HVP lattice QED and strong IB corrections effects

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Outline

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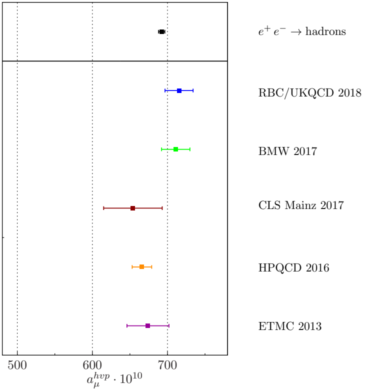
HVP from the **R**-ratio ↔ Lattice

- (published) HVP results from lattice calculations



- **R**-ratio $\mathbf{a_\mu^{hvp} = (692.3 \pm 4.2 \pm 0.3) \times 10^{-10}}$ [Davier et al., Eur.Phys.J. **C71**, 1515 (2011)]
- lattice result to be competitive with **R**-ratio requires precision of $\lesssim$ **1%**
- comparable upcoming experiment precision of $\lesssim$ **0.2%**

→ Isospin Breaking Corrections need to be included

Sources of IB corrections

- different masses for up- and down quark (of $\mathcal{O}((\mathbf{m_d} - \mathbf{m_u})/\Lambda_{\text{QCD}})$)
- Quarks have electrical charge (of $\mathcal{O}(\alpha)$)

Status IB corrections to HVP

- QED and strong IB at unphysical quark masses [V.G. *et al.*,JHEP **09**, 153 (2017)]
- QED for **s** and **c**; extrapolated to physical masses [D. Giusti *et al.*, JHEP **10**, 157 (2017)]
- strong IB at physical (valance + sea) masses [B. Chakraborty *et al.* Phys. Rev. Lett. **120** 152001 (2018)]
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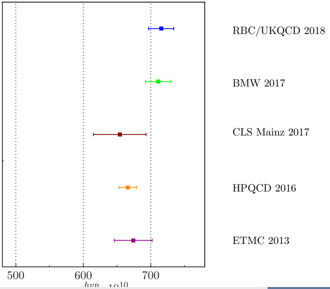

RBC/UKQCD 2018 $\longrightarrow$ QED/sIB calculation included

BMW 2017 $\longrightarrow$ phenomenology estimate for IB

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- plus work in progress

Strong IB corrections

- lattice calculations usually done with $\mathbf{m_u = m_d}$
- different masses for up- and down quark

[PDG] $\mathbf{m_u = 2.2^{+0.5}_{-0.4}}$ MeV $\mathbf{m_d = 4.7^{+0.5}_{-0.3}}$ MeV at $\overline{\text{MS}}$(2 GeV)

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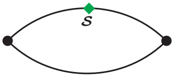
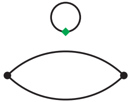
- strong Isospin Breaking on the lattice
  - use different up, down quark masses
    sea quark effects: configurations with different up, down masses
  - perturbative expansion in $\mathbf{\Delta m = (m_u - m_d)}$ [G.M. de Divitiis *et al*, JHEP 1204 (2012) 124]

$$\langle O \rangle_{m_u \neq m_d} = \langle O \rangle_{m_u = m_d} + \Delta m \frac{\partial}{\partial m} \langle O \rangle \Big|_{m_u = m_d} + \mathcal{O}\left(\Delta m^2\right)$$



sea quark effects:
quark-disconnected diagrams



QED corrections from the lattice

- same order in α as light-by-light
- Euclidean path integral including QED

$$\langle \mathbf{O} \rangle = \frac{1}{Z} \int \mathcal{D}[\Psi, \overline{\Psi}]\mathcal{D}[U]\mathcal{D}[A]\ O\ e^{-S_F[\Psi,\overline{\Psi},U,A]}\, e^{-S_G[U]}\, e^{-S_\gamma[A]}$$

