Hadronic vacuum polarization contribution to muon g-2 from four flavors of HISQ quarks

Ruth Van de Water
(for the Fermilab Lattice, HPQCD,
& MILC Collaborations)

Second plenary workshop of Muon g-2 Theory Initiative
Johannes Gutenberg Universität Mainz
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Participants

Fermilab Lattice Collaboration

- Aida El Khadra (Illinois)
- Andreas Kronfeld (Fermilab)
- Ethan Neil (Colorado)
- Ruth Van de Water (Fermilab)

HPQCD

- Daniel Hatton (Glasgow)
- Christine Davies (Glasgow)
- Peter Lepage (Cornell)
- Craig McNeile (Plymouth)

MILC Collaboration

- Carleton DeTar (Utah)
- Steve Gottlieb (Indiana)
- Jack Laiho (Syracuse)
- Yuzhi Liu (Indiana)
- Doug Toussaint (Arizona)
- Alejandro Vaquero (Utah)
- Shuhei Yamamoto (Utah)

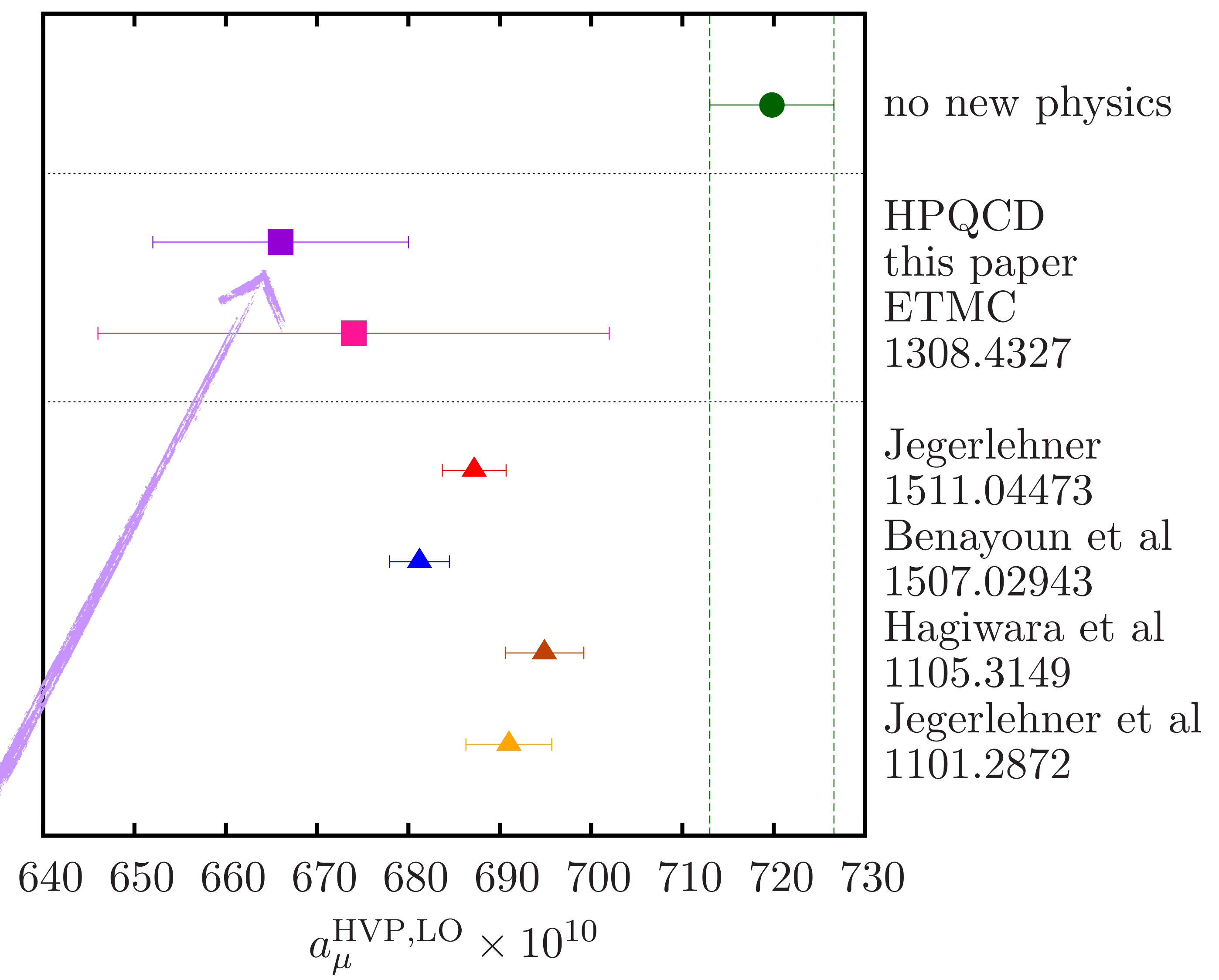
Motivation

- Muon anomalous magnetic moment (g-2) provides sensitive probe of physics beyond the Standard Model:
  - Mediated by quantum-mechanical loops
  - Known to very high precision of 0.54ppm
- Measurement from BNL E821 disagrees with Standard-Model theory expectations by more than 3σ
- Muon g-2 Experiment at Fermilab aims to reduce the experimental error by a factor of four, and *expect first results in 2019*!
- **Theory error must be reduced to a commensurate level to identify definitively whether any deviation observed between theory and experiment is due to new particles or forces**

> We are using *ab-initio* lattice-QCD to target the hadronic vacuum-polarization (HVP) contribution, which is the largest source of theory error

Background

- Build upon HPQCD's published work
  - Introduced "time-moments" method obtain a_μ^{HVP} from zero-momentum vector-current correlation functions [PRD89, no. 11, 114501 (2014)]
  - Obtained **precise s, c-, and b-quark connected contributions** [PRD 91, no. 7, 074514 (2015)] and **bound on quark-disconnected contribution** [HPQCD+HadSpec, PRD 93, no. 7, 074509 (2016)]
  - Calculated **light-quark connected contribution & total $a_\mu^{HVP,LO}$ with ~2% uncertainty** [HPQCD+RV, PRD96 (2017) no. 3, 034516]



Dominant errors from omission of EM & isospin breaking, and from quark-disconnected contributions

Ensembles & parameters

- Employ large set of MILC ensembles with four flavors of dynamical HISQ sea quarks with four lattice spacings, multiple spatial volumes, & physical light-quark masses
- Vector-current correlator data from PRD96 (2017) no.3, 034516 plus:
  - a~0.15 fm physical-mass ensemble with better-tuned quark masses & ~15×statistics
  - a~0.09 & 0.06 fm physical-mass data

$\approx a$ (fm)	$am_l^{\rm sea}/am_s^{\rm sea}/am_c^{\rm sea}$	w_0/a	$Z_{V,\bar{s}s}$	M_{π_5} (GeV)	$\left(\frac{L}{a}\right)^3 \times \left(\frac{T}{a}\right)$	$N_{\rm conf.}$
0.15	0.01300/0.065/0.838	1.1119(10)	0.9881(10)	0.3025(16)	$16^3 \times 48$	9947
0.15	0.0064/0.064/0.828	1.12720(70)	0.9881(10)	0.2155(11)	$24^3 \times 48$	1000
0.15	0.00235/0.0647/0.831	1.13670(50)	0.9881(10)	0.13304(70)	$32^3 \times 48$	997
0.15	0.002426/0.0673/0.8447	1.13215(35)	0.9881(10)	0.13473(71)	$32^3 \times 48$	**7746**
0.12	0.01020/0.0509/0.635	1.3826(11)	0.99220(40)	0.3012(16)	$24^3 \times 64$	1053
0.12	0.0050724c/0.0507/0.628	1.40290(90)	0.99220(40)	0.2178(12)	$24^3 \times 64$	1020
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0.12	0.0050740c/0.0507/0.628	1.40290(90)	0.99220(40)	0.2163(11)	$40^3 \times 64$	331
0.12	0.00184/0.0507/0.628	1.41490(60)	0.99220(40)	0.13273(70)	$48^3 \times 64$	998
0.09	0.00740/0.037/0.440	1.9006(20)	0.99400(50)	0.3076(17)	$32^3 \times 96$	1000
0.09	0.00363/0.0363/0.430	1.9330(20)	0.99400(50)	0.2191(12)	$48^3 \times 96$	298
0.09	0.00120/0.0363/0.432	1.95180(70)	0.99400(50)	0.12834(68)	$64^3 \times 96$	1557
0.06	0.0008/0.022/0.260	3.0170(23)	0.9941(11)	0.13495(72)	$96^3 \times 192$	945

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NEW!

Analysis updates & improvements

(1) Employ only 5 physical-mass ensembles in main analysis

- Comparison with fit to all available data provides check

(2) Drop ρ rescaling from analysis

- Number of ππ levels near or below the ρ resonance increases with decreasing lattice spacing ➜ ground-state energy of correlator not the ρ mass
- **Unnecessary now that analysis does not use ensembles with heavy pions**

(3) Include estimate of low-energy ππ contributions from times t>1.5 fm

(4) Calculate slope and curvature of vacuum polarization function

- Enables comparison of more intermediate quantity between lattice-QCD calculations and with experiment

Noise-reduction strategy

- Reduce statistical errors in light-quark connected contribution to a_μ as in 1601.03071 by:

(1) Simultaneous fit of 4 combinations of (local, smeared) correlators G_{ij}

- Employ cosh parameterization that accounts for periodic temporal boundary conditions
- **Fit between (t_{min}, T-t_{min})** to ensure that fit describes correlator over entire lattice time extent
- Choose number of states and t_{min} based on stability of leading energies and amplitudes & goodness-of-fit
- Employ same t_{min}~0.45 fm for all lattice spacings

(2) Replacing $G_{data}(t)$ with $G_{fit}(t)$ for $t > t^*$

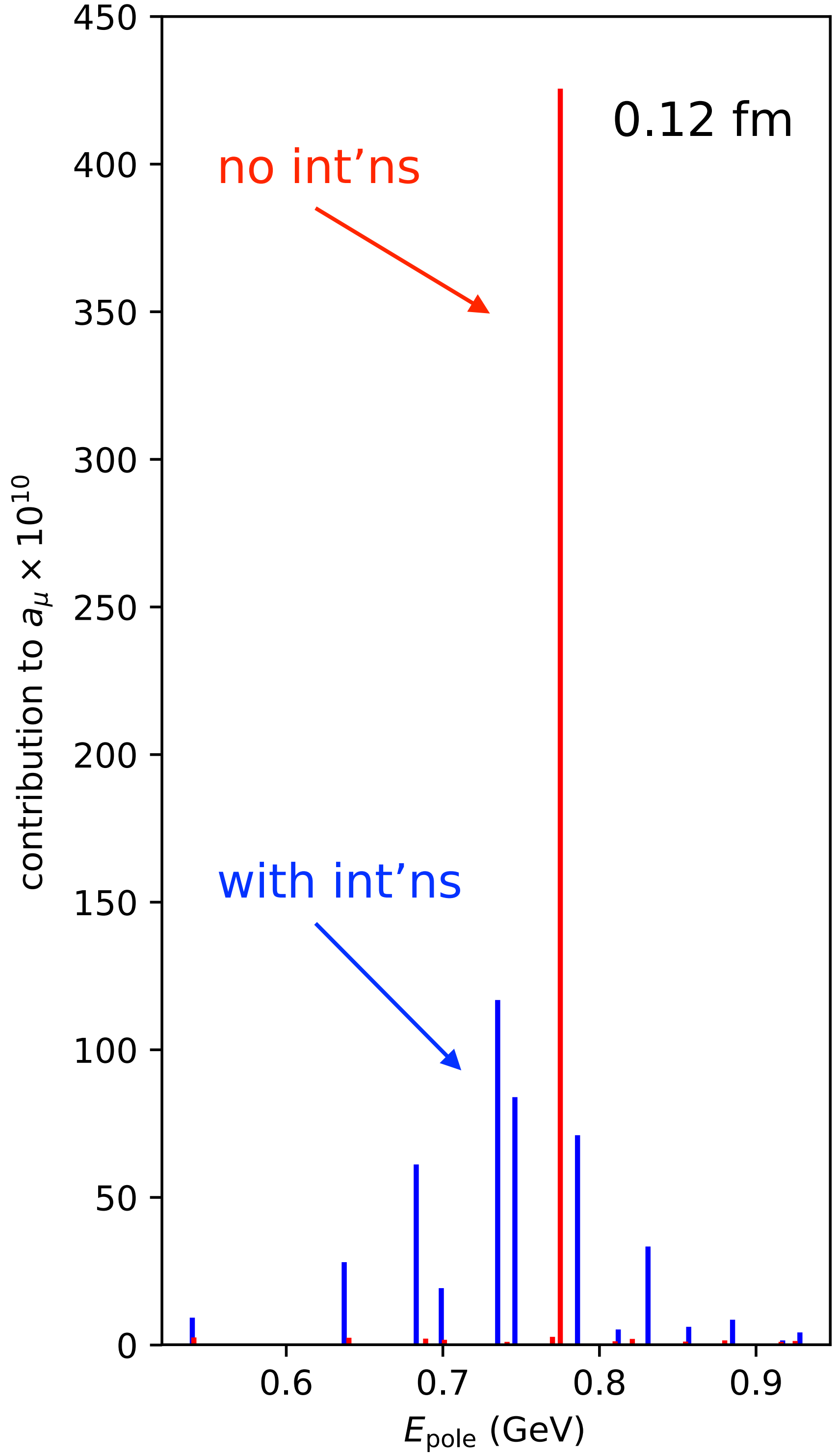
- Correct for finite temporal extent by calculating $G_{fit}(t)$ using 2-point fit parameters in exponential parameterization and extending times in $G_{fit}(t)$ to 2 x T

$$G(t) = \begin{cases} G_{\rm data}(t) & t \leq t^* \\ G_{\rm fit}(t) & t > t^* \end{cases}$$

Impact of low-energy ππ states

- True spectrum more complicated than simple fit parameterization employed in our analysis
  - (multiple ρ and ππ levels and mixing between the two)
- **Study effects of low-energy ππ states on a_μ^{HVP} within extended chiral perturbation theory that includes π's, ρ's, and γ's** [Jegerlehner & Szafron, EPJC71 1632 (2011)]
  - (same EFT used to compute finite-volume + staggered discretization corrections)
  - Compute ππ levels numerically from poles and residues of $\Pi(q^2)$ in Appendix B of Chakraborty et al., PRD96 (2017) no.3, 034516
  - **Include leading interactions that couple ρ and ππ channels**

$\pi\pi$–ρ Contributions to a_μ (Model)



- Without interactions: ρ dominates, $\pi\pi$ negligible (due to finite volume, staggered pions).
- With interactions: ρ mixes with $\pi\pi$ and its contribution spread over many states.
- Total a_μ same in both cases (<0.5%).
- Noisy data ⇒ fitter can't tell difference.

