HVP Lattice Status Report BMW

At Physical Point Mass with Full Systematics

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Muon $g-2$, June 21, 2018

Budapest-Marseille-Wuppertal Collaboration
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$a_\mu^{exp.}$ vs. a_μ^{SM}

SM contribution	$a_\mu^{\mathrm{contrib.}} \times 10^{10}$	Ref.
QED [5 loops]	11658471.8951 ± 0.0080	[Aoyama et al '12]
HVP-LO (pheno.)	692.6 ± 3.3	[Davier et al '16]
	694.9 ± 4.3	[Hagiwara et al '11]
	681.5 ± 4.2	[Benayoun et al '16]
HVP-NLO	-9.84 ± 0.07	[Hagiwara et al '11]
		[Kurz et al '11]
HVP-NNLO	1.24 ± 0.01	[Kurz et al '11]
HLbyL	10.5 ± 2.6	[Prades et al '09]
Weak (2 loops)	15.36 ± 0.10	[Gnendiger et al '13]
SM tot [0.42 ppm]	11659180.2 ± 4.9	[Davier et al '11]
[0.43 ppm]	11659182.8 ± 5.0	[Hagiwara et al '11]
[0.51 ppm]	11659184.0 ± 5.9	[Aoyama et al '12]
Exp [0.54 ppm]	11659208.9 ± 6.3	[Bennett et al '06]
Exp − SM	28.7 ± 8.0	[Davier et al '11]
	26.1 ± 7.8	[Hagiwara et al '11]
	24.9 ± 8.7	[Aoyama et al '12]

$$a_\mu^{\text{LO-HVP}}|_{NoNewPhys} \simeq 720 \pm 7,$$

FNAL E989 (2017): 0.14-ppm, J-PARC E34: 0.1-ppm

Really $a_\mu^{exp.} \neq a_\mu^{SM}$?

- $\sigma(e^+e^- \to \pi^+\pi^-)$ and $a_\mu^{\mathrm{LO}-\pi\pi}$ is under discussion (talks yesterday). It is challenging to control systematics in the integral of R-ratio:

$$\hat{\Pi}(Q^2) = \int_0^\infty ds \frac{Q^2}{s(s+Q^2)} \frac{\mathrm{Im}\Pi(s)}{\pi} = \frac{Q^2}{12\pi^2} \int_0^\infty ds \frac{R_{had}(s)}{s(s+Q^2)} \, ,$$

$$R_{had}(s) \equiv \frac{\sigma(e^+e^- \to had.)}{4\pi\alpha^2(s)/(3s)} \, . \quad (1)$$

- Independent cross-checks by Lattice QCD is demanded. Permil-Level determination of Leading-Order (LO) Hadronic Vauccum Polarization (HVP) contribution to muon g-2 ($a_\mu^{\text{LO-HVP}}$) is required in terms of the on-going /forth-coming experiments.

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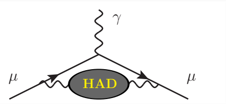
Objective in This Work

LO-HVP contribution to muon g-2 for all leptons by lattice QCD:

$$a_{\ell=e,\mu,\tau}^{\text{LO-HVP},f} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2\ \omega(Q^2/m_\ell^2)\hat{\Pi}^f(Q^2)\ .$$

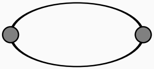


where suffix f stands for a flavor $f = l(u,d), s, c, disc$, and

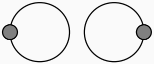
$$\hat{\Pi}^f(Q^2) = \Pi^f(Q^2) - \Pi^f(0) = \sum_t t^2 \left[1 - \left(\frac{\sin(z/2)}{z/2}\right)^2\right]_{z=Qt} \frac{1}{3}\sum_{i=1}^{3} C_{ii}^f(t)\ , \quad (2)$$

with

$$C_{\mu\nu}^{f=l,s,c}(t) = q_{f=l,s,c}^2 \sum_{\vec{x}} \langle\ j_\mu^f(x) j_\nu^f(0)\ \rangle|_{conn}\ ,$$

