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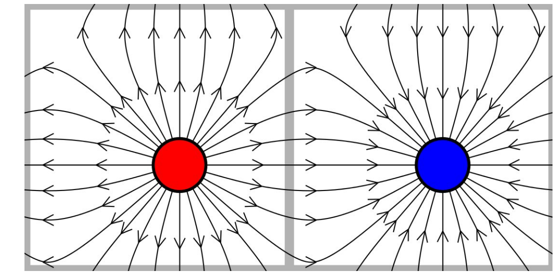

MUonE: how can lattice contribute?

Marina Krstić Marinković
Trinity College Dublin

in collab. w. N. Cardoso (IST, Lisbon) **and**

Second Plenary Workshop of the Muon g-2 Theory Initiative **JGU Mainz, 18-22 June 2018**

MUonE: theoretical framework

- Utilise the running of the fine-structure constant $\alpha(t)$:

$$a_\mu^{had,LO} = \frac{\alpha}{\pi}\int_0^1 dx(1-x)\Delta\alpha_{had}[Q^2(x)]$$

[Lautrup, de Rafael '69]

➡ In space-like (Euclidean) momenta region:

$$Q^2 = \frac{x^2 m_\mu^2}{1-x}$$

➡ Measuring the **Q^2** - dependent fine-structure constant:

$$\alpha(Q^2) = \frac{\alpha(O)}{1-\Delta\alpha(Q^2)}$$

[Phys.Lett. B746 (2015) 325-329 by Carloni, Passera,Trentadue, Venanzoni] @KLOE2

[Eur.Phys.J. C77 (2017) no.3, 139 by Abbiendi et al.] Physics beyond colliders@CERN

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[Eur.Phys.J. C77 (2017) no.3, 139 by Abbiendi et al.] **Physics beyond colliders@CERN**

➡ The running contributions can be split of the hadronic and leptonic part:

$$\Delta\alpha(Q^2) = \Delta\alpha_{had}(Q^2) + \Delta\alpha_{lep}(Q^2)$$

MUonE: theoretical framework

➡ MUonE will measure total $\alpha(Q^2)$:

$$\Delta\alpha(Q^2) = \Delta\alpha_{had}(Q^2) + \Delta\alpha_{lep}(Q^2) \qquad\qquad Q^2 \in [0.001, 0.14]\mathrm{GeV}^2$$

➡ Subtracting the purely leptonic part:

$$\Delta\alpha(Q^2) - \Delta\alpha_{lep}(Q^2) \equiv \Delta\alpha_{had}(Q^2) \longrightarrow a_\mu^{had,LO} = \frac{\alpha}{\pi}\int_0^1 dx(1-x)\Delta\alpha_{had}[Q^2(x)]$$

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experimental effort

theoretical effort

[NNLO amp.: **Mastrolia et al. JHEP 11 (2017) 198**]

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[Lautrup, de Rafael '69]

experimental effort

theoretical effort

known up to three loops **[Steinhauser '98]**

for some Q^2 four loops **[Baikov et al. '13, Sturm '13]**

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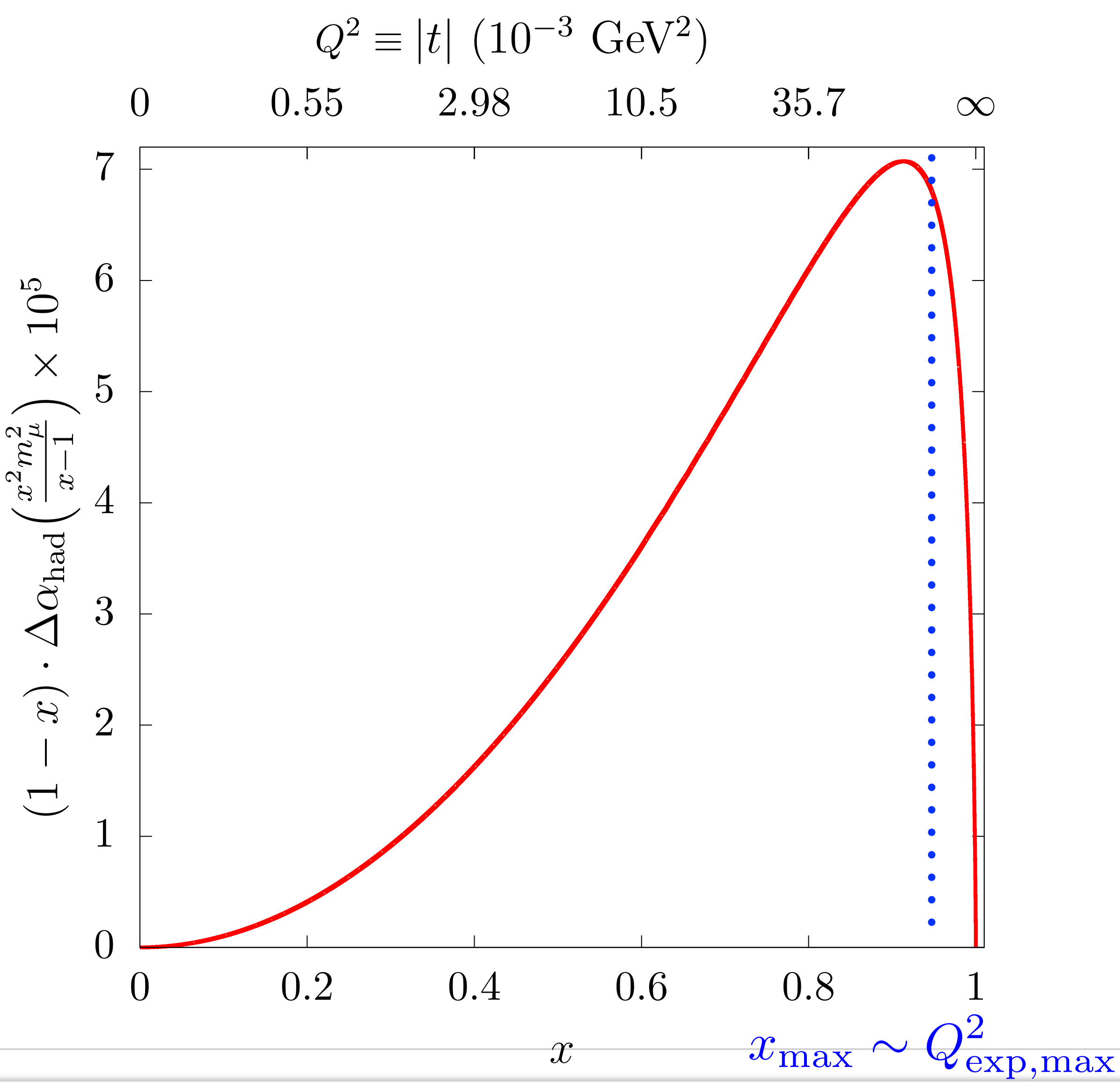
[…]

