Treatment of uncertainties and correlations in combinations of e^+e^- annihilation data

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Content of the talk

- Sources of uncertainties and their correlations
- Treatment in various combination approaches
- Indications of uncertainties on uncertainties and on correlations & their implications for combinations
- Example of the combination of 3 KLOE measurements

Treatment of uncertainties: requirements

- Properly propagate uncertainties and correlations

- *Between measurements (data points/bins) of a given experiment* (covariance matrices and/or detailed split of uncertainties in sub-components)
- *Between experiments* (common systematic uncertainties, e.g. VP) – based on detailed information provided in publications
- *Between different channels* – motivated by understanding of the meaning of systematic uncertainties and identifying the common ones:

BABAR luminosity (ISR or BhaBha), efficiencies (photon, Ks, Kl, modeling);
BABAR radiative corrections; $4\pi 2\pi^0$-$\eta\omega$
CMD2 $\eta\gamma - \pi^0\gamma$; CMD2/3 luminosity; SND luminosity;
FSR; hadronic VP (old experiments)

- Optimize g-2 integral uncertainty *without overestimating the precision with which the uncertainties of the measurements are known*

χ^2 definitions and properties

$$\chi^2(\mathbf{d};\mathbf{t}) = \sum_{i,j}(d_i - t_i)\cdot\left[C^{-1}(\mathbf{t})\right]_{ij}\cdot(d_j - t_j) \qquad C_{ij} = C_{ij}^{stat} + \sum_k s_i^k \cdot s_j^k$$

$$\chi^2(\mathbf{d};\mathbf{t}) = \min_{\beta_a}\left\{\sum_{i,j}\left[d_i - \left(1+\sum_a \beta_a \cdot \left(\epsilon_a^{\pm}(\beta_a)\right)_i\right)t_i\right]\cdot\left[C_{\mathrm{su}}^{-1}(\mathbf{t})\right]_{ij} \cdot\left[d_j - \left(1+\sum_a \beta_a \cdot \left(\epsilon_a^{\pm}(\beta_a)\right)_j\right)t_j\right] + \sum_a \beta_a^2\right\},$$

→ Two χ^2 definitions, with systematic uncertainties included in covariance matrix or treated as fitted "nuisance parameters"

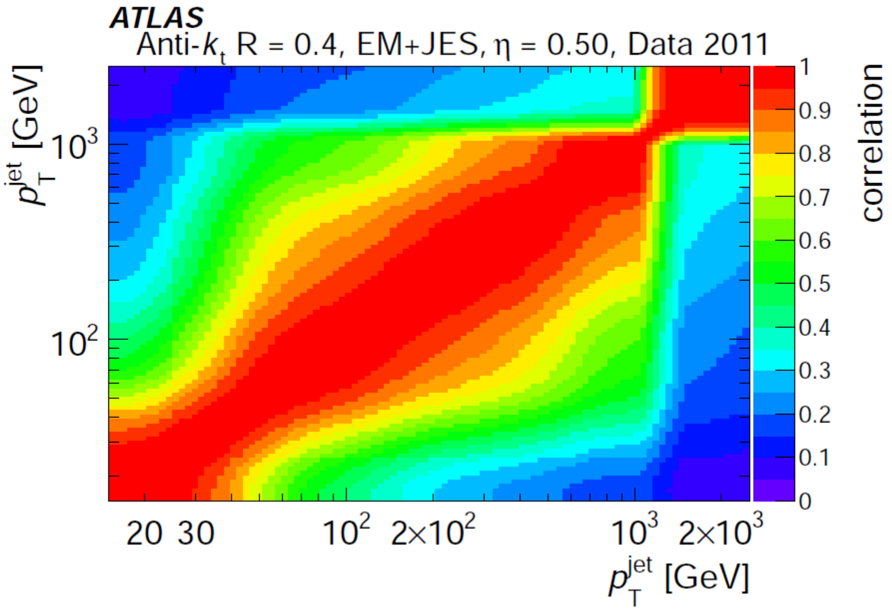
→ Equivalent for symmetric Gaussian uncertainties

(1312.3524 - ATLAS)