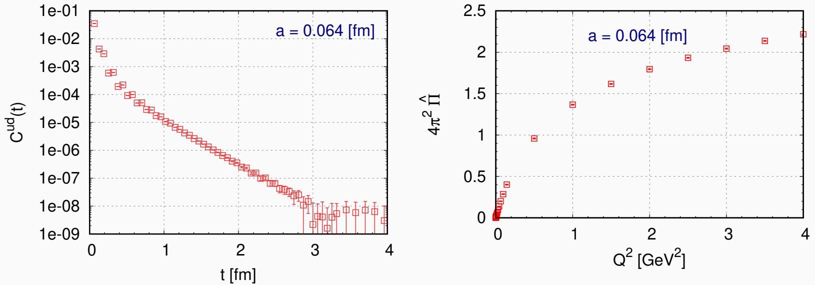


$$C_{\mu\nu}^{f=disc}(t) = q_{f=disc}^2 \sum_{\vec{x}} \langle(\bar{l}\gamma_\mu l - \bar{s}\gamma_\mu s)(\bar{l}\gamma_\nu l - \bar{s}\gamma_\nu s)\rangle|_{disc}\ .$$



Here, charge factors are given by $(q_l^2, q_s^2, q_c^2, q_{disc}^2) = (5/9,\ -1/9,\ 4/9,\ 1/9)$.

Correlator and HVP: Example



$$Left : C^{ud}(t) = \frac{5}{9} \sum_{\vec{x}} \frac{1}{3} \sum_{i=1}^{3} \langle j_i^{ud}(\vec{x}, t) j_i^{ud}(0) \rangle \ , \qquad (3)$$

$$Right : \hat{\Pi}^{ud}(Q^2) = \sum_t t^2 \left[1 - \left(\frac{\sin(z/2)}{z/2} \right)^2 \right]_{z=Qt} C^{ud}(t) \ . \qquad (4)$$

Bounding I

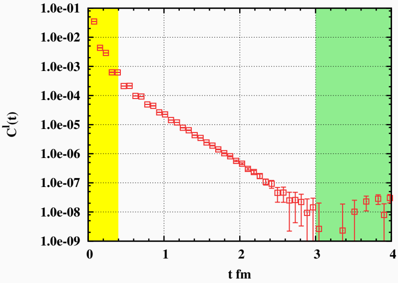
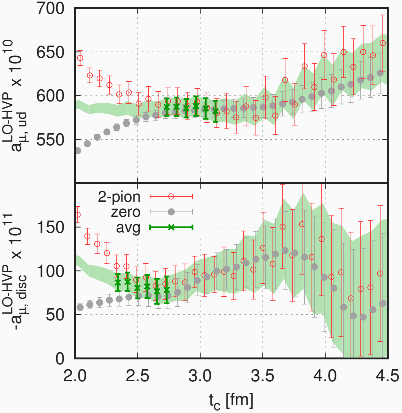


Figure:
$C^{ud}(t) = \frac{5}{9}\sum_{\vec{x}}\frac{1}{3}\sum_{i=1}^{3}\langle j_i^{ud}(\vec{x},t)j_i^{ud}(0)\rangle$

- The connected-light correlator $C^{ud}(t)$ loses signal for $t > 3fm$. To control statistical error, consider $C^{ud}(t > t_c) \to C^{ud}_{\rm up/low}(t, t_c)$, where $C^{ud}_{\rm up}(t, t_c) = C^{ud}(t_c)\ \varphi(t)/\varphi(t_c)$, $C^{ud}_{\rm low}(t, t_c) = 0.0$, with $\varphi(t) = \cosh[E_{2\pi}(T/2 - t)]$, and $E_{2\pi} = 2(M_\pi^2 + (2\pi/L)^2)^{1/2}$.
- Similarly, $C^{disc}(t) \to C^{disc}_{\rm up/low}(t, t_c)$, $-C^{disc}_{\rm up}(t > t_c) = 0.1C^{ud}(t_c)\ \varphi(t)/\varphi(t_c)$, $-C^{disc}_{\rm low}(t > t_c) = 0.0$.
- $C^{ud,disc}_{low}(t, t_c) \le C^{ud,disc}(t) \le C^{ud,disc}_{up}(t, t_c)$.

