

Transition form factors and HLBL from Dyson-Schwinger equations

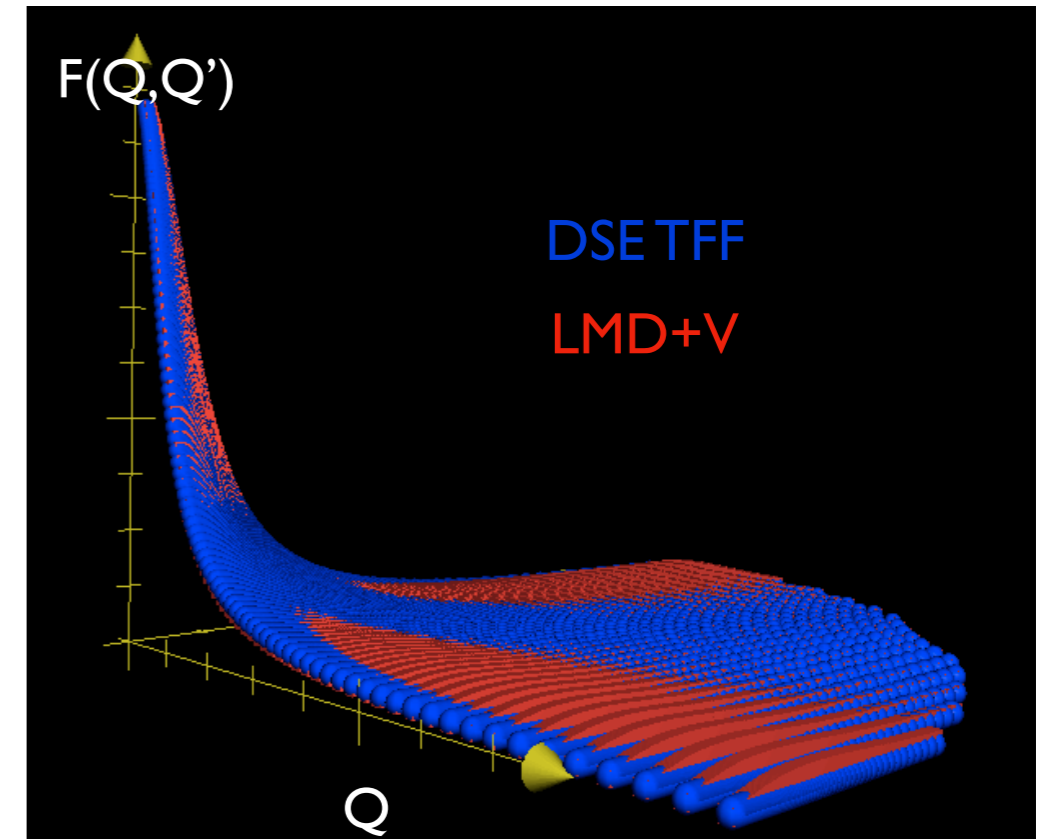
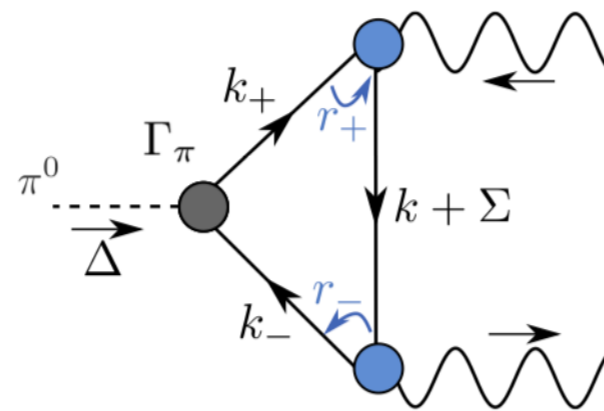
Christian S. Fischer

Justus Liebig Universität Gießen

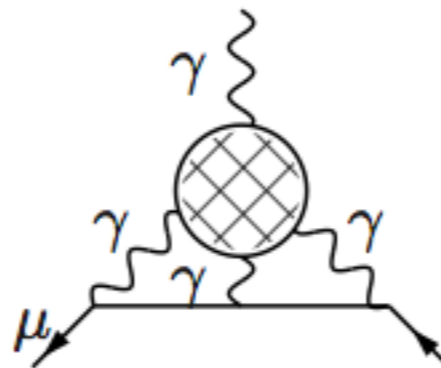
June 2018

Together with
Gernot Eichmann, Esther Weil, Richard Williams

1. (Transition-) form factors



2. Hadronic light by light



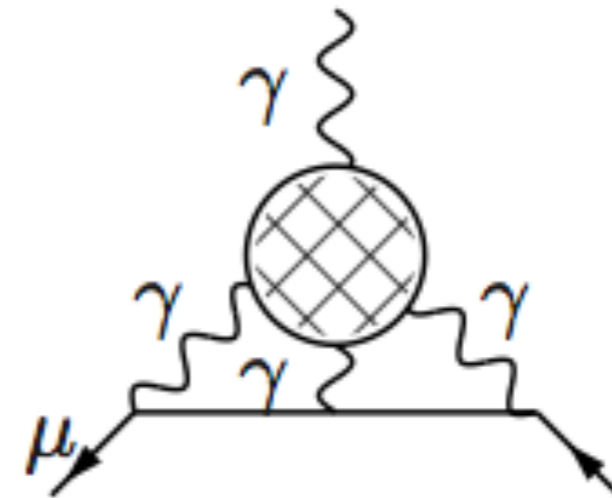
Nonperturbative QCD: Complementary approach

Quarks and gluons

- Lattice simulations
 - Ab initio
 - Gauge invariant
- Dyson-Schwinger Equations
 - Physical quark masses
 - Full momentum dependencies
 - Multi-scale problems feasible

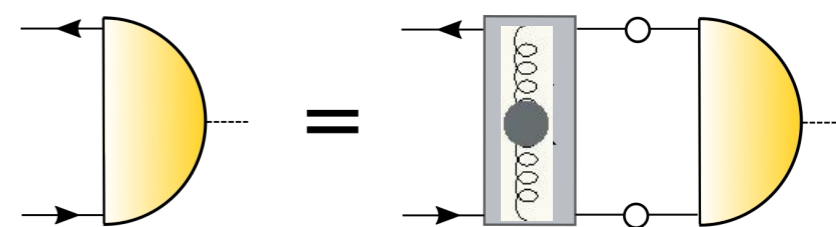
Hadrons

- Effective theories and models (χ PT, chiral models,...)
- Dispersive approach
 - Physical degrees of freedom

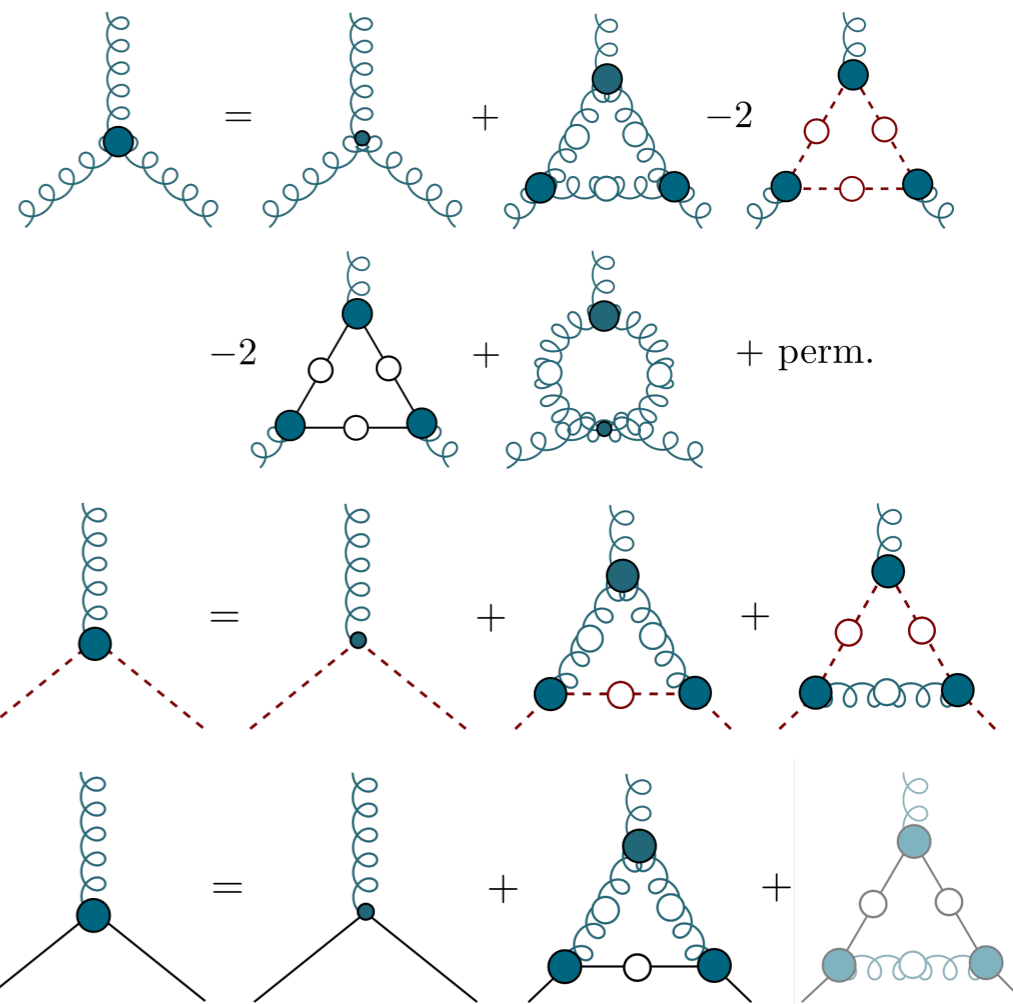
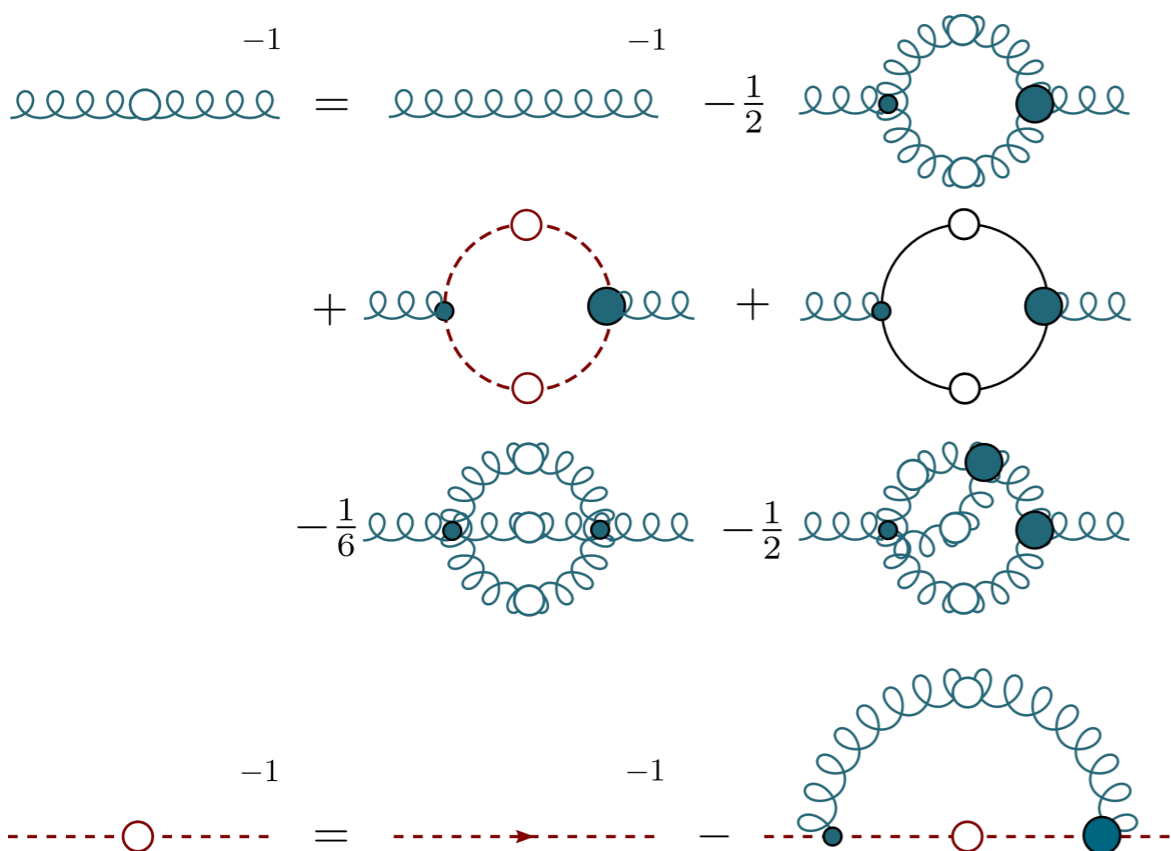


$$[S(p)]^{-1} = [-ip + M(p^2)]/Z_f(p^2)$$

Rainbow-Ladder (RL):



Beyond the rainbow (BRL):

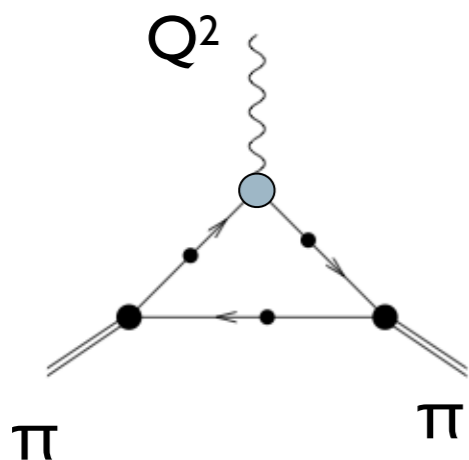


Williams, CF, Heupel, PRD 93 (2016) 034026
 CF, Williams, PRL 103 (2009) 122001

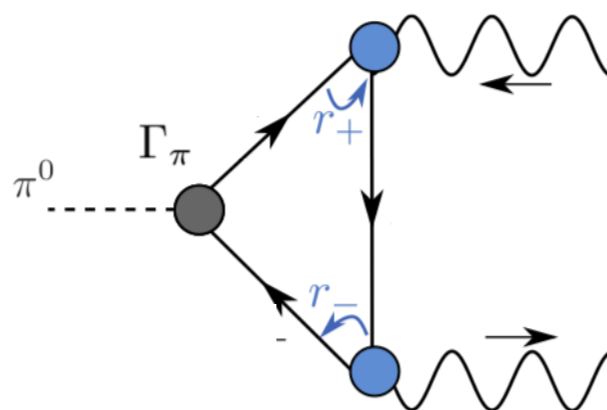
→ backup slides

(Transition-) Form factors and decays

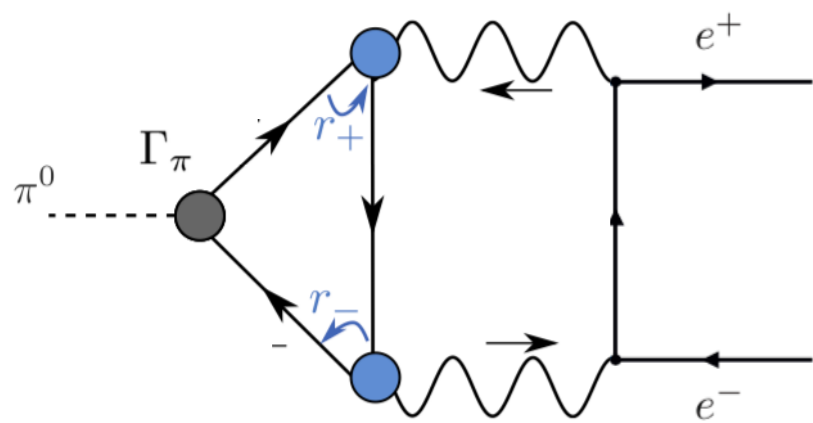
Pion form factor:



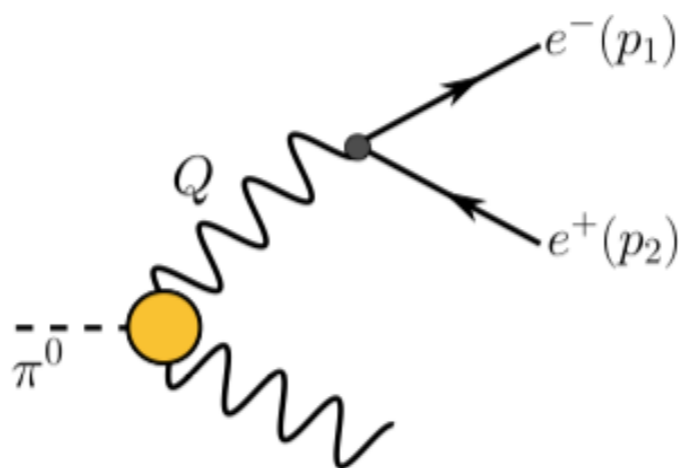
Pion transition FF



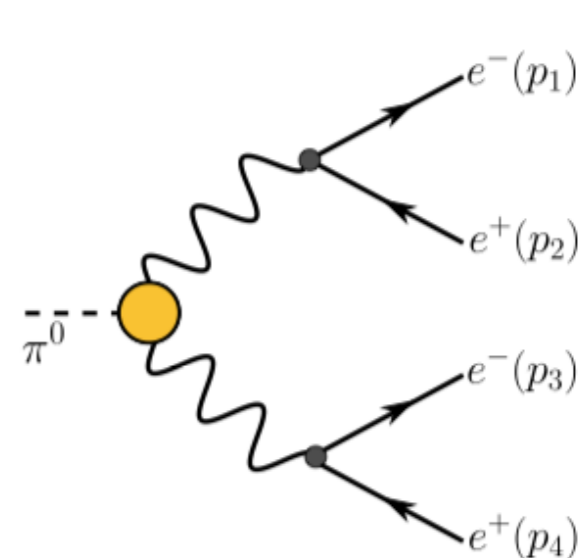
Rare pion decay $\pi^0 \rightarrow e^+e^-$:



$\pi_0 \rightarrow e^+e^-\gamma$

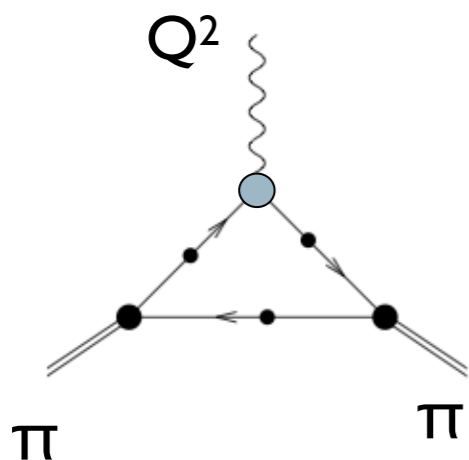


$\pi_0 \rightarrow e^+e^-e^+e^-$

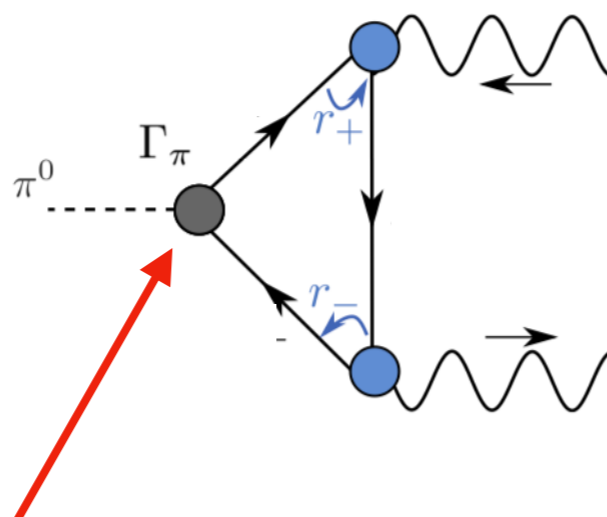


(Transition-) Form factors and decays

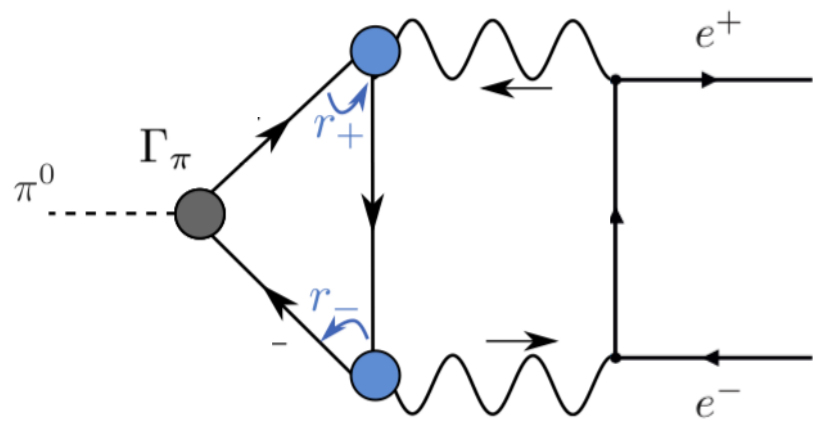
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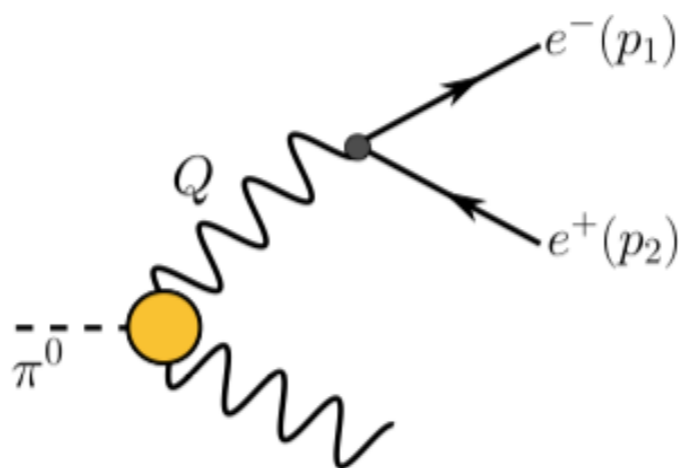
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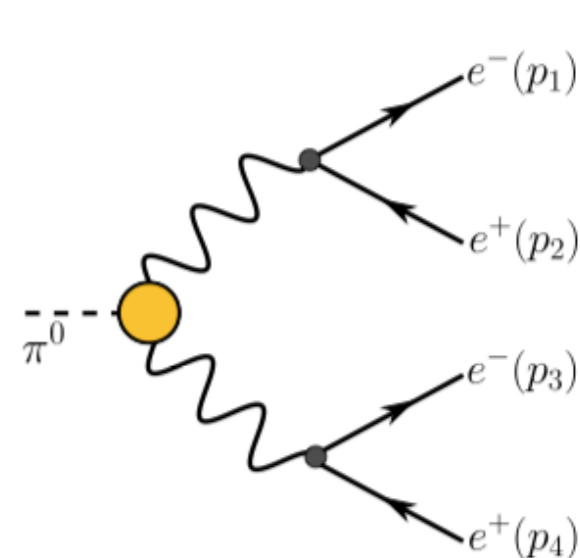
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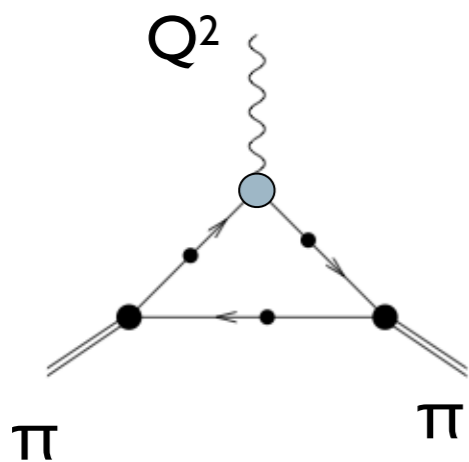