Model in B. Chakraborty *et al* 1601.03071
Phys. Rev. D96 (2017) 034516 (App. B)

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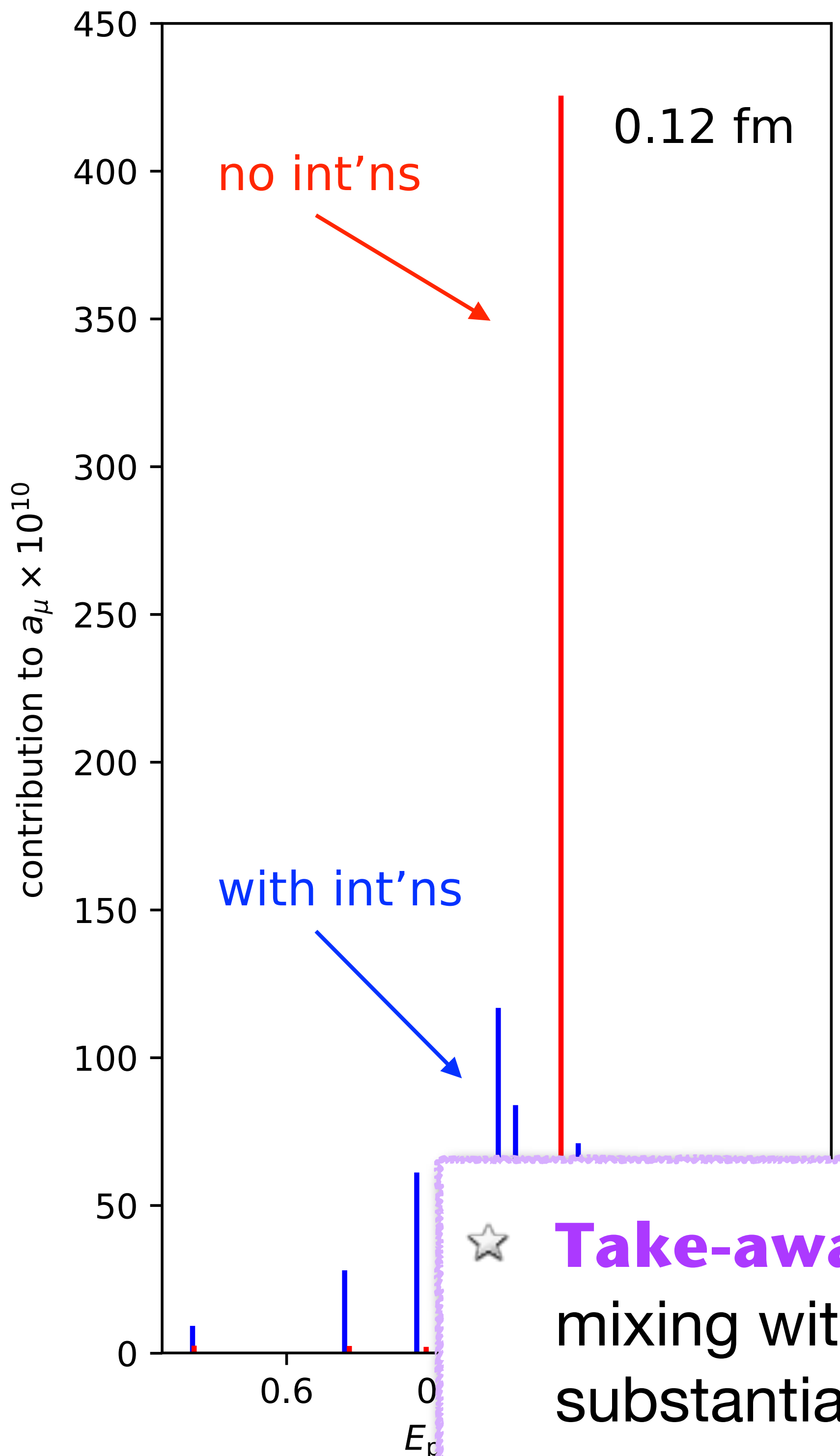

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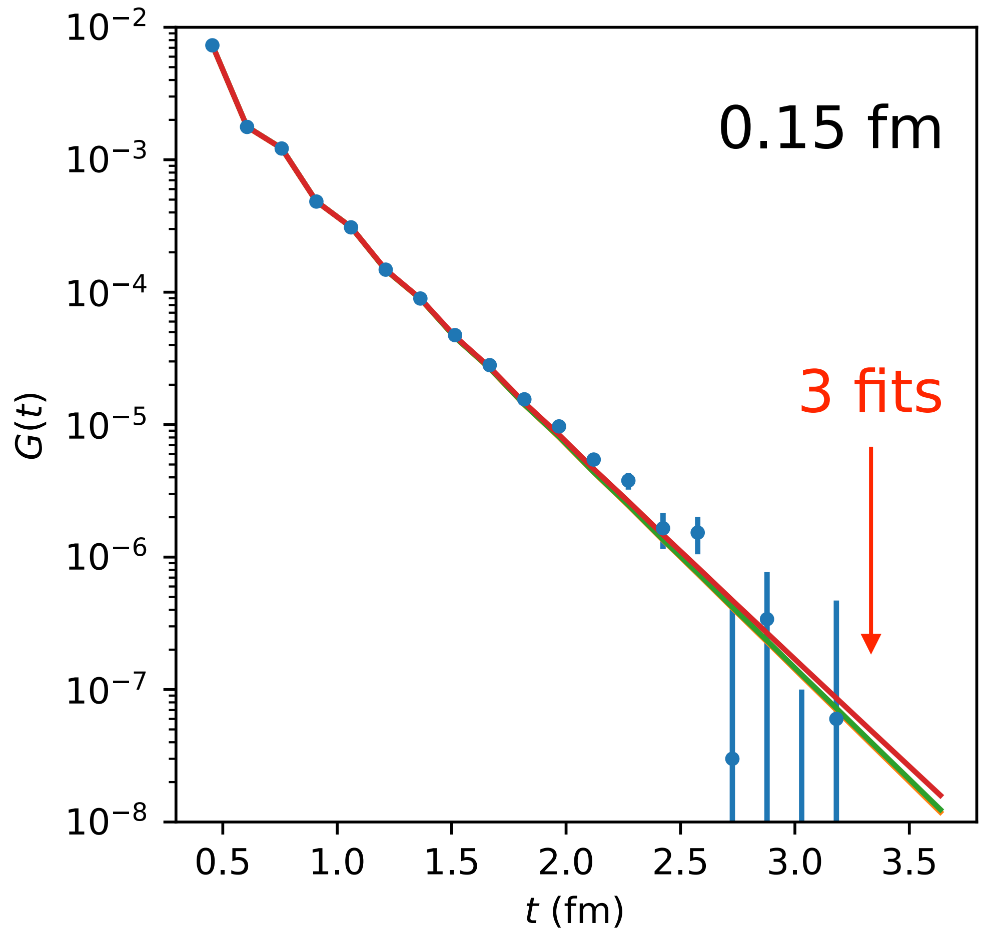
- With interactions: ρ mixes with $\pi\pi$ and its contribution spread over many states.
- Total a_μ same in both cases (<0.5%).
- Noisy data ⇒ fitter can't tell difference.

☆ **Take-away:** ππ interactions and mixing with the ρ change the spectrum substantially, but not the total a_μ^{HVP}

al 1601.03071

4516 (App. B)

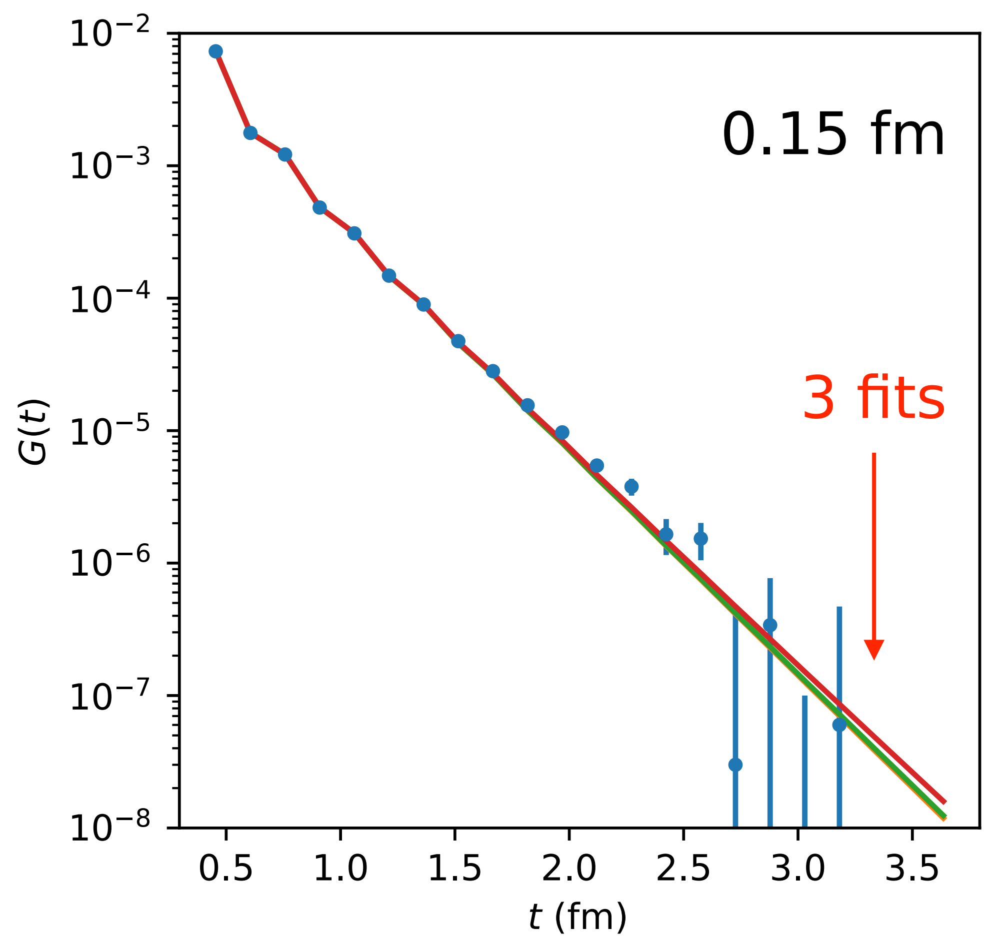
Equivalent Fits (χ^2/dof ≤ 1)



- Fit unable to resolve difference between one or multiple rho mesons; spreads contribution over multiple terms.
- Negligible difference for t<4fm, and irrelevant for a_μ.

	E_0 \ a_0	E_1 \ a_1	E_2 \ a_2	E_3 \ a_3	a_μ x 10^{10}
Fit 1	0.770 \ 0.132	1.72 \ 0.35	2.5 \ 0.1		602(7)
Fit 2	0.75 \ 0.09	0.80 \ 0.09	1.75 \ 0.36		602(7)
Fit 3	0.71 \ 0.08	0.78 \ 0.04	0.84 \ 0.10	1.76 \ 0.36	608(8)

Equivalent Fits (χ^2/dof ≤ 1)



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	$E_0 \backslash a_0$	$E_1 \backslash a_1$	$E_2 \backslash a_2$	$E_3 \backslash a_3$	$a_\mu \times 10^{10}$
Fit 1	0.770 \ 0				602(7)
Fit 2	0.75 \ 0				602(7)
Fit 3	0.71 \ 0				608(8)

☆ **Take-away:** can parameterize data over a finite t range (<3-4fm) by a fit with a single ρ

Missing ππ contributions

- correlator fits will catch large contributions from low-energy ππ states that mix substantially with the ρ
  - Calculate smaller missing contributions from ππ states below M_ρ within (non-interacting) chiral perturbation theory, and add $G_{\pi\pi}(t)$ to $G_{fit}(t)$ for $t>t^*$ fm

$$G_{\pi\pi}(t) = \sum_{\substack{\{\pi_1,\pi_2\}\in \\ \text{taste } V}} \frac{1}{3L^3} \sum_{\boldsymbol{k}} \frac{\boldsymbol{k}^2 \exp[-(E_1+E_2)t]}{E_1 E_2}$$

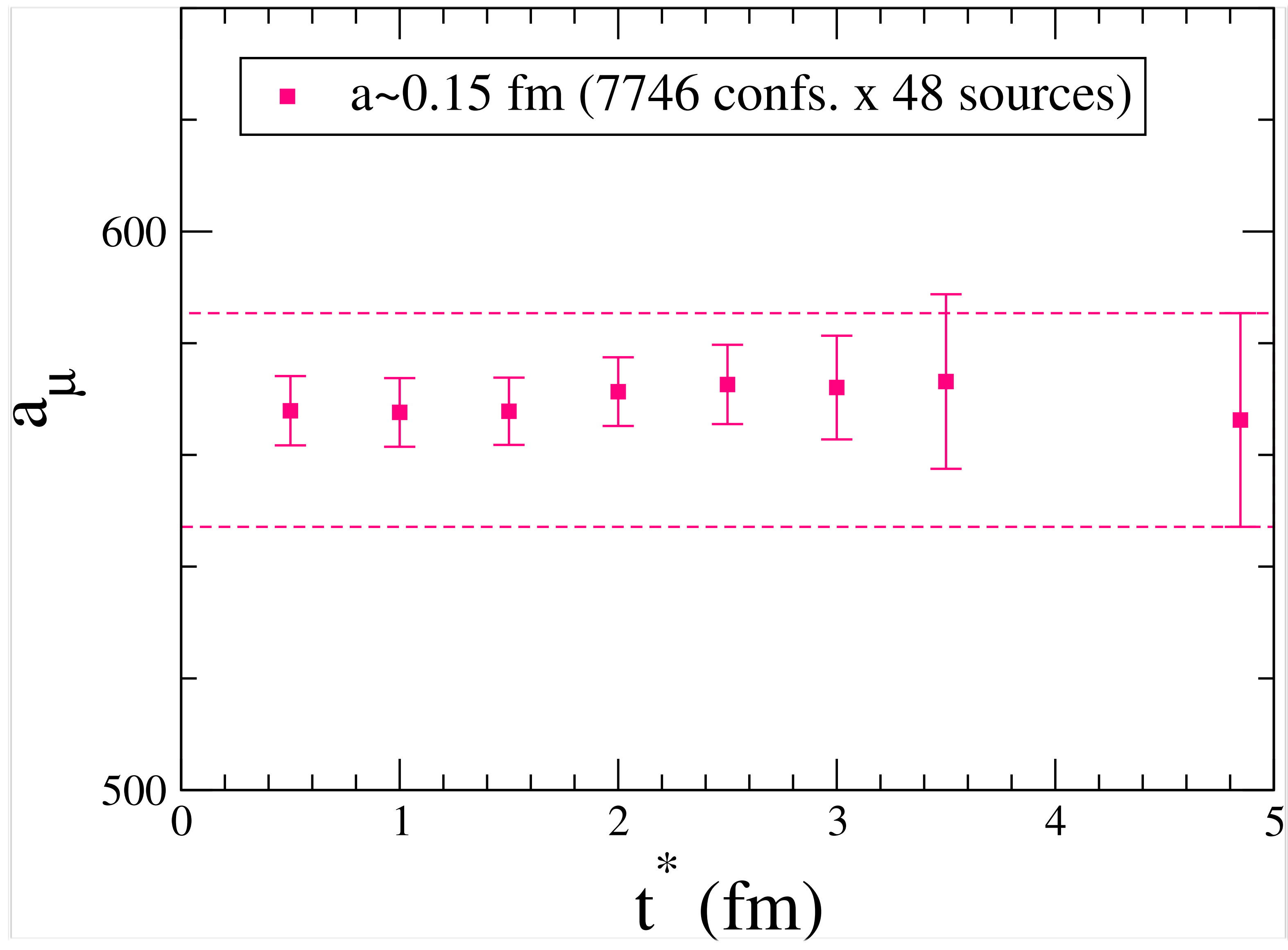
$$E_i = \sqrt{\boldsymbol{k}^2 + m_{\pi_i}^2}$$

