Isospin Breaking in τ input for $(g-2)$ from Lattice QCD

Mattia Bruno

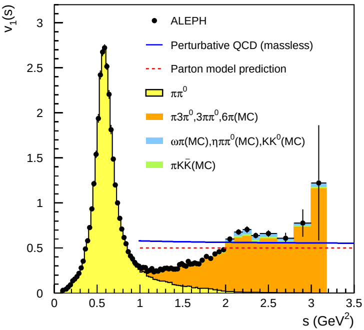
in collaboration with
T. Izubuchi, C. Lehner and A. Meyer

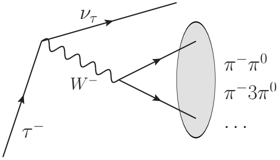
for the RBC/UKQCD Collaboration

BROOKHAVEN NATIONAL LABORATORY

Second Plenary Workshop of the Muon g-2 Theory Initiative
June 21st, 2018

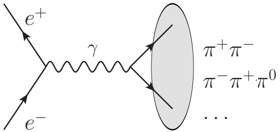
Motivations





$V - A$ current

Final states $I = 1$ charged



EM current

Final states $I = 0, 1$ neutral

τ data can improve $a_\mu[\pi\pi]$

→ $E \in [2m_\pi, m_\tau]$

→ 72% of total Hadronic LO

BROOKHAVEN NATIONAL LABORATORY

Isospin Corrections

Restriction to $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^-\pi^0\,\nu_\tau$

$$v_0(s) = \frac{s}{4\pi\alpha^2}\sigma_{\pi^+\pi^-}(s)$$

$$v_-(s) = \frac{m_\tau^2}{6|V_{ud}|^2}\frac{\mathcal{B}_{\pi\pi^0}}{\mathcal{B}_e}\frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds}\left(1-\frac{s}{m_\tau^2}\right)^{-1}\left(1+\frac{2s}{m_\tau^2}\right)^{-1}\frac{1}{S_{\rm EW}}$$

Isospin correction $v_0 = R_{\rm IB} v_-$ $\quad R_{\rm IB} = \frac{\rm FSR}{G_{\rm EM}}\frac{\beta_0^3|F_\pi^0|^2}{\beta_-^3|F_\pi^-|^2}$ [Alemani et al. '98]

0. $S_{\rm EW}$ electro-weak radiative correct. [Marciano, Sirlin '88][Braaten, Li '90]

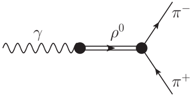
1. Final State Radiation of $\pi^+\pi^-$ system [Schwinger '89][Drees, Hikasa '90]

2. $G_{\rm EM}$ (long distance) radiative corrections in τ decays
Chiral Resonance Theory [Cirigliano et al. '01, '02]
Meson Dominance [Flores-Talpa et al. '06, '07]

3. Phase Space ($\beta_{0,-}$) due to $(m_{\pi^\pm} - m_{\pi^0})$

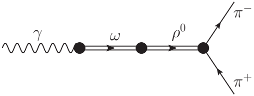
Pion form factors

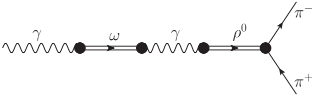
$$F_\pi^0(s) \propto \frac{m_\rho^2}{D_\rho(s)}$$



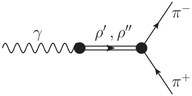
[Gounaris, Sakurai '68]
[Kühn, Santamaria '90]

$$\times \left[1 + \delta_{\rho\omega} \frac{s}{D_\omega(s)}\right]$$

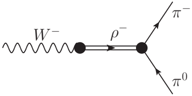




$$+ \frac{m_X^2}{D_X(s)} \qquad X = \rho' \,, \rho''$$



$$F_\pi^-(s) \propto \frac{m_{\rho^-}^2}{D_{\rho^-}(s)} + (\rho' \,, \rho'')$$



Sources of IB breaking in phenomenological models

$m_{\rho^0} \neq m_{\rho^\pm}$, $\Gamma_{\rho^0} \neq \Gamma_{\rho^\pm}$, $m_{\pi^0} \neq m_{\pi^\pm}$

$\rho - \omega$ mixing $\delta_{\rho\omega} \simeq O(m_\mathrm{u} - m_\mathrm{d}) + O(e^2)$