$\pi_0 \rightarrow e^+e^-e^+e^-$

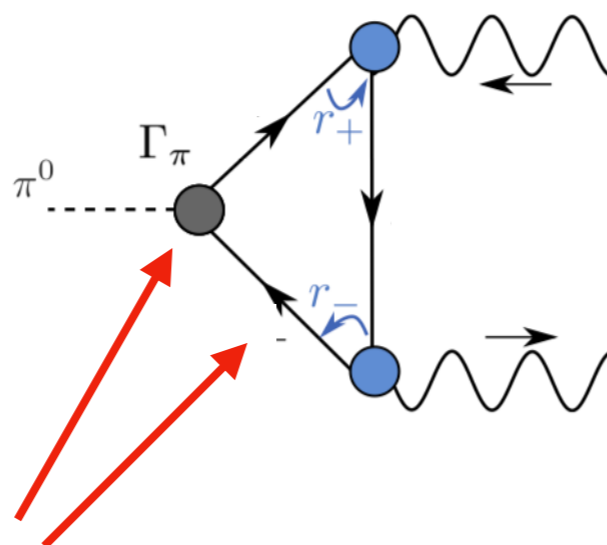


(Transition-) Form factors and decays

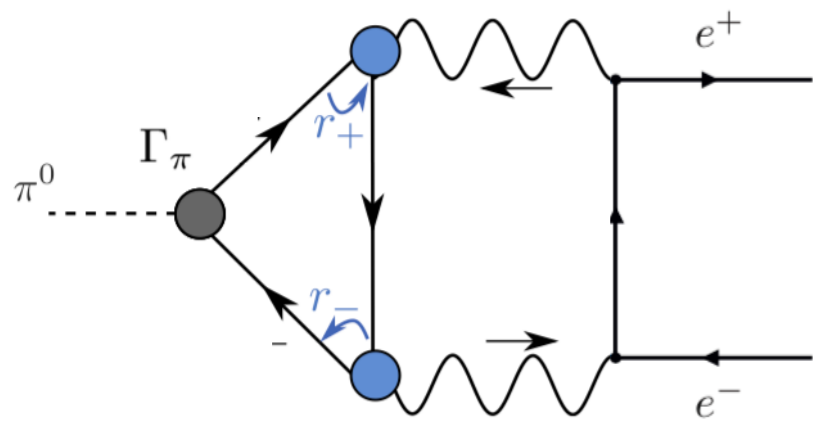
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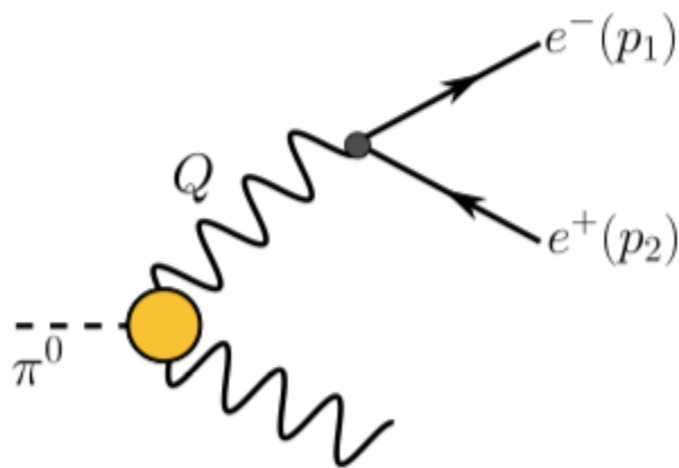
Pion transition FF



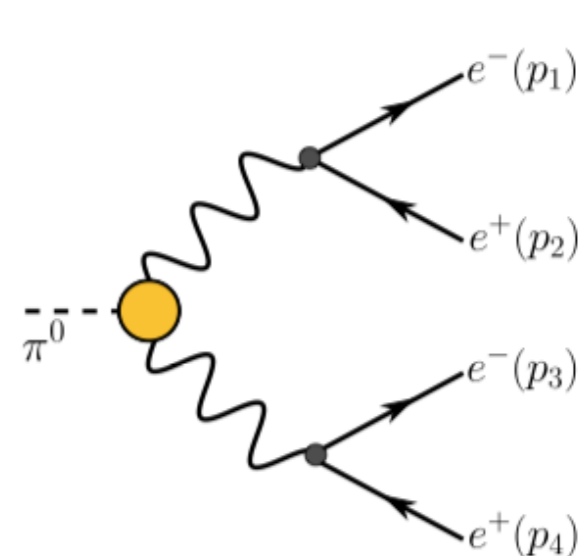
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$\pi_0 \rightarrow e^+e^- \gamma$

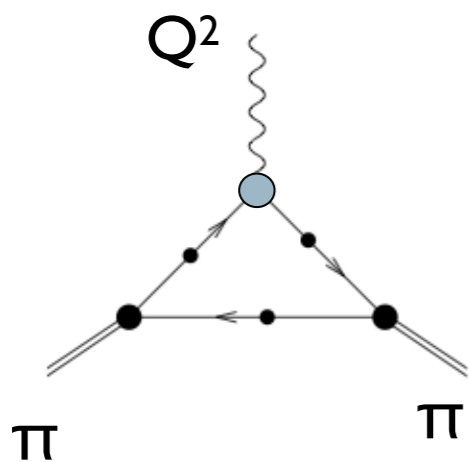


$\pi_0 \rightarrow e^+e^-e^+e^-$

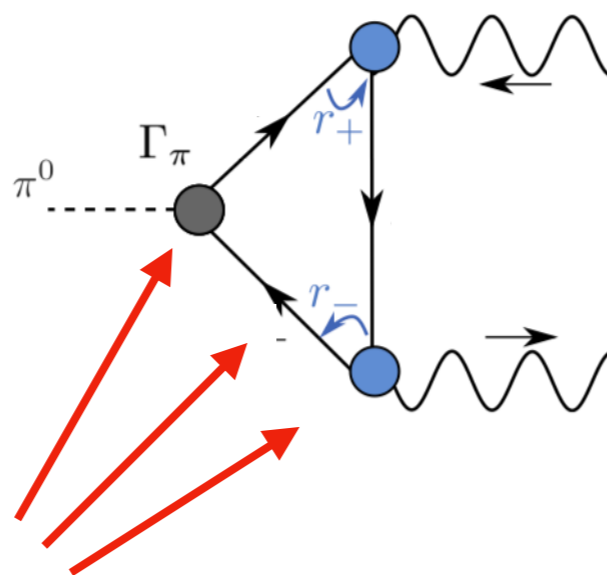


(Transition-) Form factors and decays

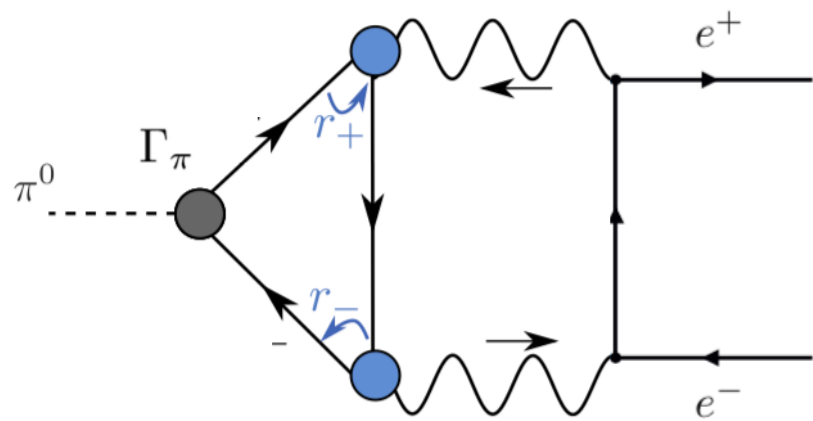
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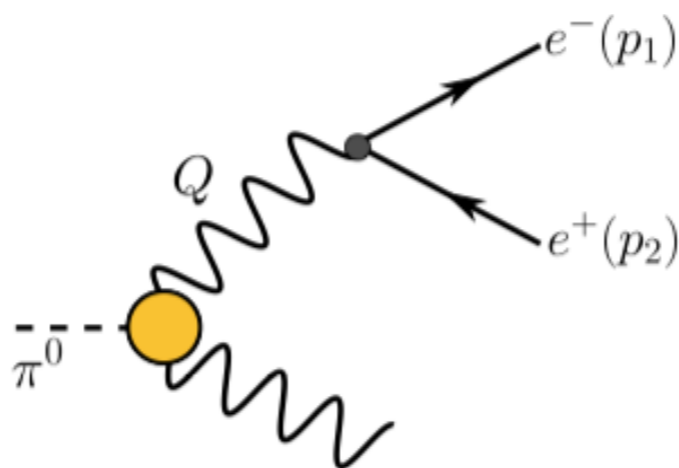
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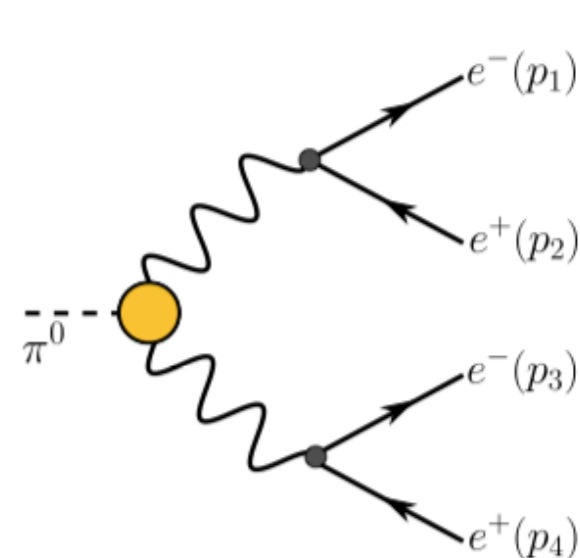
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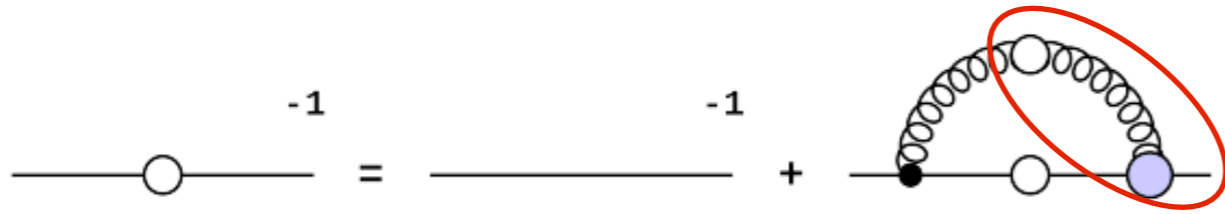
$\pi_0 \rightarrow e^+e^-\gamma$



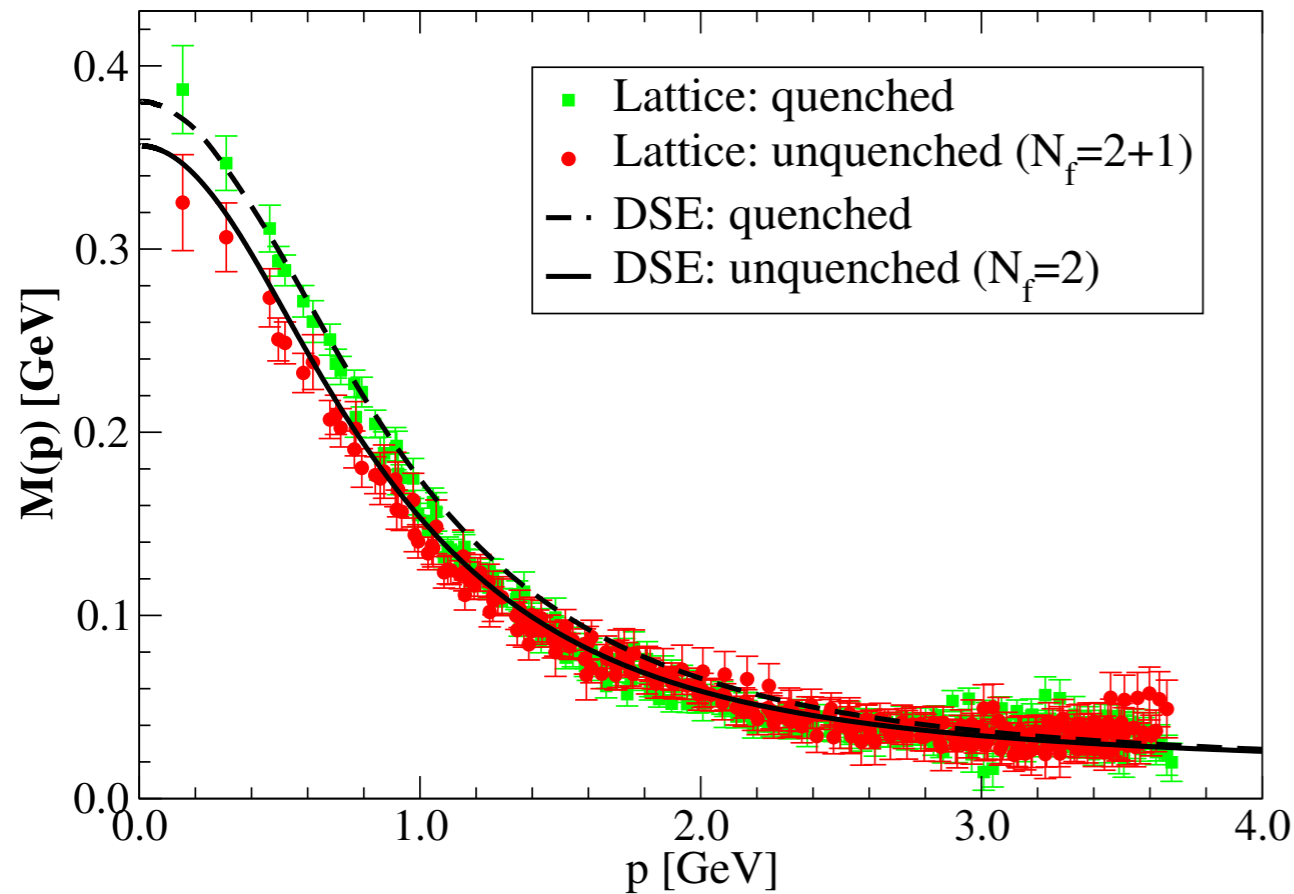
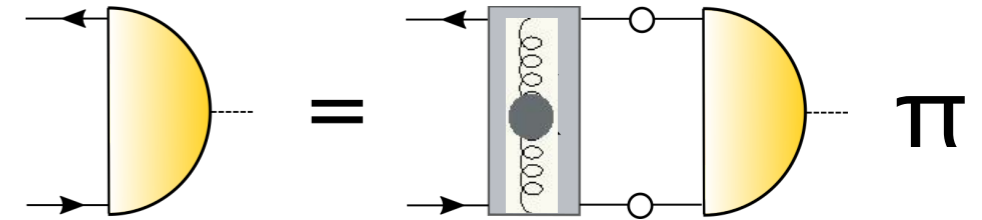
$\pi_0 \rightarrow e^+e^-e^+e^-$



Quark mass and pion BSE



$$[S(p)]^{-1} = [-i\not{p} + M(p^2)]/Z_f(p^2)$$

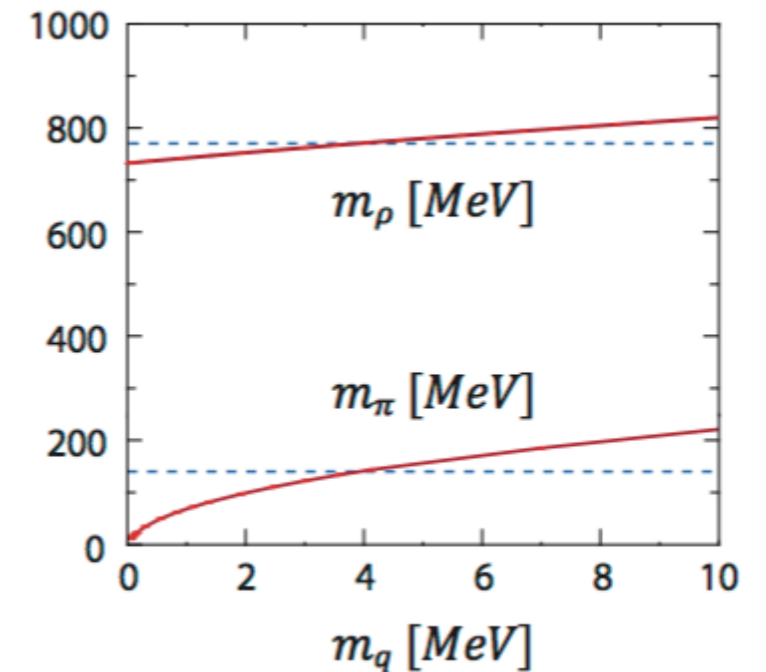


CF, Nickel, Williams, EPJ C 60 (2009) 47

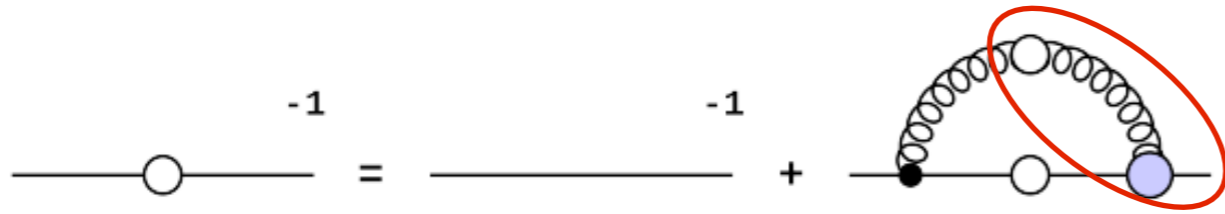
● Dynamical mass: $M \approx 350 \text{ MeV}$

