HVP lattice status report RBC/UKQCD

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Outline

1.

Calculation of the hadronic vacuum polarization contribution to the muon anomalous magnetic moment

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2.

Improved methods for reduced statistical and systematic errors

Time-Moment Representation

Starting from the vector current $J_\mu(x) = i\sum_f Q_f \overline{\Psi}_f(x)\gamma_\mu \Psi_f(x)$ we may write

$$a_\mu^{\mathrm{HVP\ LO}} = \sum_{t=0}^{\infty} w_t C(t)$$

with

$$C(t) = \frac{1}{3}\sum_{\vec{x}}\sum_{j=0,1,2}\langle J_j(\vec{x},t)J_j(0)\rangle$$

and w_t capturing the photon and muon part of the HVP diagrams (Bernecker-Meyer 2011).

The correlator $C(t)$ is computed in lattice QCD+QED at physical pion mass with non-degenerate up and down quark masses including up, down, strange, and charm quark contributions. The missing bottom quark contributions are computed in pQCD.

Diagrams – Isospin limit

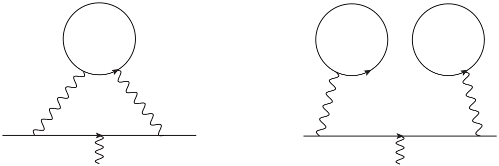


FIG. 1. Quark-connected (left) and quark-disconnected (right) diagram for the calculation of $a_\mu^{\mathrm{HVP\ LO}}$. We do not draw gluons but consider each diagram to represent all orders in QCD.

Diagrams – QED corrections

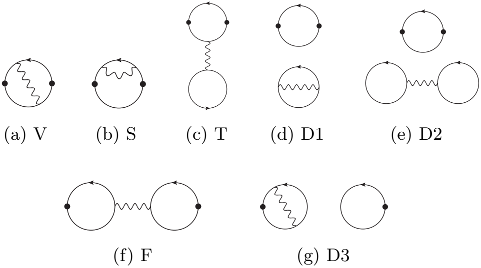


FIG. 2. QED-correction diagrams with external pseudo-scalar or vector operators.

For diagram F we enforce exchange of gluons between the quark loops as otherwise a cut through a single photon line would be possible. This single-photon contribution is counted as part of the HVP NLO and not included for the HVP LO.

Diagrams T, D1, D2, D3 are not included for the central value of the current calculation. They are suppressed by SU(3), $1/N_c$, or both and we estimate their contribution in our uncertainty. Diagrams V, S, and F are included.

Diagrams – Strong isospin breaking

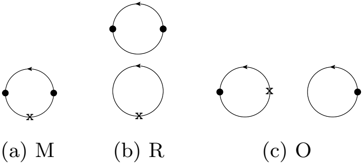


FIG. 3. Strong isospin-breaking correction diagrams. The crosses denote the insertion of a scalar operator.

For the HVP R is negligible since $\Delta m_u \approx -\Delta m_d$ and O is SU(3) and $1/N_c$ suppressed. Therefore we do not include R and O for the current calculation and only estimate their contribution in our uncertainty. The leading diagram M is included.

Regions of precision (R-ratio data here is from **Fred Jegerlehner** 2017)

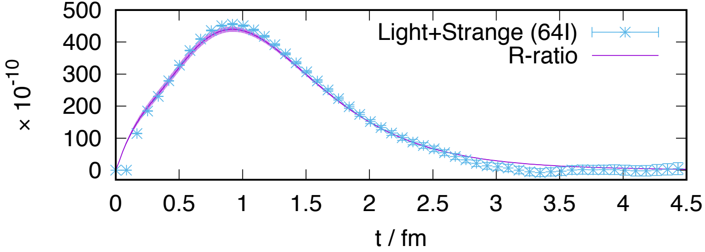


FIG. 4. Comparison of $w_t C(t)$ obtained using R-ratio data [1] and lattice data on our 64I ensemble.

The precision of lattice data deteriorates exponentially as we go to large t, however, is precise at intermediate distances. The R-ratio is very precise at long distances.

Note: in this plot a direct comparison of R-ratio and lattice data is not appropriate. Continuum limit, infinite-volume corrections, charm contributions, and IB corrections are missing from lattice data shown here.