  - Estimated low-energy ππ contribution to a_μ^{HVP} from t>1.5 fm *that is not included in the fit* ranges from about 0.2% at a~0.15 fm to 2.5% at a~0.06 fm

High-statistics *test*

- Test method to reduce noise and include low-energy ππ contributions on new high-statistics ensemble where a_μ^{HVP} is known precisely from data

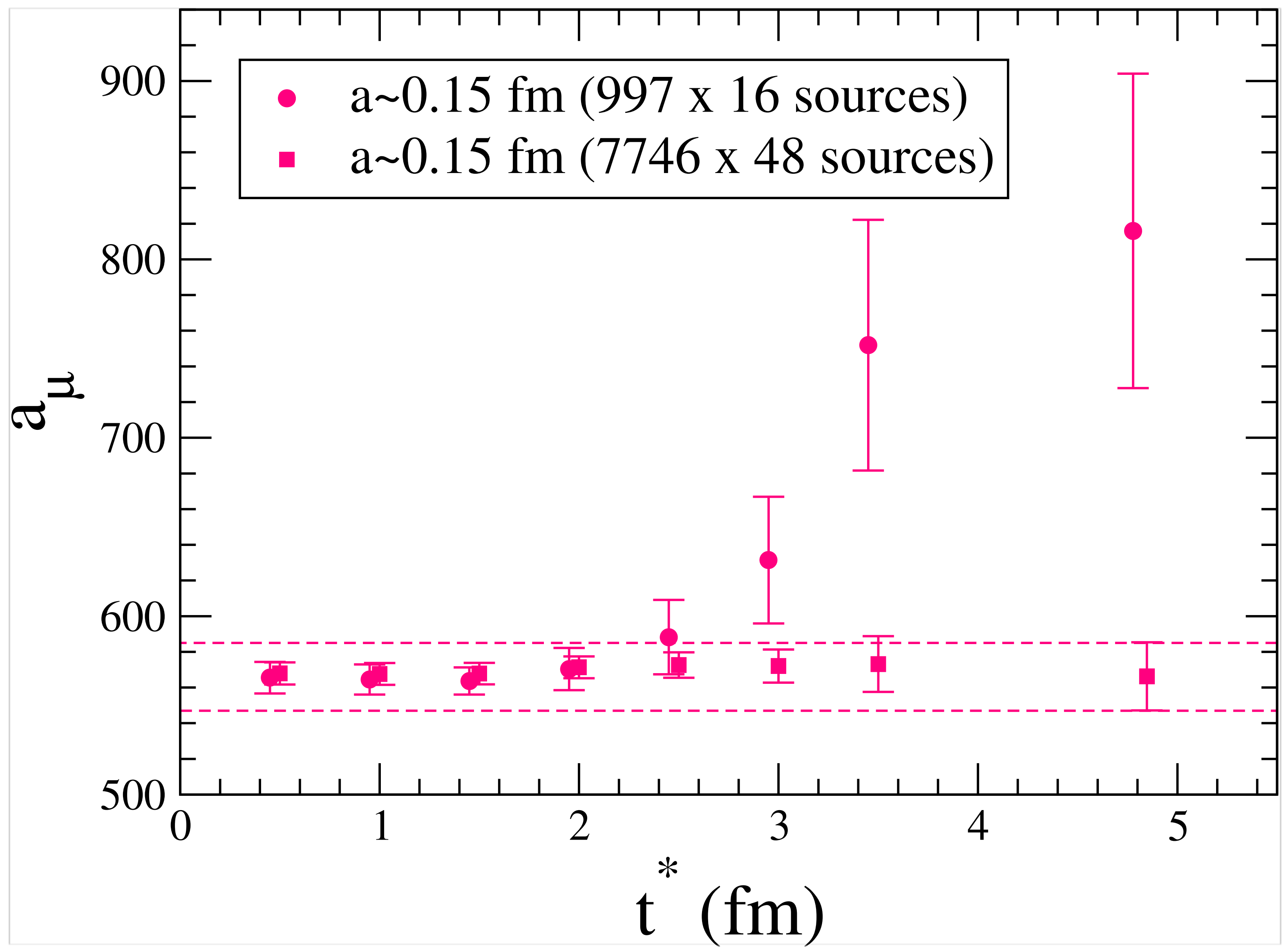
☆ a_μ^{HVP} obtained from data + fit + estimated ππ contributions consistent with data-only value for all values of t^*



Comparison with noisy data

- Compare with results on independent ensemble with same lattice spacing and slightly different quark masses, but substantially lower statistics
  - *Answers consistent until t*~2 fm, but diverge substantially beyond t*~3 fm*

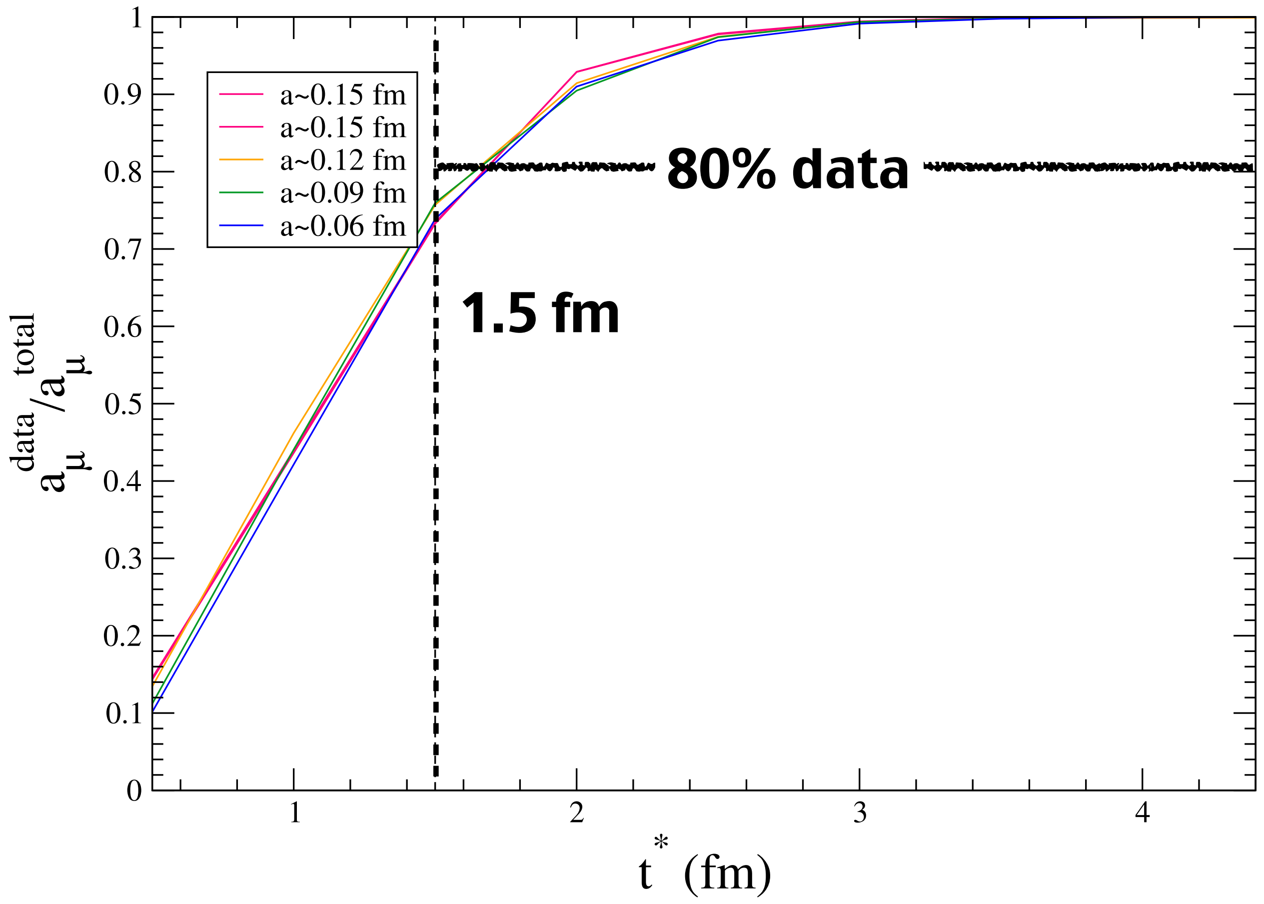
> ☆ Can obtain a_μ^{HVP} from data + fit for t^* below ~2–2.5 fm



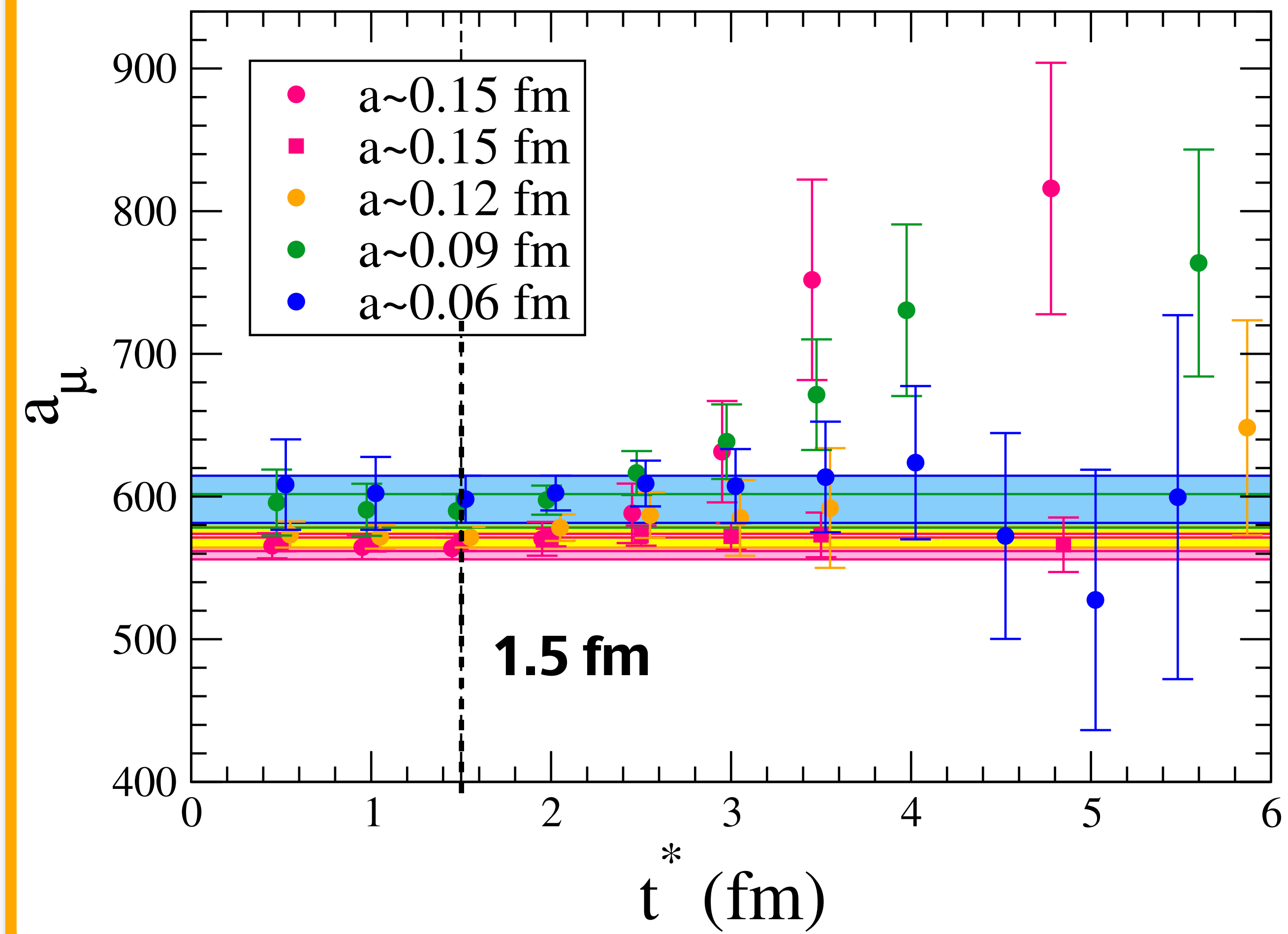
Selection of t*

✦ For **t* = 1.5 fm**, a_μ^{HVP} comes primarily from data region, and errors still controlled