● $\langle \bar{\Psi}\Psi \rangle \approx (250 \text{ MeV})^3$

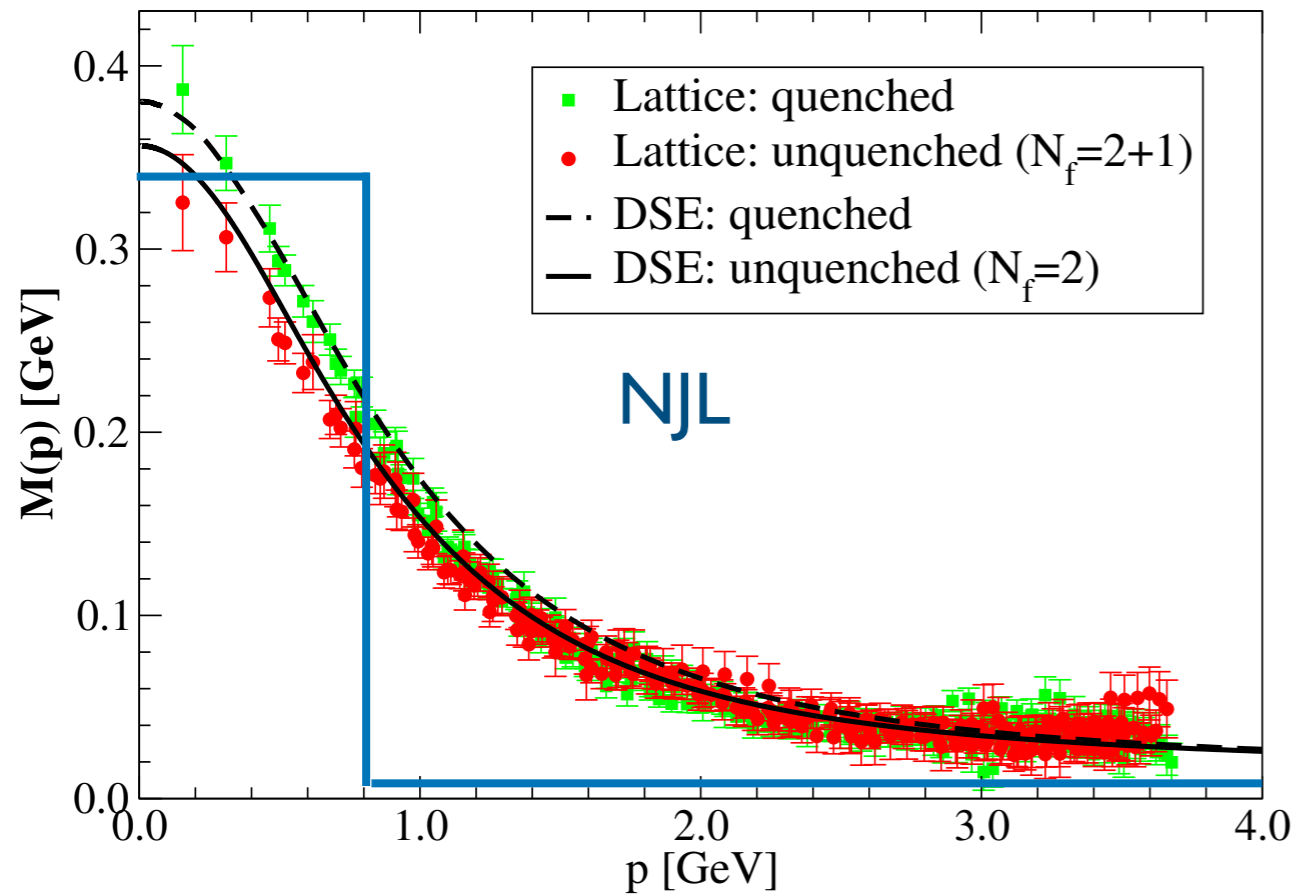
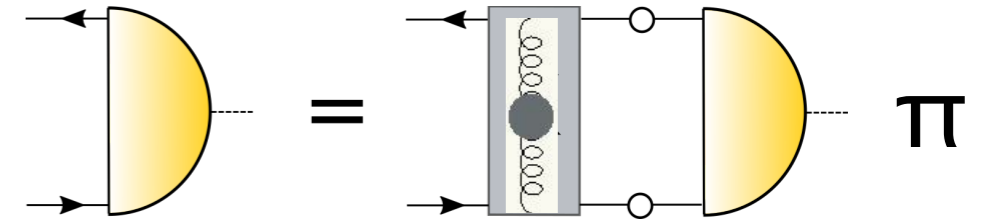
● GMOR



Quark mass and pion BSE



$$[S(p)]^{-1} = [-i\not{p} + M(p^2)]/Z_f(p^2)$$

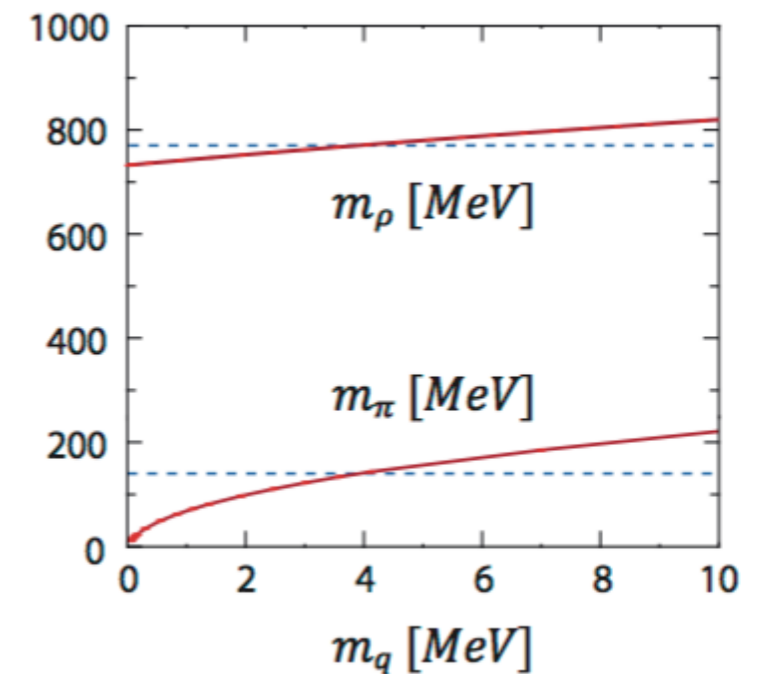


CF, Nickel, Williams, EPJ C 60 (2009) 47

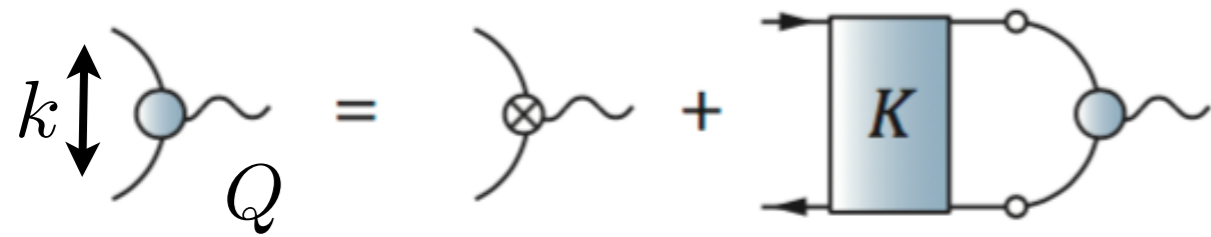
● Dynamical mass: $M \approx 350 \text{ MeV}$

● $\langle \bar{\Psi}\Psi \rangle \approx (250 \text{ MeV})^3$

● GMOR



Quark-photon vertex and dynamical vector mesons



Basis:

$$\{\gamma^\mu, Q^\mu, k^\mu\} \otimes \{\mathbb{1}, \not{Q}, \not{k}, \not{Q}\not{k}\}$$

→ 12 elements

$$\Gamma^\mu(k, Q) = \Gamma_{\text{BC}}^\mu(k, Q) + \Gamma_{\text{T}}^\mu(k, Q) = \sum_{i=1,4} \lambda_i L_i^\mu + \sum_{i=1,8} \tau_i T_i^\mu$$

gauge part
‘Ball-Chiu’

transverse part
→ vector-mesons

Ball and Chiu, PRD 22 (1980) 2542.

WTI: $Q^\mu \Gamma^\mu(k, Q) = S^{-1}(k + Q/2) - S^{-1}(k - Q/2)$

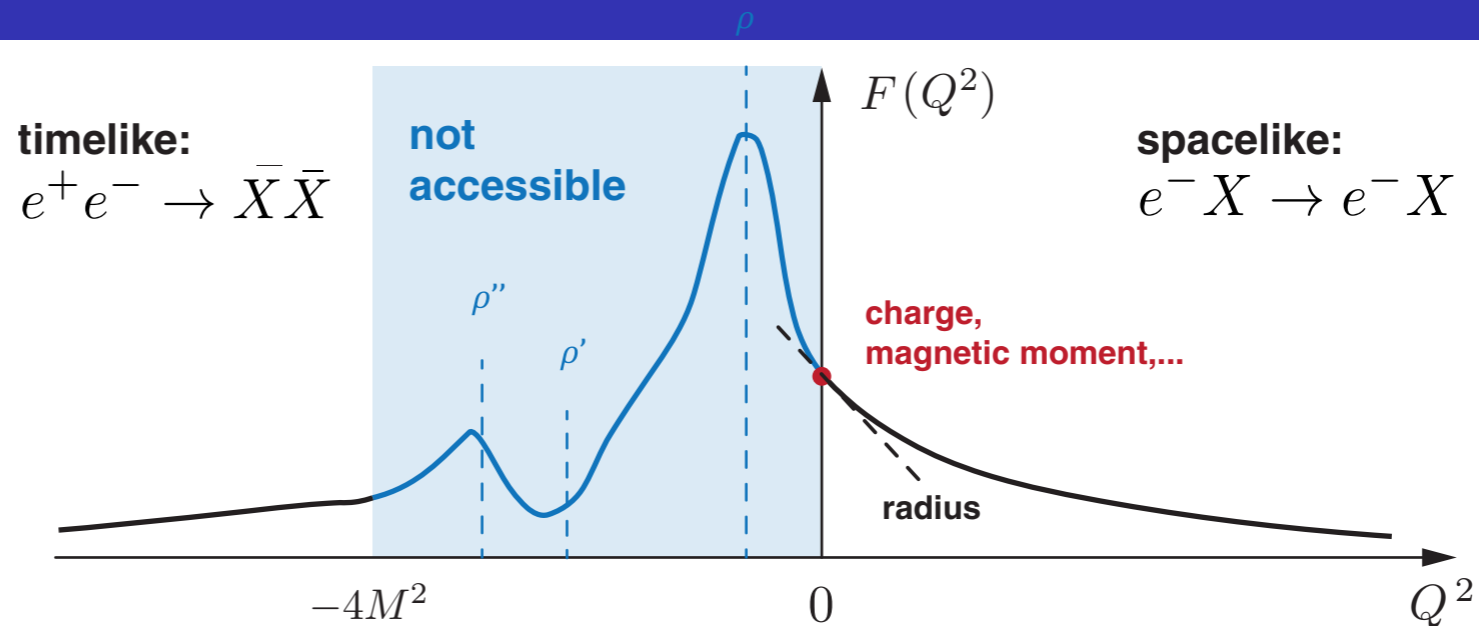
EM gauge invariance satisfied

✓

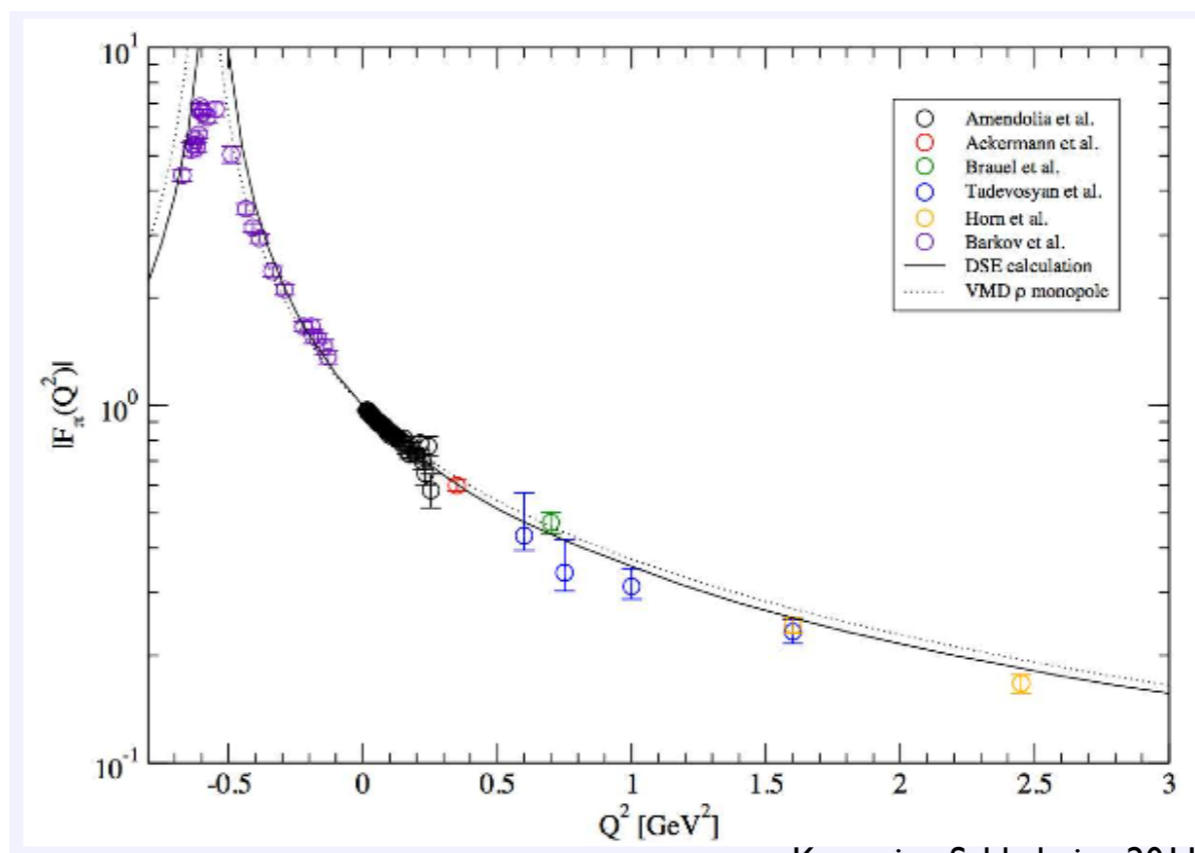
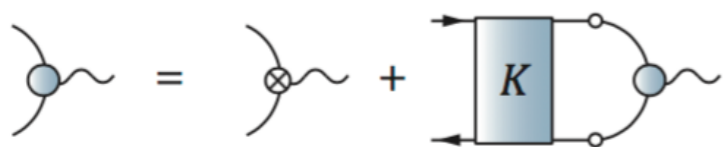
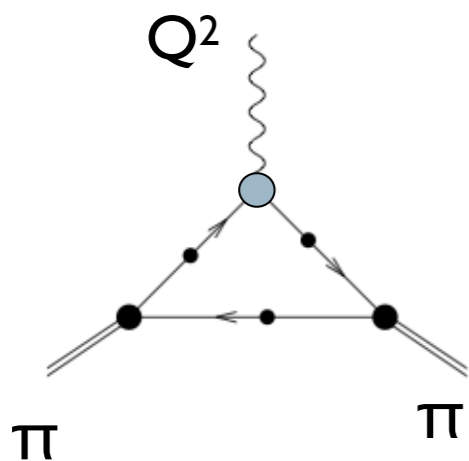
Vector mesons: dynamically generated

✓

Pion form factor



Pion form factor:



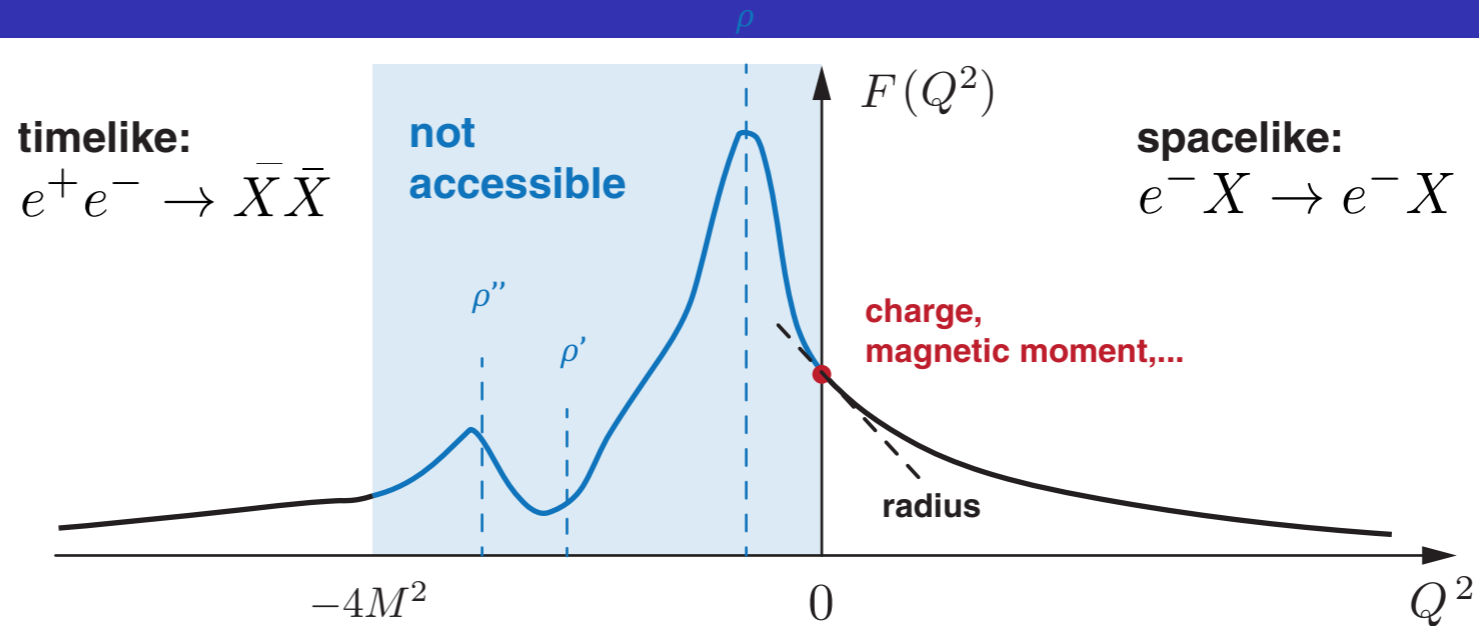
Krassnigg, Schladming 2011; Maris, Tandy NPPS 161, 2006

Vector meson poles dynamically generated!

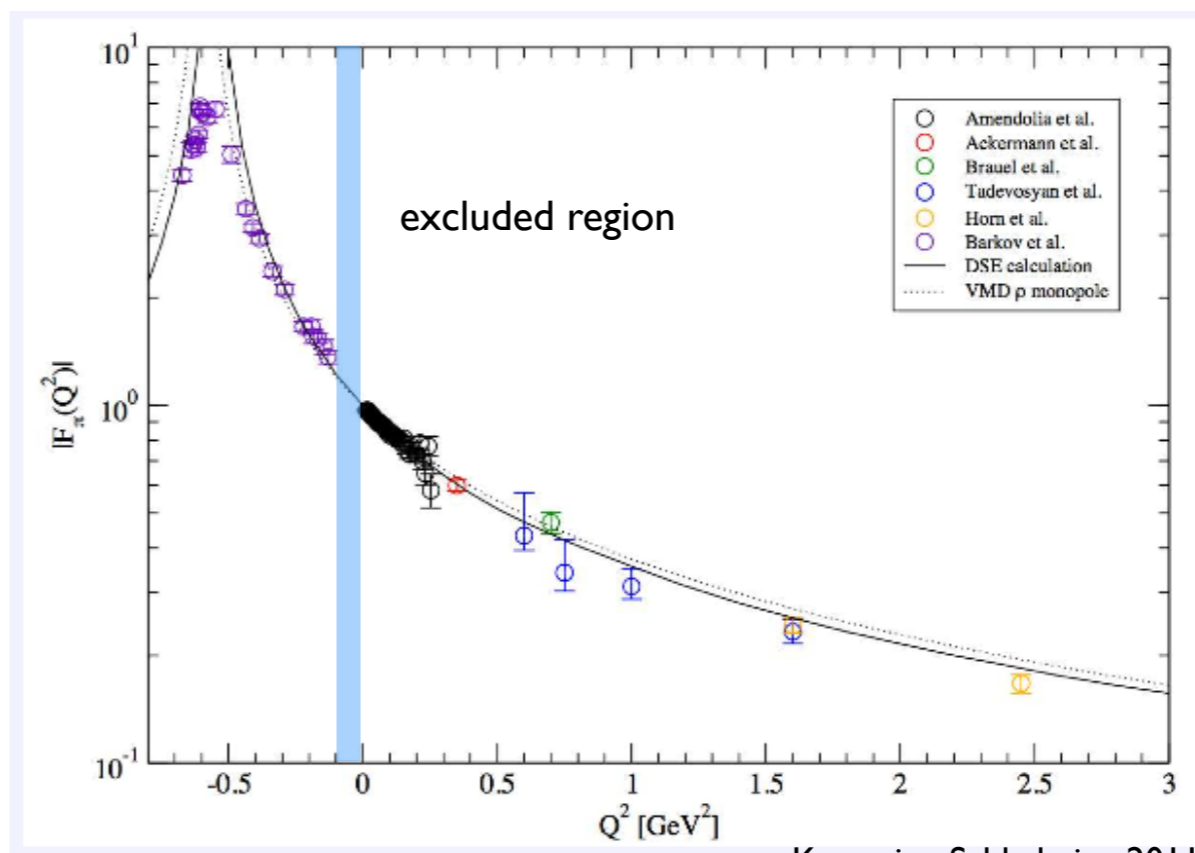
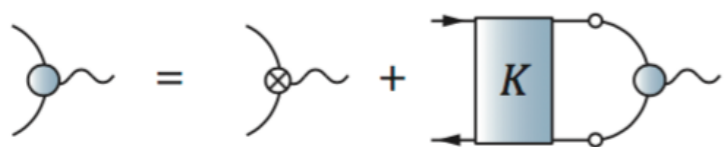
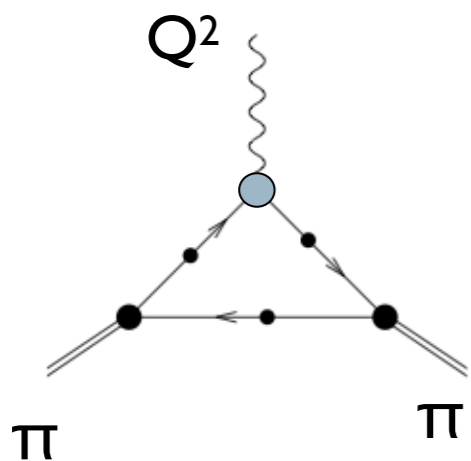
$\rho \rightarrow \pi\pi$: backup slides

