HVP lattice long-distance contributions and sQED vs. GEVP

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Second Workshop of the Muon $g - 2$ Initiative
Mainz, 18–22 June 2018

Motivation

Time-momentum representation [Meyer and Bernecker, 2011]

$$a_\mu^{\mathrm{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \mathrm{d}t\, \widetilde{K}(m_\mu, t) G(t)$$

with a known kernel $\widetilde{K}$, and $G(t) = \sum_{\boldsymbol{x}} \langle j_3(t, \boldsymbol{x}) j_3(0) \rangle$ the vector correlator.

- Integration runs to infinity, lattice data do not
- Noise rapidly increases at large times
  - particularly at small pion mass and large volume
- Some modelling of long-distance behaviour is required

Large-time modelling

Large-time modelling of the vector correlator:

- Extract naive ground-state mass from fit to a (smeared) correlator, model $G(t)$ as single exponential at large t;
- Perform multi-exponential fits to (smeared and/or local) correlator(s), model $G(t)$ by fitted correlator at large t;
- Use the large-time approximation

$$G_{\infty}^{\rho\rho}(t) = \frac{1}{48\pi^2} \int_0^{\infty} \mathrm{d}\omega\, \omega^2 \left(1 - \frac{4m_\pi^2}{\omega^2}\right)^{\frac{3}{2}} |F_\pi(\omega)|^2 \mathrm{e}^{-\omega t}$$

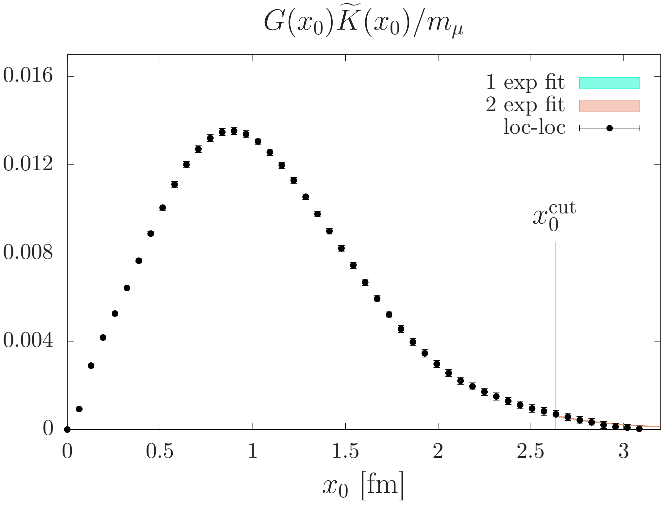
  by either

  - modelling F_π using the Gounaris-Sakurai parameterization with a mass and width extracted semi-naively from smeared and local correlators [Mainz $N_f = 2$, 2017] , or
  - extracting F_π using the Lüscher method [B. Hörz et al.]

  This also allows for correcting finite-volume effects

  → **talk/discussion by D. Giusti**

Modelling the integrand ($m_\pi = 280$) MeV



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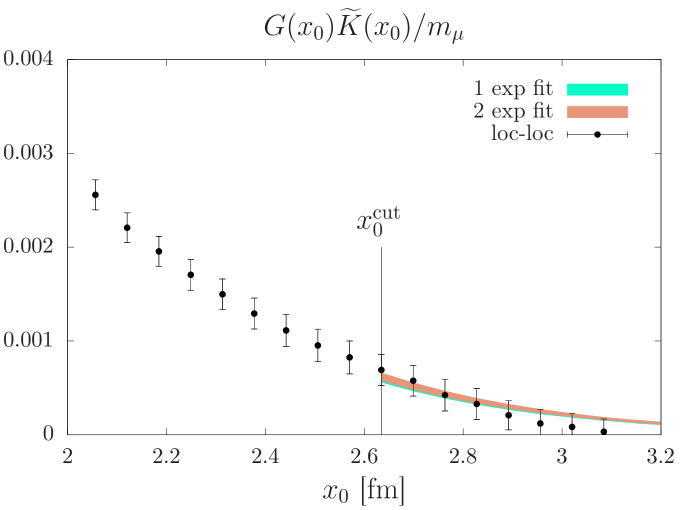
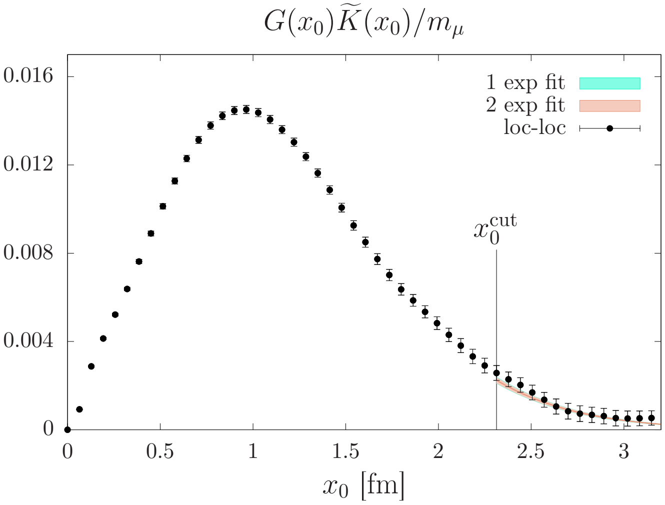


