Dyson-**S**chwinger **E**quations approach to pseudoscalar poles contribution to HLbL piece of a_μ

work done in collaboration with ***Adnan Bashir*** & ***Khépani Raya***

(to appear soon)

Pablo Roig

π^0 TFF

DSE input from

K. Raya, L. Chang, A. Bashir, J. J. Cobos-Martínez, L. X. Gutiérrez-Guerrero, C. D. Roberts and P. C. Tandy, Phys. Rev. D **93**, no. 7, 074017 (2016) doi:10.1103/PhysRevD.93.074017.

K. Raya, M. Ding, A. Bashir, L. Chang and C. D. Roberts, Phys. Rev. D **95**, no. 7, 074014 (2017) doi:10.1103/PhysRevD.95.074014.

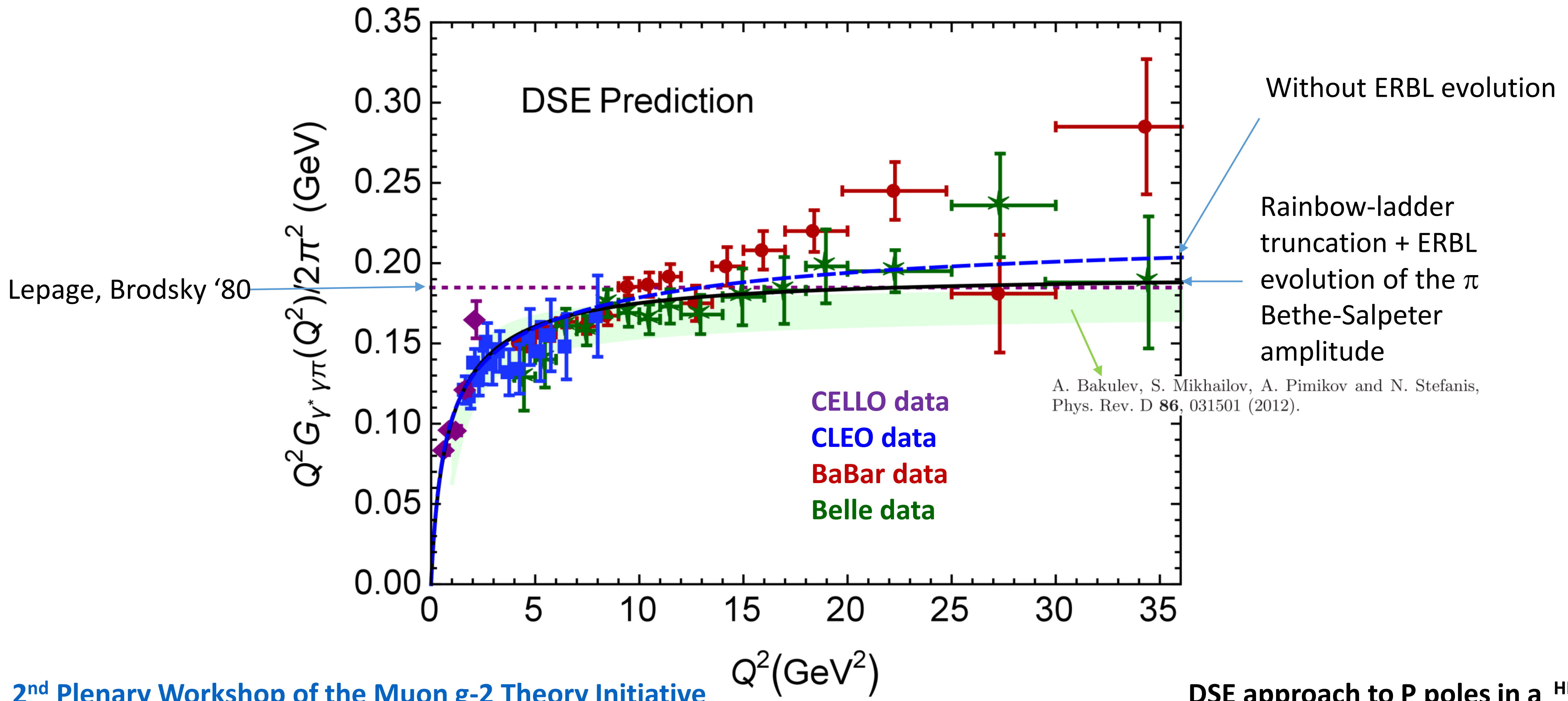
$\eta_{c/b}$ TFF

2nd Plenary Workshop of the Muon g-2 Theory Initiative

18-22 June 2018, Helmholtz-Institut Mainz, Germany

π^0 TFF from Dyson-Schwinger equations

K. Raya, L. Chang, A. Bashir, J. J. Cobos-Martínez, L. X. Gutiérrez-Guerrero, C. D. Roberts, P. C. Tandy Phys.Rev. D93 (2016) no.7, 074017



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(K. Raya, A. Bashir & P. Roig)

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LMD+V+V' can be of interest for lattice Colls. in order to parametrize their data.