Williams, arXiv:1804.11161

Pion form factor



Pion form factor:



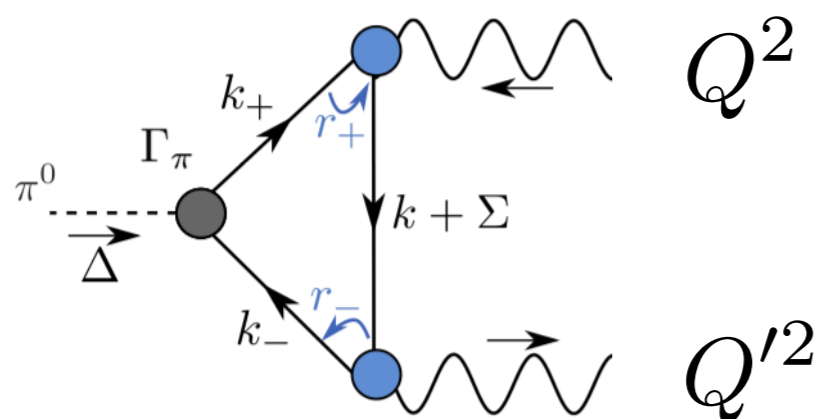
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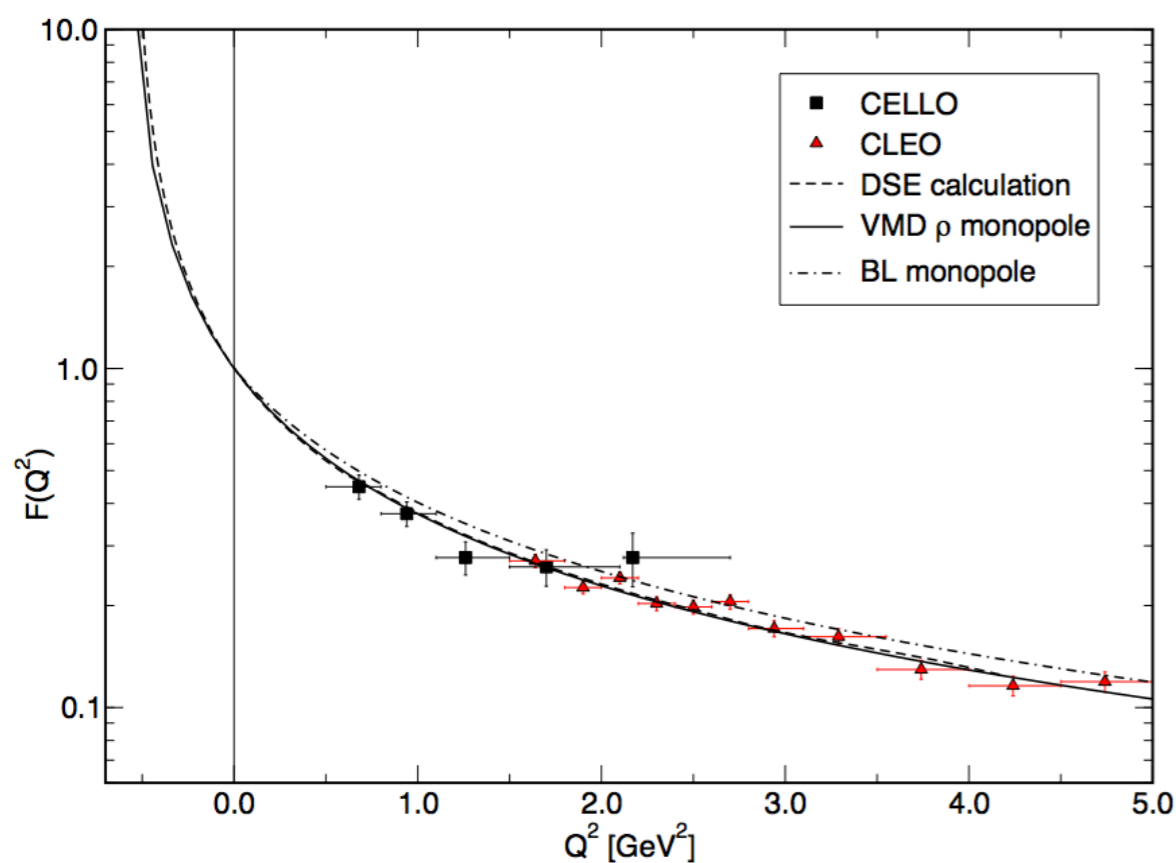
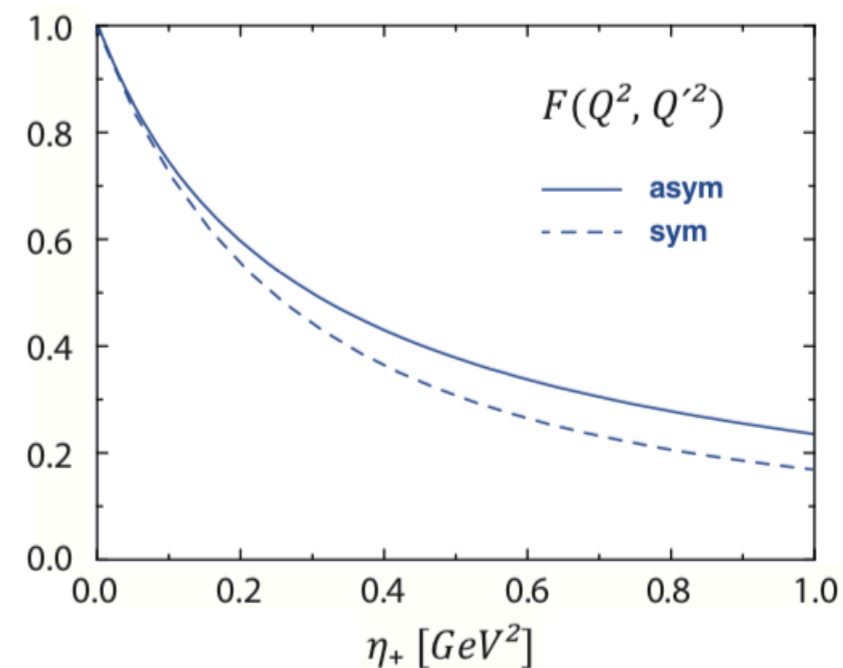
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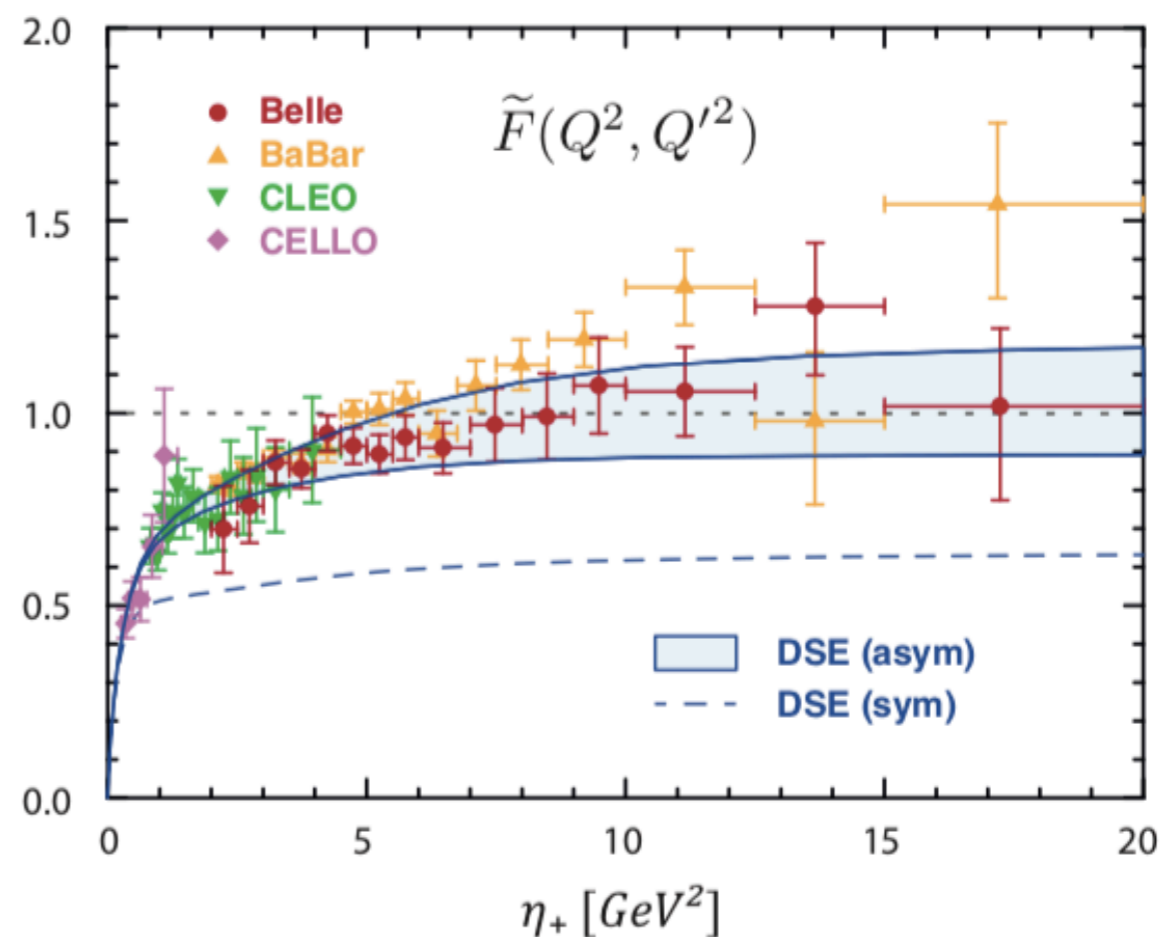
Pion transition form factor



$$\eta_+ = \frac{Q^2 + Q'^2}{2}$$



Maris, Tandy, Phys. Rev. C 65 045211 (2002)



Eichmann, CF, Weil and Williams, PLB 774 (2017) 425

Pion transition form factor

$$F(Q^2, Q'^2) = \frac{\mathcal{A}(w) + w(1 - z^2) \mathcal{B}_1(w) (1 + \mathcal{B}_2(w) z^2)}{(1 + w)^2 - w^2 z^2}$$

$$w = (Q^2 + Q'^2)/2$$

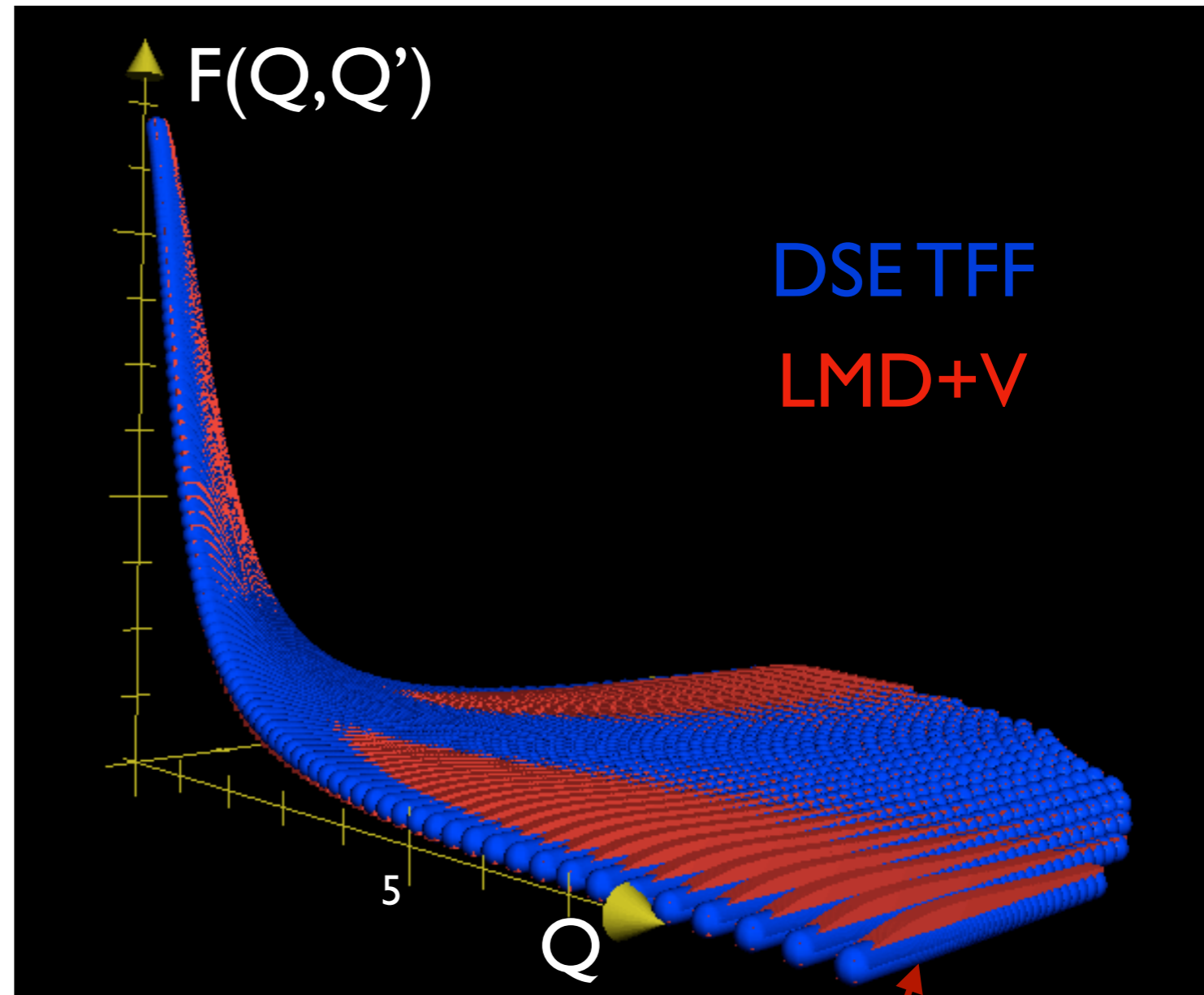
$$z = (Q^2 - Q'^2)/(Q^2 + Q'^2)$$

$$\mathcal{A}(w) = \frac{a_0 + \xi (a_1 b_1 w + a_2 b_2 w^2 + a_3 b_3 w^3)}{1 + b_1 w + b_2 w^2 + b_3 w^3},$$

$$\mathcal{B}_i(w) = \frac{c_i e_i w^2}{1 + d_i w + e_i w^2}$$

with fit parameters $a_0 = 0.996$ and

$a_1 = 0.735,$	$b_1 = 0.089,$
$a_2 = 1.214,$	$b_2 = 0.133,$
$a_3 = 1.547,$	$b_3 = 0.0002,$
$c_1 = 0.384,$	$c_2 = 0.430,$
$d_1 = 2.010,$	$d_2 = 0.024,$
$e_1 = 1.540,$	$e_2 = 0.00005.$



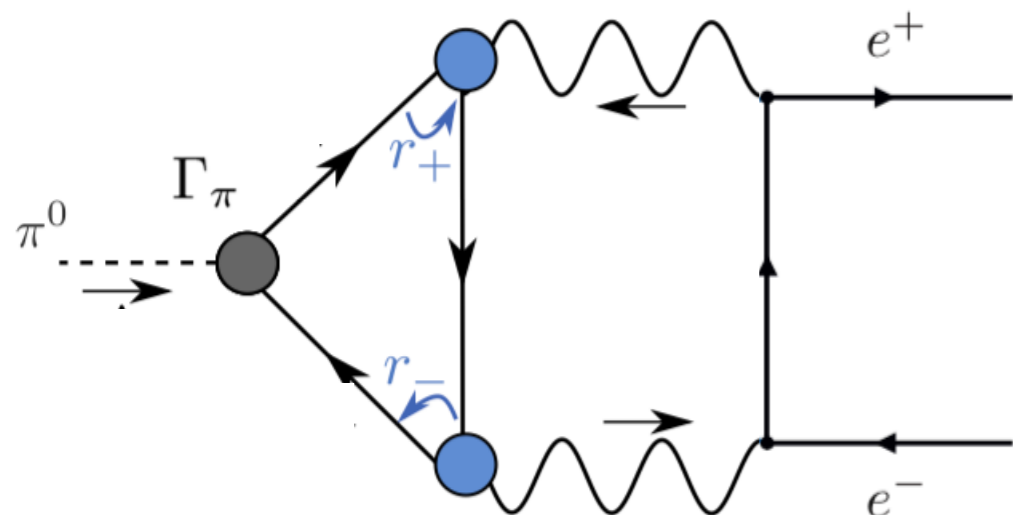
Eichmann, CF, Weil and Williams, PLB 774 (2017) 425

- agrees with LMD+V at small Q
- correct symm./asymm. UV limits
- no 'wiggles' at large Q in contrast to LMD+V

Scalar + axial vector transition from factors

Rare pion decay

Rare pion decay $\pi^0 \rightarrow e^+e^-$:



$$\frac{B(\pi_0 \rightarrow e^+e^-)}{B(\pi_0 \rightarrow \gamma\gamma)} = 2 \left(\frac{m\alpha_{em}}{\pi m_\pi} \right)^2 \sqrt{1 - 4 \frac{m^2}{m_\pi^2}} |A(-m_\pi^2/4)|^2$$

Usual: dispersive approach
DSE: direct calculation

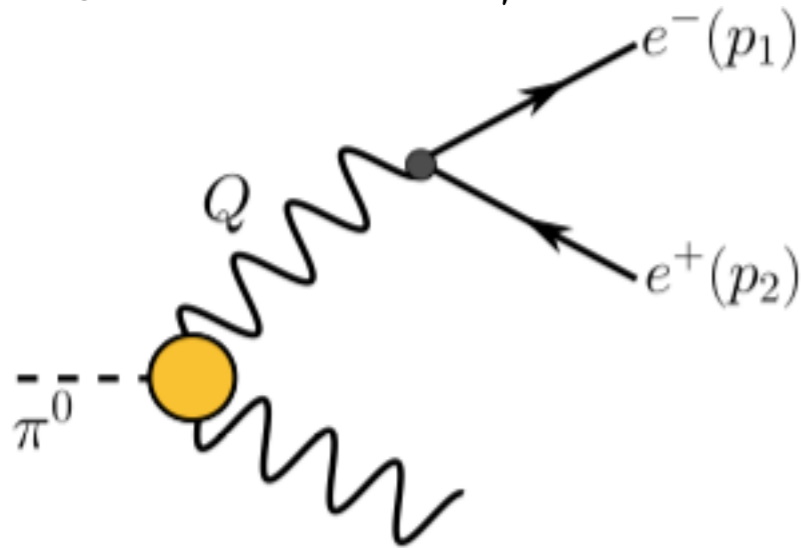
Collaboration	$B(\pi^0 \rightarrow e^+e^-) [10^{-8}]$
Experiment [1, 27, 28]	6.87(36)
Dorokhov et al. [2, 26]	6.23(9)
Husek et al. [35](THS)	6.14(8)
Masjuan et al. [29]	6.23(5)
Our result (DR)	6.21(3)
Our result (direct)	6.22(3)

- same result as everybody else
- discrepancy with exp. remains

Weil, Eichmann, CF and Williams, PRD96 (2017) no.1, 014021

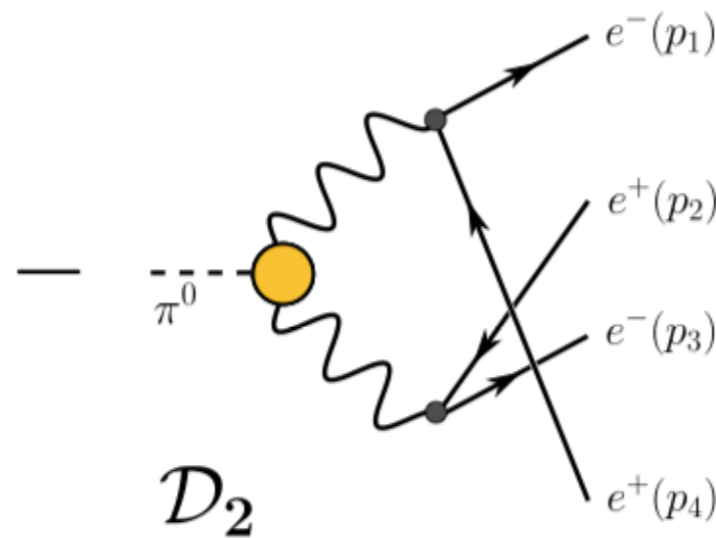
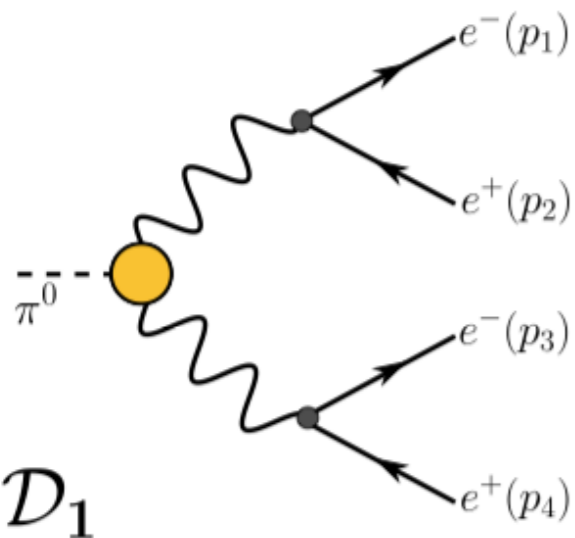
More rare decays

$$\pi_0 \rightarrow e^+ e^- \gamma$$



Collaboration	$\Gamma_{\pi^0 \rightarrow e^+ e^- \gamma} [10^{-11} \text{ GeV}]$
PDG [21]	9.06(18)
Terschlüsen et al. [22]	9.26
Hoferichter et al. [23]	9.065
Our result	9.11(4)