BROOKHAVEN NATIONAL LABORATORY

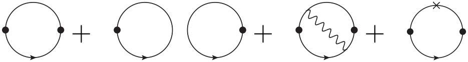
Contribution to a_μ

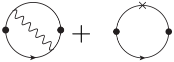
Time-momentum representation [Bernecker, Meyer, '11]

$$G^\gamma(t) = \frac{1}{3}\sum_k \int d\vec{x}\, \langle j_k^\gamma(x) j_k^\gamma(0)\rangle \quad \to \quad a_\mu = 4\alpha^2 \sum_t w_t G^\gamma(t)$$

Isospin decomposition of u, d current

$$j_\mu^\gamma = \frac{i}{6}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d\right) + \frac{i}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right) = j_\mu^{(0)} + j_\mu^{(1)}$$

$G_{00}^\gamma \leftarrow \langle j_k^{(0)}(x) j_k^{(0)}(0)\rangle =$  $\ldots$

$G_{01}^\gamma \leftarrow \langle j_k^{(0)}(x) j_k^{(1)}(0)\rangle =$  $\ldots$

$G_{11}^\gamma \leftarrow \langle j_k^{(1)}(x) j_k^{(1)}(0)\rangle =$  $\ldots$

Decompose $a_\mu = a_\mu^{(0,0)} + a_\mu^{(0,1)} + a_\mu^{(1,1)}$

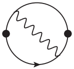
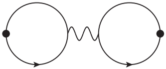
BROOKHAVEN NATIONAL LABORATORY

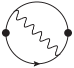
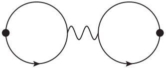
Neutral vs Charged

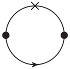
$$\frac{i}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right),\begin{bmatrix} I=1 \\ I_3=0 \end{bmatrix} \quad \rightarrow \quad j_\mu^{(1,-)} = \frac{i}{\sqrt{2}}\left(\bar{u}\gamma_\mu d\right),\begin{bmatrix} I=1 \\ I_3=-1 \end{bmatrix}$$

Isospin 1 charged correlator $G_{11}^{W} = \frac{1}{3}\sum_k \int d\vec{x}\,\langle j_k^{(1,+)}(x) j_k^{(1,-)}(0)\rangle$

$$\delta G^{(1,1)} \equiv G_{11}^{\gamma} - G_{11}^{W}$$

$$= Z_V^4 (4\pi\alpha)\frac{(Q_{\mathrm{u}} - Q_{\mathrm{d}})^4}{4}\Big[$$  $+$  $\Big]$

$$G_{01}^{\gamma} = Z_V^4 \frac{(Q_u^2 - Q_d^2)^2}{2}(4\pi\alpha)\Big[$$  $+\,2\times$  $+$  $+\dots\Big]$

$$+Z_V^2 \frac{Q_u^2 - Q_d^2}{2}(m_u - m_d)\Big[2\times$$  $+\dots\Big]$

$\dots$ = subleading diagrams currently not included

$\Delta a_\mu[\pi\pi, \tau]$

Restriction to 2π $\rightarrow$ neglect pure $I=0$ part $a_\mu^{(0,0)}[\pi^0\gamma, 3\pi\,, \ldots]$

$$\text{Lattice: } \Delta a_\mu[\pi\pi,\tau] = 4\alpha^2 \sum_t w_t \;\times\; \left[G^\gamma_{01}(t) + G^\gamma_{11}(t) - G^W_{11}(t) \right]$$

$$\text{Pheno: } \Delta a_\mu[\pi\pi,\tau] = \int_{4m_\pi^2}^{m_\tau^2} ds K(s) \;\left[\qquad v_0(s) \qquad - v_-(s) \right]$$

Conversion to Euclidean time for direct comparison

$$\Delta a_\mu[\pi\pi,\tau] = 4\alpha^2 \textstyle\sum_t w_t \times \left\{ \frac{1}{12\pi^2} \int d\omega\; \omega e^{-\omega t} \left[R_{\rm IB}(\omega^2) - 1 \right] v_-(\omega^2) \right\}$$