(K. Raya, A. Bashir & P. Roig)

$m_\pi^2 \sim 0$

π^0 TFF & LMD+V+V'

$$F_{\pi\gamma\gamma}(Q_1^2, Q_2^2) = \frac{f_\pi}{N_c} \frac{P(Q_1^2, Q_2^2)}{D(Q_1^2, Q_2^2)} \,, \qquad D(Q_1^2, Q_2^2) = \prod_{i=1}^{N} (Q_1^2 + M_{V_i}^2)(Q_2^2 + M_{V_i}^2)$$

$$P(Q_1^2, Q_2^2) = \sum_{\alpha,\beta} c_{\alpha,\beta} (Q_1^2 + Q_2^2)^\alpha (Q_1^2)^\beta (Q_1^2)^\beta$$

(LMD+V corresponds to N=2)

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$$\begin{aligned} P(Q_1^2, Q_2^2) \quad = \quad & c_{11}(Q_1^2 Q_2^2)(Q_1^2 + Q_2^2) + c_{20}(Q_1^2 + Q_2^2)^2 + c_{01}(Q_1^2 Q_2^2) + c_{10}(Q_1^2 + Q_2^2) + c_{00} \\ + \quad & c_{02}(Q_1^2 Q_2^2)^2 + c_{21}(Q_1^2 Q_2^2)(Q_1^2 + Q_2^2)^2 + c_{12}(Q_1^2 Q_2^2)^2 (Q_1^2 + Q_2^2) \\ + \quad & c_{30}(Q_1^2 + Q_2^2)^3 + c_{31}(Q_1^2 Q_2^2)(Q_1^2 + Q_2^2)^3 \, . \end{aligned}$$

It corresponds to N=3, **LMD+V+V'**

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First row corresponds to LMD+V

(K. Raya, A. Bashir & P. Roig)

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π^0 TFF & LMD+V+V'

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It corresponds to N=3, LMD+V+V'

First row corresponds to LMD+V

It can be rewritten as $F(Q_1^2, Q_2^2) = \frac{f_\pi}{N_c} \left[f(Q_1^2) + \sum_{M_{V_i}} \frac{1}{Q_2^2 + M_{V_i}^2} g_{M_{V_i}}(Q_1^2) \right]$ (Knecht & Nyffeler'02)

(K. Raya, A. Bashir & P. Roig)

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\end{aligned}
$$

$c_{30} \to 0$, $c_{31} \to 0$

(to fulfil **QCD asymptotics**)

(K. Raya, A. Bashir & P. Roig)

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ABJ anomaly: $c_{00} = N_c \dfrac{(M_{V_1} M_{V_2} M_{V_3})^4}{4\pi^2 f_\pi^2}$

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Allows for (seeming) small violations due to *data non* being *asymptotic*

Fully asymmetric BL: $c_{20} = 2N_c(M_{V_1}M_{V_2}M_{V_3})^2(1+\delta_{BL})$

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If *subleading corrections* are taken into account:

$$\frac{F_{\pi\gamma\gamma}(Q^2,Q^2)}{F(0,0)} = \frac{8}{3}\pi^2 f_\pi^2\left\{\frac{1}{Q^2}-\frac{8}{9}\frac{\delta_S^2}{Q^4}+\cdots\right\} \Rightarrow c_{21} = -\left(\frac{1}{4}c_{02}+\frac{4N_c}{27}\delta_S^2\right)$$

$\delta_S^2 = (0.2 \pm 0.02)\ \mathrm{GeV}^2$

We obtain a larger (and opposite in sign) value

(K. Raya, A. Bashir & P. Roig)

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The **BL limit** must not change *when the non-asymptotic photon is virtual*: $c_{21} + c_{12}Q_0^2 \approx 0\ \forall Q_0^2$, which is not possible.