- Finite Volume corrections [Talk by A. Portelli]
- two approaches for including QED
  - stochastic QED using **U(1)** gauge configurations
    [A. Duncan, E. Eichten, H. Thacker, Phys.Rev.Lett. **76**, 3894 (1996)]
  - perturbative QED by expanding the path integral in α
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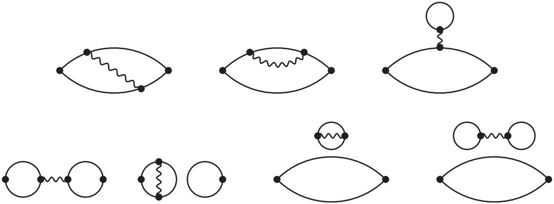

+ tadpole contributions, + diagrams from conserved current expansion

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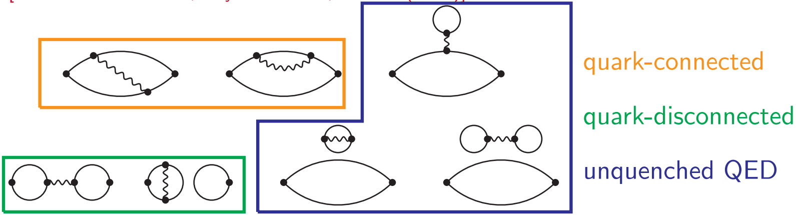

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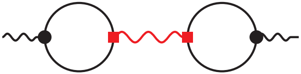
QED correction disconnected HVP

- QED correction to the disconnected HVP



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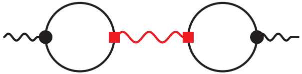

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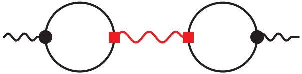

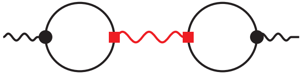

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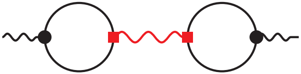

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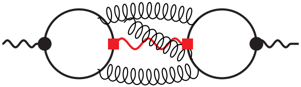
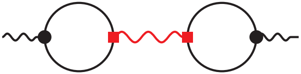
gluons between the quarks lines

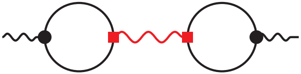




$\rightarrow$ QED correction to LO HVP

QED correction disconnected HVP

- QED correction to the disconnected HVP



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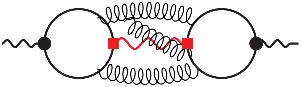
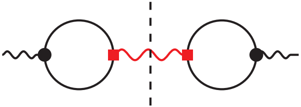


→ QED correction to LO HVP

no gluons between the quarks lines



→ included in NLO HVP

Outline

Results IB corrections - Fermilab/HPQCD/MILC

- strong IB corrections at the physical point [B. Chakraborty *et al.* Phys. Rev. Lett. **120** 152001 (2018)]
- HISQ action, $\mathbf{32^3 \times 48}$, $\mathbf{a \approx 0.15}$ fm
- two physical mass ensembles, differ only by light sea with or without sIB

$$\mathbf{N_f = 2 + 1 + 1}$$
$$\mathbf{N_f = 1 + 1 + 1 + 1}$$

where bare masses $\mathbf{m_\ell^{2+1+1} = (m_u + m_d)^{1+1+1+1}/2}$

- allows for testing effects of IB in sea-quark

- quark mass tuning:
  tune quark masses to experimental values with removed QED effects [S. Basek *et al.* PoS **Lattice2015**, 259 (2016)]

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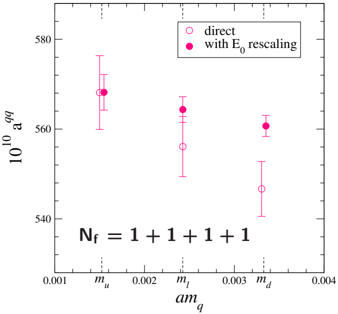
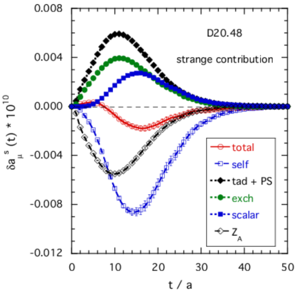
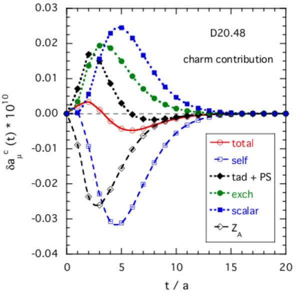


