Estimate of the muon g-2 : Update of the HLS Model Prediction

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LPNHE Paris 6/7

Mainz WG g-2 Theory Initiative (June 2018)

OUTLINE

- Why an Update ?
- $e^+e^- \rightarrow K^+K^-$, within HLS : CMD2/BaBar/CMD3
- Analysis of $e^+e^- \rightarrow K^+K^-$: CMD2
- Analysis of $e^+e^- \rightarrow K^+K^-$: BaBaR
- Analysis of $e^+e^- \rightarrow K^+K^-$: CMD3
- Modified $(g-2)_\mu$ estimates
- Conclusions

Update of the HLS Results

- **Main Focus : the K^+K^- Sector**

➢ **Use of Exponentiated Coulomb Term & FSR_K**

A.Hoefer *et al*. EPJC 24 (2002) 51

➢ **Reexamining the K^+K^- Data within HLS :**

K+K- : CMD2 Phys. Lett. B669 (2018) 217

K+K- : BaBaR Phys. Rev. D88 (2013) 032013

K+K- : CMD3 Phys. Lett. B779 (2018) 64

K_LK_S: CMD3 Phys. Lett. **B760 (2016) 314**

Data Handled via Global Fits

- Almost all (≈50 - 60) existing data samples used
- e+e- annihil. & τ spectra & π & K spacelikeFF's
- Devoted treatment of normalization errors (unbiasing method)

M. Benayoun *et al.* EPJ C75 (2015) 613

- Update : Exponentiated Coulomb Term & FSR_K
- Main focus of this update : the $K^+ K^-$ sector

π+ π- : KLOE 2 JHEP 1803 (2018) 17

Revisiting $e^+e^- \to K^+ K^-$ within BHLS2

- **Method : global fit of the HLS channels/Data**

$e^+e^- \to \pi\pi / K_L K_S / \pi\gamma / \eta\gamma / \pi\pi\pi$ & $\tau \to \pi\pi\nu_\tau$
& PVγ, Pγγ decays & π^+ & K^+ FF's at $s<0$

Together with CMD2/CMD3/BaBar $e^+e^- \to K^+ K^-$ data samples each in turn

Presently : discard SND $K^+ K^-$ spectra (≈ 7% syst)

ρ & ω background beneath the ϕ : 100% controlled by the $e^+e^- \to \pi^+\pi^-$ ($K_L K_S$) data/channel

Systematics in $e^+e^- \rightarrow K^+ K^-$ Data

Systematics (1.00 to 1.03 GeV Region)

- **CMD2 : σ = 2.2 %**
- **BaBar : σ = (E< 1.01 GeV) 0.82 % (E> 1.01 GeV) 0.72 %**
- **CMD3 : σ = 2.0 %**

1/Systematics : point to point correlated

(normalization uncertainties)

2/ Normalization correction :

sampled on a gaussian pdf $D_{norm}(m=0,\sigma)$

3/ Each Experiment : one sampling λ of $D_{norm.}$

Scale Uncertainty & Correction

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- **Normalization correction :**

$$\lambda = \left\{ M^T V^{-1} [\, m - M \,] \right\} \Big/ \left\{ M^T V^{-1} M + \frac{1}{\sigma^2} \right\}$$

Unbiasing data m vs Model /Ref. M : m→ m - λ M

V= covariance of uncorrelated errors

- **(m vs M) Consistency Condition: λ/σ < 3**

CMD2 $e^+e^- \to K^+ K^-$ & global FIT

CMD2 $K^+ K^-$: (incl. 1995)

Rescalings : 1.005/1.013

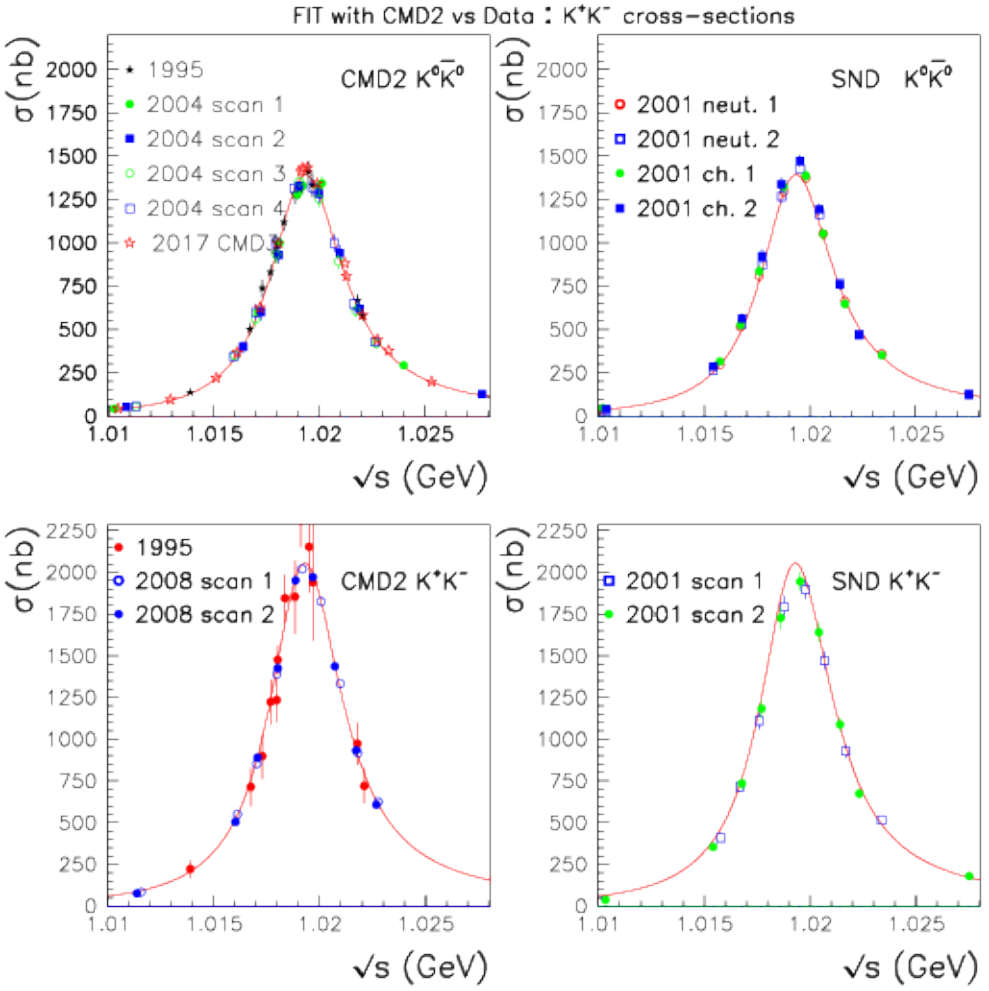
- **χ^2/N_p=31.28/36 (95.4%)**

photon VP from DN

FSI (FSR+Coulomb): FJ

E<1.05 GeV :

$a_\mu(K^+ K^-)$=17.28 ±0.29



$e^+e^- \rightarrow K^+ K^-$:CMD3 & BaBaR vs CMD2

Bulk of data samples for several channels

in the ϕ region ($K_L K_S$ /πγ /ηγ /πππ) almost solely collected by CMD2 & SND :

Natural choice for Reference Energy : CMD2 & SND

Assumption on Energy calibrations around the ϕ :

- $E_{CMD2} = E_{CMD3} + \delta E_{CMD3}$
- $E_{CMD2} = E_{BaBar} + \delta E_{BaBaR}$

Energy shifts δE_{CMD3} & δE_{BaBaR} submitted to fit

BaBaR $e^+e^- \rightarrow K^+ K^-$ & global fit

BaBar Data :

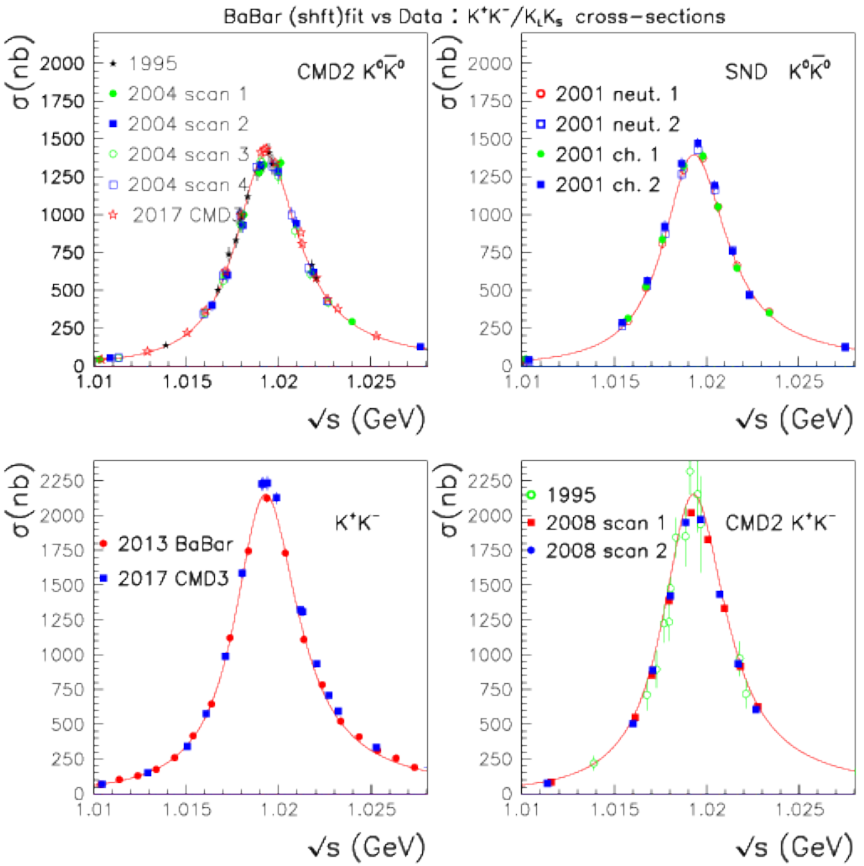
Fit up to 1.025 GeV

DN's VP:

- χ^2/N_p=47.5/27 (89.2%)

E<1.05 GeV :

- $a_\mu(K^+ K^-)$=18.12 ±0.21



a_μ(BaBaR <1.06)=18.64±0.16$_{stat}$±0.13$_{syst}$

BaBaR e^+e^- → $K^+ K^-$ & global fit II

(Meas-Fit)/Fit

BaBar Data :

Fit up to 1.025 GeV

Unshifted Data :

