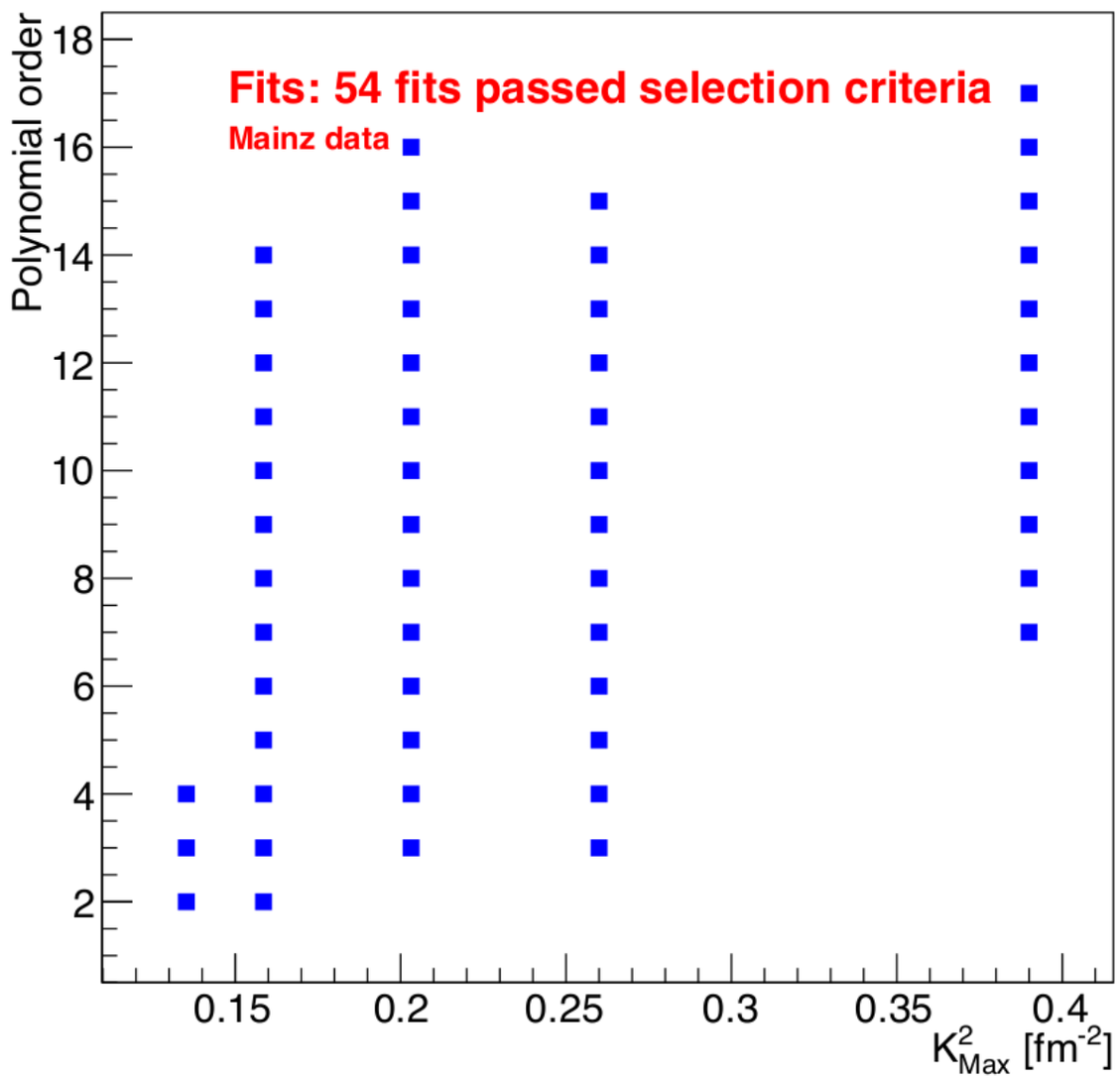
Functional form, the radius and convergence radii

- Consider a spatial probability density of the form

$$\rho(r) = \mathcal{N}\, r^{\alpha} exp^{-\lambda r}$$



Over fitting the data!



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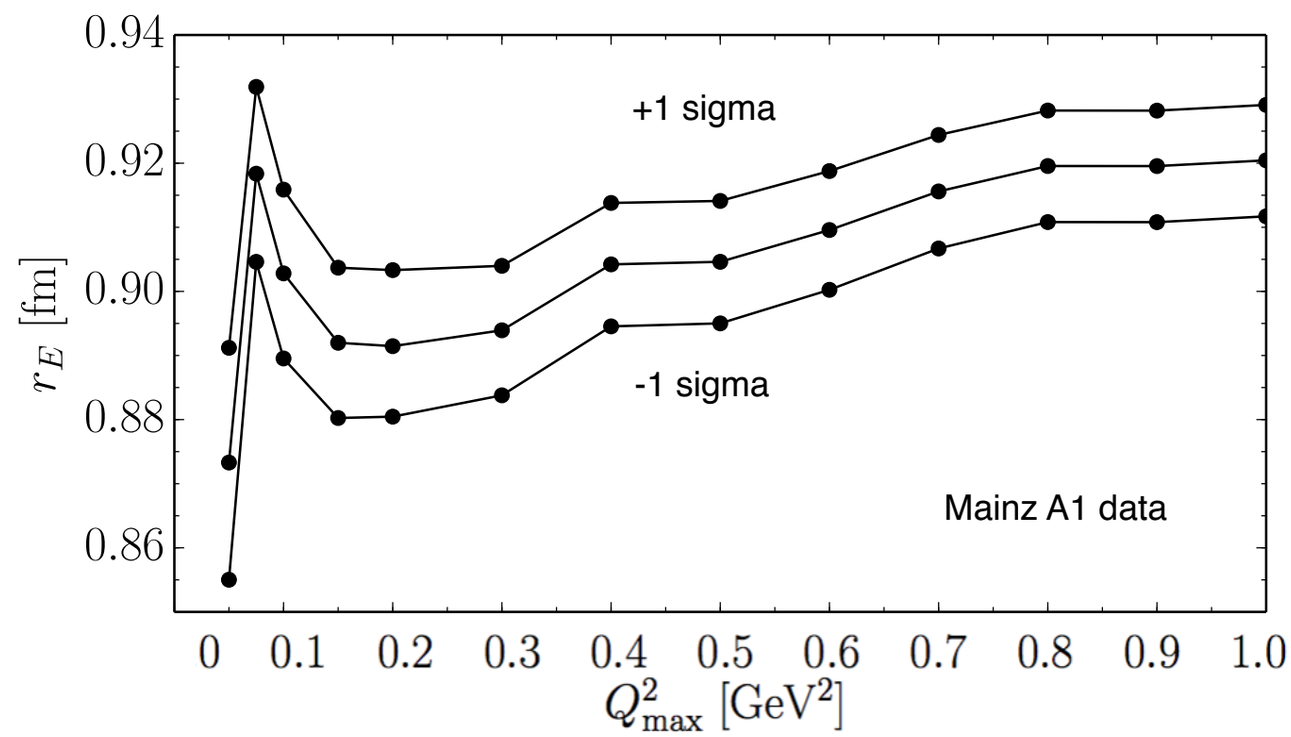

Over fitting the data!

- Toy study: over fitting the data is an issue, choosing arbitrary the fitting functional is tricky
  - Recover R
  - Does not recover higher order moments of ρ(r)