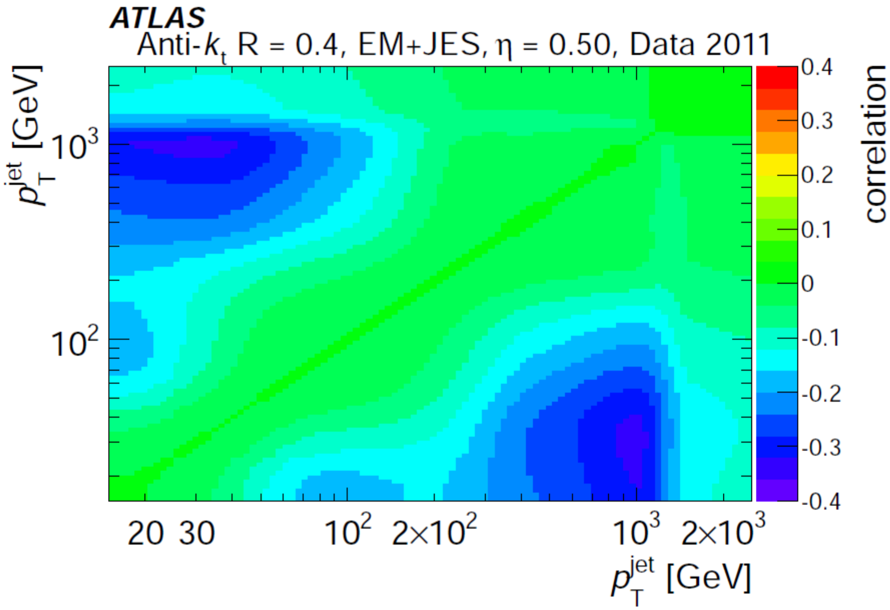
→ *Both approaches assume the knowledge of the amplitude, shape (phase-space dependence) and correlations of systematic uncertainties*

Example: published uncertainties on correlations

1406.0076 – ATLAS jet energy scale uncertainties



Nominal correlation scenario



Weaker - stronger correlation scenarios

Two different approaches for combining (e^+e^-) data

DHMZ:

→ χ^2 computed locally (in each fine bin), taking into account correlations between measurements (see previous slides)

→ used to determine the weights on the measurements in the combination and their level of agreement

→ uncertainties and correlations propagated using pseudo-experiments or ±1σ shifts of each uncertainty component

KNT:

→ χ^2 computed globally (for full mass range)

$$\chi_I^2 = \sum_{i=1}^{N_{\rm tot}} \sum_{j=1}^{N_{\rm tot}} \left(R_i^{(m)} - \mathcal{R}_m^{i,I}\right) \mathbf{C}_I^{-1}\left(i^{(m)}, j^{(n)}\right)\left(R_j^{(n)} - \mathcal{R}_n^{j,I}\right)$$

KNT (1802.02995)

$$\chi^2 = \sum_{i=1}^{195}\sum_{j=1}^{195}\left(\sigma^0_{\pi\pi(\gamma)}(i) - \bar{\sigma}^0_{\pi\pi(\gamma)}(m)\right)\mathbf{C}^{-1}\left(i^{(m)}, j^{(n)}\right)\left(\sigma^0_{\pi\pi(\gamma)}(j) - \bar{\sigma}^0_{\pi\pi(\gamma)}(n)\right)$$

KLOE-KMT (1711.03085)

→ relies on description of correlations on long ranges

→ *One of the main sources of differences for the uncertainty on a_μ (slide 14)*

Evaluation of uncertainties and correlations (e^+e^-)

	$\sigma_{\pi\pi\gamma}$	$\sigma^0_{\pi\pi}$	F_π	$\Delta^{\pi\pi}a_\mu$
Reconstruction Filter	negligible			
Background subtraction	Tab. 1			0.3%
Trackmass	0.2%			
Pion cluster ID	negligible			
Tracking efficiency	0.3%			
Trigger efficiency	0.1%			
Acceptance	Tab. 2			0.2%
Unfolding	Tab. 3			negligible
L3 filter	0.1%			
$\sqrt{s}$ dependence of H	-	Tab. 4		0.2%
Luminosity	0.3%			
Experimental systematics				0.6%
FSR resummation	-	0.3%		
Radiator function H	-	0.5%		
Vacuum Polarization	-	0.1%	-	0.1%
Theory systematics				0.6%