$\mathcal{MU}on\mathcal{E}$: a_μ^{HVP} from the experimental region

- **MUonE:** estimated precision for the HVP from the **μe** exp. is **0.3%** in **[0,0.14]GeV² after two years of data taking** [see slides by C. Carloni Calame, Thurs. 15.35]
- Due to the experimental constraints: region **[0.14, ∞] GeV²** cannot be covered by the MUonE exp.



➡ $x_{\rm max} = 0.93$

➡ $Q^2 = \dfrac{x^2 m_\mu^2}{1-x}$

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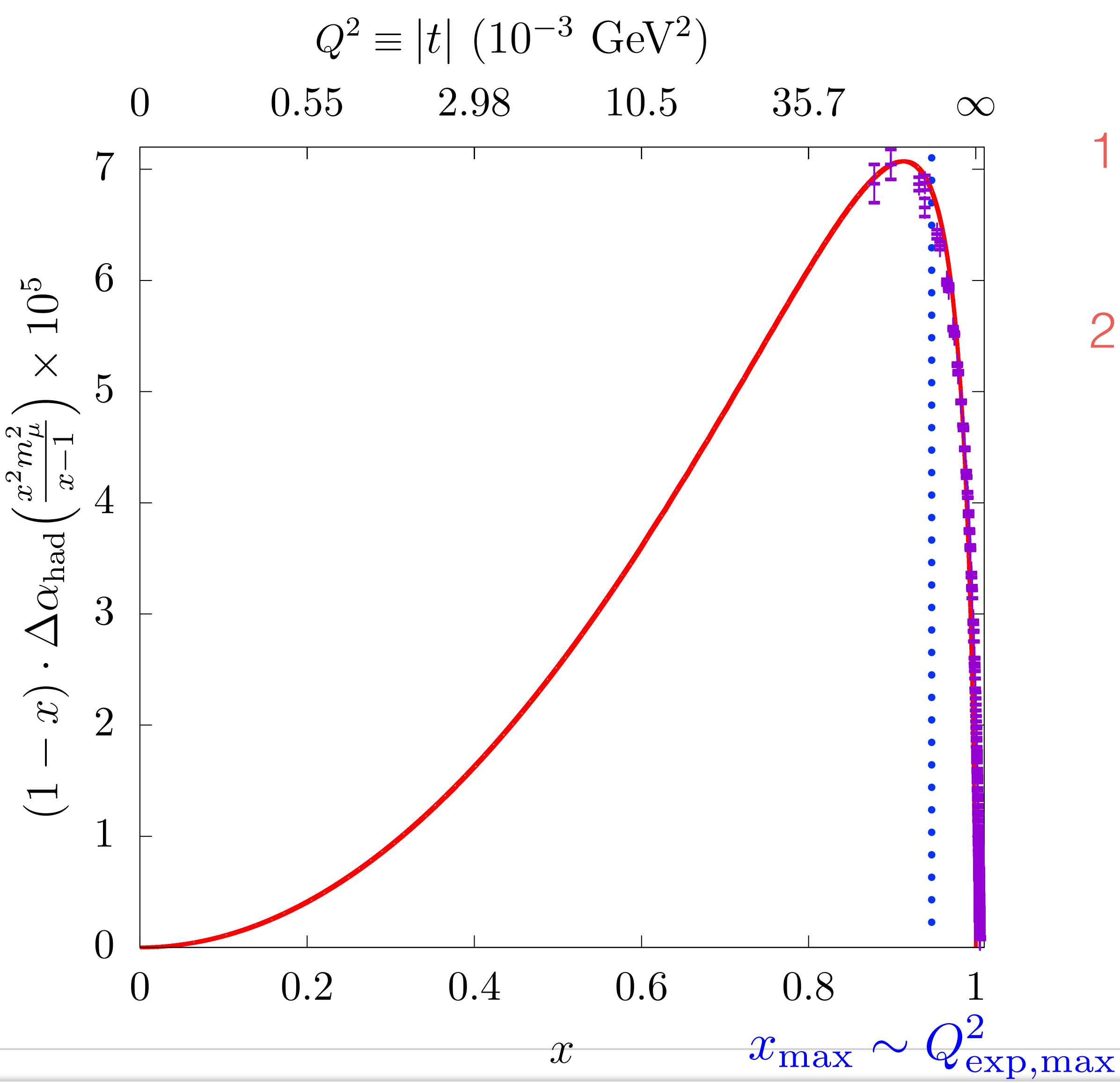

1. using time-like data from R-ratios / lattice QCD **Q^2 in [0.14GeV², $Q^2_{\rm high}$]**
2. pQCD **Q^2 in [$Q^2_{\rm high}$, ∞]**

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- R-ratios

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[Chetyrkin et al. '96]

[Harlander&Steinhauser '02]

