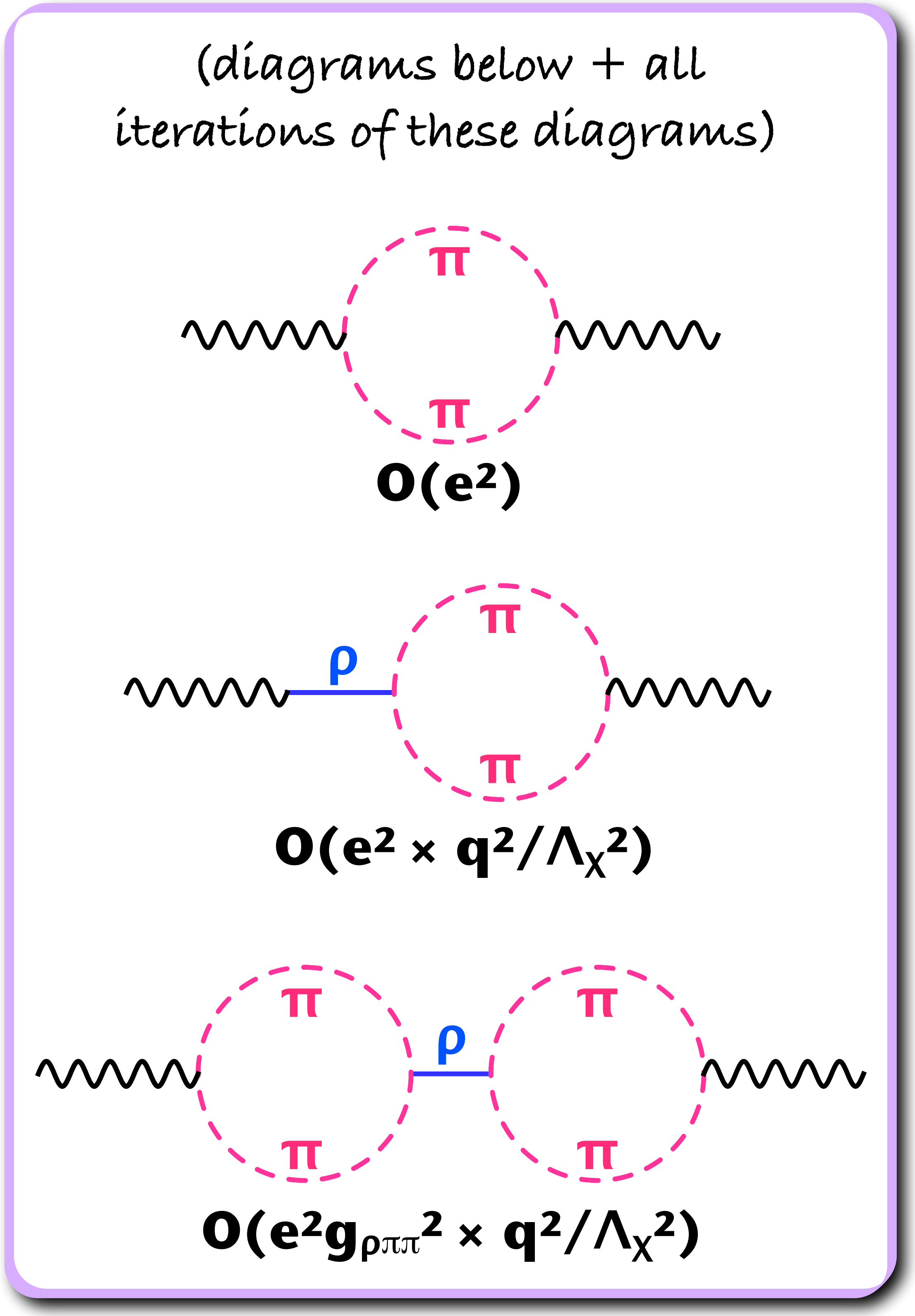
Notes on Low-Energy $\pi\pi$

HPQCD Collaboration

(Peter Lepage)

Extended scalar QED model

- Study effects of low-energy $\pi\pi$ states on a_μ^{HVP} within extended chiral perturbation theory /scalar QED that includes π's, ρ's, and γ's [Jegerlehner & Szafron, EPJC71 1632 (2011)]
  - (same theory used to compute finite-volume + staggered discretization corrections)
  - Include leading interactions that couple ρ and $\pi\pi$ channels with g_ρ=5.4, $g_{\rho\pi\pi}$ =6.0, f_ρ=0.21 GeV