Window method

We therefore also consider a window method. Following Meyer-Bernecker 2011 and smearing over t to define the continuum limit we write

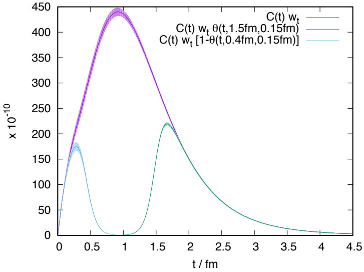
$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

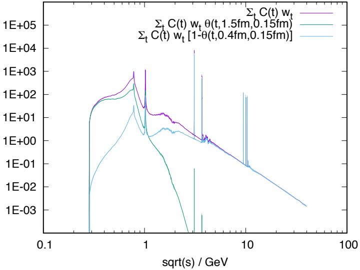
with

$$\begin{aligned} a_\mu^{\mathrm{SD}} &= \sum_t C(t) w_t [1 - \Theta(t, t_0, \Delta)]\,, \\ a_\mu^{\mathrm{W}} &= \sum_t C(t) w_t [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)]\,, \\ a_\mu^{\mathrm{LD}} &= \sum_t C(t) w_t \Theta(t, t_1, \Delta)\,, \\ \Theta(t, t', \Delta) &= [1 + \tanh[(t - t')/\Delta]]/2\,. \end{aligned}$$

In this version of our calculation, we use $C(t) = \frac{1}{12\pi^2}\int_0^\infty d(\sqrt{s}) R(s) s e^{-\sqrt{s}t}$ with $R(s) = \frac{3s}{4\pi\alpha^2}\sigma(s, e^+e^- \to \mathrm{had})$ to compute a_μ^{SD} and a_μ^{LD}.

How does this translate to the time-like region?





Most of $\pi\pi$ peak is captured by window from $t_0 = 0.4$ fm to $t_1 = 1.5$ fm, so replacing this region with lattice data reduces the dependence on BaBar versus KLOE data sets.

Results (**Fred's alphaQED17 results used for window result**)