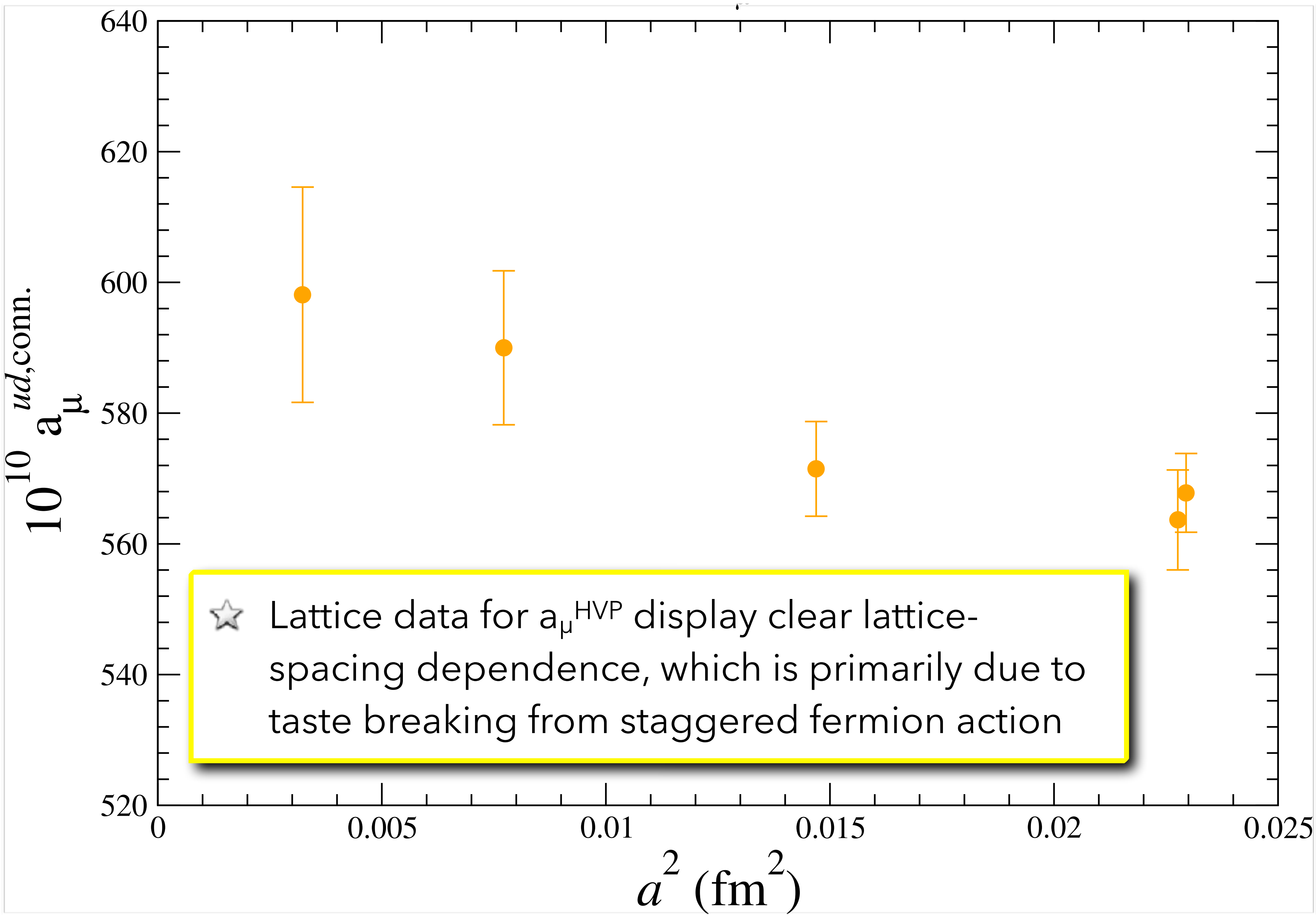
Data contribution



Stability



Approach to continuum

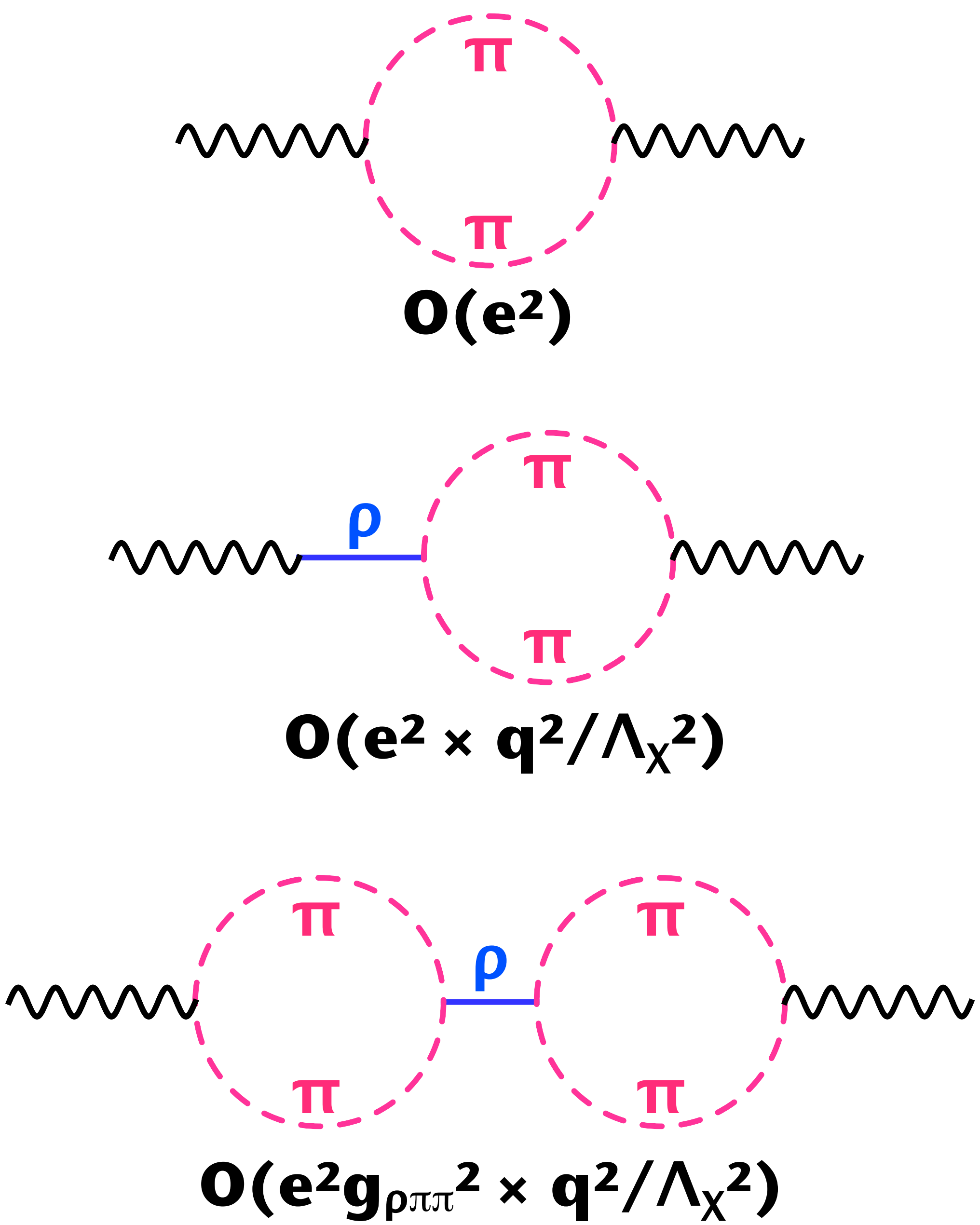


☆ Lattice data for a_μ^{HVP} display clear lattice-spacing dependence, which is primarily due to taste breaking from staggered fermion action

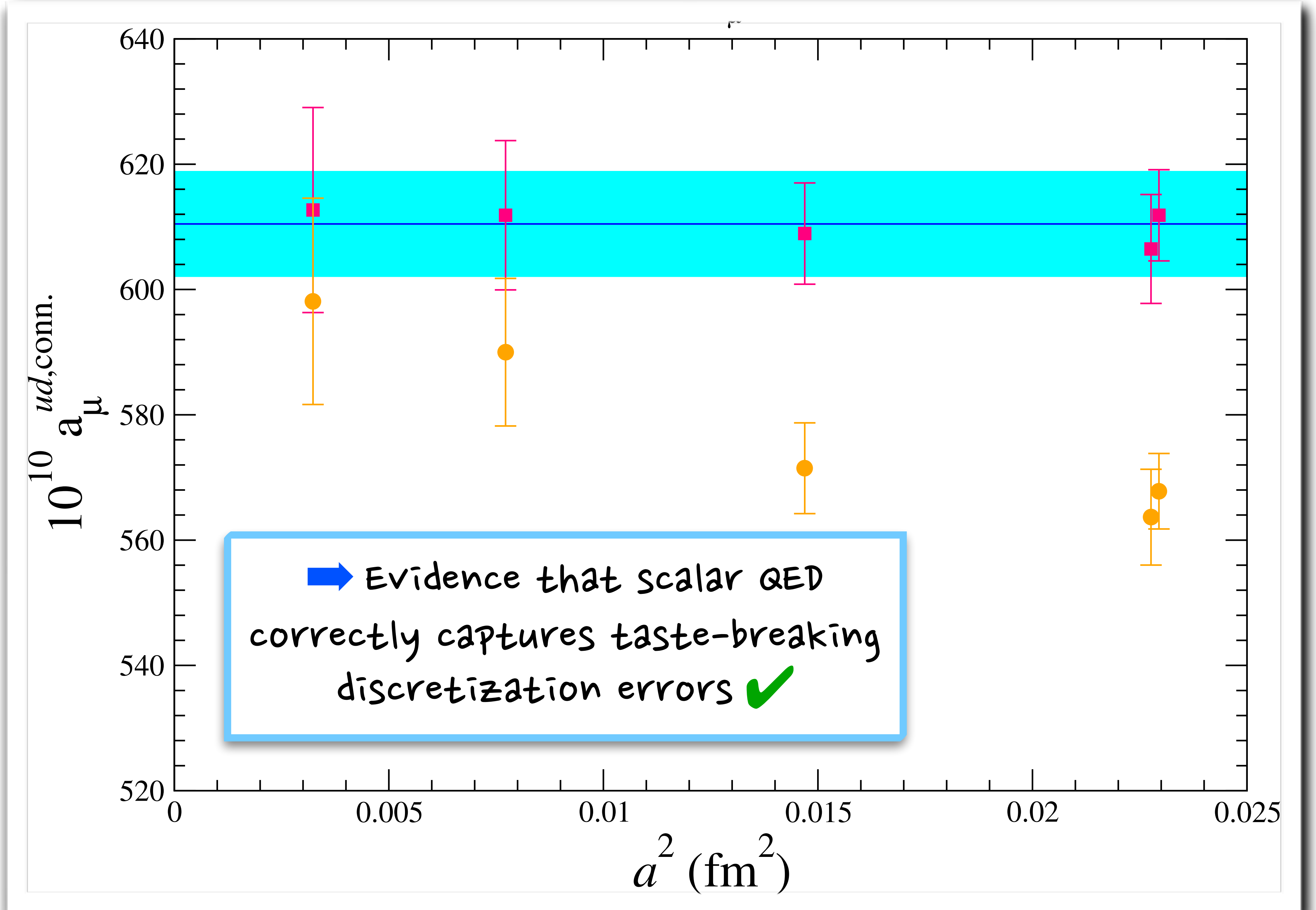
Lattice corrections

- Correct a_μ^{HVP} for finite-volume and taste-breaking discretization effects before continuum extrapolation
- Calculate 1-pion-loop contributions to a_μ^{HVP} to all orders in leading interactions that couple ρ^0-γ-$\pi^+\pi^-$ channels
  - Corrections given by difference between results in continuum, infinite volume, and in finite volume with lattice artifacts
  - corrections largely from taste splittings, and decrease with lattice spacing

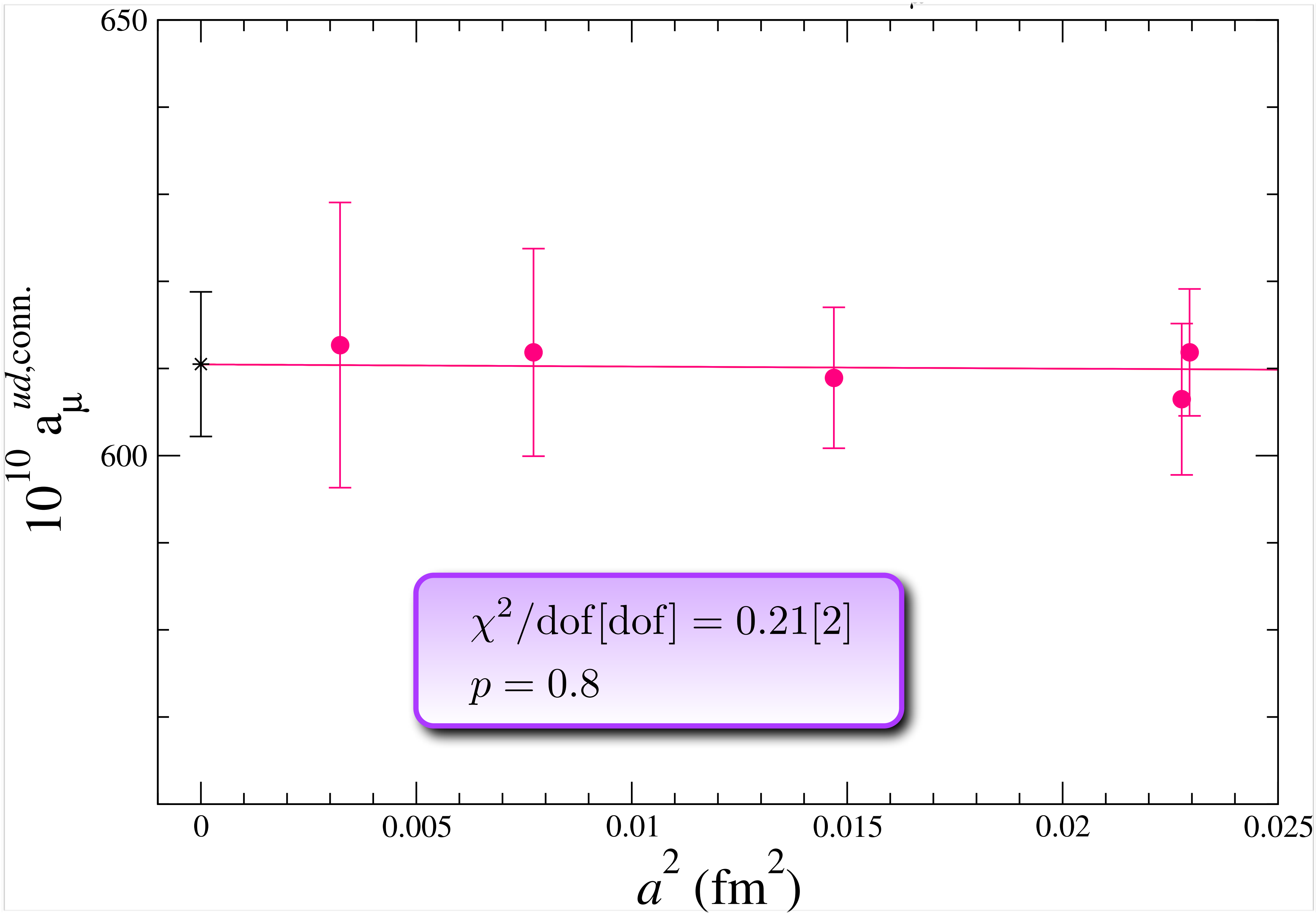
(diagrams below + all iterations of these diagrams)



Residual a^2 dependence



Continuum extrapolation



Error budget for $a_\mu^{ud,conn.}$

- Shown for isospin-limit quantity ➡ include QED & isospin-breaking errors only in total LO HVP contribution

	$a_\mu^{ud,\mathrm{HVP}}$ (%)	
	HPQCD+RV 1601.03071	Fermilab-MILC-HPQCD (*preliminary*)
Continuum ($a \to 0$) extrapolation	0.2	0.8
$\pi\pi$ states (t^*)	0.5	0.6
Finite-volume & discretization corrections	0.7	0.4
Statistics + 2pt fit	0.4	0.3
Sea (m_s) adjustment	0.2	0.1
Current renormalization (Z_V)	0.2	0.1
Lattice-spacing (a^{-1}) uncertainty	<0.05	0.8
Padé approximants	0.4	0.0
Fit total	1.1%	1.4%

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No E_0 rescaling

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Two finer lattice spacings

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Include missing $\pi\pi$ states directly in $G(t)$