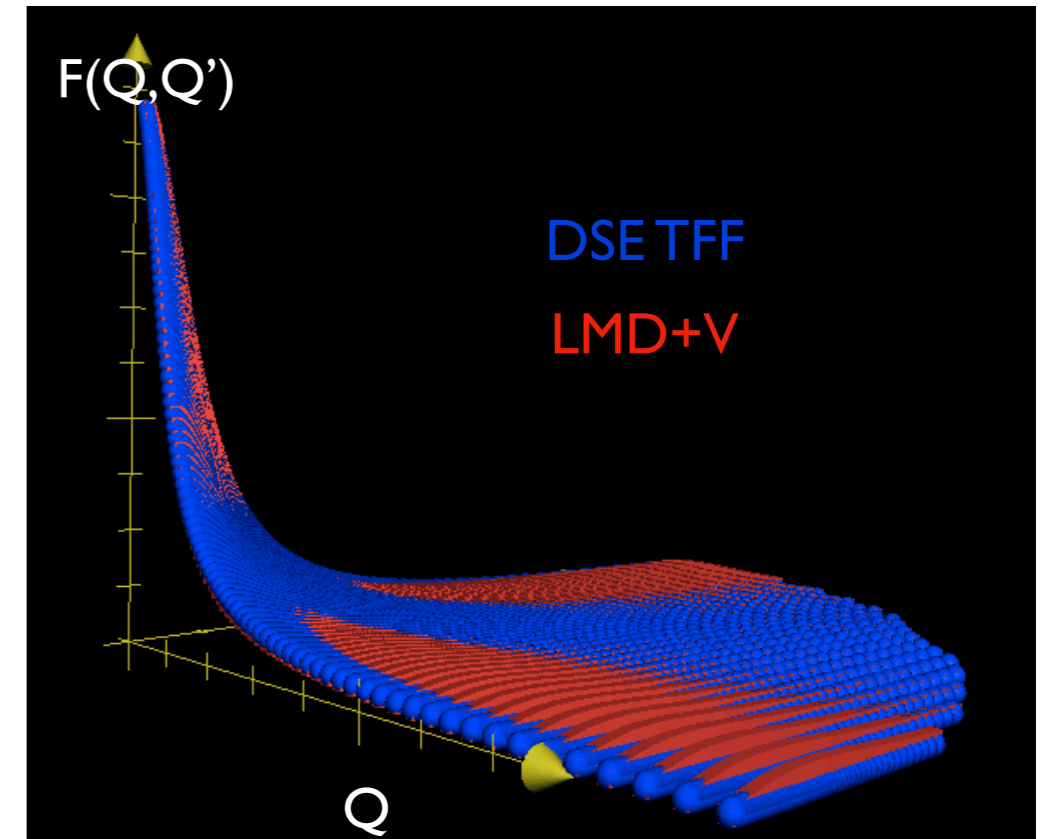
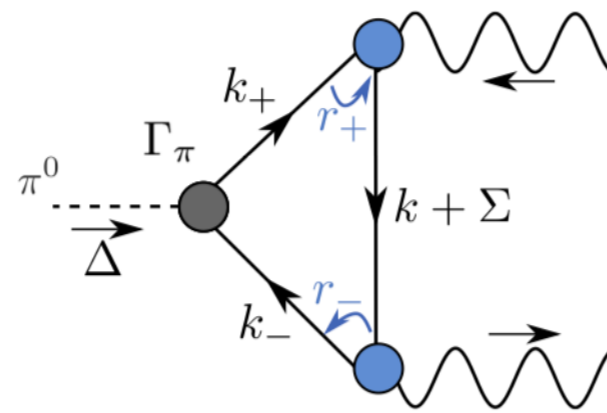
$$\pi_0 \rightarrow e^+ e^- e^+ e^-$$



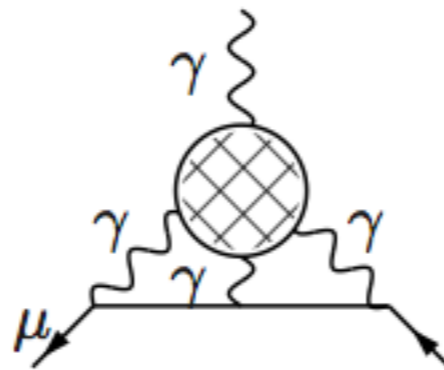
Collaboration	$\Gamma_{\pi^0 \rightarrow 2e^+ 2e^-} [10^{-13} \text{ GeV}]$
PDG [21]	2.58(12)
Terschlüsen et al. [22]	2.68
Escribano et al. [24]	2.62
Our result	2.63(1)

Weil, Eichmann, CF and Williams, Phys.Rev. D96 (2017) no.1, 014021

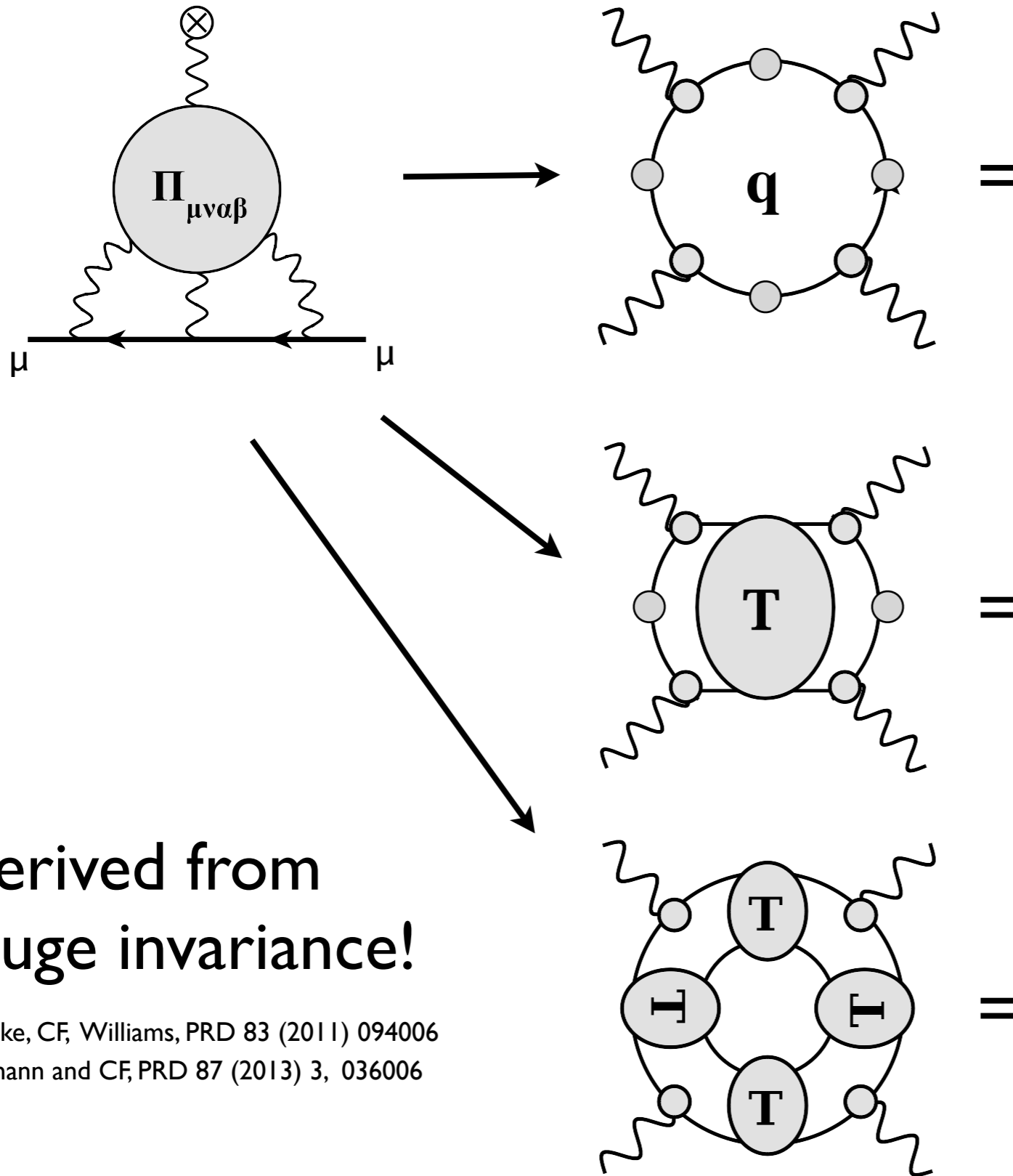
1. (Transition-) form factors



2. Hadronic light by light



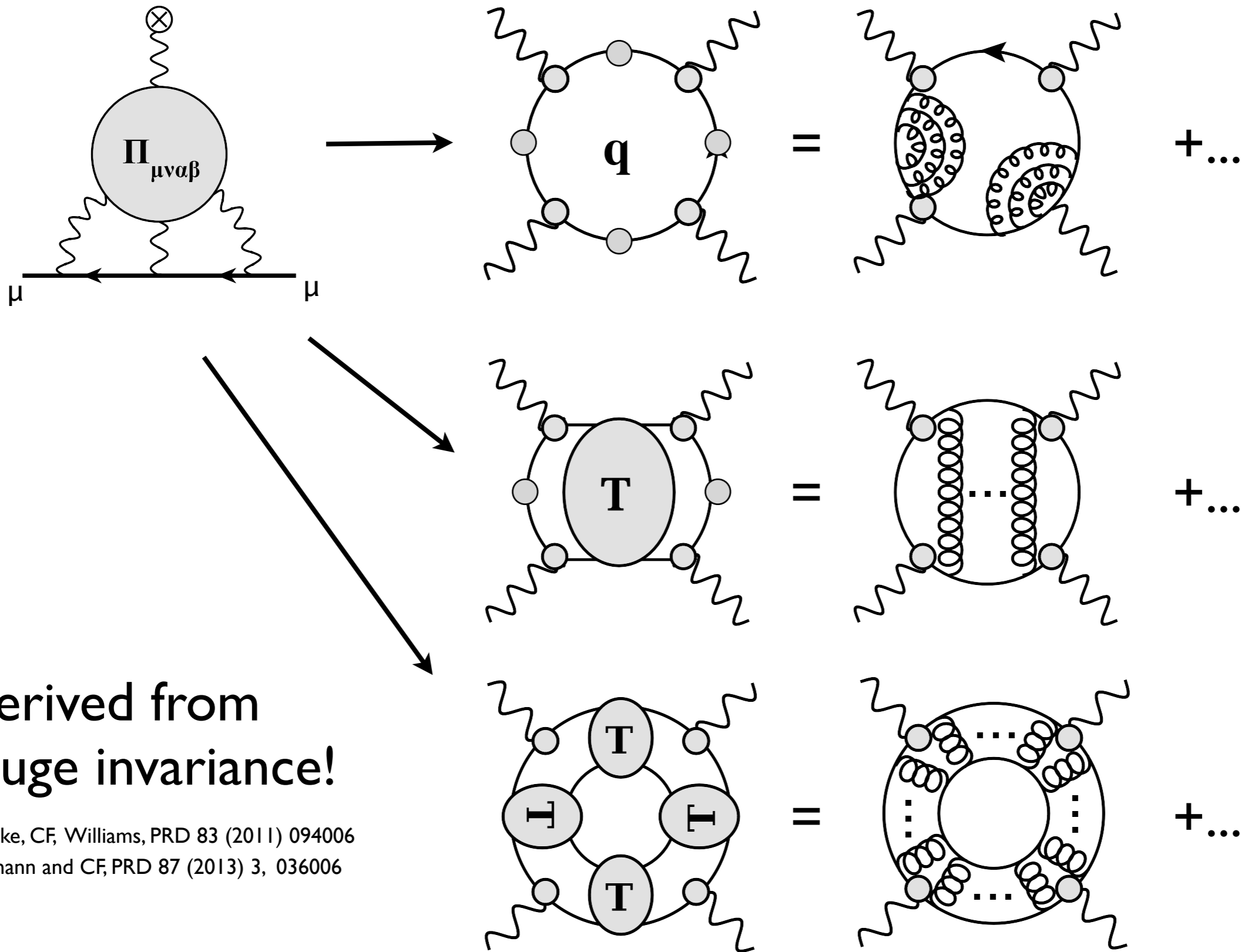
Hadronic Light-by-Light scattering



Goecke, CF, Williams, PRD 83 (2011) 094006

Eichmann and CF, PRD 87 (2013) 3, 036006

Hadronic Light-by-Light scattering

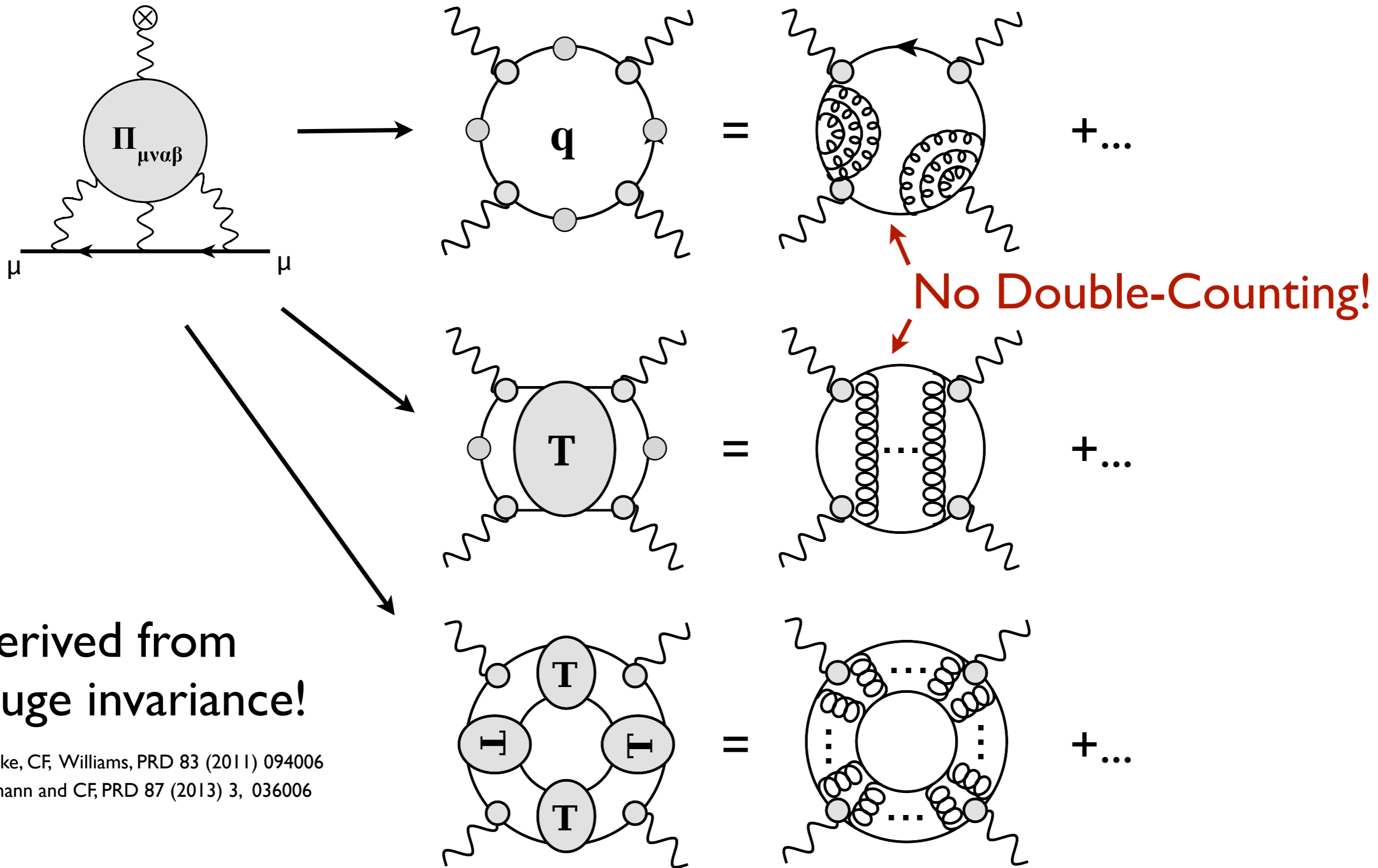


Derived from
gauge invariance!

Goecke, CF, Williams, PRD 83 (2011) 094006

Eichmann and CF, PRD 87 (2013) 3, 036006

Hadronic Light-by-Light scattering

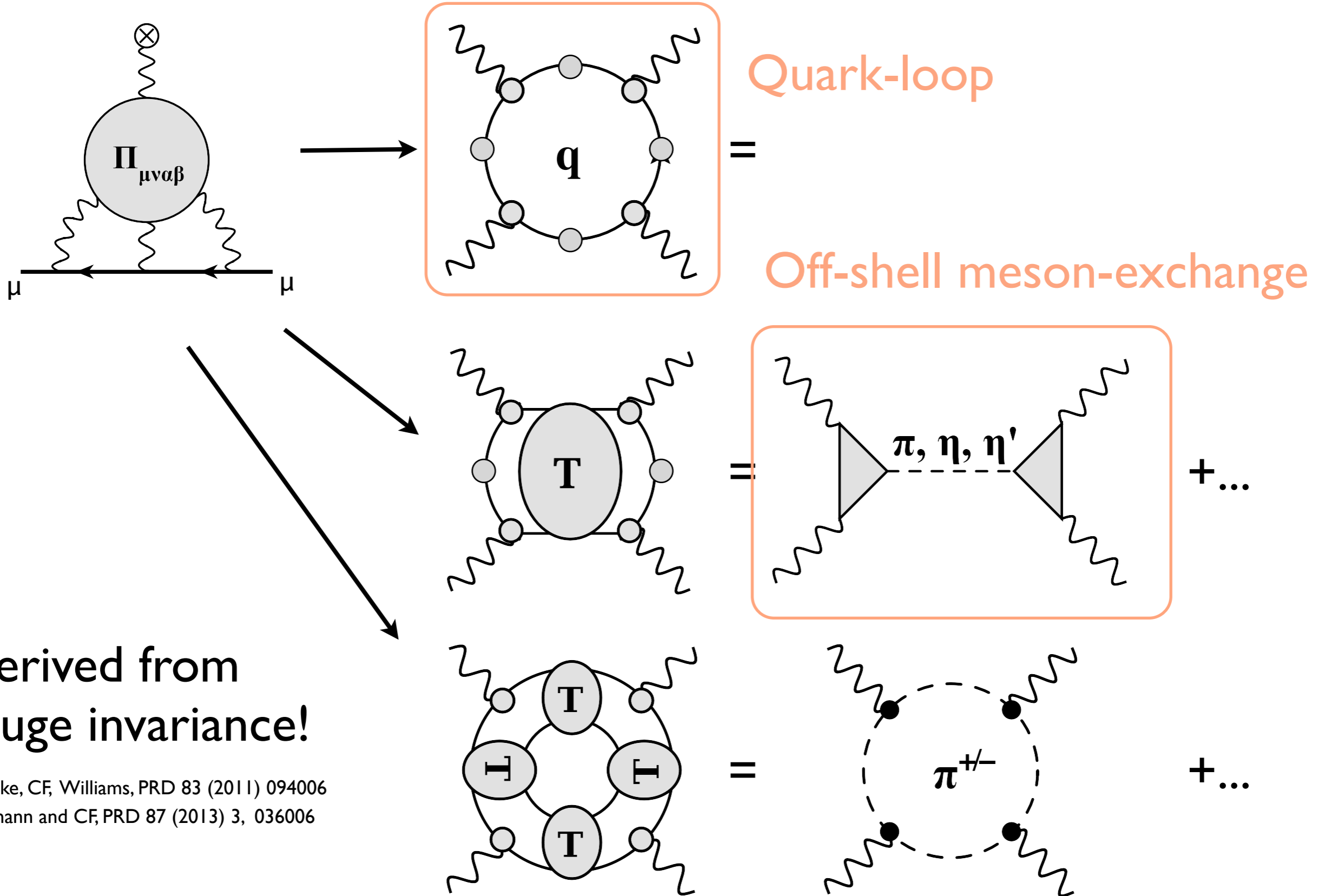


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Eichmann and CF, PRD 87 (2013) 3, 036006

Hadronic Light-by-Light scattering

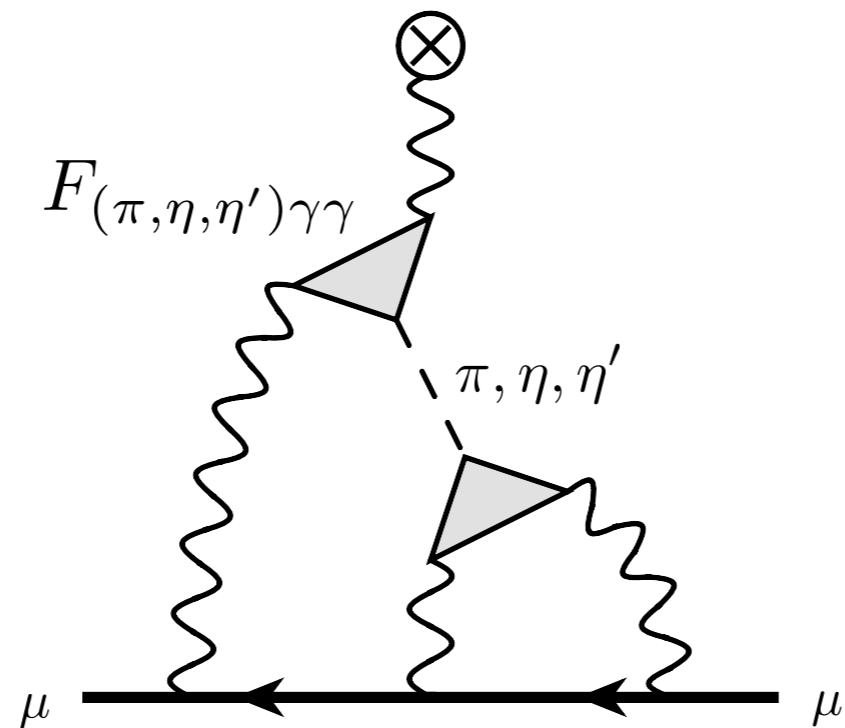


Derived from
gauge invariance!