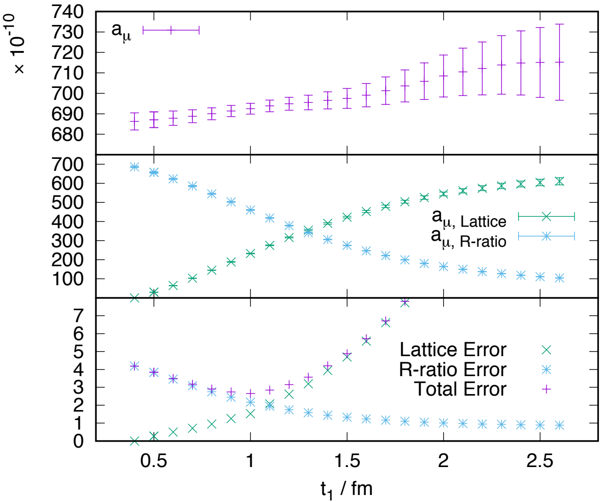
$a_\mu^{\rm ud,\ conn,\ isospin}$	$202.9(1.4)_{\rm S}(0.2)_{\rm C}(0.1)_{\rm V}(0.2)_{\rm A}(0.2)_{\rm Z}$	$649.7(14.2)_{\rm S}(2.8)_{\rm C}(3.7)_{\rm V}(1.5)_{\rm A}(0.4)_{\rm Z}(0.1)_{\rm E48}(0.1)_{\rm E64}$
$a_\mu^{\rm s,\ conn,\ isospin}$	$27.0(0.2)_{\rm S}(0.0)_{\rm C}(0.1)_{\rm A}(0.0)_{\rm Z}$	$53.2(0.4)_{\rm S}(0.0)_{\rm C}(0.3)_{\rm A}(0.0)_{\rm Z}$
$a_\mu^{\rm c,\ conn,\ isospin}$	$3.0(0.0)_{\rm S}(0.1)_{\rm C}(0.0)_{\rm Z}(0.0)_{\rm M}$	$14.3(0.0)_{\rm S}(0.7)_{\rm C}(0.1)_{\rm Z}(0.0)_{\rm M}$
$a_\mu^{\rm uds,\ disc,\ isospin}$	$-1.0(0.1)_{\rm S}(0.0)_{\rm C}(0.0)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}$	$-11.2(3.3)_{\rm S}(0.4)_{\rm V}(2.3)_{\rm L}$
$a_\mu^{\rm QED,\ conn}$	$0.2(0.2)_{\rm S}(0.0)_{\rm C}(0.0)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(0.0)_{\rm E}$	$5.9(5.7)_{\rm S}(0.3)_{\rm C}(1.2)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(1.1)_{\rm E}$
$a_\mu^{\rm QED,\ disc}$	$-0.2(0.1)_{\rm S}(0.0)_{\rm C}(0.0)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(0.0)_{\rm E}$	$-6.9(2.1)_{\rm S}(0.4)_{\rm C}(1.4)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(1.3)_{\rm E}$
$a_\mu^{\rm SIB}$	$0.1(0.2)_{\rm S}(0.0)_{\rm C}(0.2)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(0.0)_{\rm E48}$	$10.6(4.3)_{\rm S}(0.6)_{\rm C}(6.6)_{\rm V}(0.1)_{\rm A}(0.0)_{\rm Z}(1.3)_{\rm E48}$
$a_\mu^{\rm udsc,\ isospin}$	$231.9(1.4)_{\rm S}(0.2)_{\rm C}(0.1)_{\rm V}(0.3)_{\rm A}(0.2)_{\rm Z}(0.0)_{\rm M}$	$705.9(14.6)_{\rm S}(2.9)_{\rm C}(3.7)_{\rm V}(1.8)_{\rm A}(0.4)_{\rm Z}(2.3)_{\rm L}(0.1)_{\rm E48}(0.1)_{\rm E64}(0.0)_{\rm M}$
$a_\mu^{\rm QED,\ SIB}$	$0.1(0.3)_{\rm S}(0.0)_{\rm C}(0.2)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(0.0)_{\rm E}(0.0)_{\rm E48}$	$9.5(7.4)_{\rm S}(0.7)_{\rm C}(6.9)_{\rm V}(0.1)_{\rm A}(0.0)_{\rm Z}(1.7)_{\rm E}(1.3)_{\rm E48}$
$a_\mu^{\rm R-ratio}$	$460.4(0.7)_{\rm RST}(2.1)_{\rm RSY}$	
a_μ	$692.5(1.4)_{\rm S}(0.2)_{\rm C}(0.2)_{\rm V}(0.3)_{\rm A}(0.2)_{\rm Z}(0.0)_{\rm E}(0.0)_{\rm E48}(0.0)_{\rm b}(0.1)_{\rm c}(0.0)_{\overline{\rm S}}(0.0)_{\overline{\rm Q}}(0.0)_{\rm M}(0.7)_{\rm RST}(2.1)_{\rm RSY}$	$715.4(16.3)_{\rm S}(3.0)_{\rm C}(7.8)_{\rm V}(1.9)_{\rm A}(0.4)_{\rm Z}(1.7)_{\rm E}(2.3)_{\rm L}(1.5)_{\rm E48}(0.1)_{\rm E64}(0.3)_{\rm b}(0.2)_{\rm c}(1.1)_{\overline{\rm S}}(0.3)_{\overline{\rm Q}}(0.0)_{\rm M}$

TABLE I. Individual and summed contributions to a_μ multiplied by 10^{10}. The left column lists results for the window method with $t_0 = 0.4$ fm and $t_1 = 1$ fm. The right column shows results for the pure first-principles lattice calculation. The respective uncertainties are defined in the main text.

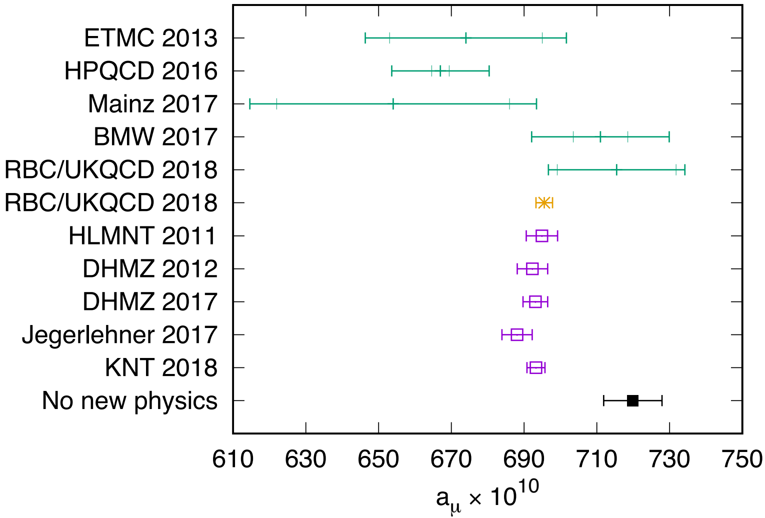
For the pure lattice number the dominant errors are (S) statistics, (V) finite-volume errors, and (C) the continuum limit extrapolation uncertainty. Updates for S,V,C in second part of talk.