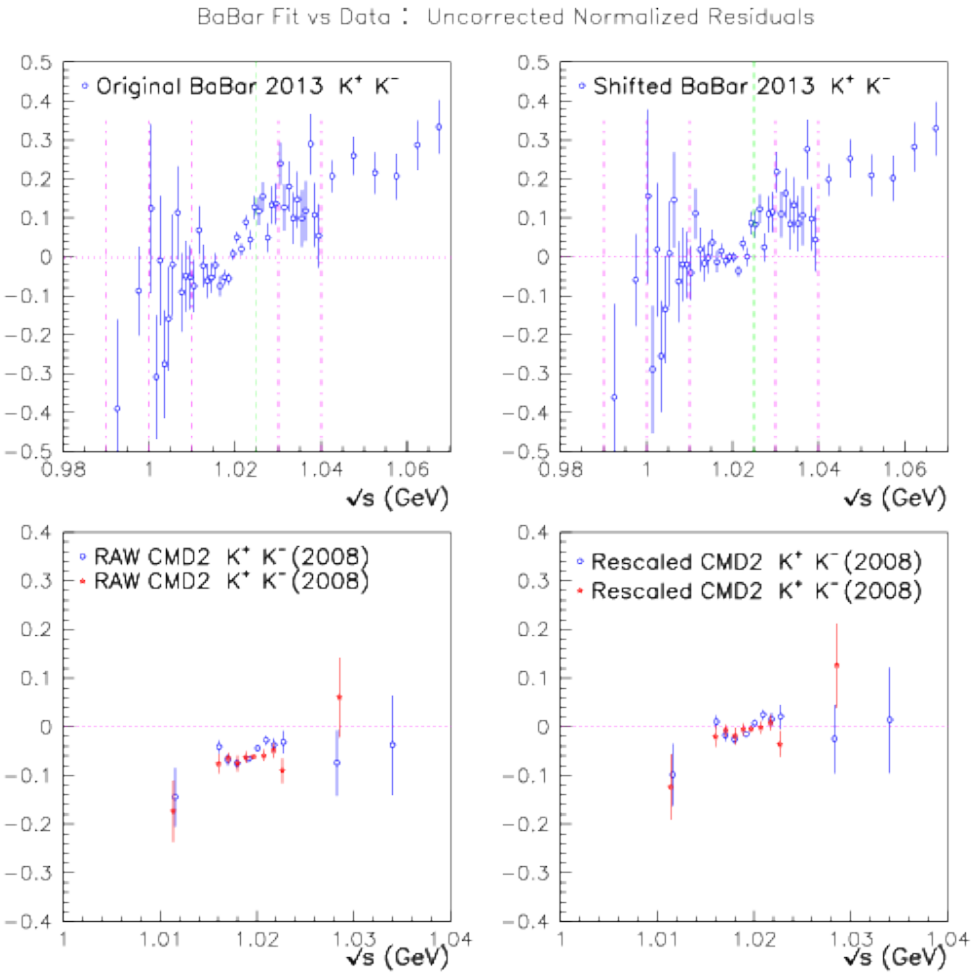
χ^2/N_p=83.0/27 (43.2%)

Shifted Data :

χ^2/N_p=47.5/27 (89.2%)

CMD2 Rescaling : ≈1.05

only a ≈ 2.2 σ effect



BaBaR $e^+e^- \rightarrow K^+ K^-$ & global fit III

BaBar Data :

Fit up to 1.025 GeV

DN's VP:

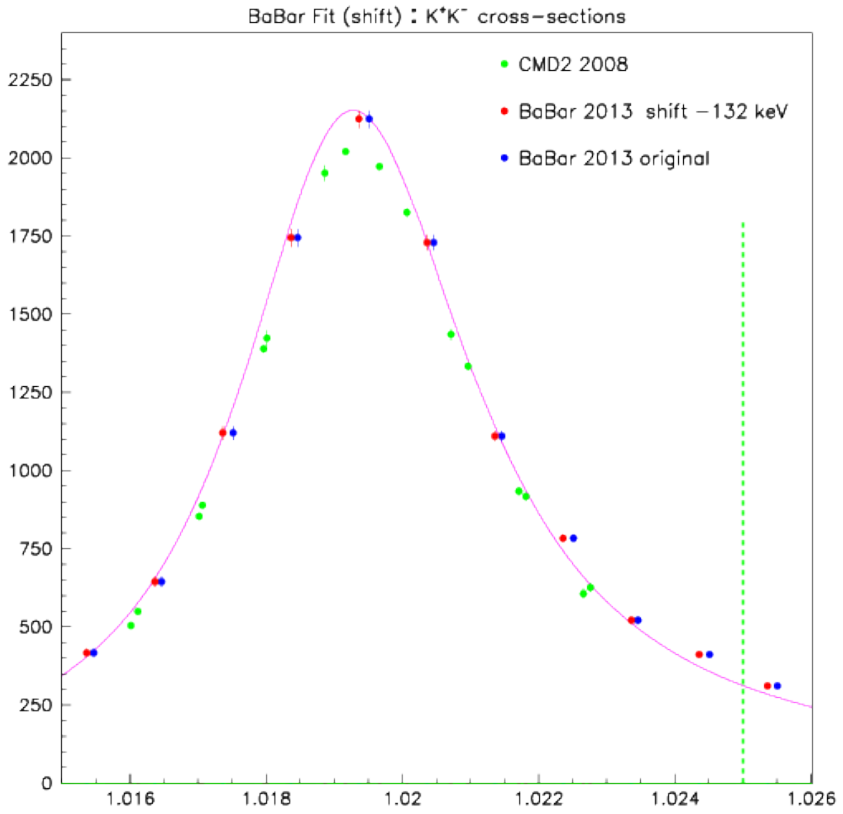
- $\delta E_{BaBaR} = -132.0 \pm 19.1$ keV

FJ's VP:

- $\delta E_{BaBaR} = -121.0 \pm 16.2$ keV

FI's VP:

- $\delta E_{BaBaR} = -135.1 \pm 21.2$ keV



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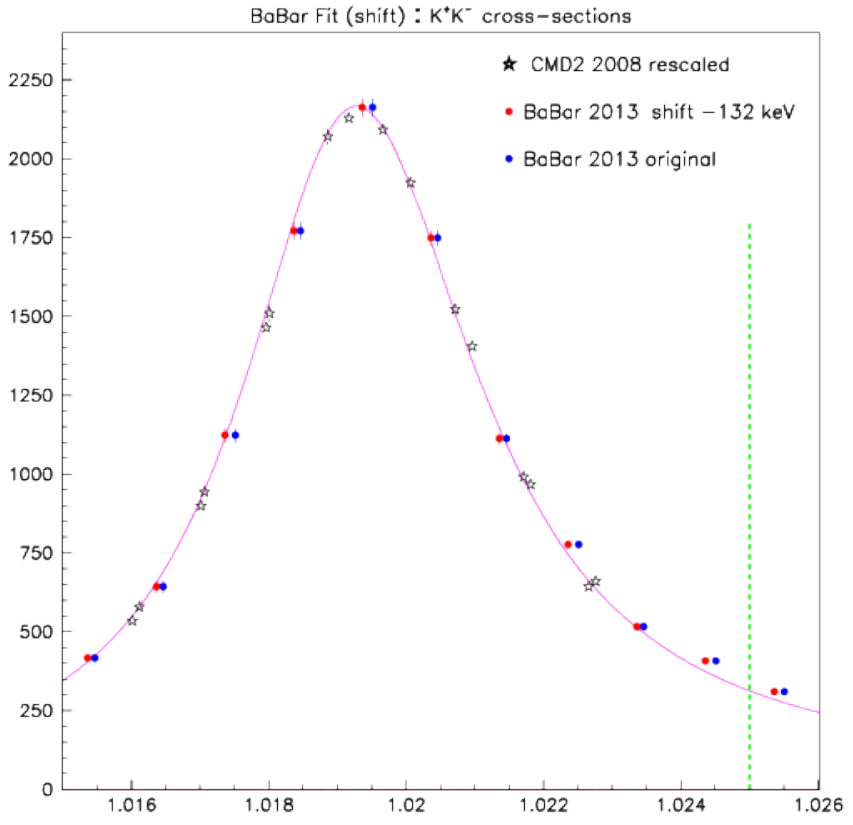


a_μ from BHLS2 Fits (update I)

HLS channels :

FSI updated

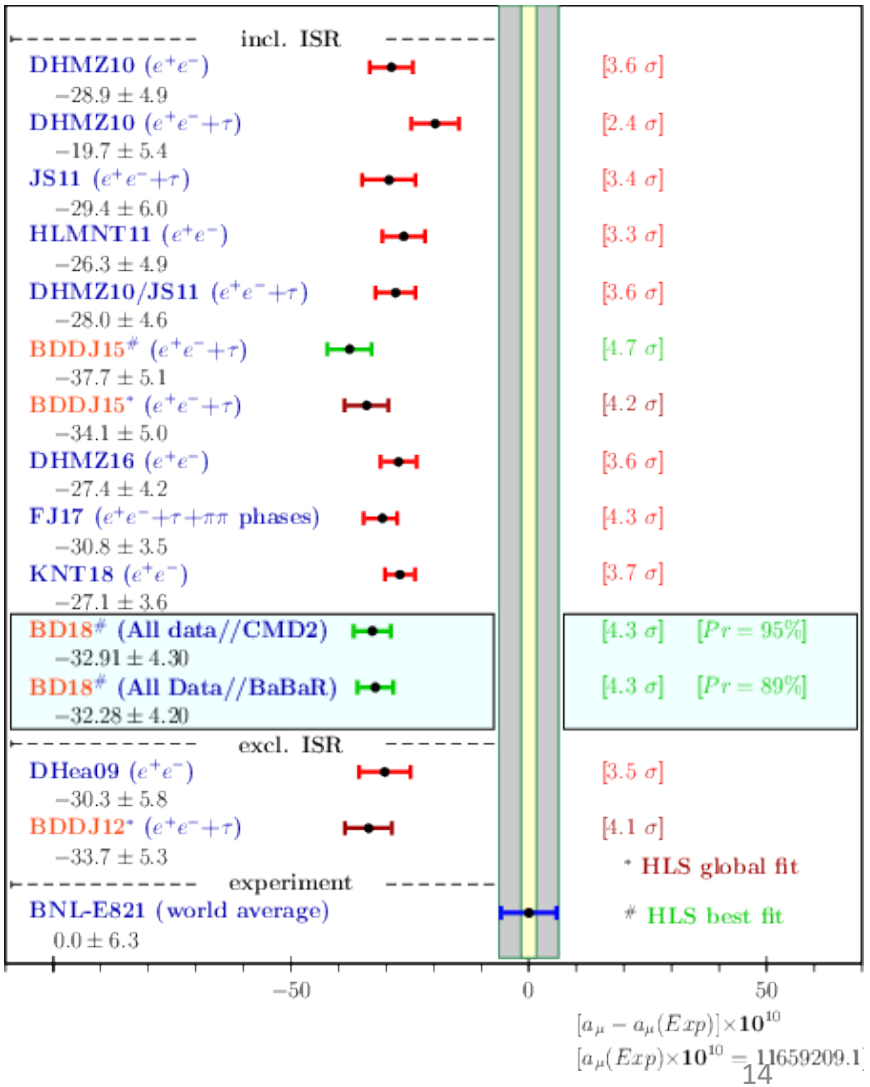
BHLS

Non HLS channels :

HVP update from FJ

- **+ ADD. SYST.**

BHLS2



Global fit : CMD2 & BaBaR $e^+e^- \rightarrow K^+K^-$

Data :