- $\Delta^{m_u \neq m_d} a_\mu = 7.7(3.7) \times 10^{-10}$ $\quad N_f = 2+1+1$
  $\Delta^{m_u \neq m_d} a_\mu = 9.0(2.3) \times 10^{-10}$ $\quad N_f = 1+1+1+1$
- sea-quark effect smaller than statistical error
- work in progress: generate dynamical QCD+QED ensemble at physical quark masses

Results IB corrections - ETMC

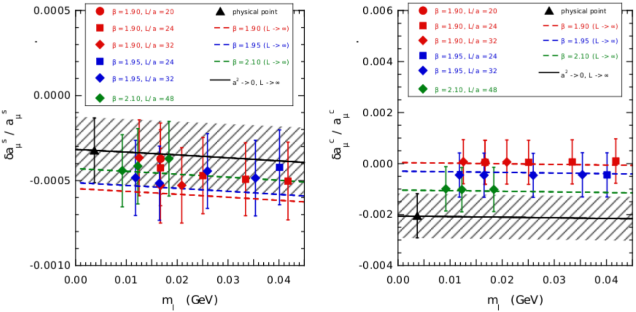
- QED corrections to strange and charm HVP [D. Giusti *et al.*, JHEP **10**, 157 (2017)]
- physical strange and charm masses; matched renormalized quark masses at $\overline{\mathbf{MS}}$**(2** GeV**)** in QCD and QED+QCD
  [N. Carrasco *et al.*, Nucl.Phys. **B887** (2014) 19-68, D. Giusti *et al.*, Phys. Rev. **D 95**, 114504 (2017)]
- perturbative expansion in α and $\mathbf{\Delta m_q}$
- results QED correction to integrand ($\mathbf{a_\mu} = \int \mathrm{dt}\ \mathbf{w_t C(t)}$)





Results IB corrections - ETMC

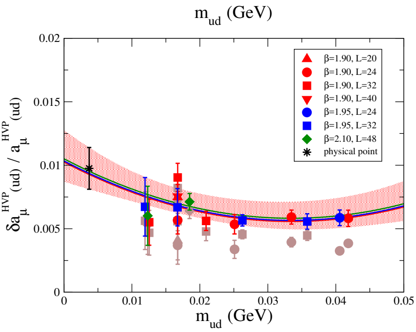
- QED corrections to strange and charm HVP [D. Giusti *et al.*, JHEP **10**, 157 (2017)]
- several ensembles: three lattice spacings, $\mathbf{m_\pi = 210 - 450}$ MeV



- $\boldsymbol{\delta a_\mu^s = (-0.018 \pm 0.011) \times 10^{-10}}$ $\boldsymbol{\delta a_\mu^c = (-0.030 \pm 0.013) \times 10^{-10}}$
- negligible within current uncertainties of $\mathbf{a_\mu}$

Results IB corrections - ETMC

- preliminary results: IB correction to light-quark contribution [see talk by S. Simula]



- $\boldsymbol{\delta a_\mu^\ell = 6.9(1.9) \times 10^{-10}}$ (strong IB and QED)

Results IB corrections - RBC/UKQCD

- QED and sIB corrections at physical quark masses [C. Lehner, V.G. *et al.* arXiv:1801.07224]
- $\mathbf{N_f = 2 + 1}$ Möbius DWF, $\mathbf{48^3} \times \mathbf{96}$ lattice, $\mathbf{a^{-1} = 1.730(4)}$ GeV
- IB corrections from perturbative expansion in α and $\mathbf{\Delta m_f}$
- tune $(\mathbf{u},\mathbf{d},\mathbf{s})$ masses to reproduce experimental π^+, $\mathbf{K^+}$ and $\mathbf{K_0}$ mass (and check π^0 mass)
- lattice spacing: fix another mass including QED, e.g. Omega-Baryon