For the window method there are additional R-ratio systematic (RSY) and R-ratio statistical (RST) errors.

Window method with fixed $t_0 = 0.4$ fm



For $t = 1$ fm approximately 50% of uncertainty comes from lattice and 50% of uncertainty comes from the R-ratio. Is there a small slope? More in a few slides!
Can use this to check experimental data sets; see my KEK talk for more details



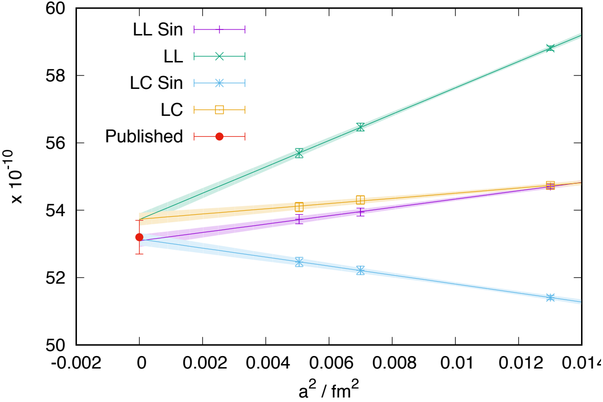
BMW and RBC/UKQCD pure lattice are compatible **both** with no new physics and R-ratio, need more precision!

Consolidate continuum limit

Adding a finer lattice

Add $a^{-1} = 2.77$ GeV lattice spacing

- Third lattice spacing for strange data ($a^{-1} = 2.77$ GeV with $m_\pi = 234$ MeV with sea light-quark mass corrected from global fit):



- For light quark need new ensemble at physical pion mass. Proposed for early science time at Summit Machine at Oak Ridge later this year ($a^{-1} = 2.77$ GeV with $m_\pi = 139$ MeV).

Statistical noise

Improved bounding method

Bounding Method BMW/RBC/UKQCD 2016

Our correlator in finite volume

$$C(t) = \sum_n |\langle 0|V|n\rangle|^2 e^{-E_n t} \, .$$

We can bound this correlator at each t from above and below by the correlators

$$\tilde{C}(t; T, \tilde{E}) = \begin{cases} C(t) & t < T \, , \\ C(T)e^{-(t-T)\tilde{E}} & t \geq T \end{cases}$$

for proper choice of $\tilde{E}$. We can chose $\tilde{E} = E_0$ (assuming $E_0 < E_1 < \ldots$) to create a strict upper bound and any $\tilde{E}$ larger than the local effective mass to define a strict lower bound.

Improved Bounding Method

RBC/UKQCD 2018

Therefore if we had precise knowledge of the lowest $n = 0, \ldots, N$ values of $|\langle 0|V|n\rangle|$ and E_n, we could define a new correlator

$$C^N(t) = C(t) - \sum_{n=0}^{N} |\langle 0|V|n\rangle|^2 e^{-E_n t}$$

which we could bound much more strongly through the larger lowest energy $E_{N+1} \gg E_0$. New method: do a GEVP study of FV spectrum to perform this subtraction.

For more details on how to determine the energies, matrix elements, and the new bounds see discussion contribution tomorrow by Aaron Meyer!

Improved Bounding Method – Update for $a^{-1} = 1.73$ GeV ensemble

Results for light-quark isospin-symmetric connected contribution:

- Original bounding method: $631.4(10.0) \times 10^{-10}$
- Improved bounding method: $625.7(3.9) \times 10^{-10}$
- Lower end of error bars still touch, more statistics under way (factor 2 more in a few weeks)
- For $a^{-1} = 2.359$ GeV ensemble, will start generating data for this method on July 1st, for $a^{-1} = 2.77$ GeV we are aiming at this fall.