$M^2_{\pi\pi}$ range (GeV^2)	Systematic error (%)
$0.35 \leq M^2_{\pi\pi} < 0.39$	0.6
$0.39 \leq M^2_{\pi\pi} < 0.43$	0.5
$0.43 \leq M^2_{\pi\pi} < 0.45$	0.4
$0.45 \leq M^2_{\pi\pi} < 0.49$	0.3
$0.49 \leq M^2_{\pi\pi} < 0.51$	0.2
$0.51 \leq M^2_{\pi\pi} < 0.64$	0.1
$0.64 \leq M^2_{\pi\pi} < 0.95$	-

KLOE 08 (0809.3950)

→ Systematics *evaluated* in ~wide mass ranges with sharp transitions

KLOE 10 (1006.5313)

	$\sigma_{\pi\pi\gamma}$	$\sigma^{bare}_{\pi\pi}$	$\|F_\pi\|^2$	$\Delta a^{\pi\pi}_\mu$
	threshold ; ρ-peak			(0.1 - 0.85 GeV^2)
Background Filter	0.5% ; 0.1%			negligible
Background subtraction	3.4% ; 0.1%			0.5%
$f_0 + \rho\pi$ bkg.	6.5% ; negl.			0.4%
Ω cut	1.4% ; negl.			0.2%
Trackmass cut	3.0% ; 0.2%			0.5%
π-e PID	0.3% ; negl.			negligible
Trigger	0.3% ; 0.2%			0.2%
Acceptance	1.9% ; 0.3%			0.5%
Unfolding	negl. ; 2.0%			negligible
Tracking	0.3%			
Software Trigger (L3)	0.1%			
Luminosity	0.3%			
Experimental syst.				1.0%
FSR treatment	-	7% ; negl.		0.8%
Radiator function H	-	0.5%		
Vacuum Polarization	-	Ref. [34]	-	0.1%
Theory syst.				0.9%

→ "For the correlation of the systematic uncertainty due to the acceptance, only half of the KLOE10 uncertainty is correlated with the KLOE08 uncertainty in order to ensure that the photon detection acceptance that enters into the KLOE10 uncertainty (that is not present in the KLOE08 analyses) is not correlated and only the correlation of the pion tracks is duly accounted for."

Is this statement (same impact of photon and pions on the acceptance) valid on the full √s range?

Evaluation of uncertainties and correlations (e^+e^-)

BABAR (1205.2228)

Sources	0.3-0.4	0.4-0.5	0.5-0.6	0.6-0.9	0.9-1.2	1.2-1.4	1.4-2.0	2.0-3.0
trigger/ filter	5.3	2.7	1.9	1.0	0.7	0.6	0.4	0.4
tracking	3.8	2.1	2.1	1.1	1.7	3.1	3.1	3.1
π-ID	10.1	2.5	6.2	2.4	4.2	10.1	10.1	10.1
background	3.5	4.3	5.2	1.0	3.0	7.0	12.0	50.0
acceptance	1.6	1.6	1.0	1.0	1.6	1.6	1.6	1.6
kinematic fit (χ^2)	0.9	0.9	0.3	0.3	0.9	0.9	0.9	0.9
correl $\mu\mu$ ID loss	3.0	2.0	3.0	1.3	2.0	3.0	10.0	10.0
$\pi\pi/\mu\mu$ non-cancel.	2.7	1.4	1.6	1.1	1.3	2.7	5.1	5.1
unfolding	1.0	2.7	2.7	1.0	1.3	1.0	1.0	1.0
ISR luminosity	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4
sum (cross section)	13.8	8.1	10.2	5.0	6.5	13.9	19.8	52.4

→ Systematics *evaluated* in ~wide mass ranges with sharp transitions (statistics limitations when going to narrow ranges)

Uncertainties on uncertainties and correlations (e^+e^-)

→ Shapes of systematic uncertainties *evaluated* in ~wide mass ranges with sharp transitions

→ One standard deviation is statistically not well defined for systematic uncertainties

→ Systematic uncertainties like acceptance, tracking efficiency, background etc. not necessarily fully correlated between low and high mass