Goecke, CF, Williams, PRD 83 (2011) 094006

Eichmann and CF, PRD 87 (2013) 3, 036006

Meson exchange contribution to LBL



$$(a_{\mu}^{\pi})_{DSE} = 57.5(0.6) \times 10^{-11}$$

$$(a_{\mu}^{\pi, \eta, \eta'})_{DSE} = 80.7(0.8) \times 10^{-11}$$

numerical ↗

Goecke, CF, Williams, PRD 83 (2011) 094006

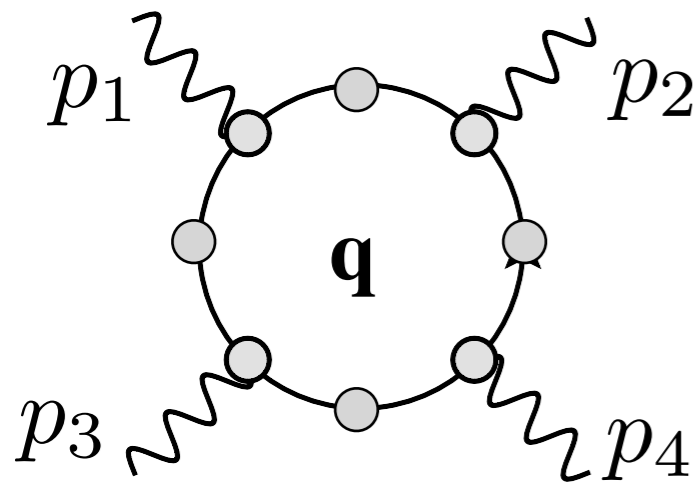
$$a_{\mu}^{\pi} = 62.6_{-2.5}^{+3.0} \times 10^{-11}$$

Hoferichter et al., arXiv:1805.01471

$$a_{\mu}^{\pi} = 65.0 \pm 8.3 \times 10^{-11}$$

Gerardin et al., PRD94 (2016) no.7, 074507

The quark loop



2013: $(10.7 \pm 0.3) \cdot 10^{-10}$

used only part of quark-photon vertex
 → calculation not complete !

$$\Gamma^\mu = \sum_{i=1,4} BC_i L_i^\mu + \sum_{i=1,8} F_i T_i^\mu$$

Goecke, CF, Williams, PRD 87 (2013) 034013

2018: Know how: Compton vertex

Eichmann, Ramalho, arXiv:1806.04579

Orthogonal basis: |36 elements - 4| transverse

$$\mathcal{M}^{\mu\nu\rho\sigma} = \sum_{i=1}^{41} a_i \tau_{\perp i}^{\mu\nu\rho\sigma} + \sum_{j=1}^{95} b_j \tau_j^{\mu\nu\rho\sigma}$$

$$\rightarrow \sum_{i=1}^4 f_i X_i^{\mu\nu\rho\sigma} + \sum_{j=1}^{11} g_j K_j^{\mu\nu\rho\sigma}$$

Eichmann, CF, Heupel, PRD 92 (2015) no.5, 056006

with $f_i(S_0), g_i(S_0)$

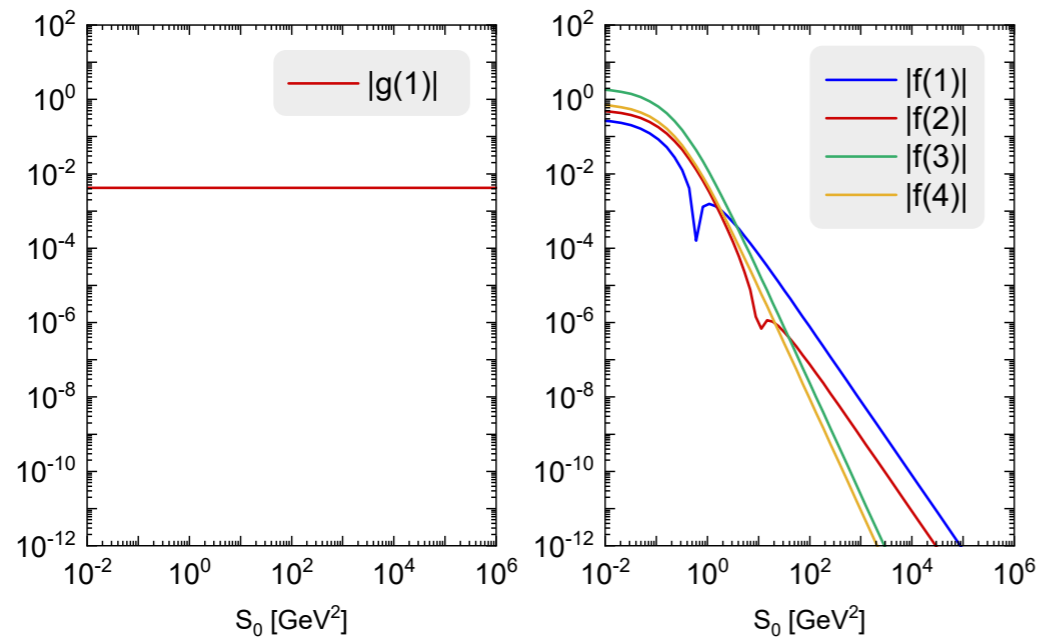
$$S_0 = \frac{p_1^2 + p_2^2 + p_3^2 + p_4^2}{4}$$

when S4-Singlet S_0 only is considered

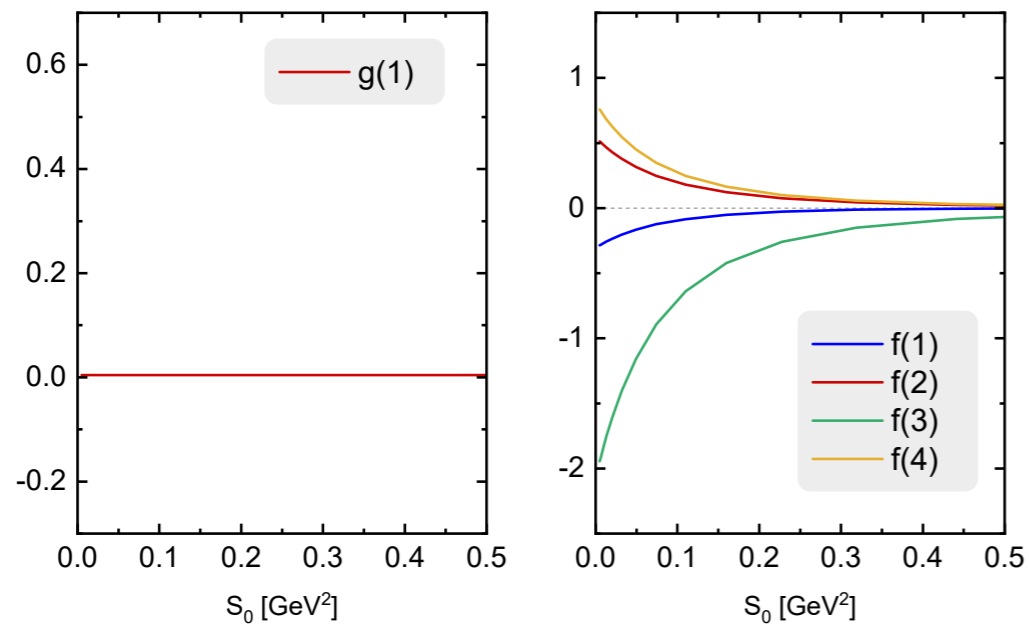
Results

NJL

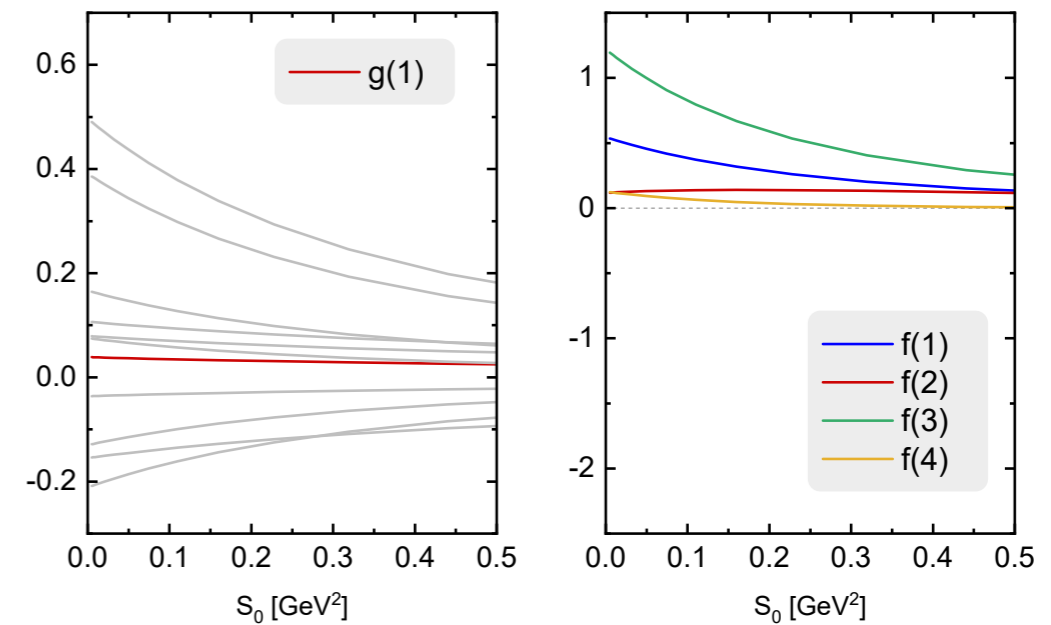
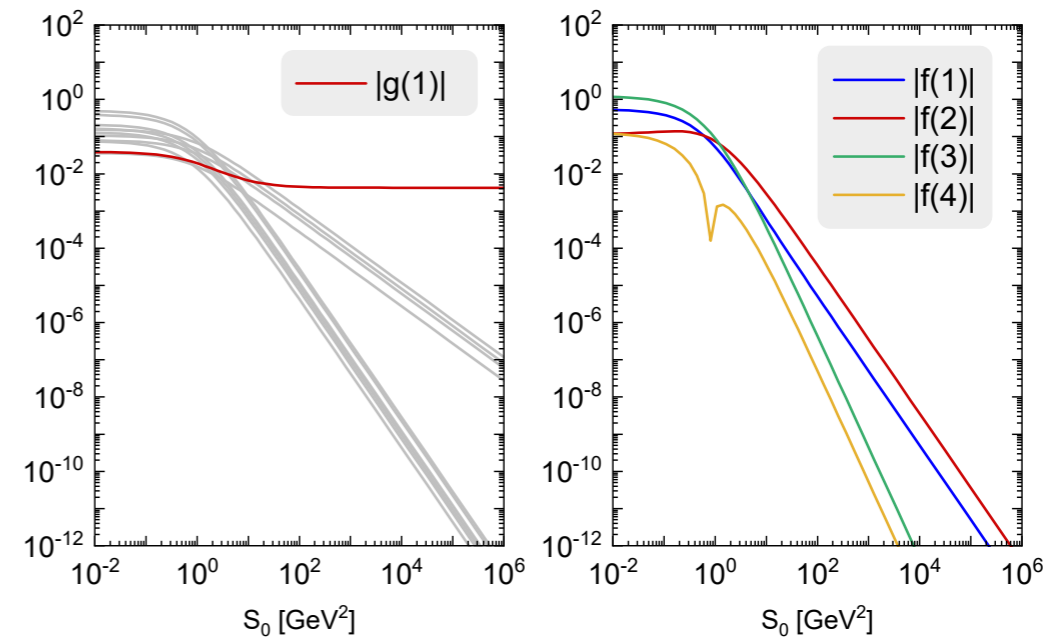
log



lin



DSE



- agree with analytic result
- serve to check S_0 -approx

- to do: use $f(i)$ to calc.
 $(a_{\mu}^{quark-loop})_{DSE}$

Summary

- (Euclidean) QCD based approach
 - renormalizable
 - full momentum dependence (unlike NJL)
 - multiscale problems feasible

- HVP

$$(a_{\mu}^{HVP})_{DSE}^{iso} = 6877(50) \times 10^{-11}$$

$$(a_{\mu}^{HVP})_{DSE}^{full} = 6951(50) \times 10^{-11}$$

Bonnet, Eichmann, CF, Williams, in prep.

- Light-by-light

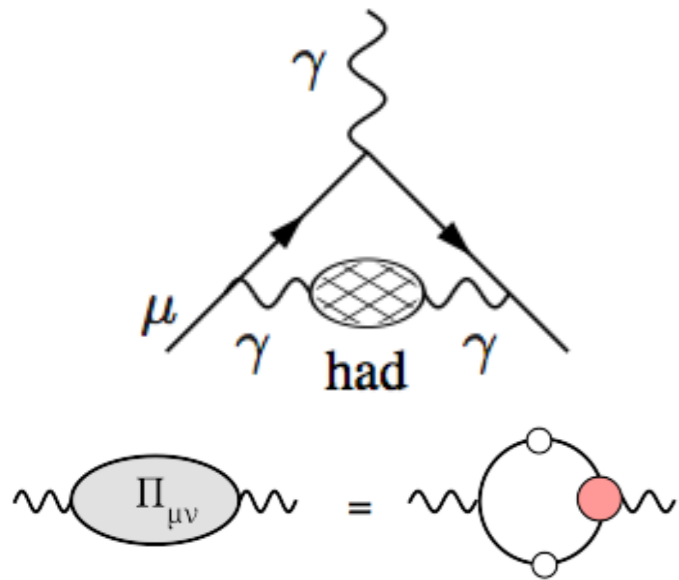
- TFF: calc. small and large Q, Q'
- TFF: exp. constraints satisfied

- $(a_{\mu}^{\pi})_{DSE} = 57.5(0.6) \times 10^{-11}$

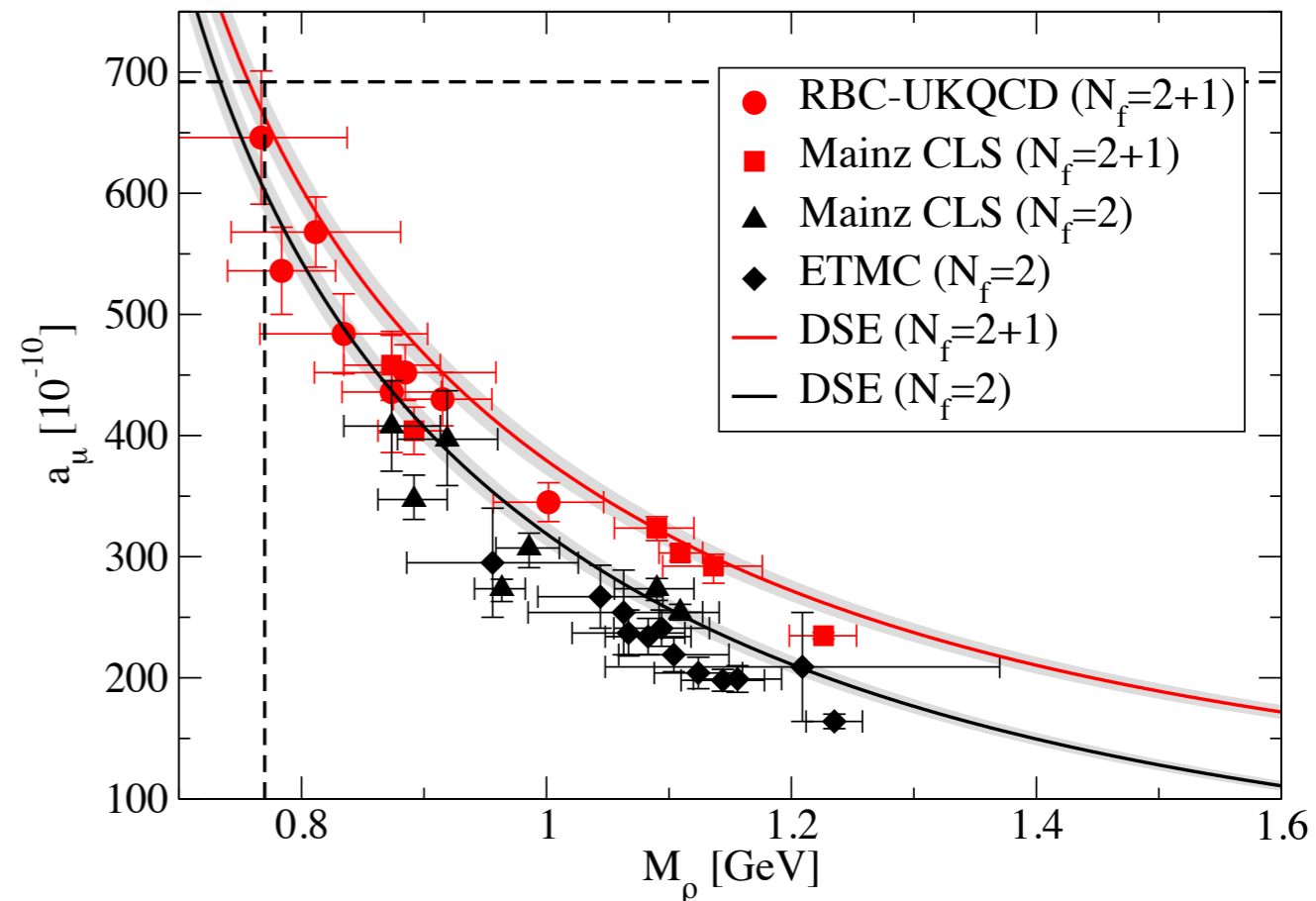
- Quarkloop: first results very encouraging

—————> **update of** Goecke, CF, Williams, PRD 87 (2013) 034013

Results: Hadronic vacuum polarisation



10^{-10}	u/d	s	c	b
a_μ	600	60	15	1



DSE:

Goecke, CF, Williams, PLB 704 (2011)

Experiment:

$$a_\mu^{had.(1)} = (744.0 \pm 2) \cdot 10^{-10} \quad (m_\pi)$$

$$a_\mu^{had.(1)} = (676.0 \pm 2) \cdot 10^{-10} \quad (m_\rho)$$

$$a_\mu^{had.(1)} = (692.1 \pm 5.6) \cdot 10^{-10}$$

Very reasonable agreement !