(K. Raya, A. Bashir & P. Roig)

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However, c21 ϵ [$-1.1\, c_{12} M_{V_1}^2$, $-0.9\, c_{12} M_{V_1}^2$] works remarkably well, since the dominant region is around $Q_0^2 \sim M_{V_1}^2 \sim M_\rho^2$.

(K. Raya, A. Bashir & P. Roig)

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From the *subleading terms*, we get

$$c_{20} + c_{11}Q_0^2 + c_{02}Q_0^4 = 6(M_{V_1}^2+Q_0^2)(M_{V_2}^2+Q_0^2)(M_{V_3}^2+Q_0^2)(1+\delta_{BL})$$

Approximately satisfied (within 9%) for $Q_0^2 \sim M_\rho^2$

(K. Raya, A. Bashir & P. Roig)

$m_\pi^2 \sim 0$

π^0 TFF & LMD+V+V'

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Approximately satisfied (within 9%) for $Q_0^2 \sim M_\rho^2$

It is seen, a posteriori, that $M_{V_{2,3}}$ can be identified with $M_{\rho'}$, $M_{\rho''}$

$m_\pi^2 \not\sim 0$

(K. Raya, A. Bashir & P. Roig)

π^0 TFF & LMD+V+V'

$$\frac{1}{4\pi^2 f_\pi} \longrightarrow \frac{1}{4\pi^2 f_\pi}(1-\Delta) \longrightarrow c_{00} = N_c \frac{(M_{V_1} M_{V_2} M_{V_3})^4}{4\pi^2 f_\pi^2}(1-\Delta)$$

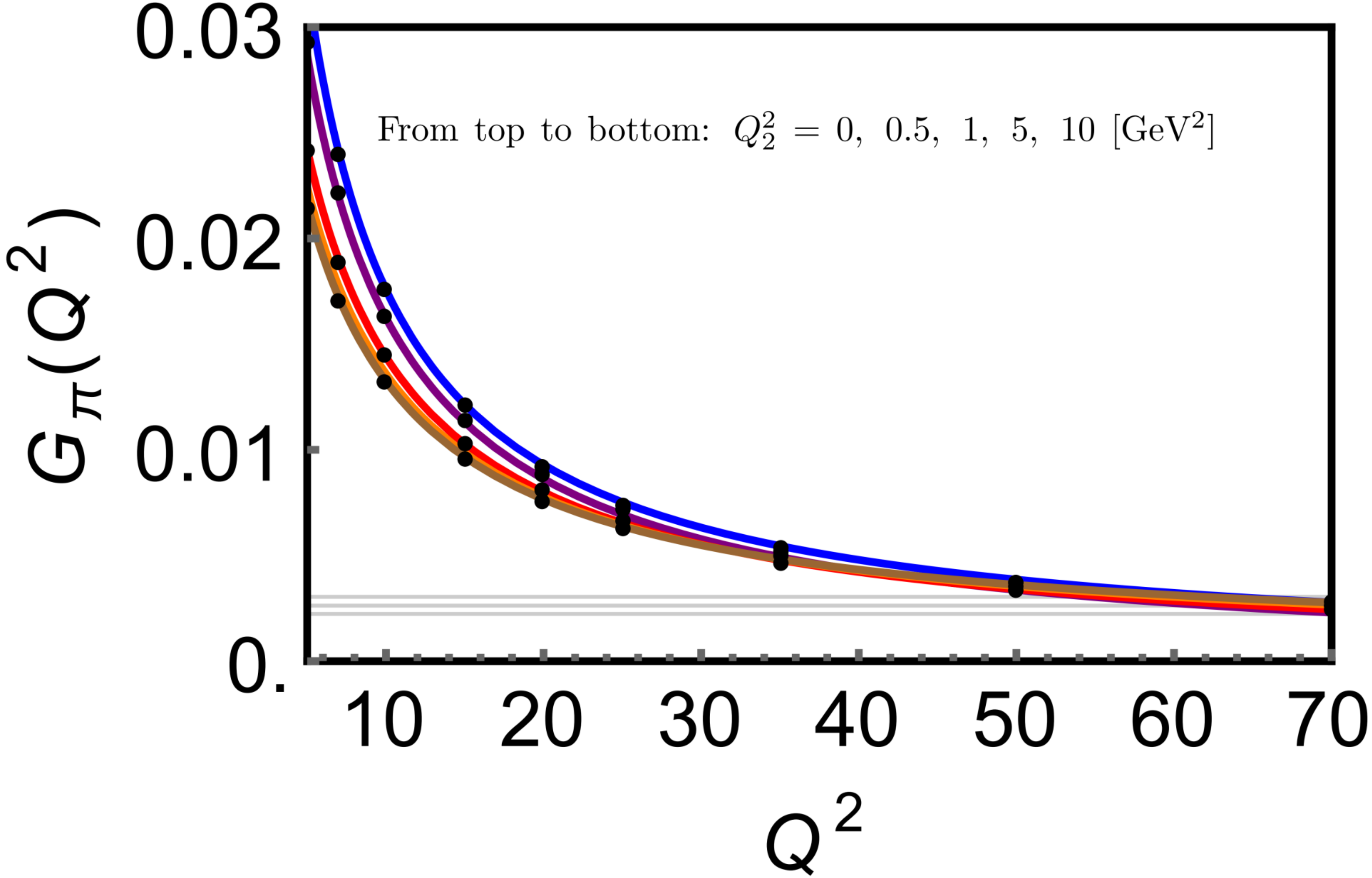
No other corrections enter $P(Q_1^2,Q_2^2)$ because we are dealing with **π^0 pole contributions**