BaBar E< 1.025 GeV

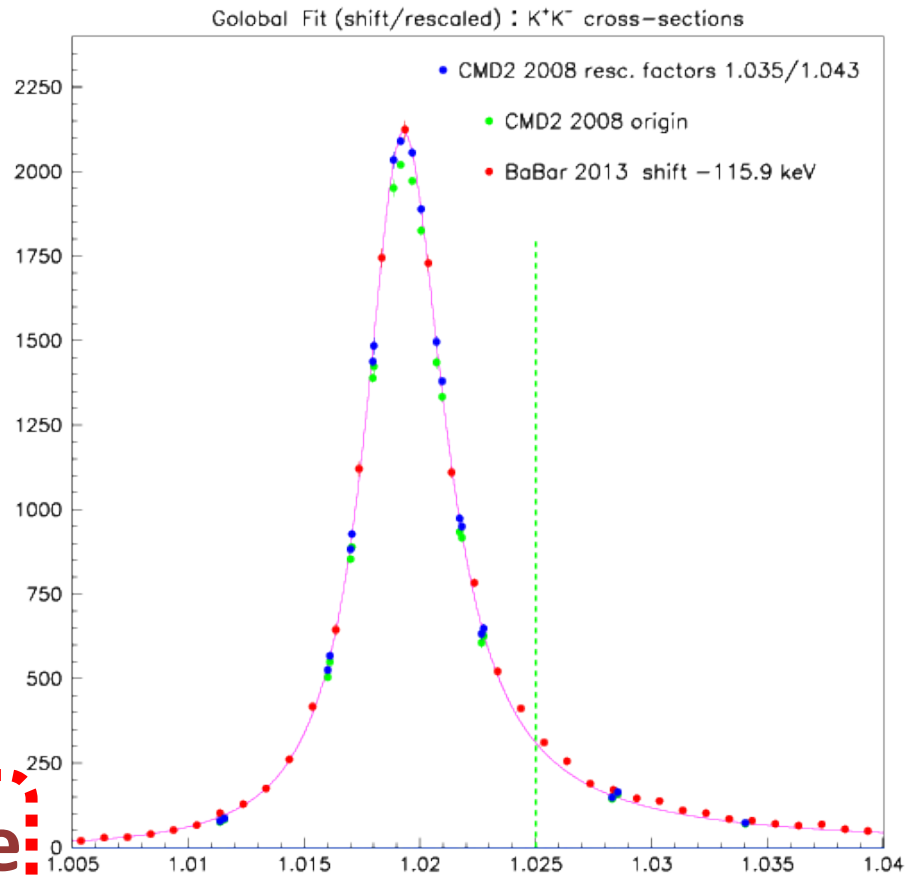
CMD2 (2 scans)

Fit with DN's VP: (85.8%)

- χ^2/N_p(CMD2)=36.4/36
- χ^2/N_p(BaBaR)=49.7/27

Reminder:

ρ–ω bkg (& ϕ pole) not free



Global fit : CMD2 & BaBar $e^+e^- \rightarrow K^+K^-$

BaBar & CMD2 Data :

$\delta E_{BaBaR} = -115.9 \pm 17.4$ keV

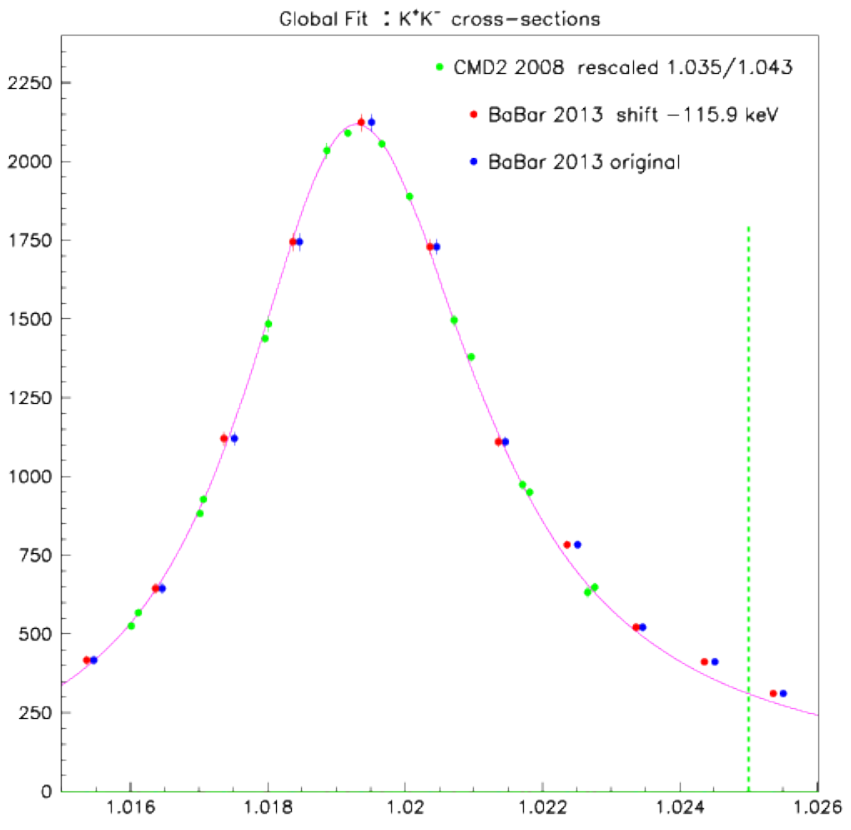
(BaBar Fit ϕ mass vs CMD2:

$\Delta m = (-)99 \pm 8_{CMD2} \pm 13_{BaBaR}$ keV

CMD2 Rescaling factor :

1.041 ±0.005

Still a ≈ 2 σ effect !



E<1.05 GeV :

- $a_\mu(K^+ K^-) = 17.83 \pm 0.20$

Global fit : CMD2 & BaBar $e^+e^- \rightarrow K^+K^-$

BaBar & CMD2 Data :

$\delta E_{BaBaR} = -115.9 \pm 17.4$ keV

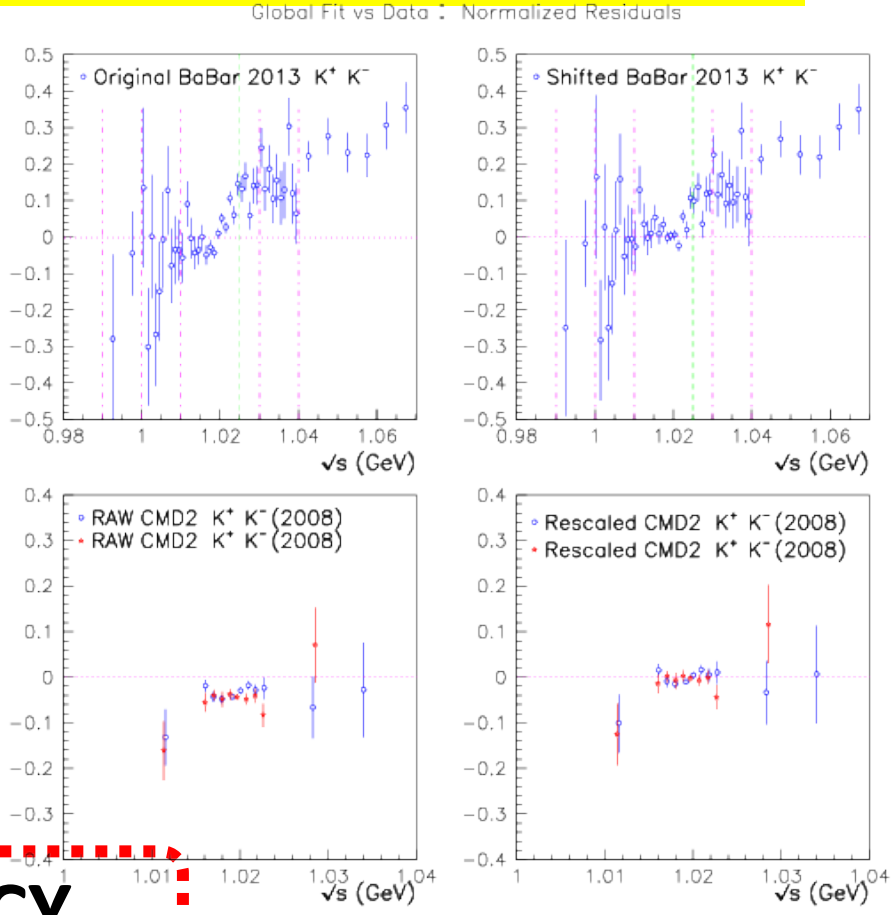
CMD2 Rescaling factor :

1.041 ±0.005

Still a ≈ 2. σ effect !

Fit Prob. 85.8%

NO OBVIOUS INCONSISTENCY



a_μ from BHLS2 Fits (update II)

HLS channels :

FSI updated

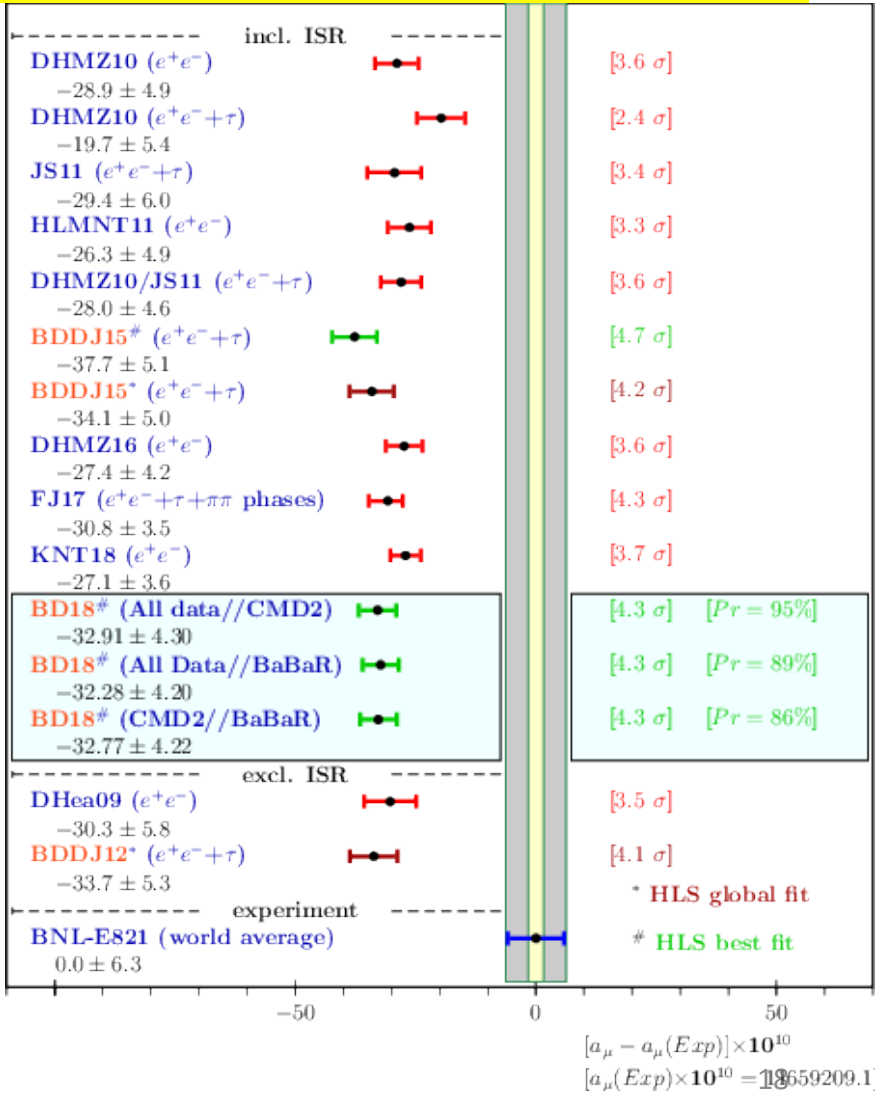
BHLS

Non HLS channels :

HVP update from FJ

- **+ ADD. SYST.**

BHLS2



CMD3 $e^+e^- \rightarrow K^+ K^-$ & global fit I

CMD3 Data :