Modelling the integrand ($m_\pi = 200$) MeV



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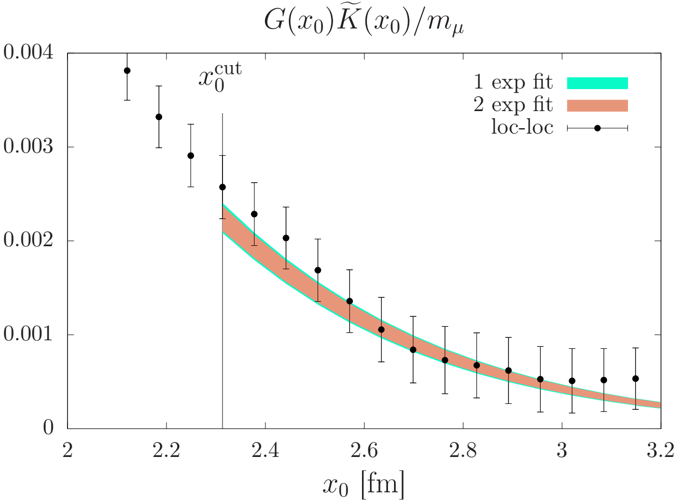
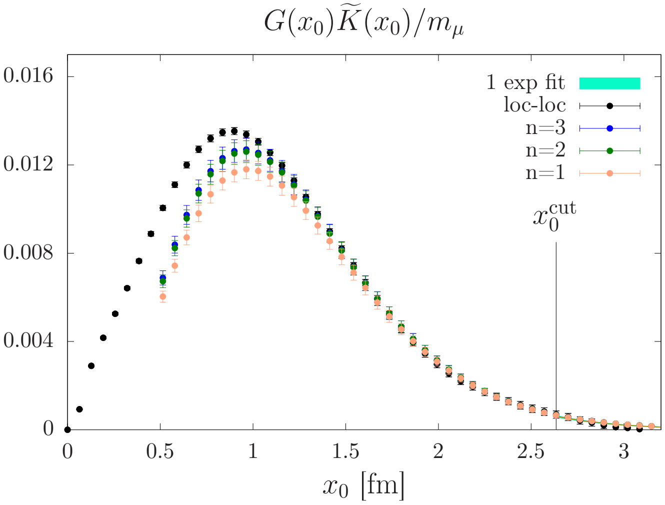


Reconstructing the integrand ($m_\pi = 280$) MeV



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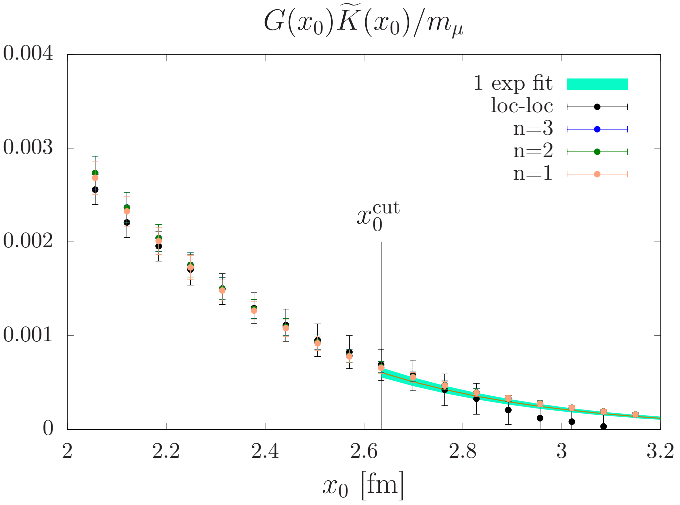