Formulae

- Compute ππ levels numerically from poles and residues of $\Pi(q^2)$ in Appendix B of Chakraborty et al., PRDD96 (2017) no.3, 034516

$$\hat{\Pi}(-q_E^2, f_\rho, m_\rho, m_\pi) = -\hat{\Sigma}(-q_E^2, m_\pi, m_\pi) + \frac{f_\rho^2}{2m_\rho^2} \frac{q_E^2 \left(1 + g_\rho g_{\rho\pi\pi}\, \hat{\Sigma}(-q_E^2, m_\pi, m_\pi)\right)^2}{q_E^2 \left(1 + g_{\rho\pi\pi}^2\, \hat{\Sigma}(-q_E^2, m_\pi, m_\pi)\right) + m_\rho^2}$$

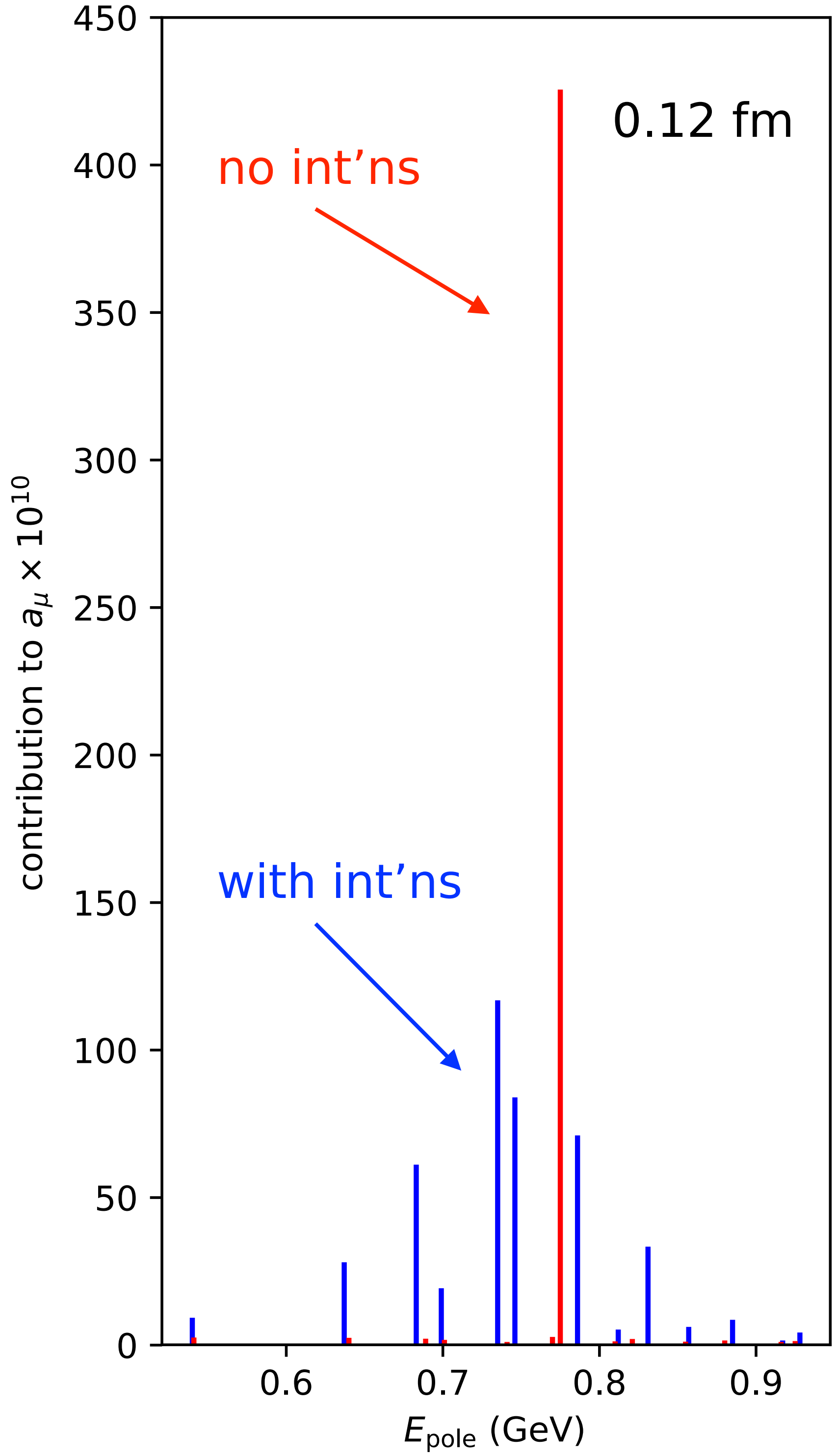
with $$-\hat{\Sigma}(-q_E^2, m_a, m_a) \equiv \frac{4q_E^2}{3} \int \frac{d^3\mathbf{k}}{(2\pi)^3 2E_a E_b} \frac{\mathbf{k}^2}{(E_a+E_b)^3(q_E^2 + (E_a+E_b)^2)}.$$