Fit up to 1.025 GeV

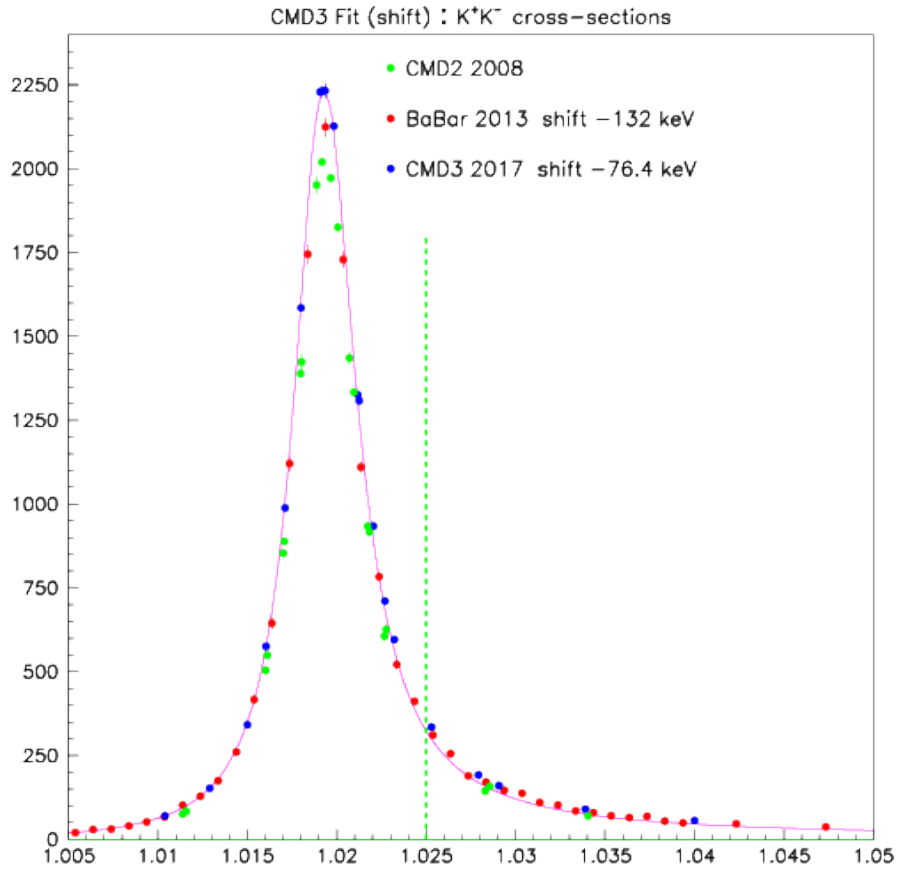
Most likely fit : FJ's VP

- χ^2/N_p=29.7/16 (87.7%)
- δE_{CMD3} = -76.4 ± 9.9 keV

E<1.05 GeV :

- $a_\mu(K^+ K^-)$=18.75 ±0.16

a_μ(CMD3<1.06)=19.33±0.40



CMD3 $e^+e^- \rightarrow K^+ K^-$ & global fit II

CMD3 Data :

Fit up to 1.025 GeV

FJ's VP:

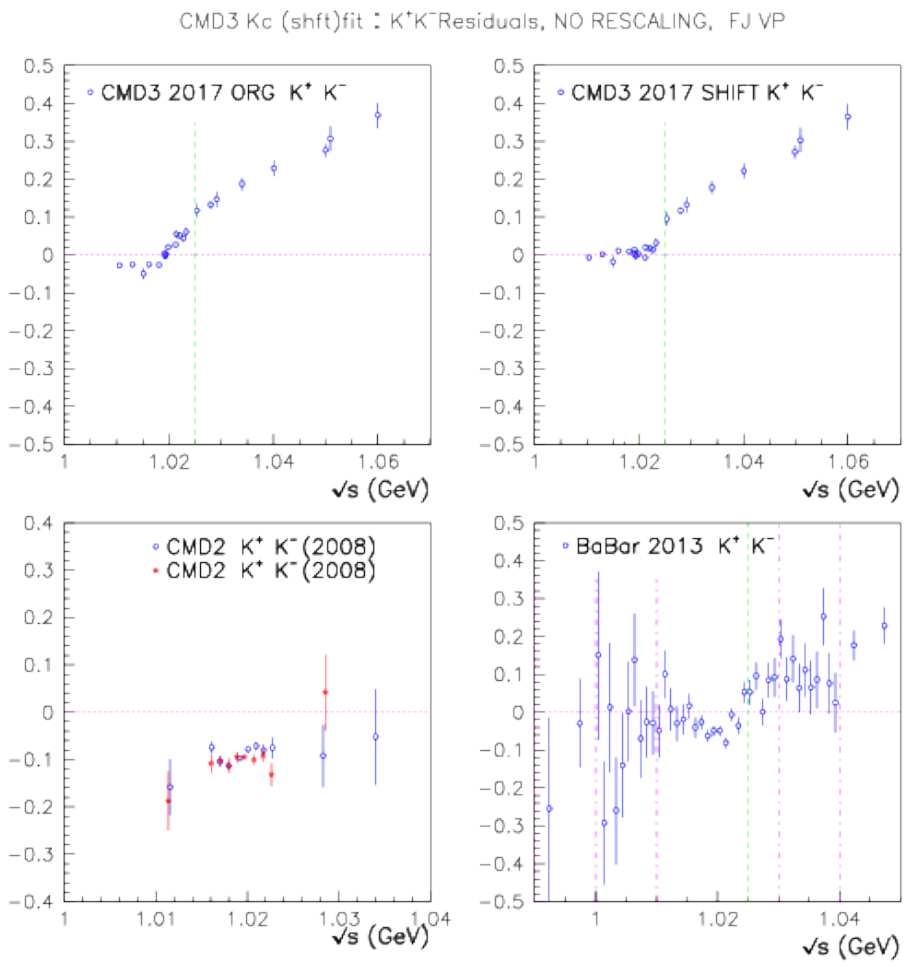
- **CMD2 scale ≈1.10**

  (≈ 4.5 σ_{CMD2})

- **BaBar scale ≈ 1.034**

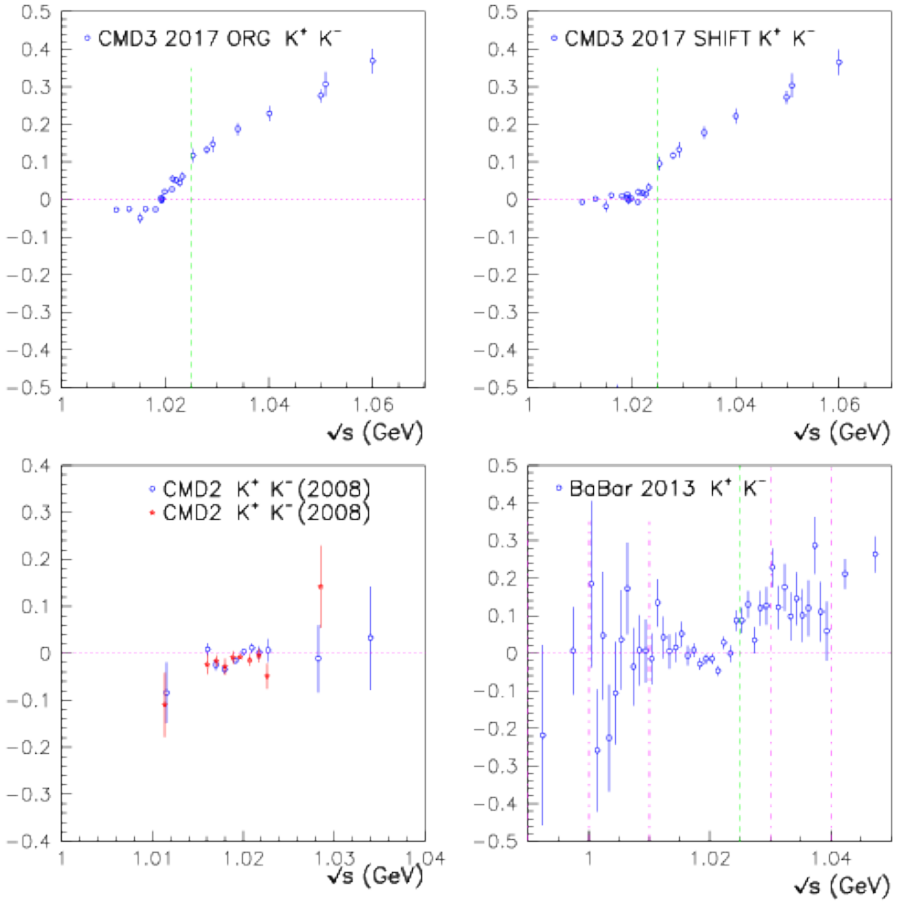
  (≈ 4. σ_{BaBaR})

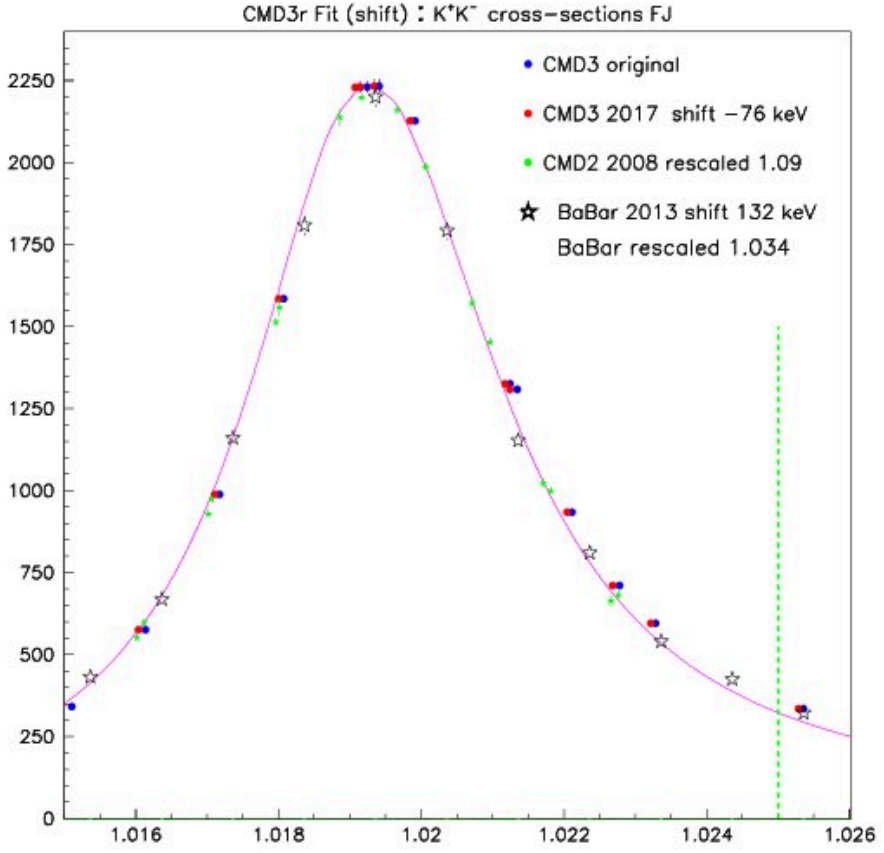
Confirm Inconsistency



CMD3 $e^+e^- \rightarrow K^+ K^-$ & global fit II

Applying Rescalings :





a_μ from BHLS2 Fits (update III)

HLS channels :