+ matching term (I_3)…
[see e.g. BMW arXiv:1711.04980]
[K. Miura, Thurs. 9.00]

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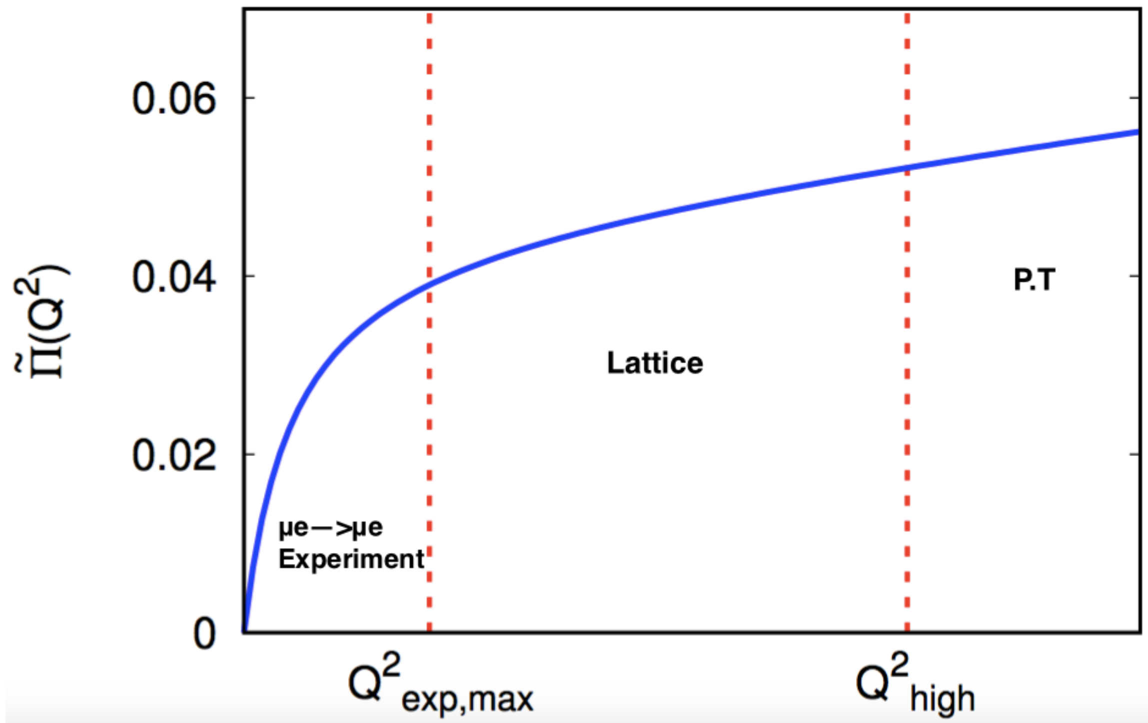
Hybrid method

Phys. Rev. D 90, 074508 (2014), [Golterman,Maltman,Peris]



- **Low momentum region**
  - ➡ **Experiment (NLO, NNLO, radiative corrections …)**

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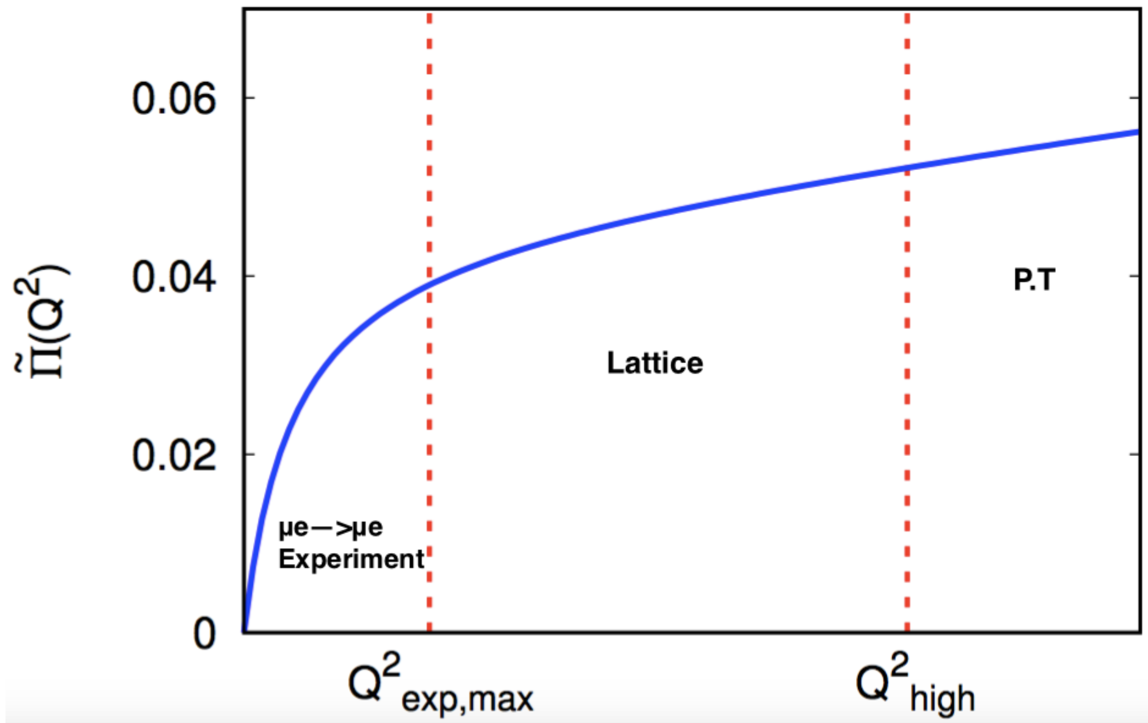