- Include taste splittings between pions by and finite lattice volume by

$$\hat{\Sigma}(-q_E^2, m_\pi, m_\pi) \to \frac{1}{16} \sum_{\xi_a, \xi_b} \hat{\Sigma}_V(-q_E^2, m_\pi(\xi_a), m_\pi(\xi_b))$$

$$\int \frac{d^3\mathbf{k}}{(2\pi)^3} \to \frac{1}{L^3} \sum_{k_x=-\infty}^{\infty} \sum_{k_y=-\infty}^{\infty} \sum_{k_z=-\infty}^{\infty}$$

$\pi\pi$–ρ Contributions to a_μ (Model)

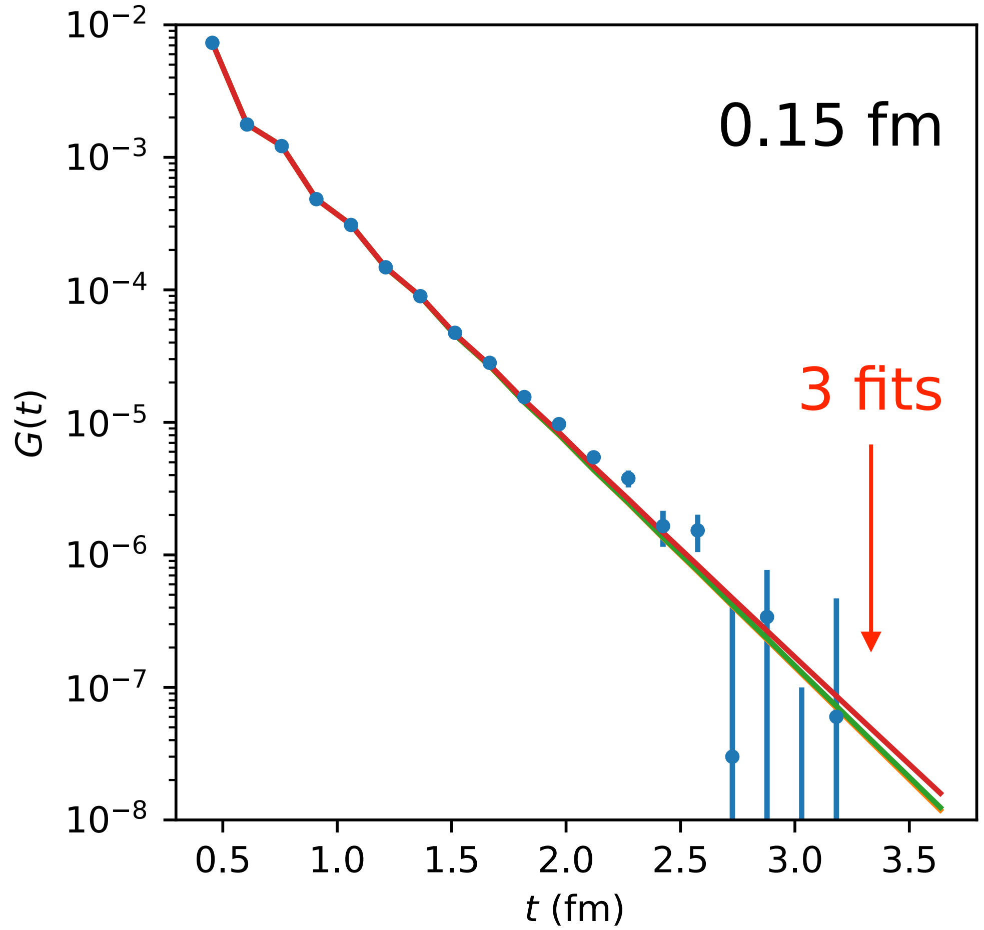


- Without interactions: ρ dominates, $\pi\pi$ negligible (due to finite volume, staggered pions).
- With interactions: ρ mixes with $\pi\pi$ and its contribution spread over many states.
- Total a_μ same in both cases (<0.5%).
- Noisy data ⇒ fitter can't tell difference.

Model in B. Chakraborty *et al* 1601.03071

Phys. Rev. D96 (2017) 034516 (App. B)