Finite-volume errors

Beyond finite-volume scalar QED

Compute finite-volume effects from first-principles

Study QCD at physical pion mass at three different volumes:

$L = 4.66$ fm, $L = 5.47$ fm (published data), $L = 6.22$ fm

Results for light-quark isospin-symmetric connected contribution:

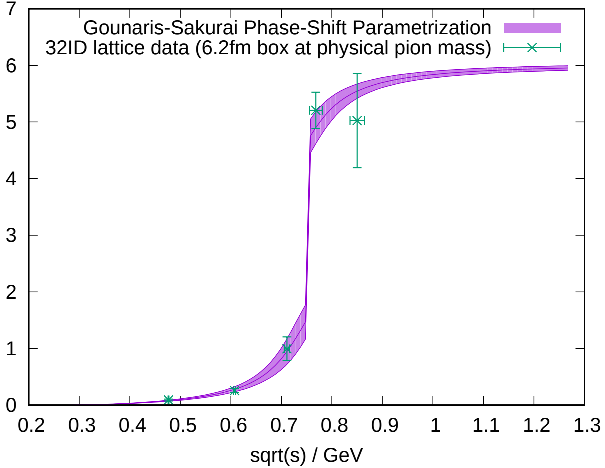
- $a_\mu(L = 6.22 \text{ fm}) - a_\mu(L = 4.66 \text{ fm}) = 12.2 \times 10^{-10}$ (sQED), $21.6(6.3) \times 10^{-10}$ (lattice QCD)
- Improved bounding method crucial for reduced statistical noise to resolve the FV effect clearly
- First time this is resolved from zero in a first-principles calculation at physical pion mass (previously bound in E. Shintani 2018)
- Need to do better than sQED in finite-volume

Gounaris-Sakurai-Lüscher method [H. Meyer 2012, Mainz 2017]

- Produce FV spectrum and matrix elements from phase-shift study (Lüscher method for spectrum and amplitudes, GS for phase-shift parametrization)

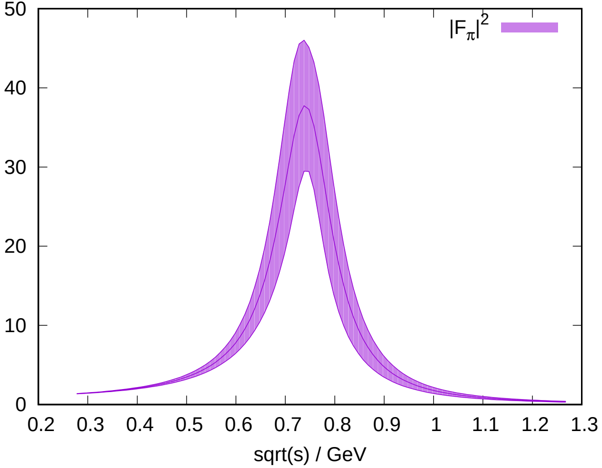
- This allows for a prediction of FV effects beyond chiral perturbation theory given that the phase-shift parametrization captures all relevant effects (can be checked against lattice data)

First constrain the p-wave phase shift from our $L = 6.22$ fm physical pion mass lattice:



$E_\rho = 0.766(21)$ GeV (PDG 0.77549(34) GeV)
$\Gamma_\rho = 0.139(18)$ GeV (PDG 0.1462(7) GeV)

Predicts $|F_\pi(s)|^2$:



We can then also predict matrix elements and energies for our other lattices; successfully checked!

GSL finite-volume results compared to sQED and lattice

Results for light-quark isospin-symmetric connected contribution:

- FV difference between $a_\mu(L = 6.22 \text{ fm}) - a_\mu(L = 4.66 \text{ fm}) = 12.2 \times 10^{-10}$ (sQED), $21.6(6.3) \times 10^{-10}$ (lattice QCD), $20(3) \times 10^{-10}$ (GSL)
- GSL prediction agrees with actual FV effect measured on the lattice, sQED is in slight tension, two-loop FV ChPT to be compared next Bijnens and Relefors 2017
- Use GSL to update FV correction of arXiv:1801.07224: $a_\mu(L \to \infty) - a_\mu(L = 5.47 \text{ fm}) = 16(4) \times 10^{-10}$ (sQED), $22(1) \times 10^{-10}$ (GSL); sQED error estimate based on Bijnens and Relefors 2017, table 1.