Bounding II

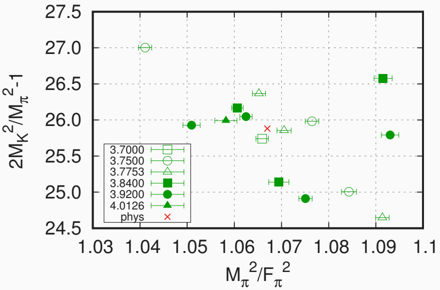


- Corresponding to $C^{ud,disc}_{up/low}(t_c)$, we obtain upper/lower bounds for $g-2$: $a^{ud,disc}_{\ell,up/low}(t_c)$.
- Two bounds meet around $t_c = 3fm$. Consider the average of bounds: $\bar{a}^{ud,disc}_{\ell}(t_c) = 0.5(a^{ud,disc}_{\ell,up} + a^{ud,disc}_{\ell,low})(t_c)$, which is stable around $t_c = 3fm$.
- We pick up such averages $\bar{a}^{ud,disc}_{\ell}(t_c)$ with $4-6$ kinds of t_c around $3fm$. The average of average is adopted as $a^{\rm LO\text{-}HVP}_{\ell,{\rm ud/disc}}$ to be analysed, and a fluctuation over selected t_c is incorporated into the systematic error.

Simulation Setup

State of The Art

- Nf=(2+1+1) simulations around Physical Mass Points.
- Large Volume: $(L, T) \sim (6, 9-12) fm$.
- Controlled Continuum Limit with 15 simulation points.



β	a[fm]	N_t	N_s	#traj.	M_π[MeV]	M_K[MeV]	#SRC (l,s,c,d)
3.7000	0.134	64	48	10000	~ 131	~ 479	$(768, 64, 64, 9000)$
3.7500	0.118	96	56	15000	~ 132	~ 483	$(768, 64, 64, 6000)$
3.7753	0.111	84	56	15000	~ 133	~ 483	$(768, 64, 64, 6144)$
3.8400	0.095	96	64	25000	~ 133	~ 488	$(768, 64, 64, 3600)$
3.9200	0.078	128	80	35000	~ 133	~ 488	$(768, 64, 64, 6144)$
4.0126	0.064	144	96	04500	~ 133	~ 490	$(768, 64, 64, -)$

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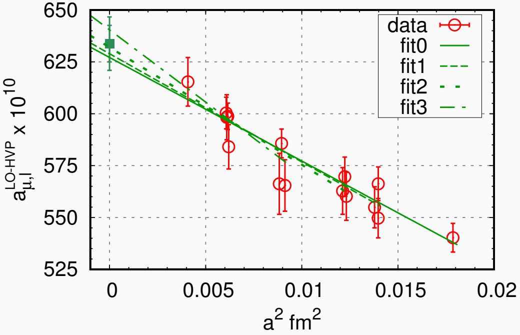
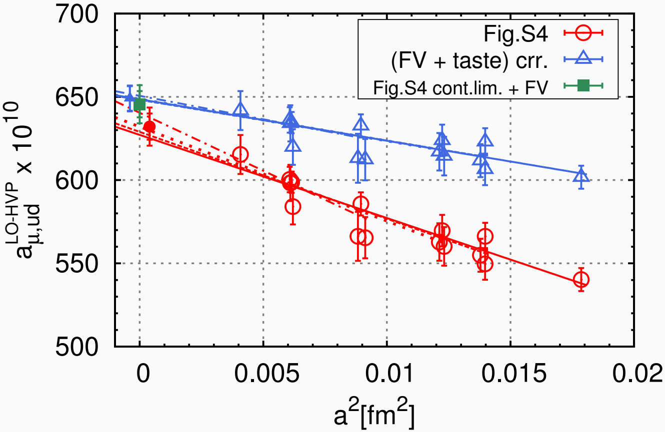
Continuum Extrap. of Light Conn. Component: $a_{\mu,ud}^{\text{LO-HVP}}$



$$F(a_{\mu,ud}^{\text{LO-HVP}}, A, C_\pi, \cdots) = a_{\mu,ud}^{\text{LO-HVP}}(1 + Aa^2) + C_\pi \Delta M_\pi^2 + \cdots .$$