$$\Delta = 0.008 \pm 0.010 \text{ matches the PDG value of } \Gamma[\pi^0 \to \gamma\gamma]$$

$m_\pi^2 \not\sim 0$

(K. Raya, A. Bashir & P. Roig)

π^0 TFF & LMD+V+V', fit results



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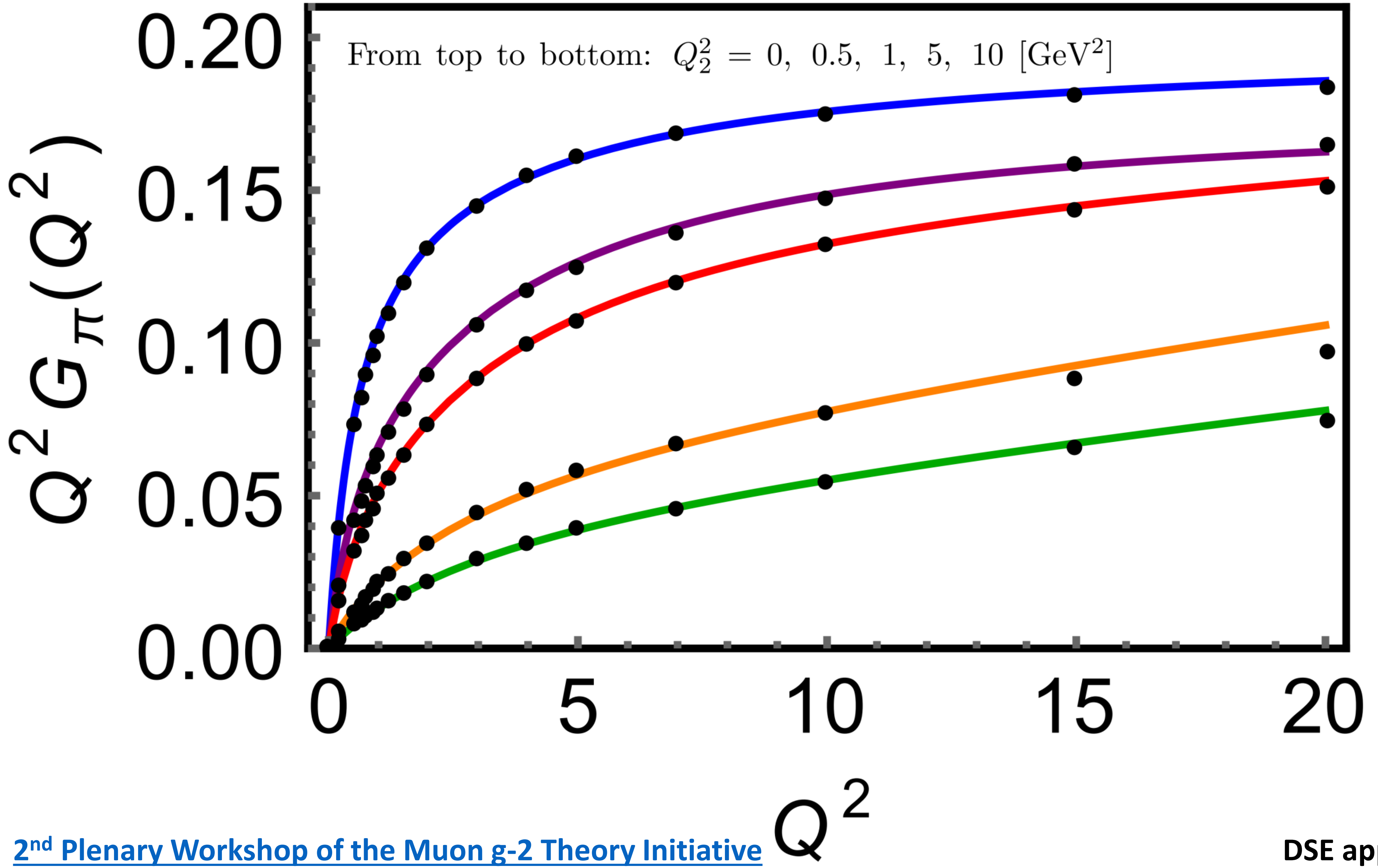
π^0 TFF & LMD+V+V', fit results