Outlook for errors

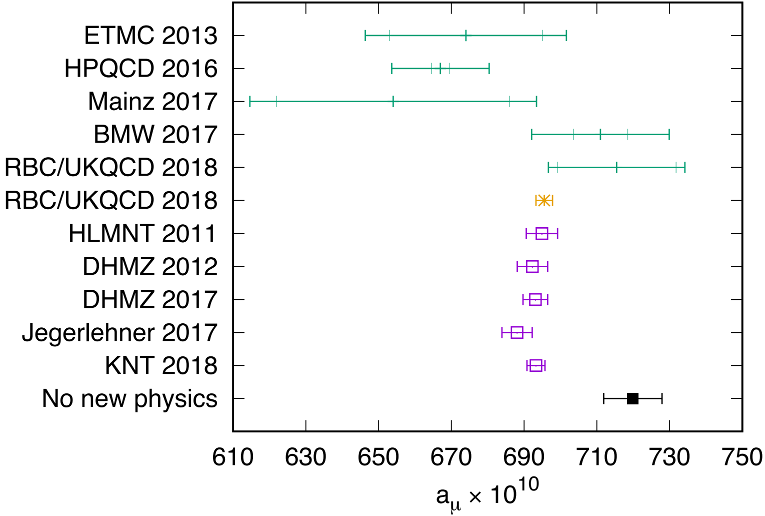
In the next 6 months we can expect:

- Statistical error reduction from improved bounding method by factor of 3 ($16.3 \times 10^{-10} \to 5 \times 10^{-10}$)
- Better control of finite-volume correction (target error of smaller than 2.5×10^{-10})
- Consolidate continuum limit (no error reduction at this point expected but more confidence)

These improvements would reduce our current error from $18.7 \times 10^{-10} \to 7.5 \times 10^{-10}$; may be able to distinguish "no new physics" and "R-ratio" scenarios.

Further work in progress on similar time-scale:

- Much better statistics for diagram M (strong-isospin breaking)
- Use HLbL data to improve QED precision, compute sub-leading diagrams M. Bruno
- Update for our 2015 disconnected diagrams result with more statistics



Error bar of RBC/UKQCD 2018 pure lattice result may be halved by end of year.

Conclusions

- We now have a lattice calculation at physical pion mass with QCD+QED and non-degenerate light quark masses.
- We have results both for a pure lattice and a combined lattice/R-ratio analysis. This can cut out a significant fraction of $\pi\pi$ data sets from R-ratio (BaBar/KLOE).
- We have a new method to tame statistical noise (Improved Bounding Method combined with GEVP study, see A. Meyer discussion tomorrow)
- We have for the first time resolved from first-principles QED the finite-volume effects on two boxes and cross-checked against sQED and GSL.
- We have a third lattice spacing for the strange quark contribution and by the end of the year hopefully also for the light quarks
- Possible that by end of the year the pure lattice result can distinguish between "no new physics" and "R-ratio" scenarios
- Similarly by end of the year we may have resolution on $\pi\pi$ BaBar/KLOE difference

Backup

We perform the calculation as a perturbation around an isospin-symmetric lattice QCD computation with two degenerate light quarks with mass $m_{\rm light}$ and a heavy quark with mass $m_{\rm heavy}$ tuned to produce a pion mass of 135.0 MeV and a kaon mass of 495.7 MeV.

The correlator is expanded in the fine-structure constant α as well as $\Delta m_{\rm up,\ down} = m_{\rm up,\ down} - m_{\rm light}$, and $\Delta m_{\rm strange} = m_{\rm strange} - m_{\rm heavy}$. We write

$$C(t) = C^{(0)}(t) + \alpha C^{(1)}_{\rm QED}(t) + \sum_f \Delta m_f C^{(1)}_{\Delta \rm m_f}(t) + \mathcal{O}(\alpha^2, \alpha\Delta m, \Delta m^2)\,.$$