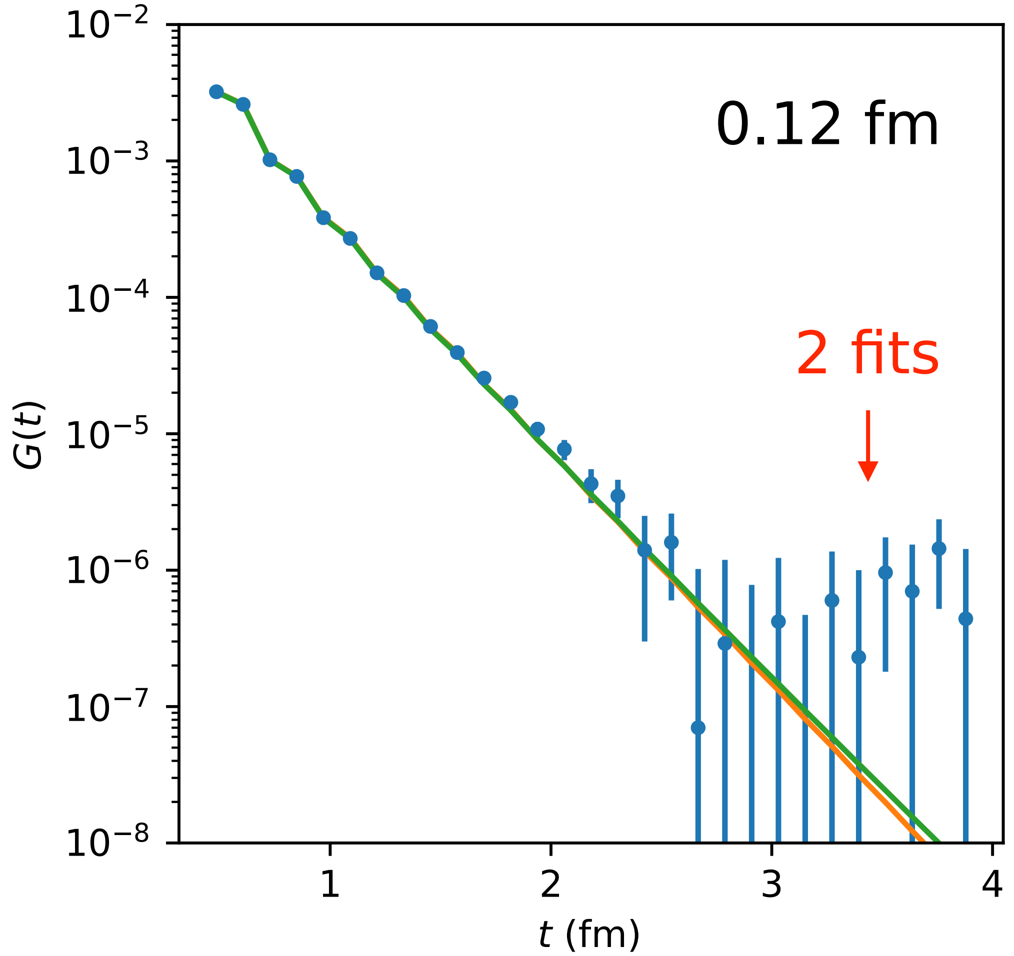
Equivalent Fits (χ^2/dof ≤ 1)



- Fit unable to resolve difference between one or multiple rho mesons; spreads contribution over multiple terms.
- Negligible difference for t<4fm, and irrelevant for a_μ.

	E_0 \ a_0	E_1 \ a_1	E_2 \ a_2	E_3 \ a_3	a_μ x 10^{10}
Fit 1	0.770 \ 0.132	1.72 \ 0.35	2.5 \ 0.1		602(7)
Fit 2	0.75 \ 0.09	0.80 \ 0.09	1.75 \ 0.36		602(7)
Fit 3	0.71 \ 0.08	0.78 \ 0.04	0.84 \ 0.10	1.76 \ 0.36	608(8)

Equivalent Fits (χ^2/dof ≤ 1)