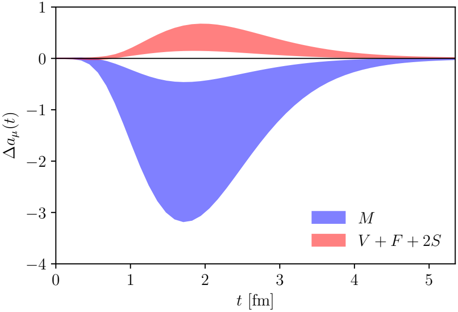
FSR, $G_{\rm EM}$ $\rightarrow$ (presently) not computed from lattice

required for direct comparison v_- vs G^W_{11}

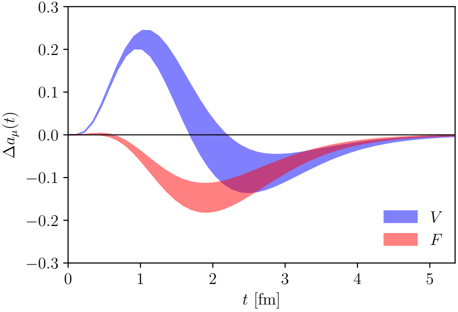
BROOKHAVEN NATIONAL LABORATORY

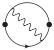
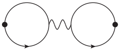
Lattice: Preliminary results

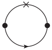
Δa_μ from G_{01}^{γ} (QED and SIB):



Pure $I = 1$ only $O(\alpha)$ terms:



$V =$  $F =$  $S =$ 

$M =$  $\rightarrow$ dominates noise

BROOKHAVEN NATIONAL LABORATORY

Conclusions

For detailed comparison lattice vs pheno:

study systematic errors $\rightarrow$ ongoing finite volume study

improvement of errs $\rightarrow$ high stat. data set from HLbL

Discussion points and prospects:

1. full lattice calculation of $\Delta a_\mu[\tau]$

2. lattice QCD calculation $\rightarrow$ various comparisons

comparison v_- with experiment requires FSR, S_{EW} and G_{EM}
$\rightarrow$ test of long distance QED corrections

study G_{01}^{γ} alone $\rightarrow$ $\rho - \omega$ mixing ?

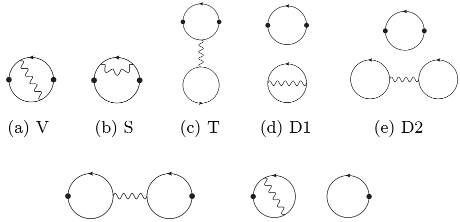
study $\delta G^{(1,1)}$ alone $\rightarrow$ ρ^0 vs ρ^- properties ?

Thanks for your attention

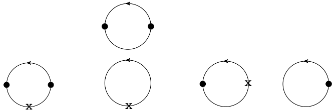
BROOKHAVEN NATIONAL LABORATORY

Full QED and SIB

[Blum et. al. '18]
[C. Lehner talk]



(a) V (b) S (c) T (d) D1 (e) D2 (f) F (g) D3



(a) M (b) R (c) O

Presently only leading diagrams are computed V, F, S, M [Blum et al. '18]

→ improving precision between 2 and 4 times

$SU(3)$ and $1/N_c$ diagrams presently not computed

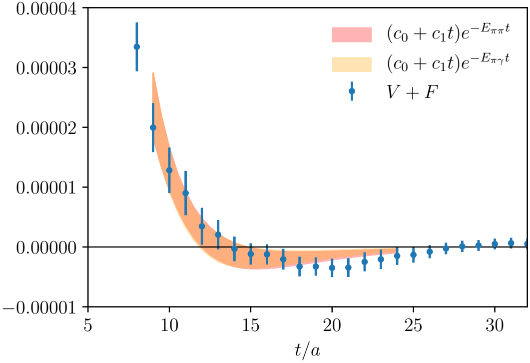
BROOKHAVEN NATIONAL LABORATORY

Peeking at the data - I

Lattice fully inclusive → comparison with v_- problematic

manipulate correlator to implement energy cut

fit lowest energy state $(c_0 + c_1 t)e^{-Et}$



lattice correlator more precise
at short distances

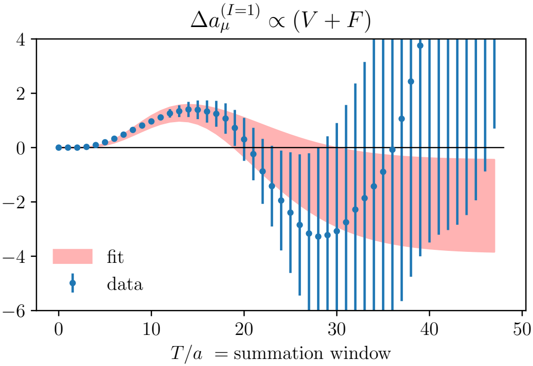
fit with fixed energy
$E_{\pi\pi^{I=1}}$, $E_{\pi\gamma}$