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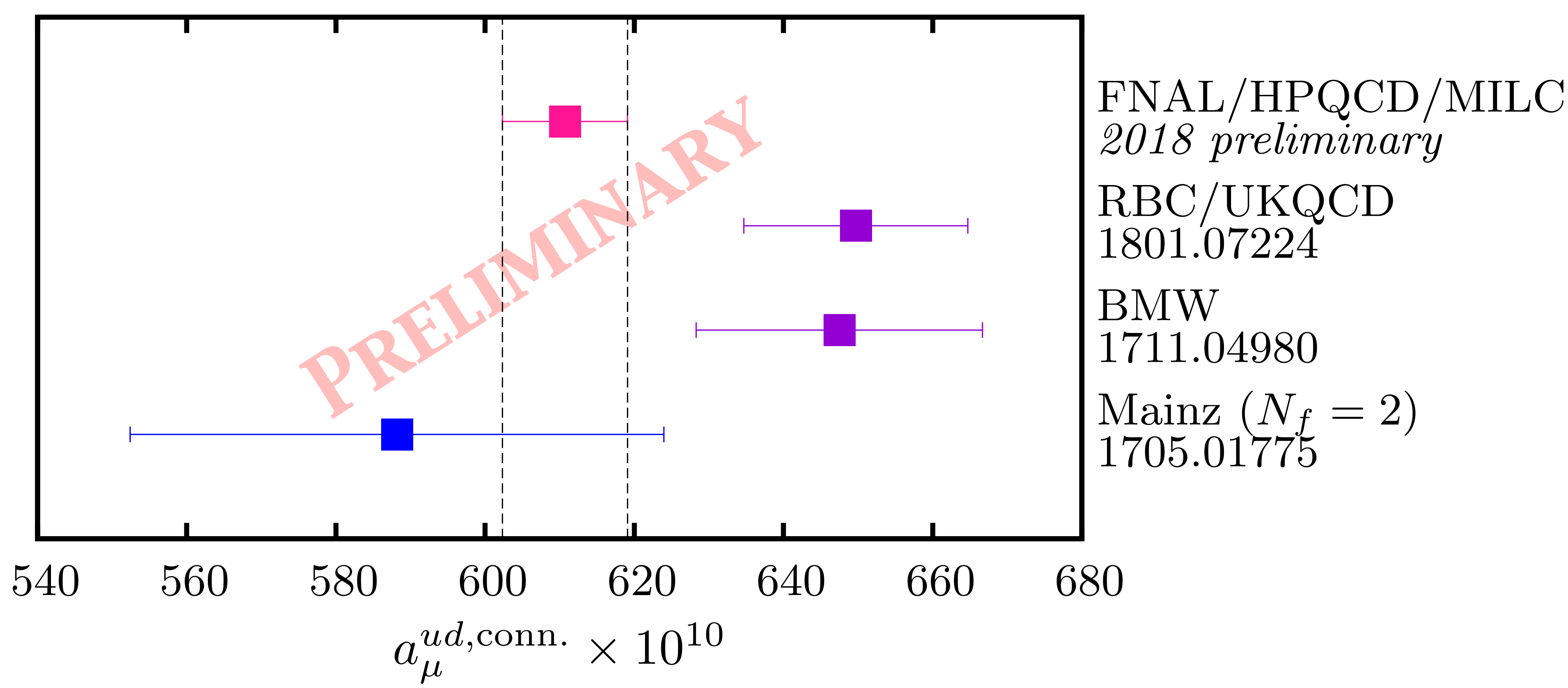
Use [3,3] Padés

- Increased (preliminary!) uncertainty primarily from dropping unphysical-mass ensembles and E_0 rescaling, but analysis still in progress

Comparison with other work

✦ Results shown are for isospin-limit quantity without QED & isospin-breaking corrections



Difference between BMW & RBC/UKQCD reduced, but still ~2σ

Total leading-order a_μ^{HVP}

- As in HPQCD+RV [PRD96 (2017) no.3, 034516], add to light-quark connected contribution:

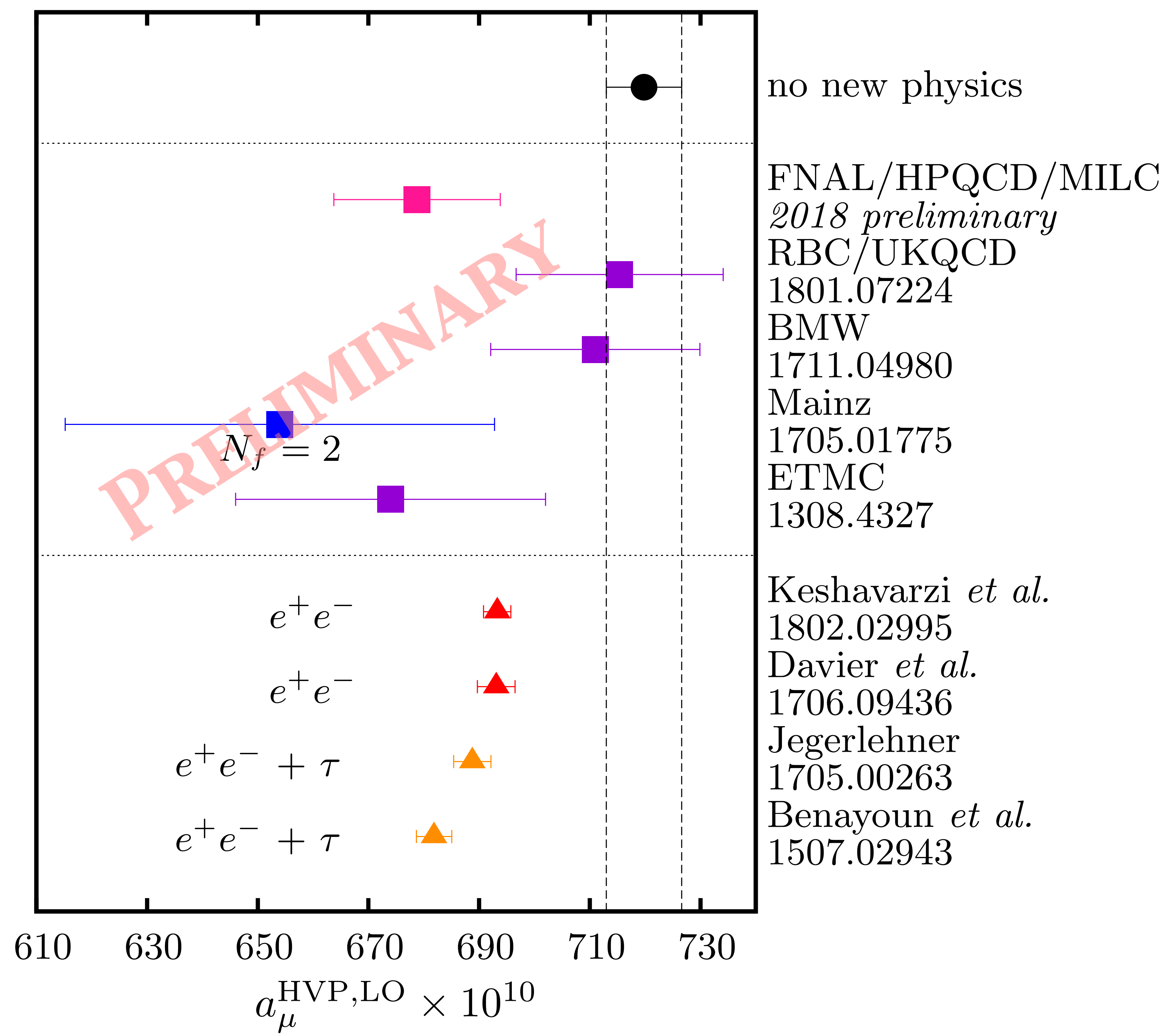
  (1) s, c-, and b-quark connected contributions from HPQCD [PRD 91, no. 7, 074514 (2015)]

  (2) Quark-disconnected contribution from HPQCD+HadSpec [PRD 93, no. 7, 074509 (2016)]

  (3) 1% ⊕ 1% = 1.4% error to account for omission of QED and isospin breaking

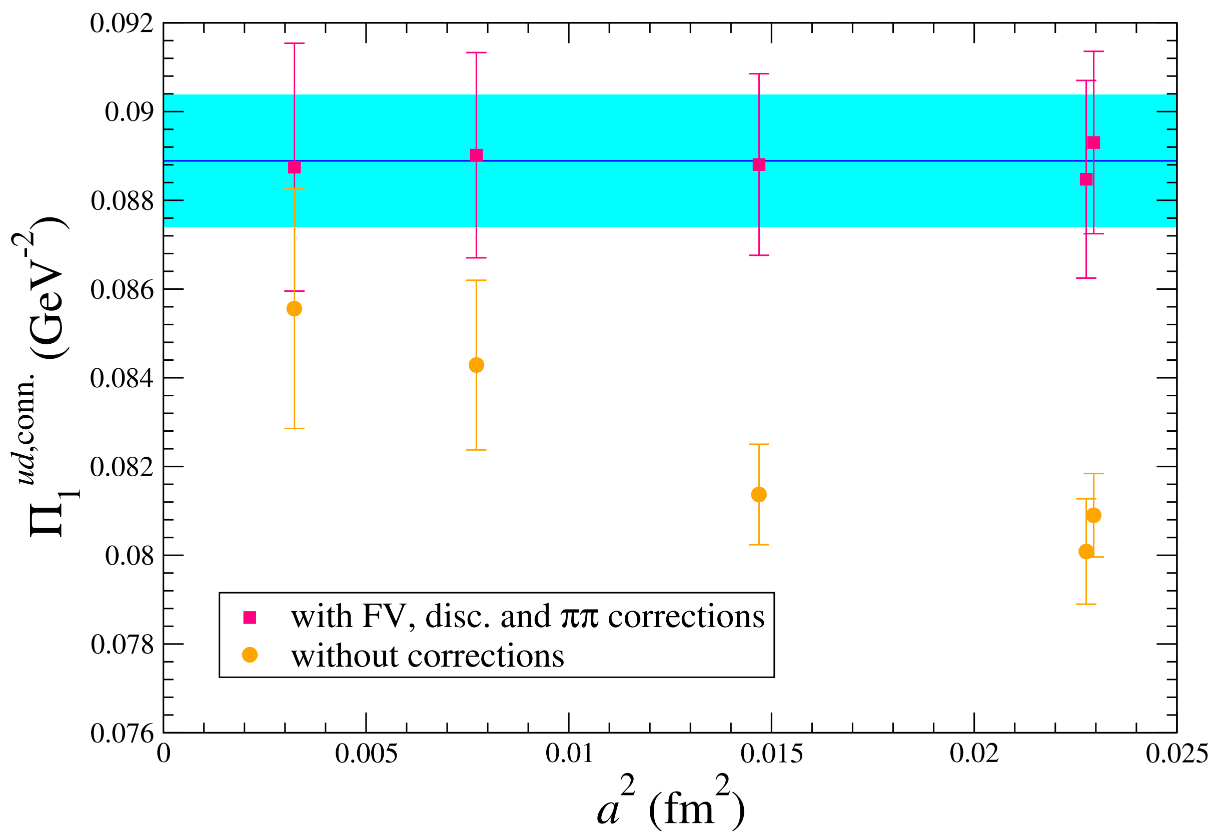
  ☆ Wait to include direct estimate of strong-isospin-breaking correction when we also have QED contribution, since cancellations are expected

Total leading-order a_μ^{HVP}



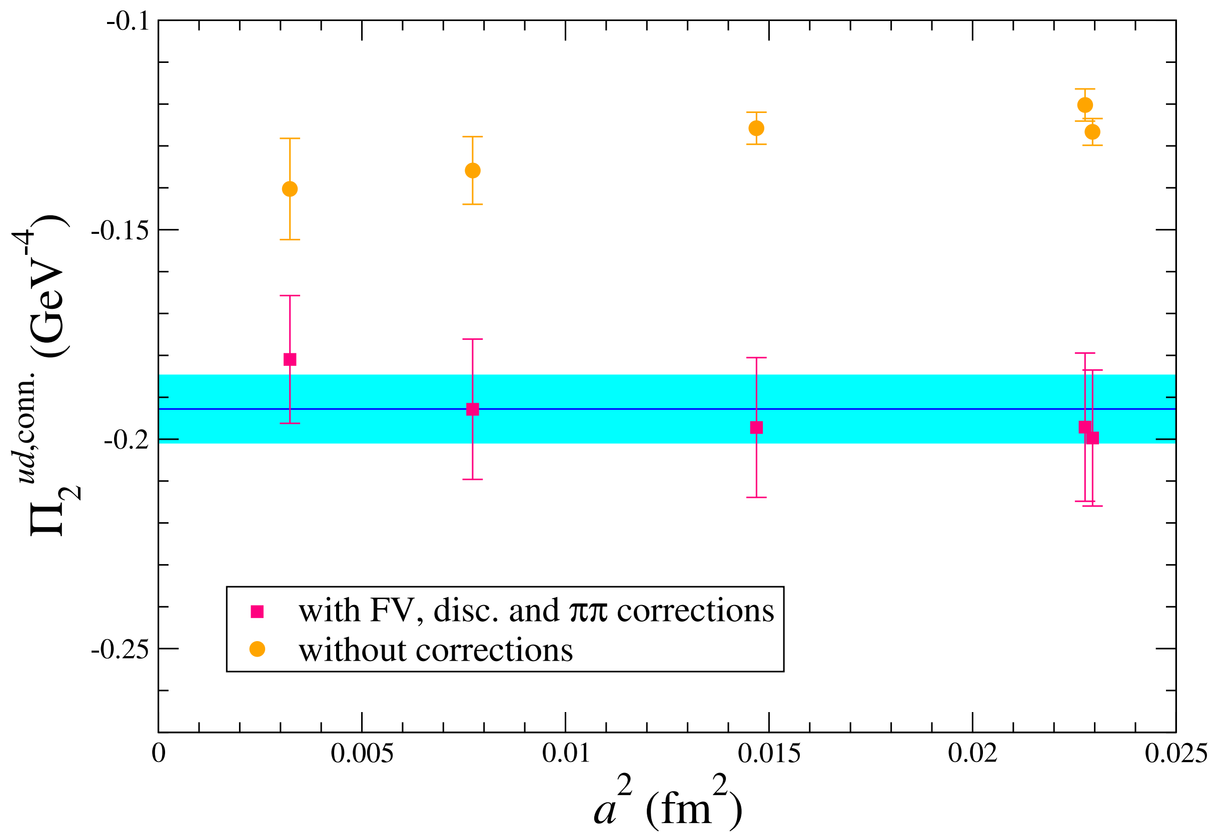
New! Taylor coefficients of Π(q²)

Slope



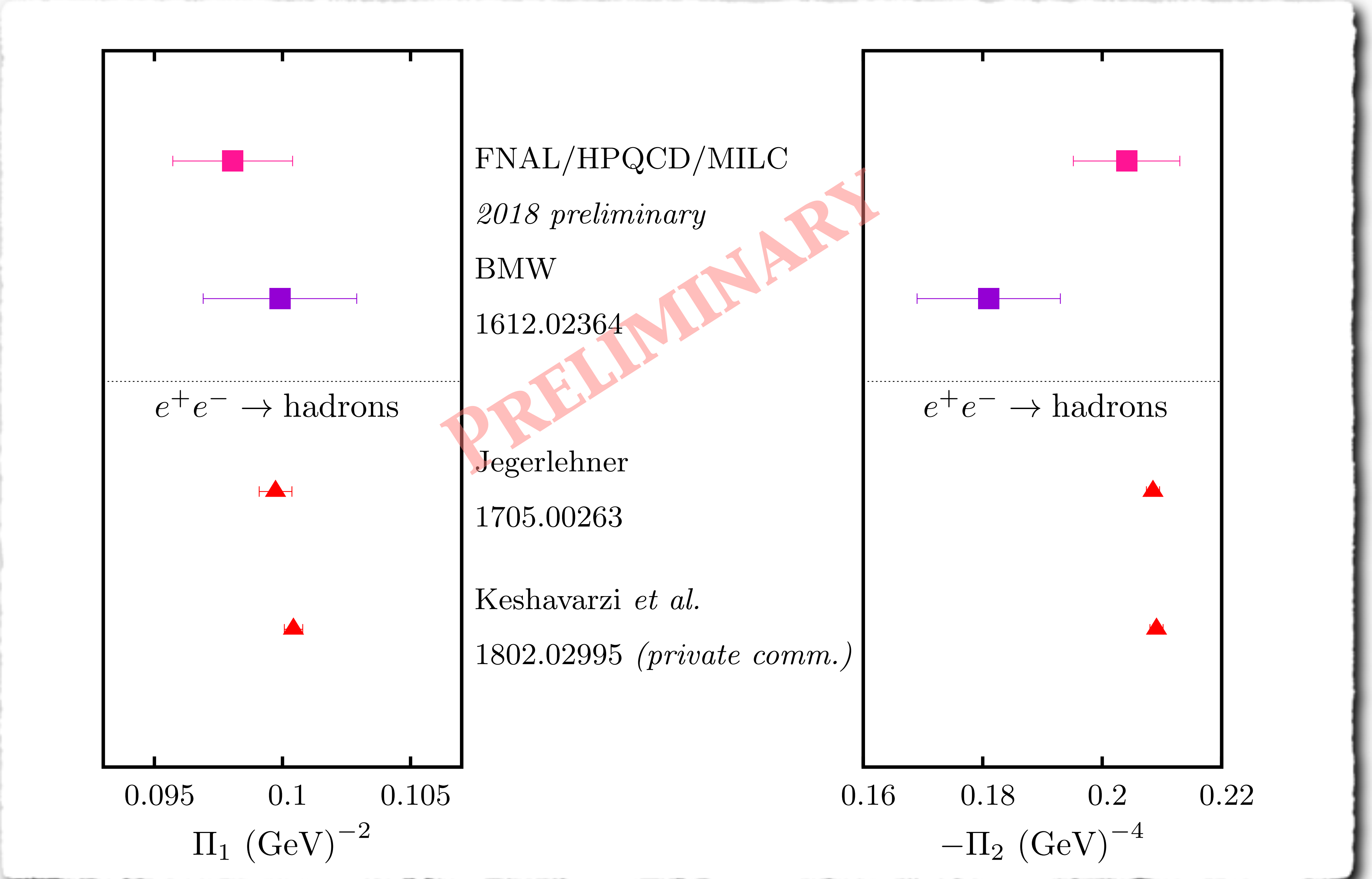
$\chi^2/\mathrm{dof[dof]} = 0.05[2]$
$p = 0.95$