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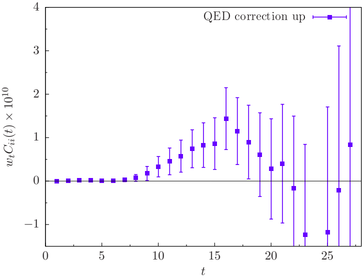
- results connected QED correction to integrand ($\mathbf{a_\mu} = \int$dt $\mathbf{w_t C(t)}$)



Results IB corrections - RBC/UKQCD

- Ansatz for $\mathcal{O}(\alpha)$-correction to correlator

$$\delta C(t) = (c_1 + c_0 t)e^{-Et}$$

- vary **E** between $\pi\gamma$ and $\pi\pi$ $\quad\longrightarrow$ systematic error
- result connected QED correction

$$a_\mu^{\mathrm{QED},\ell} = 5.9(5.7)(1.7) \times 10^{-10}$$

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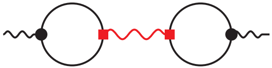
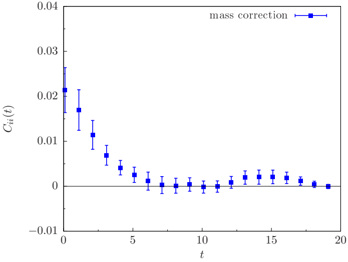
- QED correction to disconnected diagram using data generated for [T. Blum *et al.* Phys. Rev. Lett. 118, 022005 (2017)]



- $a_\mu^{\text{QED, disc}} = -6.9(2.1)(2.7) \times 10^{-10}$

Results IB corrections - RBC/UKQCD

- strong IB (connected)



- $\delta \mathbf{C(t) = (c_1 + c_0 t) e^{-Et}}$
  with lowest lying state $\pi\pi$
- result sIB $\mathbf{a_\mu^{sIB} = 10.6(4.3)(6.8) \times 10^{-10}}$

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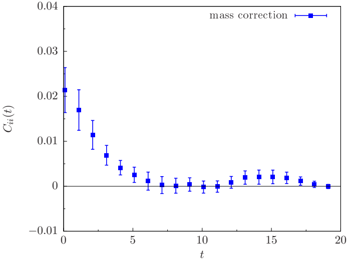

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Work in progress

- re-use LbL data to [+ M. Bruno]
  - increase statistics for QED diagrams
  - calculate the QED-unquenched diagrams
- strong IB: effects from sea quark mass shift, second lattice spacing
- second lattice spacing for QED corrections

Estimate IB corrections - BMW

- phenomenology estimate for IB effects [BMW collaboration, arXiv:1711.04980]
- estimate of different missing contributions

	$\ell = e$ $[\times 10^{-14}]$	$\ell = \mu$ $[\times 10^{-10}]$	$\ell = \tau$ $[\times 10^{-8}]$
$\pi^0\gamma$	1.05 ± 0.04	4.64 ± 0.04	1.77 ± 0.07
$\eta\gamma$	0.14 ± 0.00	0.65 ± 0.01	0.29 ± 0.01
$\rho - \omega$ mixing	0.74 ± 0.37	2.71 ± 1.36	0.72 ± 0.36
FSR	1.17 ± 0.59	4.22 ± 2.11	1.40 ± 0.70
M_π vs $M_{\pi^\pm}$	-1.45 ± 1.45	-4.47 ± 4.47	-0.83 ± 0.83
total	1.7 ± 1.6	7.8 ± 5.1	3.4 ± 1.1