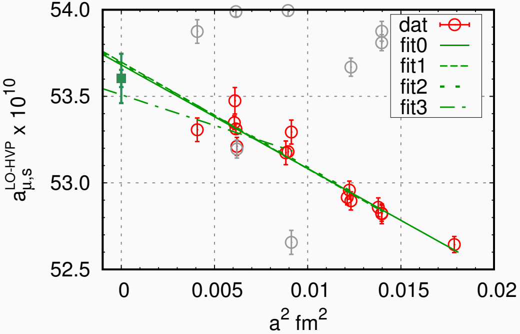
$$a_{\mu,ud}^{\text{LO-HVP}} = 632.1(7.9)(8.3) \ , \quad \chi^2/\text{dof} = 7.8/12 \text{ (fit1 case)}.$$

Crosscheck of Continuum Extrapolation



c.f. HPQCD PRD2017

Continuum Extrap. of Strange Conn. Component: $a_{\mu,s}^{\text{LO-HVP}}$

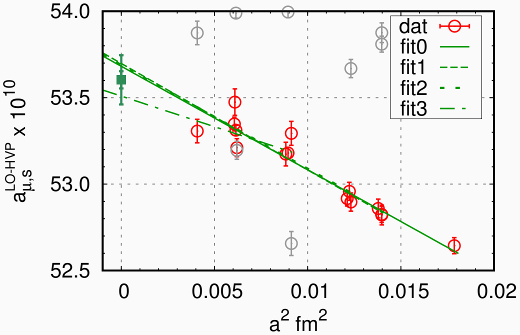


$$F(a_{\mu,s}^{\text{LO-HVP}}, A, C_K) = a_{\mu,s}^{\text{LO-HVP}}(1 + Aa^2) + C_K \Delta M_K^2 \ . \quad (5)$$

$a_{\mu,s}^{\text{LO-HVP}} = 53.64(04)(14)$, $\chi^2/\text{dof} = 16.7/11$ (fit1 case).

c.f. Mainz Group: $M_\pi^2 \log M_\pi^2$ collection.

Continuum Extrap. of Strange Conn. Component: $a_{\mu,s}^{\text{LO-HVP}}$



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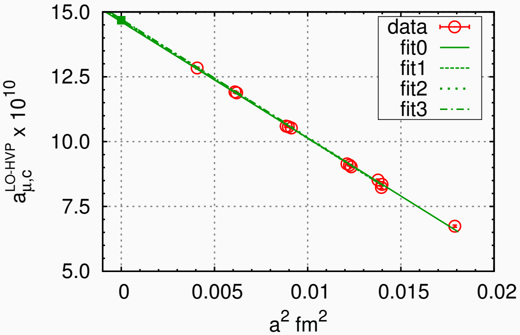
Continuum Extrap. of Charm Conn. Component: $a_{\mu,c}^{\text{LO-HVP}}$

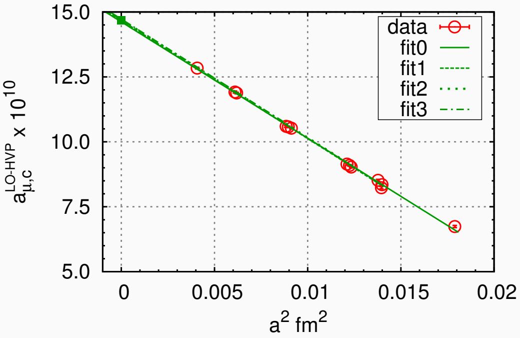


$$F(a_{\mu,c}^{\text{LO-HVP}}, A, C_{\pi,\eta_c}) = a_{\mu,c}^{\text{LO-HVP}}(1 + Aa^2) + C_\pi \Delta M_\pi^2 + C_{\eta_c} \Delta M_{\eta_c} \ .$$

$a_{\mu,c}^{\text{LO-HVP}} = 14.68(03)(06)$, $\chi^2/\text{dof} = 1.4/7$ (fit2 case).