FSI updated

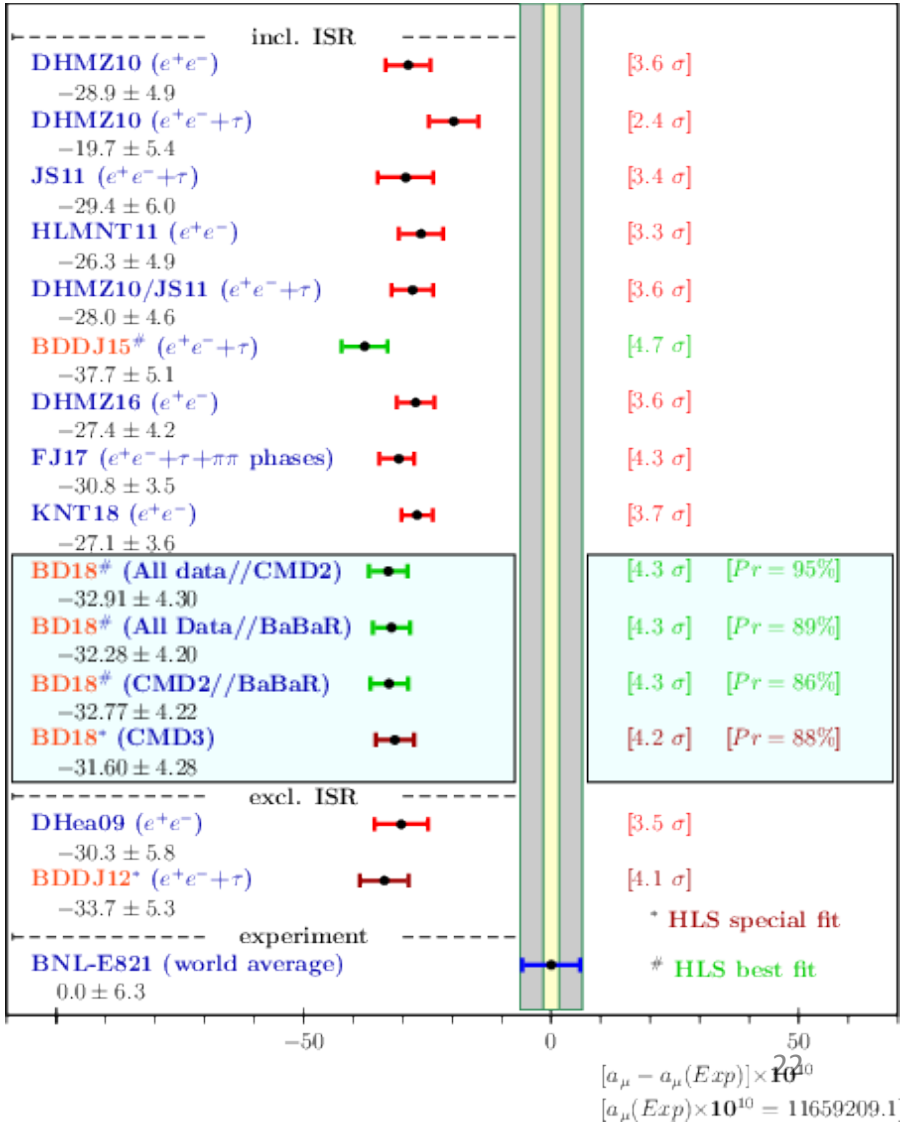
BHLS

Non HLS channels :

HVP update from FJ

- **+ ADD. SYST.**

BHLS2



Additional syst. to a_μ^{HVP}

- **Updated value for $10^{10}\, a_\mu^{HVP}$:**

$$a_\mu = 11659176.8 \pm 4.20$$

with additional systematics

$$\left[a_\mu^{\exp} - a_\mu^{th} \right] = 32.28 \pm 0.5_{\phi\pi\pi} \pm 0.4_{\alpha_{em}} \pm 0.7_{CMDx} \pm 4.20_{th} \pm 6.30_{\exp}$$

- $0.5_{\phi\pi\pi} + 0.4_{\alpha_{em}} + 0.7_{CMDx}$ **may contribute as bias**
- **$\Delta a_\mu^{HVP} \geq 30.68 \pm 4.20 \pm 6.30$ (4.1 σ)**
- **CMD2 update ?**

Conclusions

- CMD2 & BaBaR distant each from CMD3 by >4 σ
- CMD2 distant from BaBaR only by ≈ 2.2 σ_{CMD2}

  CMD2 & BaBaR fit well within the HLS framework
- New Reference central value (using BaBaR) :

$$\left[a_\mu^{\exp} - a_\mu^{th} \right] = 32.28 \pm 0.5_{\phi\pi\pi} \pm 0.4_{\alpha_{em}} \pm 0.7_{CMDx} \pm 4.20_{th} \pm 6.30_{\exp}$$

- Additional syst. account for the CMD2/3 fits
- CMD2 update : welcome!

BACKUP

The HLS Phenomenology

- **Once broken, the Hidden Local Symmetry model:**

  a unified VMD framework for

$e^+e^- \to \pi^+\pi^-$ /K^+K^- /K_LK_S / $\pi^0\gamma$ /$\eta\gamma$ / $\pi^+\pi^-\pi^0$ & $\tau \to \pi\pi\,\nu_\tau$

& PVγ, Pγγ decays & (η /$\eta' \to \gamma\pi\pi$) & F_π/F_K at $s<0$

(HLS Correlated via their underlying physics)

[83% of the muon HVP]

- ➢ **Limit : One vector nonet (the best constrained)**
- ✓ **Up to the ≈ ϕ mass region (≈ 1.05 GeV)**

Scale Uncertainty

- **Minimize :**

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$$\chi^2 = \left[\, m - M - \lambda A \,\right]^T V^{-1} \left[\, m - M - \lambda A \,\right] + \lambda^2 / \sigma^2$$

- **Solve for λ (A= m or M) gives:**

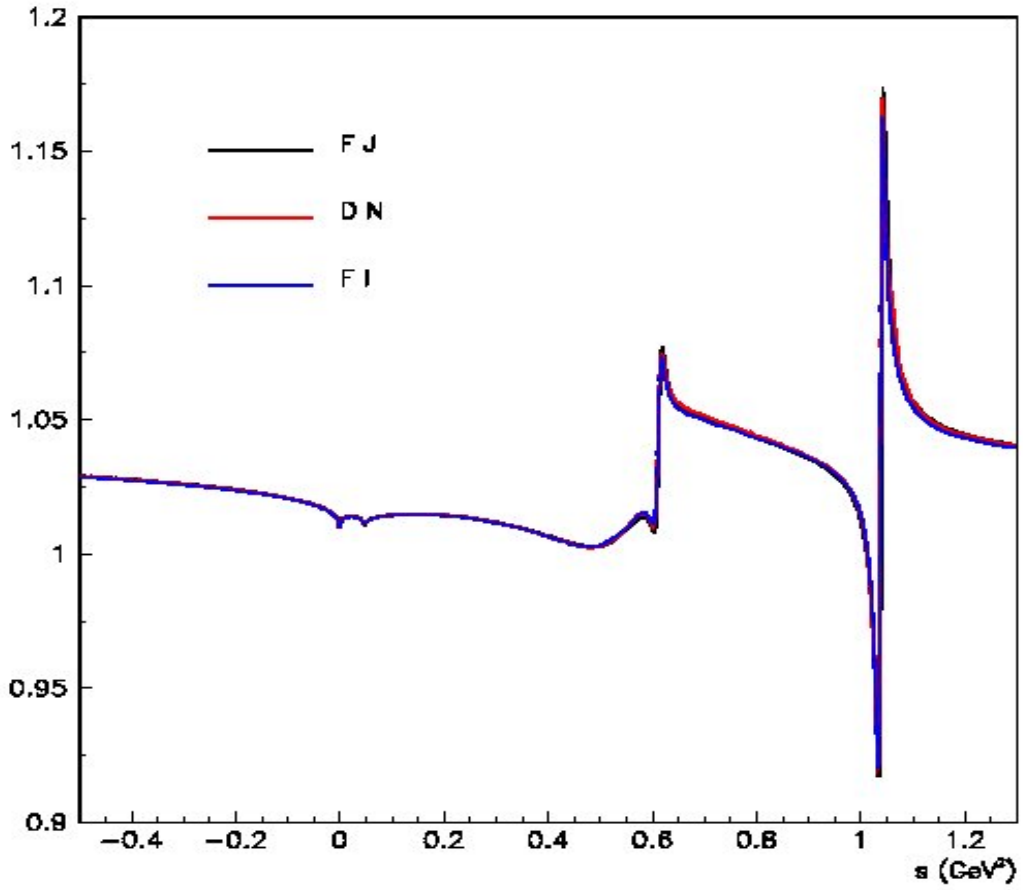
$$\chi^2 \equiv \left[\, m - M \,\right]^T \left[\, V + \sigma^2 A A^T \,\right]^{-1} \left[\, m - M \,\right]$$

- **Unbiasing data vs Model m→ m - λ M :**

$$\lambda = \left\{ A^T V^{-1} \left[\, m - M \,\right] \right\} / \left\{ A^T V^{-1} A + \frac{1}{\sigma^2} \right\}$$