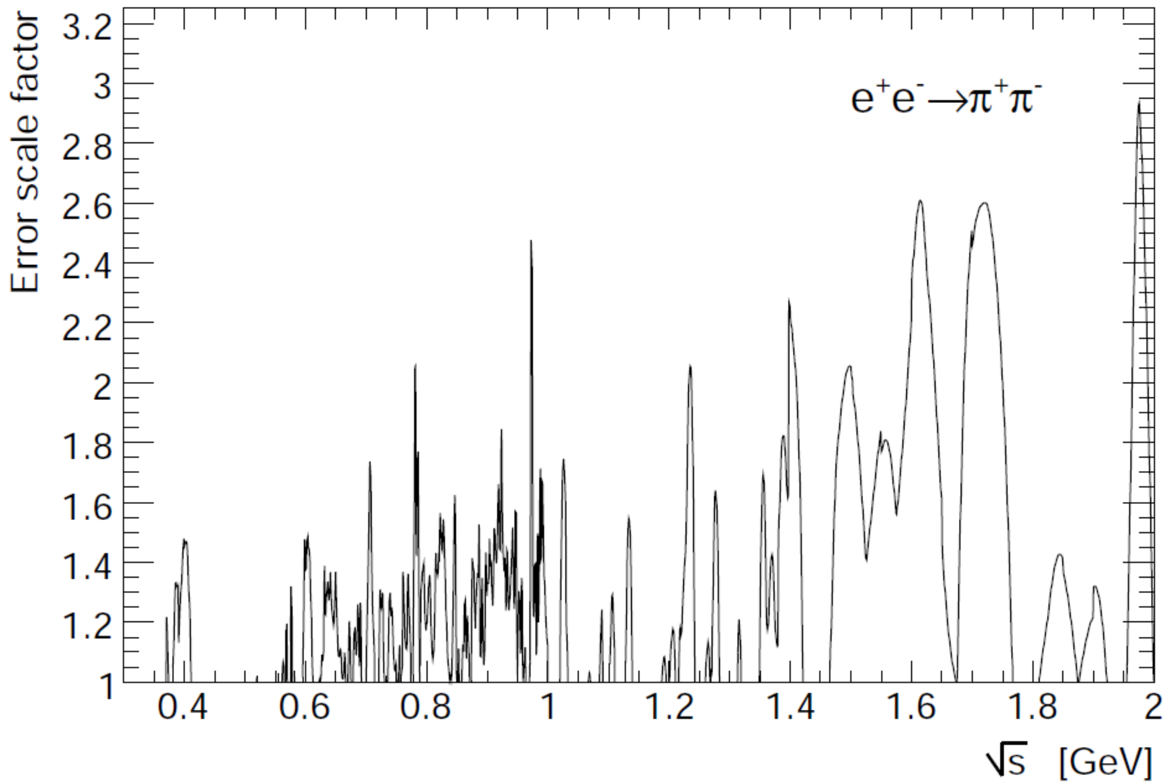
→ Are all systematic uncertainty components fully independent between each-other? (e.g. tracking and trigger)

→ *Yield uncertainties on uncertainties and on correlations*

Combination procedure: compatibility between measurements

For each final bin:

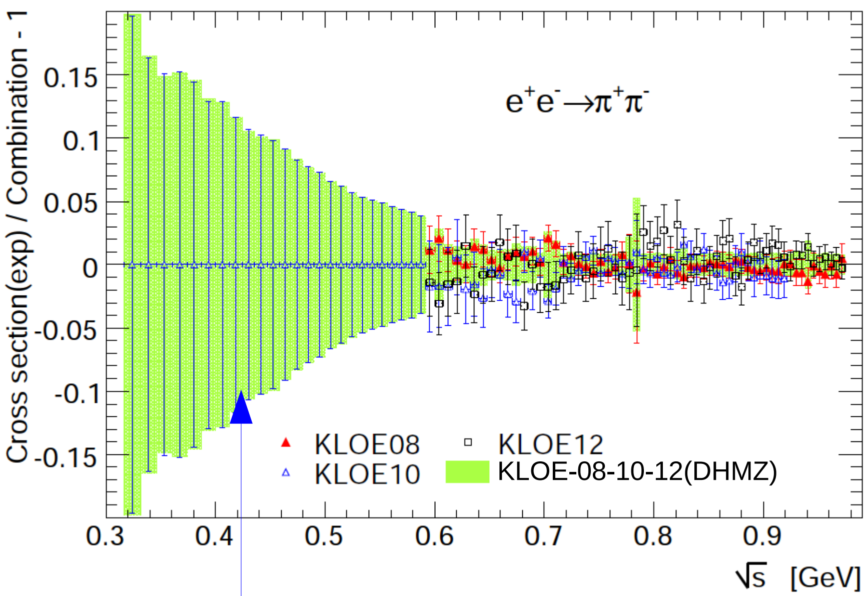
→ χ^2 /ndof: test locally the level of agreement between input measurements, *taking into account the correlations*