(K. Raya, A. Bashir & P. Roig)

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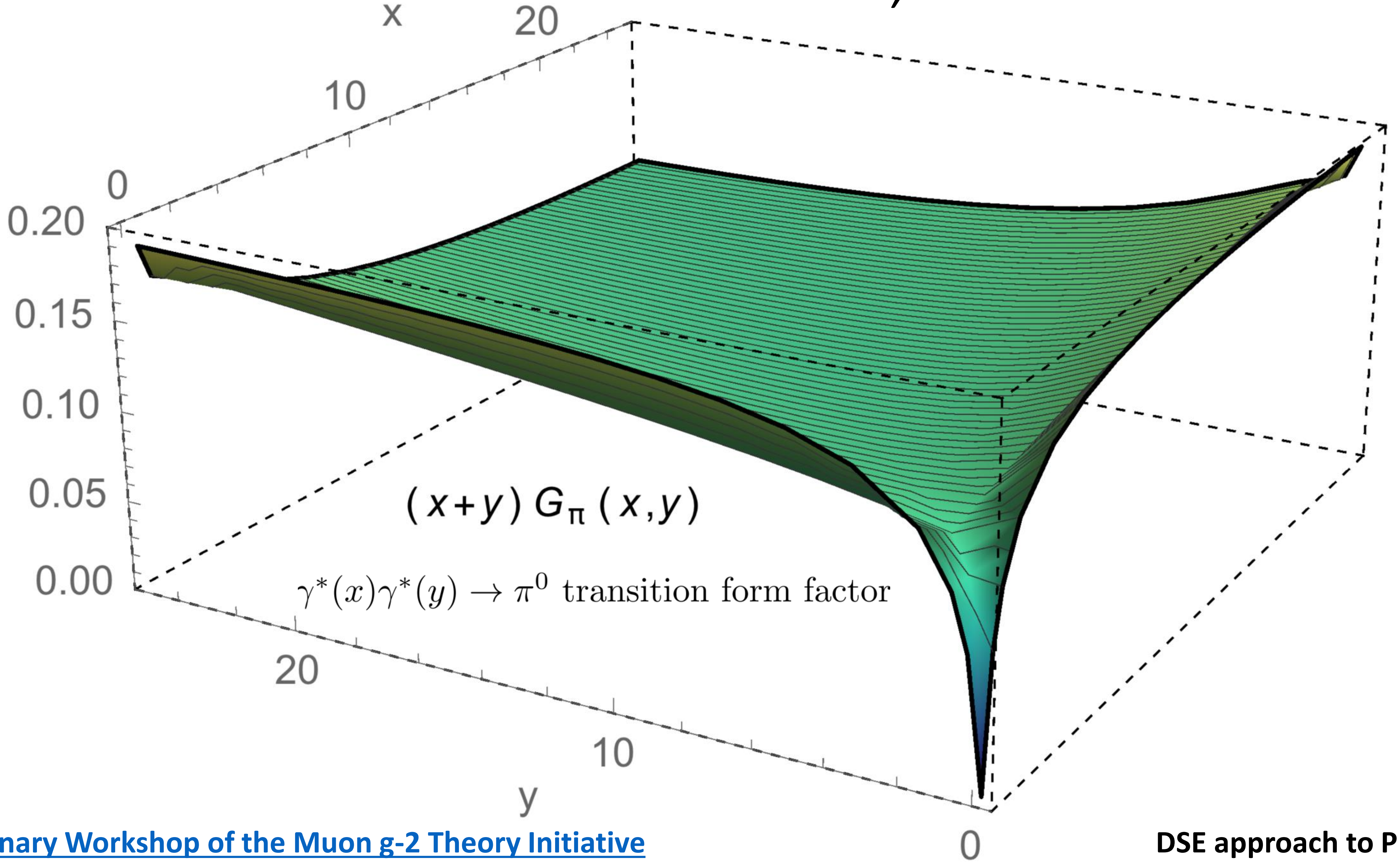
π^0 TFF & LMD+V+V', fit results