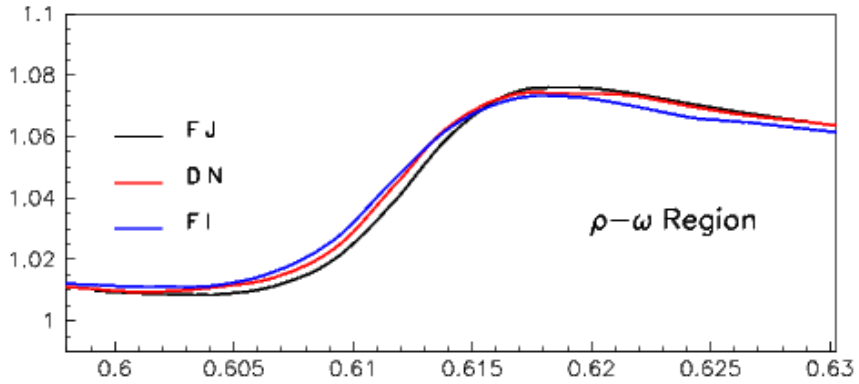
- **(m vs M) Consistency : λ/σ < 3**

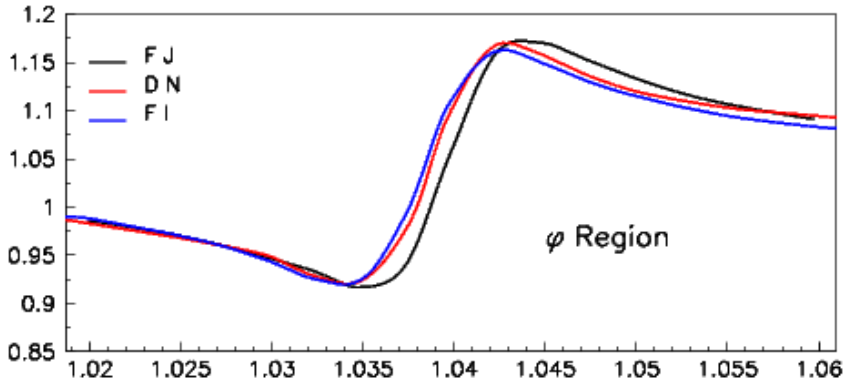
The photon HVP functions



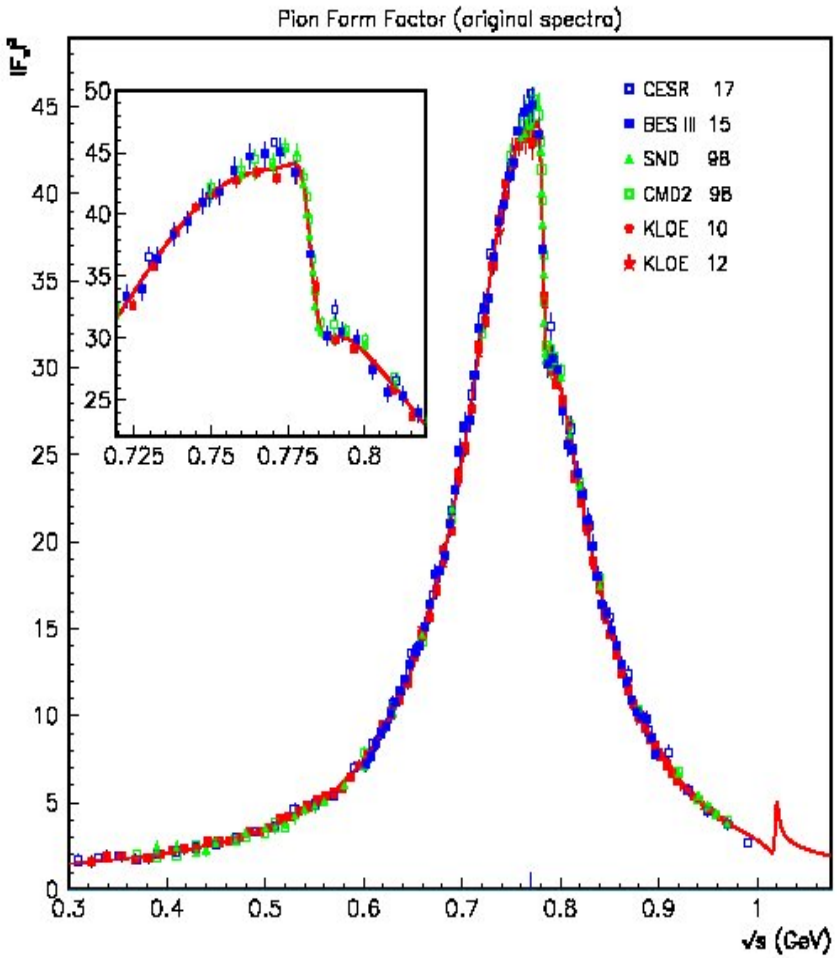
Uncertainty on photon HVP

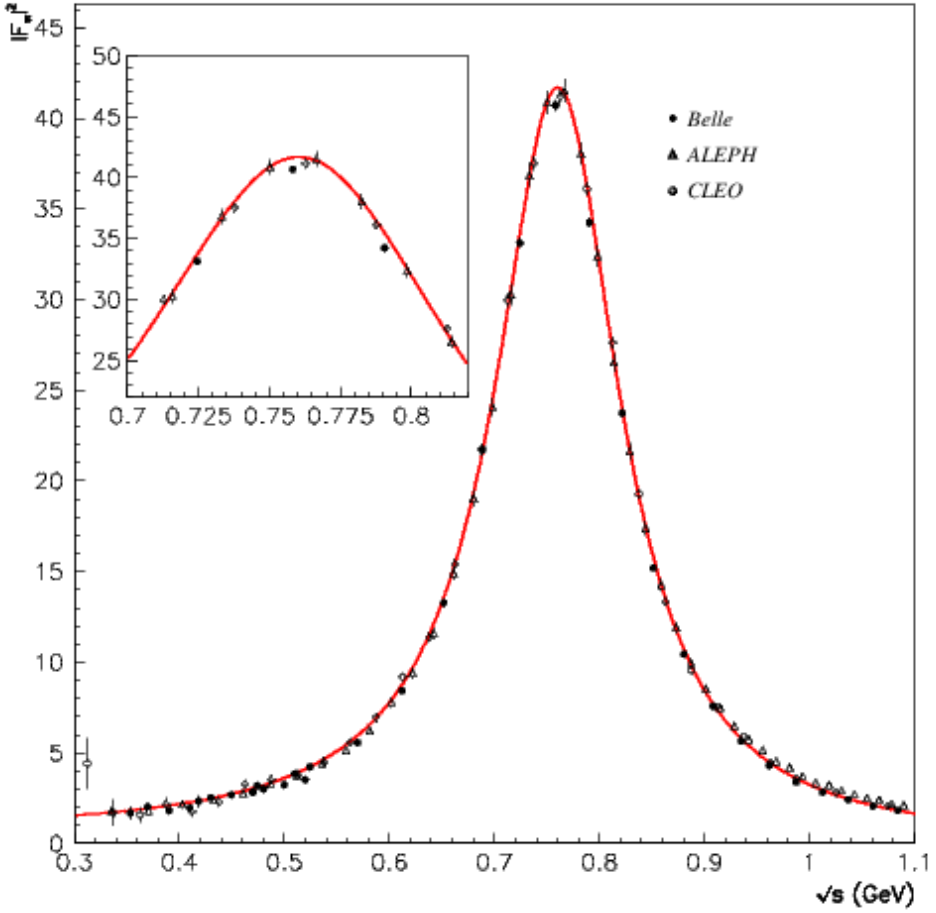
- No **unique** way to treat narrow resonances



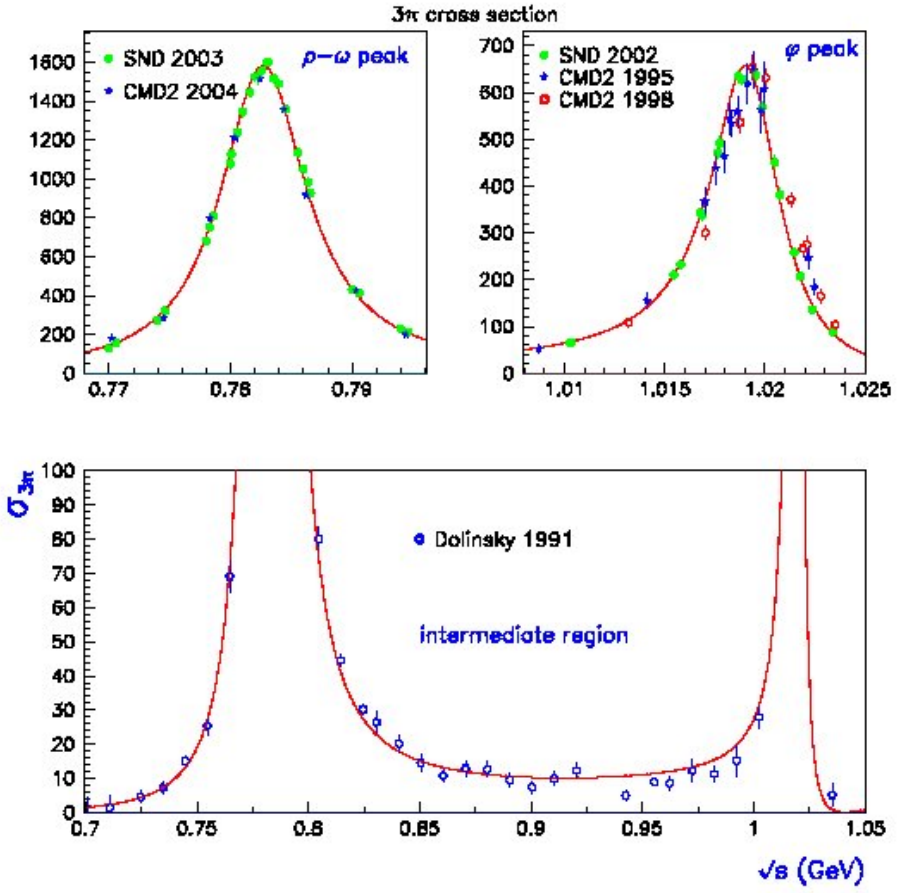


ππ Data


Pion Form Factor (original spectra)
$|F_\pi|^2$
CESR 17
BES III 15
SND 98
CMD2 98
KLOE 10
KLOE 12
√s (GeV)


$|F_\pi|^2$
Belle
ALEPH
CLEO
√s (GeV)

3 pion Data

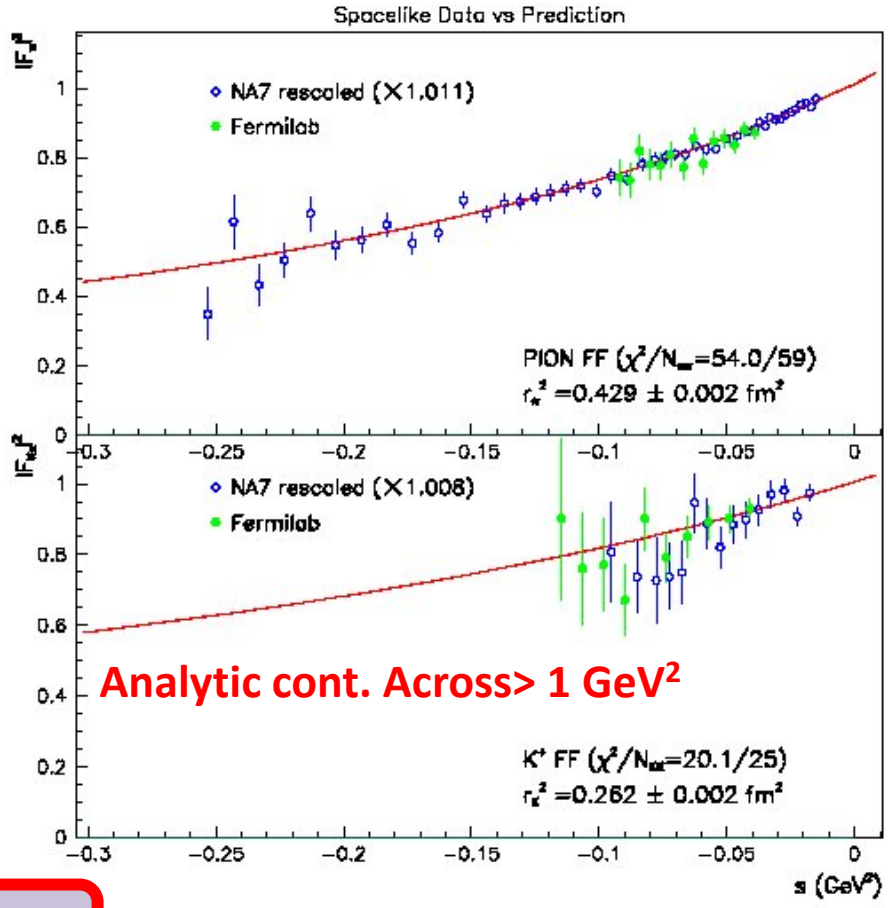

3π cross section
SND 2003
CMD2 2004
ρ−ω peak
SND 2002
CMD2 1995
CMD2 1998
φ peak
Dolinsky 1991
intermediate region
√s (GeV)

Spacelike Regions : BHLS2 Predictions

- NA7 scale err : σ=0.9%

R_π^2 (fm^2)	
NA7	0.440 ±0.007 (scale)
Fermilab	0.440 ±0.031
SELEX	0.42 ± $[0.06]_{stat}$±$[0.06]_{syst}$
1706.04020	0.432 ± 0.004
BHLS 2018	0.430 ± 0.001

R_k^2 (fm^2)	
NA7	0.336 ± 0.046 (scale)
Fermilab	0.281 ± 0.053
BHLS 2018	0.261 ± 0.001