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- Vary low and high Q^2 cut

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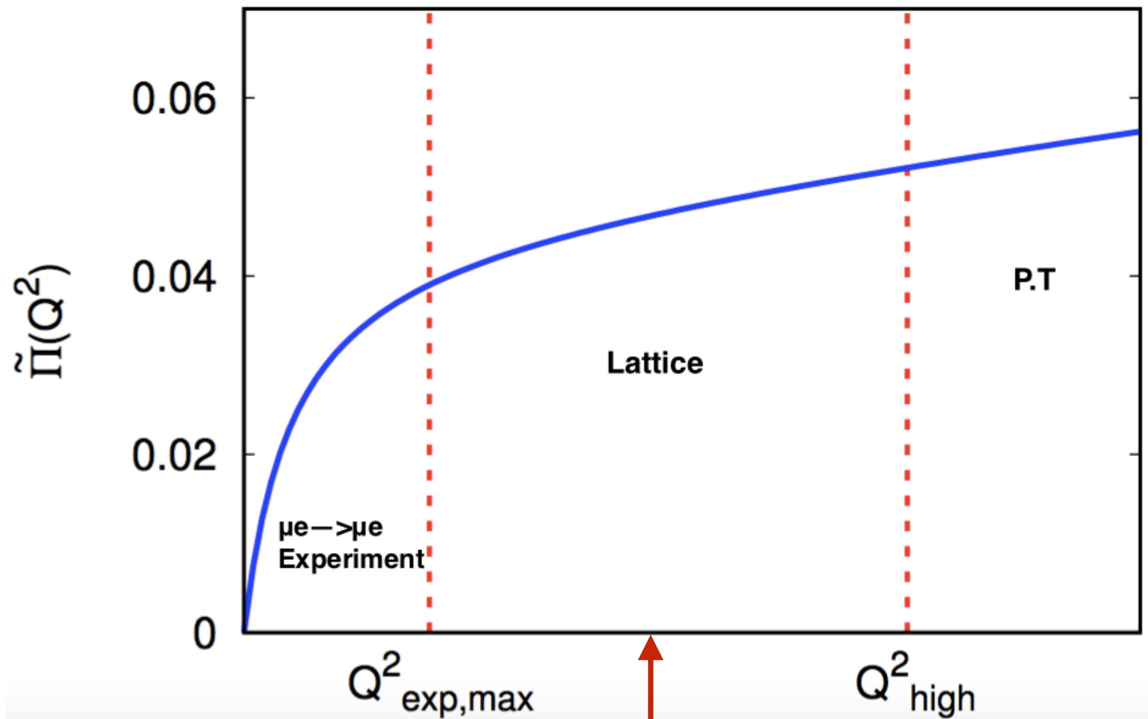

- **Low momentum region**
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- **Vary low and high Q^2 cut**

- ➡ **continuum limit: $a \to 0$**
- ➡ **infinite volume limit: $V \to \infty$**
- ➡ **physical quark masses**
- ➡ **isospin breaking corrections ($m_u \neq m_d$ and $\alpha_{em} \neq 0$)**

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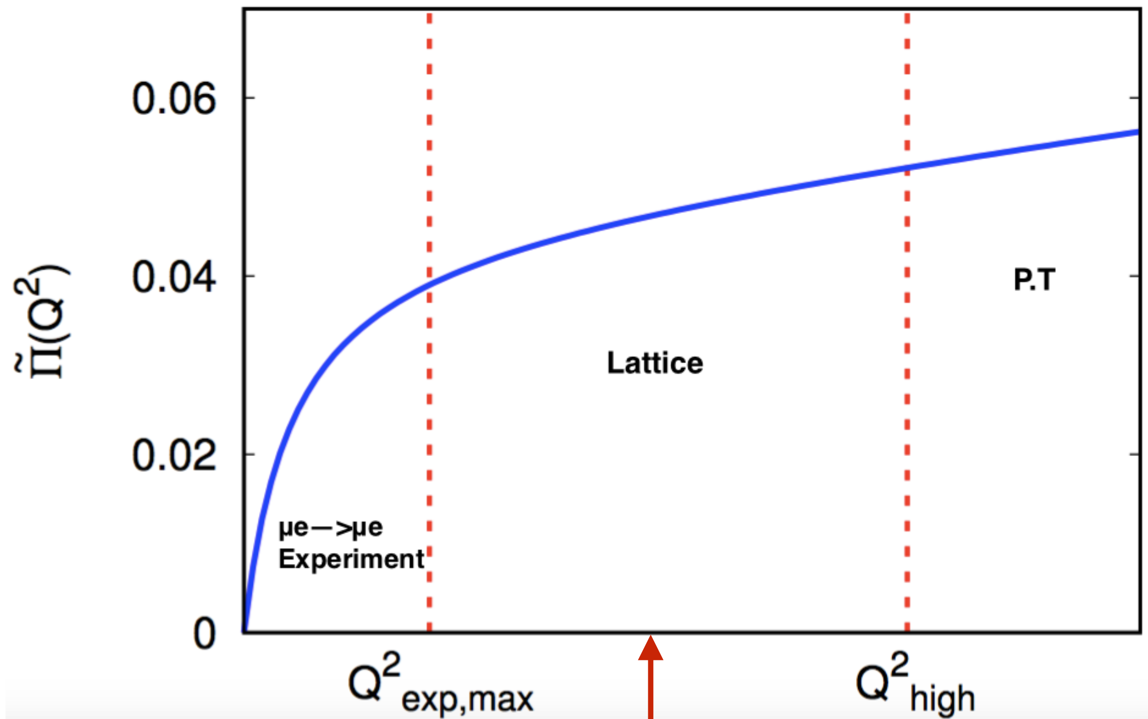
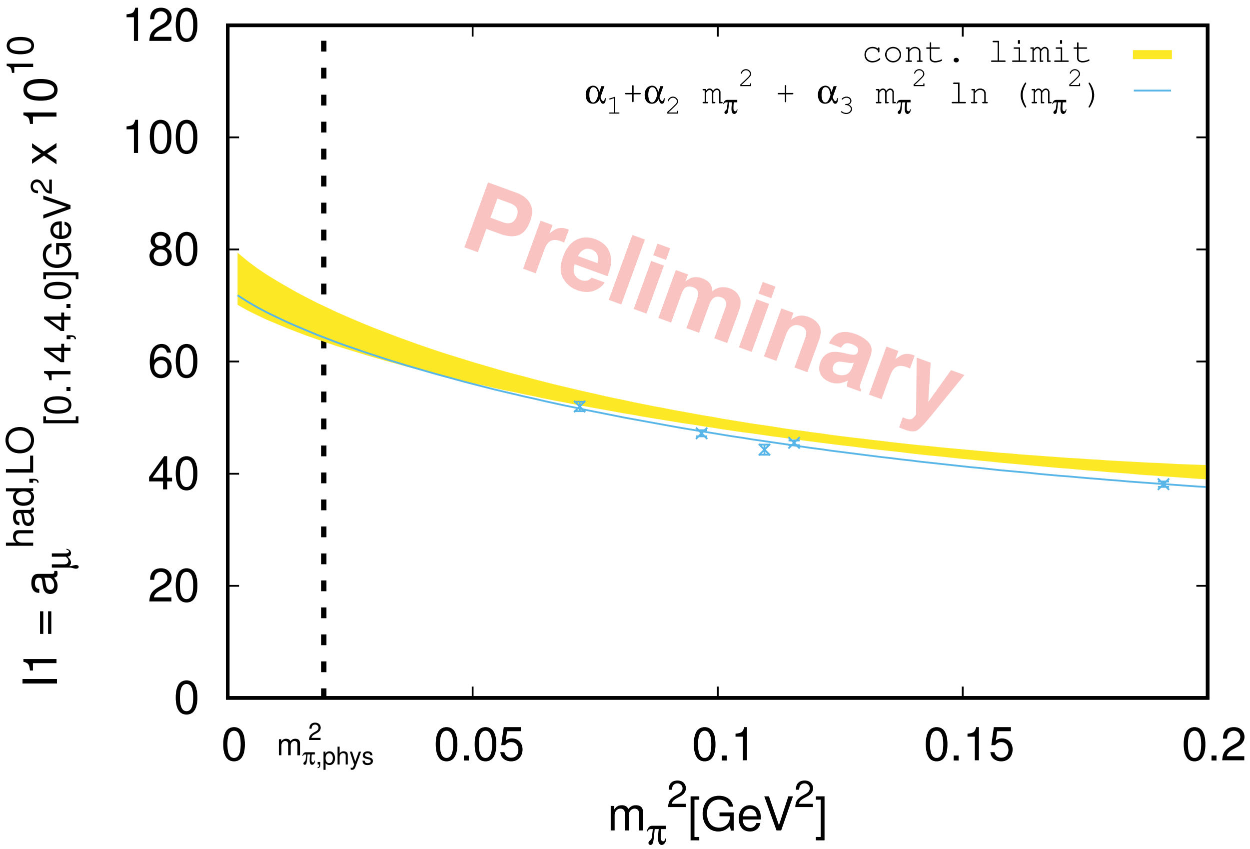