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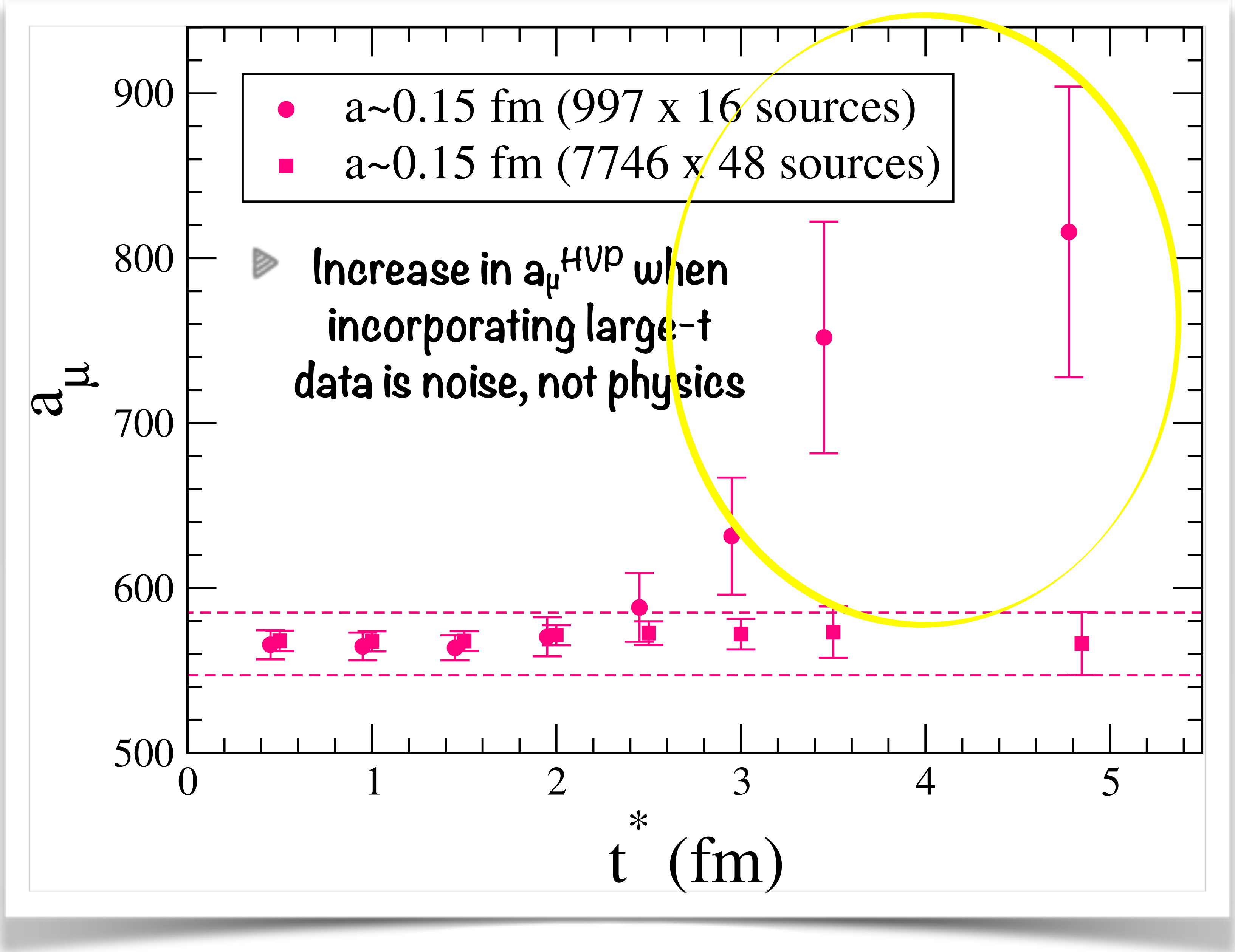
	E_0 \ a_0	E_1 \ a_1	E_2 \ a_2	E_3 \ a_3	a_μ x 10^{10}
Fit 1	0.770 \ 0.133	1.91 \ 0.42	3.3 \ 0.1		604(8)
Fit 2	0.69 \ 0.06	0.76 \ 0.04	0.83 \ 0.12	1.95 \ 0.43	606(9)

Comparison with noisy data

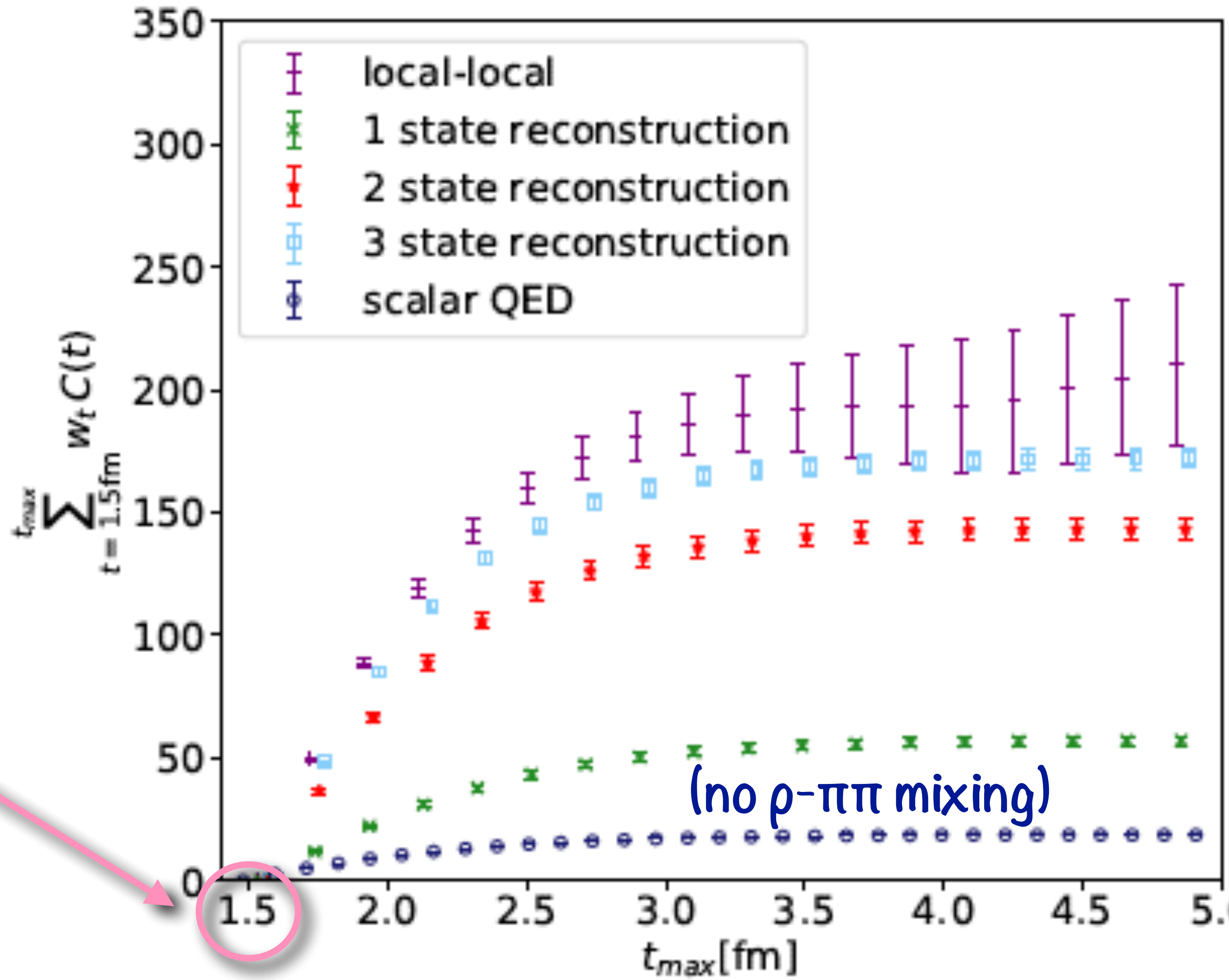
- Compare with results on independent ensemble with same lattice spacing and slightly different quark masses, but substantially lower statistics
  - *Answers consistent until t*~2 fm, but diverge substantially beyond t*~3 fm*