$m_\pi^2 \not\sim 0$

(K. Raya, A. Bashir & P. Roig)

π^0 TFF & LMD+V+V', fit results

Case	c_{10}	c_{01}	c_{11}	c_{02}
$\Delta = 0$	94.833	68.885	62.941	17.245
$\Delta = 0.008$	94.517	71.082	62.760	17.238
$\Delta = 0.018$	94.360	73.590	62.466	17.266

Used to set

$$\delta_{BL} = 0.088 \pm 0.060$$

$$c_{21} = -0.660 \pm 0.060 \approx -1.1 m_\rho^2$$

$m_\pi^2 \not\sim 0$

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With correlations:

Case $\Delta = 0$	c_{10}	c_{01}	c_{11}	c_{21}	c_{02}	δ_{BL}
c_{10}	1.00	-0.48	0.10	-0.02	0.07	-0.18
c_{01}	-0.48	1.00	-0.76	0.16	0.26	0.07
c_{11}	0.10	-0.76	1.00	-0.30	-0.61	-0.10
c_{21}	-0.02	0.16	-0.30	1.00	-0.13	0.00
c_{02}	0.07	0.26	-0.61	-0.13	1.00	0.04
δ_{BL}	-0.18	0.07	-0.10	0.00	0.04	1.00

Case $\Delta \neq 0$	c_{10}	c_{01}	c_{11}	c_{02}
c_{10}	1.00	-0.48	0.09	0.07
c_{01}	-0.48	1.00	-0.76	0.28
c_{11}	0.09	-0.76	1.00	-0.69
c_{02}	0.07	0.28	-0.69	1.00

(K. Raya, A. Bashir & P. Roig)

π^0 TFF & LMD+V+V', P pole contributions to a_μ^{HLbL}

Δ	$a_\mu^{\pi^0,Hlbl}$ $(\times 10^{10})$
0	$5.94^{+0.21}_{-0.34}$
0.008	$5.84^{+0.19}_{-0.22}$
0.018	$5.71^{+0.22}_{-0.28}$

$$a_\mu^{\pi^0,Hlbl} = \left(5.84^{+0.34}_{-0.41}\right) \cdot 10^{-10}$$

(K. Raya, A. Bashir & P. Roig)

π^0 TFF & LMD+V+V', P pole contributions to a_μ^{HLbL}

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$$a_\mu^{\pi^0,Hlbl} = \left(5.84^{+0.34}_{-0.41}\right) \cdot 10^{-10}$$

We proceeded similarly for the η_c, η_b contributions (still some work needs to be done with η, η') to obtain

(We used a slight variation of LMD for them)

Meson	$a_\mu^{\eta_{c,b},Hlbl}$ $(\times 10^{10})$
η_c	0.087 ± 0.005
η_b	0.00026 ± 0.00001

η_c contribution is negligible until 1% precisión is reached on a_μ^{HLbL}

This does not need to be the case for the $\eta(1295)$, $\eta(1405)$, $\eta(1475)$,…

Update on '*Pseudoscalar pole light-by-light contributions to the muon (g-2) in Resonance Chiral Theory*'. e-Print: arXiv: 1803.08099 [hep-ph], A. Guevara, P. Roig & J. J. Sanz-Cillero, to be published in JHEP.

Systematic theory errors
(overlooked before)