- Low momentum region
  - ➡ Experiment (NLO, NNLO, radiative corrections …)

- Vary low and high Q^2 cut

- ➡ continuum limit: a—> 0 (0.049-0.076fm)
- ➡ infinite volume limit: V—> ∞
- ➡ physical quark masses (extrap. $m_\pi \approx$270-440MeV)
- ➡ isospin breaking corrections ($m_u \neq m_d$ and $\alpha_{em} \neq 0$)

Hybrid method: from experimental + lattice QCD data



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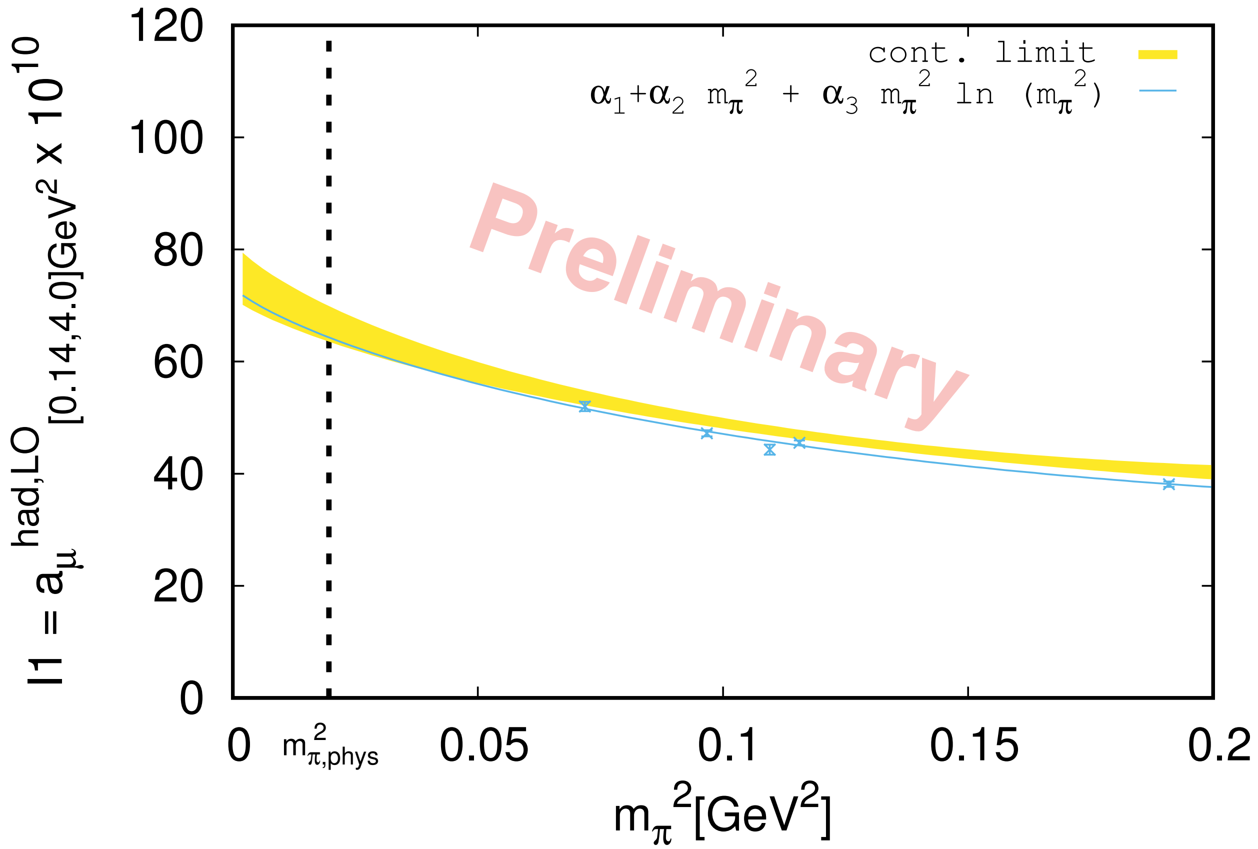