Curvature



$\chi^2/\mathrm{dof[dof]} = 0.46[2]$
$p = 0.63$

Comparison with other results



consistent with R-ratio determinations, but tension with BMW for Π_2

Work in progress

- Current and proposed running focused on reducing leading sources of error from:

(1) Statistics

- Better statistics will allow increased t^* and reduce low-energy $\pi\pi$ uncertainty
- More precise a~0.09 fm and 0.06 fm data will reduce (*dominant*) continuum-extrapolation error

(2) Electromagnetic effects

- Need direct calculation to replace (*and reduce error from*) phenomenological estimate

(3) Quark-disconnected contributions

- Contributes roughly half of total uncertainty on $a_\mu^{HVP,LO}$!

(4) Low-energy ππ contributions

- BlueWaters allocation to calculate $\langle\rho|\rho\rangle$, $\langle\rho|\pi\pi\rangle$, and $\langle\pi\pi|\pi\pi\rangle$ on a~0.15 fm physical-mass ensemble and calculate levels using GEVP
- Will enable check of chiral-pertubation-theory estimate

QED+QCD ensemble

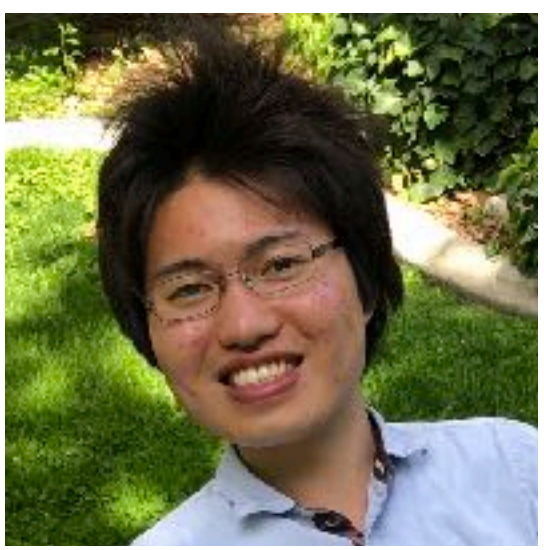
- Generation of dynamical HISQ QCD + QED ensemble in progress
  - Noncompact QED_L with formulation of Hayakawa & Uno [Prog.Theor.Phys.120:413-441,2008] for the zero-mode subtraction
  - a~0.15 fm with physical quark masses & EM charges (same parameters as $N_f = 1+1+1+1$ QCD ensemble)
  - About 300 thermalized configurations available
- Presently checking spectrum code; will then compare with previous quenched-QED results
- **Theoretical work to understand and estimate finite-volume corrections still needed**

Yuzhi Liu

Quark-disconnected contribution

- Calculating quark-disconnected contributions on a~0.15 fm physical-mass ensemble
- One-link-split HISQ current density
- To reduce statistical errors, employ:

  (1) **Deflation**
  (2) **Dilution**
  (3) **Truncated Solver Method**



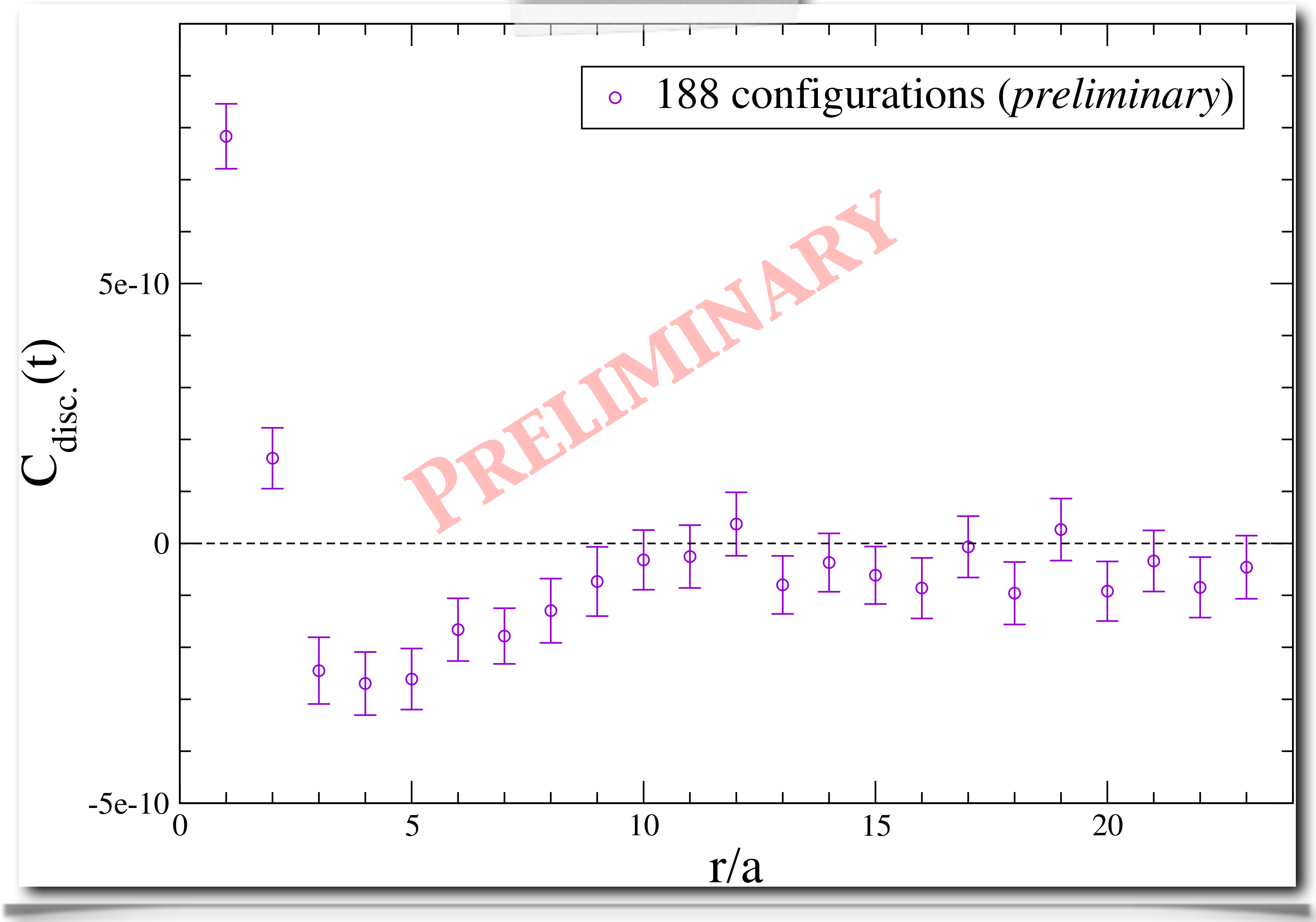
Shuhei Yamamoto

Craig

Carleton

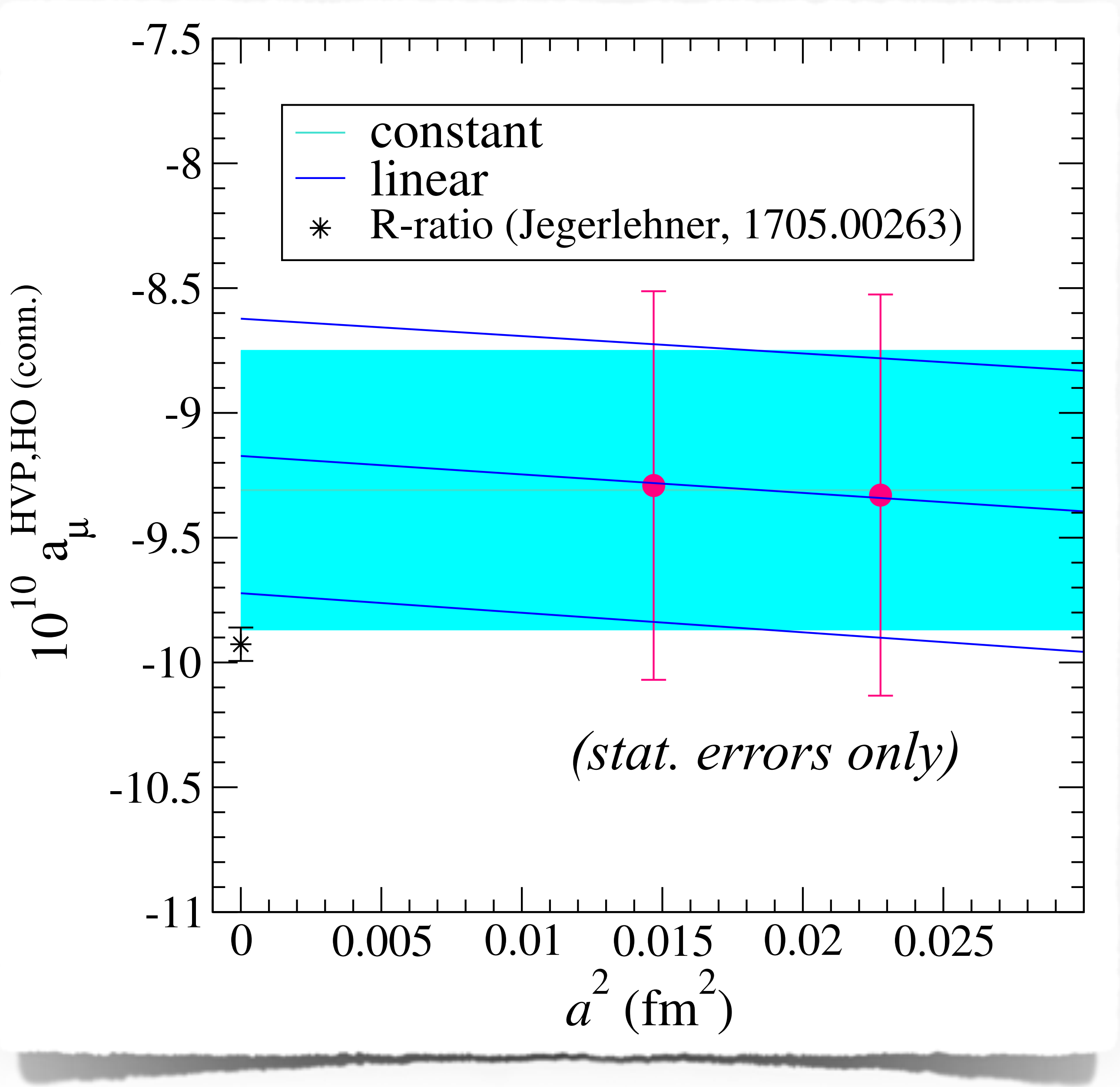
First results

- Include contributions of u, d, s, and c quarks
- **Obtain statistically-significant signal with ~200 configurations:** hope to increase by factor of four within few weeks
- *Next will employ same methodology on a~0.12 physical-mass ensemble to take continuum limit*



Advertisement! NLO HVP contribution

- **Lattice QCD can also provide higher-order HVP contributions** (inspired by first Muon g-2 Theory Initiative workshop a year ago)
  - Not critical to reach target experimental uncertainty, but good check of lattice methods
- Paper with first lattice-QCD result for NLO HVP contribution to appear soon
  - Employs published quark-connected Taylor coefficients at two lattice spacings from HPQCD
  - Provides new expressions suitable to compute O(α_{EM}^3) HVP contributions from renormalized $\Pi(q^2)$ or Euclidean vector-current correlator



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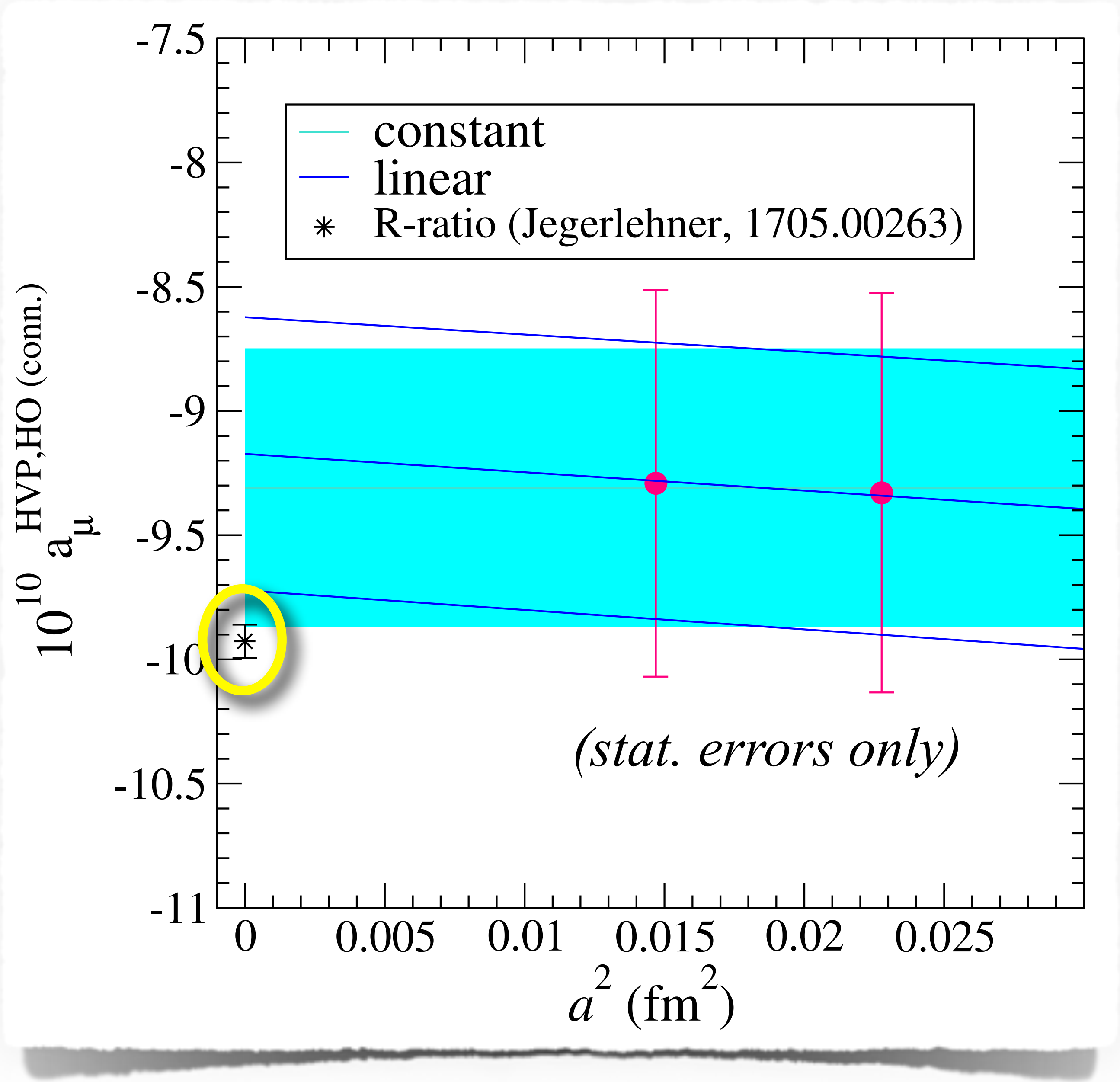