temporary solution: not
required with better precision

BROOKHAVEN NATIONAL LABORATORY

Peeking at the data - II

$\Delta a_\mu = 4\alpha^2 \sum_t w_t \; \delta G(t) \to$ weights suppress short distance



lattice correlator more precise at short distances

fit $(c_0 + c_1 t)e^{-Et}$

$E \to \pi\pi$ or $\pi\gamma$

reduction of stat. noise

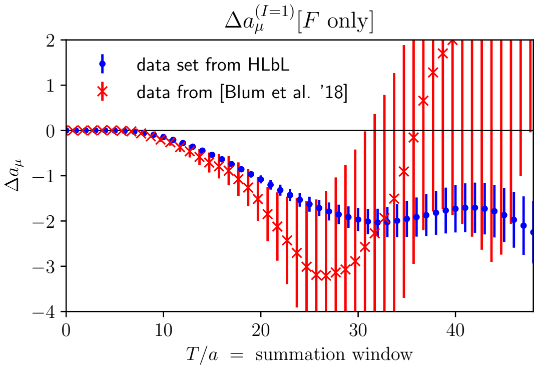
temporary solution: not required with better precision

BROOKHAVEN NATIONAL LABORATORY

Lattice improvements

Stat. improvements from data of HLbL project [Phys.Rev.Lett. 118 (2017)]

contribution of diagram F to pure $I = 1$ part of Δa_μ



$O(1000)$ point-src per conf.
$5 \cdot 10^5$ combinations
80 configurations

$\times 4$ reduction in error

finite volume errs relevant
→ dedicated study

data from [Blum et al. '18]: $O(500)$ point-src per conf.
76 configurations

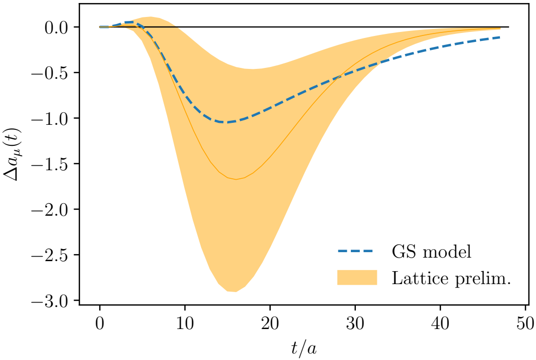
BROOKHAVEN NATIONAL LABORATORY

Example of comparison

Lattice complete contribution $\Delta a_\mu \propto (G^\gamma_{01} + \delta G^{(1,1)})$

For $[R_{\rm IB} - 1]v_-$ we use:

1. $\mathrm{FSR}\beta_0^3|F_\pi^0|^2 - \beta_-^3|F_\pi^-|^2$
2. we use GS model for $F_\pi^{0,-}$ inspired from [Davier at al. '10]



illustrative example:

comparison in Euclidean time

more accurate analysis required

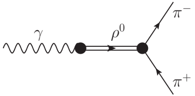
BROOKHAVEN NATIONAL LABORATORY

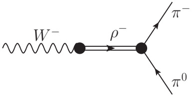
Separate studies

Potential to study $\rho^{\pm,0}$ properties and $\rho - \omega$ mixing: discussion points

---

$\delta G^{(1,1)} = G^{\gamma}_{11} - G^{W}_{11} \leftrightarrow$

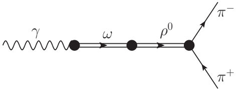


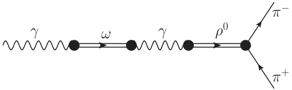


$$F_\pi^{0,-} = m_{\rho^{0,-}} D^{-1}_{\rho^{0,-}} + (\rho', \rho')\,; \quad D^{-1} \text{ à la Gounaris-Sakurai}$$

$\rightarrow$ can we expect $\delta_{\rho\omega} = 0$ to approximate $\delta G^{(1,1)}$

$\langle j_k^{(0)}(x) j_k^{(1)}(0) \rangle \leftrightarrow$





$\rightarrow$ can we expect $\dfrac{|F_\pi^0|^2}{|F_\pi^-|^2} = [1 + \delta_{\rho\omega} s D^{-1}_{KS,\omega}]$ to approximate G^{γ}_{01}?

BROOKHAVEN
NATIONAL LABORATORY