☆ Can obtain a_μ^{HVP} from data + fit for t^* below ~2–2.5 fm



Long-distance contributions from GEVP

- Plot shown by Blum at USQCD All-Hands' meeting in April shows contributions to a_μ^{HVP} from from individual lattice ππ energy levels obtained from GEVP analysis [https://indico.fnal.gov/event/16470/session/1/contribution/13/material/slides/0.pdf]



contributions only from t>1.5 fm

2nd & 3rd levels straddle M_ρ

Lowest level well below M_ρ

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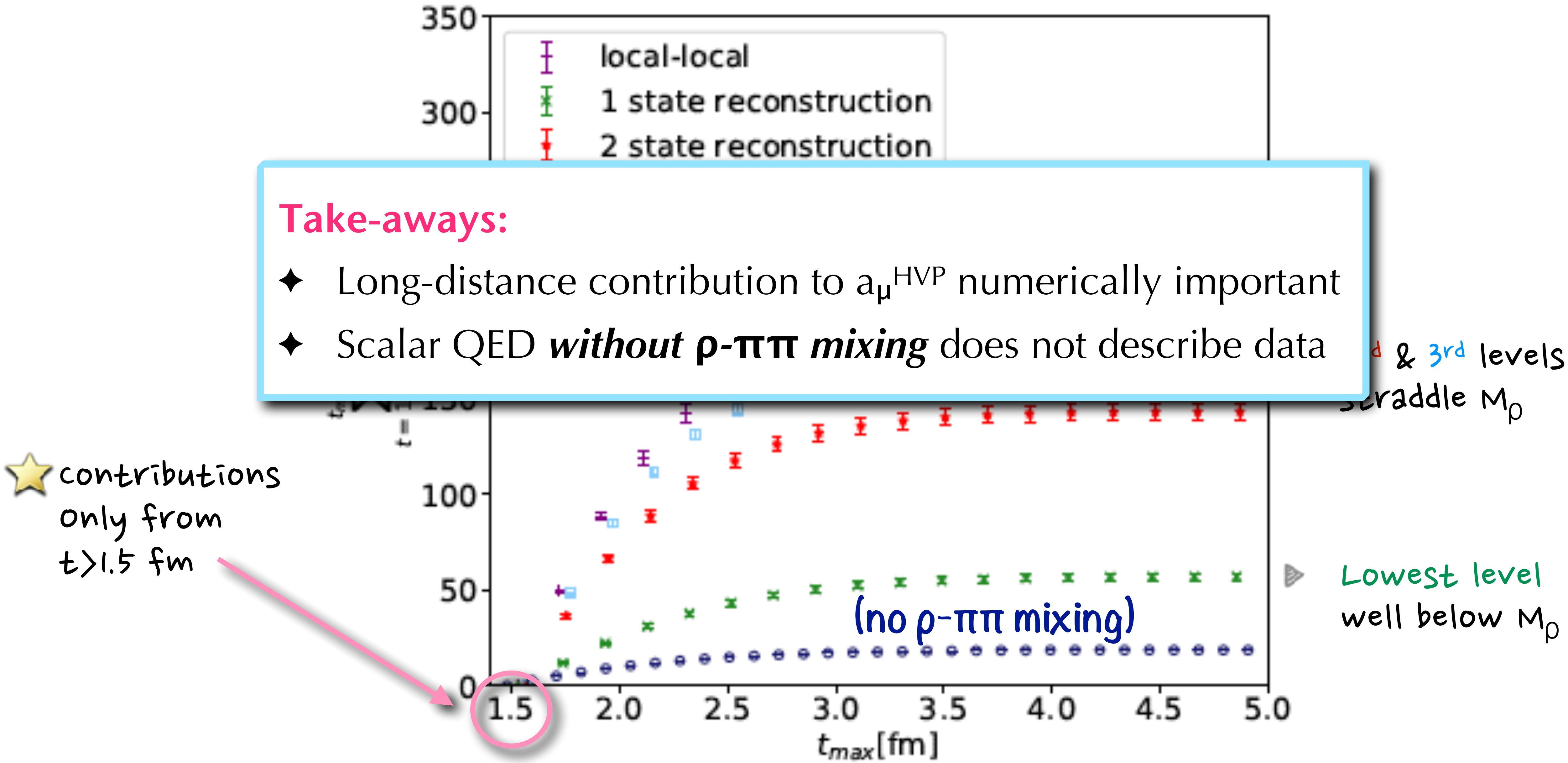
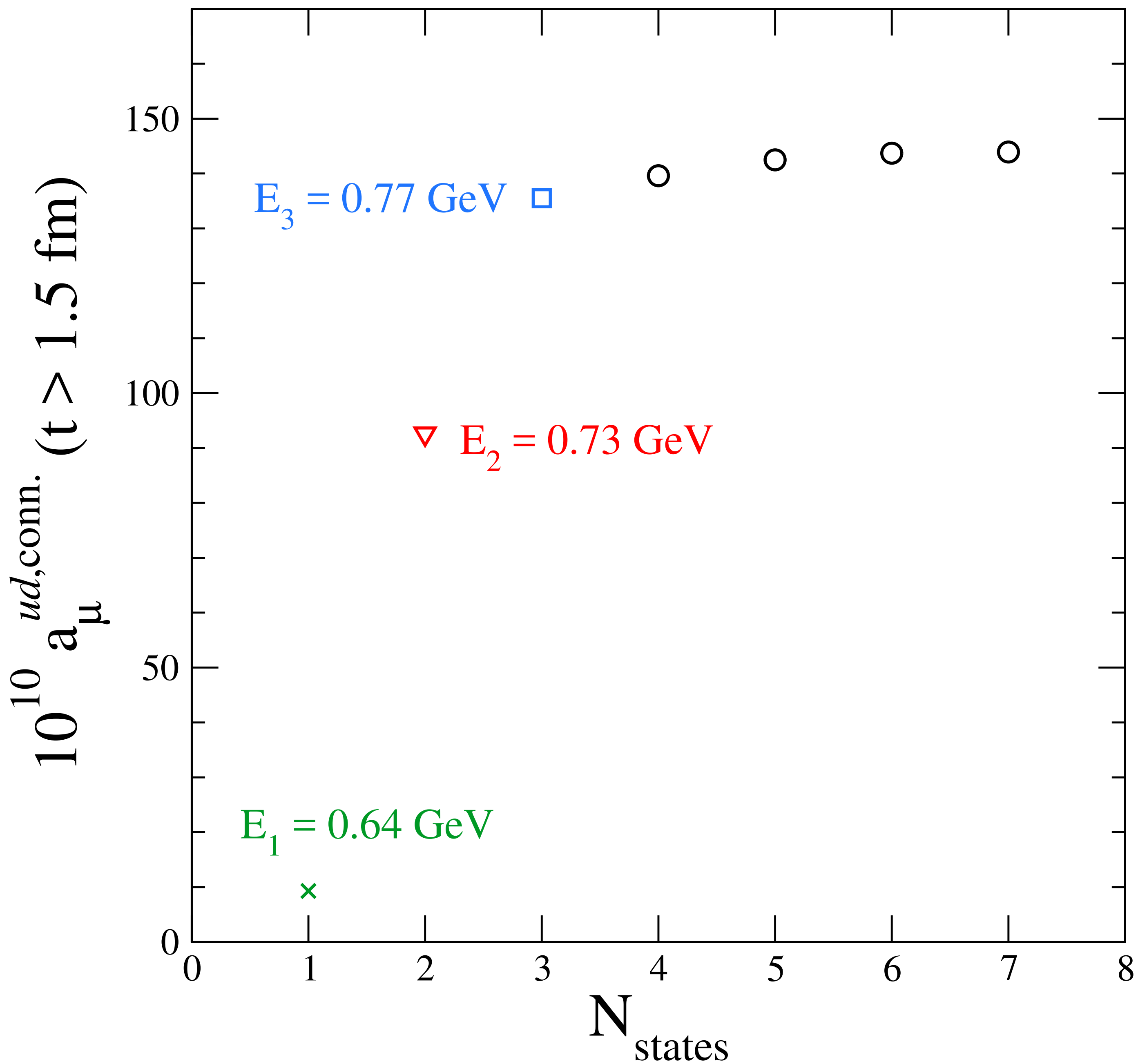