consistent — but not (yet) competitive — with experiment

Summary

- Since last year, paper on first lattice-QCD calculation of strong isospin-breaking effects in LO HVP contribution at the physical pion mass [PRL120 (2018) no.15, 152001]
- Analysis of light-quark connected HVP contribution from 5 physical-mass ensembles with a~0.15 – 0.06 fm is in progress
  - New high-statistics a~0.15 fm ensemble enables explicit check of replacing data with fit and estimate of missing low-energy $\pi\pi$ contribution above t*=1.5 fm
  - Both a_μ^{HVP} & leading Taylor coefficients of vacuum polarization function agree with determinations from R-ratio data (**preliminary**)
- Current focus is improving statistics & reducing leading sources of error in 2016 result for $a_\mu^{HVP,LO}$ from **omission of electromagnetism & quark-disconnected contribution**
  - First signal for quark-disconnected contribution!

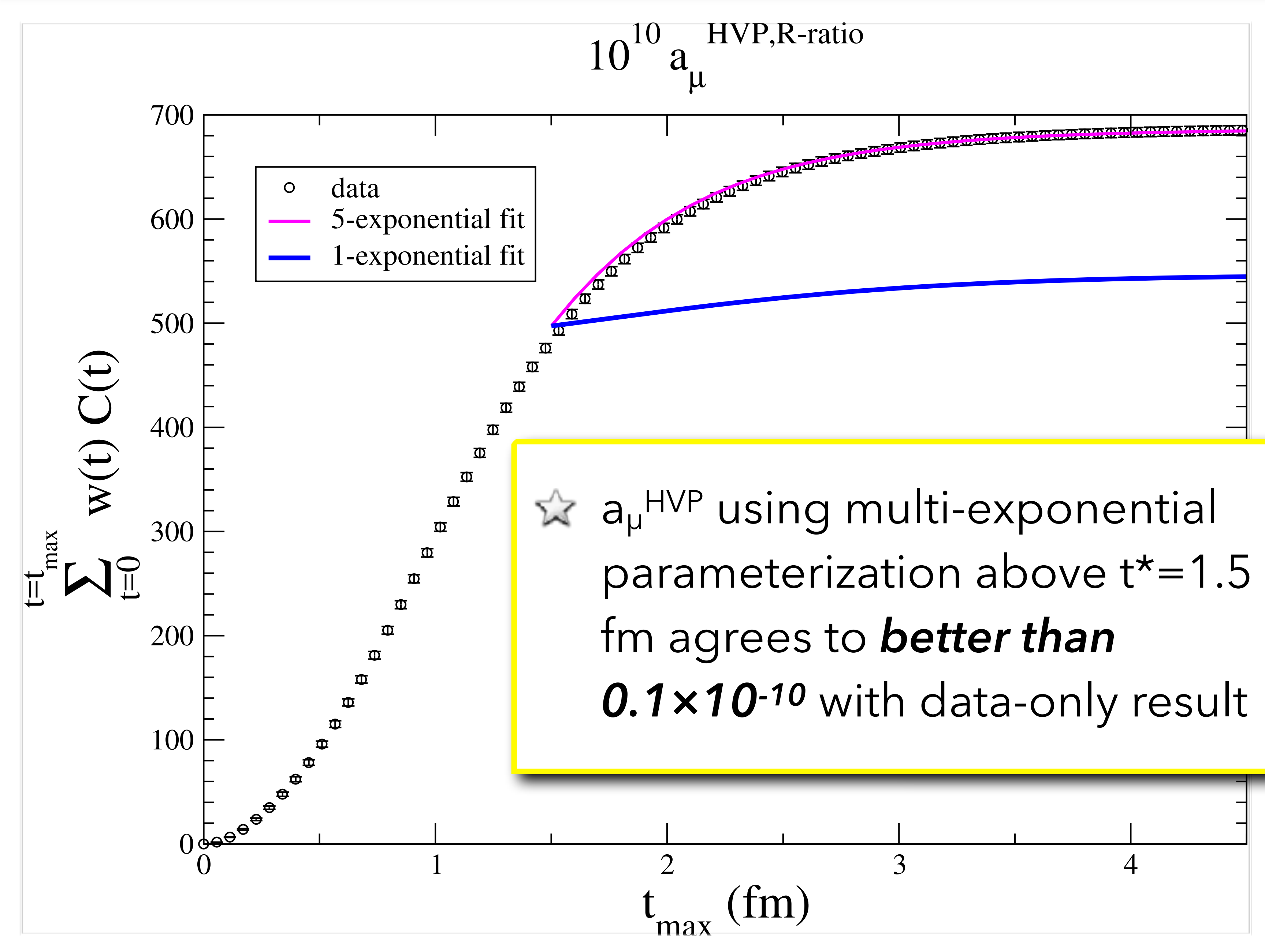
> ▸ With direct determinations of these corrections / contributions in hand, plus additional statistics, hope to obtain sub-per cent precision for total LO HVP contribution to muon g-2!

Tests with R-ratio data

Multi-exponential 2-point fits

- Fit e^+e^- data to multi-exponential parameterization and compute a_μ using data below 1.5 fm and fit above 1.5 fm

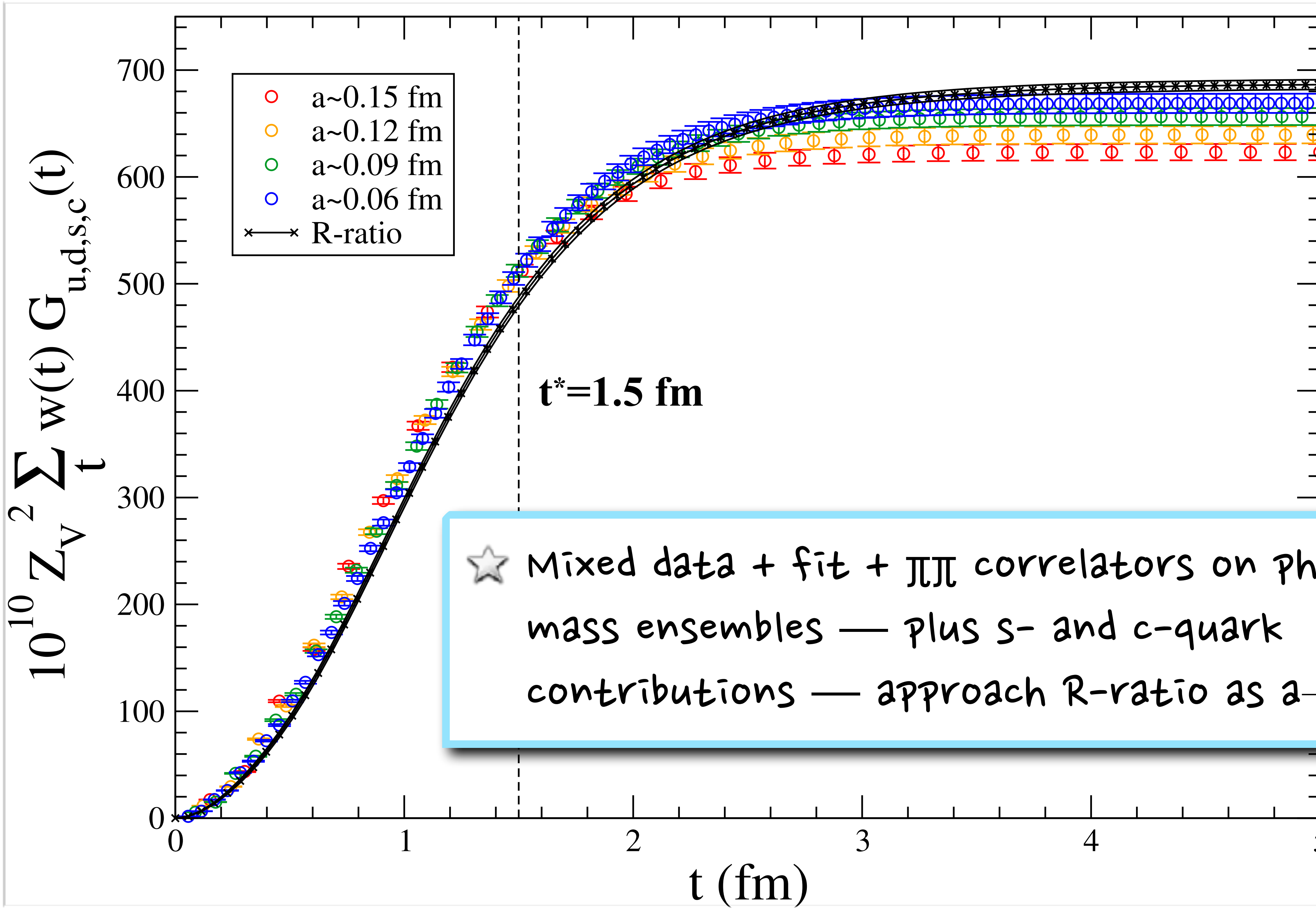
➡ **Method yields correct result provided use of multi-exponential parameterization that accurately describes data above transition time**



☆ a_μ^{HVP} using multi-exponential parameterization above t*=1.5 fm agrees to ***better than 0.1×10^-10*** with data-only result

☆ Thanks to F. Jegerlehner for compiling e^+e^- data in public [alphaQED fortran package]!

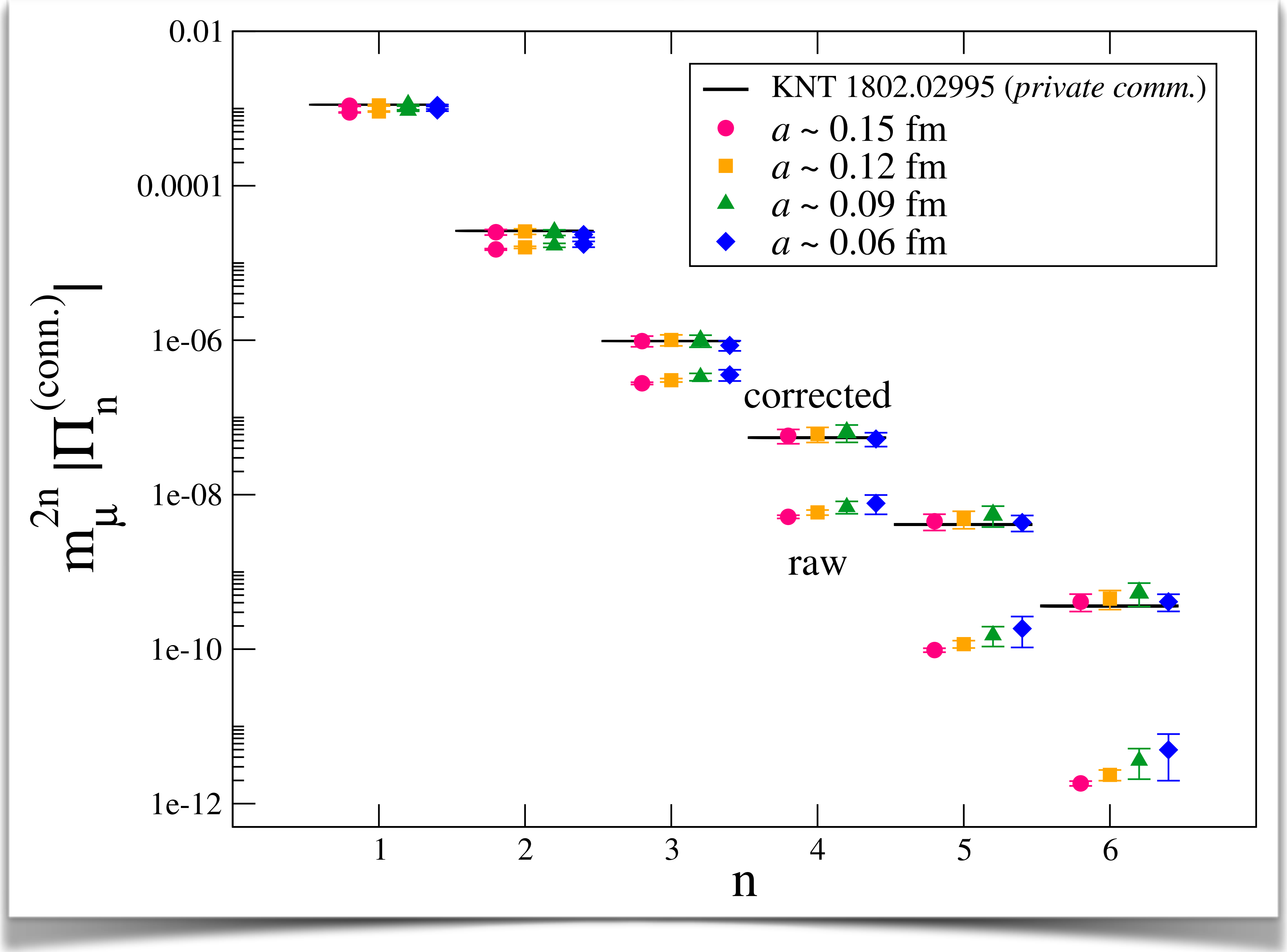
Mixed data + fit correlators



☆ Mixed data + fit + ππ correlators on physical-mass ensembles — plus s- and c-quark contributions — approach R-ratio as a→0