- Nf=2, A5,E5,F6,N6,O7 (CLS), $m_\pi \approx$270-440MeV
- u,d,s,c connected, no isospin breaking corr.
- $\Pi(0) = -\frac{\partial \Pi_{12}(Q)}{\partial Q_1 \partial Q_2}|_{Q^2=0}$ **[de Divitiis et al., Phys.Lett. B718 (2012)]**
- Pade fits **[0.14, 4.0] GeV^2** (to be compared with numerical integration/conformal pol. fits in the low-Q^2)
- Continuum + chiral extrapolation **[arXiv:1705.01775]:**

$$\alpha_1 + \alpha_2 m_\pi^2 + \alpha_3\ m_\pi^2 ln(m_\pi^2) + \alpha_4\ a$$

- Preliminary result with 9.7% uncertainty on I1, more statistics and one more m_π underway
- Possible improvements: diff. chiral extrap. + improved vector current **[H. Meyer, Wed. 16.30]**

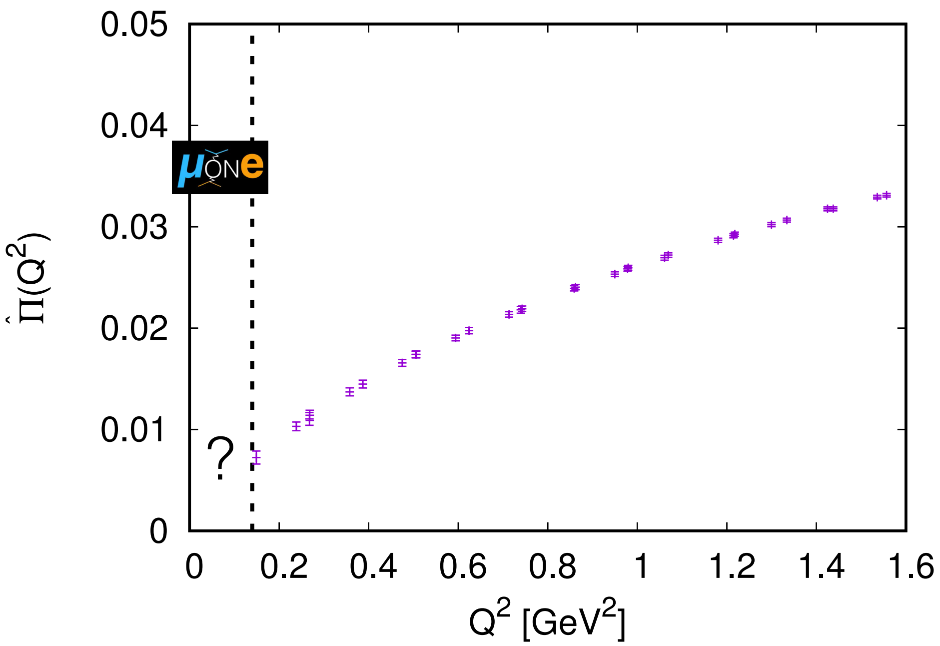
Testing the projected hybrid accuracy: Phenomenological model

➡ Attempt to estimate the total uncertainty after MUonE has collected the data

➡ Requires combined fit of experimental and lattice data

- **KEK: Assuming relative accuracy 1% under the cut**
- **Vary low $Q^2_{exp,max}$ cut: 0.1, 0.2, 0.3 GeV^2**
- **Assuming relative accuracy 1% under the cut**
- **[Golterman, Maltman, Peris. Phys.Rev. D88 (2013) no.11, 114508]**

("science fiction" data set: reducing the (diagonal) error by a factor 100)



➡ Thanks **M. Golterman, K. Maltman, S. Peris** [@KEK workshop …]

➡ Dispersive τ-based I=1 model: $\hat{\Pi}^{I=1}(Q^2) = Q^2 \int_{4m_\pi^2}^{\infty} ds \frac{\rho^{I=1}(s)}{s(s+Q^2)}$