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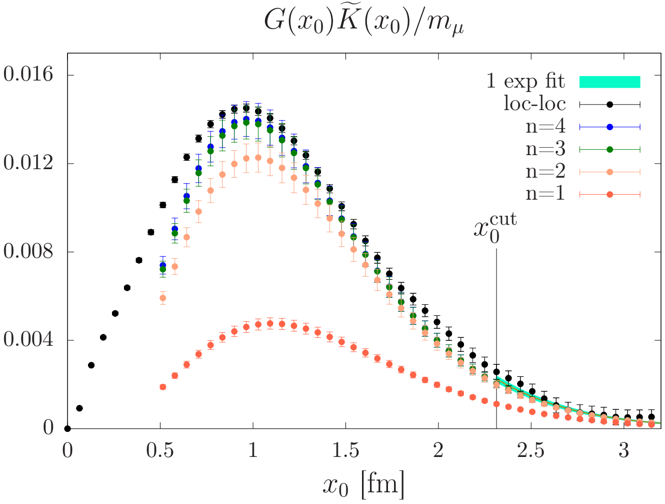

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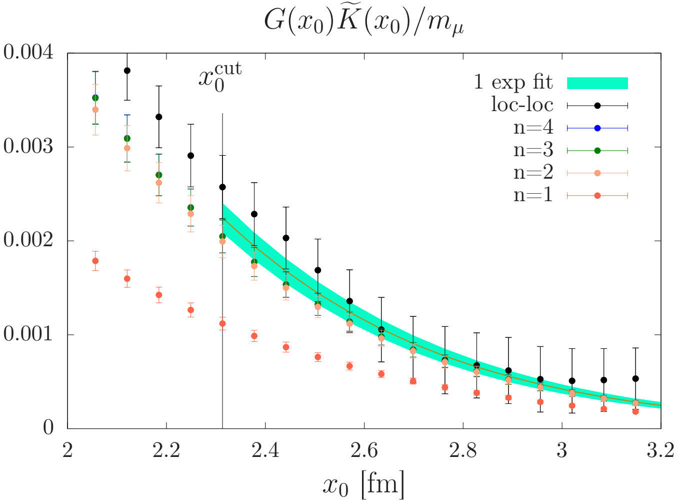
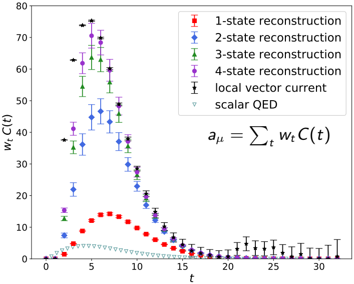


Correlation Function Reconstruction



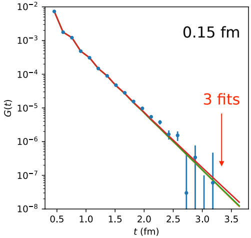
Correlated fit to reconstruct long-distance behavior of local vector current correlation function $C(t)$

More states $\implies$ better reconstruction

Scalar QED underestimates lowest state ($\sim \pi\pi$)

→ **talk by A. Meyer in the discussion session**

Equivalent Fits (χ^2/dof ≤ 1)



- Fit unable to resolve difference between one or multiple rho mesons; spreads contribution over multiple terms.
- Negligible difference for t<4fm, and irrelevant for a_μ.

	$E_0 \backslash a_0$	$E_1 \backslash a_1$	$E_2 \backslash a_2$	$E_3 \backslash a_3$	$a_\mu \times 10^{10}$
Fit 1	0.770 \ 0				602(7)
Fit 2	0.75 \ 0				602(7)
Fit 3	0.71 \ 0				608(8)

☆ **Take-away:** can parameterize data over a finite t range (<3-4fm) by a fit with a single ρ

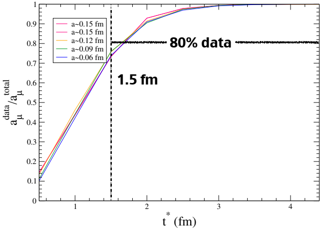
[R. Van de Water]

FNAL/MILC/HPQCD

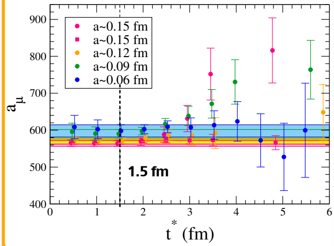
Selection of t*

- For **t* = 1.5 fm**, a_μ^{HVP} comes primarily from data region, and errors still controlled

Data contribution



Stability



R. Van de Water HVP contribution to muon g-2 with (2+1+1) HISQ quarks 14

[R. Van de Water]

Some observations

- Multi-exponential fits may not be able to properly resolve low-lying states
- Scalar QED seems not to be a good model for the two-pion contribution
- At small pion mass, the ground state ($\sim \pi\pi$) is weakly coupled to the vector correlator and only dominates at $t \gtrsim 3\ \mathrm{fm}$
- A dedicated spectroscopic study (with multi-particle states) appears to be a requirement in order to fully understand the large-distance behaviour

The End = The Beginning

The floor is open for discussion . . .