B. Ananthanarayan et al. PRL 119(2017) 132002

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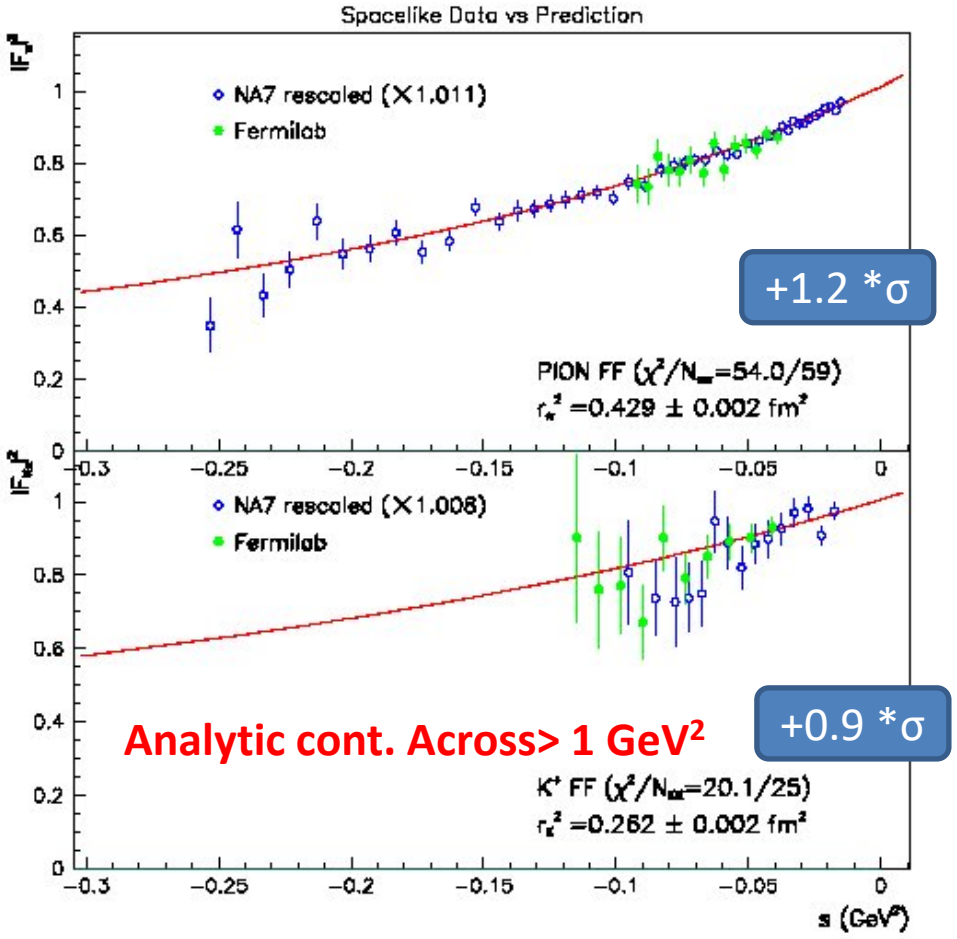
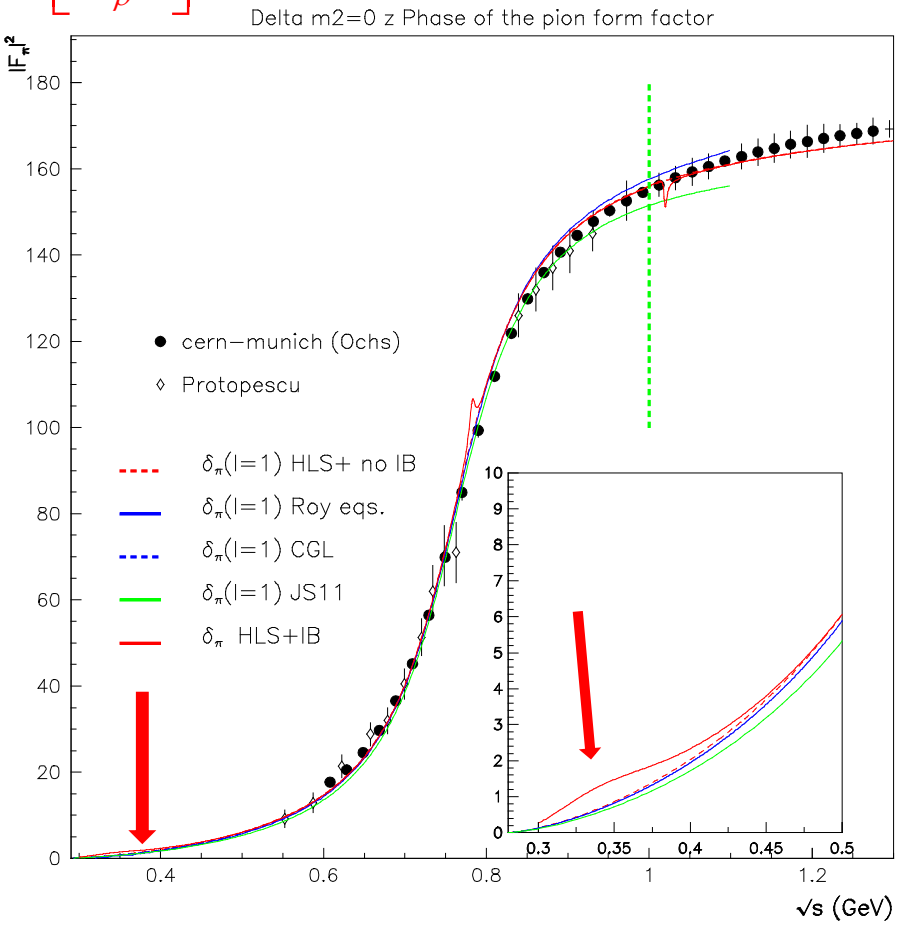
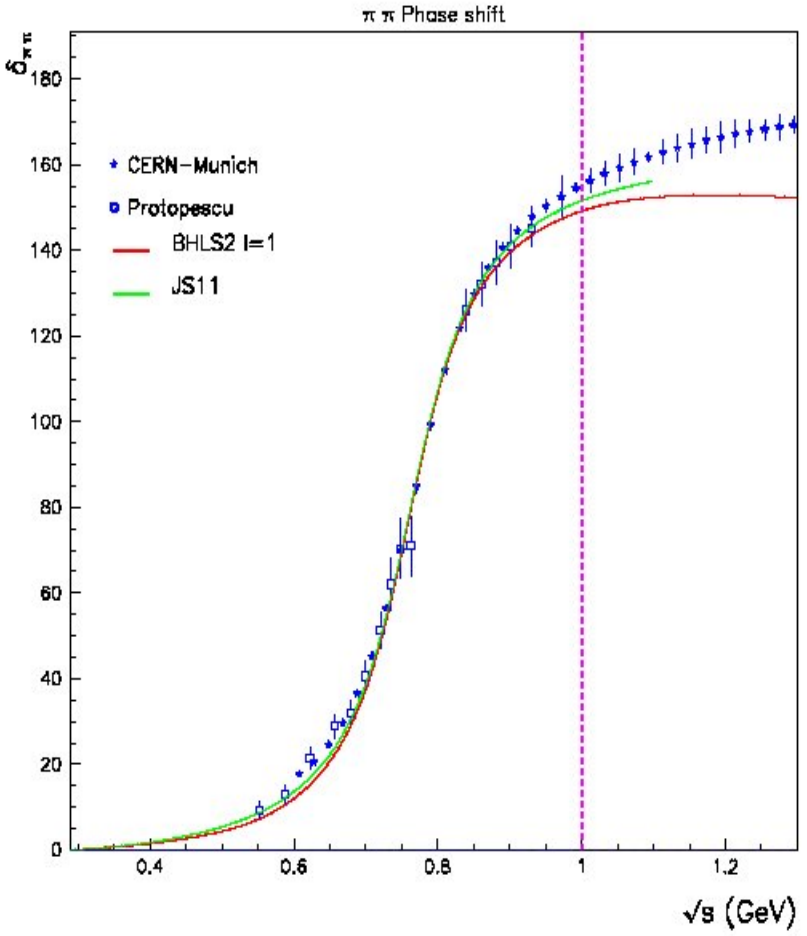


≈ **Unbiased FF's**

ππ Phase shift BHLS vs BHLS2

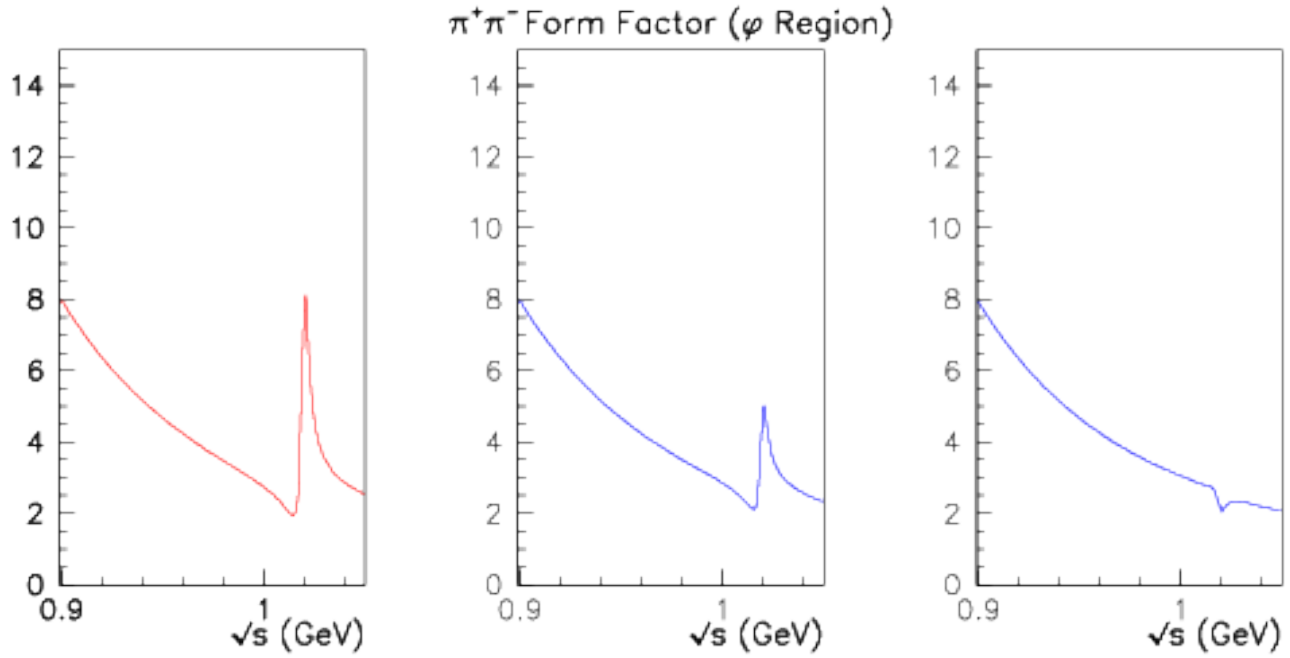
$$\left[\frac{m_{\omega}^{HK}}{m_{\rho}^{HK}}\right]^2 = 1.00$$