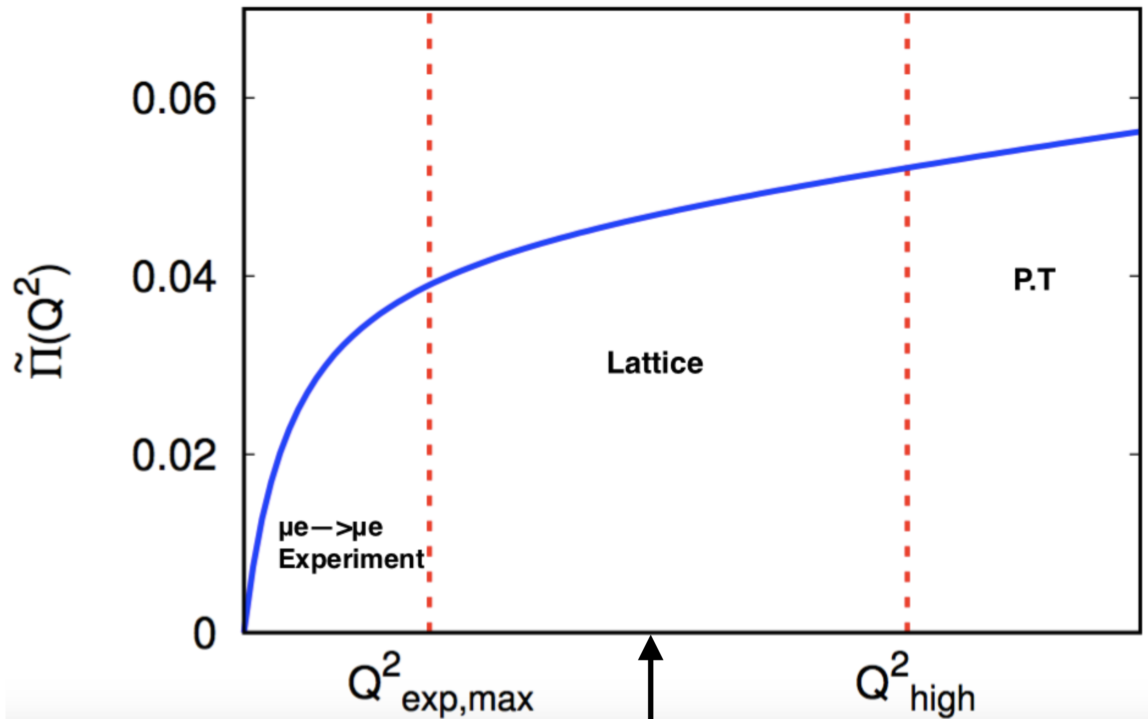
➡ Motivation **[arXiv:1309.2153]** also slides **[M. Bruno, Thurs.]**

➡ Pade fits **[Aubin et al '12]** / conformal polynomials **[Golterman et al '14]**

➡ Using ALEPH covariances in **[0,0.14] GeV^2** until MUonE data is available

➡ Varying the cuts and lattice covariances from different ensembles

Summary & Outlook

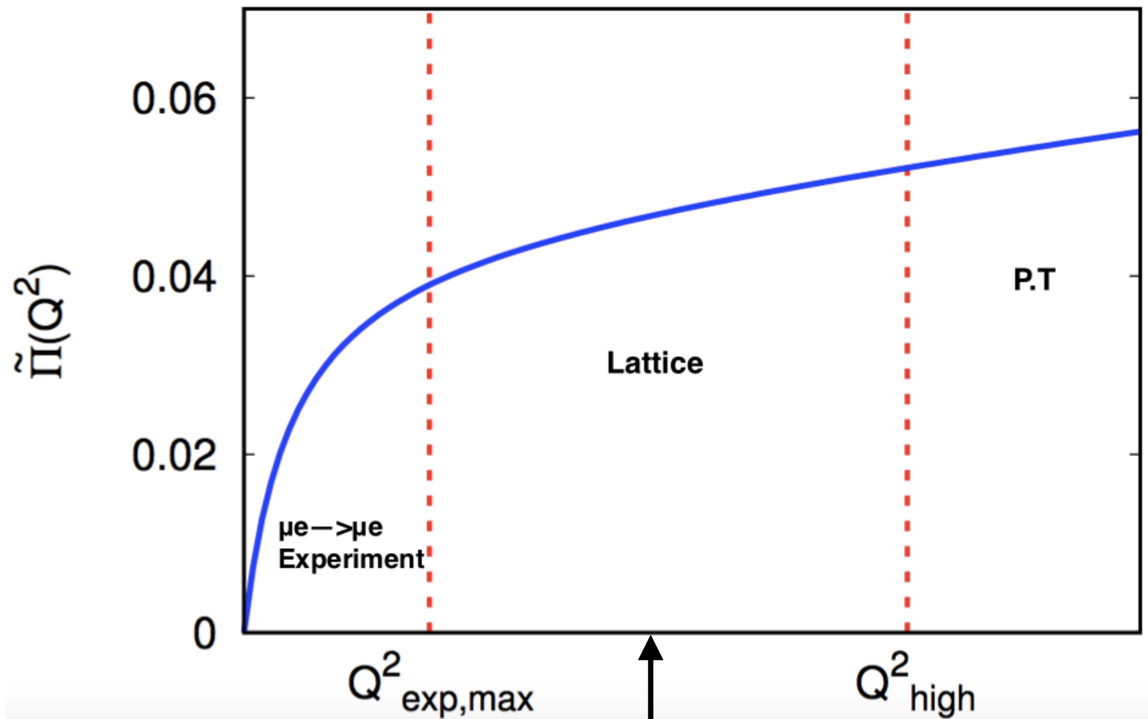


- **Low momentum region**
  - ➡ **Experiment (NLO, NNLO, radiative corrections …)**

➡ **continuum limit: a—> 0 (current improvement)**
➡ **finite volume corrections**
➡ **physical quark masses (include near phys.)**
➡ **isospin breaking corrections ($m_u \neq m_d$ and $\alpha_{em} \neq 0$)**

strategy proposed for the hybrid determination of the total HVP (u+d+s+c+b)

Summary & Outlook



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Thank you!