Individual contributions

$$a_\mu^{\pi^0,LbL} = (5.81 \pm 0.09) \cdot 10^{-10}$$

$$a_\mu^{\eta,LbL} = (1.51 \pm 0.06) \cdot 10^{-10}$$

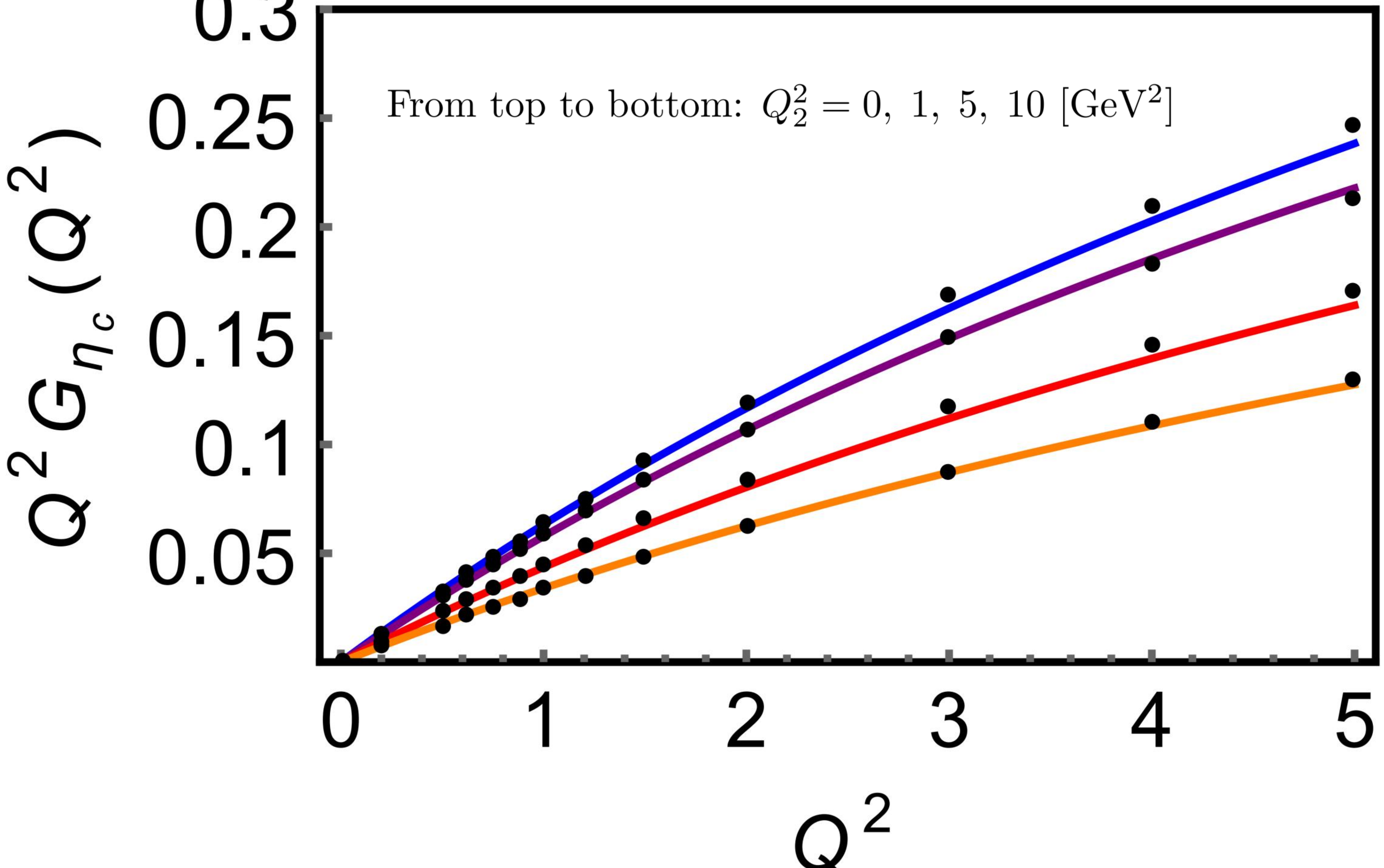
$$a_\mu^{\eta',LbL} = (1.15 \pm 0.07) \cdot 10^{-10}$$