Take-aways:

- Long-distance contribution to a_μ^{HVP} numerically important
- Scalar QED ***without ρ-ππ mixing*** does not describe data

Comparison with extended sQED

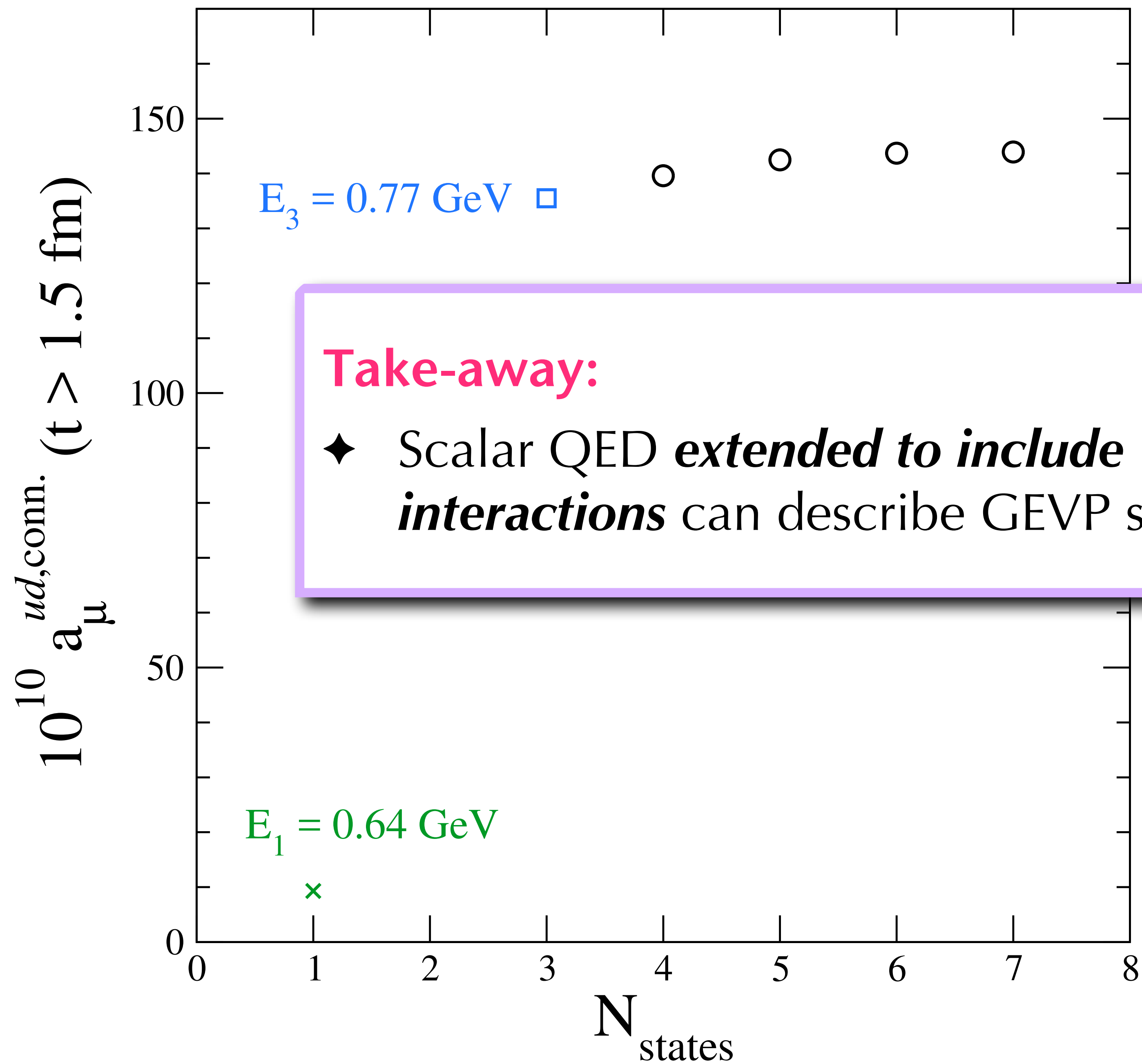
- Compute ππ levels and contributions to a_μ^{HVP} on a~0.15 fm physical-mass ensemble, which has similar spectrum to RBC/UKQCD example



- Numerical value for long-distance contribution to a_μ^{HVP} sensitive to exact lattice parameters, but same picture as RBC/UKQCD GEVP study

Comparison with extended sQED

- Compute ππ levels and contributions to a_μ^{HVP} on a~0.15 fm physical-mass ensemble, which has similar spectrum to RBC/UKQCD example



Take-away:

- Scalar QED ***extended to include ρ-ππ interactions*** can describe GEVP spectrum

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