FV + discretization corrections

- Compare Taylor coefficients on physical-mass ensembles with R-ratio values



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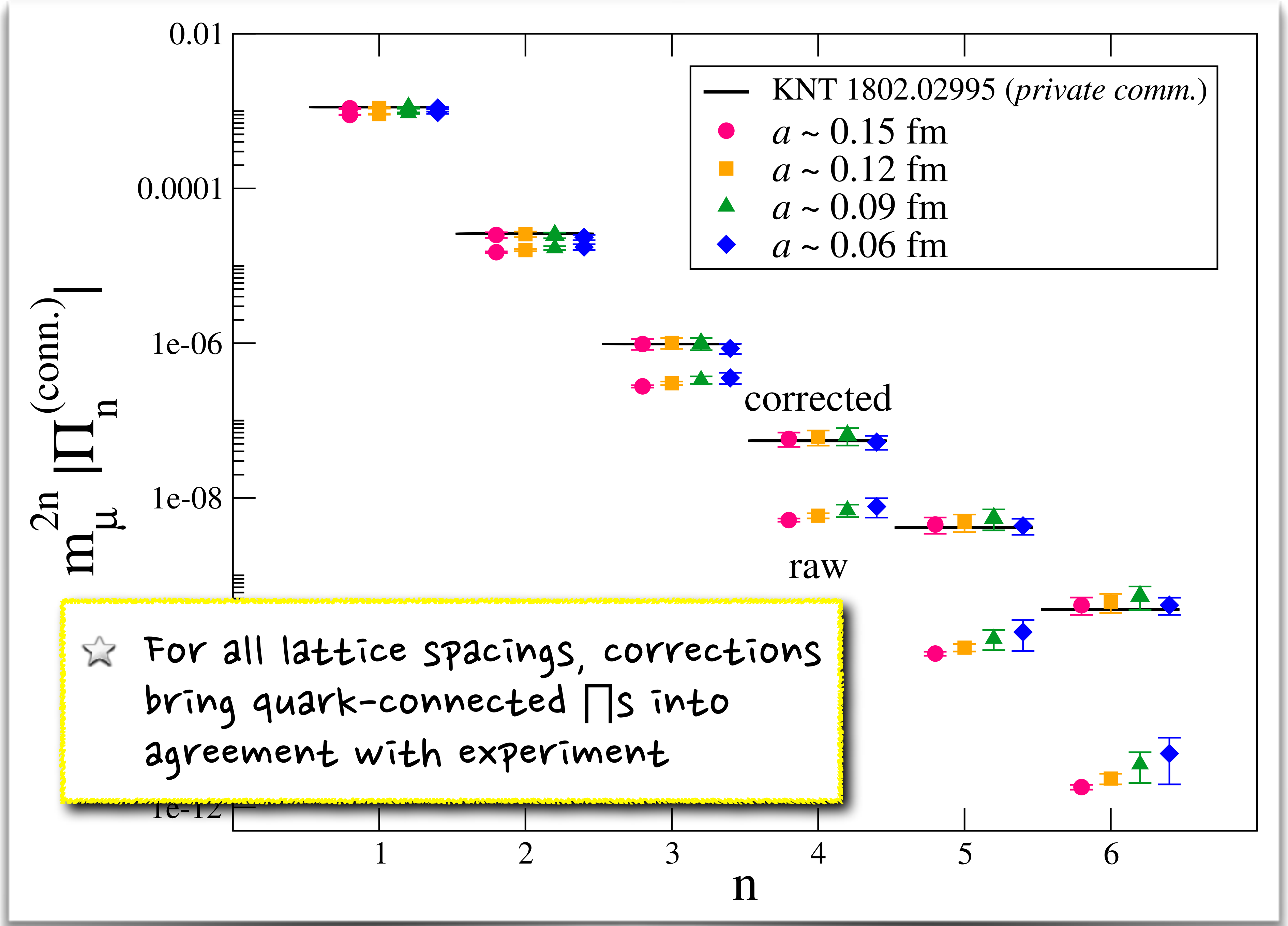


☆ For all lattice spacings, corrections bring quark-connected Πs into agreement with experiment

Supplementary material

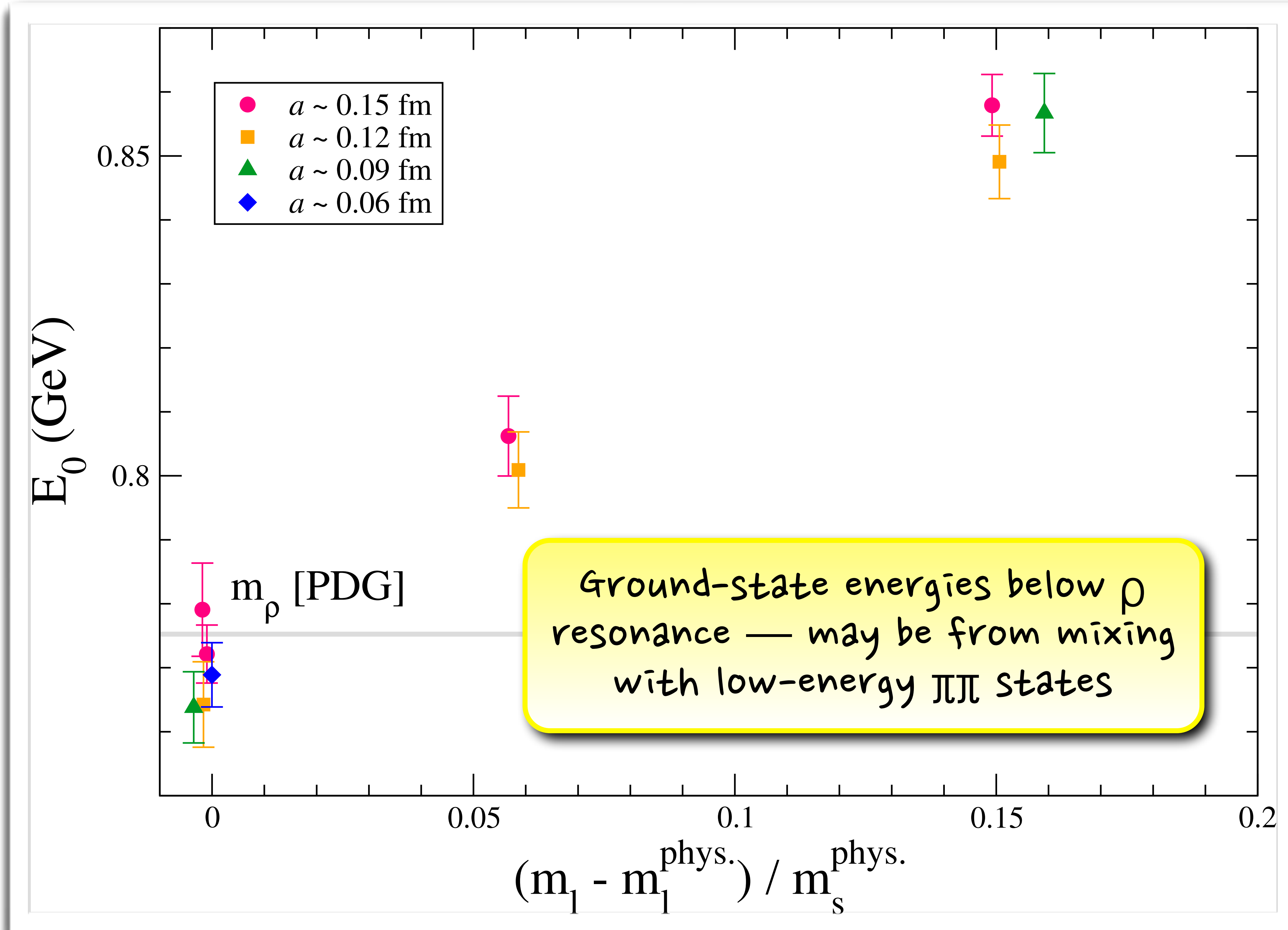
Correlator fits

Simultaneous fit of four combinations of (local, smeared) correlators

$$G_{ij}(t) = a^3 \sum_{k=0}^{N-1} b_i^{(k)}\, b_j^{(k)} \left(e^{-E^{(k)}t} + e^{-E^{(k)}(T-t)}\right)$$
$$- (-1)^t a^3 \sum_{k=0}^{N-1} d_i^{(k)}\, d_j^{(k)} \left(e^{-\tilde{E}^{(k)}t} + e^{-\tilde{E}^{(k)}(T-t)}\right)$$

- ✦ Constrain energies & amplitudes with Gaussian priors
- ✦ Employ SVD cuts to reduce d.o.f. and improve reliability of correlation matrix
  - ❖ Conservative approach replaces eigenvalues of correlation matrix below SVD cut by the SVD cut times the maximum eigenvalue, thereby increasing fit error
- ✦ Choose number of states & fit range based on stability of E_0, A_0, E_1, & goodness-of-fit
  - ❖ **Fit between (t_{min}, T-t_{min})** to ensure that fit describes correlator over entire lattice time extent
  - ❖ Employ same t_{min}~0.45 fm for all lattice spacings
  - ❖ Obtain good correlated fits and stable results with $N_{states} \geq 3$

Ground-state energies



Chiral-continuum fit function

$$a_\mu = a_\mu^{\rm LO} \times \left(1 + c_\ell \frac{\delta m_\ell}{\Lambda} + c_s \frac{\delta m_s}{\Lambda} + \tilde{c}_\ell \frac{\delta m_\ell}{m_\ell} + c_{a^2} \frac{(a\Lambda)^2}{\pi^2}\right)$$

$$\delta m_f \equiv m_f - m_f^{\rm phys.}, \qquad \Lambda = 500 \text{ MeV}$$

Constraints

$$a_\mu^{\rm LO} = 600(200) \times 10^{-10}$$

$$c_\ell, c_{a^2} = 0(1), \quad c_s = 0.0(0.3), \quad \tilde{c}_\ell = 0.00(0.03)$$

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- Correct for quark-mass mistuning
- c_s small because only enters through sea, and m_s well tuned

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- Correct for small residual quark-mass dependence after $\pi^+\pi^-$ correction

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- c_s small because on[…] enters through sea, m_s well tuned

- Correct for small […] correction

- Account for residual […]eneric and taste-breaking […]scretization errors

☆ No ml dependent terms with only physical-mass analysis

Constraints

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Isospin-breaking & QED effects

- Data-driven phenomenological analyses & model calculations suggest that contributions to connected u/d contribution to leading-order a_μ^{HVP} are at or below the ~1% level
  - Dominant EM effect from π^0-γ vacuum polarization bubbles estimated from $e^+e^- \to \pi^0\gamma$ data to be $\mathbf{\Delta(a_\mu^{HVP}) = 4.6(2) \times 10^{-10}}$ [Hagiwara et al., PRD69, 093003 (2004)]
  - Dominant isospin-breaking contribution from ρ-ω mixing estimated from $e^+e^- \to \pi^+\pi^-$ data to be $\mathbf{\Delta(a_\mu^{HVP}) \sim 2\text{–}5 \times 10^{-10}}$ [Wolfe & Maltman, PRD83 (2011) 077301]

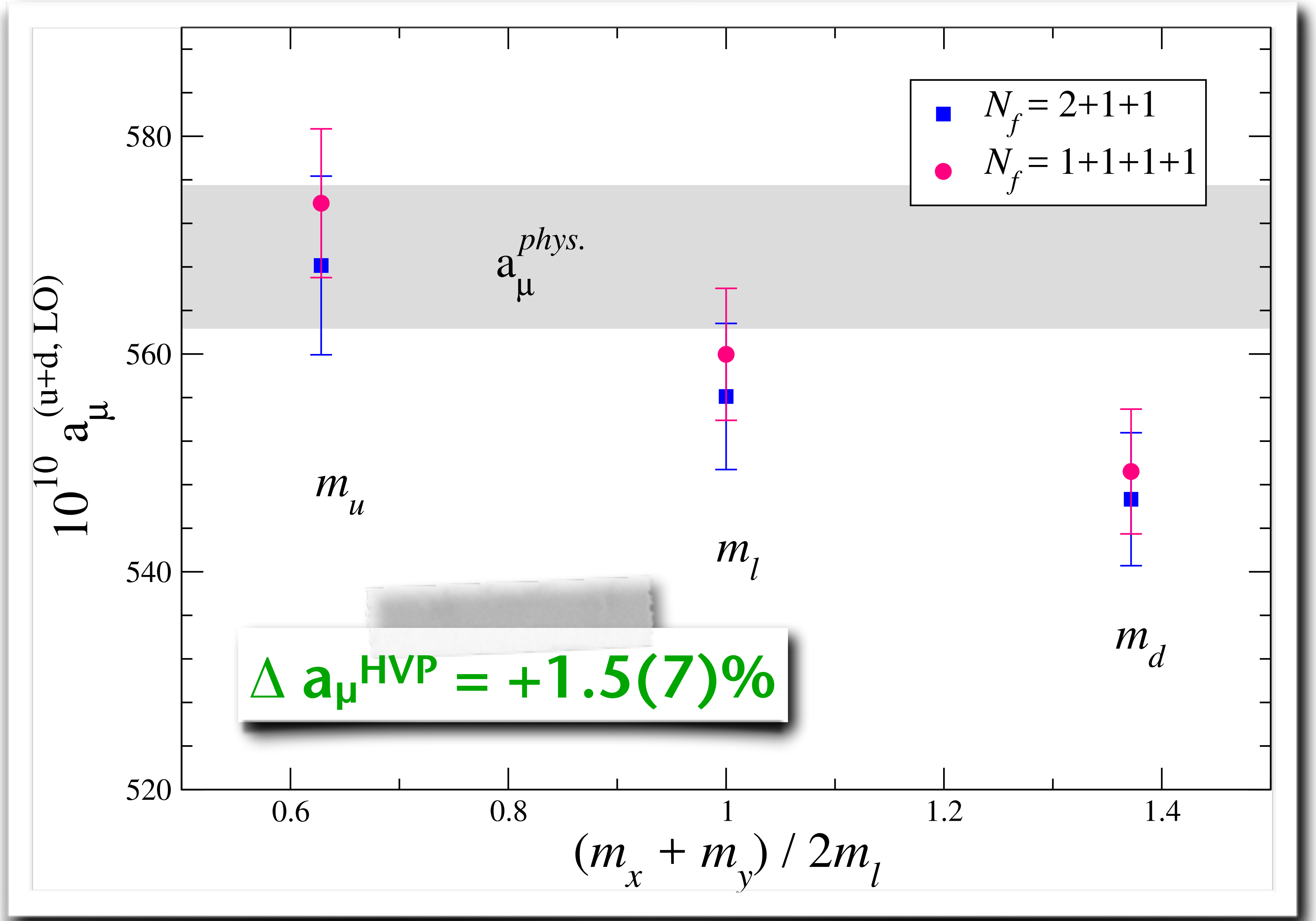
- **Consistent with Jegerlehner compilation** of missing effects in lattice QCD simulations performed in the isospin limit and without QED effects [*private communication*, to appear in "Springer Tracts in Modern Physics"]

Missing effect	$10^{10}\delta a_\mu^{\rm HVP,LO}$
ρ-ω mixing	+2.71
ρ-γ mixing	-2.74
Final-state radiation	+4.22
EM effects on M_π, M_ρ, Γ_ρ	+1.12
$\pi^0\gamma$ channel	+4.64
$\eta\gamma$ channel	+0.16
Total	+10.11

Strong isospin-breaking correction

- Directly calculate correction to a_μ^{HVP} from $m_u \neq m_d$ on a~0.15 fm $N_f = 1+1+1+1$ (and $N_f = 2+1+1$) physical-mass ensemble [PRL120 (2018) no.15, 152001]

    - Fix ratio $m_u / m_d = 0.4582$ to physical value from **MILC quenched-QED analysis** [1606.01228]
    - Compute a_μ with valence-quark masses m_u, $m_l = (m_u + m_d)/2$, & m_d
    - *c.f.* subsequent RBC/UKQCD preprint $a_\mu^{SIB}/a_\mu^{ud,conn.}$ ~1.6% [1801.07224]



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