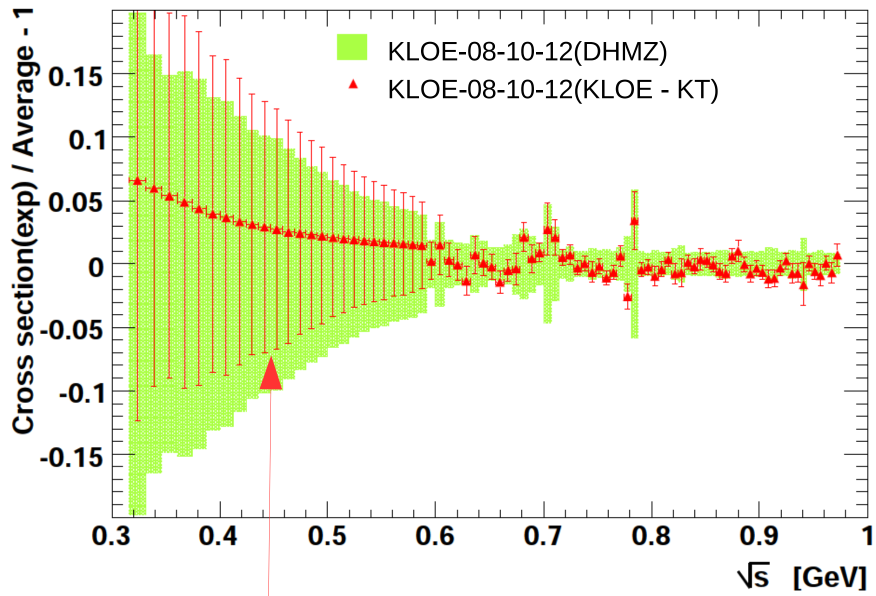


→ Tension between measurements: *indication of underestimated uncertainties*

→ Motivates conservative uncertainty treatment in evaluation of weights

Combining the 3 KLOE measurements

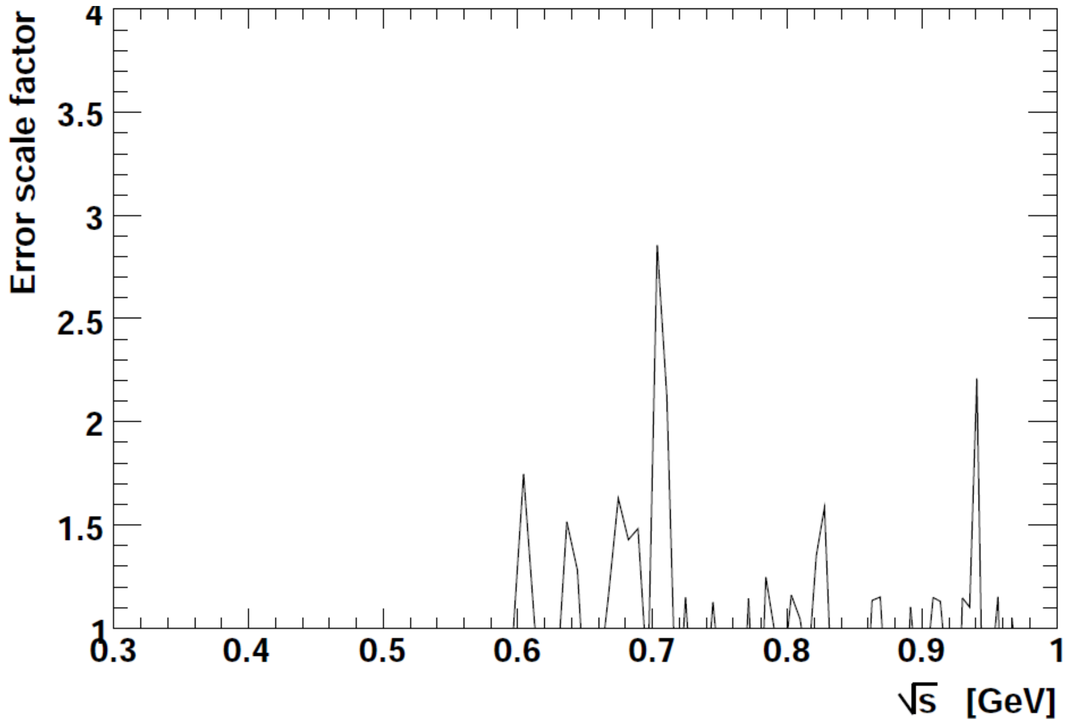




Local combination (DHMZ)