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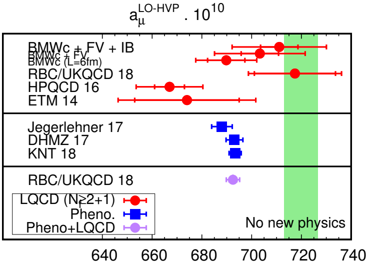
- comparison plot [L. Lellouch]



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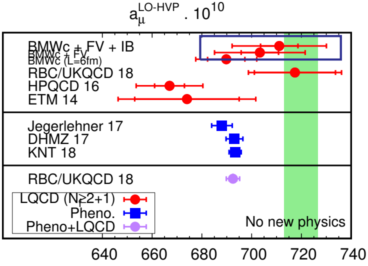


$\longrightarrow$ shift from IB corrections

Outline

Summary

- Lattice calculation with precision of $\lesssim$ **1%** require inclusion of IB and QED
- first calculations of IB Breaking effects $\rightarrow$ IB corrections at level of **1%**
  - Fermilab/HPQCD/MILC:
    strong IB quenched: $\mathbf{7.7(3.7) \times 10^{-10}}$
    strong IB unquenched: $\mathbf{9.0(2.3) \times 10^{-10}}$
  - ETMC (preliminary):
    QED (quenched) + strong IB (quenched): $\mathbf{6.9(1.9) \times 10-10}$
  - RBC/UKQCD:
    QED (quenched): $\mathbf{5.9(5.9) \times 10^{-10}}$
    disconnected QED: $\mathbf{-6.9(3.4) \times 10^{-10}}$
    strong IB (quenched): $\mathbf{10.6(8.0) \times 10^{-10}}$
- possible within $\lesssim$ **1%** precision of total $\mathbf{a}_{\mu}^{\mathrm{HVP}}$
- FV effects for the QED correction [see talk by A. Portelli]
- unquenched QED?

Outline

Backup

Muon $\mathbf{a}_\mu$ and the hadronic vacuum polarisation (HVP)

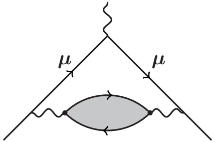
- experiment: polarized muons in a magnetic field [Bennet et al., Phys.Rev. **D73**, 072003 (2006)]

$$\mathbf{a}_\mu = \mathbf{11659208.9(5.4)(3.3)} \times \mathbf{10^{-10}}$$

- Standard Model [PDG]

$$\mathbf{a}_\mu = \mathbf{11659180.3(0.1)(4.2)(2.6)} \times \mathbf{10^{-10}}$$

- Comparison of theory and experiment: $\mathbf{3.6\sigma}$ deviation
- largest error on SM estimate from HVP



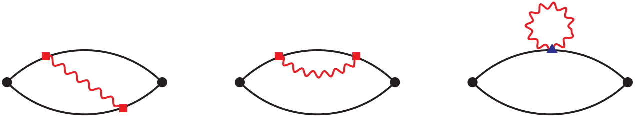
- current best estimate from $\mathbf{e^+e^-} \longrightarrow$ hadrons [Davier et al., Eur.Phys.J. **C71**, 1515 (2011)]

$$\mathbf{(692.3 \pm 4.2 \pm 0.3)} \times \mathbf{10^{-10}}$$

tadpole contributions to QED correction

- expand path integral expansion [RM123 Collaboration, Phys.Rev. **D87**, 114505 (2013)]

$$\langle O \rangle = \langle O \rangle_0 + \frac{1}{2} e^2 \frac{\partial^2}{\partial e^2} \langle O \rangle \bigg|_{e=0} + \mathcal{O}(\alpha^2)$$



- HVP from vector-vector correlation function

$$C_{\mu\nu}(x) = \langle V_\mu(x) V_\nu(0) \rangle$$

- conserved vector current depends on link variables

$$U_\mu(x) \rightarrow e^{ieA_\mu(x)} U_\mu(x) \qquad \text{and thus} \quad V^c_\mu(x) \rightarrow V^{c,e}_\mu(x)$$