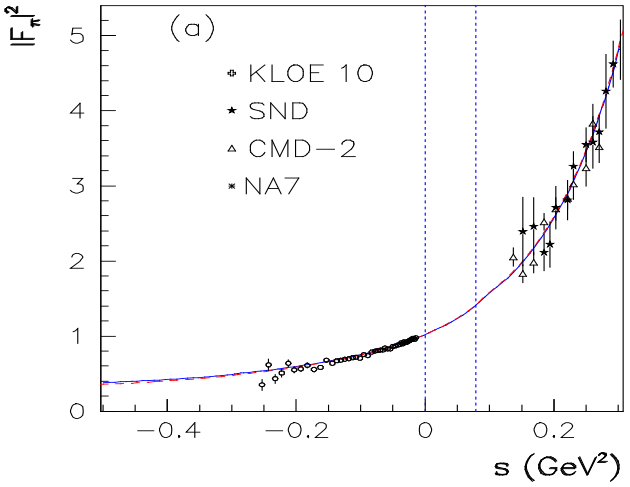
Lack of constraints : F_π at ϕ

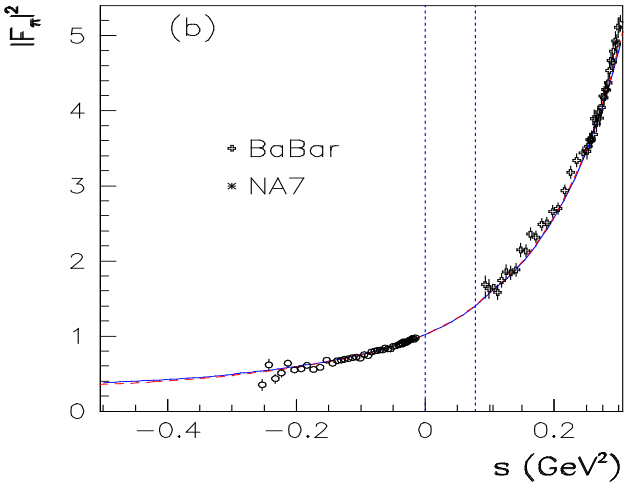
- **Missing** exp. spectrum for $F_\pi(s)$ around ϕ
- Constraints : $Br(\phi \rightarrow \pi^+ \pi^-)$ &$Br(\phi \rightarrow e^+ e^-) * Br(\phi \rightarrow \pi^+ \pi^-)$



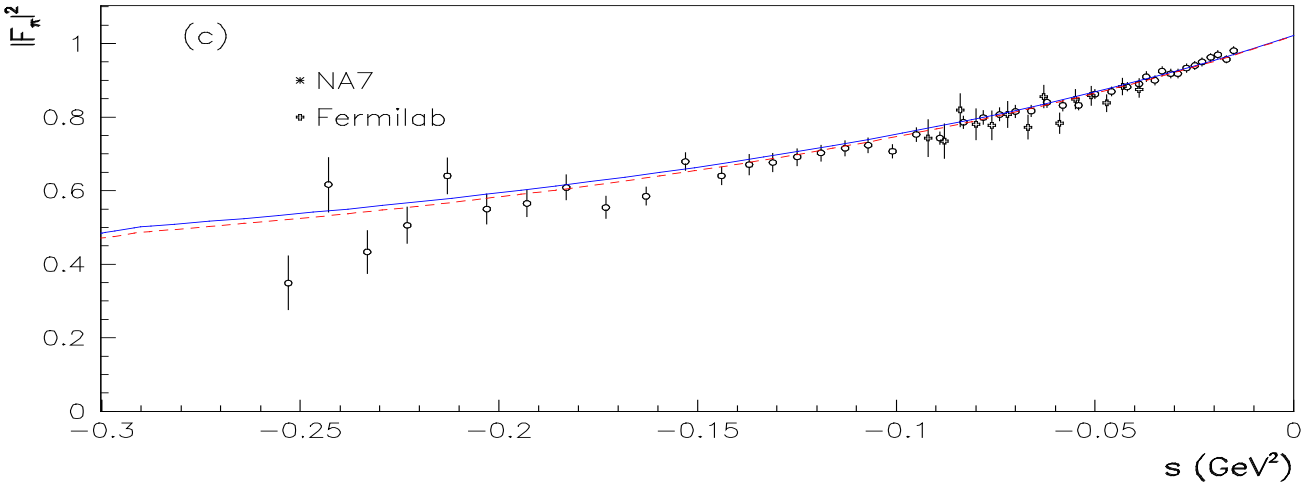
BACKUP’s

Spacelike & Threshold Regions



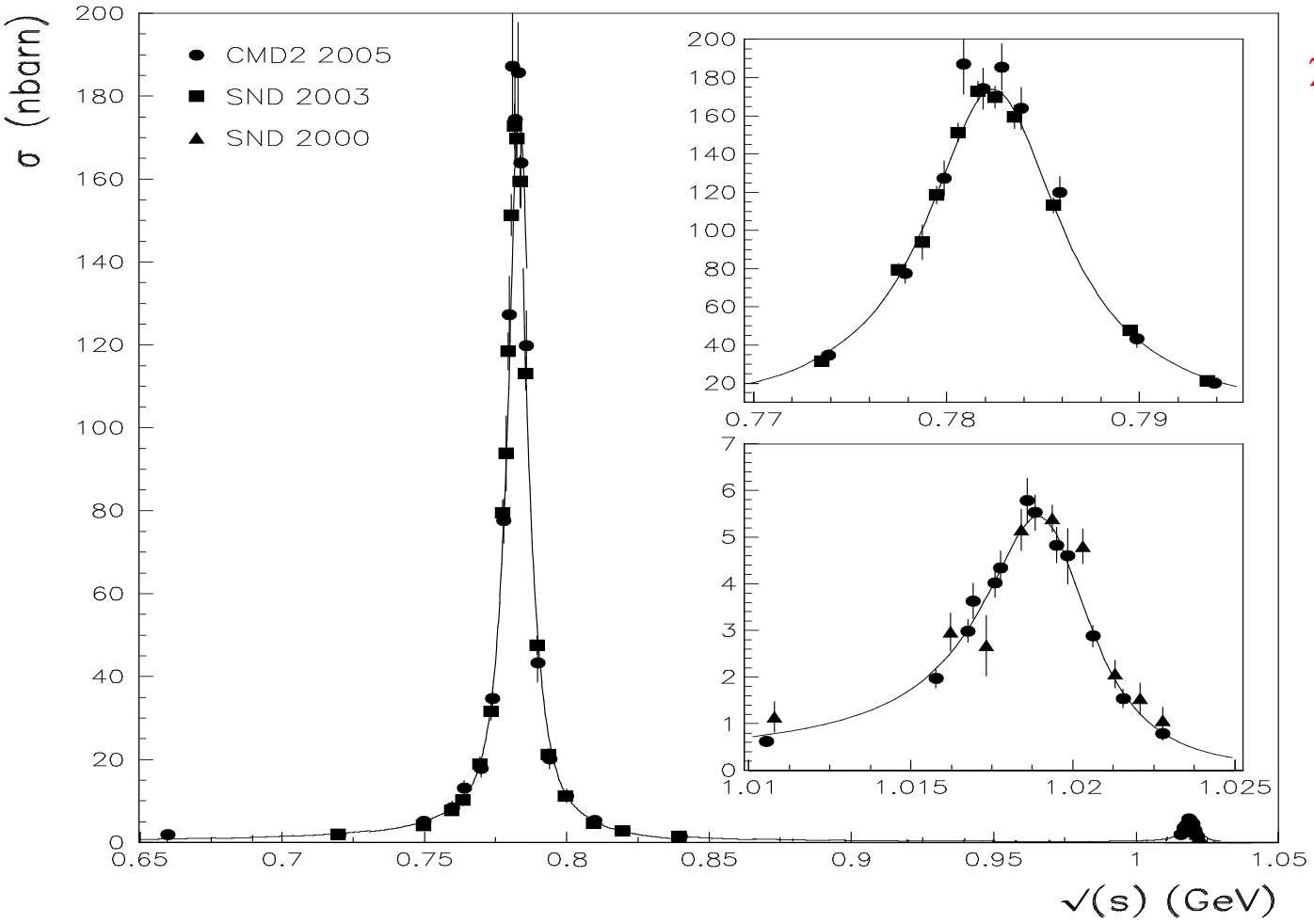


Connection at threshold



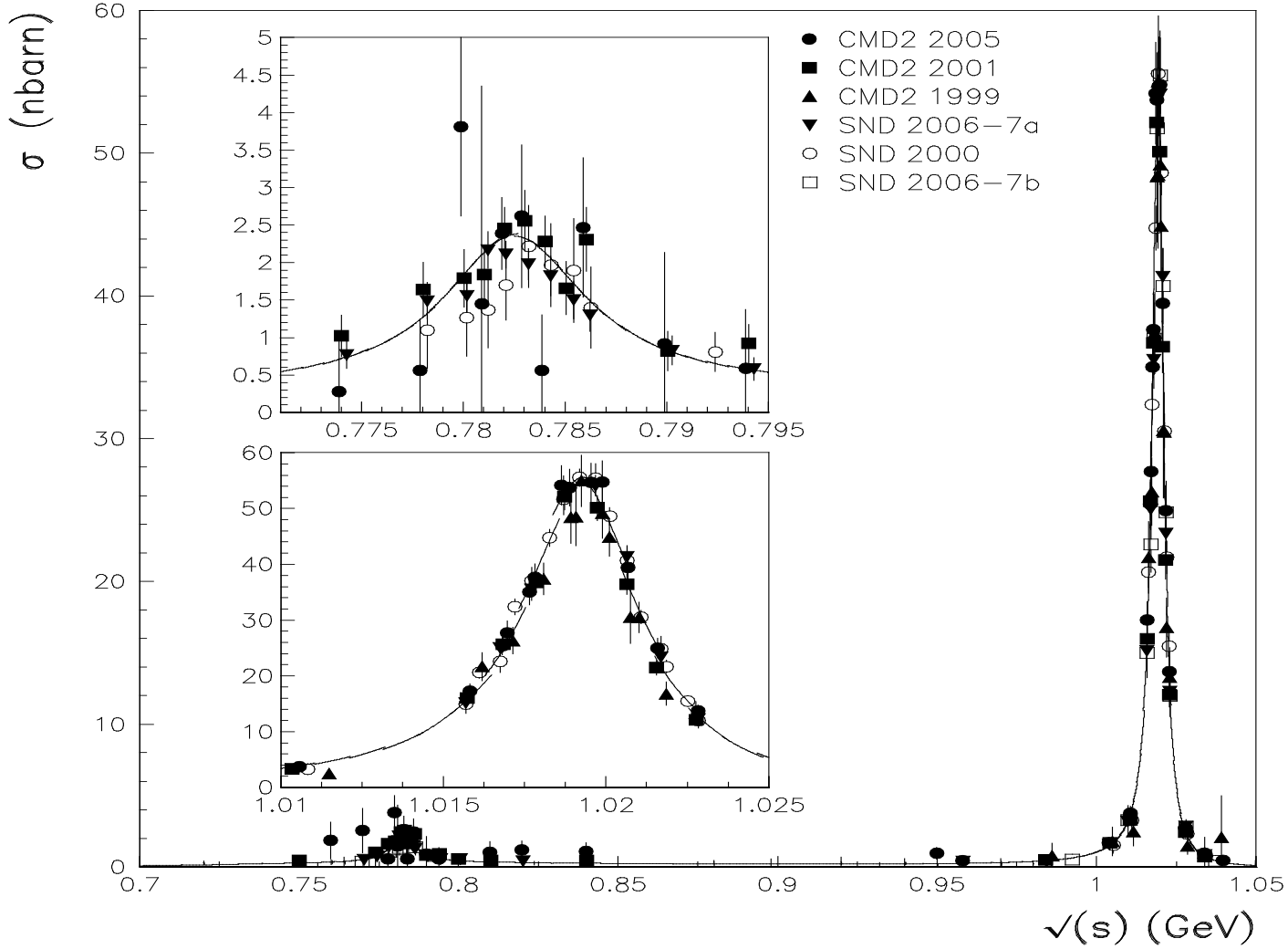
Extrapolation to s<0

$e^+e^- \to \pi^0 \gamma$



$$\chi^2/N_p = 68/86$$

$e^+e^- \to \eta\gamma$



$$\chi^2/N_p = 123/182$$