The fit model works in M_{D_s} determination.

Continuum Extrap. of Charm Conn. Component: $a_{\mu,c}^{\text{LO-HVP}}$

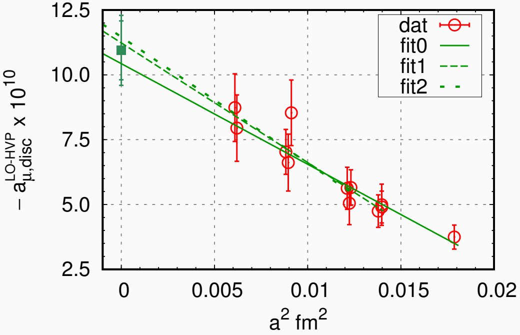


$$F(a_{\mu,c}^{\text{LO-HVP}}, A, C_{\pi,\eta_c}) = a_{\mu,c}^{\text{LO-HVP}}(1 + Aa^2) + C_\pi \Delta M_\pi^2 + C_{\eta_c} \Delta M_{\eta_c} \; .$$

$$a_{\mu,c}^{\text{LO-HVP}} = 14.68(03)(06) \; , \quad \chi^2/\text{dof} = 1.4/7 \text{ (fit2 case).}$$

The fit model works in M_{D_s} determination.

Continuum Extrap. of Disc. Component: $a_{\mu,disc}^{\text{LO-HVP}}$

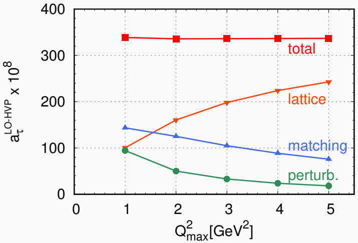


$$F(a_{\mu,disc}^{\text{LO-HVP}}, A, C_\pi, \cdots) = a_{\mu,disc}^{\text{LO-HVP}}(1 + Aa^2) + C_\pi \Delta M_\pi^2 + \cdots .$$

$$a_{\mu,disc}^{\text{LO-HVP}} = -11.0(1.1)(0.6) , \quad \chi^2/\text{dof} = 2.4/10 \text{ (fit1 case)}.$$

Various Corrections

- **High Q^2 Control:**
  The lattice data have enough overlap to perturbative regime even in tau case.
  $a^{\text{LO-HVP}}_{\ell,f} = a^{\text{LO-HVP}}_{\ell,f}(Q \leq Q_{max}) + (\gamma_\ell \hat{\Pi}^f)(Q_{max}) + \Delta^{pert} a^{\text{LO-HVP}}_{\ell,f}(Q > Q_{max})$.



- **Isospin/QED Collections:**
  Model estimates amounts to 1.1% corrections (table thanks to F.Jegerlehner (& M. Benayoun)).

Effect	$\delta a_\mu^{\text{LO-HVP}} \times 10^{10}$
ρ–ω mix.	2.71 ± 1.36
FSR	4.22 ± 2.11
$M_\pi \to M_{\pi\pm}$	-4.47 ± 4.47
$\pi^0\gamma$	4.64 ± 0.04
$\eta\gamma$	0.65 ± 0.01
Total	$\mathbf{7.8 \pm 5.1}$

- **FV Collections:**
  The dominant FV in $I = 1$, $\pi^+\pi^-$ loop channel is estimated by XPT (Aubin et al '16): $(a^{\text{LO-HVP}}_{\mu,I=1}(\infty) - a^{\text{LO-HVP}}_{\mu,I=1}(6fm))|_{\text{XPT}} = 13.42(13.42) \times 10^{-10}$, (1.9%) .