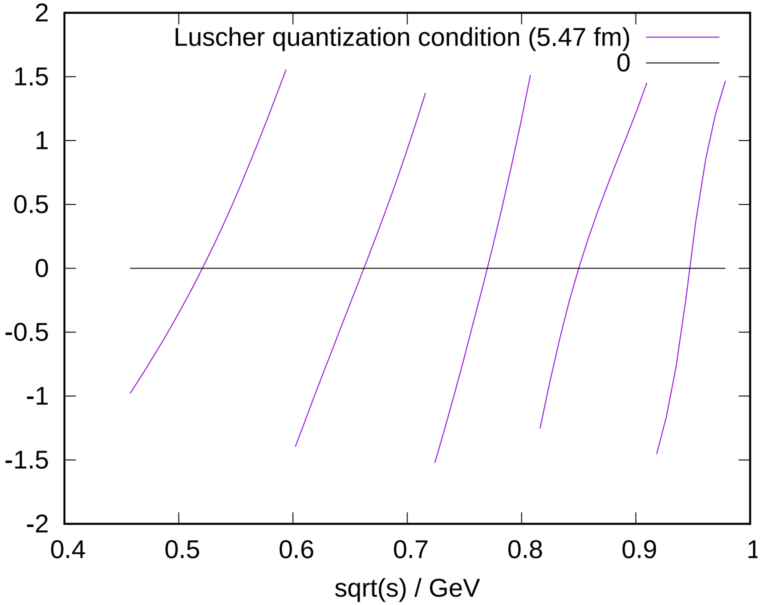
The correlators of this expansion are computed in lattice QCD with dynamical up, down, and strange quarks. We compute the missing contributions to a_μ from charm sea quarks in perturbative QCD (RHAD) by integrating the time-like region above 2 GeV and find them to be smaller than 0.3×10^{-10}.

We tune the bare up, down, and strange quark masses m_{up}, m_{down}, and m_{strange} such that the π^0, π^+, K^0, and K^+ meson masses computed in our calculation agree with the respective experimental measurements. The lattice spacing is determined by setting the Ω^- mass to its experimental value.

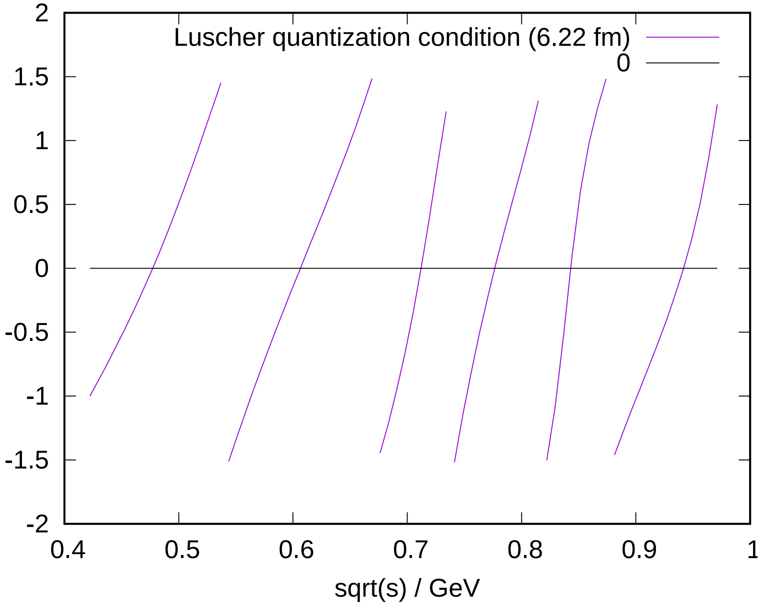
We perform the lattice calculations for the light quark contributions using RBC/UKQCD's 48I and 64I lattice configurations with lattice cutoffs $a^{-1} = 1.730(4)$ GeV and $a^{-1} = 2.359(7)$ GeV and a larger set of ensembles with up to $a^{-1} = 2.774(10)$ GeV for the charm contribution.

From the parameter tuning procedure on the 48I we find $\Delta m_{\mathrm{up}} = -0.00050(1)$, $\Delta m_{\mathrm{down}} = 0.00050(1)$, and $\Delta m_{\mathrm{strange}} = -0.0002(2)$.

The shift of the Ω^- mass due to the QED correction is significantly smaller than the lattice spacing uncertainty and its effect on $C(t)$ is therefore not included separately.



Luscher quantization condition (5.47 fm)
0
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0.4
0.5
0.6
0.7
0.8
0.9
1
sqrt(s) / GeV


Luscher quantization condition (6.22 fm)
0
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0.4
0.5
0.6
0.7
0.8
0.9
1
sqrt(s) / GeV