Rainbow-ladder model for quark-gluon interaction



Combine **gluon** with **quark-gluon vertex**:

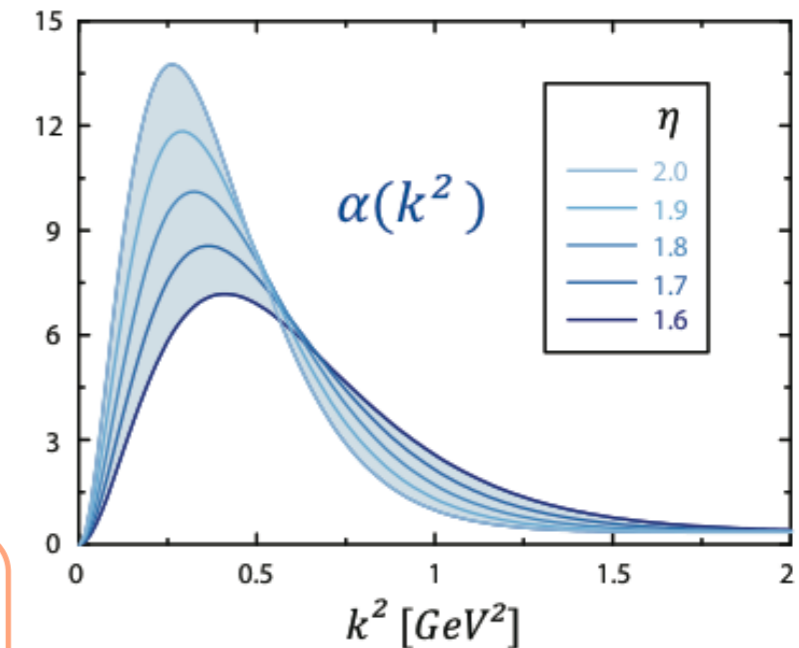
Rainbow-ladder model for quark-gluon interaction



Combine **gluon** with **quark-gluon vertex**:

effective coupling

$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2} \right) e^{-\eta^2 \left(\frac{k^2}{\Lambda^2} \right)} + \alpha_{UV}(k^2)$$

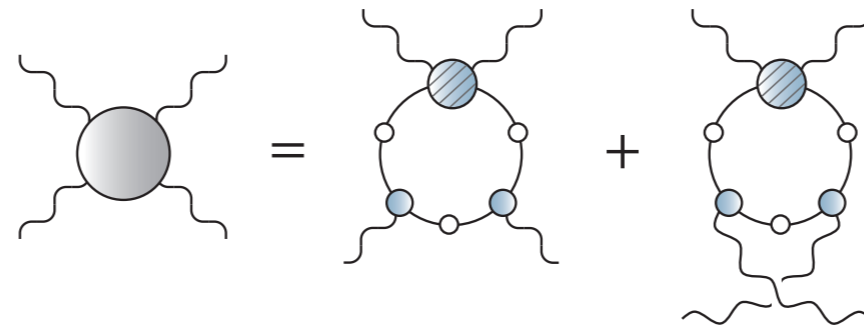


Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

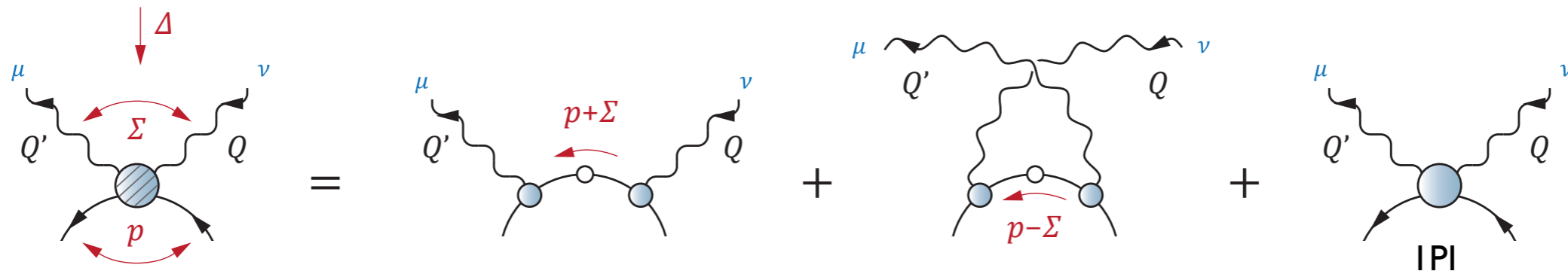
- scale Λ from f_π , masses $m_u=m_d, m_s$ from m_π, m_K
- α_{UV} from perturbation theory
- parameter η : results for spectrum and FF almost independent
- qualitatively similar to explicit calc.

Williams, EPJA 51 (2015) 5, 57.
 Sanchis-Alepuz, Williams, PLB 749 (2015) 592;
 Mitter, Pawłowski and Strodthoff, PRD 91 (2015) 054035
 Williams, CF, Heupel, PRD93 (2016) 034026, and refs. therein

Quark-Compton-vertex



Compton-vertex:



Quark-Loop incl. "meson-exchange"

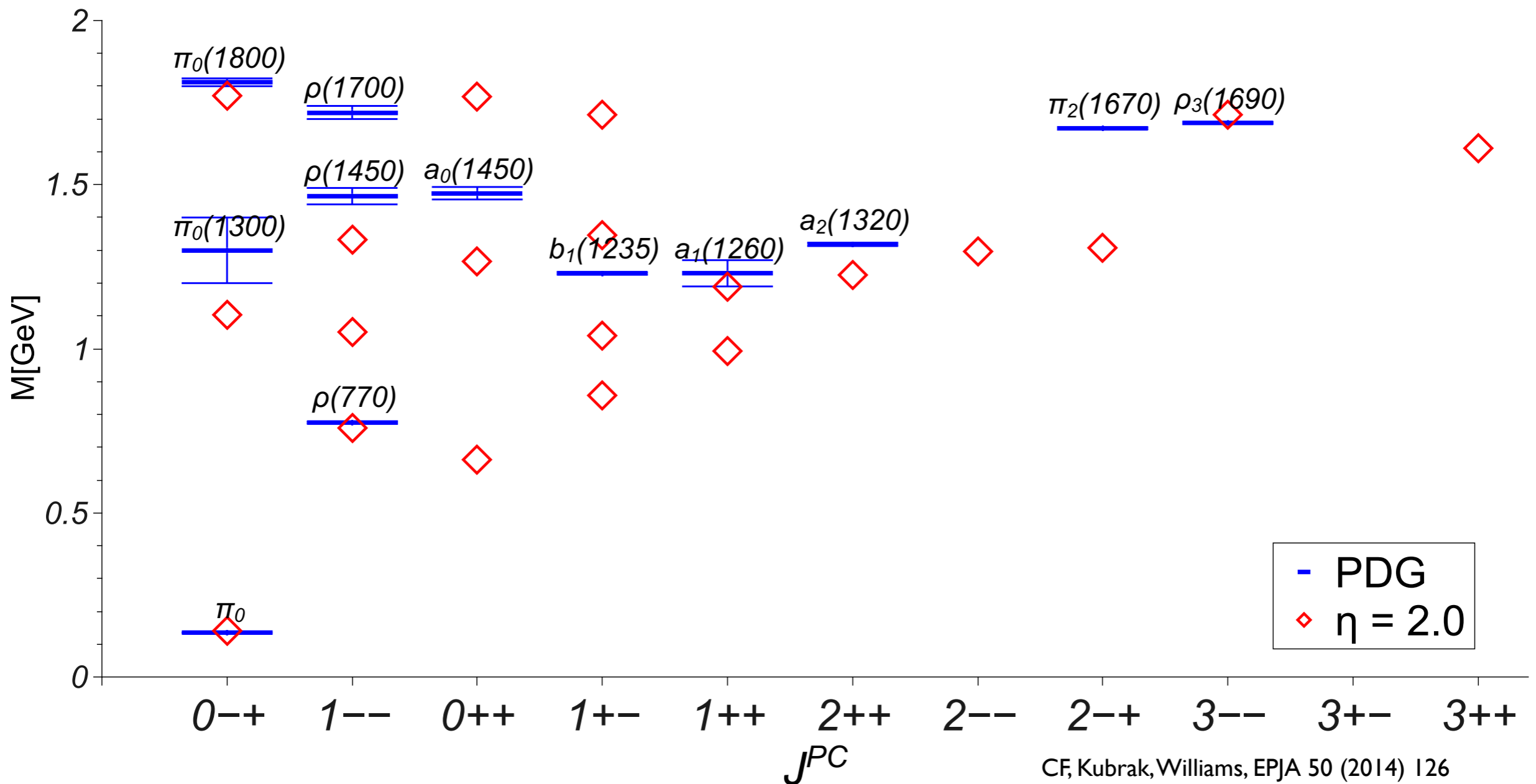
- basis constructed (128 elements)
- meson exchange contributions isolated and $F_{\pi\gamma\gamma}$ recovered

Eichmann and CF, PRD 87 (2013) 3, 036006

- applied to determine resonance contributions to nucleon Compton scattering

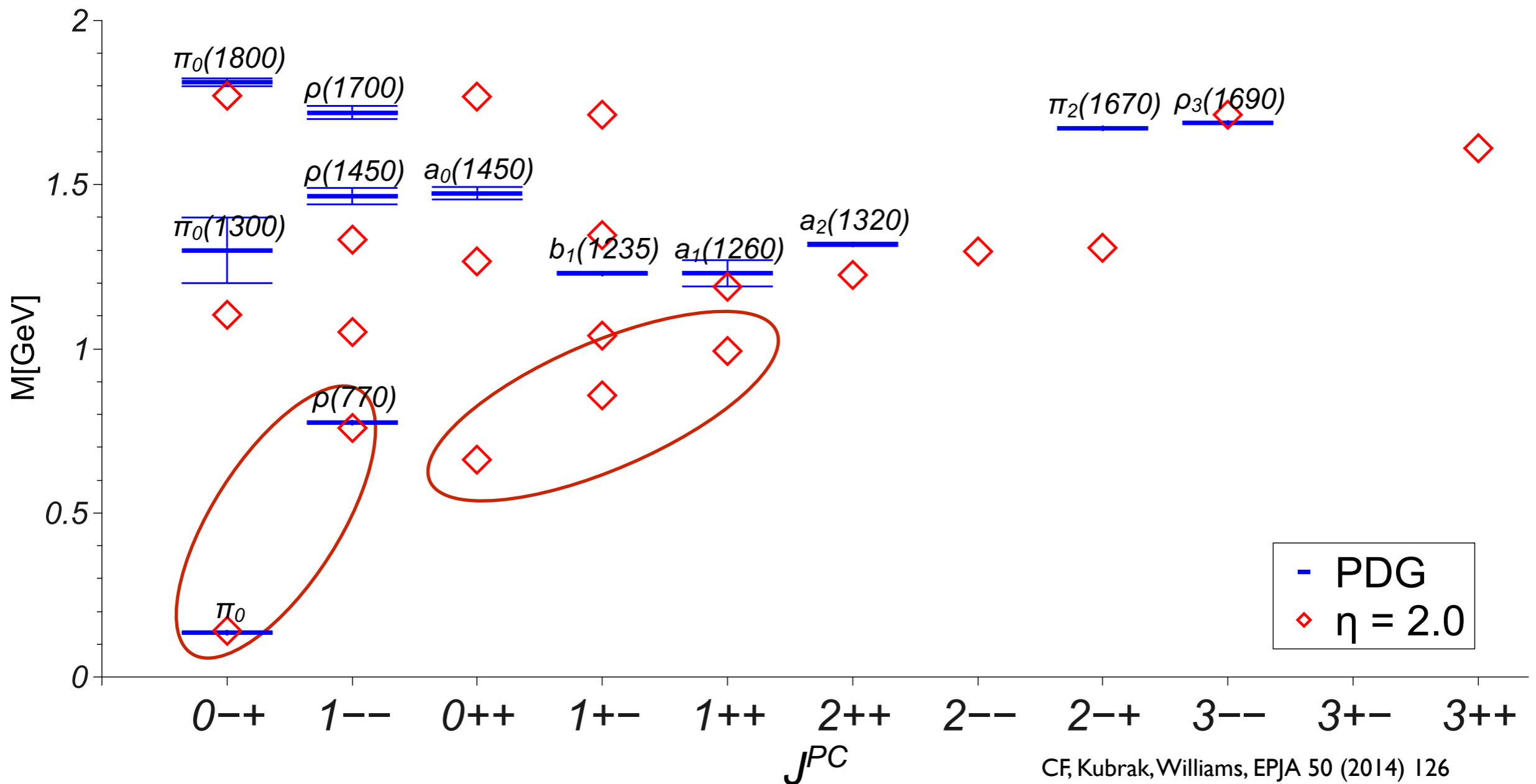
Eichmann, Ramalho, arXiv:1806.04579

Light meson spectrum



- good channels (ground state): 0^{-+} , 1^{-}
- acceptable channels (ground state) : 2^{++} , 3^{-} , ...
- clear deficiencies in other channels and excited states

Light meson spectrum

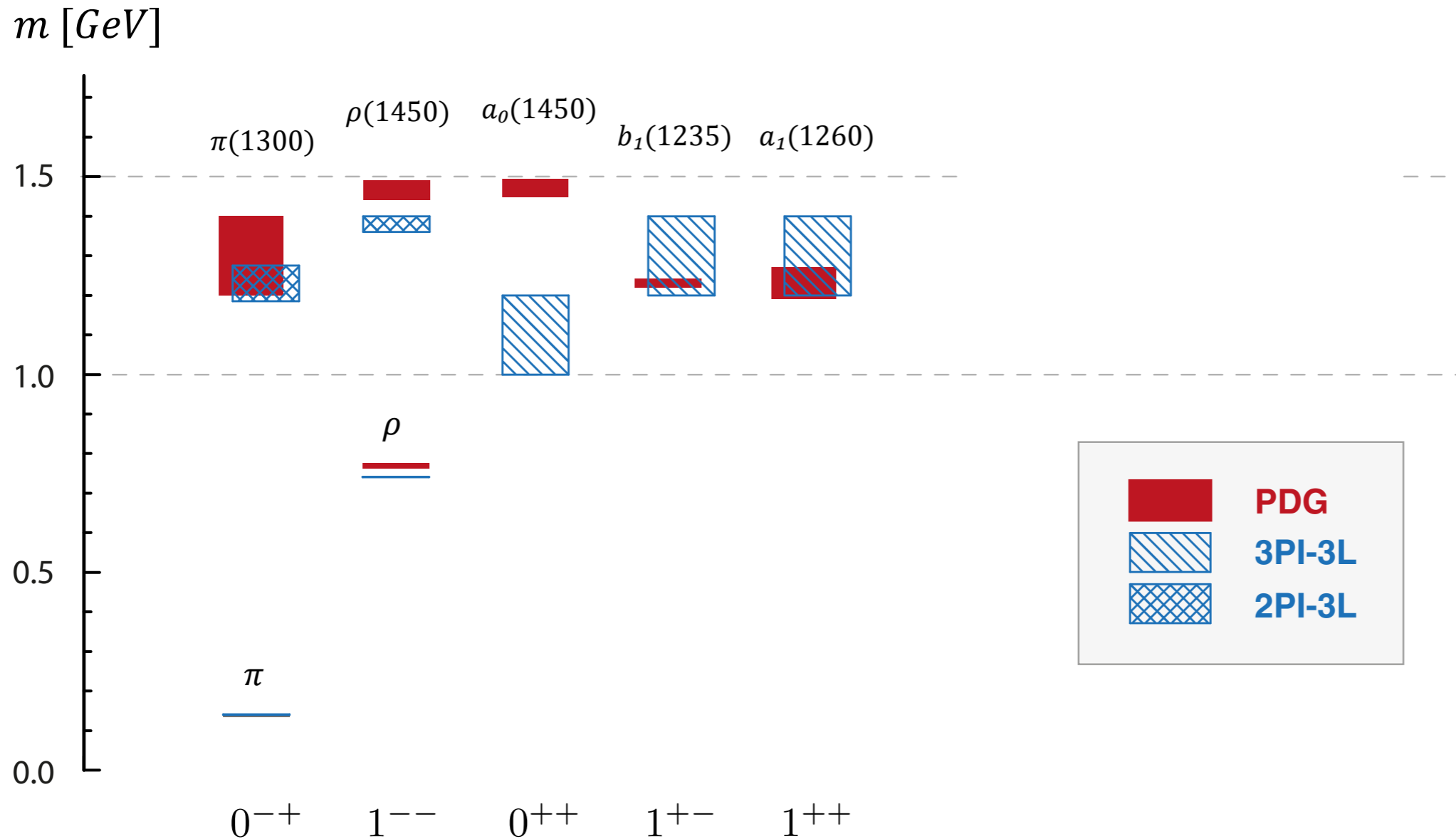


CF, Kubrak, Williams, EPJA 50 (2014) 126

Williams, CF, Heupel, PRD93 (2016) 034026

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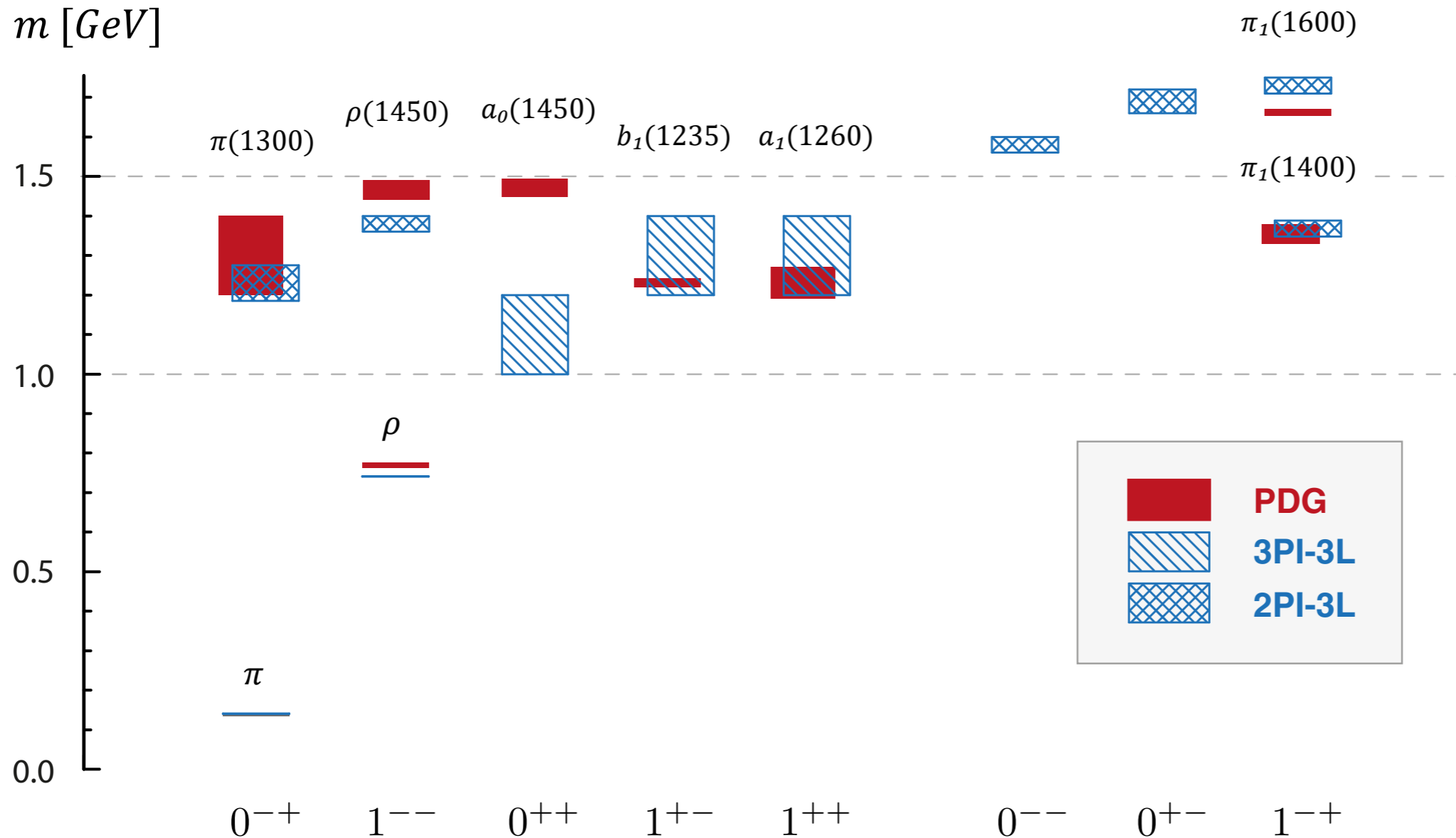


CF, Kubrak, Williams, EPJA 50 (2014) 126

Williams, CF, Heupel, PRD93 (2016) 034026

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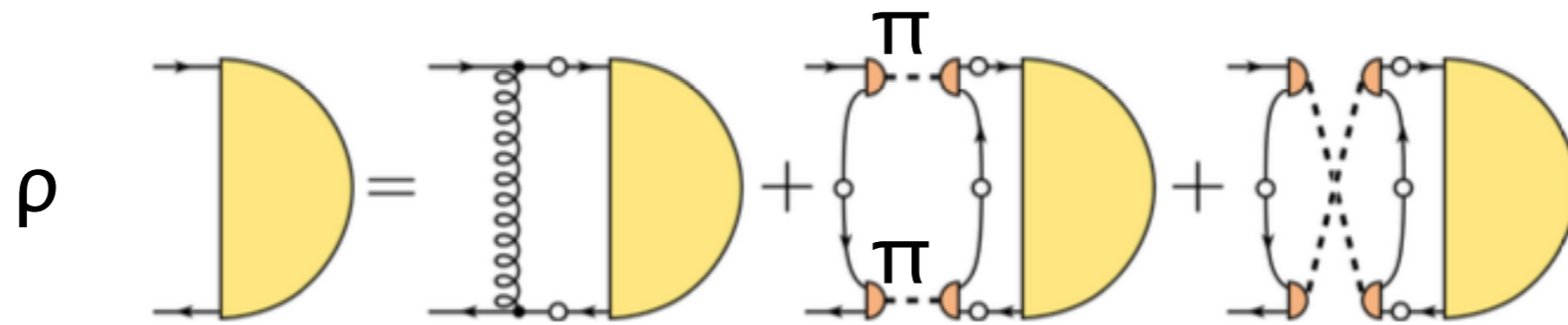
CF, Kubrak, Williams, EPJA 50 (2014) 126