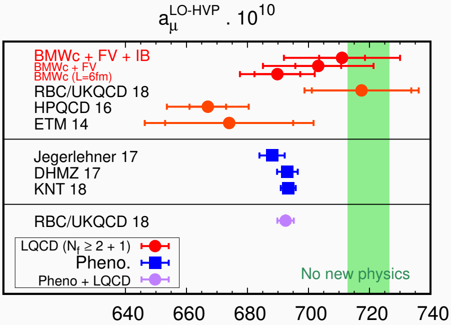
Summary on $a_\mu^{\text{LO-HVP}}$ PRL2018

$a_\mu^{\text{LO-HVP}}$ BMWc

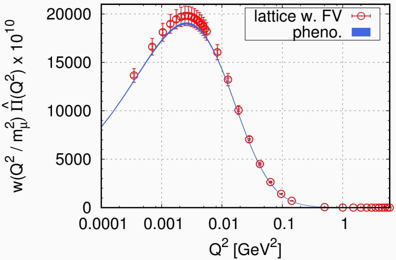
$I = 1$	$582.9(6.7)_{st}(7.2)_{acut}(0.1)_{tcut}(0.0)_{qcut}(4.5)_{da}(13.5)_{fv}$
$I = 0$	$120.5(3.4)_{st}(3.5)_{acut}(0.2)_{tcut}(0.0)_{qcut}(1.0)_{da}$
total	$711.1(7.5)_{st}(8.0)_{acut}(0.2)_{tcut}(0.0)_{qcut}(5.5)_{da}(13.5)_{fv}(5.1)_{iso}$

Remarks

- Our Lattice QCD results are consistent with both "No New Physics" and Dispersive Method.
- Total error of our LQCD is 2.6%, dominated FV effects.



$\hat{\Pi}^{lat}(Q^2)$ vs $\hat{\Pi}^{pheno}(Q^2)$ for Various Q^2 Preliminary



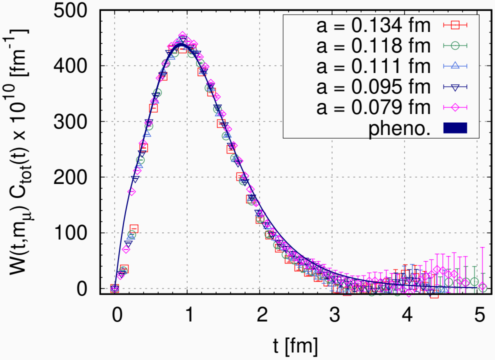
$$\hat{\Pi}^{lat}(Q^2) = \lim_{a\to 0} \sum_{t=0}^{T/2} \left[t^2 - \left(\frac{\sin Qt/2}{Qt/2} \right)^2 \right] \frac{C^f_{ii}(t)}{3} ,$$

$$\hat{\Pi}^{pheno}(Q^2) = \frac{Q^2}{12\pi^2} \int_0^\infty ds \frac{R_{had}(s)}{s(s+Q^2)} .$$

Lat (BMWc) vs Pheno (alphaQEDc17 by Jegerlehner) for $\omega(Q^2/m_\mu^2)\hat{\Pi}(Q^2)$

- The contributions at $Q^2 \sim (m_\mu/2)^2$ are dominant, and the lattice and phemenology are consistent within the error-bars there.
- However, the lattice error gets larger at $Q^2 \sim (m_\mu/2)^2$. More precise estimates are demanded and in progress.