Information propagated between mass regions, through shifts of systematics - relying on correlations, amplitudes and shapes of systematics (KLOE-KT)

Direct comparison of the 3 KLOE measurements



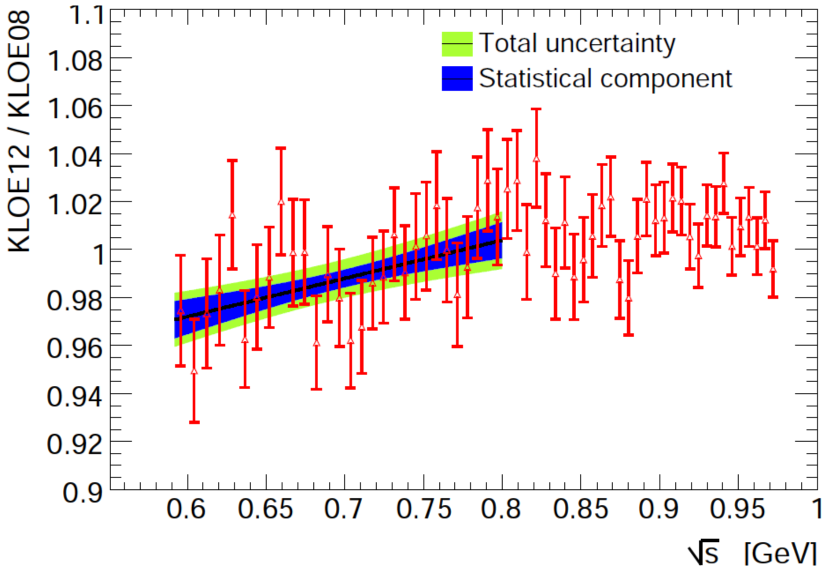
→ Local χ^2/ndof test of the local compatibility between KLOE 08 & 10 & 12, taking into account the correlations: some tensions observed

→ Does not probe general trends of the difference between the measurements (e.g. slopes in the ratio)

Direct comparison of the 3 KLOE measurements

→ Fitting the ratio taking into account correlations

→ Full propagation of uncertainties and correlations – 3 methods yielding consistent results: ±1σ shifts of each uncertainty, pseudo-experiments and fit uncertainties from Minuit



χ^2 [p0 + p1√s]: 20.7 / 27(DOF)

p-value= 0.80

p0 : 0.876 ± 0.056

p1 : 0.159 ± 0.081

→ Significant shift and slope (~2σ) at low √s

→ Should motivate conservative treatment of uncertainties and correlations in combination

Combining the 3 KLOE measurements - $a_\mu^{\pi\pi}$ contribution

KLOE08 a_μ[0.6 ; 0.9] : 368.3 ± 3.2 [10^{-10}]

KLOE10 a_μ[0.6 ; 0.9] : 365.6 ± 3.3

KLOE12 a_μ[0.6 ; 0.9] : 366.8 ± 2.5

→ Correlation matrix:

	08	10	12
08	1	0.70	0.35
10	0.70	1	0.19
12	0.35	0.19	1

→ Amount of independent information provided by each measurement

→ KLOE-08-10-12(DHMZ) - a_μ[0.6 ; 0.9] : 366.5 ± 2.8 (Without χ^2 rescaling: ± 2.2)

→ Conservative treatment of uncertainties and correlations (not perfectly known) in weight determination

→ KLOE-08-10-12(KLOE-KT) - a_μ[0.6 ; 0.9]GeV : 366.9 ± 2.2

→ Assuming perfect knowledge of the correlations to minimize average uncertainty

Summary and outlook

→ Indications of uncertainties on uncertainties and on correlations, with a direct impact on combinations

Proposal:

1) Short term (with current experimental inputs)
→ use combination approaches that do not exploit assumptions on long-range correlations that are experimentally not under control

2) Long term
→ provide measurements with information on uncertainties on uncertainties and on their correlations

Backup Slides

Combining the 3 KLOE measurements