Williams, CF, Heupel, PRD93 (2016) 034026

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Decays: $\rho\pi\pi$

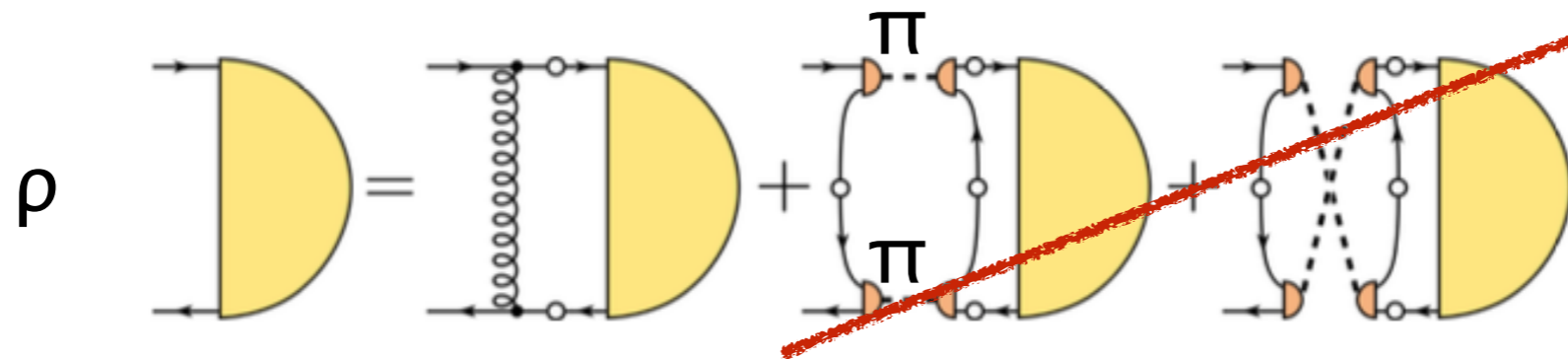
Beyond rainbow-ladder: pion contributions in BSE-kernel:



Williams, arXiv:1804.11161

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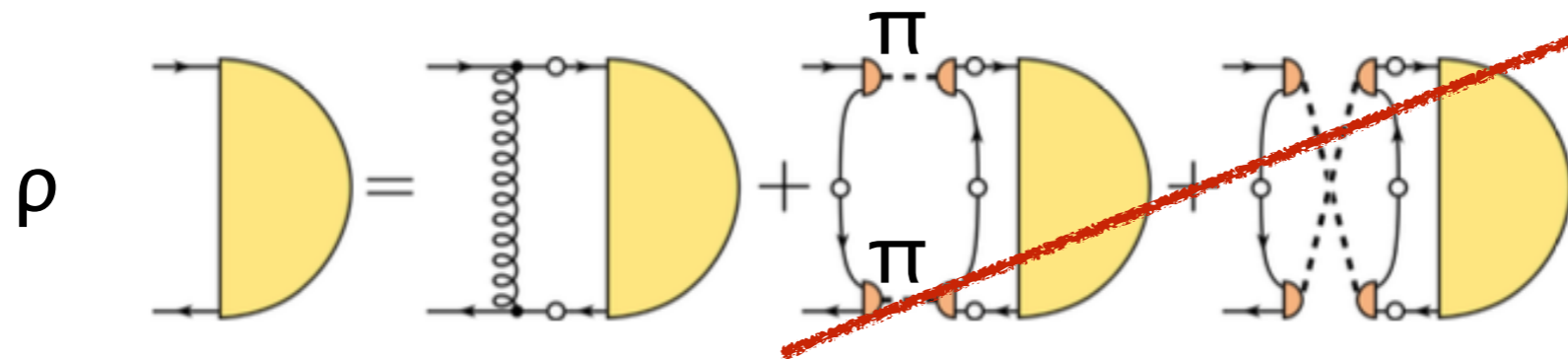
Beyond rainbow-ladder: pion contributions in BSE-kernel:



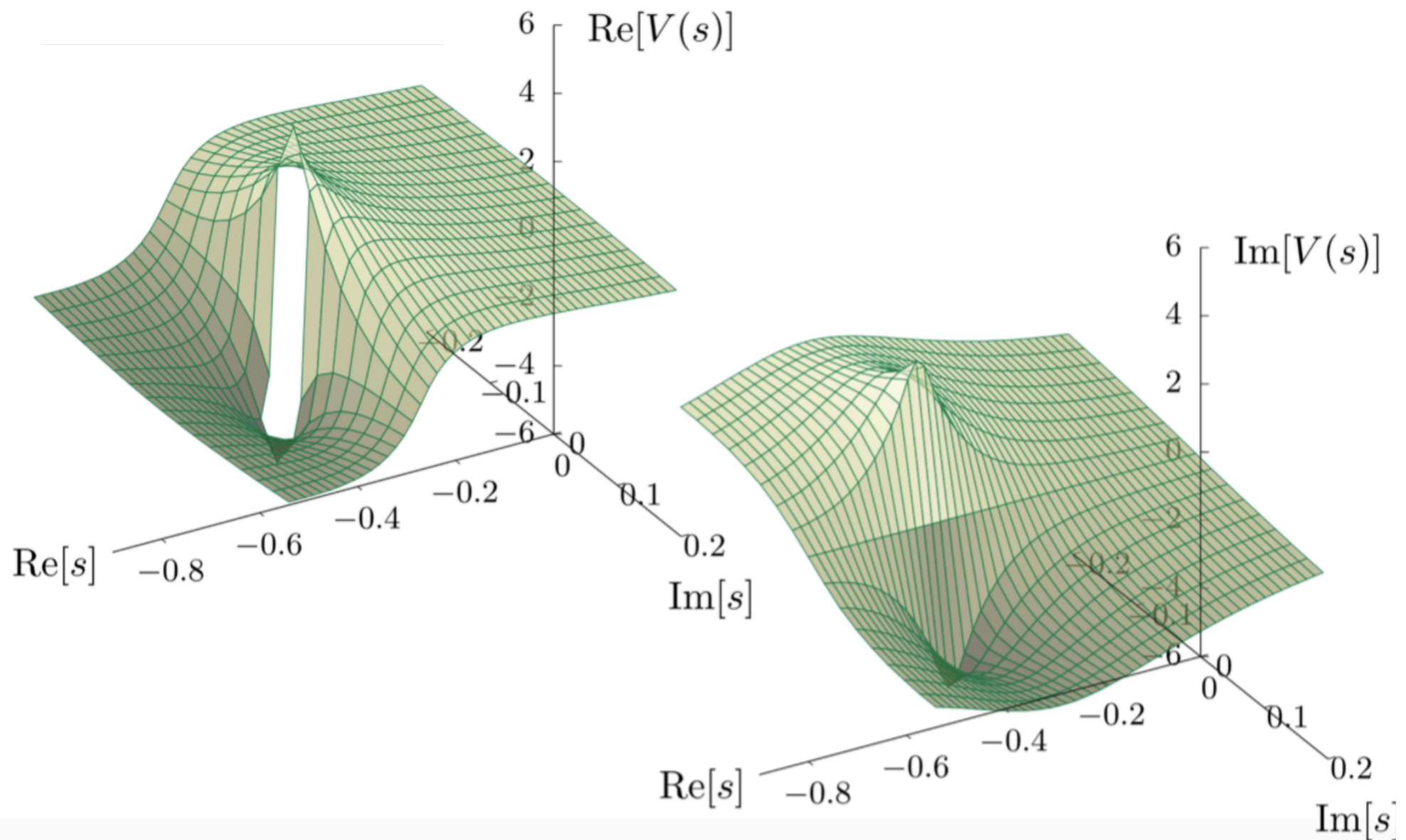
Williams, arXiv:1804.11161

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Beyond rainbow-ladder: pion contributions in BSE-kernel:

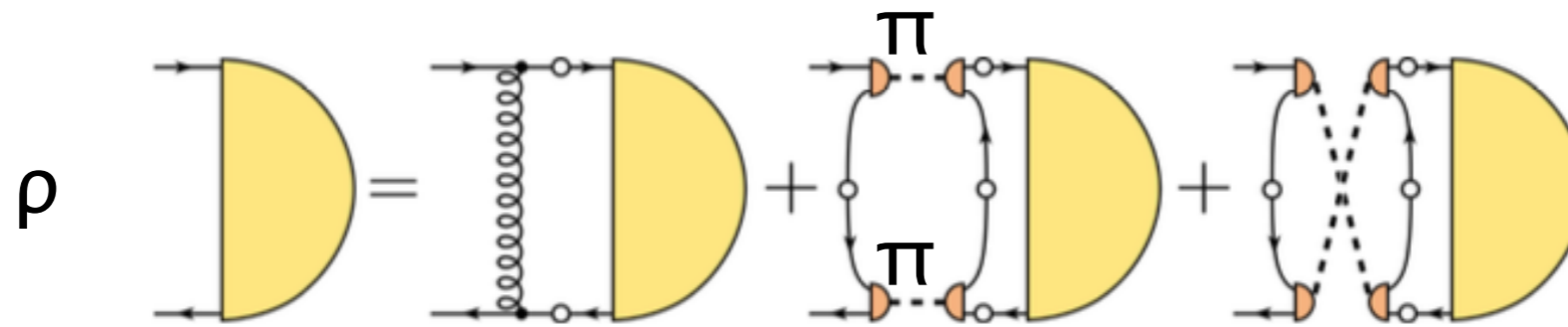


Williams, arXiv:1804.11161

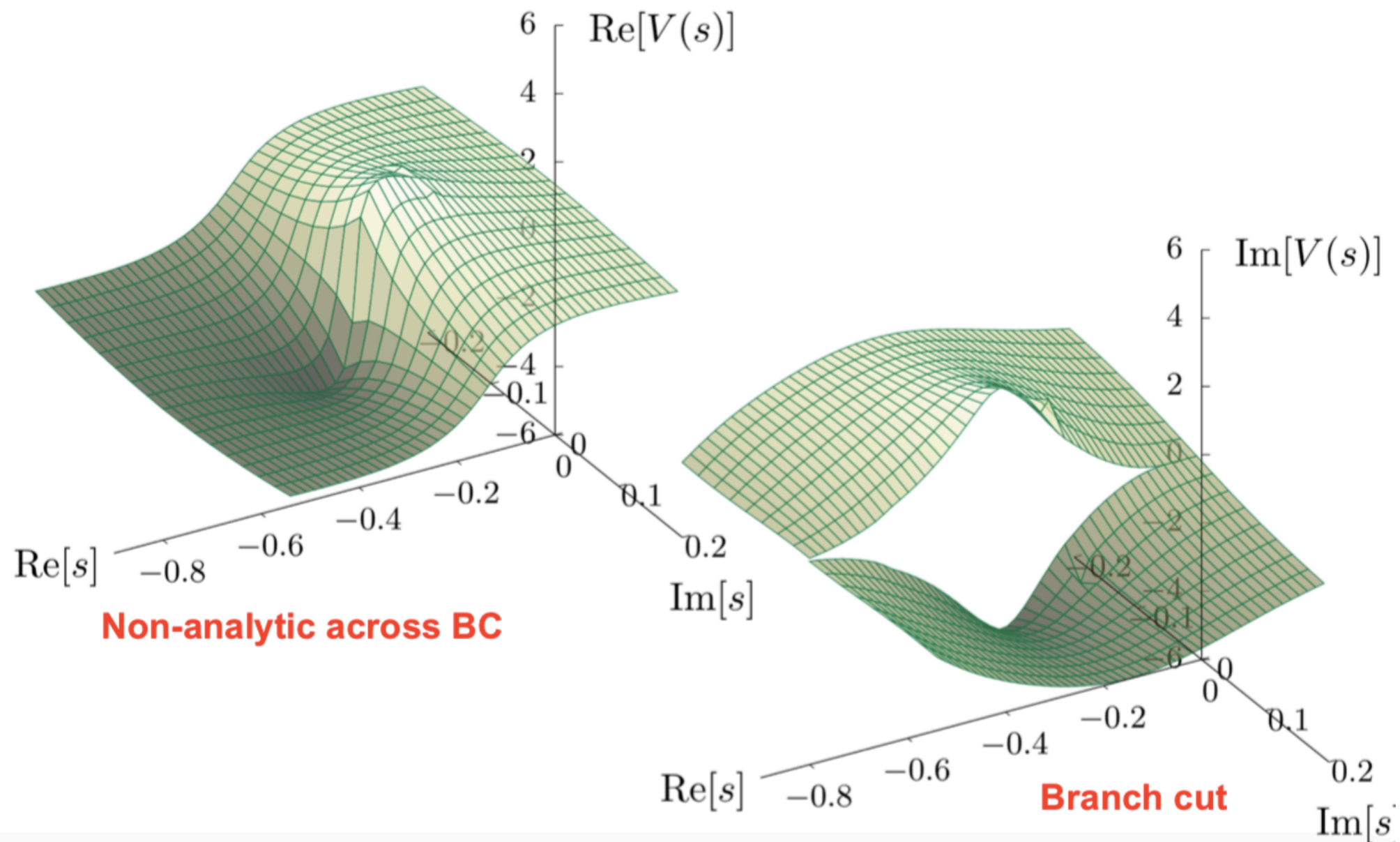


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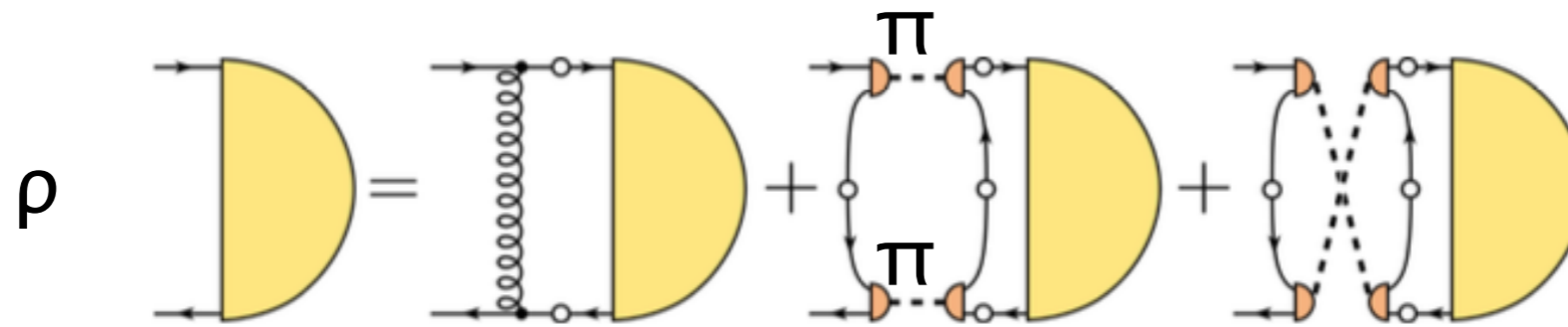


Williams, arXiv:1804.11161

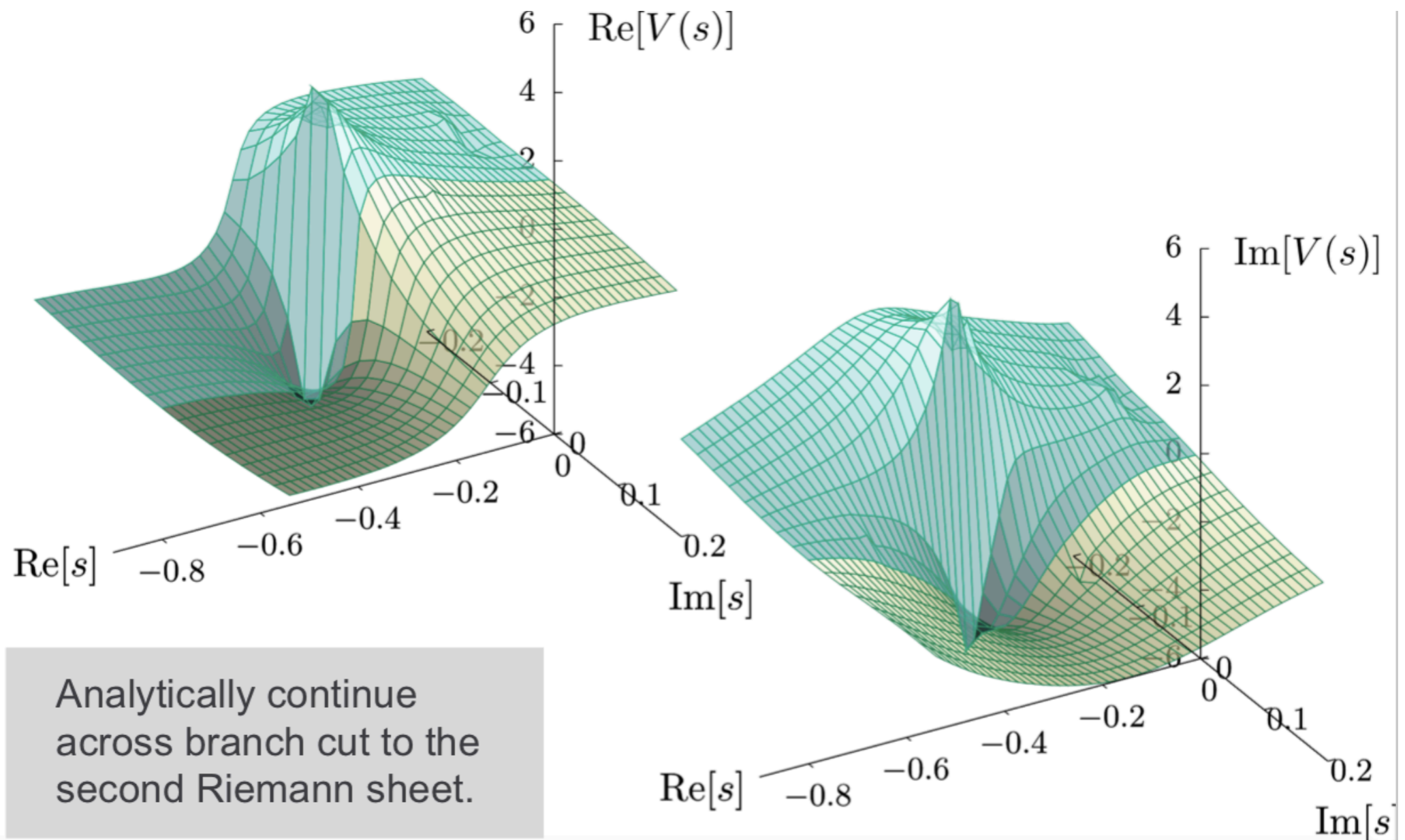


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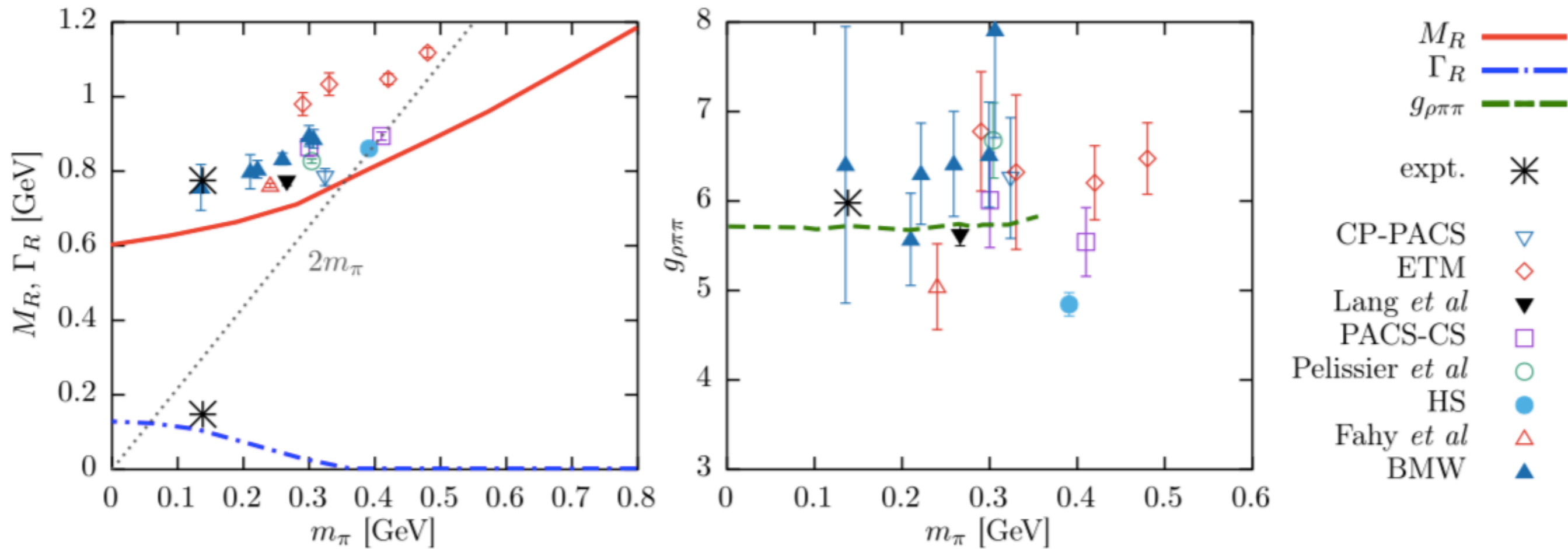
Beyond rainbow-ladder: pion contributions in BSE-kernel:



Williams, arXiv:1804.11161



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Williams, arXiv:1804.11161

- Additional corrections known to increase mass by $O(100)$ MeV

CF and Williams, PRL 103 (2009), 122001