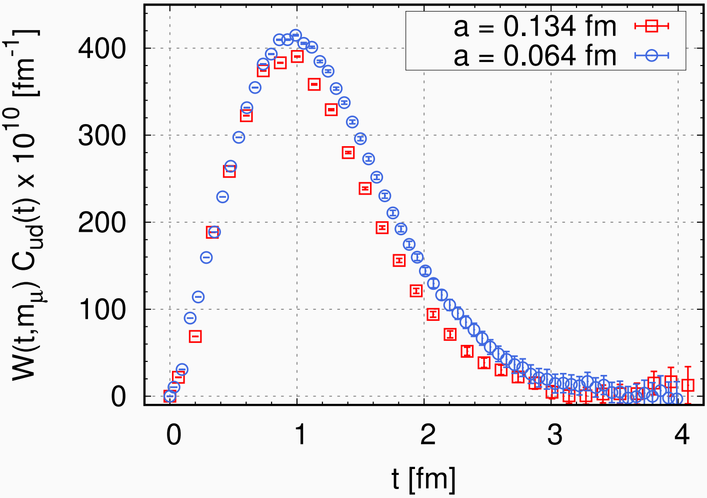
Integrand of $a_{\mu,ud}^{\text{LO-HVP}}$ I



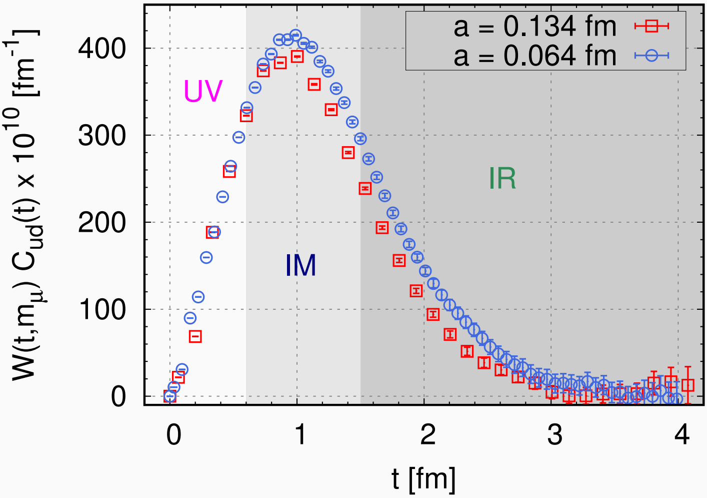
$$a_{\mu,ud}^{\text{LO-HVP}} = \sum_t W(t, m_\mu) C_{tot}(t) \ , \tag{6}$$

$$c.f. \ C_{tot}^{pheno}(t) = \int_0^\infty ds \sqrt{s} R_{had}(s) e^{-\sqrt{s}|t|} \ . \tag{7}$$

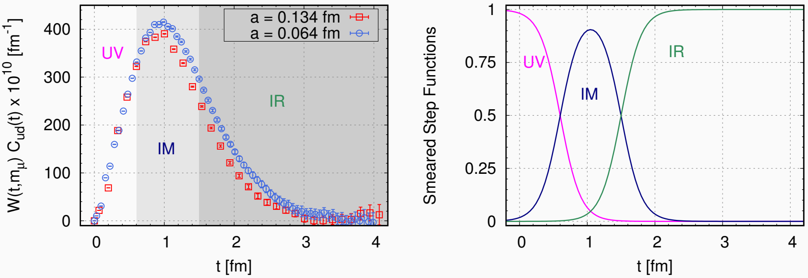
Integrand of $a_{\mu,ud}^{\text{LO-HVP}}$ II



Integrand of $a_{\mu,ud}^{\text{LO-HVP}}$ III



Window Method



$$\text{UV: } S_{UV}(t) = 1.0 - (1.0 + \tanh\left[(t - t_0)/\Delta\right])/2\ , \quad (8)$$