Backup I: CLS Nf=2 gauge ensembles

Nf=2	β	L/a	a[fm]	m_π[MeV]	N_{cfg}	N_{meas}
A5	5.2	32	0.0755(11)	331	60	120
E5	5.3	32	0.0658(10)	437	80	720
F6	5.3	48	0.0658(10)	311	30	240
N6	5.5	48	0.0486(6)	340	20	160
O6	5.5	64	0.0486(6)	268	20	640

Backup II: Statistical error ⟷ Systematic error

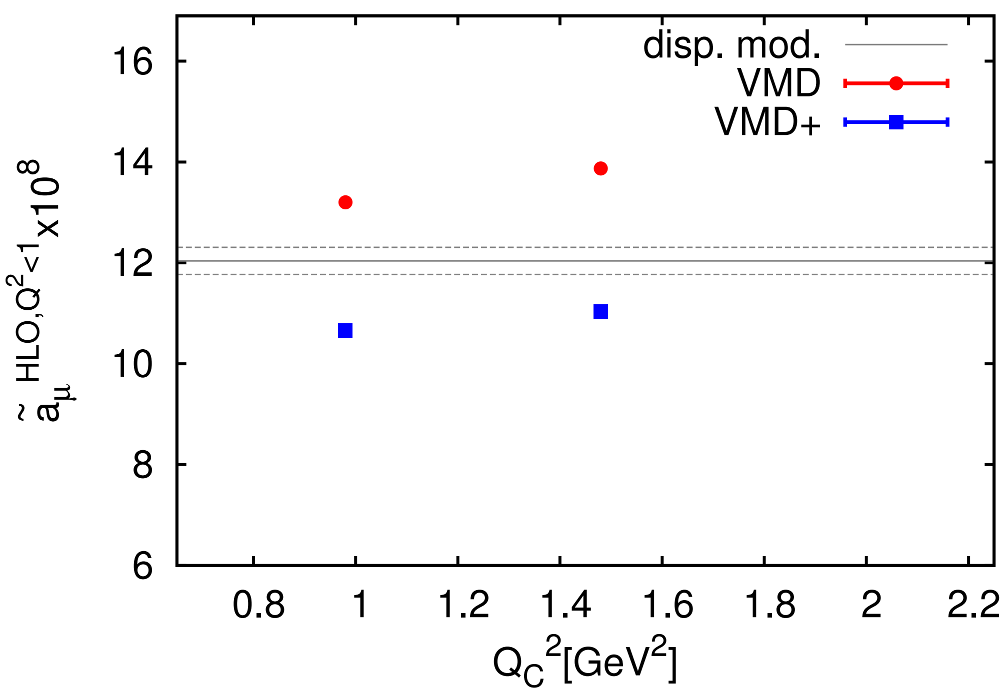
- Understanding the systematics is extremely important and usually challenging
- Dominant sources of errors
  - ➡ deterioration of signal at **Q^2 —> 0**
  - ➡ disconnected diagrams
  - ➡ isospin breaking effects
  - ➡ scale setting error
  - ➡ finite volume effects
  - ➡ discretization effects
  - ➡ scale setting uncertainty …

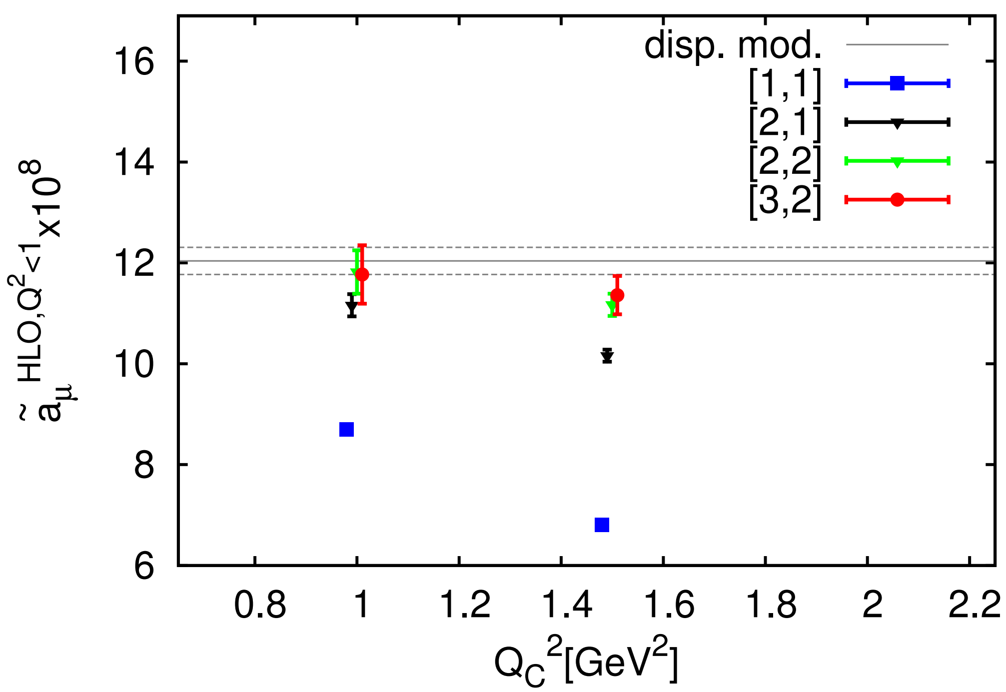
➡ New proposals for the **space-like** experimental measurements of HVP

- ➡ [**Phys.Lett. B746 (2015) 325-329 by Carloni, Passera,Trentadue, Venanzoni**] **@KLOE2**
- ➡ [**Eur.Phys.J. C77 (2017) no.3, 139 by Abbiendi et al.**] **@CERN (?)**

Backup III: Phenomenological model of HVP [Golterman, Maltman, Peris '13]

- A method to quantitatively examine the systematics of lattice computations
- Dispersive τ-based $I=1$ model: $\hat{\Pi}^{I=1}(Q^2) = Q^2 \int_{4m_\pi^2}^{\infty} ds\, \frac{\rho^{I=1}(s)}{s(s+Q^2)}$
- Fake lattice data for $\Pi(Q^2) - \Pi(0)$ & compared with true answer from model





- Outcome:
  - Fitting until high Q^2 dangerous, unless higher order Padés used
  - Better focus on low-Q^2 region needed