Error due to subleading 1/Nc corrections: $\pm 0.09 \times 10^{-10}$

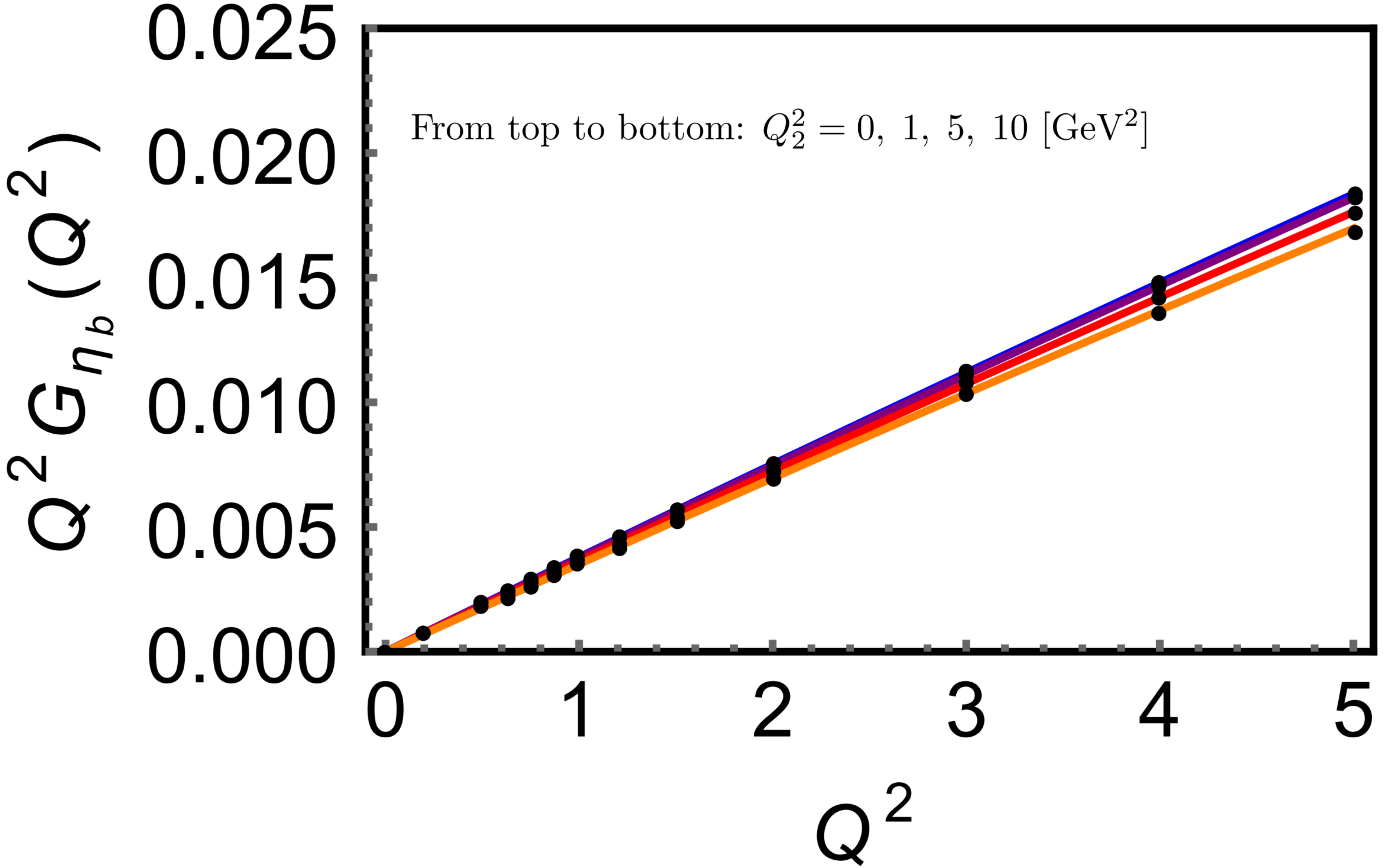
Error due to falling as $1/Q^4$ in the doubly asymptotic limit (instead of as $1/Q^2$): $+0.5 \times 10^{-10}$

$$a_\mu^{P,LbL} = (8.47 \pm 0.16) \cdot 10^{-10}$$

BACKUP



From top to bottom: $Q_2^2 = 0,\ 1,\ 5,\ 10$ [GeV^2]
$Q^2 G_{\eta_c}(Q^2)$
Q^2
0.3
0.25
0.2
0.15
0.1
0.05
0
1
2
3
4
5



0.025
0.020
0.015
0.010
0.005
0.000
$Q^2 G_{\eta_b}(Q^2)$
From top to bottom: $Q_2^2 = 0,\ 1,\ 5,\ 10$ [GeV^2]
0
1
2
3
4
5
Q^2

Some caveats on **Fischer, Goecke & Williams** Eur.Phys.J. A47 (2011) 28; Phys.Rev. D83 (2011) 094006, Erratum: Phys.Rev. D86 (2012) 099901 & Phys.Rev. D87 (2013) no.3, 034013

- Their off-shell prescription is based on an axial-vector WTI which holds only for the leading amplitude (Si-Xue Qin, Craig D. Roberts, S. M. Schmidt Phys.Lett. B733 (2014) 202-208)

- Use of PTIRs or extrapolations?

- Consistency with axial anomaly in the study of η/η' TFFs?

- Use of phenomenology to constrain dressing functions?

- Double-counting?

- …?