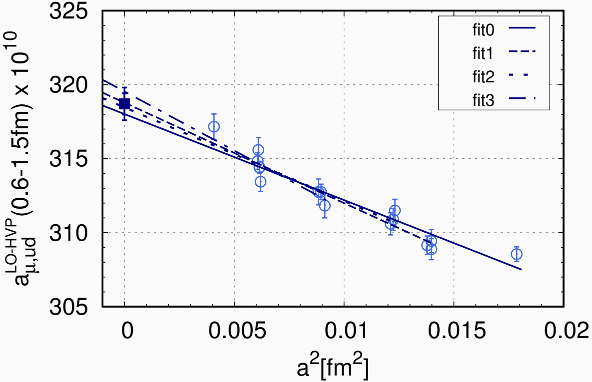
$$\text{IM: } S_{IM}(t) = \frac{1}{2}\left(\tanh\left[(t - t_0)/\Delta\right] - \tanh\left[(t - t_1)/\Delta\right]\right)\ , \quad (9)$$

$$\text{IR: } S_{IR}(t) = (1.0 + \tanh\left[(t - t_1)/\Delta\right])/2\ , \quad (10)$$

$$\text{We shall adopt } t_0 = 0.6fm\ , \quad t_1 = 1.5fm\ , \quad \Delta = 0.3fm\ . \quad (11)$$

c.f. RBC-UKQCD, arXiv: 1801.07224

Continuum Extrapolation in Dominant Window Preliminary



For the most important window (0.6 – 1.5 fm), the lattice QCD provides very precise data with per-mil level precision.

Summary on $a_{e,\tau}^{\text{LO-HVP}}$ PRL(2018)

$a_e^{\text{LO-HVP}}$ BMWc

$I=1$	$156.9(2.4)_{st}(2.1)_{acut}(0.0)_{tcut}(0.0)_{qcut}(1.2)_{da}(4.6)_{fv}$
$I=0$	$30.7(1.2)_{st}(1.0)_{acut}(0.1)_{tcut}(0.0)_{qcut}(0.2)_{da}$
total	$189.3(2.6)_{st}(2.3)_{acut}(0.1)_{tcut}(0.0)_{qcut}(1.5)_{da}(4.6)_{fv}(1.6)_{iso}$

$a_\tau^{\text{LO-HVP}}$ BMWc

$I=1$	$253.2(0.7)_{st}(1.4)_{acut}(0.0)_{tcut}(0.1)_{qcut}(1.2)_{da}(1.8)_{fv}$
$I=0$	$84.4(0.4)_{st}(0.7)_{acut}(0.0)_{tcut}(1.1)_{qcut}(3.4)_{da}$
total	$341.0(0.8)_{st}(1.6)_{acut}(0.0)_{tcut}(1.1)_{qcut}(1.5)_{da}(1.8)_{fv}(1.1)_{iso}$

Burger et.al.('15): $a_e^{\text{LO-HVP}} = 178.2(6.4)(8.6)$, $a_\tau^{\text{LO-HVP}} = 341(8)(6)$
HPQCD ('16): $a_e^{\text{LO-HVP}} = 177.9(3.9)$
Jeherlehner('16): $a_e^{\text{LO-HVP}} = 185.11(1.24)$.
Eidelman et.al.('07): $a_\tau^{\text{LO-HVP}} = 338(4)$.

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3 Summary and Perspective

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- We have obtained $a_\mu^{\text{LO-HVP}}$ directly at physical point masses: $a_\mu^{\text{LO-HVP}} = 711.1(7.5)(17.4) \times 10^{-10}$.
- Full controlled continuum extrapolation and matching to perturbation theory. Model assumptions are put on only for small corrections from FV/QED/isospin breaking. Total error is 2.6%, dominated by FV.
- Our Lattice QCD results are consistent with "No New Physics" as well as Phenomenological Dispersive Methods with a conservative systematic errors.
- Lat-Pheno. comparisons are made for HVP: consistent at small Q^2, but lattice tends to be larger, leading to larger $a_{\mu,lat}^{\text{LO-HVP}}$.
- Need $\sim 0.2\%$ precision to match Fermilab/J-PARC experiments!!
  1. lat-pheno combined analyses: window method (per-mil level precision at present statistics).
  2. new technique to reduce statistical errors (per-mil level precision at present statistics).
  3. control FV effects directly based on the first-principle.
  4. simulations with QED and isospin breaking corrections taken account.